Occupational Hazards and Social Disability Insurance

PRELIMINARY AND INCOMPLETE. COMMENTS ARE WELCOME

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Abstract

Disability risk is a large portion of life-time income risk and the level of exposure is tied to occupation. As governments can affect the level of risk individual's bear, we analyze disability insurance (DI) in a general equilibrium model of occupational choice. DI provides welfare gains through two channels: (i) risk sharing, and (ii) reallocating labor into risky occupations. Reallocation improves productive efficiency and increases all workers' welfare, even those without disability risk. However, the risk sharing channel can compromise efficiency gains. In a calibrated economy resembling the US, welfare gains from socially optimal DI are reduced by reallocation: the risk sharing provided leads to more labor in risky occupations than is productively efficient.

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1 Introduction

Social insurance against disability risk is a substantial and growing part of Federal expenditures in the United States. Disability insurance provided benefits to 4.7% of working age adults and cost \$200 billion (1.7% of GDP) in 2012. For comparison, the next largest non-employment social insurance program, unemployment insurance, benefited 2.5% of the working age population at a cost of \$78.9 billion in 2012.[1]. To evaluate the social benefits of this program, we take a critical look at the nature of disability risk. We provide evidence that lifetime occupation choices are an important component of individuals' non-employment risk. We then seek to answer the question: what are the welfare effects of social insurance when workers choose occupations considering these risks?

Evaluation of social insurance programs often focuses on the tradeoff between insurance and incentives. The idea is that disability insurance benefits society by providing a valuable transfer to workers who suffer an adverse shock[2]. However, insurance is costly because it provides disincentives to gain and maintain employment and incentives to misrepresent one's disability status. This paper considers a different incentive disability insurance creates. When the nature of work impacts disability risk, social insurance alters the composition of occupations in the US economy, shifting workers into occupations with higher risk.

We evaluate social disability insurance in a theoretical model of occupation choice and occupation-specific disability risk, highlighting the importance of general equilibrium effects. We show that, in a world with no insurance (incomplete markets), the competitive allocation puts fewer workers in risky occupations than the social planner would. This is because workers in high risk jobs demand a sufficiently high wage to self-insure against future inability to work. In the presence of decreasing marginal product of additional workers, these high wages are maintained by shifting workers to less risky occupations. If occupations are not

[1]UI beneficiares ranged from 1.7-4.5% of covered employment from 2000-2013. UI statistic from DOL ET Financial Data Handbook 394 Report and Congressional Budget Office; SSDI statistics from Administration (2013).

[2]For example, Gruber (1997) predicts consumption would fall three-times more (22% in total) during unemployment in the absence of the U.S. unemployment insurance system.

perfect substitutes, this imbalance makes production inefficient. Equilibrium prices cannot achieve efficiency because self-insurance is too expensive.

The introduction of a social disability insurance is a Pareto improvement over the world with no insurance. Even workers with zero disability risk are better off paying a flat tax to fund the scheme. This is because every dollar of disability benefits paid reduces the wage premium required to work in a risky occupations by more than one dollar. As a result: more workers take risky jobs, the allocation of workers across occupations becomes more efficient, output increases, and consumption becomes cheaper. These benefits are larger than the cost to fund the scheme because social insurance is more efficient than self-insurance. Under self-insurance some workers who accumulate savings never become disabled. With social insurance, funds only need to be provided for workers who actually become disabled.

Our quantitative analysis highlights the importance of including occupation-specific disability risk in the evaluation of social disability programs. We find a natural grouping of high risk and low risk occupations in the data. Using the Health and Retirement Survey, we estimate occupation experience accounts for 29% differences in disability across these occupation groups. We use these facts to calibrate our model of occupation risk. We find the optimal social disability insurance program is 8-9% of GDP and improves welfare by 0-10% (consumption). Further, optimal social disability insurance changes total output by -10% to 10%, depending on the elasticity of substitution between occupations, but still always provides a Pareto improvement.

Topically, this paper builds on literature evaluating social insurance under the assumption of i.i.d. risk. The economic mechanism of this paper is most related to Acemoglu and Shimer (1999). They show unemployment insurance can raise output by inducing workers to search for higher productivity jobs which are rarer, and therefore, more risky to pursue. In this paper, disability insurance also increases output by inducing workers to take on more risky occupations; specifically those with greater disability risk. A main theoretical distinction of our mechanism is the added feature that, since occupations are imperfect substitutes, this reallocation provides a Pareto improvement for all workers. Our specification also allows for direct measurement of non-employment risk, classified by occupation, in the data. We also provide results illustrating how the elasticity of substitution across occupations determines the reallocation of workers across occupations and welfare gains of the policy.

2 Data on occupations and disability

In this section, we present data about the connection between one's occupation and the risk of disability. First, we construct a measure of lifetime exposure to an occupation using the University of Michigan's Health and Retirement Study. These same respondents provide detailed information on their health conditions, from which we infer whether they are "disabled." We see that occupations have quite disparate disability rates, much of which can be attributed to the occupations' effects themselves.

2.1 Constructing the dataset

We begin with the RAND Corporation's curated version the University of Michigan's Health and Retirement Study (HRS). The HRS surveys households whose head is older than 50 and also includes data on their "spouses," who may be a non-married cohabitant. The survey spans 10 waves, biannually from 1992-2010, and tracks respondents. The individual listed as a spouse may change though the survey and we drop these observations. Everyone lists both their current occupation and "longest held occupation" as well as their tenure in each of these. To measure disability, we use one of two measures. First, there is a direct question, whether a health problem limits one's ability for paid work. Second, we can look more restrictively, at whether they report limitations to activities of daily living (ADLs). After excluding observations for non-response, we have between 16,128 and 21,623 observation per wave with a total of 184,541.

We will associate an individual with his or her longest-held occupation. Were we to use the current occupation, we would mistakenly consider an occupation dangerous if workers in poor health switched into that occupation later in life. Each wave of the survey, workers report their longest-held occupation and the tenure there. For each individual, we take the occupation that is longest of these longest. The longest-held occupation may change from wave to wave because the individual's tenure in the current occupation overtakes the prior longest-held occupation or because of coding error. The latter case never happens 4% of the time in the first wave, but then less than 1% of the time. We overwrite these coding errors. The median occupational tenure is 19 years and the bottom quartile is 11 years.

To describe someone's health, we will use two types of measures. For all of the, we only count health problems if they occur before 65. This is because we are looking for health conditions that have a direct effect on one's ability to work. Though it is likely that a problem that appears at 66 may have had manifestations earlier, we will try to be conservative.

Our primary measure is based on the self-reported limitations to activities of daily living (ADL). In each wave, individuals report how much, if any, difficulty they have with a number of ADLs. These include walking across a room, getting dressed, bathing, eating using the toilet and getting in and out of bed. We look across waves for consistency, so if a worker reports a difficulty, we check whether they continue to report that difficulty in subsequent waves. If so then we record two variables: one is an indicator of whether the individual reports any difficulties with an ADL and the other is a metric over how many and how severe are the difficulties. For this metric, for each difficulty we give a half point if there is "a little" difficulty and a whole point for any more than that.

For our other measure, we use an HRS question about whether a health condition limits the individual's ability to work. Again, we look for consistency across waves: if the individual reports a health limitation to work, we check that it is still reported in all subsequent waves. If so, we record that as an indicator variable.

2.2 The distribution of disability across occupations

The occupations differ quite a bit in their average disability rates. Particularly, there is quite a long tail of high risk occupations while most of the population is in occupations clustered at fairly low levels of risk. The distinction between high risk and low-risk occupations is great enough that the distribution looks bi-modal. Of course, some of these differences are due to observable differences between the populations in each of these occupations. So, we take some steps to disentangle the occupations' "treatment effect" from the other observables that may contribute to disability and also may differ across occupations.

We first present the raw distributions across occupations. In Figure 1 we present the distribution across occupations of the incidence of a disability, as measured by having a difficulty with an ADL. In this kernel density estimate, each data point is an occupation, and within that occupation, we construct the average rate of ADL difficulties. We then weight the occupations by their population using the appropriate weights provided by the HRS. Along the horizontal axis we plot dots representing the position of occupations.

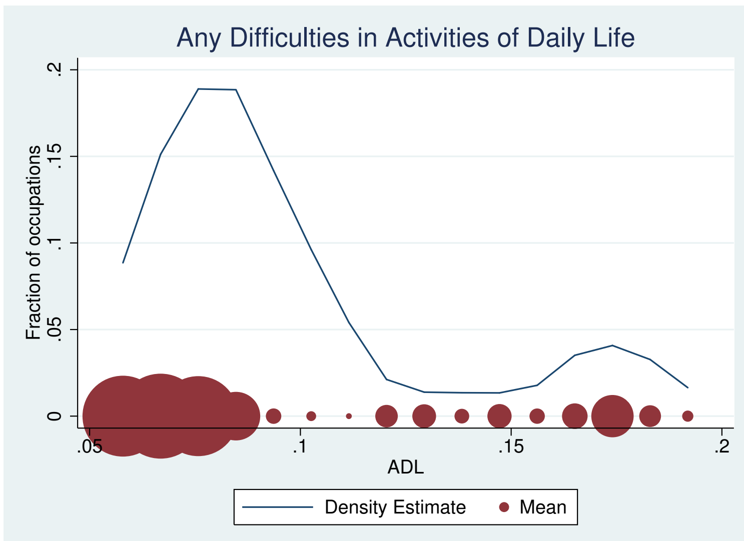

Figure 1: The density across occupations of the incidence of difficulties with ADLs

Notice the large clustering of occupations at the low end of the distribution and the long tail of occupations with more than twice the average rate of difficulties. It turns out the figure looks remarkably similar if we use a weighted average of the number of disabilities instead of just an indicator for the presence of a disability. In Figure 2 we count the number of disabilities difficult ADLs an individual lists and count a half point for ADLs the individual has "some difficulty" doing. We take the average across waves and then the average within an occupation.

The largest high risk occupations are machine and transport operators along. Construction and extraction also represent a large portion of the high risk occupations. Cleaning also carries quite a high rate of disability. The lowest risk occupations are professional and management occupations along with administrative support and sales.

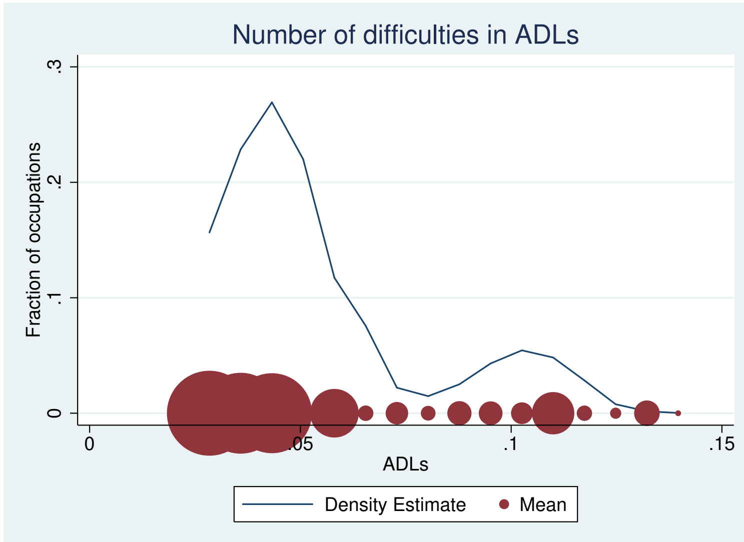

Figure 2: The density across occupations of the weighted number of difficulties with ADLs

To check robustness, rather than the presence of an ADL difficulty, we might use a self-reported limitation to work based on health grounds. In this figure too, we tell almost exactly the same story. In fact, the same occupations with rates of ADL difficulty that are are in the top third are the same occupations in the top third of limitations to work. Figure 3 shows this distribution, constructed in the same way as our previous two distributions. Again, the tail of high risk occupations is quite long with large occupations, transportation and other operators more than 200% above the clustering of lower-risk occupations.

We estimate a mixture of two normal distributions to get a sense of how to classify occupations as "risky" or "safe." We will concentrate on our favored measure of disability, an indicator as to whether a worker has any limitations to ADLs. The parameters of these distributions are in Table 1. Note that again the high risk occupations have about twice the risk of disability as the lower risk group. Figure 4 super-imposes these two distributions onto the kernel density estimate.

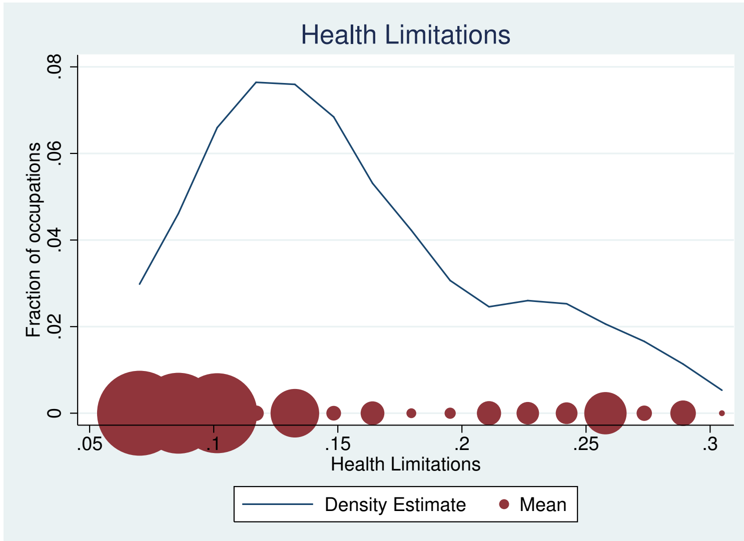

Figure 3: The density across occupations of the rate reporting a health limitation to work

	Low Risk	High Risk
Mean	0.08	0.16
StdDev	0.01	0.02
Fraction	0.83	0.17

Table 1: Parameters of the disability risk mixture distribution

2.3 Extracting the occupation effect

How much of this difference is simply due to differences between populations in the high risk occupations and lower-risk occupations? Potentially, there are systematic differences between those who choose what look like "risky" occupations and those who choose safe occupations. We will attempt to find an effect of occupation on disability in two ways. We use a Oaxaca-Blinder decomposition to pull out the residual effect of occupation on disability from observable differences between workers in different occupations. However, this decomposition is problematic because the grouping is endogenous: people choose their occupations. Then we will use instrumental variable methods to get the marginal effect of being in a risky occupation.

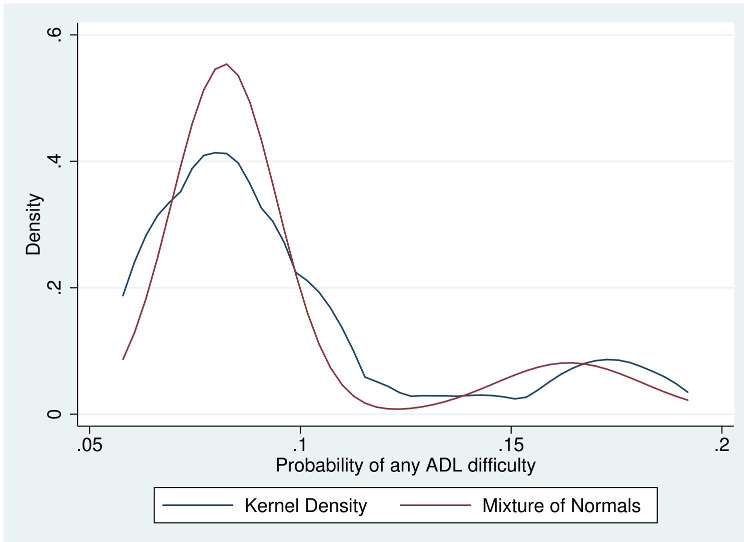

Figure 4: Any limitations to ADLs, fit with a mixture of normals

In our first method to extract the occupation effect on disability, we segment occupations into those at whose risk level there more density from the risky or safe distribution. Then we use an Oaxaca-Blinder decomposition to isolate the difference due to observable characteristics and the residual differences in disability rates, or as Fortin et al. (2011) interpret it, the "treatment" effect of a high-risk occupation.

There is some nuance to this regression. On the right-hand side, our regressors are a cubic for potential experience, BMI, time, dummies for education level, gender, marital status, race and smoker. However, at what time should we measure these variables, which may change over the course of the survey? If we include every observation for every wave, then we are implicitly putting less weight on individuals who drop out of the sample because of death or non-response. Instead, we collapse the panel to a single observation for each individual. For workers who report a disability, we take a snapshot at the wave in which they become disabled. We take the mean value across waves for workers who never report a disability. The results are summarized in Table 2.

Columns (1) and (2) of Table 2 show the decomposition with the dependent as a difficulty with an ADL, while (3) and (4) have a reported health limitation to work. Columns (1) and (3) use only one observation per individual, with covariates chosen as described above and

	(1)	(2)	(3)	(4)
Safe	0.189	0.106	0.152	0.069
Risky	0.264	0.157	0.206	0.094
Difference	-0.076	-0.051	-0.053	-0.025
Observables	-0.044	-0.030	-0.034	-0.017
% Difference	57.9	58.8	64.2	68.0
Occupation	-0.031	-0.021	-0.020	-0.008
% Difference	42.1	41.2	35.8	32.0
N	20,328	127,298	20,328	127,298

Table 2: The decomposition between higher and lower disability risk

(2) and (4) pool all of the data. In our baseline case, Column (1), the occupation effect accounts for 42% of the difference in risk. The finding is robust to using health limitations, where the riskier occupations have much higher rates of limitations, and 35% of that is due to the residual effect of the occupations themselves.

As alluded above, we realize that the groupings are endogenous. Perhaps there are unobservable characteristics that lead workers to choose certain occupations and also contribute to their probability of becoming disabled. To address this problem we will use instrumental variables that are set before a worker ever makes a career decision. To instrument for the occupation choice, we use census birth region, birth cohort and race so that the first stage assigns a worker the probability of being in a risky occupation given by cells defined on birth characteristics. Table 3 gives results for the regression

$$\Pr[\text{Disabled}] = F(\gamma \mathbb{I}_{\text{risky}} + x'\beta)$$

Where x is a vector of potential determinants, F is either the Normal CDF or a linear function in the case of the linear probability model. Columns (1)-(3) present results for the linear probability model and (4)-(5) give results of a probit regression. Columns (1) and (4) give IV regression results where the first stage is on a set of dummies defining cells for birth place, cohort and race. Column (2) gives the control function estimates, where the control function is a probit regression on the same set of dummies. Columns (3) and (5) treat the occupation choice as exogenous, a hypothesis which the Wald statistic cannot reject at the 1% level of significance.

	(1)	(2)	(3)	(4)	(5)
Risky	0.499***	0.040***	0.013**	2.006***	0.012**
	(0.047)	(0.015)	(0.006)	(0.260)	(0.005)
Potential Experience	0.019***	0.021***	0.021***	0.097***	0.021***
	(0.000)	(0.000)	(0.000)	(0.003)	(0.000)
BMI	0.009***	0.010***	0.011***	0.036***	0.009***
	(0.001)	(0.000)	(0.000)	(0.003)	(0.000)
Woman	0.149***	0.060***	0.060***	0.578***	0.039***
	(0.010)	(0.005)	(0.005)	(0.056)	(0.004)
Self-employed	0.003	-0.014***	-0.019***	-0.020	-0.023***
	(0.007)	(0.005)	(0.005)	(0.039)	(0.005)
College	0.175***	0.075***	0.082***	0.721***	0.079***
	(0.012)	(0.006)	(0.006)	(0.066)	(0.007)
No HS	-0.073***	-0.020***	-0.021***	-0.355***	-0.034***
	(0.009)	(0.006)	(0.006)	(0.050)	(0.005)
Not married	0.106***	0.103***	0.094***	0.368***	0.084***
	(0.007)	(0.006)	(0.007)	(0.038)	(0.007)
Not white		0.049***	0.054***		0.046***
		(0.006)	(0.006)		(0.006)
Smoker	0.012*	0.028***	0.028***	0.096***	0.031***
	(0.006)	(0.005)	(0.005)	(0.035)	(0.006)
Observations	21,268	21,262	21,651	21,232	21,651

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3: The marginal effect of being in a risky occupation

3 A Model Economy with Occupation-specific Disability Risk

To prove our theoretical results, we use a two-period overlapping generations model where disability is a discrete event. Workers who receive a disability shock cannot work.

Time is discrete. There is a single consumption good produced with labor of workers in different occupations $j \in 1, 2...J$. The production technology exhibits constant elasticity of substitution across occupations with elasticity of substitution $\rho = \frac{1}{1-\gamma} < \infty$. Each occupation also defines a probability that the worker will be disabled, θ_j where $\theta_1 = 0$ and indexed such that $k > j \Rightarrow \theta_k > \theta_j$.

There is a continuum of competitive firms. The representative firm hires labor n_j in occupation specific spot markets to solve the following maximization problem:[3]

$$\max_{n_j} y - w_j n_j \tag{3.1}$$

$$\text{st } y = \left(\sum_{j=1}^{J} n_j^{\gamma} \right)^{1/\gamma} \tag{3.2}$$

Each period a unit measure of workers is born. Workers are identical at birth and live for two periods. They have strictly risk averse, time separable preferences over consumption in both periods $U(c_1, c_2) = u(c_1) + u(c_2)$. The utility function $u(\cdot)$ is assumed strictly increasing, strictly concave, and continuously differentiable.

We start with the case where occupation and disability are perfectly verifiable. In the first period, workers choose a single occupation, which persists their entire life-time. They then work, collecting wage w_j, pay occupation-specific taxes τ_j, and choose consumption and savings a, which has a gross rate of return R.

[3] Since firms hire in spot markets, their problem is static. This means they cannot design efficient wage contracts. We make this assumption because it is unrealistic a firm could commit to a lifetime path of wages and disability payments. The assumption also implies firms not internalize the fact that hiring workers in risky occupations decreases labor available in the next period. This consideration enters workers' problems only.

In the second period, disability shocks occur with occupation-specific probability $\theta_j \in [0,1)$, $\theta_j < \theta_{j'}\ \forall J \geq j' > j > 0$. Workers that receive a disability shock cannot work and report themselves as disabled. Workers who do not receive a disability shock cannot report themselves as disabled and work. Workers then pay occupation-specific taxes, consume whatever savings and occupation-specific disability benefits b_j or wage income they have, then die. The subscript 1 denotes period 1 consumption, d is period 2 consumption when disabled and n is for period 2 when not disabled. This problem can be represented as $\max_{j\in\{1,\ldots,J\}} E[U_j(c_{j,1}, c_{j,n}, c_{j,d})]$, where:

$$E[U_j(c_{j1}, c_{j,n}, c_{j,d})] = \max_{c_{j,1}, c_{j,n}, c_{j,d}, a} u(c_{j,1}) + (1-\theta_j)u(c_{j,n}) + \theta_j u(c_{j,d}) \ :$$
$$c_{j,1} \leq w_j - \tau_j - a$$
$$c_{j,n} \leq w_j - \tau_j + aR$$
$$c_{j,d} \leq b_j - \tau_j + aR$$

The solution to this problem gives occupation-specific savings decision a_j^*. Also, define ℓ_j^*, the measure of workers choosing occupation j.

Definition 3.1 (Competitive Equilibrium with Discrete Disability)**.** An equilibrium consists of allocations $\{c_{j,1}^*, c_{j,d}^*, c_{j,n}^*, a_j^*, \ell_j^*, n_j^*\}_{j=1}^J$ and prices $\{\{w_j^*\}_{j=1}^J, R\}$ such that: (i) given prices, allocations solve the workers' and firm's problems; (ii) feasibility is satisfied in the labor market: $n_j^* \leq (2-\theta_j)\ell_j^*$; and market clearing in (iii) goods and (iv) asset markets.

Optimality in the labor market requires two conditions are satisfied:

- Firms: Wage equals marginal product: $w_j = \left(\frac{y}{n_j}\right)^{1-\gamma}$
- Workers: Indifferent between entering any occupation: $U_j = U_k$ for all $j, k \in \{1, 2 \ldots J\}$.

Market clearing in the asset market requires that bonds be in net-zero supply

$$0 \geq \sum_{j=1}^{J} a_j^* \ell_j^*$$

Goods market clearing requires that

$$y \geq \sum_{j=1}^{J} c_{j,1} + (1-\theta_j)c_{j,n} + \theta_j c_{j,d}$$

Our first proposition shows that the competitive allocation is inefficient. This is not surprising, as the competitive equilibrium with only one asset restricts the set of assets needed to span the risks faced by workers.

Proposition 3.2 (The Competitive Allocation without Insurance is Inefficient). *Let* $\{c^*_{j,1}, c^*_{j,n}, c^*_{j,d}, a^*_j, n^*_j, \ell^*_j\}_{j=1}^J$ *satisfy Definition 3.1 for the case* $b_j = \tau_j = 0\ \forall j$*. There exists an alternative feasible allocation* $\{\hat{c}_{j,1}, \hat{c}_{j,d}, \hat{c}_{j,n}, \hat{\ell}_j\}_{j=1}^J$ *such that*

$$\begin{aligned} E[U_j(\hat{c}_{j,1}, \hat{c}_{j,n}, \hat{c}_{j,d})] &\geq E[U_j(c^*_{j,1}, c^*_{j,n}, c^*_{j,d})]\ \forall j \in \{1,\ldots,J\} \\ \exists k\ : E[U_k(\hat{c}_{k,1}, \hat{c}_{k,n}, \hat{c}_{k,d})] &> E[U_k(c^*_{k,1}, c^*_{k,n}, c^*_{k,d})] \end{aligned}$$

and

$$\sum_j \hat{\ell}_j\left(\hat{c}_{j,1} + \theta_j \hat{c}_{j,d} + (1-\theta_j)\hat{c}_{j,n}\right) \leq \left(\sum_j (\hat{\ell}_j(2-\theta_j))^\gamma\right)^{\frac{1}{\gamma}}$$

Proof. See Appendix A.2 □

Furthermore, the efficient, complete markets allocation is a strict Pareto improvement over the competitive allocation. This implies a role for welfare gains of policy beyond the standard redistribution/insurance motive. The following proposition states this formally:

Proposition 3.3. *Let* $\{c^{cm}_{j,1}, c^{cm}_{j,d}, c^{cm}_{j,n}, \ell^{cm}_j\}_{j=1}^J$ *be the efficient, planner's allocation*[4] *solving*

$$\max_{\{c_{j,1}, c_{j,d}, c_{j,n}, \ell_j\}_j} \sum_j \ell_j\left(u(c_{j,1}) + \theta_j u(c_{j,d}) + (1-\theta_j)u(c_{j,n})\right)\ : \tag{3.3}$$

$$\left(\sum_j ((2-\theta_j)\ell_j)^\gamma\right)^{\frac{1}{\gamma}} \geq \sum_j \ell_j\left(c_{j,1} + \theta_j c_{j,d} + (1-\theta_j)c_{j,n}\right) \tag{3.4}$$

$$1 \geq \sum_j \ell_j \tag{3.5}$$

Then it is the case that $\{c^{cm}_{j,1}, c^{cm}_{j,d}, c^{cm}_{j,n}, \ell^{cm}_j\}_{j=1}^J$ *strictly Pareto dominates* $\{c^*_{j,1}, c^*_{j,d}, c^*_{j,n}, \ell^*_j\}_{j=1}^J$

[4] It is easy to show directly that this corresponds to the complete markets equilibrium

Proof. See Appendix A.3 □

So far, our discussion has not been specific about the allocation of labor under incomplete markets relative to complete markets. With CES production (indeed even with linear, $\gamma = 1$), the efficient allocation of labor requires marginal product to be constant across inputs. A competitive wage cannot maintain this allocation because of the risk premium necessary in risky jobs when workers cannot fully insure. Thus, it is inefficient. The following corollary shows inefficiency of the competitive allocation is associated with too few workers in risky occupations.

Proposition 3.4. *[The Competitive Allocation without Insurance Puts Too Few Workers in Risky Occupations] Let $\{\ell_j^*\}$ satisfy Definition 3.1 for the case $b_j = \tau_j = 0\ \forall j$. Let $\{\ell_j^{cm}\}$ be the feasible, output maximizing allocation. Then,*

$$\sum_{j=1}^{t} \ell_j^* \leq \sum_{j=1}^{t} \ell_j^{cm} \quad \forall t = 1, \dots, J$$

This is to say, the efficient distribution of labor across occupations first-order stochastically dominates the distribution in the competitive allocation.

Proof. See Appendix A.3 □

It is intuitive that the degree of this inefficiency depends both on the extent of risk aversion of workers and the elasticity of substitution across occupations in production. As risk aversion increases, the competitive allocation becomes less efficient by concentrating even more workers in less risky occupations. Comparably, if workers are risk neutral, the competitive allocation is efficient. Figures 5 illustrate this for Constant Relative Risk Aversion preferences given by: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $\gamma = 0.5$.

As the elasticity of substitution between occupations increases, both the competitive and the efficient allocations put fewer workers in the risky occupations. While the efficient allocation always puts (weakly) more workers in the risky occupations, the distance between the two distributions is non-monotone. When occupations are perfect substitutes, the distributions are equal: both allocations put all of the workers in the safest occupation.

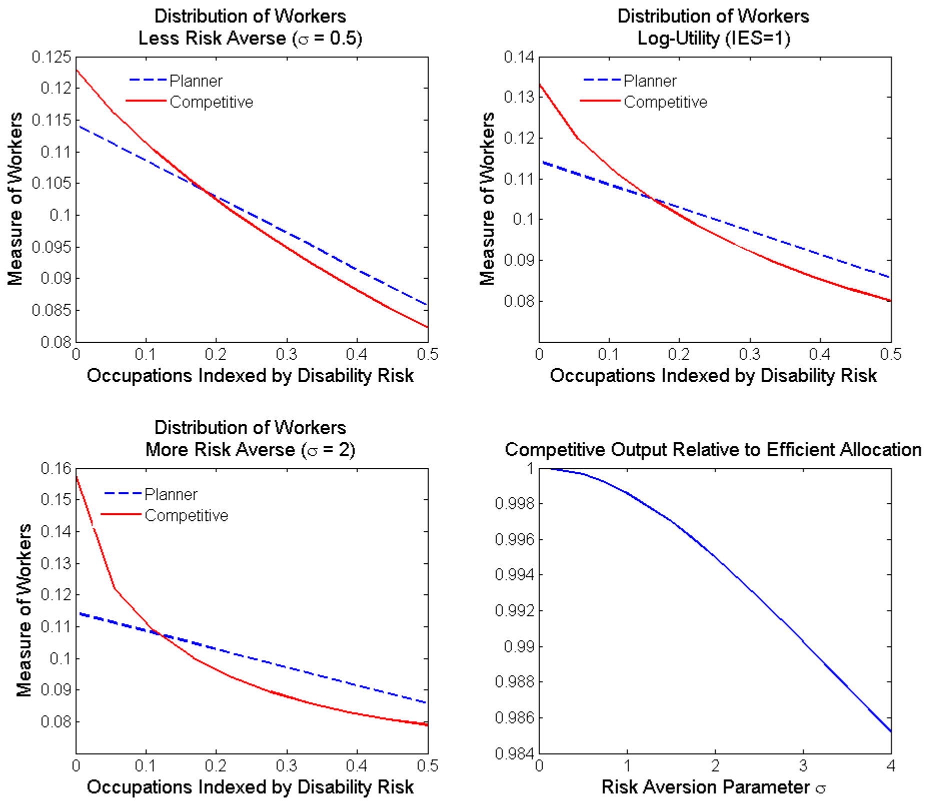

Figure 5: Risk aversion affects the competitive allocation.

When occupations are perfect complements (production is Leontief), both allocations evenly distribute workers across occupations. Figures 6 illustrate this, fixing risk aversion at$\sigma = 2$.

Since inefficiency in the incomplete markets competitive allocation comes from too few workers in risky occupations, social insurance can improve efficiency. When transfers are made to disabled workers, the wage premium required to work in risky occupations decreases because the need for self-insurance is reduced. Therefore, under social insurance the distribution of workers shifts towards more risky occupations.

A key insight is that the redistribution of workers across occupations under social insurance increases the real wage in *all* occupations. This implies social insurance, at least on the margin, is Pareto improving; all workers, even those in the second generation who know

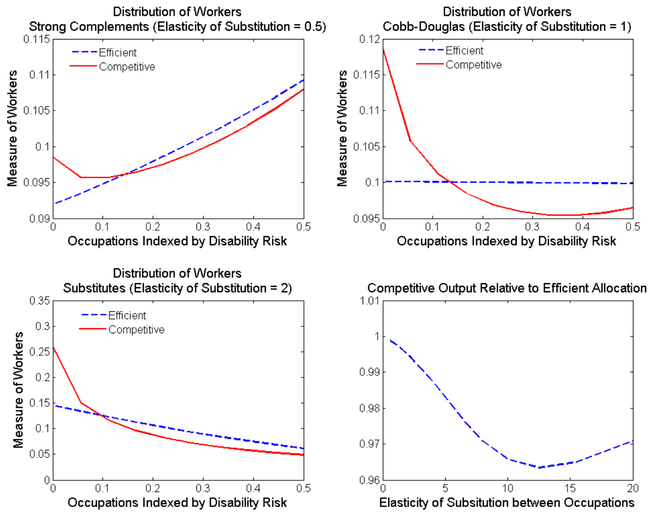

Figure 6: Elasticity of substitution affects the allocations.

they will never be disabled are better off. The cost to each worker to fund the program is less than the benefit because taxes only need to cover workers who are ex-post disabled. Self-insurance, on the other hand, wastes resources by reserving too much funds for workers who never become disabled.

Proposition 3.5 (Social Insurance is Pareto Improving (on the margin))**.** *Let $EU_j(b)$ be the expected utility $E[U_j(c^*_{j1}, c^*_{j2}, c^{d*}_{j2})]$ derived from the allocation in competitive equilibrium $\{c^*_{j1}, c^*_{j2}, c^{d*}_{j2}, a^*_j, n^*_j, \ell^*_j\}_{j=1}^{J}$ satisfying Definition 3.1 for arbitrary occupation independent benefit b funded by occupation independent lump sum tax τ. Then,*

$$EU'_j(0) > 0 \quad \forall j$$

It is obvious that, if taxes and transfers are occupation specific, the Pareto optimal allocation can be achieved. This will be our benchmark for the maximum welfare gain from social insurance.

As a corollary we show that there exists an optimal level of flat benefits when funded by a flat tax. This is the level at which we will evaluate the flat scheme at in the quantitative section to provide a lower bound of the welfare gains of social insurance.

Corollary 3.6 (There Exists an Optimal Level of Social Insurance with Occupation Independent Taxes and Transfers)**.** *Let $EU_j(b)$ be the expected utility $E[U_j(c^*_{j1}, c^*_{j2}, c^{d*}_{j2})]$ derived from allocation $\{c^*_{j1}, c^*_{j2}, c^{d*}_{j2}, a^*_j, n^*_j, \ell^*_j\}_{j=1}^J$ satisfying Definition 3.1 for arbitrary occupation independent benefit b funded by occupation independent lump sum tax τ. Then $\exists! b^*$ such that,*

$$EU'_j(b^*) = 0 \quad \forall j$$

Figure 7 illustrates the following features of welfare and output gains from the flat tax and transfer social insurance system. First, starting with no social insurance, the welfare and output gains are always positive. Second, there exists unique output maximizing tax level and a unique welfare maximizing tax level, but these tax levels are not necessarily the same. This is because welfare include both output gains and gains from insurance (consumption smoothing). For this parametrization (IES =3), the welfare maximizing system actually puts more labor in risky occupations than the output maximizing (efficient) allocation. This means that at the output maximizing allocation, there are greater gains from providing higher benefits for consumption smoothing than the output loss incurred as these benefits move more workers into risky occupations than is efficient.

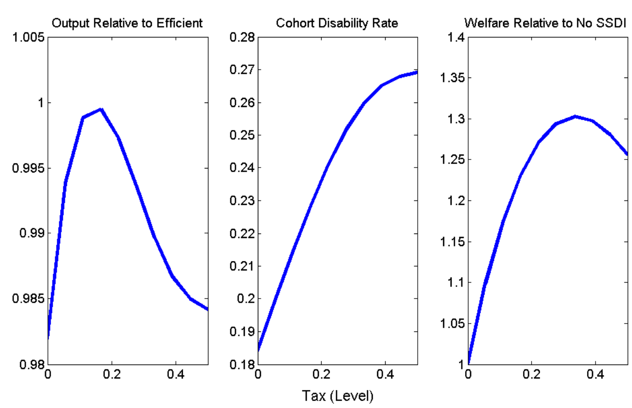

Figure 7: Output, disability, and welfare for different tax levels.

Figure 8 illustrates that the welfare gains of the optimal insurance system are decreasing in the elasticity of substitution across occupations. Behind the monotonicity of welfare gains are non-monotonicities in the dynamics of output. For very low elasticity of substitution, the market already pays a large premium to workers in the risky occupation and little reallocation is needed. Insurance brings such large welfare gains in this case because this is where it both reduces the risk premium the most and has the largest share of workers in the risky occupation enjoying the risk sharing. As the elasticity of substitution increases, greater worker reallocation occurs, up to a point and then declines. This dynamic hinges on the market ability to provide insurance. Welfare gains fall as the elasticity of substitution increases mainly through the risk sharing channel as there are fewer workers in the risky occupations.

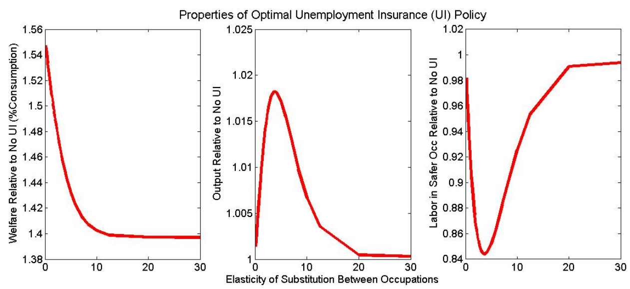

Figure 8: Welfare, output, and labor share in the safer occupation at the optimal tax.

4 Decomposing Welfare Gains from SSDI

In our model, all agents have the same welfare gain in a steady state with SSDI because the equilibrium requires indifference across occupations. To understand where the welfare gains come from, it is interesting to consider the how SSDI affects welfare in different realized states: disabled and non-disabled. To understand these welfare effects, we take an old-generation from the no SSDI world, keeping their previous asset choices, and then give them the wages, taxes, and benefits of various sized SSDI programs. The welfare gains for this experiment, across occupations and disability outcome, are shown in the figure below.

The first panel in Figure 9 shows disabled workers always collect a windfall gain from unexpected disability benefits in all cases. The second panel shows occupation-specific risk is the only scenario in which workers who know they will never be disabled have a welfare

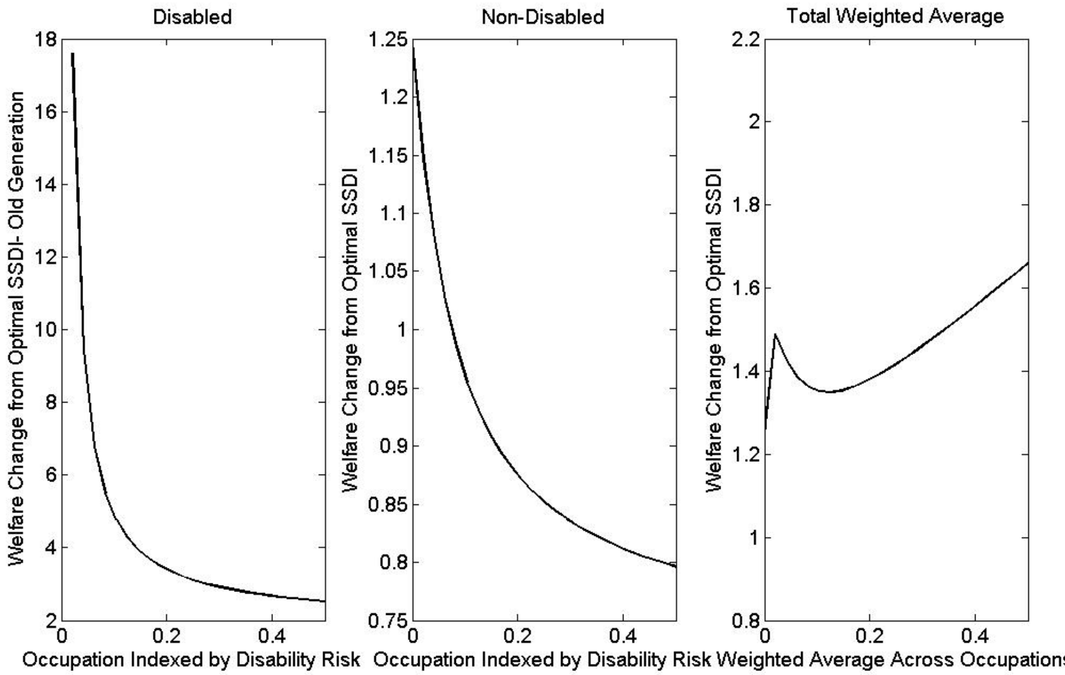

Figure 9: Output, disability, and welfare for different tax levels.

gain. This is entirely the gain through wages: rising wages in less risky occupations and lower wages in more risky occupations as the distribution of workers shifts towards risky occupations. The final panel shows the total average benefit for workers in each occupation weighted by the composition of disabled and non-disabled.

5 Quantitative Evaluation of Social Insurance Considering Occupation Hazards

We now explore the quantitative properties of our model applied to a realistic model of disability in the US. The quantitative model features two occupations calibrated to match a partition of the data into two occupation groups: high differential disability risk and low. We consider a production technology using both labor in low risk occupations n_ℓ and in high risk occupations n_h:

$$Y = Q(\alpha n_\ell^\gamma + (1-\alpha)n_h^\gamma)^{1/\gamma}$$

Hazard rates of non-employment risk in each occupation are taken directly from the data. We then calibrate α and Q *for each* of a range of $\gamma \in [-5, .99]$. Q is chosen such that $Y = 1$. Share α is chosen such that the percent of workers in each occupation matches the empirical equivalent. Mechanically, this means that α will be much lower for high elasticities of substitution in order to raise employment in the risky occupation to match the data. This is crucial to keep in mind when interpreting our results. We are not providing a comparative statics for γ. We are instead viewing the true value of γ as unknown and recalibrating the model to replicate US data for each potential γ.

Preferences specify instantaneous utility as $u(c_t) = ln(c_t)$ and discount rate $\beta = 1$. The incomplete markets feature a single asset with an interest rate of zero (as in a backyard storage technology). Aggregate population is normalized to a unit measure.

For each γ, we calibrate to a world with no insurance and then solve for the optimal fully funded insurance system. The system pays a flat benefit in non-employment and is funded by a proportional tax on labor income. This is a second best policy. An occupation-specific tax and subsidy can implement the Pareto efficient solution to the Planner's problem.

5.1 Quantitative Evaluation of Disability Insurance

We now evaluate the quantitative implications of our model for the US economy. We maintain the two-generation overlapping generations structure of the theory section, but we consider only two occupations, guided by the natural break in occupational risks observed in the Health and Retirement Survey sample. As described in the empirical section, after controlling for demographics, the base risk in the low-risk occupation is 9% probability of disability before retirement and the additional risk in the high risk occupations amounts to 13% probability of disability before retirement.

The production technology uses low disability risk labor n_ℓ and high risk labor n_h: $Y = Q(\alpha n_\ell^\gamma + (1-\alpha)n_h^\gamma)^{1/\gamma}$. Preferences are CES $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ with no discounting the future. For our exercise we choose $\sigma = 1$. Our preferred calibration of the elasticity of substitution is 2, but we also display results for elasticities of substitution equal to one-half and ten. For each elasticity, we calibrate α to match 20% of workers choosing the high risk occupation, as in the HRS. Lastly, we choose Q to normalize output to one to allow comparison over each scenario.

The below table presents results for how aggregate statistics change at the optimal, ie: most welfare improving, social disability system.[5] Welfare is measured in percent consumption subsidy necessary for indifference between the no-SSDI and optimal policy worlds.

Effects of Optimal SSDI compared to No SSDI			
Elasticity of Substitution	$\frac{1}{3}$	1	2.5
Calibrated α	0.98	0.77	0.62
Welfare			
($\Delta X\%$)	1.054	1.054	1.053
Output			
($\Delta X\%$)	0.99	0.99	0.99
SSDI/GDP			
level	0.096	0.097	0.098

Figure 10 outcomes at the optimal policy for a large range of elasticities of substitution. The first panel shows welfare gains from SSDI are approximately equal across elasticities; though they are slightly higher when occupations are closest to perfect complements or perfect substitutes. This result is driven by the relative magnitudes of the two channels through which SSDI improves welfare: (i) improving the efficiency of production; and (ii) providing insurance. When occupations are close to perfect complements, the market pays a high risk premium to put many workers in the risky occupation. Here, welfare gain of SSDI comes mostly through reallocation. Social insurance lowers the cost to place many workers in risky occupations and raises output. But balancing this, as the elasticity rises, fewer workers are needed in the risky occupation. Everybody has similar risks and the market also

[5] The values for a system with a lump-sum tax and replacement and for a wage- proportional tax/replacement are quantitatively identical to the second decimal place, so we present only the lump-sum tax.

provides no insurance. Here the welfare gain of SSDI comes mostly through the insurance channel, not reallocation.

On this welfare pane we also plotted the line for the fixed and flexible occupations cases. In the fixed occupations, we forced workers to stay in the same, socially optimal occupation and then computed the optimal tax. This shuts down the reallocation forces in the model and only allows disability insurance to fulfil a risk-sharing motive. As we can see from the figure, each accounts for about half of the welfare gains.

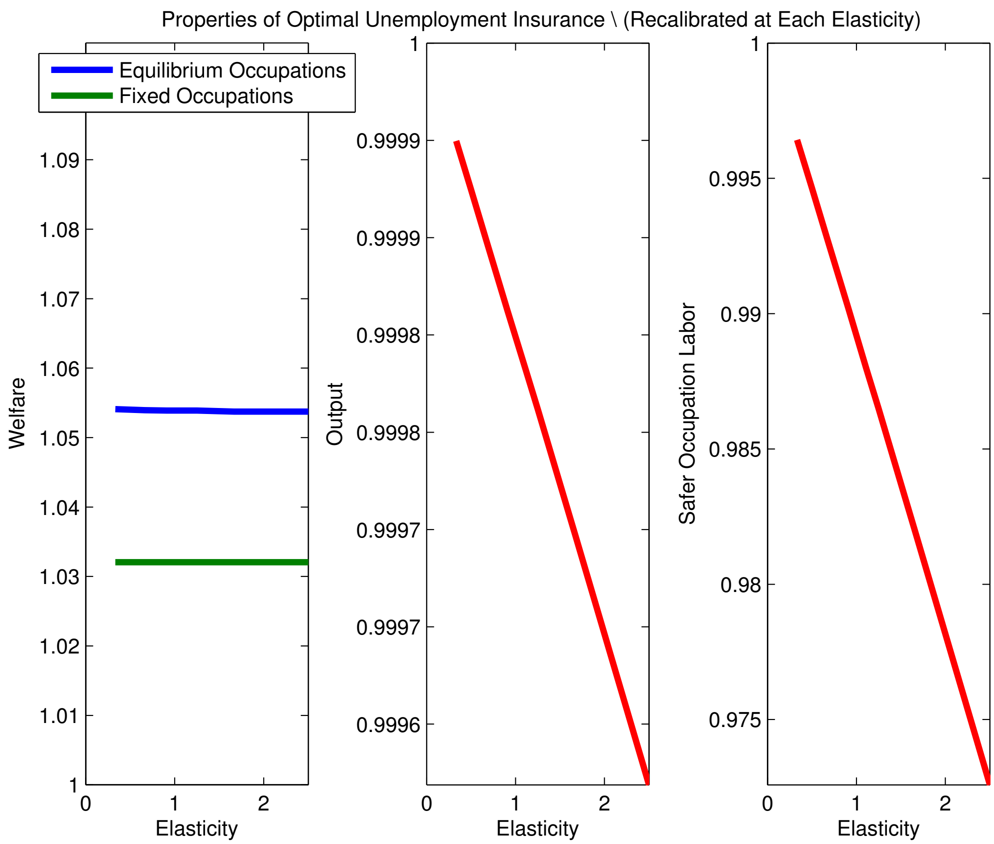

Figure 10: Analysis of Optimal Unemployment Insurance, calibrated at different Elasticities of Substitution

Conclusion This paper has addressed an important aspect of disability risk: occupational choice. We have documented occupational choice provides 29% of the variation in overall disability risk. We find inclusion of occupational choice in evaluating social disability insurance is important because 1) welfare gains are Pareto improving; and 2) the optimal level of social insurance is smaller compared to assuming occupations have no causal affect on disability.

This idea could also be promising for other questions. Why don't firms provide insurance? It also provides a new justification for social insurance.

Caveat: We have abstracted from other things. Investment in health, other heterogeneity in risks and preferences. Important for more serious quantitative evaluation of these policies.

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Appendices

A Theoretical Results and Proofs

A.1 Setting up tax free economy

We first set up versions of our tax-free economy ($\tau_j = b_j = 0 \ \forall j \in \{1, \ldots, J\}$) with complete and incomplete markets. In both cases, the firms' problem is the same.

Firms give spot contracts, meaning the representative firm solves

$$\max_{\{n_j\}} \left(\sum_{j=1}^{J} n_j^{\gamma} \right)^{1/\gamma} - \sum_j w_j n_j$$

Where in equilibrium $n_j = (2 - \theta_j)\ell_j$. From this problem, the FOC is

$$w_j = \left(\frac{y}{n_j} \right)^{1-\gamma} \tag{A.1}$$

And for any θ_j, θ_k

$$\frac{w_j}{w_k} = \left(\frac{n_j}{n_k} \right)^{\gamma-1} \tag{A.2}$$

Complete Markets In this section, we identify the endogeneous variables of the competitive equilibrium with the superscript cm. We begin by setting up the households' problem with the complete set of Arrow securities. It is trivial to show directly that these allocations correspond to those in from the social planner's problem, as with the full set of assets, both welfare theorems holds and the efficient allocation, $\{c_{j,1}^{cm}, c_{j,d}^{cm}, c_{j,n}^{cm}, \ell_j^{cm}\}_{j=1}^{J}$ is also unique.[6]

[6] We can equally set up time-zero trading, in which case the problem is:

$$\max_{\{c_{j,s}\},\{\ell_j\}_j} \sum_{j=1}^{J} \{u(c_{j,1}) + \theta_j u(c_{j,d}) + (1-\theta_j) u(c_{j,n})\} \ell_j \tag{A.3}$$

$$0 \leq \sum_j \{(w_j - c_{j,1}) + q_{j,n}(w_j - c_{j,n}) + q_{j,d} c_{j,d}\} \ell_j \tag{A.4}$$

$$1 \geq \sum_j \ell_j \tag{A.5}$$

Their possible states are $s = \{1, d, n\}$ for periods 1, or 2 if they become disabled, or not disabled and j for the occupation they choose. The full set of Arrow securities, $\{a_{j,d}, a_{j,n}\}_{j=1}^{J}$ spans states j, s.

$$\max_{\{c_{j,s}\},\{\ell_j\}_j,\{a_{j,n},a_{j,d}\}} \sum_{j=1}^{J} \{u(c_{j,1}) + \theta_j u(c_{j,d}) + (1-\theta_j) u(c_{j,n})\} \ell_j \tag{A.6}$$

$$c_{j,1} \leq w_j - \sum_{k=1}^{J} a_{k,n} - a_{k,d} \tag{A.7}$$

$$c_{j,n} \leq w_j + \sum_{k} R_{k,n} a_{k,n} \tag{A.8}$$

$$c_{j,d} \leq \sum_{k} R_{k,d} a_{k,d} \tag{A.9}$$

$$1 \geq \sum_{j} \ell_j \tag{A.10}$$

Then it is easy to show that the solution to this problem sets each interest rate $R_{j,n} = \frac{1}{1-\theta_j}$, $R_{j,d} = \frac{1}{\theta_j}$ and the saving policies are $a_{j,n}^{cm} = \frac{-\theta_j}{2}(1-\theta_j) w_j^{cm}$, $a_{j,d}^{cm} = \frac{2-\theta_j}{2} \theta_j w_j^{cm}$ for j such that $\ell_j = 1$ and $a_{k,s}^{cm} = 0 \ \forall k \neq j$. This implies consumption is smoothed across time and states:

$$c_{j,1}^{cm} = c_{j,n}^{cm} = c_{j,d}^{cm} = \frac{2-\theta_j}{2} w_j^{cm}$$

The occupation choice ℓ_j requires that expected earnings must be equalized across occupations j, k:

$$w_j^{cm}(2-\theta_j) = w_k^{cm}(2-\theta_k) \tag{A.11}$$

Combining the wage condition with consumption, we get that

$$w_j^{cm} = \frac{2c}{2-\theta_j} \tag{A.12}$$

Then, we normalize the price of the first period to 1 and the rest of the state-dependent prices are, as per usual, easily solved for from first order conditions, $q_{j,n}^{cm} = 1 - \theta_j, q_{j,d}^{cm} = \theta_j$. And this implies consumption is equalized across states as in the case with arrow securities.

where c is the consumption level of every worker. From the firms' side, Equation A.1, and substituting market clearing that $y = 2c$, and $\ell_j^{cm}(2-\theta_j) = n_j$ we also have

$$w_j^{cm} = \left(\frac{2c}{(2-\theta_j)\ell_j^{cm}}\right)^{1-\gamma} = \frac{2c}{2-\theta_j}$$

Solving for $\ell(\theta)$, we have

$$\ell_j^{cm} = \left(\frac{2c}{2-\theta_j}\right)^{\frac{\gamma}{\gamma-1}} \tag{A.13}$$

Incomplete Markets With only one asset, households solve

$$\max_{j\in\{1,\ldots,J\},a} u(w_j - a) + \theta u(aR) + (1-\theta_j)u(w_j + aR) \tag{A.14}$$

$$\{a\} : u'(w_j^* - a_j^*)/R^* = \theta u'(a_j^* R^*) + (1-\theta_j)u'(w_j^* + a_j^* R^*) \tag{A.15}$$

$$\begin{aligned}\{j\} : &u(w_j^* - a_j^*) + \theta_j u(a_j^* R^*) + (1-\theta_j)u(w_j^* + a_j^* R^*) \\ &= u(w_k^* - a_k^* R^*) + \theta_k u(a_k^* R^*) + (1-\theta_k)u(w_k^* + a_k^* R^*)\end{aligned} \tag{A.16}$$

The FOC on occupation choice that households are indifferent between choosing any risk level $\theta_j, \theta_k,$. We have denoted the optimal savings policy of an agent choosing j as a_j^*, which is distinct from use of subscript in the complete markets case where we indexed potential $\{a_{k,s}\}_{k=1}^J$ by the level of risk assumed and the occupation that would buy them.

Consider the case $\theta_k = 0$, then the two first-order-conditions imply

$$u(w_j^* - a_j^*) + \theta u(a_j^* R^*) + (1-\theta_j)u(w_j^* + a_j^* R) = 2u(w_k^*) \tag{A.17}$$

Note also that the Inada condition on utility guarantees that for $j > 1$ and $\theta_j > 0$, $a_j^* > 0$ otherwise in the disabled state marginal utility is not finite.

A.2 Inefficiency of the incomplete markets equilibrium

Proof. Proof of 3.2 (reprinted here): Let $\{c_{j,1}^*, c_{j,n}^*, c_{j,d}^*, a_j^*, n_j^*, \ell_j^*\}_{j=1}^J$ satisfy Definition 3.1 for the case $b_j = \tau_j = 0\ \forall j$. There exists an alternative feasible allocation $\{\hat{c}_{j,1}, \hat{c}_{j,d}, \hat{c}_{j,n}, \hat{\ell}_j\}_{j=1}^J$

such that

$$E[U_j(\hat{c}_{j,1}, \hat{c}_{j,n}, \hat{c}_{j,d})] \geq E[U_j(c^*_{j,1}, c^*_{j,n}, c^*_{j,d})] \ \forall j \in \{1, \ldots, J\}$$
$$\exists k \ : E[U_k(\hat{c}_{k,1}, \hat{c}_{k,n}, \hat{c}_{k,d})] > E[U_k(c^*_{k,1}, c^*_{k,n}, c^*_{k,d})]$$

and

$$\sum_j \hat{\ell}_j \left(\hat{c}_{j,1} + \theta_j \hat{c}_{j,d} + (1-\theta_j)\hat{c}_{j,n}\right) \leq \left(\sum_j (\hat{\ell}_j(2-\theta_j))^\gamma\right)^{\frac{1}{\gamma}}$$

It is easy to construct one such, Pareto dominating economy to the one in the incomplete markets competitive equilibrium. Let $\hat{\ell}_j = \ell^*_j \ \forall j = 1, \ldots, J$. Next, note that if we solve for $\{c^*_j\}$, it is the same problem as in complete markets except that we restrict $R_{j,n} = R_{j,d} = R$, while they are not in equilibrium. If instead we allow workers to solve with two assets. Agents take prices $\{\hat{w}_j\}$ and $\{\hat{R}_{j,n}, \hat{R}_{j,d}\}$ and solve:

$$\max_{c_{j,1}, c_{j,n}, c_{j,d}, a_{j,d}, a_{j,n}} u(\hat{w}_j - a_{j,d} - a_{j,n}) + \theta_j u(a_{j,d}\hat{R}_{j,d}) + (1-\theta_j)u(\hat{w}_j + a_{j,n}\hat{R}_{j,n})$$

then we get fully smooth consumption consumption, $\hat{c}_{j,1} = \hat{c}_{j,n} = \hat{c}_{j,d} = \frac{2-\theta_j}{2}\hat{w}_j$. Here $\hat{w}_j = w^*_j$, which we solved for in the competitive equilibrium and $\hat{R}_{j,d} = \frac{1}{\theta_j}$, $\hat{R}_{j,n} = \frac{1}{1-\theta_j}$ This is clearly feasible, as we have only redistributed the same output among workers who choose occupation j. This is strictly preferred for each $j = 2, \ldots, J$ because of strict concavity and for $j = 1$, the worker in the risk-free occupation is indifferent. □

A.3 Comparing the complete and incomplete markets allocations

Lemma A.1. *Compared to the incomplete markets allocation, the complete markets allocation has more mass in every risky occupation relative to the zero-risk occupation:* $\forall j = 1, \ldots, J$, $\frac{\ell_j}{\ell_1} \leq \frac{\ell^{cm}_j}{\ell^{cm}_1}$.

Proof. To prove this, we use contradiction. Hence, we begin with the counter factual,[7]

$$\exists \ j \ : \ \frac{\ell_j}{\ell_1} > \frac{\ell^{cm}_j}{\ell^{cm}_1} \tag{A.18}$$

[7]It is trivial to show the case of $\theta_1 = 0$ because $\frac{\ell_1}{\ell_1} = 1 = 1 = \frac{\ell^{cm}_1}{\ell^{cm}_1}$. Hence, we prove for cases in which $\theta_j \in (0, 1]$

We can then take this to the wage space using Equation A.2. First multiply both sides, $\frac{(2-\theta_j)\ell_j}{2\ell_1} > \frac{(2-\theta_j)\ell_j^{cm}}{2\ell_1^{cm}}$. Raising both the power $\gamma - 1$ we get, subject to a parameter restriction that $\gamma < 1$

$$\left(\frac{(2-\theta_j)\ell_j}{2\ell_1}\right)^{\gamma-1} < \left(\frac{(2-\theta_j)\ell_j^{cm}}{2\ell_1^{cm}}\right)^{\gamma-1}$$

This implies, by Equation A.2

$$\frac{w_j}{w_1} < \frac{w_j^{cm}}{w_1^{cm}} \tag{A.19}$$

Next, from the occupation choice indifference condition, workers must be indifferent between either occupation

$$u(w_j - a_j) + \theta u(Ra_j) + (1-\theta)(w_j + Ra_j) = 2u(w_1)$$

And, by Equation A.19 and monotonicity

$$2u\left(w_1\right) > 2u\left(\frac{w_1^{cm}}{w_j^{cm}} w_j\right)$$

Then, because $\frac{w_1^{cm}}{w_j^{cm}} = \frac{2-\theta}{2}$ as shown in Equation A.11, we get that

$$2u\left(\frac{w_1^{cm}}{w_j^{cm}} w_j\right) = 2u\left(w_j \frac{2-\theta}{2}\right)$$

where the right-hand side is the expected earnings from occupation θ. Combining these,

$$u(w_j - a_j) + \theta u(Ra_j) + (1-\theta)(w_j + Ra_j) = 2u(w_1) > 2u\left(w_j \frac{2-\theta}{2}\right)$$

But this means that the incomplete markets, risky allocation is strictly preferred to the expected earnings, which violates Jensen's inequality with strictly concave preferences. Hence, our assumption in Equation A.18 could not be and we have established our Lemma A.1.

□

Note that a corollary of this is that $\frac{\ell_1}{\ell_1^{cm}} \geq 1$. This is easy to see, using the fact that both ℓ, ℓ^{cm} are densities and hence $\sum_j \ell_j = \sum_j \ell_j^{cm} = 1$.

Corollary A.2. *There is more mass at the risk-free occupation, where $\theta_1 = 0$, in the incomplete markets allocation: $\frac{\ell_1^*}{\ell_1^{cm}} \geq 1$*

Proof. To see this, begin with

$$\sum_j \ell_j^{cm} \frac{\ell_1^*}{\ell_1^{cm}} \geq \sum_j \ell_j^*$$

which holds because by Lemma A.1, at any θ, $\ell_j^{cm} \frac{\ell_1^*}{\ell_1^{cm}} \geq \ell_j^*$ and hence it must also hold when we sum over all of them. But because ℓ^*, ℓ^{cm} are both distributions, it must be the case that

$$\sum_j \ell_j^{cm} \frac{\ell_1^*}{\ell_1^{cm}} \geq \sum_j \ell_j^* = 1 = \sum_j \ell_j^{cm}$$

Moving the fraction $\frac{\ell_1^*}{\ell_1^{cm}}$ out of the summation, we get:

$$\frac{\ell_1^*}{\ell_1^{cm}} \sum_j \ell_j^{cm} \geq \sum_j \ell_j^{cm}$$

And just dividing gives us $\frac{\ell_1^*}{\ell_1^{cm}} \geq 1$ □

Proof. Proof of 3.3 (reprinted here): Let $\{c_{j,1}^{cm}, c_{j,d}^{cm}, c_{j,n}^{cm}, \ell_j^{cm}\}_{j=1}^J$ be the efficient, planner's allocation[8] solving

$$\max_{\{c_{j,1}, c_{j,d}, c_{j,n}, \ell_j\}_j} \sum_j \ell_j \left(u(c_{j,1}) + \theta_j u(c_{j,d}) + (1-\theta_j)u(c_{j,n})\right) \ : \tag{A.20}$$

$$\left(\sum_j ((2-\theta_j)\ell_j)^\gamma\right)^{\frac{1}{\gamma}} \geq \sum_j \ell_j \left(c_{j,1} + \theta_j c_{j,d} + (1-\theta_j)c_{j,n}\right) \tag{A.21}$$

$$1 \geq \sum_j \ell_j \tag{A.22}$$

Then it is the case that $\{c_{j,1}^{cm}, c_{j,d}^{cm}, c_{j,n}^{cm}, \ell_j^{cm}\}_{j=1}^J$ strictly Pareto dominates $\{c_{j,1}^*, c_{j,d}^*, c_{j,n}^*, \ell_j^*\}_{j=1}^J$

First, note that both allocations satisfy feasability because of the problems from which they are defined. That is

$$\sum_j \ell_j \left(c_{j,1} + \theta_j c_{j,d} + (1-\theta_j)c_{j,n}\right) \leq \left(\sum_j (\ell_j(2-\theta_j))^\gamma\right)^{\frac{1}{\gamma}}$$

[8] It is easy to show directly that this corresponds to the complete markets equilibrium

for both $\{c^{cm}_{j,1}, c^{cm}_{j,d}, c^{cm}_{j,n}, \ell^{cm}_j\}$, $\{c^*_{j,1}, c^*_{j,d}, c^*_{j,n}, \ell^*_j\}$

Then, from Corollary A.2 $\ell^*_1 \geq \ell^{cm}_1 \Leftrightarrow w^*_1 \leq w^{cm}_1$ and because the utility function is strictly increasing $2u(w^*_1) \leq 2u(w^{cm}_1)$. Adding in the occupational choice indifference conditions gives, for arbitrary j:

$$u(w^*_j - a^*_j) + \theta_j u(a^*_j) + (1-\theta_j)u(w^*_j + a^*_j) = 2u(w^*_1) \leq 2u(w^{cm}_1) = 2u(w^{cm}_j \frac{2-\theta_j}{2})$$

or equivalently:

$$EU(c^*_{j,1}, c^*_{j,d}, c^*_{j,n}) = EU(c^*_{1,1}, c^*_{1,d}, c^*_{1,n}) \leq EU(c^{cm}_{j,1}, c^{cm}_{j,d}, c^{cm}_{j,n}) = EU(c^{cm}_{1,1}, c^{cm}_{1,d}, c^{cm}_{1,n})$$

This means that $\forall j$, $EU(c^{cm}_{j,1}, c^{cm}_{j,d}, c^{cm}_{j,n}) \geq EU(c^*_{j,1}, c^*_{j,d}, c^*_{j,n})$ just as we required.

Now, suppose that for any k the weak inequality holds with equality, $EU(c^{cm}_{k,1}, c^{cm}_{k,d}, c^{cm}_{k,n}) = EU(c^*_{k,1}, c^*_{k,d}, c^*_{k,n})$. Then the indifference condition for occupational choice on both sides again holds that

$$EU(c^{cm}_{j,1}, c^{cm}_{j,d}, c^{cm}_{j,n}) = EU(c^{cm}_{k,1}, c^{cm}_{k,d}, c^{cm}_{k,n}) = EU(c^*_{k,1}, c^*_{k,d}, c^*_{k,n}) = EU(c^*_{j,1}, c^*_{j,d}, c^*_{j,n})$$

But if that is the case than, agents are equally well off with the competitive equilibrium allocation, $\{c^*_{j,1}, c^*_{j,n}, c^*_{j,n}, \ell^*_j\}_j$ as they are with the efficient allocation $\{c^{cm}_{j,1}, c^{cm}_{j,n}, c^{cm}_{j,n}, \ell^{cm}_j\}_j$. That cannot be because it contradicts Proposition 3.2. Hence, $\nexists k$ such that $EU(c^{cm}_{k,1}, c^{cm}_{k,d}, c^{cm}_{k,n}) = EU(c^*_{k,1}, c^*_{k,d}, c^*_{k,n})$ and we have that

$$EU(c^{cm}_{k,1}, c^{cm}_{k,d}, c^{cm}_{k,n}) > EU(c^*_{k,1}, c^*_{k,d}, c^*_{k,n}) \; \forall k = 1, \ldots, J$$

$\square$

Lemma A.3. *The ratio of labor in any two occupations, one riskier than the other, is greater in the complete markets allocation than the equilibrium allocation with incomplete markets:* $\forall L, H$ *such that* $\theta_L, \theta_H \in (0,1]$ *and* $\theta_L \leq \theta_H$, $\frac{\ell^{cm}_H}{\ell^{cm}_L} \geq \frac{\ell^*_H}{\ell^*_L}$

Proof. First, suppose this is not the case, then we assume

$$\frac{\ell^{cm}_H}{\ell^{cm}_L} < \frac{\ell^*_H}{\ell^*_L} \tag{A.23}$$

Which is equivalent to

$$\frac{\ell_H^{cm}}{\ell_H^*} < \frac{\ell_L^{cm}}{\ell_L^*}$$

and by the same reasoning as in Lemma A.1(and imposing $\gamma < 1$)

$$\frac{w_H^*}{w_H^{cm}} < \frac{w_L^*}{w_L^{cm}}$$

And because we have established that $\frac{w_H^{cm}}{w_L^{cm}} = \frac{2-\theta_L}{2-\theta_H}$ that gives us that that

$$w_H^*(2-\theta_H) < w_L^*(2-\theta_L) \tag{A.24}$$

This is to say, the expected return to occupation L is higher than that in H and θ_L also has more risk. The lottery associated with θ_L, w_L first-order stochastic dominates that of lottery θ_H, w_H. Clearly, this cannot be consistent with household's indifference

To put this more formally, use our earlier notation $a_X^* = \text{argmax}_{a_X}\, u(w_X^* - a_X) + \theta_X u(\theta_X) + (1-\theta_X)u(w_X^* + a_X)$. Now consider the sequence of inequalities:

$$\begin{aligned} u(w_L^* - a_L^*) + &\theta_L u(a_L^*) + (1-\theta_L)u(w_L^* + a_L^*) && \text{(A.25)} \\ &\geq u(w_L^* - a_H^*) + \theta_L u(a_H^*) + (1-\theta_L)u(w_L^* + a_H^*) && \text{(A.26)} \\ &> u(w_H^* - a_H^*) + \theta_H u(a_H^*) + (1-\theta_H)u(w_H^* + a_H^*) && \text{(A.27)} \end{aligned}$$

Where the first holds by definition of a_L^*, a_H^* and the second holds because of our assumption. However, by our occupational-choice indifference condition, A.17, $u(w_L^* - a_L^*) + \theta_L u(a_L^*) + (1-\theta_L)u(w_L^* + a_L^*) = u(w_H^* - a_H^*) + \theta_H u(a_H^*) + (1-\theta_H)u(w_H^* + a_H^*)$

□

Proof. Proof of 3.4 (reprinted here): [The Competitive Allocation without Insurance Puts Too Few Workers in Risky Occupations] Let $\{\ell_j^*\}$ satisfy Definition 3.1 for the case $b_j = \tau_j = 0\ \forall j$. Let $\{\ell_j^{cm}\}$ be the feasible, output maximizing allocation. Then,

$$\sum_{j=1}^{t} \ell_j^* \leq \sum_{j=1}^{t} \ell_j^{cm} \quad \forall t = 1, \ldots, J$$

This is to say, the efficient distribution of labor across occupations first-order stochastically dominates the distribution in the competitive allocation.

Again, the converse will imply a contradiction. Suppose that

$$\exists T \; : \; \sum_{j=1}^{T} \ell_j^{cm} > \sum_{j=1}^{T} \ell_j^* \tag{A.28}$$

We first consider $1 < T < J$, as the end points are already established. For our supposition, Equation A.28, to hold, $\exists L < T \; : \; \ell_L^{cm} > \ell_L$.

Because both distributions sum to 1, our assumption implies that $sum_{j=T+1}^{J} \ell_j^{cm} < \sum_{j=T+1}^{J} \ell_j^*$ and hence there exists a $H > T \; : \; \ell_H^* > \ell_H^{cm}$.

Combining our two inequalities we have:

$$\frac{\ell_H^{cm}}{\ell_H^*} < 1 < \frac{\ell_L^{cm}}{\ell_L^*}$$

Rearranging this, we get

$$\frac{\ell_H^{cm}}{\ell_L^{cm}} < \frac{\ell_H^*}{\ell_L^*}$$

But this is a contradiction to Lemma A.3

As earlier alluded, if $T = 1$ then our contradictory supposition, equation A.28, implies $\ell_1^{cm} > \ell_1^*$ which directly contradicts Corollary A.2. If $T = J$, then because $\{\ell_j^{cm}\}, \{\ell_j^*\}$ are distributions obviously $\sum_{j=1}^{J} \ell_j^* = \sum_{j=1}^{J} \ell_j^{cm} = 1$

□