The allocation of interest rate risk 
and the financial sector

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Motivation

- Financial institutions are exposed to interest rate risk
  - maturity transformation (loans & securities vs deposits)
  - derivatives

- How to describe bank positions & compare them?
  - to match to economic models
  - to assess how & why riskiness of banks varies over time
    - do derivatives hedge other on-balance-sheet exposures?
    - do bank strategies differ in the cross section?
Notional Values of Interest Rate Derivatives

- All contracts
- Swaps

Graph showing the notional values of interest rate derivatives from 1995 to 2010.
Concentrated Holdings of Interest Rate Derivatives

- For trading
- Not for trading
- Top 3 dealers

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Overview

- data: bank positions in many fixed income instruments
  - organized by contract form (loan, deposit, swap etc.)

- size and direction of interest rate exposures?

- strategy: approximate positions as portfolios in a few bonds
  - works because values of instruments have a factor structure
  - makes instruments comparable & provides risk measures

- estimate portfolios from observables on each bank
  (separately for loans/deposits, securities, derivatives)

- compare across positions & banks
Related literature

- Interest exposure & derivative use
  - Flannery-James 84, Venkatachalam 96, Hirtle 97,…

- Derivatives positions
  - interest rate: Gorton-Rosen 95
  - credit: Stulz et al. 08, Hirtle 08

- Nonfinancial firms
  - who uses derivatives: Hentschel-Kothari 01, Jermann & Yue 12
  - market value of hedging: Alayanis-Weston 01, Jorion-Jin 06
  - economic magnitudes: Guay-Kothari 03

- Bank balance sheets
  - Adrian & Shin 08, Shin 11, He & Krishnamurthy 11

- Risk measurement for institutions
  - measures of tail risk (e.g., VaR literature)
    Acharya-Pederson-Philippon-Richardson 10, Kelly-Lustig-van Nieuwerburgh 11
  - “stress tests”: Brunnermeier-Gorton-Krishnamurthy 12, Duffie 12,…
Outline

- Replication
  - basic argument
  - one factor implementation
  - loan & security replication

- Interest rate swaps
  - terminology & data
  - estimation of replicating portfolio

- Example results for large US banks
Replication with spanning securities

- Factor structure with normal shocks
  - consider payoff stream with value $\pi(f_t, t)$
  - factors $f_t = \phi f_{t-1} + \sigma \epsilon_t$, $\epsilon_t \sim \mathcal{N}(0, I_K \times K)$

- Change in value of payoff stream $\pi$ between $t$ and $t+1$
  \[ \pi(f_{t+1}, t+1) - \pi(f_t, t) \approx a_t^\pi + b_t^\pi \epsilon_{t+1} \]

- form replicating portfolio from $K + 1$ spanning securities
  - always include $\theta_1^t$ 1 period bonds (= cash) with price $e^{-i_t}$
  - use $\hat{\theta}_t$ other securities, e.g. longer bonds

- choose $\theta_1^t, \hat{\theta}_t$ to match change in value $\pi$ for all $\epsilon_{t+1}$:
  \[
  \begin{pmatrix}
  \theta_1^t \\
  \hat{\theta}_t
  \end{pmatrix}
  \begin{pmatrix}
  e^{-i_t} i_t & 0 \\
  \hat{a}_t & \hat{b}_t
  \end{pmatrix}
  \begin{pmatrix}
  1 \\
  \epsilon_{t+1}
  \end{pmatrix}
  =
  \begin{pmatrix}
  a_t^\pi & b_t^\pi
  \end{pmatrix}
  \begin{pmatrix}
  1 \\
  \epsilon_{t+1}
  \end{pmatrix}.
  \]

- no arbitrage: value of replicating portfolio at $t = \text{value } \pi(f_t, t)$
Riskless (solid) & risky (dotted) zero coupon bond yields
Riskless (solid) & risky (dotted) zero coupon bond yields
Implementation with one factor

- **single factor** $f_t = 2$ year risky zero coupon yield from swap curve
- to relate value of other payoff streams $\pi$ to $f$, estimate one factor model of risky & riskless bonds jointly
- pricing kernel

$$M_{t+1} = \exp \left( -i_t - \frac{1}{2} \lambda_t^2 - \lambda_t \epsilon_{t+1} \right)$$

$$\lambda_t = \lambda_0 + \lambda_1 f_t$$

- riskless zero coupon bonds

$$P_t^{(n)} = E_t \left[ M_{t+1} P_{t+1}^{(n-1)} \right], \quad P_t^{(0)} = 1$$

$$P_t^{(n)} = \exp (A_n + B_n f_t)$$

- find $B_n < 0$ (hi interest rates, low prices)
- also $\lambda_t < 0$ so $E_t[\text{excess return on } n \text{ period bond}] = B_{n-1} \sigma \lambda_t > 0$
Credit risk

- Risky bonds default; recovery value proportional to price
- Knockoff per dollar invested

\[ \Delta_{t+1} = \exp \left( -d_0 - d_1 f_t - \frac{1}{2} d_2 - d_2 \varepsilon_{t+1} \right) \]

- Risky zero coupon prices

\[ \tilde{P}_t^{(n)} = E_t \left[ M_{t+1} \Delta_{t+1} \tilde{P}_t^{(n-1)} \right], \quad \tilde{P}_t^{(0)} = 1 \]
\[ \tilde{P}_t^{(n)} = \exp (\tilde{A}_n + \tilde{B}_n f_t) \]

- Parameters \( d \) describe spreads, e.g. risky short rate

\[ \tilde{i}_t = i_t + d_0 + d_1 f_t - d_2 \lambda_t \]

- Estimation finds
  - \( d_1 > 0 \) (spreads high when rates high)
  - \( d_2 < 0 \) (\( d_2 \lambda_t > 0 \) compensates for risk of drop in rates)
Replication with cash & 20 qtr riskless
with one factor model $a_t^\pi + b_t^\pi \varepsilon_{t+1} = \pi_t (i_t + \hat{\alpha}^\pi (\lambda_t + \varepsilon_{t+1}))$
Replication of bank positions

- Loans
  - derive stream of promised payments from book value & interest rate data
  - view stream of promises as bundle of risky zero coupon bonds
  - replicate as above

- Securities
  - observe fair values by maturity & issuer (private, government)
  - use risky, riskless prices to compute book values
  - replicate as above
  - bonds held for trading: rough assumptions on maturity

- Deposits & money market funds
  - mostly short term (≡ cash)

- Next: replication of interest rate derivatives
**Interest rate swaps**

- **Terminology**
  - swap = agreement to exchange streams of interest rate payments
  - payments ∝ notional amount $N$, up to maturity $m$
  - fixed leg = stream of payments at *swap rate* $s +$ notional at maturity
  - floating leg = market short rate
  - "pay-fixed" swap: pay fixed payments, receive floating payments
  - conversely, "pay-floating" swap

- Value of fixed leg: discount with bond prices $\tilde{P}_t^{(n)}$

  \[
  PV(\text{fixed leg}) = N \cdot \left( \sum_{i=1}^{m} s \cdot \tilde{P}_t^{(i)} + \tilde{P}_t^{(m)} \right)
  \]

- Value of floating leg:

  \[
  PV(\text{floating leg}) = N
  \]

- Fair value of pay fixed swap = $PV(\text{floating leg}) - PV(\text{fixed leg})$
Fair values of interest rate swaps

- Fair value of pay fixed (receive floating) swap with notional 1:

\[ F_t(m, s) = 1 - \left[ \sum_{i=1}^{m} s \cdot \tilde{P}_t^{(i)} + 1 \cdot \tilde{P}_t^{(m)} \right] \]

- At inception date, swap rate set s.t. \( F_t(m, s) = 0 \)
- Once \( s \) locked in & fair value moves with bond prices:
  - fair value falls if bond prices rise
    (interest rates fall, lower floating payments received)
  - fair value rises if bond prices fall
    (interest rates rise, higher floating payments received)

- Available data
  - bank notionals \( N_t \) & fair value \( FV_t \)
  - market swap rates \( s_t \)
  - do not observe direction (pay fixed or floating)

- Use state space model to infer swap position from fair values & history of interest rates
Gross versus net positions

- Dealers incorporate bid ask spread $\xi$ into swap rates
- $s =$ “midmarket” swap rate; quotes are $s \pm \xi/2$
- Dealer’s fair value on $N^{fix}$ notionals pay fixed

$$F_t (m, s - \xi/2) \cdot N^{fix} = \left(1 - \sum_{i=1}^{m} (s - \xi/2) \cdot \tilde{P}_t^{(i)} - \tilde{P}_t^{(m)}\right) N^{fix}$$

- Dealer’s fair value on $N^{fl}$ notional pay floating

$$-F_t (m, s + \xi/2) \cdot N^{fl} = \left(\sum_{i=1}^{m} (s + \xi/2) \cdot \tilde{P}_t^{(i)} + \tilde{P}_t^{(m)} - 1\right) N^{fl}$$

- Add up for net net pay-fixed position

$$\left(N^{fix} - N^{fl}\right) F_t (s, m) + \left(N^{fix} + N^{fl}\right) \xi \sum_{i=1}^{m} \tilde{P}_t^{(i)}$$

- Get 2nd term from data on total notionals & bidask spreads
- Now estimate first term
State space model of net swap positions

- Replicate net position with portfolio of cash and “spanning swap”
- State variables:
  \[ \omega_t = \text{net spanning swap position (pay-fixed!)} / \text{notionals} \]
  \[ K_t = \text{cash position} / \text{notionals} \]
  \[ \bar{s}_t = \text{locked swap rate} \]
- Given last period’s strategy
  \[ \text{net fair value}(t) = \left[ \omega_{t-1} F_t \left( m - 1, \bar{s}_{t-1}, (\tilde{P}_t^{(n)}) \right) + K_{t-1} \right] N_{t-1} \]
- State space representation of ratio net fair value/notionals
  \[ \frac{\text{net fair value}(t)}{\text{notionals}(t-1)} = \omega_{t-1} F_t \left( m - 1, \bar{s}_{t-1}, (\tilde{P}_t^{(n)}) \right) + K_{t-1} + u_t \]
  \[ (\omega_t, K_t, \bar{s}_t) = T_t (\omega_{t-1}, K_{t-1}, \bar{s}_{t-1}) \]
- Transition captures
  - updating of maturities: from \( \omega_{t-1} \) (end of t-1) to \( \tilde{\omega}_t \) (beginning of t)
  - swap trading: from \( \tilde{\omega}_t \) (beginning of t) to \( \omega_t \) (end of t)
Swap trading

- Transition equation for state variables
  \[(\omega_t, \bar{s}_t, K_t) = T_t (\omega_{t-1}, \bar{s}_{t-1}, K_{t-1})\]

- From \((\tilde{\omega}_t, \tilde{K}_t)\) (beginning of period) to \((\omega_t, K_t)\) (end of period)

- Two possible swap trades
  1. Increasing exposure: start new swaps \(\omega_{t,new}^{\text{notionals}(t)}\)
     adjust swap rate proportionately to share of new swaps
  2. Decreasing exposure: offset a fraction \(\gamma_t\) of the existing old swaps
     swap rate does not change

- Swap position after trading at \(t\)
  \[\omega_t \text{ notionals}(t) = (1 - \gamma_t) \tilde{\omega}_t \text{ notionals}(t-1) + \omega_{t,new}^{\text{notionals}(t)}\]

- Assume only one type of trade occurs every period...
  .. except when switching direction \((\gamma_t = 1 \text{ and } \omega_t = \omega_{t,new})\)
State space representation: estimation

- State space representation for bank’s ratio fair value / notional value

\[
\frac{\text{net fair value}(t)}{\text{notionals}(t-1)} = \omega_{t-1} F_t (m - 1, \bar{s}_{t-1}) + K_{t-1} + u_t
\]

\[
(\omega_t, \bar{s}_t, K_t) = T_t (\omega_{t-1}, \bar{s}_{t-1}, K_{t-1})
\]

- Transition equation implies

sequence \( \{\omega_t\}_{t=1}^T, \bar{s}_1, K_1 \Rightarrow \) unique sequences \( \{\bar{s}_t, K_t\}_{t=2}^T \)

- Bayesian estimate of \( \{\omega_t\} \) derived by MCMC
  - \( \omega = \) random walk with iid normal innovations
  - uninformative gamma priors for variances of innovations, measurement error
  - initial conditions for 1995 \( K_1 = 0, \bar{s}_1 = \) current swap rate
JP Morgan Chase: swap position

notionals

$ trillions

1995 2000 2005 2010

0 20 40 60

net fair value / notional (%)

1995 2000 2005 2010

-0.1 -0.05 0 0.05 0.1

5 year swap rate (% p.a.)

1995 2000 2005 2010

2 4 6

current
JP Morgan Chase: swap position

- **Notionals**
  - Y-axis: $ trillions

- **Net fair value / notional (%)**
  - Y-axis: -0.1 to 0.1

- **Omega (%)**
  - Y-axis: -2 to 6

- **5 year swap rate (% p.a.)**
  - Y-axis: 2 to 6
  - Two lines: current (gray), locked in (blue)

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JPMorgan Chase (blue) & BofA (green): swap positions

Notionals

Net fair value / notional (%)

Omega (%)

5 year swap rate (% p.a.)

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JP Morgan Chase: replicating portfolios

![Graph showing portfolio replication](image)

Legend:
- Cash, old FI
- 5 year, old FI
- Cash, deriv
- 5 year, deriv

Trillions $US

-0.5
0
0.5
1
1.5

1995 2000 2005 2010
JP Morgan Chase: replicating portfolios
Summary

- Methodology to measure exposures in bank positions:
  Accounting data (e.g. in Bank call reports, SEC filings)
  Loans, deposits, securities etc.:
  maturity information, face/fair values
  Derivatives (mostly swaps):
  Bayesian estimation of net exposure under assumptions on trading

- Results for top dealer banks
  Derivatives further increase exposure to interest rate risk
  when rates fell, gains from floating positions

- Currently working on
  - investment banks (annual report data before 2009)
  - idiosyncratic default & role of CDS
  - refine valuation (trading assets, prepayment options)

- Future work
  - other factors (slope, credit, liquidity)