Taxation, redistribution, and debt in incomplete market economies with aggregate shocks.

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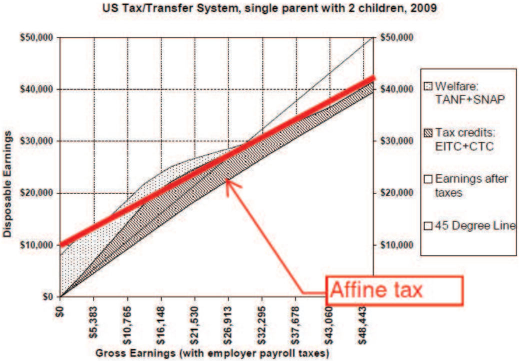
Questions

- How should distorting taxes, government transfers, and debt respond to exogenous government expenditure shocks?
- How do answers depend on agents' heterogeneity, incompleteness of markets, and shapes of schedules for taxes on labor income?

Forces at play

- Consumers want to smooth consumption over time and across states.
- Workers want to smooth labor supply.
- Government wants to redistribute and finance purchases.
- Nonlinear taxes.
  - Affine – linear minus transfer: $\tau_t \theta \ell_t - T_t$.
  - More general nonlinear.

U.S. taxes and transfers



Earlier environments with $\tau_t \theta \ell_t - T_t$

Lucas and Stokey (1983)

- Representative agent, complete markets.
- $T_t \geq 0$ always binds.
- State contingent government debt important.

AMSS (2002)

- Representative agent, incomplete markets (Bewley).
- $T_t \geq 0$ imparts precautionary *saving* motive to government.
- $T_t \geq 0$ eventually slack, implying that $\tau_t = 0$ for $t \geq S$.
- Government debt a key state variable.

Werning (2007)

- Heterogenous agents, complete markets, affine taxes, T_t unrestricted.
- Distorting taxes balance redistribution vs. efficiency.
- Debt and lump sum taxes used to smooth govt. expenditures.

Our environment with $\tau_t \theta_i \ell_{it} - T_t$

Golosov-Sargent (2012)

- Heterogeneous agents, incomplete markets (Bewley).
- Government debt *not* a key state variable.
- No precautionary motive for government to acquire assets.
- Asset distribution across agents is key state vector.
- T_t not restricted, but ....
- If we had imposed $T_t \geq 0$, it might never bind.

The Model

- $t = 0, 1, ...$
- I types of infinitely lived agents. Mass π_i, $\sum_{i=1}^{I} \pi_i = 1$.
- Endowments: $\bar{l} > 0$ + productivity $\theta_i \geq 0$ fixed $\forall t$.
- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t}\right)$$

U^i concave in $(c, -l)$, twice continuously differentiable.
$\lim_{x \to \bar{l}} U_l^i \left(c, x\right) = \infty$ and $\lim_{x \to 0} U_l^i \left(c, x\right) = 0 \; \forall c, i$.

The Model

- Exogenous govt. purchases $\{g_t\}$ follow an irreducible finite state Markov process.

- Affine taxes: $\tau_t \theta_i l_{i,t} - T_t \ \forall t \geq 0$.
- Pareto weights $\alpha_i \geq 0$, $\sum_{i=1}^{I} \alpha_i = 1$.

$$\mathbb{E}_0 \sum_{i=1}^{i} \pi_i \alpha_i \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t} \right).$$

- One-period risk free bond in 0 net supply. Gross return $R_{t-1}, R_{-1} \equiv 1$.

- Feasibility
$$\sum_{i=1}^{I} \pi_i c_{i,t} + g_t = \sum_{i=1}^{I} \pi_i \theta_i l_{i,t}.$$
- Government budget constraint
$$g_t + B_t = \tau_t \sum_{i=1}^{I} \pi_i \theta_i \ell_{i,t} - T_t + R_{t-1} B_{t-1}.$$

Agent's Problem

Choose $\{c_{i,t}, b_{i,t}, l_{i,t}\}_{s^t,t\geq 0}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, l_{i,t}\right)$$

s.t. $\quad \forall t \geq 0$

$$\begin{aligned} c_{i,t} + b_{i,t} &= (1-\tau_t)\,\theta_i l_{i,t} + T_t + R_{t-1} b_{i,t-1} \\ b_{i,t} &\geq \underline{b} \\ & \quad b_{i,-1} \text{ given}, R_{-1} \equiv 1 \end{aligned}$$

Competitive Equilibrium with Affine Taxes

Given $\left\{\{b_{i,-1}\}_i, B_{-1}\right\}$ and $\{\tau_t, T_t\}_t$, a competitive equilibrium is a sequence $\left\{\{c_{i,t}, l_{i,t}, b_{i,t}\}_i, B_t, R_t\right\}_t$ such that

1. $\{c_{i,t}, b_{i,t}, l_{i,t}\}_t$ solves agent $i's$ problem for all i,
2. allocation is feasible, i.e.,
$$\sum_i \pi_i c_{i,t} + g_t = \sum_i \pi_i \theta_i l_{i,t}, \forall t \geq 0,$$
3. government's budget constraint is satisfied, i.e., $\forall t \geq 0$
$$g_t + B_t = \sum_i \pi_i \tau_t \theta_i l_{i,t} - T_t + R_{t-1} B_{t-1},$$
4. bond market clears
$$B_t + \sum_i \pi_i b_{i,t} = 0, \forall t \geq -1.$$

Agent i's first-order conditions

$$(1-\tau_t)\,\theta_i U^i_{c,t} = -U^i_{l,t}$$

$$U^i_{c,t} = \beta R_t \mathbb{E}_t U^i_{c,t+1}.$$

Implementability Constraints (IC)

FOCs + agent $i's$ budget constraint

$$c_{i,t} + b_{i,t} = -\frac{U^i_{l,t}}{U^i_{c,t}} l_{i,t} + T_t + \frac{U^i_{c,t-1}}{\beta \mathbb{E}_{t-1} U^i_{c,t}} b_{i,t-1} \ \forall i, t.$$

Substracting IC for agent 1

$$(c_{i,t} - c_{1,t}) + \widetilde{b}_{i,t} = -\frac{U^i_{l,t}}{\theta_i U^i_{c,t}} (\theta_i l_{i,t} - \theta_1 l_{1,t}) + \frac{U^i_{c,t-1}}{\beta \mathbb{E}_{t-1} U^i_{c,t}} \widetilde{b}_{i,t-1}$$

$\forall i > 1, \forall$, where $\widetilde{b}_{i,t} = b_{i,t} - b_{1,t}$.

$\Longrightarrow$ Equilibrium $\left\{ \{b_{i,t}\}_i , B_t, T_t \right\}_{s^t,t}$ is indeterminate by Ricardian-Modigliani-Miller reasoning.

Implementabilty measurability constraints

Lucas-Stokey (1983):

$$(*) \qquad B_{-1} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U^1_{c,t+j}}{U^1_{c,0}} \left[g_t - \left(1 + \frac{U_{l,t}}{U_{c,t}}\right) \theta_1 \ell^1_t \right]$$

AMSS (2002): Lucas-Stokey's $(*)$ plus

$$\mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \frac{U^1_{c,t+j}}{U^1_{c,t}} \left[g_{t+j} - \left(1 + \frac{U^1_{l,t+j}}{U^1_{c,t}}\right) \theta_1 \ell^1_{t+j} \right]$$

must be measurable with respect to g^{t-1} $\forall t \geq 0$.

Implementability measurability constraints

Golosov-Sargent (2012): Not Lucas-Stokey's equation ($*$) but

$$\mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \frac{U^i_{c,t+j}}{U^i_{c,t}} \left[(c_{i,t+j} - c_{1,t+j}) + \frac{U^i_{\ell,t+j}}{\theta_i U^i_{c,t+j}} (\theta_i \ell^i_{t+j} - \theta_1 \ell^1_{t+j}) \right]$$

must be measurable with respect to g^{t-1} $\forall t \geq 0$ and $\forall i > 1$.
These differ from AMSS and are more numerous.

Ricardian Equivalence

Proposition:

(i) We are free to set B_t or $b_{i,t}$ for some i to zero $\forall t$.

$\implies$ Any c.e. allocation is supported by a $B_t = 0$ fiscal policy in which the government always runs a balanced budget.

(ii) For all distributions of assets $\left\{ \{b_{i,-1}\}_i \, , B_{-1} \right\}$ for which $\left\{ \widetilde{b}_{i,-1} \right\}_{i \geq 2}$ is fixed, optimal allocations are identical.

$\implies$ Initial level of government assets B_{-1} irrelevant. Distribution of assets across agents *is* relevant.

Ramsey problem

Choose $\{c_{i,t}, \ell_{i,t}, \tilde{b}_{i,t}\}_{i,t}$ to maximize

$$\mathbb{E}_0 \sum_{i=1}^{I} \pi_i \alpha_i \sum_{t=0}^{\infty} \beta^t U^i\left(c_{i,t}, l_{i,t}\right) \quad \mathrm{s}ubject\ to$$

$$\begin{aligned} & (c_{i,t} - c_{1,t}) + \tilde{b}_{i,t} \\ = \quad & -\frac{U^i_{l,t}}{\theta_i U^i_{c,t}}\left(\theta_i l_{i,t} - \theta_1 l_{1,t}\right) + \frac{U^i_{c,t-1}}{\beta \mathbb{E}_{t-1} U^i_{c,t}} \tilde{b}_{i,t-1} \text{ for all } i > 1, t. \end{aligned}$$

$$\sum_i \pi_i c_{i,t} + g_t = \sum_i \pi_i \theta_i \ell_{it} \forall t \geq 0$$

$$\frac{\mathbb{E}_t c_{i,t+1}}{ci, t} = \frac{\mathbb{E}_t c_{j,t+1}}{c_{j,t}} \forall i, j, t \geq 0$$

$$\frac{U^i_{\ell,t}}{\theta_i U^i_{c,t}} = \frac{U^j_{\ell,t}}{\theta_i U^j_{c,t}} \forall i, j, t \geq 0$$

Ramsey plan

- $\{c_{i,t}, \ell_{it}\}_{i=1}^{I} = A_t(g^t, \tilde{b}_{-1}) \quad t \geq 0$
- $(\tau_t, \tilde{b}_t) = P_t(g^t, \tilde{b}_{-1}) \quad t \geq 0$
- $\tau_t = 1 + \frac{U^i_{c,t}}{\theta_i U^i_{c,t}}$ for any i.
- $R_t = \frac{U^i_{c,t}}{\beta \mathbb{E}_t U^i_{c,t+1}}$ for any i.
- Name any stochastic process $\{B_{t-1}(g^{t-1}), T_t(g^t)\}_{t=0}^{\infty}$ that satisfies

$$B_t = \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j \frac{U^i_{c,t+j}}{U^i_{c,t}} \left[g_{t+j} + T_{t+j} - \tau_{t+j} \sum_{i=1}^{I} \pi_i \theta_i \ell_{i,t+j} \right]$$

.

Recursive Formulation of Ramsey problem, $t > 0$

Let $\widetilde{b} = \left(\widetilde{b}_2, \ldots, \widetilde{b}_I\right)$ and $u = \left(U_c^1, \ldots, U_c^I\right)$.

$$V\left(\widetilde{b}, u, g_-\right) = \max_{(\widetilde{b}', c, l)(g)} \sum_g \Pr(g|g_-)\left[\sum_i \pi_i \alpha_i U^i(g) + \beta V\left(\widetilde{b}', u', g\right)\right]$$

$$(c_i(g) - c_1(g)) + \widetilde{b}_i'(g) + \frac{U_l^i(g)}{U_c^i(g)}\left(l_i(g) - \frac{\theta_1}{\theta_i} l_1(g)\right) = \frac{u_i}{\beta E_{g_-} U_c^i(g)} \widetilde{b}_i$$

$$\forall g, \forall i > 1$$

$$\begin{aligned}
\frac{\sum_g \Pr(g|g_-) U_c^i(g)}{u_i} &= \frac{\sum_g \Pr(g|g_-) U_c^j(g)}{u_j}, \forall i, j \\
\frac{U_l^i(g)}{\theta_i U_c^i(g)} &= \frac{U_l^j(g)}{\theta_j U_c^j(g)}, \forall i, j, g \\
\sum_i \pi_i c_i(g) + g &= \sum_i \pi_i \theta_i l_i(g), \forall g
\end{aligned}$$

Recursive Formulation of Ramsey problem, $t = 0$

Given $\tilde{b}_{-1}$ and g_0, the planner chooses $(c_0, \tilde{b}_0)$

$$W(\tilde{b}_{-1}) = \max_{c_0, \tilde{b}_0} \sum_{i=1}^{I} \alpha_i U^i(c_{i,0}, l_{i,0}) + \beta V(\tilde{b}_0, u_{c,0}, g_0)$$

subject to

$$\begin{aligned}
(c_{i,0} - c_{1,0}) + \tilde{b}_{i,0} \quad &+ \quad \frac{U^i_{\ell,0}}{\theta_i U^i_{c,0}}(\theta_i \ell_{i,0} - \theta_1 \ell_{1,0}) = \tilde{b}^i_{-1}, \quad \forall i > 1 \\
\frac{U^i_{\ell,0}}{\theta_i U^i_{c,0}} \quad &= \quad \frac{U^j_{\ell,0}}{\theta_j U^j_{c,0}}, \quad \forall i, j \\
\sum_{i=1}^{I} \pi_i c_{i,0} + g_0 \quad &= \quad \sum_{i=1}^{I} \pi_i \theta_i \ell_{i,0}.
\end{aligned}$$

Recursive Representation of Ramsey plan

- $g_t = G(g_{t-1}, \epsilon_t)$
- $(\tilde{b}_t, u_t) = \tilde{Z}(\tilde{b}_{t-1}, u_{t-1}, g_t)$
- $\{c_{it}, \ell_{it}\}_{i=1}^{I} = \tilde{P}(\tilde{b}_{t-1}, u_{t-1}, g_t)$
- $(\tau_t, T_t) = \tilde{S}(\tilde{b}_{t-1}, u_{t-1}, g_t)$

Martingales

- Mathematical structure similar to AMSS and Farhi; economic outcomes quite different.
- Like AMSS, Lagrange multipliers $\psi_{i,t}$ on implementability constraints follow a "random-walk-like" process:

$$\psi_{i,t} = \mathbb{E}_t \psi_{i,t-1} + \left(\mathbb{E}_t \left(U^i_{c,t+1}\right)\right)^{-1} Cov_t \left(U^i_{c,t+1}, \psi_{i,t+1}\right).$$

- Because we don't impose $T_t \geq 0$, these martingale do not impart AMSS-like upward drift to government asset holdings.

Four example economies

- Quasi-linear preferences.
- AMSS-like economy with type 1 agent quasi-linear, risk-averse disabled $\theta_2 = 0$ type 2 agent.
- A nonstochastic stationary economy.
- Minimally stochastic economy.

Quasi-linear preferences in AMSS

AMSS example.

- Representative agent with preferences

$$U^i(c, l) = c - h(l)$$

- Bound on consumption: $c \geq \underline{c}$
- Unleashes AMSS precautionary motive for government to accumulate assets.
- Eventually, the government finances all revenue needs with asset earnings. $\tau_t \to 0$

Quasi-linear preferences in our economy

Proposition: Suppose that preferences are quasi-linear for all i and that the equilibrium is interior. Suppose that h_i satisfies

$$0 \leq h_i''' \leq \left(h_i''\right)^2 / h_i' \text{ for all } i.$$

Then the optimal tax, τ_t^*, satisfies $\tau_t^* = \tau^*$. An optimum debt pattern $\left\{b_{i,t}^*, B_t^*\right\}_{i,t}$ can be chosen to satisfy $b_{i,t}^* = b_{i,-1}$ for all $i,\ t \geq 0$ and $B_t^* = B_{-1}$ for all $t \geq 0$.

Quasi-linear preferences, II

Remarks:

- Fluctuations in lump sum taxes and transfers do all the work.
- If the planner wants to redistribute enough towards low skilled types, T_t can be positive at all dates and states.
- Then even if we had imposed the AMSS constraint $T_t \geq 0$, it would never bind.

Comparison with representative agent AMSS model

- Relative to standard representative agent model we made two departures
  - heterogeneity, $I > 1$.
  - $T_t \geq 0$
- Which matter?
- Suppose we had exogenously imposed $T_t \geq 0$
  - Let χ_t be Lagrange multiplier on $T_t \geq 0$
  - **Proposition**: With quasi-linear preferences, for any $I \geq 1$, $\chi_t \to 0$
  - **Proposition**: With quasi-linear preferences, if $I > 1$ and low type is sufficiently poor, $\chi_t = 0$ for all t.

Comparison with representative agent AMSS model (2)

- This explains stark difference in dynamics of two economies
- Constraint $T_t \geq 0$ gives the government a precautionary motive to accumulate assets to relax future constraints.
  - when interest rate equals discount rate, continue until all future constraints are relaxed.
  - in representative agent model, $T_t \geq 0$ is slack only in the first best $\Longrightarrow$ no taxes in the long run
  - in heterogeneous agent models constraint $T_t \geq 0$ is always slack for sufficiently redistributory government $\Longrightarrow$ no need to accumulate assets precautionary or change profile of distorting taxes over time.

AMSS-like $I = 2$ economy

There are two types of agents. A type 1 agent has quasilinear preferences with $\theta_1 = 1$, while a disabled type 2 agent is risk averse and has $\theta_2 = 0$; his preferences can be represented with a strictly concave, twice differentiable utility function $u(c_{2,t})$ that satisfies Inada conditions. Higher curvature of u makes fluctuations in $c_{2,t}$, and hence in transfers T_t, more costly.

AMSS-like $I = 2$ economy, II

Proposition: Suppose that there is unique $\hat{l}$ that solves $h''(\hat{l})\hat{l} + h'(\hat{l}) = 1$. Let $c^*_{1,t}$ be an optimal allocation of consumption of the risk-neutral agent 1 in the *AMSS-like economy*. Then $c^*_{1,t} = \underline{c}$ infinitely often almost surely.

AMSS-like $I = 2$ economy, III

The government wants (a) to smooth labor distortions caused by taxes, and (b) to smooth consumption of the risk-averse agent. Can't do both perfectly. There is always a long enough sequence of bad shocks so that either (i) the government hits its borrowing limit and must adjust the distorting tax rate to raise revenues; (ii) the risk-averse type 2 agent hits his borrowing limit and can no longer smooth his consumption, in which case the government must adjust the distorting tax rate to help the type 2 risk-averse agent smooth consumption.

Therefore, optimal allocations are generally *history dependent*: allocation in period t depends not only on current realization of g_t but also on history of shocks g^{t-1}. Distorting taxes are also history dependent.

Nonstochastic stationary example

- Constant government expenditures $g_t = 1 \; \forall t$
- 2 types of agents, $\theta_1 \geq \theta_2$, with preferences

$$U^i(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$$

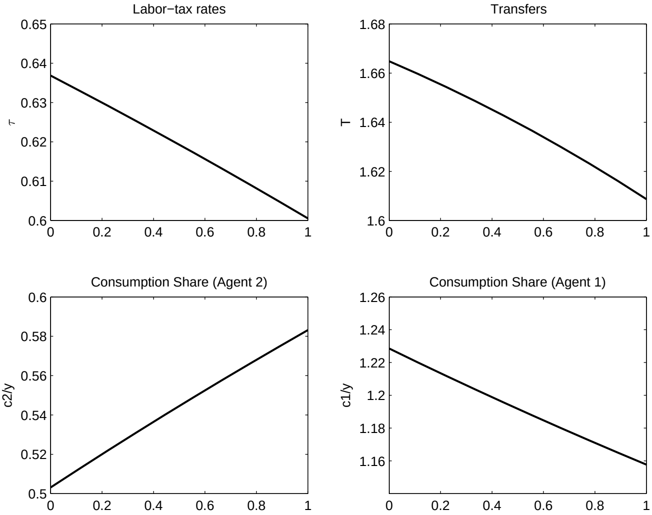
- B_{-1} is twice undistorted full employment GDP.
- The PO allocation has constant taxes and consumption.

Nonstochastic stationary example

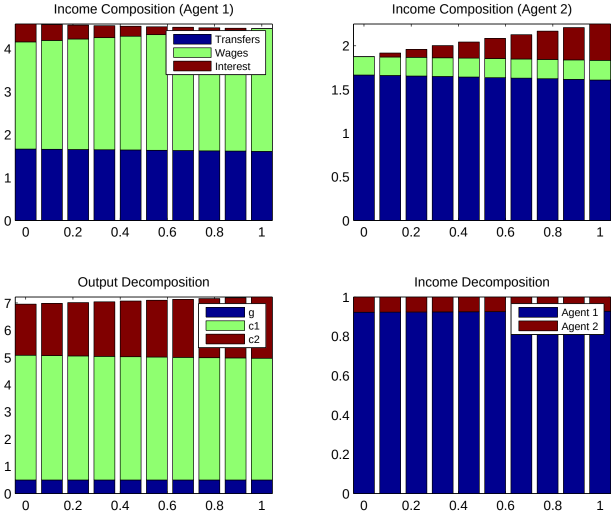
Parameter values:

- Preferences: $\sigma = 1, \gamma = .3$
- Endowments: $\theta_1 = 8, \theta_2 = 1$. (yields after tax $90 - 10$ labor earnings ratio of 5).
- Government expenditures: $g = 1$, government debt $= 2 \times$ GDP.
- Pareto weights: $\alpha_1 = \alpha_2 = .5$.
- Measures of types: $\pi_1 = \pi_2 = .5$.
- Fraction of debt owned by low skilled type 2 worker is $x \in [0, 1]$, displayed on x axis in graphs.

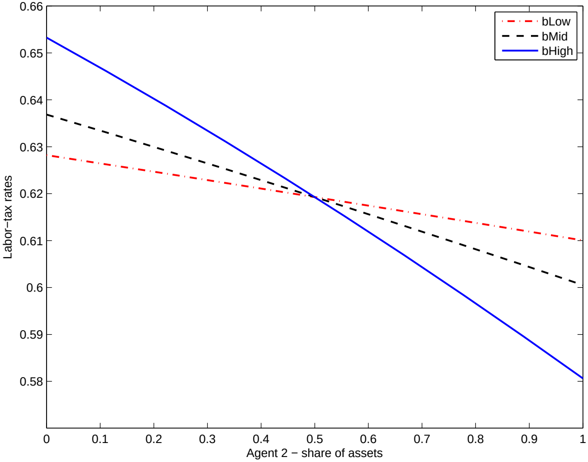
Taxes, transfers, and consumptions



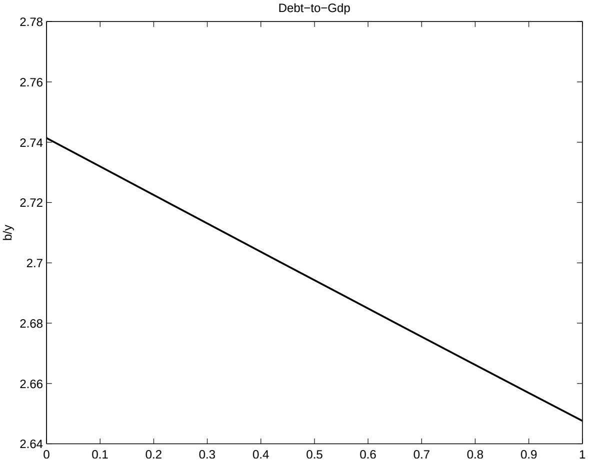
Income and output shares



Sensitivity of tax to type 2's share of assets



Debt to GDP ratio



Nonstochastic example: outcomes

The government sets affine taxes to finance g and to transfer from the high θ type to the low θ type through two types of transfers: T, the constant in the affine tax schedule, and $x\pi_1(R-1)B$, interest payments on the government debt received by the low θ type. When x is higher, the government sets a lower distorting tax rate τ and a lower explicit transfer T_t. Higher levels of initial government debt steepen the slopes of the τ on x curve because the larger is B, the more potent interest payments become as a means of subsidizing the low θ type.

Stochastic example

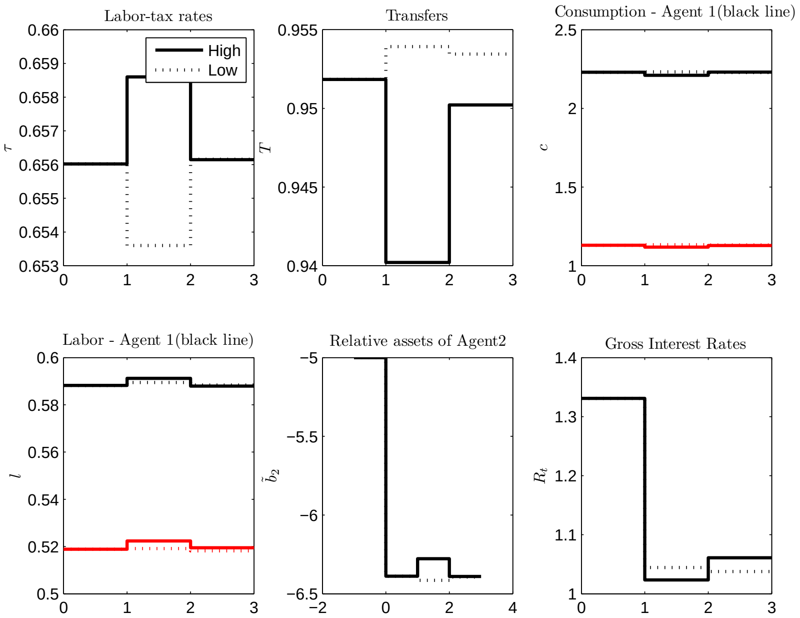
- I=2. $\theta_1 = 5, \theta_2 = 1, \gamma = 2, \sigma = 2, g_l = .1, g_h = .2.$
- $\alpha_1 = \alpha_2 = .5$; $\pi_1 = \pi_2 = .5$.
- Government expenditure path

$$g_t = \begin{cases} g_l & \text{if } t \neq 2 \\ g_l \quad \text{or} \quad g_h & \text{with prob .5 if } t = 2. \end{cases}$$

Outcomes, stochastic example

Distorting taxes rise and transfers fall in response to the high expenditure shock. Labor supplies increase enough to finance the extra output associated with the high government expenditure shock. Denote $-B_t = b_{1,t}$ as the total government *debt*, all held by Agent 1, so we normalize $b_{2,t} = 0$ always. The graphs take $-B_{-1} = 5$ as an initial condition. Whether Agent 1 accumulates assets in anticipation of the expenditure shock depends on how B_{-1} compares the steady state level B_{SS}.

Outcomes



Labor-tax rates
High
Low
τ
0.66
0.659
0.658
0.657
0.656
0.655
0.654
0.653
0
1
2
3
Transfers
T
0.955
0.95
0.945
0.94
Consumption - Agent 1(black line)
c
2.5
2
1.5
1
Labor - Agent 1(black line)
l
0.6
0.58
0.56
0.54
0.52
0.5
Relative assets of Agent2
$\tilde{b}_2$
−5
−5.5
−6
−6.5
−2
0
2
4
Gross Interest Rates
R_t
1.4
1.3
1.2
1.1
1

Outcomes, stochastic example

The level of debt is in its steady state from period 3 onwards. For a given $-B_{-1}$, $-B_{SS}$ is decreasing in γ. In period 0, agent 1 adjusts his savings to be approximately around this level. Then if the high government expenditure shock occurs in period 1, agent 1 draws down his savings, while he accumulates assets if the low shock is realized. This allows him partially to smooth consumption, since his after tax labor earnings and transfers are low if high government expenditures materialize. The government uses transfers to smooth tax distortions by reducing transfers when government expenditures are high.

Simple nonlinear taxes

$$c_{i,t} + b_{i,t} = \theta_i l_{i,t} - T_t(\theta_i l_{i,t}) + R_{t-1} b_{i,t-1}$$

$$g_t + B_t = \sum_{i=1}^{I} \pi_i T_t(\theta_i l_{i,t}) + R_{t-1} B_{t-1}.$$

Similar messages with affine taxes:

- Net distribution of initial assets $\left\{\tilde{b}_{i,-1}\right\}_{i>1}$ rather than $\{b_{i,-1}\}_{i=1}^{I}$ determines welfare under the optimal allocation.
- Optimal allocations are generally history dependent.

Constrained optima implemented with more general taxes

Mechanism design (NDPF):

$$\max_{\{c_{i,t}, y_{i,t}\}} \mathbb{E}_0 \sum_{i=1}^{I} \alpha_i \pi_i \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, \frac{y_{i,t}}{\theta_i} \right)$$

subject to incentive constraints

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{i,t}, \frac{y_{i,t}}{\theta_i} \right) \geq \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^i \left(c_{j,t}, \frac{y_{j,t}}{\theta_i} \right) \text{ for all } i, j$$

and feasibility constraints

$$\sum_{i=1}^{I} \pi_i c_{i,t} + g_t = \sum_{i=1}^{I} \pi_i y_{i,t}. \tag{1}$$

Implementation

Proposition: (i) Constrained optimal allocations can be decentralized as a competitive equilibrium with tax function $T_t\left(y_t, b_{t-1}, F\left(\{y_s\}_{s=0}^{t-1}\right)\right)$ where $F\left(\{y_s\}_{s=0}^{t-1}\right)$ is a function of previous labor earnings. (ii) The marginal tax on debt $\frac{\partial T_t\left(y_t, b_{t-1}, F\left(\{y_s\}_{s=0}^{t-1}\right)\right)}{\partial b}$ must be either a function of $\left(y_t, F\left(\{y_s\}_{s=0}^{t-1}\right)\right)$ or a non-linear function of b_{t-1}. We are free to set $T_t\left(y_t, b_{t-1}, F\left(\{y_s\}_{s=0}^{t-1}\right)\right) = y_t + \max\{R_t b_t, 0\}$ if $b_t \neq 0$.

Implementation

Remarks:

- In the optimal allocation, agents' marginal rates of substitution are not equalized:

$$\frac{\beta \mathbb{E}_t U_c^i \left(c_{i,t+1}^{sp}, y_{i,t+1}^{sp}/\theta_i \right)}{U_c^i \left(c_{i,t}^{sp}, y_{i,t}^{sp}/\theta_i \right)} \neq \frac{\beta \mathbb{E}_t U_c^j \left(c_{j,t+1}^{sp}, y_{j,t+1}^{sp}/\theta_j \right)}{U_c^j \left(c_{j,t}^{sp}, y_{j,t}^{sp}/\theta_j \right)}.$$

- Therefore, marginal returns on assets $\frac{\partial T_t\left(y_t, b_{t-1}, F\left(\{y_s\}_{s=0}^{t-1}\right)\right)}{\partial b}$ cannot be linear in asset holdings and must either depend on an agent's income, which typically will make them be state-contingent, or else be non-linear.
- Debt plays no useful role.
- The government can implement the optimum by taxing away *all* of an agent's income from assets if his debt differs from zero.
- The planner effectively completes asset markets either by making returns state-contingent or by shutting them.

Big picture speculation about roles of debt

- If a tax system is sufficiently flexible (i.e., sufficiently history dependent), debt markets play no useful role.
- If a tax system isn't sufficiently flexible, debt markets play a useful role.
- What role they play depends on many details of the environment including heterogeneity of agents, limits on transfers, and completeness of markets.