A Model of Mortgage Renegotiation*

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May 13, 2014

Abstract

Lenders are sometimes willing to renegotiate mortgage contracts with homeowners. This paper models renegotiation as a simple sequential-move game in which the homeowner seeks renegotiation and the lender decides whether to modify the mortgage. The model is used to examine the effect of incentives like those given to homeowners and lenders during the foreclosure crisis. Results show that, without incentives, lenders renegotiate with a subset of homeowners who avoid foreclosure as a result. Incentives expand the set of homeowners who receive modifications and avoid foreclosure. However, under certain conditions, incentives lead lenders to renegotiate with homeowners who subsequently end up in foreclosure. (JEL codes: G21, H81, R31)

1 Introduction

In response to the foreclosure crisis that began in 2006, the U.S. federal government introduced several programs to help homeowners avoid foreclosure. For example, the Home

*The idea for this paper germinated from a collaboration with the Community Development staff at the Federal Reserve Bank of Richmond, whom I acknowledge with gratitude. I thank Andreas Horstein, Ned Prescott and seminar participants at the Federal Reserve Bank of Richmond for helpful comments. I am solely responsible for any errors. The views expressed in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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Affordable Modification Program (HAMP) encouraged servicers to work with homeowners to modify the terms of their mortgage. HAMP offered servicers $1,000 for each modification completed under the program (Making Home Affordable, 2010). Additional incentives were offered to homeowners and servicers for up to three years for loans that remained in good standing.

The goal of this paper is examine the effect of such incentives on mortgage renegotiation or modification (I use the terms interchangeably). Adelino, Gerardi, and Willen (2009) point out that renegotiation is risky from the lenders' perspective in that it potentially exposes them to homeowners whose mortgages they would not want to modify. They group such homeowners into two categories — those who would "self-cure," that is, make their mortgage payments on time without renegotiation, and those who would "redefault," that is, fail to make their payments on time despite renegotiated terms. I formalize this idea in a sequential move game. I compare outcomes from a model with monetary incentives for homeowners and lenders to one without.

Results show that, in the absence of incentives, lenders renegotiate with a subset of homeowners who would neither i) redefault despite receiving modified terms nor ii) self-cure without modified terms. The renegotiation enables this subset of homeowners to avoid foreclosure. In the model with incentives, this subset is larger than in the model without incentives. However, for certain parameter values, incentives induce lenders to renegotiate with homeowners who subsequently redefault.

Mortgage modification programs are sometimes gauged by comparing "success rates" — defined as the fraction of homeowners who avoid foreclosure — across homeowners who do and do not receive modifications. I show that this comparison is not necessarily informative because success rates among those who do not receive modifications may be high if this group includes a large proportion of homeowners who self-cure.

This paper complements a body of research that studies mortgage default and modifications as well as more recent work that focuses specifically on the challenges of mortgage modification programs such as HAMP. As in Adelino, Gerardi, and Willen (2009) and Wang, Young, and Zhou (2002), the model in this paper treats mortgage renegotiation as a sequential move game between the homeowner and lender. While these papers focus on the role of information asymmetry as a barrier to successful renegotiations, this paper highlights issues that might arise even with full information. Papers that focus on recent modification programs find that these programs attract homeowners who might otherwise self cure (Chang and Xiao, 2013; Mayer et al., 2011). Modifications are offered only to a small fraction of applicants (Adelino, Gerardi, and Willen, 2009) and lead only to a small reduction in foreclosures (Agarwal et al., 2012). The model in this paper can deliver results that are consistent with all of these observations, as will be discussed in the next section.

2 The Model

In my model of strategic interaction, the players are a single lender and a continuum of homeowners of type α, where α is uniformly distributed on the interval $[0, 1]$. Let M denote the mortgage balance and P the market price of the home. I assume that $M - P > 0$, based on the literature that finds that negative equity is a trigger for default, e.g., Foote, Gerardi, and Willen (2008).

Figure 1 illustrates the payoffs of the possible outcomes of the interaction between the lender and an individual homeowner. The homeowner moves first and decides whether to seek renegotiation (denoted by action s) or not seek renegotiation (ns). If he does not seek renegotiation and does not default on his mortgage (denoted by action nd), his payoff is 0. (I calculate payoffs as changes in net worth.) If he defaults (denoted by action d), he is foreclosed upon and his payoff is $M - P - \alpha D$. This expression reflects the assumption that homeowners differ in their cost of mortgage default. Specifically, homeowners of type α face a cost αD of defaulting. If the homeowner does not seek renegotiation and does not default, the lender receives the mortgage amount M as per the original contract. If he defaults and is foreclosed upon, the lender's payoff is the price P of the home less the cost associated with foreclosing on the home, F.

Once the homeowner decides to seek renegotiation, the lender has to decide whether or not to agree. If the lender does not agree to renegotiate (na), the homeowner's payoffs are the same as in the case where he chose not to seek renegotiation. Thus the payoff to the homeowner of seeking but not receiving a modification and then not defaulting is 0 while the payoff from defaulting is $M - P - \alpha D$. There is no change to the lender's payoff either; she receives M if the homeowner does not default and $P - F$ if he does.

If the lender agrees, denoted by action a, the modification leads to the homeowner being paid an amount A. If the homeowner does not default, his payoff is A. In this case, the lender receives $M - A$. If the homeowner receives A and still defaults, his payoff is $M - P - \alpha D + \rho A$. Since there is no time dimension in the model, ρ loosely captures what might occur during the modification process. Consider an example in which a homeowner receives a lower interest rate. We can think of the total amount A as the difference between the original payments and the new, lower payments under the new interest rate over the full length of the loan term. However, if the homeowner defaults and is foreclosed upon after making a few of the new payments, he receives in effect only a fraction of the amount, i.e., ρA. In this case, the lender's payoff is $P - F - \rho A$.

2.1 Model with No Incentives

We first assume that there is no government program in place. In other words, renegotiations between the lender and homeowner are purely bilateral with no externally funded incentives.

In principle, it is possible for the lender to choose both whether or not to renegotiate and how much to offer the homeowner. However, to avoid the complexities associated with a continuum of strategies, we assume for now that the lender has only two choices — not renegotiate (na) or agree to renegotiate and offer a specific amount $A = M - P$. The payoffs under this specific assumption are shown in Figure 2.

In solving this game backwards, we observe that homeowners can be grouped into types. Some homeowners would not default at any of the terminal nodes. For these homeowners, $\alpha \in [\bar{\alpha}, 1]$, where

$$\bar{\alpha} = \frac{M - P}{D}.$$

Also observe that there are homeowners who would get a higher payoff from defaulting even when offered the maximum amount. For these homeowners, $\alpha \in [0, \underline{\alpha})$, where

$$\underline{\alpha} = \frac{\rho(M - P)}{D}.$$

We assume that $0 < \underline{\alpha} < \bar{\alpha} < 1$. In other words, homeowners can be grouped into three categories: (i) those with $\alpha \in [0, \underline{\alpha})$ who would default even if they received a modification, (ii) those with $\alpha \in [\bar{\alpha}, 1]$ who would not default even if they received no modification and (iii) those with $\alpha \in [\underline{\alpha}, \bar{\alpha})$ who would default if they received no modification but not if they received a modification.

In the absence of any renegotiation between the lender and homeowners, all homeowners with $\alpha \in [0, \bar{\alpha})$ would default on their mortgages and be foreclosed upon while all homeowners with $\alpha \in [\bar{\alpha}, 1]$ would not. The lender's payoff in this case would be

$$\bar{\alpha}(P - F) + (1 - \bar{\alpha})M. \tag{1}$$

We now formally describe the solution to the model by characterizing the subgame perfect Nash equilibrium. This requires specifying the strategy profile that includes strategies of every player. Since there is a continuum of homeowners, we describe strategy profiles over intervals within $[0, 1]$.

Proposition 1. *Assume full information (the homeowners' type and the lenders' actions* *are observable). Let* $\underline{\alpha} = \frac{\rho(M-P)}{D}$ *and* $\bar{\alpha} = \frac{M-P}{D}$*. Then the strategy profile*[1]

$$
\begin{aligned}
\{(s \text{ } \textit{Always choose } d), na\} \quad &\forall \quad \alpha \in [0, \underline{\alpha}) \\
\{(s \text{ } nd|A = M - P \text{ } d|\textit{otherwise}), a\} \quad &\forall \quad \alpha \in [\underline{\alpha}, \bar{\alpha}) \\
\{(s \text{ } \textit{Always choose } nd), na\} \quad &\forall \quad \alpha \in [\bar{\alpha}, 1]
\end{aligned}
$$

is a subgame perfect Nash equilibrium of the game in Figure 2.[2]

Proof. See Appendix. □

The above result shows that there is an equilibrium in which all types of homeowners choose to seek renegotiation. This illustrates the point that Adelino, Gerardi, and Willen (2009) make: renegotiation exposes the lender to homeowners who would self-cure (those with $\alpha \in [\bar{\alpha}, 1]$ in our model) or redefault (those with $\alpha \in [0, \underline{\alpha})$). The lender does not renegotiate with homeowners of type $\alpha \in [0, \underline{\alpha})$ because they would default even if they received a modification. As a result, the lender's payoff from renegotiating, $P - F - \rho A$, would be strictly less than her payoff from not doing so, $P - F$. The lender also does not renegotiate with homeowners of type $\alpha \in [\bar{\alpha}, 1]$ because her payoff from not modifying the terms, M, is strictly higher than her payoff from modifying terms, $M - A$. In this equilibrium, the only homeowners whose mortgage terms are modified are of type $\alpha \in [\underline{\alpha}, \bar{\alpha})$. These are homeowners who would have gone through foreclosure in the absence of the modification but avoid foreclosure because they receive it.

It can be shown that the payoff to the lender from the above solution exceeds the payoff from the solution with no renegotiation as described by (1).

Certain parameterizations of the model can yield results consistent with empirical observations. For example, Adelino, Gerardi, and Willen (2009) point out that lenders renegotiate

[1] The strategy profile is of the form {(Homeowner's strategy at initial node Homeowner's conditional strategy at terminal nodes), Lender's strategy}

[2] Note that the subgame perfect Nash equilibrium is not unique. To be specific, strategy profiles in which homeowners with $\alpha \in [0, \underline{\alpha})$ and $\alpha \in [\bar{\alpha}, 1]$ always chose action ns, or randomize between s and ns, would also be subgame perfect Nash equilibria because the payoffs from the two are the same.

only a small fraction of delinquent loans. Our model can obtain a qualitatively similar result if the interval $[\underline{\alpha}, \bar{\alpha})$ is small, that is, if the number of homeowners who would successfully avoid foreclosure with a modification is small relative to the number who would redefault or self-cure.[3]

2.2 Model with Incentives

We now solve the model in the presence of a government program that gives incentives to homeowners and lenders. We are particularly interested in comparing the solutions from this model to the model without the program to see whether the former is more effective in terms of preventing foreclosure.

The model of homeowner and lender renegotiation in the presence of incentives is shown in Figure 3. I model the incentives around the rules that were prevalent in 2010. Specifically, the HAMP program offered incentive compensation of \$1,000 to servicers for each successful permanent modification completed (Making Home Affordable, 2010). In addition, it offered up to \$1,000 each to the homeowner and servicer for every year that the loan remained in good standing (or \$83.33 monthly), for a maximum of three years. We introduce this incentive compensation structure into our model as follows. The lender receives I_1 for offering a modification, regardless of whether or not the homeowner subsequently defaults. If the homeowner does not default and thereby avoids foreclosure, the lender receives an additional I_2 as "pay-for-success".

To compare the solution from this model to the model with no incentives, assume that all other variables are the same as before. I first show that an equilibrium exists in which a larger fraction of homeowners receives modifications and avoids foreclosure. The incentives thus have the effect of preventing some foreclosures that would have occurred in the absence of the program. The following result characterizes the equilibrium.

[3] Data on the HAMP program suggests that this might be the case: as of February 2014, servicers had processed over 7.7 million applications but have approved less than one-third of them (Making Home Affordable, 2014)

Proposition 2. *Assume full information. Let* $\underline{\alpha}' = \frac{\rho(M-P)-I_2}{D}$ *and* $\bar{\alpha} = \frac{M-P}{D}$. *Assume that* $\rho(M-P) \geq I_2 > I_1$ *and that* $I_1 + I_2 < M - P$. *Then the strategy profile*

$$\begin{aligned} \{(s \ \textit{Always choose } d), na\} &\quad \forall \quad \alpha \in [0, \underline{\alpha}') \\ \{(s \ nd|A = M-P \ d|otherwise), a\} &\quad \forall \quad \alpha \in [\underline{\alpha}', \bar{\alpha}) \\ \{(s \ \textit{Always choose } nd), na\} &\quad \forall \quad \alpha \in [\bar{\alpha}, 1] \end{aligned}$$

is a subgame perfect Nash equilibrium of the game in Figure 3.[4]

Proof. See Appendix. □

Comparing Proposition 2 to Proposition 1, we see that the results are qualitatively similar. All homeowners seek renegotiation, but the lender offers it only to the subset of homeowners who can successfully avoid foreclosure as a result. The key difference is that the subset of homeowners who receive a modification and avoid foreclosure is larger in this case.[5] This follows from the fact that $\underline{\alpha}' < \underline{\alpha}$. Intuitively, the homeowners' payoff from receiving a modification and not defaulting is increased by the incentive payment I_2, which makes this option attractive to a larger fraction of homeowners.

The next result shows that, under different assumptions about the incentive structure, lenders may be induced to also renegotiate with homeowners of type $\alpha \in [0, \underline{\alpha}')$, and that these homeowners will subsequently default.

Proposition 3. *Assume full information. Let* $\underline{\alpha}' = \frac{\rho(M-P)-I_2}{D}$. *Assume that* $I_1 \geq \rho(M-P) \geq I_2$ *and that* $I_1 + I_2 < M - P$. *Then the strategy profile*

$$\begin{aligned} \{(s \ \textit{Always choose } d), a\} &\quad \forall \quad \alpha \in [0, \underline{\alpha}') \\ \{(s \ nd|A = M-P \ d|otherwise), a\} &\quad \forall \quad \alpha \in [\underline{\alpha}', \bar{\alpha}) \\ \{(ns \ \textit{Always choose } nd), na\} &\quad \forall \quad \alpha \in [\bar{\alpha}, 1] \end{aligned}$$

[4] For the same reasons as described for Proposition 1, the equilibrium is not unique.

[5] This is consistent with Agarwal et al. (2012), who find that HAMP led to a modest reduction in foreclosures.

is a subgame perfect Nash equilibrium of the game in Figure 3.

Proof. See Appendix. □

As in Proposition 2, a larger fraction of homeowners receives modifications and avoid foreclosure compared to the no incentive case. The key difference between this result and Proposition 2 is that the lender now also renegotiates with homeowners of type $\alpha \in [0, \underline{\alpha})$. Homeowners of this type subsequently default and are foreclosed upon. The reason for the difference is the incentive structure. In particular, the incentive payment given to the lender simply for renegotiating, I_1, is higher than in the previous case, making it worthwhile for the lender to renegotiate even with those homeowners who default.[6]

Finally observe that it is possible in theory but unlikely in practice to have incentives large enough to induce lenders to renegotiate with homeowners who would otherwise self-cure. This can be seen if the proof of Proposition 2 was reworked under the assumption that $I_1 + I_2 \geq M - P$. This is an unlikely assumption in practice because it requires that the incentive payments exceed the modification amount that the lender offers.

To summarize, our models show that in the absence of incentives, the lender renegotiates the mortgage terms of a subset of homeowners who avoid foreclosure as a result. In the presence of incentives, the lender renegotiates with a larger subset of homeowners who avoid foreclosure as a result. However, under certain assumptions about the incentive structure, the lender may also renegotiate with homeowners who subsequently default and are foreclosed upon.

2.3 Mortgage Modifications and Success Rates

Mortgage modifications are often evaluated by comparing "success rates" — defined as the fraction of homeowners who avoid foreclosure — across homeowners who do and do not receive modifications. Our models show that this comparison is not necessarily informative

[6]Mayer, Morrison, and Piskorski (2009) propose an incentive fee structure that would avoid this scenario by rewarding servicers only for successful modifications.

about the effectiveness of mortgage modifications. This is because success rates among those who do not receive modifications may be high if this group includes a large proportion of homeowners who self-cure. The solutions described by Proposition 1 and Proposition 2 illustrate this. In those solutions, the success rate conditional on not receiving a modification is $\frac{1-\bar{\alpha}}{1-\bar{\alpha}+\underline{\alpha}}$. This number can be close to 1 if the interval $[\bar{\alpha}, 1]$ is large relative to the interval $[0, \underline{\alpha}]$. Recent research suggests that this is indeed the case. Chang and Xiao (2013) report that the top reason that modification applications were rejected was that the credit quality of the applicants was too good, and further that these applicants self-cured without a modification at relatively high rates. Mayer et al. (2011) find that borrowers who became delinquent following a program announcement to help seriously delinquent borrowers were "those who appear to have been least likely to default otherwise."[7] As a result, cure rates or success rates can end up being high among those who do apply but do not receive modifications. The conclusion is that success rate comparisons should be interpreted with caution when judging the effectiveness of mortgage modification programs.

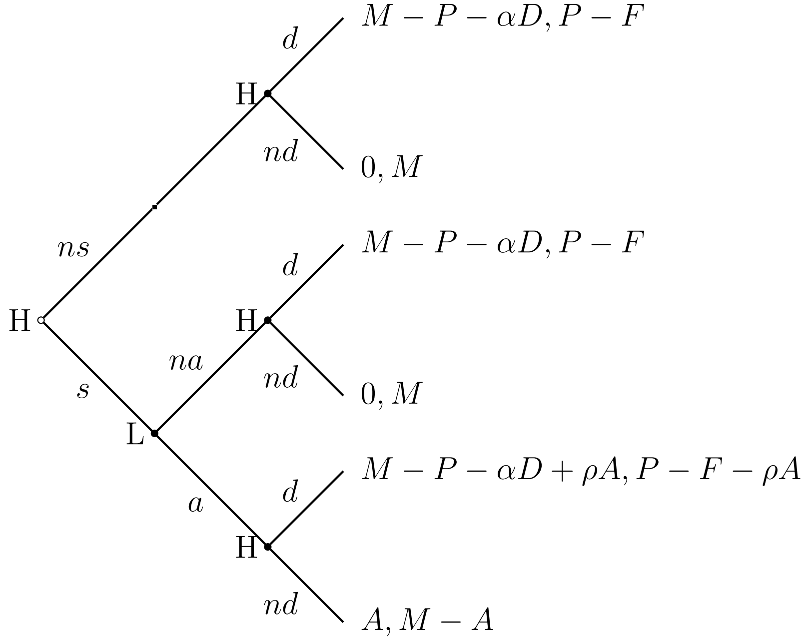
3 Conclusion

The models in this paper provide a framework to analyze mortgage renegotiation between homeowner and lender. They compare outcomes in the absence of incentives to outcomes in the presence of externally-funded incentives to homeowners and lenders. The results show that, in the absence of incentives, lenders renegotiate only with those homeowners who would successfully avoid foreclosure upon receiving a modification but would default without it. In other words, lenders do not renegotiate with homeowners who would self-cure without a modification, or with homeowners who would redefault despite receiving it. The share of homeowners who receive modifications and avoid foreclosure is larger in the presence of incentives. It is beyond the scope of this paper to determine whether the benefit exceeds the cost of providing such incentives; however, I show that incentives might also induce lenders

[7] Andersson et al. (2013) also suggest that HAMP may have made default on mortgage debt more attractive

to renegotiate with homeowners who subsequently default.[8] An important caveat is that I abstract from information asymmetry between the lender and homeowner. I think this is a reasonable assumption; as Agarwal et al. (2012) describe, HAMP, for example, had extensive screening criteria. However, to the extent that this is an issue, it may overstate how much lenders are able to target the "right" homeowners. Nonetheless, the point we illustrate is that even if lenders are able to target the right homeowners, incentives such as those given through recent government programs may lead them to also renegotiate with homeowners who cannot be protected from foreclosure.

Figure 1: Homeowner and Lender Payoffs



[8]For an assessment of the costs and benefits of the HAMP program in particular, see Hembre (2013).

Figure 2: Homeowner and Lender Payoffs With $A = M - P$

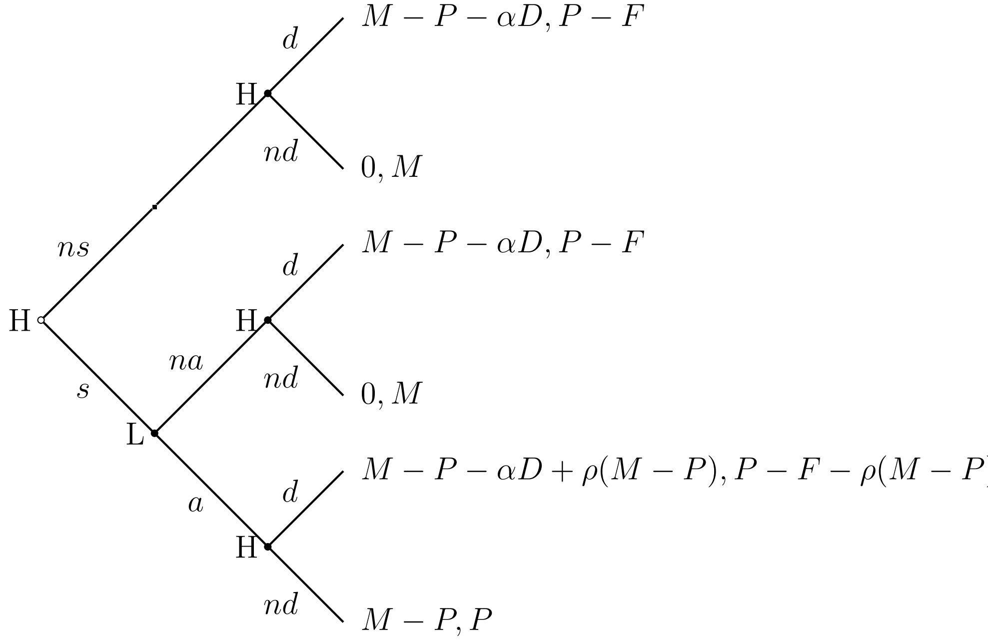
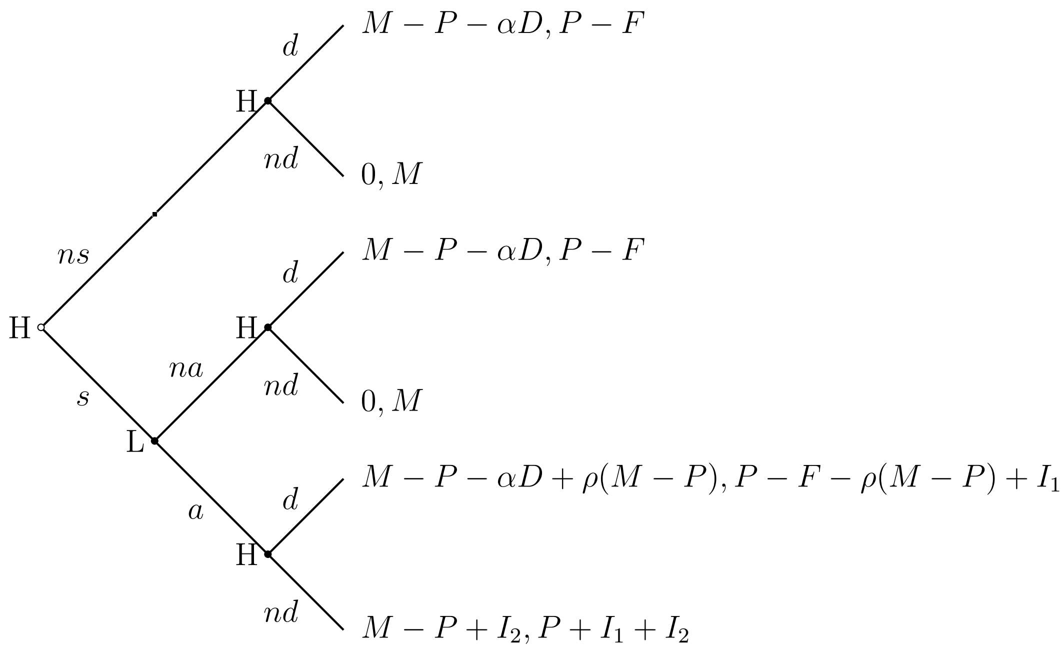


Figure 3: Homeowner and Lender Payoffs With Incentives



A Proofs

A.1 Proof of Proposition 1

Proof. We show that the strategy profile is a subgame perfect Nash equilibrium by solving the game in Figure 2 by backwards induction.

For homeowners of type $\alpha \in [0, \underline{\alpha})$, we show that the payoff from action d exceeds the payoff from action nd at each of the three terminal nodes in Figure 2, working from top to bottom.

1. $M - P - \alpha D > 0$ by assumption

2. $M - P - \alpha D > 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P$$
$$\text{that is} \Leftrightarrow \alpha < \frac{\rho(M - P)}{D}$$

which is true because in this case $\alpha \in [0, \underline{\alpha})$ and $\underline{\alpha} = \frac{\rho(M-P)}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ always choose action d, the lender will choose action na because her payoff from doing so, $P - F$ strictly exceeds her payoff from offering a, $P - F - \rho(M - P)$. By backwards induction, knowing that the lender will choose na, the homeowner will be indifferent between choosing s and ns at the initial node because the payoff is $M - P - \alpha D$ in each case.

For homeowners of type $\alpha \in [\underline{\alpha}, \bar{\alpha})$, we show that the payoff from action d exceeds the payoff from action nd at the top two terminal nodes and the payoff from nd exceeds the payoff from d at the bottom terminal node:

1. $M - P - \alpha D > 0$ by assumption

2. $M - P - \alpha D > 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P$$
$$\Leftrightarrow \alpha < \frac{\rho(M - P)}{D}$$

which is false because in this case $\alpha \in [\underline{\alpha}, \bar{\alpha})$ and $\underline{\alpha} = \frac{\rho(M-P)}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ choose action $nd|A = M - P$ and d otherwise, the lender will choose action a because her payoff from doing so, P, strictly exceeds her payoff from na, $P - F$. By backwards induction, knowing that the lender will choose a, the homeowner will choose s at the initial node because $M - P > M - P - \alpha D$.

For homeowners of type $\alpha \in [\bar{\alpha}, 1]$, we show that the payoff from action nd exceeds the payoff from action d at each terminal node in Figure 2, working from top to bottom.

1. $M - P - \alpha D < 0$ by assumption

2. $M - P - \alpha D < 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P$$
$$\text{that is} \Leftrightarrow \alpha < \frac{\rho(M - P)}{D}$$

which is false because in this case $\alpha \in [\bar{\alpha}, 1]$ and $\bar{\alpha} = \frac{(M-P)}{D} > \frac{\rho(M-P)}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ always choose action nd, the lender will choose action na because her payoff from doing so, M strictly exceeds her payoff from offering a, P. By backwards induction, knowing that the lender will choose na, the homeowner will be indifferent between choosing s and ns at the initial node because the payoff is 0 in each case. □

A.2 Proof of Proposition 2

Proof. We show that the strategy profile is a subgame perfect Nash equilibrium by solving the game in Figure 3 by backwards induction. The assumption that $\rho(M - P) \geq I_2$ ensures that $\underline{\alpha}' \in [0, \underline{\alpha})$.

For homeowners of type $\alpha \in [0, \underline{\alpha}')$, we show that the payoff from action d exceeds the payoff from action nd at each terminal node in Figure 3, working from top to bottom.

1. $M - P - \alpha D > 0$ by assumption
2. $M - P - \alpha D > 0$ by assumption
3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P + I_2$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$
\begin{aligned}
M - P - \alpha D + \rho(M - P) &> M - P + I_2 \\
\text{that is} \Leftrightarrow \alpha &< \frac{\rho(M - P) - I_2}{D}
\end{aligned}
$$

   which is true because in this case $\alpha \in [0, \underline{\alpha}')$ and $\underline{\alpha}' = \frac{\rho(M-P)-I_2}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha}')$ always choose action d, the lender will compare her payoff from a, which is $P - F - \rho(M - P) + I_1$, to her payoff from choosing action na

which is $P - F$. The lender will choose a if and only if

$$P - F - \rho(M - P) + I_1 \geq P - F,$$
$$\text{that is, } \Leftrightarrow I_1 \geq \rho(M - P)$$

which is false by assumption. Hence the lender will choose na. By backwards induction, knowing that the lender will choose na, the homeowner will be indifferent between choosing s and ns at the initial node because the payoff is $M - P - \alpha D$ in either case.

For homeowners of type $\alpha \in [\underline{\alpha}', \bar{\alpha})$, we show that the payoff from action d exceeds the payoff from action nd at the top two terminal nodes and the payoff from nd exceeds the payoff from d at the bottom terminal node:

1. $M - P - \alpha D > 0$ by assumption

2. $M - P - \alpha D > 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P + I_2$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P + I_2$$
$$\Leftrightarrow \alpha < \frac{\rho(M - P) - I_2}{D}$$

which is false because in this case $\alpha \in [\underline{\alpha}', \bar{\alpha})$ and $\underline{\alpha}' = \frac{\rho(M-P)-I_2}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ choose action $nd|A = M - P$ and d otherwise, the lender will choose action a because her payoff from doing so, $P + I_1 + I_2$ strictly exceeds her payoff from na, $P - F$. By backwards induction, knowing that the lender will choose a, the homeowner will choose s at the initial node because $M - P + I_2 > M - P - \alpha D$.

For homeowners of type $\alpha \in [\bar{\alpha}, 1]$, we show that the payoff from action nd exceeds the payoff from action d at each terminal node in Figure 2, working from top to bottom.

1. $M - P - \alpha D < 0$ by assumption

2. $M - P - \alpha D < 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P + I_2$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P + I_2$$
$$\text{that is} \Leftrightarrow \alpha < \frac{\rho(M - P) - I_2}{D}$$

which is false because in this case $\alpha \in [\bar{\alpha}, 1]$ and $\bar{\alpha} = \frac{(M-P)}{D} > \frac{\rho(M-P)-I_2}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ always choose action nd, the lender will compare her payoff from a, which is $P + I_1 + I_2$, to her payoff from choosing action na which is M. The lender will choose a if and only if

$$P + I_1 + I_2 \geq M,$$
$$\text{that is} \Leftrightarrow I_1 + I_2 \geq M - P,$$

which is false by assumption. Thus the lender will choose na in this case. By backwards induction, knowing that the lender will choose na, the homeowner will be indifferent between choosing s and ns at the initial node because his payoff is 0 in either case. □

A.3 Proof of Proposition 3

Proof. We show that the strategy profile is a subgame perfect Nash equilibrium by solving the game in Figure 3 by backwards induction. The assumption that $\rho(M - P) \geq I_2$ ensures that $\underline{\alpha}' \in [0, \underline{\alpha})$.

For homeowners of type $\alpha \in [0, \underline{\alpha}')$, we show that the payoff from action d exceeds the payoff from action nd at each terminal node in Figure 3, working from top to bottom.

1. $M - P - \alpha D > 0$ by assumption

2. $M - P - \alpha D > 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P + I_2$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P + I_2$$
$$\text{that is} \Leftrightarrow \alpha < \frac{\rho(M - P) - I_2}{D}$$

which is true because in this case $\alpha \in [0, \underline{\alpha}')$ and $\underline{\alpha}' = \frac{\rho(M-P)-I_2}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha}')$ always choose action d, the lender will compare her payoff from a, which is $P - F - \rho(M - P) + I_1$, to her payoff from choosing action na, which is $P - F$. The lender will choose a if and only if

$$P - F - \rho(M - P) + I_1 \geq P - F,$$
$$\text{that is,} \Leftrightarrow I_1 \geq \rho(M - P)$$

which is true by assumption. Hence the lender will choose a. By backwards induction, knowing that the lender will choose a, the homeowner will compare choosing ns with choosing s. He will choose the latter if and only if

$$M - P - \alpha D \geq M - P - \alpha D + \rho(M - P)$$

which is true. Hence the homeowner will indeed choose s.

For homeowners of type $\alpha \in [\underline{\alpha}', \bar{\alpha})$, we show that the payoff from action d exceeds the payoff from action nd at the top two terminal nodes and the payoff from nd exceeds the payoff from d at the bottom terminal node:

1. $M - P - \alpha D > 0$ by assumption

2. $M - P - \alpha D > 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P + I_2$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will choose d if and only if

$$\begin{aligned} M - P - \alpha D + \rho(M - P) > M - P + I_2 \\ \Leftrightarrow \alpha < \frac{\rho(M - P) - I_2}{D} \end{aligned}$$

which is false because in this case $\alpha \in [\underline{\alpha}', \bar{\alpha})$ and $\underline{\alpha}' = \frac{\rho(M-P)-I_2}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ choose action $nd|A = M - P$ and d otherwise, the lender will choose action a because her payoff from doing so, $P + I_1 + I_2$ strictly exceeds her payoff from na, $P - F$. By backwards induction, knowing that the lender will choose a, the homeowner will choose s at the initial node because $D > \varepsilon \Rightarrow M - P + I_2 > M - P - \alpha D$. For homeowners of type $\alpha \in [\bar{\alpha}, 1]$, we show that the payoff from action nd exceeds the payoff from action d at each terminal node in Figure 3, working from top to bottom.

1. $M - P - \alpha D < 0$ by assumption

2. $M - P - \alpha D < 0$ by assumption

3. Given that the lender is offering $A = M - P$, the homeowner's payoff from choosing action nd is $M - P + I_2$ and from d is $M - P - \alpha D + \rho(M - P)$. The homeowner will

choose d if and only if

$$M - P - \alpha D + \rho(M - P) > M - P + I_2$$
$$\text{that is} \Leftrightarrow \alpha < \frac{\rho(M-P) - I_2}{D}$$

which is false because in this case $\alpha \in [\bar{\alpha}, 1]$ and $\bar{\alpha} = \frac{(M-P)}{D} > \frac{\rho(M-P)-I_2}{D}$.

Knowing that homeowners with $\alpha \in [0, \underline{\alpha})$ always choose action nd, the homeowner will compare his payoff from a, which is $P + I_1 + I_2$, to his payoff from choosing action na which is M. The lender will choose a if and only if

$$P + I_1 + I_2 \geq M,$$
$$\text{that is} \Leftrightarrow I_1 + I_2 \geq M - P,$$

which is false by assumption. Thus the lender will choose na in this case. By backwards induction, knowing that the lender will choose na, the homeowner will be indifferent between choosing s and ns at the initial node because the payoff from either action is 0. □

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