Reputation and TFP shocks

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How does reputation investment respond to aggregate shocks?

Reputation relates to

Brand value, advertising,
intangible capital,
Tobin’s q
Reputation may also help explain

1. news-shocks effects
2. Great Moderation vs. the rise in idiosyncratic volatility
3. Durable sectors – why they lead GDP.
Model: Holmström (99) + TFP shocks + asset market + representative family

Reputation about firm efficiency
Firm’s output in efficiency units

\[ y_t = z_t (\theta_t + a_t + \varepsilon_t), \]

\[ a_t = \text{effort} \]

\[ \text{cost of } a \text{ in goods} = g(a_t) \]

\[ z_t = \text{TFP (aggregate variable)} \]

\[ \theta_t = \text{“quality” (firm specific)} \]

\[ \theta_t = \theta_{t-1} + \nu_t, \]

\[ \varepsilon_t \sim \mathcal{N}(0, \sigma^2_{\varepsilon}) \]

\[ \nu_t \sim \mathcal{N}(0, \sigma^2_{\nu}) \]

Histories \( (y_t, z_t) \) are public info.
Learning. — No one knows $\theta$, and the common prior is $\mathcal{N} \left( m_0, \sigma^2_\theta \right)$. Let

$$x_t \equiv \frac{y_t}{z_t} - a^*_t = \theta + \varepsilon_t + a_t - a^*_t,$$

(1)

Posterior $\theta_t \sim \mathcal{N} \left( m_t, \sigma^2_{\theta, t} \right)$.

$$m_{t+1} = \lambda_t m_t + (1 - \lambda_t) x_t$$

$$= m_t + (1 - \lambda_t) \left( \theta - m_t + \varepsilon_t + a_t - a^*_t \right),$$

and

$$\sigma^2_{\theta, t+1} = \lambda_t \sigma^2_{\theta, t} + \sigma^2_v.$$
\[ y_t = z_t (\theta_t + a_t + \varepsilon_t) \]

Timing during period \( t \):

1. Everyone sees \( z_t \)
2. Customers diversify their goods purchases. Firms get up front revenue:
   \[ R_t = z_t (m_t + a_t^*) \]
3. Firm chooses \( a_t \)
4. \( y_t \) is realized and publicly observed
5. Firm pays dividend
   \[ D_t = R_t - g(a_t) \]
   then dies with Prob. \( \delta \).
6. Assets trade
First best.—

\[ z_t = g'(a_t) . \] \hspace{1cm} (2)

You would get it if

1. you had contracts contingent on \( a \) or even on \( y \)
2. policy can reward \( y \) retroactively and tax lump sum
\[ y_t = z_t (\theta + a_t + \epsilon_t) \]

**Learning on the equilibrium path.**

\[ a^*_t (z_t, x^t) = \text{equilibrium action} \]

\[ x^t \equiv (x_0, ..., x_{t-1}) \] and where

\[ x_t \equiv \frac{y_t}{z_t} - a^*_t (z_t, x^t) = \theta + \epsilon_t. \] (3)

**Law of motion of beliefs.**— Let \( \Delta \) denote a deviation from the equilibrium action, then

\[

m_{t+1} = m_t + (1 - \lambda_t) \left[ \epsilon_t + \Delta_t - S_t \right], \quad \lambda_t \equiv \frac{\sigma^2_\epsilon}{\sigma^2_\epsilon + \sigma^2_{\theta,t}},
\]

\[
S_{t+1} = \lambda_t S_t + (1 - \lambda_t) \Delta_t.
\]

\( S_t \) weighted sum of past deviations: *Persistence* of private information.
Steady state.—

\[
\bar{\sigma}_\theta^{-2} \equiv \lim_{t \to \infty} \sigma_{\theta,t}^{-2} = \frac{1}{2} \left( \sqrt{\frac{1}{\sigma_\varepsilon^4} + \frac{4}{\sigma_\varepsilon^2 \sigma_v^2}} - \frac{1}{\sigma_\varepsilon^2} \right)
\]

For tractability, assume that:

1. Firms die with prob. \( \delta \). Replaced by new firms with \( \sigma_{\theta,0} = \bar{\sigma}_\theta \)
2. Priors’ mean \( m = 0 \) \( \Rightarrow \bar{m} = 0 \).
Preferences.—Large representative family:

\[ E \left\{ \sum \beta^t U(c_t) \mid z_0, s_0 = 1 \right\}. \]  \hspace{1cm} (4)

Income identity.—Every firm chooses the same \( a_t \).

\[ D_t = \int_0^1 y_i \, di - g(a_t) = z_t a_t - g(a_t), \]  \hspace{1cm} (5)
\[ c_t = D_t - \delta k. \]

Budget constraint.—At equilibrium, \( s(m, z) = 1 \). Starting from equilib.
initial holdings (i.e., one-shot deviations only)

\[ c = \delta \left[ p(\bar{m}, z) - k \right] + D(z) + (1 - \delta) P(z) - \int p(m, z) \, s(m, z) \, \Phi \left( \frac{dm}{\bar{\sigma}_\theta} \right). \]  \hspace{1cm} (6)

where

\[ P(z) = \int p(m, z) \, \Phi \left( \frac{dm}{\bar{\sigma}_\theta} \right). \]
Family FOC.—

\[ p(m, z) = \beta (1 - \delta) E \left[ \frac{U'(c(z'))}{U'(c(z))} \left( z'm' + D(z') + p(m', z') \right) \right] | m, z \]  

(7)

Tobin's q.—Aggregate and individual Tobin's q

\[ Q = \frac{P(z)}{k} \quad \text{and} \quad q = \frac{p(m, z)}{k} \]  

(8)
The firm manager’s problem.— “Maximize $p(m, z)$.”

\[
V(m, z, S) = \max_{a+\Delta} \left\{ \begin{array}{l}
-g(a + \Delta) + (1 - \delta) \beta E \frac{U'(c(z'))}{U'(c(z))} \times \\
\times [z'(m' + a(z')) - g(a^*(z')) + V(m', z', S')] \end{array} \right\}
\]

(9)

\[
m' = m + (1 - \lambda) \left[ \Delta - S + \epsilon \right],
\]

(10)

\[
S' = \lambda S + (1 - \lambda) \Delta.
\]

FOC

\[
g'(a^*_t) = \frac{1 - \lambda}{\lambda} \sum_{s=t+1}^{\infty} [(1 - \delta) \beta \lambda]^{s-t} \mathbb{E}_t \left[ \frac{U'(c_s)}{U'(c_t)} z_s \right].
\]

(11)

Then

\[
p(m, z) = V(m, z, 0) + g(a^*(z))
\]

(12)
SIMPLER DEFINITION OF EQ.

An equilibrium is a pair \( \{ a, P \} \) that solves the Incentive Constraint and Asset Pricing equations

\[
\begin{align*}
(\text{IC}) \quad & g'(a) = (1 - \delta) \beta E \left[ \frac{U'(c(z'))}{U'(c(z))} \left( (1 - \lambda)z' + \lambda g'(a') \right) \right], \\
(\text{AP}) \quad & P(z) = (1 - \delta) \beta E \left[ \frac{U'(c(z'))}{U'(c(z))} (c(z') + P(z')) \right],
\end{align*}
\]

where

\[
c(z) = z \left[ \bar{m} + a(z) \right] - g \left[ a(z) \right] - \delta k.
\]

- Strategic complementarity to be explained verbally
- when \( z_t = 1 \) all \( t \), there is a constant solution and there are cycles
The expression for $g'(a_t^*)$ is given by:

$$g'(a_t^*) = \frac{1 - \lambda}{\lambda} \sum_{s=t+1}^{\infty} [(1 - \delta) \beta \lambda]^{s-t} E_t \left[ \frac{U'(c_s)}{U'(c_t)} z_s \right]. \quad (13)$$

Stock prices and news shocks.

The $E_t \left[ \frac{U'(c_s)}{U'(c_t)} z_s \right]$ channel is present in other models. But: Bigger effect here – amplified via reaction of a which (to be shown below) is probably below first best.
Solved example.—Let \( k = 0 \). Then \( C = D \)

\[
U(c) = \frac{C^{1-\gamma}}{1-\gamma}, \text{ and } g(a) = a^2/2 .
\]

\[
\log(z_{t+1}) = \log(z_t) + \varepsilon_t^z, \text{ where } \varepsilon_t^z \sim \mathcal{N}\left(\mu - \frac{\sigma_z^2}{2}, \sigma_z^2\right)
\]

Then

\[
a(z) = Az,
\]

\[
c(z) = \left(A - \frac{A^2}{2}\right) z^2 ,
\]

where

\[
A = \frac{(1 - \delta) \beta (1 - \lambda) E \left[\exp(\varepsilon^z)^{1-2\gamma}\right]}{1 - (1 - \delta) \beta \lambda E \left[\exp(\varepsilon^z)^{1-2\gamma}\right]} . \quad (14)
\]

where

\[
E \left[\exp(\varepsilon^z)^{1-2\gamma}\right] = \exp \left((2\gamma - 1) \left(\gamma \sigma_z^2 - \mu\right)\right)
\]
News Shocks

\[ \log (z') = \log (z) + n + \epsilon_t^z. \]

"MIT" shock \( n \) is a one time increase in the TFP growth rate.
MARGINAL EFFECT OF NEWS ON EFFORT AS A FUNCTION OF RISK AVERSION

\[ A'(n) > 0 \]
\[ A'(n) < 0 \]
\[ A'(n) > 0 \]

\[ C/Y = 0.85 \]
The aggregate share price as a function of \( z \) and \( n \) reads

\[
P(z, n) = P(n)z^2,
\]

where \( P(n) = \)

\[
\frac{(A - A^2/2)^{1-\gamma}}{(A(n) - A(n)^2/2)^{-\gamma}} \left( \frac{\beta(1-\delta)E\left[\exp\left(\varepsilon z'\right)^{2-2\gamma}\right] \exp\left(n\right)^{2-2\gamma}}{1 - \beta(1-\delta)E\left[\exp\left(\varepsilon z'\right)^{2-2\gamma}\right] \exp\left(n\right)^{2-2\gamma}} \right).
\]
EFFECT OF NEWS ON STOCK PRICE AND PRICE/DIVIDEND RATIO
AS A FUNCTION OF RISK AVERSION

\[ \frac{\partial P}{\partial n} > 0 \quad \frac{\partial (P/D)}{\partial n} > 0 \]
\[ \frac{\partial P}{\partial n} < 0 \quad \frac{\partial (P/D)}{\partial n} < 0 \]

C/Y = 0.85
Since

\[ c(z) > 0 \iff A < 2 \]

This requires

\[ (1 - \delta) \beta (1 + \lambda) \exp(\sigma_z^2) < 2 \]  

(15)

Comparison to first best

\[ a^{FB} = z \text{ and } c^{FB} = \frac{1}{2} z^2 \]

It immediately follows from (14) that

\[ (1 - \delta) \beta < \exp(-\sigma_z^2) \iff a^* < a^{FB} \]
For our example

\[ p(m, z) = \left( \frac{\beta (1 - \delta) \exp(\sigma_z^2)}{1 - \beta (1 - \delta) \exp(\sigma_z^2)} \right) mz + \frac{\beta (1 - \delta)}{1 - \beta (1 - \delta)} \left( A - \frac{A^2}{2} \right) z^2. \]

First best \((A = 1)\) maximizes stock price.
Extension 1: An “investment-specific” shock

\[ a = \zeta g^{-1} \text{ (consumption goods)} \]

Then

\[ \text{cost} = \frac{1}{\zeta} g(a). \]

write

\[ a = \zeta g^{-1}(I) \]

where \( I \) is hidden investment.

Then you cannot reverse engineer \( a \) from \((y, D)\)....
Extension 2: A Mehra Prescott version

What if growth is AR1? E.g.,

$$\Delta \ln z_t \in \{n_1, n_2\}$$

with a first-order transition probability matrix

$$n_t \begin{bmatrix} 0 & n_{t+1} \\ 1 & 1 - \alpha & \alpha \end{bmatrix}$$

(16)

with $$\alpha > 1/2.$$
Similar to LBD, except doing yields only aggregate gains, no individual gains
Literature:

1. Signal confusion models.
   - Li & Weinberg *IER* 03. Confusing $z$ and local shocks
   - Lucas JET 72: Confusing $z$ and $m$

2. Atkeson, Hellwig Ordonez 12
   - only one hidden action at entry

3. Fishman and Rob *JPE* 05
   - multiple equilibria – no types $\theta$ to anchor things.

4. Advertising and pricing as a signal Milgrom Roberts *JPE* 96

5. Customer switching costs Gourio & Rudanko

   - When $\sigma^2_{\epsilon} \uparrow$ firm pays less attention to $x$
Data implications

1. Reputation yields a positive effect of news on stock prices and on activity.

2. Idiosyncratic volatility reduces response to news and shocks.

3. Great moderation as a result of a rise in idiosyncratic volatility?

4. Durables lead the cycle?