Reputation and TFP shocks

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How does reputation investment respond to aggregate shocks?

Reputation relates to

Brand value, advertising,
intangible capital,
Tobin's q

Reputation may also help explain

1. news-shocks effects
2. Great Moderation vs. the rise in idiosyncratic volatility
3. Durable sectors – why they lead GDP.

Model: Holmström (99) + TFP shocks + asset market + representative family

Reputation about firm efficiency

Firm's output in efficiency units

$$y_t = z_t \left(\theta_t + a_t + \varepsilon_t\right),$$

a_t = effort

$$\text{cost of } a \text{ in goods} = g\left(a_t\right)$$

z_t = TFP (aggregate variable)

θ_t = "quality" (firm specific)

$$\theta_t = \theta_{t-1} + \nu_t \ ,$$

$$\begin{aligned} \varepsilon_t &\sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right), \\ \nu_t &\sim \mathcal{N}\left(0, \sigma_\nu^2\right). \end{aligned}$$

Histories (y_t, z_t) are public info.

Learning.—No one knows θ, and the common prior is $\mathcal{N}\left(m_0, \sigma_\theta^2\right)$. Let

$$x_t \equiv \frac{y_t}{z_t} - a_t^* = \theta + \varepsilon_t + a_t - a_t^* \ , \tag{1}$$

Posterior $\theta_t \sim \mathcal{N}\left(m_t, \sigma_{\theta,t}^2\right)$.

$$\begin{aligned} m_{t+1} &= \lambda_t m_t + (1 - \lambda_t)\, x_t \\ &= m_t + (1 - \lambda_t)\left(\theta - m_t + \varepsilon_t + a_t - a_t^*\right), \end{aligned}$$

and

$$\sigma_{\theta,t+1}^2 = \lambda_t \sigma_{\theta,t}^2 + \sigma_\nu^2.$$

where

$$\lambda_t \equiv \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\theta,t}^2}.$$

Output

$$y_t = z_t \left(\theta_t + a_t + \varepsilon_t\right)$$

Timing during period t:

1. Everyone sees z_t
2. Customers diversify their goods purchases. Firms get up front revenue:
$$R_t = z_t \left(m_t + a_t^*\right)$$
3. Firm chooses a_t
4. y_t is realized and publicly observed
5. Firm pays dividend
$$D_t = R_t - g\left(a_t\right)$$
then dies with Prob. δ.
6. Assets trade

First best.—

$$z_t = g'(a_t). \tag{2}$$

You would get it if

1. you had contracts contingent on a or even on y
2. policy can reward y retroactively and tax lump sum

$$y_t = z_t \left(\theta + a_t + \varepsilon_t \right)$$

Learning on the equilibrium path.

$$a_t^* \left(z_t, x^t \right) = \text{equilibrium action}$$

$$x^t \equiv \left(x_0, ..., x_{t-1} \right) \text{ and where}$$

$$x_t \equiv \frac{y_t}{z_t} - a_t^* \left(z_t, x^t \right) = \theta + \varepsilon_t . \tag{3}$$

Law of motion of beliefs.— Let Δ denote a deviation from the equilibrium action, then

$$\begin{aligned} m_{t+1} &= m_t + \left(1 - \lambda_t \right) \left[\varepsilon_t + \Delta_t - S_t \right] , \quad \lambda_t \equiv \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_{\theta,t}^2} , \\ S_{t+1} &= \lambda_t S_t + \left(1 - \lambda_t \right) \Delta_t . \end{aligned}$$

S_t weighted sum of past deviations: *Persistence* of private information.

Steady state.—

$$\bar{\sigma}_\theta^{-2} \equiv \lim_{t \to \infty} \sigma_{\theta,t}^{-2} = \frac{1}{2}\left(\sqrt{\frac{1}{\sigma_\varepsilon^4} + \frac{4}{\sigma_\varepsilon^2 \sigma_v^2}} - \frac{1}{\sigma_\varepsilon^2}\right) .$$

For tractability, assume that:

1. Firms die with prob. δ. Replaced by new firms with $\sigma_{\theta,0} = \bar{\sigma}_\theta$
2. Priors' mean $m = 0 \Rightarrow \bar{m} = 0$.

Preferences.—Large representative family:

$$E\left\{\sum \beta^t U(c_t) \mid z_0, s_0 = 1\right\}. \tag{4}$$

Income identity.—Every firm chooses the same a_t.

$$\begin{aligned} D_t &= \int_0^1 y_i di - g(a_t) = z_t a_t - g(a_t), \\ c_t &= D_t - \delta k. \end{aligned} \tag{5}$$

Budget constraint.—At equilibrium, $s(m,z) = 1$. Starting from equilib. initial holdings (i.e., one-shot deviations only)

$$c = \delta\left[p(\bar{m}, z) - k\right] + D(z) + (1-\delta) P(z) - \int p(m,z) s(m,z) \Phi\left(\frac{dm}{\bar{\sigma}_\theta}\right) \tag{6}$$

where

$$P(z) = \int p(m,z) \Phi\left(\frac{dm}{\bar{\sigma}_\theta}\right) .$$

Family FOC.—

$$p(m,z) = \beta(1-\delta) E\left[\frac{U'(c(z'))}{U'(c(z))} \left(z'm' + D(z') + p(m',z')\right) \mid m,z \right] \tag{7}$$

Tobin's q.—Aggregate and individual Tobin's q

$$Q = \frac{P(z)}{k} \quad \text{and} \quad q = \frac{p(m,z)}{k} \tag{8}$$

The firm manager's problem.— "Maximize $p(m,z)$."

$$
\begin{aligned}
V(m,z,S) &= \max_{a+\Delta}\left\{\begin{array}{c} -g(a+\Delta)+(1-\delta)\beta E\frac{U'(c(z'))}{U'(c(z))}\times \\ \times\left[z'(m'+a(z'))-g(a^*(z'))+V(m',z',S')\right]\end{array}\right\} \\
m' &= m+(1-\lambda)[\Delta-S+\varepsilon]\ , & (9) \\
S' &= \lambda S+(1-\lambda)\Delta\ . & (10)
\end{aligned}
$$

FOC

$$g'(a_t^*)=\frac{1-\lambda}{\lambda}\sum_{s=t+1}^{\infty}[(1-\delta)\beta\lambda]^{s-t}E_t\left[\frac{U'(c_s)}{U'(c_t)}z_s\right]\ . \qquad (11)$$

Then

$$p(m,z)=V(m,z,0)+g(a^*(z)) \qquad (12)$$

SIMPLER DEFINITION OF EQ.

An equilibrium is a pair $\{a, P\}$ that solves the Incentive Constraint and Asset Pricing equations

$$
\begin{aligned}
(IC) \quad &: \quad g'(a) = (1-\delta)\,\beta E\left[\frac{U'(c(z'))}{U'(c(z))}\left((1-\lambda)z' + \lambda g'(a')\right)\right], \\
(AP) \quad &: \quad P(z) = (1-\delta)\,\beta E\left[\frac{U'(c(z'))}{U'(c(z))}\left(c(z') + P(z')\right)\right],
\end{aligned}
$$

where

$$
c(z) = z\left[\overline{m} + a(z)\right] - g\left[a(z)\right] - \delta k.
$$

- Strategic complementarity to be explained verbally
- when $z_t = 1$ all t, there is a constant solution and there are cycles

FOC again.—

$$g'\left(a_t^*\right) = \frac{1-\lambda}{\lambda}\sum_{s=t+1}^{\infty}\left[\left(1-\delta\right)\beta\lambda\right]^{s-t} E_t\left[\frac{U'\left(c_s\right)}{U'\left(c_t\right)} z_s\right] . \qquad (13)$$

Stock prices and news shocks.

The $E_t\left[\frac{U'(c_s)}{U'(c_t)} z_s\right]$ channel is present in other models

But: Bigger effect here – amplified via reaction of a which (to be shown below) is probably below first best.

Solved example.—Let $k = 0$. Then $C = D$

$$U(c) = \frac{C^{1-\gamma}}{1-\gamma}, \text{ and } g(a) = a^2/2 \; .$$

$$\log(z_{t+1}) = \log(z_t) + \varepsilon_t^z, \text{ where } \varepsilon_t^z \sim \mathcal{N}\left(\mu - \frac{\sigma_z^2}{2}, \sigma_z^2\right)$$

Then

$$a(z) = Az,$$

$$c(z) = \left(A - \frac{A^2}{2}\right) z^2 \; ,$$

where

$$A = \frac{(1-\delta)\beta(1-\lambda) E\left[\exp(\varepsilon^z)^{1-2\gamma}\right]}{1-(1-\delta)\beta\lambda E\left[\exp(\varepsilon^z)^{1-2\gamma}\right]} \; . \tag{14}$$

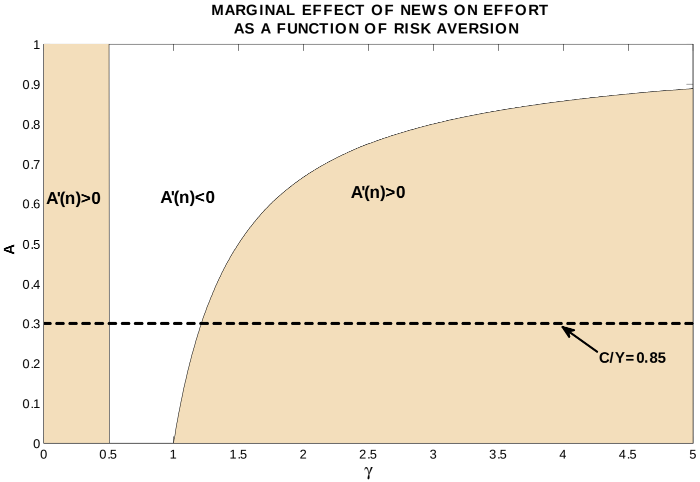
where

$$E\left[\exp(\varepsilon^z)^{1-2\gamma}\right] = \exp\left((2\gamma-1)\left(\gamma\sigma_z^2 - \mu\right)\right)$$

News Shocks

$$\log(z') = \log(z) + n + \varepsilon_t^z \ .$$

"MIT" shock n is a one time increase in the TFP growth rate.

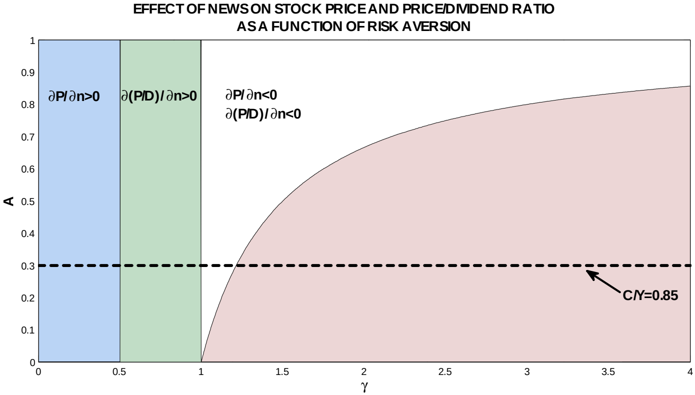

MARGINAL EFFECT OF NEWS ON EFFORT
AS A FUNCTION OF RISK AVERSION
A'(n)>0
A'(n)<0
A'(n)>0
C/Y=0.85
A
γ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

The aggregate share price as a function of z and n reads

$$P(z,n) = P(n)z^2 \ ,$$

where $P\left(n\right) =$

$$\frac{\left(A - A^2/2\right)^{1-\gamma}}{\left(A\left(n\right) - A\left(n\right)^2/2\right)^{-\gamma}} \left(\frac{\beta\left(1-\delta\right) E\left[\exp\left(\varepsilon^{z\prime}\right)^{2-2\gamma}\right] \exp\left(n\right)^{2-2\gamma}}{1 - \beta\left(1-\delta\right) E\left[\exp\left(\varepsilon^{z\prime}\right)^{2-2\gamma}\right] \exp\left(n\right)^{2-2\gamma}} \right) \ .$$


EFFECT OF NEWS ON STOCK PRICE AND PRICE/DIVIDEND RATIO
AS A FUNCTION OF RISK AVERSION
∂P/∂n>0
∂(P/D)/∂n>0
∂P/∂n<0
∂(P/D)/∂n<0
C/Y=0.85
A
γ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.5
1
1.5
2
2.5
3
3.5
4

Since

$$c(z) > 0 \Longleftrightarrow A < 2$$

This requires

$$(1-\delta)\,\beta\,(1+\lambda)\exp\left(\sigma_z^2\right) < 2 \tag{15}$$

Comparison to first best

$$a^{FB} = z \text{ and } c^{FB} = \frac{1}{2}z^2 \ .$$

It immediately follows from (14) that

$$(1-\delta)\,\beta < \exp\left(-\sigma_z^2\right) \Leftrightarrow a^* < a^{FB} \ .$$

For our example

$$p(m,z) = \left(\frac{\beta(1-\delta)\exp\left(\sigma_z^2\right)}{1-\beta(1-\delta)\exp\left(\sigma_z^2\right)} \right) mz + \frac{\beta(1-\delta)}{1-\beta(1-\delta)} \left(A - \frac{A^2}{2} \right) z^2 \ .$$

First best $(A=1)$ maximizes stock price.

Extension 1: An "investment-specific" shock

$$a = \zeta g^{-1} \left(\text{consumption goods}\right)$$

Then

$$\text{cost} = \frac{1}{\zeta} g\left(a\right).$$

write

$$a = \zeta g^{-1}\left(I\right)$$

where I is hidden investment.

Then you cannot reverse engineer a from (y, D)....

Extension 2: A Mehra Prescott version

What if growth is AR1? E.g.,

$$\Delta \ln z_t \in \{n_1, n_2\}$$

with a first-order transition probability matrix

$$n_t \quad \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \quad\quad n_{t+1} \\ \begin{array}{cc} 0 & \quad\quad\quad 1 \end{array} \\ \left[\begin{array}{cc} \alpha & 1-\alpha \\ 1-\alpha & \alpha \end{array} \right] \end{array} \tag{16}$$

with $\alpha > 1/2$.

Similar to LBD, except doing yields only aggregate gains, no individual gains

Literature:

1. Signal confusion models.
   Li & Weinberg *IER* 03. Confusing z and local shocks
   Lucas JET 72: Confusing z and m

2. Atkeson, Hellwig Ordonez 12
   only one hidden action at entry

3. Fishman and Rob *JPE* 05
   multiple equilibria – no types θ to anchor things.

4. Advertising and pricing as a signal Milgrom Roberts JPE 96

5. Customer switching costs Gourio & Rudanko

6. Bounded rationality: Mackowiak & Wiederholt *AER* 09.
   When $\sigma_\varepsilon^2 \uparrow$ firm pays less attention to x

Data implications

1. Reputation yields a positive effect of news on stock prices and on activity
2. Idiosyncratic volatility reduces response to news and shocks.
3. Great moderation as a result of a rise in idiosyncratic volatility?
4. Durables lead the cycle?