Political Economy and the Structure of Taxation*

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Abstract

We study the constrained Pareto efficient allocations in a dynamic production economy in which the group in political power decides the allocation of resources. We show that Pareto efficient allocations take a quasi-Markovian structure and can be represented recursively as a function of the identity of the group in power and updated Pareto weights. For high discount factors, the economy ultimately converges to a first-best allocation in which there may be transfers between groups, but labor supply decisions are not distorted and the levels of labor supply and consumption do not fluctuate over time. When discount factors are low, the economy converges to an invariant stochastic distribution in which distortions do not disappear and labor supply and consumption levels fluctuate over time. In these allocations with distortions, the labor supply of individuals from groups that are not in power are taxed, while the labor supply of those in power is subsidized. The subsidies are useful to relax the political economy/sustainability constraints.

We also show that the set of sustainable first-best allocations for high enough discount factors are independent of the Markov process for power change. This result contradicts a common conjecture that there will be fewer distortions when the political system creates a "stable ruling group". The reason why this conjecture is incorrect is that social groups can be rewarded not only when they hold power, but also when they engage in production. Consequently, the probability of power switches do not directly affect "effective discount factors". Nevertheless, it remains true that distortions decrease along sample paths where a particular group remains in power for a longer span of time. Finally, we demonstrate that the constrained efficient allocation can only be decentralized using distortionary taxes (even when the political system has access to lump-sum taxes), so that the results about fluctuations of distortions, consumption and labor supply levels correspond to fluctuations in taxes and redistribution.

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1 Introduction

What determines the variation of taxes over time and across societies is a central question of public finance. The traditional approach in public finance posits the existence of a benevolent (welfare-maximizing) social planner with full commitment power and derives differences in taxes as a result of differences or changes in parameters and shocks (e.g., Ramsey, 1927, Barro, 1979, Diamond, 1975, Diamond and Mirrlees, 1971a,b, Lucas and Stokey, 1983, Samuelson, 1986). In practice, however, taxes, like other policies, are decided by self-interested politicians responding to political incentives rather than simply maximizing aggregate welfare. This has motivated the political economy approach to public finance, which attempts to understand differences in the structure of taxes and policies across societies as a result of differences in the structure of political institutions and the extent of social conflict (see, among others, Persson and Tabellini, 2000, Persson, Roland and Tabellini, 1997, 2000, Besley and Coate, 1997, 1998, Baron, 1998, Grossman and Helpman, 1994, 1996, Dixit and Londergan, 1995, Bueno de Mesquita et al., 2003).

In this paper, we investigate the level and dynamics of taxes and distortions in a production economy, where political power fluctuates between distinct social groups, with different preferences. These groups may correspond to social classes with different incomes or to citizens living in different regions. The Markov process for power fluctuation is taken as given. The group that is in power determines the allocation of resources, including how much labor is supplied by different agents in the economy. There are two basic constraints. First, the participation constraint of each individual has to be satisfied. Second, the "sustainability constraint" of the group in power has to be met so that this group does not expropriate the available resources.

More specifically, we consider an infinite-horizon economy with N groups, each consisting of a large number of individuals. At each date, one group is in power. All individuals, including those that belong to the group in power, decide how much labor to supply. Then the group in power allocates total output among individuals as a function of the group-identity of the individual and the amount of labor he (or she) has supplied. At the end of the period, nature determines the identity of the group that will hold power at the next date.

We characterize the (constrained) Pareto efficient allocations in this economy. Pareto efficient allocations correspond to those where no individual (or group) can be made better off without any others being made worse off. These allocations can be identified as the solution to an optimization problem subject to the participation and sustainability constraints, with different Pareto weights given to the utilities of different groups.

Our focus on (constrained) Pareto efficient allocations is motivated by our interest in studying the impact of political conflict among different groups on the allocation of resources and taxes.[1] Pareto efficiency does not necessarily ensure "first best" allocations, because of the presence of the *sustainability* constraints (which, in our model, represent the constraints resulting from political economy). First-best allocations involve no distortions and full consumption smoothing, meaning that each individual supplies the same amount of labor and receives the same level of consumption at every date (irrespective of which group is in power). First-best allocations are not always *sustainable*, because the group in power could prefer to deviate from a first-best allocation. In this case, Pareto efficient allocations involve distortions (in the sense that marginal utility of consumption and disutility of labor are not equalized) and consumption and labor fluctuate over time.

The main results of our analysis are as follows:

1. Pareto efficient allocations have a quasi-Markovian structure and can be characterized recursively, conditional on the identity of the group that is in power and Pareto weights. Dynamics are determined by updating the Pareto weights recursively.

2. We show that as long as a first-best allocation is not sustainable (at the current date), the labor supply (and production) of individuals who belong to groups that are not in power will be distorted *downwards*. This results from the fundamental political economy considerations (i.e., from the *sustainability constraints*); an increase in production raises the amount that the group in power can allocate to itself for consumption rather than allocating it among the entire population. By reducing the amount of production, the political economy constraints are relaxed and thus the group in power receives fewer

[1] The alternative would have been to focus on Pareto dominated equilibria that may emerge either in our game or in some related institutional setting. While Pareto dominated allocations will naturally induce further distortions relative to the allocations we characterize, these distortions result not from political economy interactions per se, but from the additional restrictions leading to Pareto dominated outcomes.

Put differently, we are implicitly assuming that specific institutional structures to prevent Pareto dominated solutions will emerge and will be implemented. Instead, our focus is on the effect of political economy constraints that determine the location and shape of the Pareto frontier, and on the conflict between different groups, which corresponds to moves along this Pareto frontier.

rents. Starting from an undistorted allocation, the gain to society from reducing production (and thus rents to the ruling group) is first order, while the loss is second-order. Consequently, away from first-best allocations, constrained Pareto efficiency requires distortions and underproduction as a way of relaxing the political economy/sustainability constraints.

3. In contrast, the labor supply and production of the individuals in the group in power are subsidized—i.e., they are distorted *upwards*. This is because, as long as leisure is a normal good, subsidizing labor supply increases utility more when the group in power pursues the Pareto efficient allocation than when it deviates from this allocation. Distortionary subsidies to the group in power thus emerge as a way of relaxing the political economy constraints.

4. When discount factors are below some level $\bar{\beta} < 1$, no first best allocation is sustainable and distortions always remain, even asymptotically. In particular, we show that in this case all Pareto allocations converge to an invariant *non-degenerate* distribution of consumption and leisure across groups, whereby distortions as well as the levels of consumption and labor supply for each group asymptotically fluctuate according to an invariant distribution.

5. For the case in which there are 2 groups, we also show that there exists a level of the discount factor $\hat{\beta} < 1$, such that when the discount factor of the two groups are greater than $\hat{\beta}$, then any Pareto efficient allocation path (meaning an efficient allocation starting with any Pareto weights) eventually reaches a first-best allocation, and both distortions and fluctuations in consumption and labor supply disappear.

6. Irrespective of whether first-best allocations are sustainable or not, distortions decrease when the group remains in power for longer. This is because the Pareto weight of the group in power increases the longer it remains in power, and when the group in power has a greater Pareto weight, there will typically be fewer distortions.

We also show that the same results obtain if we consider a game form in which the group in power sets lump-sum and linear (and also potentially non-linear) group-specific taxes and transfers. This game form can also be thought as as a way of decentralizing the Pareto efficient allocation described above. The interesting result here is that all Pareto efficient allocations can be decentralized by linear taxes, and when first-best allocations are not sustainable, taxes on labor supply can never become zero. The interesting implication here is that (when the first-best allocations are not sustainable) the decentralization of the Pareto efficient allocations involves distortionary linear taxes, even though the political system has access to non-distortionary, lump-sum taxes. This result is a direct consequence of the fact that, in this case, the Pareto efficient allocations involve lower levels of labor supply and production than an undistorted first-best allocation, because high levels of productions make it more attractive for the group in power to deviate and allocate more of the output to itself. Since each member of a social group is small, he does not take into account the contribution of his own production to the level of overall output. Distortionary taxes then play a useful role by regulating the level of production in the economy and reducing it to the Pareto efficient level.

Though stylized, this environment enables us to discuss a central issue of political economy; whether a more stable distribution of political power—as opposed to frequent power switches between groups—leads to "better public policies," meaning policies involving lower distortions and generating greater total output. A natural conjecture is that a stable distribution of political power should be preferable because it serves to increase the "effective discount factor" of the group in power, thus making "cooperation" easier. This conjecture receives support from a number of previous political economy analyses. For example, Olson (1993) and McGuire and Olson (1996) contrast an all-encompassing long-lived dictator to a "roving bandit" and conclude that the former will lead to better public policies than the latter. The standard principal-agent models of political economy, such as Barro (1972), Ferejohn (1986), Persson, Roland and Tabellini (1997, 2000), also reach the same conclusion, because it is easier to provide incentives to a politician who is more likely to remain in power.[2]

Our analysis shows that this conjecture is not necessarily correct. The conjecture is based on the presumption that incentives can be given to groups in power only when they remain in power. Once a politician or a social group leaves power, it can no longer be punished or rewarded for past actions. This naturally leads to the result that there is a direct link between the effective discount factor of a political agent and his likelihood of staying in power. This

[2]Interestingly, in practice, countries with the longest-lived political dynasties, such as the Belgian Congo under Mobutu or Haiti under the Duvalier's appear to have disastrous performances.

presumption is not necessarily warranted, however, when we are dealing with social groups. Members of a social group can be rewarded or punished not only when they are in power, but also *after they have left power*. Our model illustrates this in a sharp way; we show that the set of first-best sustainable allocations is *independent* of the Markov process for power switches. The reason for this result is that first-best allocations do not involve fluctuations in consumption and labor supply, thus when they are sustainable, they are independent of the Markov process for power switches. In our baseline economy, deviation from an implicitly-agreed allocation triggers a switch to the worst subgame perfect equilibrium, which involves zero labor supply and zero consumption for all groups. This implies that deviation payoffs are also independent of the Markov process. Consequently, the set of first-best allocations that are sustainable does not depend on the Markov process for power switches. This result implies that if first best is sustainable under the dictatorship of a specific group, it is also sustainable when power fluctuates between different groups.[3]

Our main analysis follows the standard repeated games literature and focuses on Pareto efficient allocations supported by the most severe punishments following a deviation from the implicitly-agreed path. One disadvantage of these punishments is that they are not "renegotiation proof," meaning that the allocations during the punishment phase are Pareto dominated. We show that our main results do not depend on the use of these most severe punishments. Instead, using movements along the Pareto frontier as a way of punishing groups that have deviated leads to the same results, 1-6, highlighted above. Interestingly, in this case, the set of sustainable first-best allocations is no longer independent of the Markov process, because the deviation payoffs of different groups depend on the Markov process. However, contrary to the conjecture discussed above, in this case the set of sustainable first-best allocations is typically maximized when there are *frequent power switches* between different groups. The reason is that when a particular group is likely to stay in power in subsequent periods, then it perceives deviation as a more attractive option because after deviation, it can still obtain relatively high returns as it is likely to remain in power. Consequently, a Markov process that makes it very likely that the group in power today will not be in power tomorrow implies more

[3] That the set of first-best sustainable allocations does not depend on the Markov process for power switches does not contradict result 5 above; that result refers to the realizations of a given sample path, while the discussion here is on the effect of the Markov process for power switches, thus on the effect of "expected" stability of political power. While the Markov process has no effect on the set of first-best sustainable allocations, when a particular group happens to stay in power longer, distortions decrease over time.

severe punishments against deviations and makes it more likely that first-best allocations are sustainable.

Our paper is related to the large and growing political economy literature already mentioned above. The distinguishing feature of our approach relative to this literature is that we emphasize dynamic political economy interactions and make relatively few assumptions about the specific institutional details that govern political decision-making. Our approach instead emphasizes the impact of the fundamental political economy constraints—that is, the fact that the level of production determines the incentives of the group in power.

A number of recent works also look at dynamic political economy issues. These include, among others, Acemoglu and Robinson (2001, 2006a), Battaglini and Coate (2006), Hassler et al. (2003), Krusell and Rios-Rull (1996, 2000), Lagunoff (2005, 2006), Roberts (1999) and Sonin (2003). The major difference of our paper from this literature is our focus on Pareto efficient allocations rather than Markov perfect equilibria. Almost all of the results in the paper are the result of this focus (since Markovian equilibria will involve zero production in this economy). Our focus on Pareto efficient allocations is also essential in enabling us to abstract from the specific political game forms or institutional details that underlie the political economy interactions.

In this respect, our work is related to previous analyses of constrained efficient allocations in political economy models or in models with limited commitment. These include, among others, the limited-commitment risk sharing models of Thomas and Worrall (1990) and Kocherlakota (1996) and the political economy models of Dixit, Grossman and Gul (2000) and Amador (2003a,b). The main difference between our paper and these works is the presence of production and labor supply in our setup. This makes the size of the pie to be redistributed between different groups endogenous and taxation an interesting margin of analysis. A number of key results of our framework are related to the presence of production; these include the dynamics of distortions and potentially slow-convergence to first-best allocations.[4]

In addition, to the best of our knowledge, no existing work has systematically analyzed the impact of the Markov process for power switches on the set of Pareto efficient allocations.[5]

[4] Acemoglu, Golosov and Tsyvinski (2006a, 2006b) and Yared (2006) also consider dynamic political economy models with production, but their models do not feature power switches between different social groups.

[5] Acemoglu and Robinson (2006b) and Robinson (2001) also question the insight that long-lived all-encompassing regimes are growth-promoting. They emphasize the possibility that such regimes may block beneficial technological or institutional changes in order to maintain their political power.

The rest of the paper is organized as follows. Section 2 introduces the basic environment and characterizes the first-best allocations. Section 3 describes the basic political economy game and characterizes the level and dynamics of distortions. Section 4 provides a complete characterization of the dynamics of distortions in the case with two parties. Section 5 shows how the constrained Pareto efficient allocations can be decentralized and links our results to fluctuations in taxes. Section 6 shows that the Markov process for power switches does not affect the set of sustainable first-best allocations. Section 7 provides an analysis of our basic political economy environment when punishments following deviations need to be renegotiation proof. Section 8 concludes, while the Appendix contains a number of technical details and proofs omitted from the text.

2 Environment and Benchmark

In this section, we introduce the basic economic environment.

2.1 Demographics, Preferences and Technology

We consider an infinite horizon economy in discrete time with a unique final good. There is a continuum of citizens, each belonging to one of N groups, $S_1, ..., S_N$. The size of group j is normalized to 1 without loss of any generality. Individual $i \in S_j$ (in group j) has utility at time $t = 0$ given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u_j(c_t^i, l_t^i), \tag{1}$$

where c_t^i is consumption, l_t^i is labor supply (or other types of productive effort), and $\mathbb{E}_0$ denotes the expectations operator at time $t = 0$. To simplify the analysis without loss of any economic insights, we assume that labor supply belongs to the closed interval $\left[0, \bar{l}\right]$ for each individual. We also impose the following assumption on utility functions.

Assumption 1 ***(utility function)*** *The instantaneous utility function*

$$u_j : \mathbb{R}_+ \times \left[0, \bar{l}\right] \to \mathbb{R},$$

for $j = 1, ..., N$ is uniformly continuous, twice continuously differentiable in the interior of its domain, strictly increasing in c, strictly decreasing in l and jointly strictly concave in c and l, *would* $u_j(0,0) = 0$ *and satisfies the following Inada conditions:*

$$\lim_{c \to 0} \frac{\partial u_j(c,l)}{\partial c} = \infty \textit{ and } \lim_{c \to \infty} \frac{\partial u_j(c,l)}{\partial c} = 0 \textit{ for all } l \in \left[0, \bar{l}\right],$$

$$\frac{\partial u_j(c,0)}{\partial l} = 0 \textit{ and } \lim_{l \to \bar{l}} \frac{\partial u_j(c,l)}{\partial l} = -\infty \textit{ for all } c \in \mathbb{R}_+.$$

The differentiability assumptions enable us to work with first-order conditions. The Inada conditions make sure that consumption and labor supply levels are not at corners, enabling us to simplify the notation for the first-order necessary conditions below. The concavity assumptions are also standard, but important for our results, since they create a desire for consumption and labor supply smoothing over time.

The economy also has access to a linear aggregate production function given by

$$\begin{aligned} Y_t &= L_t \\ &= \sum_{j=1}^{N} \int_{i \in S_j} l_{j,t}^i di, \end{aligned} \tag{2}$$

where the second line defines the aggregate labor supply L_t at time t as the sum of individual labor supplies, whereby $l_{j,t}^i$ denotes the labor supply of individual i from group j at time t.

2.2 Efficient Allocation without Political Economy

As a benchmark, let us start with the efficient allocation without political economy constraints. This is simply an allocation that maximizes a weighted average of different groups' utilities, with *Pareto weights* vector denoted by $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_N)$ where $\alpha_j \geq 0$ for $j = 1, ..., N$ denotes the weight given to all agents in group j, and we adopt the normalization $\sum_{j=1}^{N} \alpha_j = 1$. Given concavity of each u_j, it is clear that all individuals in a given group will be treated symmetrically, so the program for the (unconstrained) efficient allocation can be written as:

$$\max_{\left\{[c_{j,t}, l_{jt}]_{j=1}^{N}\right\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{N} \alpha_j u_j(c_{j,t}, l_{j,t}) \right] \tag{3}$$

subject to the resource constraint

$$\sum_{j=1}^{N} c_{j,t} \leq \sum_{j=1}^{N} l_{j,t} \text{ for all } t. \tag{4}$$

Standard arguments immediately establish the following result:

Theorem 1 *Suppose Assumptions 1 and 2 hold. Then without political economy constraints, the efficient allocation, i.e., the solution to maximizing (3) subject to (4), satisfies the following conditions:*

$$\textit{no distortions: } \frac{\partial u_j(c_{j,t}^{fb}, l_{j,t}^{fb})}{\partial c} = -\frac{\partial u_j(c_{j,t}^{fb}, l_{j,t}^{fb})}{\partial l} \textit{ for } j = 1, ..., N \textit{ and all } t, \tag{5}$$

$$\textit{perfect smoothing: } c_{j,t}^{fb} = c_j^{fb} \textit{ and } l_{j,t}^{fb} = l_j^{fb} \textit{ for } j = 1, ..., N \textit{ and all } t. \tag{6}$$

Proof. The result follows from the first-order conditions, which are necessary in view of the differentiability assumptions and the boundary conditions imposed by the Inada assumptions imposed in Assumption 1. Assigning Lagrange multiplier $\beta^t \zeta_t$ to (4) at time t, we have the following first-order conditions

$$\alpha_j \frac{\partial u_j(c_{j,t}^{fb}, l_{j,t}^{fb})}{\partial c} = \zeta_t \text{ and } -\alpha_j \frac{\partial u_j(c_{j,t}^{fb}, l_{j,t}^{fb})}{\partial c} = \zeta_t$$

for $j = 1, ..., N$ and all t. Combining the two conditions gives (5). Moreover, the determination of date t variables is independent of the determination of the variables and all past and future dates. This combined with the strict concavity of u_j's implies (6). ■

We refer to an efficient allocation without political economy as a *first-best allocation*. The structure of the first best allocations is standard: efficiency requires the marginal benefit from additional consumption to be equal to the marginal cost of labor supply for each individual, and also requires perfect consumption and labor supply smoothing.

Note, however, that different groups can be treated differently in the first-best allocation. Depending on the Pareto weight vector $\boldsymbol{\alpha}$, a particular group may supply a high level of labor and receive only limited consumption.

3 Political Economy

3.1 Basics

We now consider a political environment in which political power fluctuates between the N groups $j \in \mathcal{N} \equiv \{1, ..., N\}$. The game form in this political environment is as follows.

1. In each period t, we start with one group, j', in power.

2. All individuals $i \in S_j$ for $j = 1, ..., N$ make their labor supply decisions $l^i_{j,t}$, and output $Y_t = L_t = \sum_{j=1}^N \int l^i_{j,t} di$ is produced.

3. One of the agents in $S_{j'}$ is randomly chosen as the dictator and chooses N functions $C_{j,t} : [0, \bar{l}] \to \mathbb{R}_+$ subject to the feasibility constraint

$$\sum_{j=1}^{N} \int C_{j,t} \left(l^i_{j,t}\right) di \leq \sum_{j=1}^{N} \int l^i_{j,t} di, \tag{7}$$

   whereby $C_{j,t}(l)$ is the consumption level of an individual from group j that has supplied labor l.

4. A first-order Markov process m determines who will be in power in the next period, such that the probability of group j being in power following group j' is $m(j \mid j')$, with $\sum_{j=1}^N m(j \mid j') = 1$ for all $j' \in \mathcal{N}$.

A number features are worth noting. First, this game form captures the notion that political power fluctuates between groups. Second, it builds in the assumption that the allocation of resources are distributed by the group in power (without any prior commitment to what the allocation will be). This is modeled by the choice of the consumption functions $C_{j,t} : [0, \bar{l}] \to \mathbb{R}_+$. Third, rather than dealing with issues of group decision-making, one random individual from the group in power is designated as the dictator and makes the allocation decisions. Note that individuals choose their labor supplies before knowing who will be the dictator, and since there is a continuum of individuals within each group, each individual assigns zero probability to this event. It is also important that allocation rules (the consumption functions) cannot be conditioned on the identity of the individual. This implies that the randomly chosen dictator cannot treat himself differently than from other group members (as long as he has exerted the same level of labor supply as other individuals).

In addition, we impose the following assumption on the Markov process for power switches.

Assumption 2 ***(Markov process)*** *The first-order Markov process $m(j \mid j')$ is irreducible, aperiodic and ergodic.*

We are interested in *subgame perfect equilibria* of this infinitely-repeated game, with preference as given in (1). More specifically, as discussed in the Introduction, we will look at subgame perfect equilibria that correspond to constrained Pareto efficient allocations, which we refer to as *Pareto efficient perfect equilibria*.[6]

To define these equilibria, we now introduce some notation. Let $h^t = (h_0, ..., h_t)$, with $h_s \in \mathcal{N}$ be the history of power holdings (i.e., the history of power holdings in the past). Let $\mathbf{L}^t = \left(\{L_{j,0}\}_{j=1}^N, ..., \{L_{j,t}\}_{j=1}^N \right)$ be the history of labor supplies, with $L_{j,s} : S_j \to [0, \bar{l}]$ denoting the cross-sectional distribution of labor supply by individuals in group j at time t. Let $\mathcal{C}$ be the set of functions from $[0, \bar{l}]$ into $\mathbb{R}_+$ and let $\mathbf{C}^t = \left(\{C_{j,0}\}_{j=1}^N, ..., \{C_{j,t}\}_{j=1}^N \right)$ with $C_{j,s} \in \mathcal{C}$ be the history of allocation rules and let $\mathcal{C}^N$ be the N-fold product of $\mathcal{C}$.[7] A *full history* of this game at time t is

$$\omega^t = \left(h^t, \mathbf{C}^{t-1}, \mathbf{L}^{t-1} \right),$$

which describes the exact history of power holdings, all labor supply decisions and all allocation rules chosen by groups in power. Let the set of all potential date t histories be denoted by Ω^t. In addition, denote an intermediate-stage full history by

$$\hat{\omega}^t = \left(h^t, \mathbf{C}^{t-1}, \mathbf{L}^t \right),$$

and denote the set of intermediate-stage full histories by $\hat{\Omega}^t$. The difference between ω and $\hat{\omega}$ lies in the fact that the former does not contain information on labor supplies at time t, while the latter does. The latter history will be relevant at the intermediate stage where the individual in power chooses the allocation rule.

We can now define strategies as follows. First define the following sequence of mappings $\hat{l} = \left(\hat{l}^0, \hat{l}^1, ..., \hat{l}^t, ... \right)$ and $\hat{C} = \left(\hat{C}^0, \hat{C}^1, ..., \hat{C}^t, ... \right)$, where

$$\hat{l}^t : \Omega^t \to [0, \bar{l}]$$

determines the level of labor an individual will supply for every given history $\omega^t \in \Omega^t$, and

$$\hat{C}^t : \hat{\Omega}^t \to \mathcal{C}^N$$

[6] Throughout, by "Pareto efficient," we mean "constrained Pareto efficient," but we drop the adjective "constrained" to simplify the terminology.

[7] Naturally not all elements of $\mathcal{C}^N$ are feasible at time t. To avoid further notation, we do not specify the subset of $\mathcal{C}^N$ that is feasible.

a sequence of allocation rules, which the individual would choose, if he were in power, for every given intermediate-stage history $\hat{\omega}^t \in \hat{\Omega}^t$, such that $\hat{C}$ satisfies the feasibility constraint (7). A date t strategy for individual is $\sigma_{i,j}^t = \left(\hat{l}^t, \hat{C}^t\right)$ for $i \in S_j$ and $j = 1, ..., N$. Denote the set of date t strategies by Σ^t. A strategy for individual i is $\sigma_i = \left(\left\{\sigma_i^t\right\} : t = 0, 1, ...\right)$ and the set of strategies is denoted by Σ. Denote the expected utility of individual i from group j at time t as a function of his own and others and others' strategies given history ω^t and intermediate-stage history $\hat{\omega}^t$ by

$$U_j\left(\sigma_i, \sigma_{-i} \mid \omega^t, \hat{\omega}^t\right).$$

This utility is conditioned on the group identity, since utility functions differ across groups and also the strategies of other players treat individuals differently depending on which group they belong to.

We next define various concepts of equilibria which we use throughout the paper.

Definition 1 *A* subgame perfect equilibrium (SPE) *is a collection of strategies* $\sigma^* = \left(\left[\sigma_{i,j}^t\right]_{i \in S_j} : j = 1, ..., N,\ t = 0, 1, ...\right)$ *such that* σ_i^* *is best response to* σ_{-i}^* *for all* $\left(\omega^t, \hat{\omega}^t\right) \in \Omega^t \times \hat{\Omega}^t$ *and for all* $i \in S_1 \times ... \times S_N$, *i.e.,* $U_j\left(\sigma_i^*, \sigma_{-i}^* \mid \omega^t, \hat{\omega}^t\right) \geq U_j\left(\sigma_i, \sigma_{-i}^* \mid \omega^t, \hat{\omega}^t\right)$ *for all* $\sigma_i \in \Sigma$, *for all* $\left(\omega^t, \hat{\omega}^t\right) \in \Omega^t \times \hat{\Omega}^t$, *for all* $t = 0, 1, ...$ *and for all* $i \in S_1 \times ... \times S_N$ *and* $j = 1, ..., N$.

Definition 2 *A* Pareto efficient perfect equilibrium *at time t (following history* ω^t*),* σ^{**}*, is a collection of strategies that form an SPE such that there does not exist another SPE* σ^{***}*, whereby* $U_j\left(\sigma_i^{***}, \sigma_{-i}^{***} \mid \omega^t, \hat{\omega}^t\right) \geq U_j\left(\sigma_i^{**}, \sigma_{-i}^{**} \mid \omega^t, \hat{\omega}^t\right)$ *for all* $\hat{\omega}^t \in \hat{\Omega}^t$ *and for all* $i \in S_1 \times ... \times S_N$ *and* $j = 1, ..., N$*, with at least one strict inequality.*

We will also refer to *Pareto efficient allocations* as the equilibrium-path allocations that result from a Pareto efficient perfect equilibrium.

Definition 3 *A* worst subgame perfect equilibrium *at time t (following history* ω^t*),* σ^W *is a collection of strategies that form an SPE such that there does not exist another SPE* σ^{***}*, whereby* $U_j\left(\sigma_i^{***}, \sigma_{-i}^{***} \mid \omega^t, \hat{\omega}^t\right) \leq U_j\left(\sigma_i^W, \sigma_{-i}^W \mid \omega^t, \hat{\omega}^t\right)$ *for all* $\hat{\omega}^t \in \hat{\Omega}^t$ *and for all* $i \in S_1 \times ... \times S_N$ *and* $j = 1, ..., N$*, with at least one strict inequality.*

3.2 Preliminary Results

The next lemma shows that the worst subgame perfect equilibrium involves all individuals supplying zero labor and receiving zero consumption.

Lemma 1 *Suppose Assumptions 1 and 2 hold. The worst SPE is given by the collection of strategies* σ^W *such that each* $\sigma^W_{i,j} = \sigma^\emptyset$ *for all* $i \in S_i$ *and for all* $j = 1, ..., N$, *where* $\sigma^\emptyset$ *involves* $\hat{l}^t\left(\omega^t\right) = 0$ *for all* $\omega^t \in \Omega^t$ *and* $\hat{C}$ *such that* $C_{j',t}(l) = 0$ *for all* $l \in \left[0, \bar{l}\right]$ *and for all* $j' \neq j$ *and* $C_{j,t}(l) = 0$ *for all* $l \in (0, \bar{l}]$ *and* $C_{j,t}(0) = Y_t$ *for all* $\hat{\omega}^t \in \hat{\Omega}^t$.

Proof. To see that $\sigma^\emptyset$ is a best response for each individual in all subgames when other individuals are playing $\sigma^\emptyset$, consider separately individuals belonging to the group in power and the rest. First, when the individual belongs to a group that is not in power at time t, given the allocation rules implied by $\hat{C}$ (that is part of $\sigma^\emptyset$), he will always receive zero consumption, and thus for individual i in group j that is not in power, we have $U_j\left(\sigma_i, \sigma^\emptyset_{-i} \mid \omega^t, \hat{\omega}^t\right) \leq 0$ for any σ_i, and for all $\omega^t, \hat{\omega}^t$, and all t. The strategy $\hat{l}^t\left(\omega^t\right) = 0$ is therefore a best response. Next, consider a group j that is in power and the individual chosen to be the decision maker for the group in power at time t. Since $l^i_{j,t} = 0$ for all $i \in S_j$ (implied by $\sigma^\emptyset$), $C_{j,t}(l) = 0$ for all $l \in (0, \bar{l}]$ and $C_{j,t}(0) = Y_t$ is a best response. Moreover, since each individual in group j has zero probability of coming to power when choosing their labor supply, $\hat{l}^t\left(\omega^t\right) = 0$ is also a best response, establishing that $\sigma^\emptyset$ is a best response against itself. To see that this is the worst SPE, note that $U_j\left(\sigma^\emptyset_i, \sigma^\emptyset_{-i} \mid \omega^t, \hat{\omega}^t\right) = 0$ for all $\omega^t, \hat{\omega}^t$, and all t and that given the assumption $u(0,0) = 0$, there cannot be any SPE in which individuals receive negative utility. ■

Next, let us partition the set of histories into $\Omega^t_1, ..., \Omega^t_N$ depending on whether group 1,...,N is in power at time t. The following proposition characterizes Pareto efficient perfect equilibria as the solutions to the maximization problem subject to a resource constraint, a set of participation constraints, and a *sustainability constraint* for the group in power. This sustainability constraint captures the political economy interactions in our model. Let us also write $c_j(\omega)$ and $l_j(\omega)$ to denote $\left\{c_j\left(\omega^t\right)\right\}_{t=0}^\infty$ and $\left\{l_j\left(\omega^t\right)\right\}_{t=0}^\infty$.

Proposition 1 *Suppose Assumptions 1 and 2 hold. Then, any Pareto efficient perfect equilibrium is a solution to the following maximization problem*

$$\max_{\{c_j(\omega), l_j(\omega)\}_{j=1}^N} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \left[\sum_{j=1}^N \alpha_j u_j(c_j\left(\omega^t\right), l_j\left(\omega^t\right))\right] \tag{8}$$

subject to

$$\sum_{j=1}^N c_j\left(\omega^t\right) \leq \sum_{j=1}^N l_j\left(\omega^t\right)) \tag{9}$$

for all $\omega^t \in \Omega^t$ *and for all* $t = 0, 1, ...,$

$$u_j(c_j\left(\omega^t\right), l_j\left(\omega^t\right)) \geq 0 \tag{10}$$

for all $j = 1, ..., N$*, for all* $\omega^t \in \Omega^t$ *and for all* $t = 0, 1, ...,$ *and*

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_j\left(c_j\left(\omega^{t+s}\right), l_j\left(\omega^{t+s}\right)\right) \geq u_j\left(\sum_{j'=1}^{N} l_{j'}\left(\omega^t\right) \;\;, l_j\left(\omega^t\right)\right) + \beta V_j^d\left(\omega^t\right) \tag{11}$$

for all $\omega^t \in \Omega_j^t$ *and for all* $t = 0, 1, ...,$ *for some Pareto weights vector* $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_N)$*, where* $V_j^d\left(\omega^t\right)$ *is the deviation continuation value of group* j *after history* ω^t*.*

Proof. The objective function (8) restricts allocations to symmetric ones, where all members of the group have the same amount of consumption and labor supply given in particular history. For the groups that are not in power, symmetric allocations are feasible and preferred by strict concavity. For the group in power, strict concavity again implies that symmetric allocations are preferred. If they are also also compatible with individual best responses, then the Pareto efficient allocation must be symmetric. To see that they are compatible with best response, note that by strict concavity and uniform continuity of u_j, we have that for ε sufficiently small, the following is true for any $n > 1$,

$$u_j\left(c, l\right) > \frac{1}{n} u_j\left(nc, l + \varepsilon\right) > \frac{1}{n} u_j\left(nc, l - \varepsilon\right).$$

Taking limits as $n \to \infty$ implies that deviating from the common level of labor supply, which would lead to zero consumption with probability 1, is never preferable. This establishes that any Pareto efficient allocation must be symmetric and we can restrict attention to functions $c_j\left(\omega^t\right), l_j\left(\omega^t\right)$.

Next, in any feasible allocation (9) must hold as the resource constraint, and (10) must hold to ensure individual participation constraints. To see the latter, note that, since each individual is infinitesimal, they would prefer $(l = 0, c = 0)$, which gives them utility $u_j\left(0, 0\right) = 0$ and can be achieved by supplying zero labor. Therefore, any feasible allocation must satisfy (10).

Condition (11) ensures that the candidate equilibrium gives sufficient utility to the group in power relative to allocating all consumption to itself and after that receiving the deviation continuation value $V_j^d\left(\omega^t\right)$. To see why this inequality is necessary, first note that, without loss of any generality, we can take the deviation continuation value, $V_j^d\left(\omega^t\right)$, to be independent of the amount by which the group in power deviates. Now suppose that condition (11) were not satisfied for given $V_j^d\left(\omega^t\right)$. Then given symmetry, individual i chosen to make the decision for group j that is in power following history ω^t can allocate all of the output, $\sum_{j'=1}^{N} l_{j'}\left(\omega^t\right)$, as consumption to group j members with labor supply $l_j\left(\omega^t\right)$ (i.e., choose $\hat{C}$ such that $C_{j',t}\left(l\right)=0$ for all $l\in\left[0,\bar{l}\right]$ and for all $j'\neq j$ and $C_{j,t}\left(l\right)=0$ for all $l\neq l_j\left(\omega^t\right)$ and $C_{j,t}\left(l_j\left(\omega^t\right)\right)=\sum_{j'=1}^{N} l_{j'}\left(\omega^t\right)$). This yields utility equal to the right hand side of (11), while utility in the candidate equilibrium-path is given by the left-hand side. Therefore, the candidate allocation can be an equilibrium if and only if (11) is satisfied.

Finally, if (8) is not maximized for some $\boldsymbol{\alpha}$ subject to (9) and (11), it means that there exists another collection of functions $\left\{c_j\left(\omega\right), l_j\left(\omega\right)\right\}_{j=1}^{N}$ that satisfy (9) and (11), and yield a higher time 0 utility for one group without reducing the utility of others, thus the equilibrium cannot be Pareto efficient. ■

Let us next separate a strategy σ into two parts, *equilibrium-path component*, σ^E, and *off-the-equilibrium-path component*, σ^O, and write $\sigma=\left\langle\sigma^E,\sigma^O\right\rangle$. Loosely speaking, σ^E applies along histories where there is no deviation from in implicitly-agreed action plan for each individual and σ^O applies following a deviation, thus determines $V_j^d\left(\omega^t\right)$. The following proposition shows that any Pareto efficient perfect equilibrium can be supported by using the worst subgame perfect equilibrium as the punishment for deviating from the "implicitly-agreed" action profile (for similar results, see, e.g., Abreu, 1988, on repeated games, or Abreu, Pierce and Stacchetti, 1990, on dynamic games with imperfect monitoring)

Proposition 2 *Suppose Assumptions 1 and 2 hold. Then any Pareto efficient perfect equilibrium can be supported by a strategy of the form* $\sigma^{**}=\left\langle\sigma^E,\sigma^{\emptyset}\right\rangle$ *and* $V_j^d\left(\omega^t\right)=0$ *for all* $j=1,...,N$ *and for all* $\omega^t\in\Omega^t$.

Proof. Lemma 1 established that $\sigma^{\emptyset}$ is the worst subgame perfect equilibrium. Suppose the continuation strategy following any subgame involving a deviation from the implicitly-agreed path of actions is $\sigma^{\emptyset}$. This would imply that $V_j^d\left(\omega^t\right)=0$, which is the minimum feasible value of the deviation continuation value. Therefore, the play of $\sigma^{\emptyset}$ off-the-equilibrium-path necessarily relaxes the set of constraints in (11) the maximization problem (8). ■

This proposition implies that in order to characterize the entire set of Pareto efficient perfect equilibria, we can restrict attention to strategies that follow a particular prescribed equilibrium play ("implicitly-agreed" plan), with the punishment phase given by $\sigma^{\emptyset}$.

Propositions 1 and 2 together enable us to reduce the determination of the Pareto efficient equilibrium to a much simpler maximization problem. To do this, recall that h^t is the history of power holdings up to and including time t, so $h^t \in H^t \equiv \mathcal{N}^t$, where recall that $\mathcal{N} \equiv \{1, ..., N\}$. From now on, we will refer to h^t (rather than ω^t) as history to simplify notation. Let us write $j = \mathbf{j}\left(h^t\right)$ if group j is in power at time t according to history h^t, and also use the notation $h^t \in H_j^t$ whenever $j = \mathbf{j}\left(h^t\right)$. Finally, let h^0 be the initial history, with one of the groups as the one designated to be in power.

Now consider the mappings

$$\begin{aligned} c_j &: H^t \to \mathbb{R}_+ \text{ for } j = 1, ..., N \\ l_j &: H^t \to \mathbb{R}_+ \text{ for } j = 1, ..., N \end{aligned}$$

which determine consumption and labor supply levels for each group as a function of history up to and including time t. The next proposition shows that the Pareto efficient allocation can be described in terms of these simpler mappings rather than those used in Propositions 1 and 2. In particular, instead of considering mappings from entire full histories, it is sufficient to condition on the history of power holdings. Notice, however, that this proposition applies to Pareto efficient allocations, *not* to the strategies that individuals use in order to support these allocations. These strategies *must be* conditioned on information that is not contained in the history of power holdings, h^t, since individuals need to switch to the worst subgame perfect equilibrium in case there is any deviation from the implicitly-agreed action profile. This information is naturally contained in ω^t. Therefore, to describe the subgame perfect equilibrium strategies we need to condition on the full histories ω^t. Nevertheless, for our purposes, which is to characterize the set of Pareto efficient allocations and their dynamics, it is more convenient to work with the much smaller history of power holdings, $h^t \in H^t$.

Proposition 3 *Suppose Assumptions 1 and 2 hold. Then, any Pareto efficient allocation is a solution to the following maximization problem*

$$\max_{\{c_j(h), l_j(h)\}_{j=1}^N} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{N} \alpha_j u_j(c_j\left(h^t\right), l_j\left(h^t\right)) \right] \tag{12}$$

subject to

$$\sum_{j=1}^{N} c_j\left(h^t\right) \leq \sum_{j=1}^{N} l_j\left(h^t\right)) \text{ for all } h^t \in H^t \text{ and all } t = 0, 1, ..., \tag{13}$$

$$u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right) \geq 0 \text{ for all } h^t \in H^t, \text{ all } j = 1, ..., N, \text{ and all } t = 0, 1, ..., \tag{14}$$

and

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_j\left(c_j\left(h^{t+s}\right), l_j\left(h^{t+s}\right)\right) \geq u_j\left(\sum_{j'=1}^{N} l_{j'}\left(h^t\right)\ , l_j\left(h^t\right)\right) \text{ for all } h^t \in H_j^t \text{ and all } t = 0, 1, ... \tag{15}$$

for some Pareto weights vector $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_N)$.

Proof. The constraint (15) is the same as (11), with the deviation continuation value set equal to 0 following Proposition 2, and allocations only conditioned on the history of power holdings. As long as this constraint holds, the group in power prefers not to deviate. Then there is no need to keep track of aspects of the full history other than the history of power holdings. In addition, strict concavity of u_j's imply that in any constrained Pareto efficient allocation all individuals within a group have to be treated the same. Given these observations, the result follows from Proposition 2. ■

The maximization (12) subject to (13), (14) and (15) is a potentially non-convex optimization problem, because (15) defines a non-convex constraint set. This implies that randomizations may improve the value of the program (see, for example, Prescott and Townsend, 1984a,b). Randomizations can be allowed by either considering correlated equilibria rather than subgame perfect equilibria, or alternatively, by assuming that there is a commonly observed randomization device on which all individuals can coordinate their actions. In the Appendix, we will formulate an extended problem by introducing a commonly-observed, independently and identically distributed random variable, which all individual strategies can be conditioned upon. We will show that this does not change the basic structure of the problem and in fact there will be randomizations over at most two points at any date, and the history of past randomizations will not play any role in the characterization of Pareto efficient allocations. Since introducing randomizations complicates the notation considerably, in the text we do not consider randomizations (thus implicitly assuming that the problem is convex for the relevant parameters). The equivalents of the main results are stated in the Appendix for the case with randomizations.

We next present our main characterization result, which shows that the solution to the maximization problem in Proposition 3 can be represented recursively.

3.3 Recursive Characterization

Let us first define $M\left(h^{t+s} \mid h^{t}\right)$ be the (conditional) probability of history h^{t+s} at time $t+s$ given history h^{t} at time according to the Markov process $m\left(j \mid j'\right)$. Moreover, define $P\left(h^{t}\right)$ be the set of all possible date $t+s$ histories for $s \geq 1$ that can follow history h^{t}. We write $M\left(h^{t+s} \mid h^{\varnothing}\right)$ for the unconditional probability of history h^{t+s}.

First note that the maximization problem in Proposition 3 can be written in Lagrangian form as follows:

$$
\begin{aligned}
\max_{\left\{c_j(h), l_j(h)\right\}_{j=1}^{N}} \quad \mathcal{L}' = & \sum_{t=0}^{\infty} \beta^{t} M\left(h^{t} \mid h^{\varnothing}\right)\left[\sum_{j=1}^{N} \alpha_j u_j\left(c_j\left(h^{t}\right), l_j\left(h^{t}\right)\right)\right] \\
& + \sum_{t=0}^{\infty} \beta^{t} M\left(h^{t} \mid h^{\varnothing}\right) \lambda_j\left(h^{t}\right) \times \\
& \left[\sum_{s=t}^{\infty} \beta^{s-t} M\left(h^{s} \mid h^{t}\right) u_j\left(c_j\left(h^{s}\right), l_j\left(h^{s}\right)\right) - u_j\left(\sum_{i=1}^{N} l_i\left(h^{s}\right), l_j\left(h^{s}\right)\right)\right]
\end{aligned} \tag{16}
$$

subject to (13) and (14), where $\beta^{t} M\left(h^{t} \mid h^{0}\right) \lambda_j\left(h^{t}\right)$ is the Lagrange multiplier on the sustainability constraint, (15), for group j for history h^{t}. The restriction that $h^{t} \in P\left(h^{t-1}\right)$ is implicit in this expression.

The proof of Theorem 2 establishes that after history history h^{t-1}, this Lagrangian is equivalent to

$$
\begin{aligned}
\max_{\left\{c_j(h), l_j(h)\right\}_{j=1}^{N}} \quad \mathcal{L} = & \sum_{s=t}^{\infty} \beta^{s} M\left(h^{s} \mid h^{t}\right)\left[\sum_{j=1}^{N}\left(\alpha_j + \mu_j\left(h^{t-1}\right)\right) u_j\left(c_j\left(h^{s}\right), l_j\left(h^{s}\right)\right)\right] \\
& + \sum_{s=0}^{\infty} \beta^{s} M\left(h^{s} \mid h^{t}\right) \times \\
& \sum_{j=1}^{N} \lambda_j\left(h^{s}\right)\left(\sum_{s'=s}^{\infty} \beta^{s'-s} M\left(h^{s'} \mid h^{s}\right) u_j\left(c_j\left(h^{s'}\right), l_j\left(h^{s'}\right)\right) - u_j\left(\sum_{i=1}^{N} l_i\left(h^{s}\right), l_j\left(h^{s}\right)\right)\right),
\end{aligned} \tag{17}
$$

subject to (13) and (14), with μ_j's defined recursively as:

$$
\mu_j\left(h^{t}\right) = \mu_j\left(h^{t-1}\right) + \lambda_j\left(h^{t}\right)
$$

with the normalization $\mu_j\left(h^{\varnothing}\right) = 0$ for all $j \in \mathcal{N}$.

The most important implication of the formulation in (17) is that for any $h^t \in P\left(h^{t-1}\right)$, the numbers

$$\alpha_j\left(h^{t-1}\right) \equiv \frac{\alpha_j + \mu_j\left(h^{t-1}\right)}{\sum_{j'=1}^{N}\left(\alpha_{j'} + \mu_{j'}\left(h^{t-1}\right)\right)} \tag{18}$$

can be interpreted as *updated Pareto weights*, so that after history h^{t-1}, the problem is equivalent to maximizing the sum of utilities with these weights (subject to the relevant constraints). Therefore, the problem of maximizing (17) is equivalent to choosing current consumption and labor supply levels for each group and also updated Pareto weights $\{\alpha_j\}_{j=1}^N$.

In addition, (17) has the attractive feature that $\mu_j\left(h^t\right) - \mu_j\left(h^{t-1}\right) = 0$ whenever $j \neq \mathbf{j}\left(h^t\right)$, i.e., whenever group j is not in power. This is because there is no sustainability constraint for a group that is not in power. This also implies that in what follows, we can drop the subscript j and refer to $\lambda\left(h^t\right)$ rather than $\lambda_j\left(h^t\right)$, since the information on which group is in power is already incorporated in h^t.

This analysis establishes the following characterization result:

Theorem 2 *Suppose Assumptions 1 and 2 hold. Then the constrained efficient allocation has a quasi-Markovian structure whereby* $\{c_j(h), l_j(h)\}_{j=1}^N$ *depend only on* $\mathbf{s} \equiv \left(\{\alpha_j(h)\}_{j=1}^N, \mathbf{j}(h)\right)$, *i.e., only on updated weights and the identity of the group in power.*

Proof. The proof of this theorem builds on the representation suggested by Marcet and Marimon (1998). In particular, observe that for any $T \geq 0$, we have

$$\begin{aligned} & \sum_{s=0}^{T} \beta^s M\left(h^s \mid h^{\varnothing}\right) \lambda_j\left(h^s\right) \sum_{s'=s}^{T} \beta^{s'-s} M\left(h^{s'} \mid h^s\right) u_j\left(c_j\left(h^{s'}\right), l_j\left(h^{s'}\right)\right) \\ = & \sum_{s=0}^{T} \beta^s M\left(h^s \mid h^{\varnothing}\right) \mu_j\left(h^s\right) u_j\left(c_j\left(h^s\right), l_j\left(h^s\right)\right) \end{aligned} \tag{19}$$

where $\mu_j\left(h^s\right) = \mu_j\left(h^{s-1}\right) + \lambda_j\left(h^s\right)$ for $h^s \in P\left(h^{s-1}\right)$ with the initial $\mu_j\left(h^{\varnothing}\right) = 0$ for all j.

Substituting (19) in $\mathcal{L}'$ in (16), we obtain that $\mathcal{L}'$ for any h^{t-1}, can be expressed as

$$
\begin{aligned}
\max_{\{c_j(h),l_j(h)\}_{j=1}^N} \mathcal{L}'' = & \sum_{s=t}^{\infty} \beta^s M\left(h^s \mid h^{t-1}\right)\left[\sum_{j=1}^{N}\left(\alpha_j+\mu_j\left(h^{t-1}\right)\right) u_j\left(c_j\left(h^s\right), l_j\left(h^s\right)\right)\right] \\
& + \sum_{s=t}^{\infty} \beta^s M\left(h^s \mid h^{t-1}\right) \sum_{j=1}^{N} \lambda_j\left(h^s\right)\left(\sum_{s'=s}^{\infty} \beta^{s-s} M\left(h^{s'} \mid h^s\right) u_j\left(c_j\left(h^s\right), l_j\left(h^s\right)\right)-u_j\left(\sum_{i=1}^{N} l_i\left(h^s\right), l_j\left(h^s\right)\right)\right) \\
& + \sum_{s=0}^{t-1} \beta^s M\left(h^s \mid h^{\varnothing}\right) \sum_{j=1}^{N} \alpha_j u_j\left(c_j\left(h^s\right), l_j\left(h^s\right)\right) \\
& + \sum_{s=0}^{t-1} \beta^s M\left(h^s \mid h^{\varnothing}\right) \sum_{j=1}^{N} \lambda_j\left(h^s\right)\left(\sum_{s'=s}^{t-1} \beta^{s'} M\left(h^{s'} \mid h^s\right) u_j\left(c_j\left(h^{s'}\right), l_j\left(h^{s'}\right)\right)-u_j\left(\sum_{i=1}^{N} l_i\left(h^s\right), l_j\left(h^s\right)\right)\right).
\end{aligned}
$$

Since after history h^{t-1} has elapsed, all terms in the last two lines are given, maximizing $\mathcal{L}''$ is equivalent to maximizing (17).

Given the structure of problem (17), the result that optimal allocations only depend on $\{\alpha_j(h)\}_{j=1}^N$ and $\mathbf{j}(h)$ then follow immediately. In the Appendix, we prove a generalize version of this theorem, Theorem 11, that allows for randomizations. ■

The result in this theorem is quite intuitive.[8] When the sustainability constraint for the group in power is binding, the utility (value) of this group needs to be increased so as to satisfy this constraint. The best way of doing so is by increasing consumption and reducing labor supply today in a way that is consistent with consumption and labor supply smoothing (subject of further distortions). This implies that we can think of the utility of the group in power increasing when the sustainability constraint is binding by moving along the constraint Pareto efficient frontier, or in other words, by giving a higher Pareto weight to this group, which translates both into higher consumption today and higher consumption in the future (and lower labor supply today and lower labor supply in the future).

Theorem 2 allows us to work with a recursive problem, in which we only have to keep track of the identity of the group that is in power and updated Pareto weights. Moreover, the analysis preceding the theorem shows that the Pareto weights are updated following the simple formula (18), which is only a function of the Lagrange multiplier on the sustainability

[8]The existence of a recursive formulation for the problem of characterizing the set of Pareto efficient allocations also has an obvious parallel to the general recursive formulation provided by Abreu, Pearce and Stacchetti (1990) for repeated games with imperfect monitoring. Nevertheless, Theorem 2 is not a direct corollary of their results, since it establishes that this recursive formulation depends on updated Pareto weights and the identity of the group in power, and shows how these weights can be calculated from past realizations of the history h^t.

constraint of the group in power at time t.

3.4 Characterization of Distortions

We now characterize the structure of distortions arising from political economy. Our first major result shows that as long as sustainability/political economy constraints are binding, the labor supply of groups that are not in power is distorted *downwards*, meaning that there is a positive wedge between their marginal utility of consumption and marginal disutility of labor. But in contrast, as long as labor is a normal good, the labor supply of individuals from the group in power is distorted *upwards*, meaning that there is a negative way (i.e., their labor supply is "subsidized"). Recall also that without political economy constraints, in the first best allocations, the distortions are equal to zero.

Theorem 3 *Suppose that Assumptions 1 and 2 hold. Then as long as $\lambda\left(h^t\right)>0$, the labor supply of all groups that are not in power, i.e., $j \neq \mathbf{j}\left(h^t\right)$, is distorted* downwards, *in the sense that*

$$\frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial c} > -\frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial l}.$$

In addition, as long as $\partial^2 u_{\mathbf{j}(h^t)}/\partial c \partial l \leq 0$ (so that marginal this utility of labor increases with higher consumption), the labor supply of the group in power, $\mathbf{j}\left(h^t\right)$, is distorted upwards, *i.e.,*

$$\frac{\partial u_{\mathbf{j}(h^t)}\left(c_{\mathbf{j}(h^t)}\left(h^t\right), l_{\mathbf{j}(h^t)}\left(h^t\right)\right)}{\partial c} < -\frac{\partial u_{\mathbf{j}(h^t)}\left(c_{\mathbf{j}(h^t)}\left(h^t\right), l_{\mathbf{j}(h^t)}\left(h^t\right)\right)}{\partial l}.$$

Proof. This results follows from the first-order conditions of the maximization problem in Lemma 9 in the Appendix. In particular, combining (30) and (31), we have

$$\frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial c} > -\frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial l} \text{ whenever } \lambda\left(h^t\right)>0 \text{ for all } j \neq \mathbf{j}\left(h^t\right).$$

A comparison with the equalities in (5) shows that these inequalities correspond to downward distortions.

Next consider the group in power $j' = \mathbf{j}\left(h^t\right)$ and suppose that $\lambda\left(h^t\right)>0$. Inspection of (32) and (33), shows that to obtain no distortion on group j', we need

$$-\frac{\partial u_{j'}\left(\sum_{j=i}^N l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial l} = \frac{\partial u_{j'}\left(\sum_{j=i}^N l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c}, \tag{20}$$

so that the right hand side of (33), becomes $\zeta\left(h^t\right)$ as the right hand side of (32). Suppose first that $u_{j'}$ is separable, so that $\partial^2 u_{j'}/\partial c \partial l$. This implies

$$\frac{\partial u_{j'}\left(\sum_{j=i}^{N} l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial l} = \frac{\partial u_{j'}\left(c_{j'}\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial l}, \tag{21}$$

but since $\sum_{j=i}^{N} l_i\left(h^t\right) > c_{j'}\left(h^t\right)$ and $u_{j'}$ is strictly concave in c,

$$\frac{\partial u_{j'}\left(c_{j'}\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c} > \frac{\partial u_{j'}\left(\sum_{j=i}^{N} l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c}. \tag{22}$$

Now suppose, to obtain a contradiction, that there are no distortions on j', so that (20) holds. Then we have

$$\begin{aligned} -\frac{\partial u_{j'}\left(\sum_{j=i}^{N} l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial l} &= -\frac{\partial u_{j'}\left(c_{j'}\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial l} \\ &= \frac{\partial u_{j'}\left(c_{j'}\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c} \\ &> \frac{\partial u_{j'}\left(\sum_{j=i}^{N} l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c}, \end{aligned}$$

where the first equality follows from (21), the second from the hypothesis that there are no distortions, i.e., equation (5) for group j' and the last inequality follows from (22). The fact that the first-term is strictly greater than the last one contradicts the necessary condition (20), establishing that there must be distortions on group j'. In fact, in the case where $u_{j'}$ is separable, it is straightforward to verify with the same steps that

$$\frac{\partial u_{j'}\left(c_{j'}\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c} < -\frac{\partial u_{j'}\left(c_{j'}\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial l},$$

so that distortions are upwards. The same argument carries through as long as $\partial^2 u_{j'}/\partial c \partial l \leq 0$, i.e., when the marginal this utility of labor increases with higher consumption. ■

The intuition for why there will be downward distortions in the labor supply of groups that are not in power is similar to that in Acemoglu, Golosov and Tsyvinski (2006a); distortions reduce the labor supply of these groups, thus the amount of output that the group in power can "expropriate" (i.e., allocate to itself as consumption). This relaxes the sustainability constraint (15). In fact, starting from an allocation with no distortions, a small distortion in labor supply creates a second-order loss. In contrast, as long as the multiplier on the sustainability constraint is positive, this small distortion creates a first-order gain in the objective function, because it enables a reduction in the rents captured by the group in power. This intuition also highlights that the extent of distortions will be closely linked to the Pareto weights given to the group in power. In particular, when α_j is close to 1 and group j is in power, there will be little gain in relaxing the sustainability constraint (15). In contrast, the Pareto efficient allocation will attempt to provide fewer rents to group j when α_j is low, and this is only possible by reducing the labor supply of all other groups, thus distorting their labor supplies.

Perhaps the most interesting implication of Theorem 3 is that there will be also be distortions on the labor supply of the group in power , but interestingly, as long as $\partial^2 u_{\mathbf{j}(h^t)}/\partial c \partial l \leq 0$, this will be upwards. This result contrasts with the downward distortions on all groups in models such as Acemoglu, Golosov and Tsyvinski (2006a), where the politicians in power did not take part in production. In particular, when $\partial^2 u_{\mathbf{j}(h^t)}/\partial c \partial l \leq 0$, an increase in labor supply makes deviation less attractive, because current consumption is higher when the group in power expropriates the entire output. Therefore, distorting (subsidizing) the labor supply of the group in power also helps relax the sustainability constraint (15). Note also that the condition $\partial^2 u_{\mathbf{j}(h^t)}/\partial c \partial l \leq 0$ is natural, since it requires the marginal this utility of labor is non-decreasing in the level of consumption. This will be the case with separable utility functions and more generally when leisure is a normal good. The intuition for this upward distortion is as follows: greater labor supply affects the utility of the group in power differentially along the equilibrium path versus when they undertake a deviation.

Two immediate but useful corollaries of Theorem 3 are as follows:

Corollary 1 *The (normalized) Lagrange multiplier on the sustainability constraint (15) given history h^t, $\lambda\left(h^t\right)$, is a measure of distortions.*

This corollary follows immediately from Theorem 3, and more explicitly from Lemma 9 in the Appendix, which shows that the wedges between the marginal utility of consumption and the marginal disutility of labor are directly related to $\lambda\left(h^t\right)$. This corollary is useful as it will enable us to link the level and behavior of distortions to the behavior of the Lagrange multiplier $\lambda\left(h^t\right)$.

A related implication of Theorem 3 is that constrained Pareto efficient allocations will be "first-best" if and only if the Lagrange multipliers associated with all sustainability constraints are equal to zero (so that there are no distortions in a first-best allocation).

Corollary 2 *A first-best allocation starting at history h^t involves $\lambda\left(h^{t+s}\right)=0$ for all $h^{t+s}\in P\left(h^t\right)$.*

3.5 Dynamics of Distortions

Theorem 3 states that when the Lagrange multipliers are positive, these distortions will be present. We now study the evolution of the Largrange multipliers and distortions resulting from the sustainability constraints.

Our first result in this subsection is an immediate implication of the recursive formulation in Theorem 2, but it will play an important role in our results. It implies that if a group is in power today, then in the next period its updated Pareto weight must be weakly higher than today.

Lemma 2 *If $\mathbf{j}\left(h^{t+1}\right)=j$, then $\alpha_j\left(h^{t+1}\right)\geq\alpha_j\left(h^t\right)$.*

Proof. This follows immediately from equation (18) observing that if $\mathbf{j}\left(h^{t+1}\right)=j$, then $\mu_{j'}\left(h^{t+1}\right)-\mu_{j'}\left(h^t\right)=0$ for all $j'\neq j$. ■

An implication of Lemma 2 is the following important result, which states that when a particular group remains in power for sufficiently long, the Lagrange multipliers on the sustainability constraints and distortions begin declining.

Theorem 4 *Suppose Assumptions 1 and 2 hold. Then there exists $\bar{K}_j\in\mathbb{N}$ such that for any $K>\bar{K}_j$, if $\mathbf{j}\left(h^t\right)=\mathbf{j}\left(h^{t+k}\right)=j$ for $k=1,...,K$, then for all $k\in\left[\bar{K}_j,K-1\right]$, we have $\lambda\left(h^{t+k}\right)\geq\lambda\left(h^{t+k+1}\right)$ with strict inequality as long as $\lambda\left(h^{t+k}\right)>0$.*

Proof. Fix j and $h^\infty\in H^\infty$, and suppose that $\mathbf{j}\left(h^t\right)=\mathbf{j}\left(h^{t+k}\right)=j$ for all $k\in\mathbb{N}$. Then from Lemma 2,$\left\{\alpha_j\left(h^{t+k}\right)\right\}_{k=0}^{\infty}$ is a non-decreasing sequence, and moreover, by construction $\alpha_j\left(h^{t+k}\right)\in[0,1]$ for each j and all h^{t+k}. Consequently, $\alpha_j\left(h^{t+k}\right)\to\bar{\alpha}_j$. Next note that $\bar{\alpha}_j<1$. To see this note that the inspection of the maximization problem (17) shows that when $\bar{\alpha}_j=1$, the constraint (15) is slack. Since the objective function is continuous in the vector of Pareto weights $\boldsymbol{\alpha}$, this implies that for $\alpha_j=1-\varepsilon$ with ε sufficiently small, the constraint is also slack and $\lambda_j\left(h^{t+k}\right)=0$. This implies that there exists $\varepsilon_j>0$ such that starting with $\alpha_j\left(h^t\right)<1-\varepsilon_j$, we cannot have $\alpha_j\left(h^{t+1}\right)=1$ for any h^t, since from equation (18) this would imply that $\lambda_j\left(h^{t+k}\right)=\infty$, which is not possible. Next equation (18) also implies that if $\lambda\left(h^{t+k}\right)=0$, then $\alpha_j\left(h^{t+k}\right)$ will remain constant (since $\mathbf{j}\left(h^t\right)=\mathbf{j}\left(h^{t+k}\right)=j$ for all $k\in\mathbb{N}$). Therefore, $\alpha_j\left(h^{t+k}\right)\to\bar{\alpha}_j<1$. Next, inspection of equation (18) shows that as $\alpha_j\left(h^{t+k}\right)\to\bar{\alpha}_j$, we have $\mu_j\left(h^{t+k}\right)\to\mu_j\left(h^{t+k-1}\right)$ and thus $\lambda\left(h^{t+k}\right)\to 0$ (by virtue of the fact that $\bar{\alpha}_j<1$). Since $\lambda\left(h^{t+k}\right)$ converges to zero, there exists some $\bar{K}_j$, such that $\lambda\left(h^{t+k}\right)\geq\lambda\left(h^{t+k+1}\right)$ for all $k\geq\bar{K}_j$. The conclusion of the theorem then follows when we restrict attention to histories such that $\mathbf{j}\left(h^t\right)=\mathbf{j}\left(h^{t+k}\right)=j$ for $k=1,...,K$ with $K>\bar{K}_j$. ∎

Intuitively, the longest sample path in which a particular group remains in power for a long time, distortions ultimately decline. This is because as a particular group remains in power for a long time, its Pareto weight increases sufficiently and the allocations do not need to be distorted to satisfy the sustainability constraint. We summarize this in the following corollary.

Corollary 3 *As a particular group remains in power for a long time, the Lagrange multipliers on the sustainability constraints and distortions start declining.*

The major result in Theorem 4 and its corollary is that as a particular group remains in power, distortions eventually decline. Intuitively, this follows from the fact that when the group in power has a higher updated Pareto weight, then there is no need to distort allocations as much. In the limit, if the group in power had a weight equal to 1, then the Pareto efficient allocation would give all consumption to individuals from this group, and therefore, there would be no reason to distort the labor supply of other groups in order to relax the sustainability constraint and reduce rents to this group. Put differently, recall that distortions (and inefficiencies) arise because the group in power does not have a sufficiently high Pareto weight and the Pareto efficient allocation reduces its consumption by reducing total output and thus relaxing its sustainability constraint. As a group remains in power for longer, its updated Pareto weight increases and as a result, there is less need for this type of distortions. This reflects itself in a reduction in the Lagrange multiplier associated with the sustainability constraint.

Figure 1 illustrates this result of for a hypothetical economy with two groups. We assume that the utility functions of both groups are identical and given by $u(c,l)=\log(c)-l^2/2$, and the discount factor is $\beta=0.35$. The Markov process for power switches is also symmetric and imposes that the group in power stays in power with probability 0.9. In Figure 1, the circles designate periods in which group 1 is in power. The figure shows that when group 1 is in

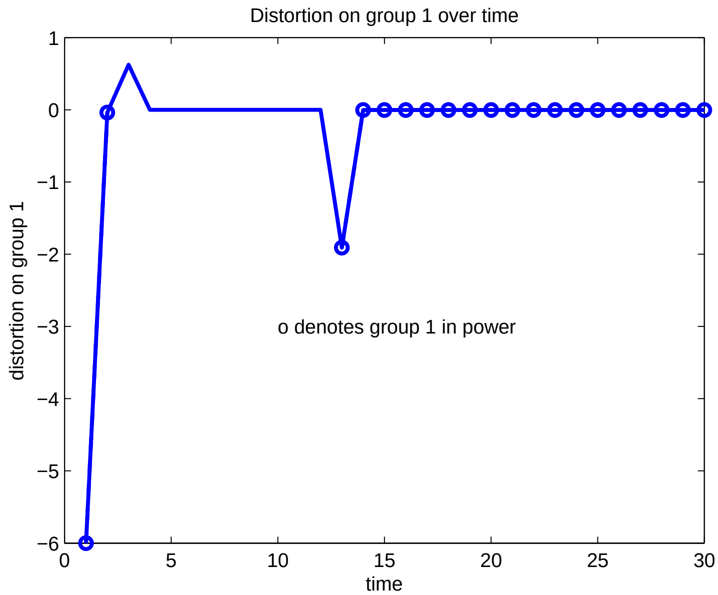

Figure 1: Distortions on the labor supply of members of group 1.

power there is a negative distortion on (subsidy to the labor supply of) the members of group 1 and a positive distortion on the members of group 2. This illustrates the results in Theorem 3. More importantly, consistent with the results in Theorem 4, as a particular group remains in power for more than one period, the extent of distortions decline.

Theorem 4 also suggests a result reminiscent to the conjecture discussed in the Introduction; greater political stability translates into lower inefficiencies and better public policies. This conclusion does not follow from the theorem, however. The theorem is for a given sample path (holding the Markov process regulating power switches fixed). The conjecture linking political stability to efficient public policy refers to a comparison of the extent of distortions for different undelying Markov processes governing power switches. We will be discuss such comparison in greater detail in Section 6.

Theorem 4 does not answer the question of whether distortions will ultimately disappear—i.e., whether we will have $\lambda\left(h^t\right) = 0$ for all h^t after some date. We next turn to this question.

Theorem 2 again considerably simplifies our analysis, since it implies that we can define a history-independent set of sustainable values for different groups (since history does not matter beyond Pareto weights and the identity of who is in power). In particular, let $\mathbf{V} \subset \mathbb{R}^N$

be the set of *expected* values, $V \equiv (V_1, ..., V_N)$ such that if $(V_1, ..., V_N) \in \mathbf{V}$, then the vector $(V_1, ..., V_N)$ designates the expected discounted utilities to the members of each of the N groups in a Pareto efficient allocation. Each Pareto efficient allocation corresponds to the value of the state variable $\mathbf{s} = \left(\boldsymbol{\alpha}\left(h^{t-1}\right), \mathbf{j}\left(h^{t-1}\right)\right)$, where $\boldsymbol{\alpha}\left(h^{t-1}\right) \equiv \left(\alpha_1\left(h^{t-1}\right), ..., \alpha_N\left(h^{t-1}\right)\right)$ is the vector of Pareto weights after history h^{t-1} and $\mathbf{j}\left(h^{t-1}\right)$ is the identity of the group in power at time $t-1$. [Put differently, each value of the state variable $\mathbf{s}$ leads to an ex ante value vector $V(\mathbf{s})$. Let $\mathbf{S} = \Delta^{N-1} \times \mathcal{N}$ be the set of possible values of $\mathbf{s}$. Then we have that $\mathbf{V} = \left\{V \in \mathbb{R}^N : V(\mathbf{s}) \text{ for some } \mathbf{s} \in \mathbf{S}\right\}$]. It is important to condition on the identity of the group in power, since future probabilities of which group will be in power depend on this information through the Markov process $m(j \mid j')$. To emphasize the dependence of V on the vector $\mathbf{s}$, we use the notation $V(\mathbf{s})$ or $V_j(\mathbf{s})$, though typically we simplify the notation by writing $V_j\left(h^{t-1}\right)$ instead of $V_j\left(\mathbf{s}\left(h^{t-1}\right)\right)$, etc.

Notice that the vector $(V_1, ..., V_N)$ refers to *ex ante* values, before the identity of the group in power is known. Thus the expectation is not only over future histories, but also over the realizations of the group that will be in power today. This way of defining expected values is convenient, since it will create a correspondence between these values and the Pareto weights. In particular, recall that a Pareto efficient allocation after some history h^{t-1} can be represented as a maximization problem with updated Pareto weights, $\boldsymbol{\alpha}\left(h^{t-1}\right) \equiv \left(\alpha_1\left(h^{t-1}\right), ..., \alpha_N\left(h^{t-1}\right)\right)$ and with some group $j \in \mathcal{N}$. For some $h^t \in P\left(h^{t-1}\right)$, let $V_j\left[h^t\right]$ be the *ex post* expected utility of a member of group j at time t given history h^t (which includes the identity of the group in power). We use the notation $V_j[\cdot]$ rather than $V_j(\cdot)$, to distinguish this ex post value from ex ante values. With this definition, we have that ex ante values are defined as

$$V_j(\mathbf{s}) = \sum_{h^t \in P(h^{t-1}) : \mathbf{s}(h^{t-1}) = \mathbf{s}} M\left(h^t \mid h^{t-1}\right) V_j\left[h^t\right]. \tag{23}$$

In addition, let $\mathbf{V}^F \subset \mathbf{V}$ be the subset of constrained efficient allocations that correspond to first best (defined is in Theorem 1). We next note the following properties of the set of first-best allocations that are sustainable as constrained Pareto efficient allocations.

Fact 1 *If $(V_1^*, ..., V_N^*) \in \mathbf{V}^F$, then the following are true:*

1. *From Theorem 1, first-best allocations involve full consumption and labor supply smooth-*

ing, thus

$$V_j^* = \frac{u_j\left(c_j^*, l_j^*\right)}{1-\beta} \text{ for } j = 1, ..., N.$$

2. *There exists* $\boldsymbol{\alpha} \in \Delta^{N-1}$*, where* Δ^{N-1} *is the* $N-1$*-dimensional simplex, such that*

$$\begin{aligned} \left\{c_j^*, l_j^*\right\}_{j=1}^N &= \arg\max_{\left\{c_j^*, l_j^*\right\}_{j=1}^N} \sum_{j=1}^N \alpha_j u_j\left(c_j, l_j\right) \\ & \quad \textit{subject to} \\ \sum_{j=1}^N l_j &= \sum_{j=1}^N c_j. \end{aligned} \tag{24}$$

3. *The sustainability constraints for all parties are satisfied, i.e.,*

$$\frac{u_j\left(c_j^*, l_j^*\right)}{1-\beta} \geq u_j\left(\sum_{j=1}^N l_j^*, l_j^*\right) \text{ for all } j = 1, ..., N. \tag{25}$$

We now have the following lemma, which shows that when the common discount factor of all the agents in the economy is less than a critical threshold $\bar{\beta}$, no first-best allocation is sustainable—i.e., the set of Pareto efficient allocations does not contain any first-best allocations.

Lemma 3 *Suppose Assumptions 1 and 2 hold. Then, there exists* $\bar{\beta} \in (0,1)$ *such that for all* $\beta \geq \bar{\beta}$, $\mathbf{V}^F \neq \varnothing$ *and* $\beta < \bar{\beta}$, $\mathbf{V}^F = \varnothing$.

Proof. By definition, $(V_1, ..., V_N)$ is a constrained Pareto efficient allocation, i.e., $(V_1, ..., V_N) \in \mathbf{V}$ if only if there exists no $(V_1', ..., V_N') \in \mathbf{V}$ such that $V_1' \geq V_1$, $V_2' \geq V_2$ and $V_N' \geq V_N$ with at least one strict inequality. Now consider an allocation $(V_1, ..., V_N) \in \mathbf{V}^F$. By Fact 1, this is a solution to (24) for some $\boldsymbol{\alpha} \in \Delta^{N-1}$. Construct the entire candidate set of first-best allocations

$$\mathbf{V}^C = \left\{ \begin{array}{c} \left(V_1\left(\boldsymbol{\alpha}\right), ..., V_N\left(\boldsymbol{\alpha}\right)\right): \;\; V_j\left(\boldsymbol{\alpha}\right) > 0 \text{ and } V_j\left(\boldsymbol{\alpha}\right) = \frac{u_j(c_j, l_j)}{1-\beta} \text{ for } j = 1,...,N. \\ \text{and } \left\{c_j, l_j\right\}_{j=1}^N \text{ is a solution to (24) for some vector } \boldsymbol{\alpha} \in \Delta^{N-1}. \end{array} \right\},$$

where we write $V_j\left(\boldsymbol{\alpha}\right)$ instead of $V_j\left(\mathbf{s}\right)$, since in first best the identity of the group in power is not relevant.

Clearly $\mathbf{V}^F \subset \mathbf{V}^C$. In particular, $\left(V_1\left(\boldsymbol{\alpha}\right), ..., V_N\left(\boldsymbol{\alpha}\right)\right) \in \mathbf{V}^F$ if they satisfy (25) from Fact 1, i.e.,

$$V_j\left(\boldsymbol{\alpha}\right) \geq u_j\left(\sum_{j=1}^N l_j, l_j\right) \text{ for } j = 1, ..., N.$$

Note also that $(V_1(\boldsymbol{\alpha}), ..., V_N(\boldsymbol{\alpha})) > (0, ..., 0)$ is always feasible for any $\boldsymbol{\alpha}$. Next, note that for each $\boldsymbol{\alpha}$, there exists $\beta(\boldsymbol{\alpha}) \in (0,1)$ such that $(V_1(\boldsymbol{\alpha}), ..., V_N(\boldsymbol{\alpha})) \in \mathbf{V}^F$. This follows because $(V_1(\boldsymbol{\alpha}), ..., V_N(\boldsymbol{\alpha}))$ is continuous in β by the Theorem of the Maximum, and because as $\beta \to 1$, (25) are satisfied for all $j = 1, ..., N$, since $(V_1(\boldsymbol{\alpha}), ..., V_N(\boldsymbol{\alpha})) > (0, ..., 0)$. Moreover, as $\alpha_j \to 0$ for any j, we have $\beta(\boldsymbol{\alpha}) \to 1$. This ensures that there exists some $\bar{\boldsymbol{\alpha}}$ such that $\beta(\bar{\boldsymbol{\alpha}}) \leq \beta(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha}$. The conclusion of the lemma follows setting $\bar{\beta} \equiv \beta(\bar{\boldsymbol{\alpha}})$. ■

The next theorem shows that with sufficiently small discount factors, distortions will not disappear and thus the results of Theorem 3 on the structure of distortions will continue to apply asymptotically

Theorem 5 *Suppose Assumptions 1 and 2 hold. Then, for all $\beta < \bar{\beta}$, distortions do not disappear in the long run.*

Proof. Lemma 3 implies that when $\beta < \bar{\beta}$, $\mathbf{V}^F = \varnothing$. Therefore, there exist no allocations that satisfy the conditions in Fact 1, and we cannot have $\lambda\left(h^t\right) = 0$ for all h^t as $t \to \infty$. This implies that the distortions characterized in Theorem 3 remain asymptotically. ■

Theorem 5 shows that distortions will not disappear, but does not characterize the asymptotic behavior of these distortions. The next theorem establishes two important features about this asymptotic behavior:

1. Values and distortions converge to a limiting invariant distribution.

2. This limiting distribution is *non-degenerate*, meaning that distortions and consumption levels will continue to fluctuate, even asymptotically.

Theorem 6 *Suppose Assumptions 1 and 2 hold. Then, for all $\beta < \bar{\beta}$, we have that as $t \to \infty$, $\left(V_1\left(h^t\right), ..., V_N\left(h^t\right)\right) \to (V_1, ..., V_N)$ almost surely, where $(V_1, ..., V_N)$ has a joint distribution F. Moreover, F is non-degenerate, meaning it does not have all of its mass at one point.*

Proof. (Part 1) Let updated weights at some point be given by the vector $\boldsymbol{\alpha} \in \Delta^{N-1}$ and $\mathbf{s} \equiv (\boldsymbol{\alpha}, \mathbf{j}) \in \mathcal{S} \equiv \Delta^{N-1} \times \mathcal{N}$. Then the constrained efficient allocation defines a Markov process for the vector $\mathbf{s}$. In particular, let $\mathcal{S}$ be the sample space and consider the σ-algebra generated by the Borel subsets of $\mathcal{S}$ and denote this by $\mathcal{D}$. Then the constrained efficient allocation defines a transition function

$$Q : \mathcal{D} \times \mathcal{S} \to [0,1],$$

where for any $A \in \mathcal{D}$ (i.e., for any measurable A), $Q(A, \mathbf{s})$is the probability that starting with the vector $\mathbf{s}$, the vector of weights and the identity of the group in power in the next period will be in the set A. Recall that Q^n stands for n-fold transitions, i.e., Q applied n times.

The transition function $Q(A, \mathbf{s})$ defines two operators. Let $M(\mathcal{S}, \mathcal{D})$ be the set of measurable functions on Δ^{N-1}. Then the first operator is $T : M(\mathcal{S}, \mathcal{D}) \to M(\mathcal{S}, \mathcal{D})$ and for each $f \in M(\mathcal{S}, \mathcal{D})$, it is defined by

$$Tf(\mathbf{s}) = \int f(\mathbf{s}') Q(d\mathbf{s}', \mathbf{s}) \text{ for all } \mathbf{s} \in \mathcal{S}.$$

The second and the more important operator is its adjoint T^*. Define $\mathcal{P}(\mathcal{S}, \mathcal{D})$ be the set of probability measures on the measurable space $(\mathcal{S}, \mathcal{D})$. Then for any $\lambda \in \mathcal{P}(\mathcal{S}, \mathcal{D})$,

$$T^*\lambda(A) = \int Q(A, \mathbf{s}) \lambda(d\mathbf{s}) \text{ for all } A \in \mathcal{D}.$$

Clearly, $T^* : \mathcal{P}(\mathcal{S}, \mathcal{D}) \to \mathcal{P}(\mathcal{S}, \mathcal{D})$, i.e., this operator takes a probability measure over $(\mathcal{S}, \mathcal{D})$ and generates another probability measure. A fixed point of this operator, λ^*, is a limiting invariant distribution.

Now recall the following condition (which is a strengthening of Doeblin's condition referred to as Condition M by Stokey, Lucas and Prescott, 1989):

Condition M: There exists $\varepsilon > 0$ and $n \geq 1$ such that for any $A \in \mathcal{D}$, either $Q^n(A, \mathbf{s}) \geq \varepsilon$ for all $\mathbf{s} \in \mathcal{S}$ or $Q^n(A^c, \mathbf{s}) \geq \varepsilon$ for all $\boldsymbol{\alpha} \in \Delta^{N-1}$, where A^c is the complement of A in $\mathcal{S}$.

The main theorem for the strong convergence of probability measures is that if a Markov process satisfies this condition, then starting with any probability distribution $\phi \in \mathcal{P}(\mathcal{S}, \mathcal{D})$, $T^{*n}\lambda \to \lambda^* \in \mathcal{P}(\mathcal{S}, \mathcal{D})$ as $n \to \infty$. In other words, there exists a unique invariant distribution to which we converge almost surely starting from any probability distribution over $(\mathcal{S}, \mathcal{D})$.

To see that this condition is satisfied for our Markov process, we can proceed as follows. For any $\beta > 0$ and for each group $j = 1, ..., N$, the argument in the proof of Theorem 4 (in particular the observation that for each j, there exists $\varepsilon_j > 0$ such that we have $\alpha_j(h^t) \leq 1 - \varepsilon_j$ for all h^t) implies that there exists $\bar{\alpha}_j < 1$ such that $\alpha_j \leq \bar{\alpha}_j$ and $\bar{\alpha}_j$ will reached in $n_j < \infty$ steps starting in $\alpha_j = 0$ if $\mathbf{j}(h^t) = \mathbf{j}(h^{t+k}) = j$ for $k = 1, ..., n_j$, which has positive probability. Consequently, starting in any $\mathbf{s} \in \mathcal{S}$, we can reach some $\bar{\boldsymbol{\alpha}}$ in $\bar{n} < \infty$ steps. Therefore, $Q^{\bar{n}}(\{\bar{\mathbf{s}}\}, \mathbf{s}) > \varepsilon$. Now consider any $A \in \mathcal{D}$. Either $\bar{\mathbf{s}} \in A$, in which case $Q^{\bar{n}}(A, \mathbf{s}) > \varepsilon$, or $\bar{\mathbf{s}} \notin A$ and thus $\mathbf{s} \in A^c$ in which case $Q^{\bar{n}}(A^c, \mathbf{s}) > \varepsilon$, so Condition M is satisfied and strong convergence to some joint distribution F applies. This completes the proof of Part 1.

(Part 2) Suppose, to obtain a contradiction, that F has all of its mass at some point. Then we must have that $\alpha_j\left(h^t\right) = \alpha_j$ for all h^t as $t \to \infty$. From (18), this is only possible if $\lambda\left(h^t\right) = 0$ for all h^t as $t \to \infty$, which from Corollary 2 would contradict the fact that $\mathbf{V}^F = \varnothing$, completing the proof of Part 2. ■

This theorem implies that for low discount factors we have complete characterization of the behavior of distortions and values asymptotically. Political economy constraints do not disappear asymptotically, and they imply not only distortions on labor supply, but also fluctuations in the level of consumption for each group. Figure 1 above illustrates the results of this theorem, since the parameter values in the example economy correspond to the case where $\beta < \bar{\beta}$, thus no first-best allocation is sustainable. Consequently, the allocations and distortions fluctuate over time according to a limiting stochastic distribution, but with no tendency to converge to constant values.

4 Dynamics of Distortions With Two Parties

In this section, we focus on the special case with two parties, $j = 1$ and 2, to characterize the solution of the problem for $\beta > \bar{\beta}$.

Our main result in this section is that as long as $\mathbf{V}^F \neq \varnothing$, or alternatively as long as $\beta \geq \hat{\beta} \in [\bar{\beta}, 1)$, the constraint Pareto efficient allocation will eventually converge to a first-best allocation. Note however that the theorem does not state (and in fact it is not true) that the Pareto efficient allocation will always correspond to a first-best allocation. On the contrary, the economy will typically start with distortions and then converge to a first-best allocation.

Theorem 7 *Suppose Assumptions 1 and 2 hold and $\partial^2 u_j/\partial c \partial l \leq 0$ for all $j \in \mathcal{N}$. There exists $\hat{\beta} \in [\bar{\beta}, 1)$ such that if $\beta \geq \hat{\beta}$, then as $t \to \infty$, $\left(V_1\left(h^t\right), V_2\left(h^t\right)\right) \to (V_1^*, V_2^*) \in \mathbf{V}^F$ almost surely.*

We will prove this theorem using a number of lemmas. The most important steps are contained in Lemmas 6, 7 and 8. Lemma 6 shows that if a particular group has a value (discounted utility) above that of a sustainable first-best level, say V_j^*, then its value will never fall below this value V_j^*. This implies that the value of a group, starting with a sufficiently high Pareto weight, will not “undershoot” the first best allocation. Lemma 7 provides a conncetion between the values and the weights assigned to a group in power and shows that if a group is in power and receives a value greater than that of a first-best allocation, then its weight is the same as the weight in the previous period. This implies that a group starting with relatively high Pareto weights will not experience further increases in its Pareto weights even if it remains in power for a long time. Finally, Lemma 8 establishes that when the group with value above first best looses power, its Pareto weight will decrease. These three lemmas are then sufficient to establish that the sequence of the updated Pareto weights for a group starting with value above the first-best allocation forms a non-increasing sequence in a compact set and thus converges. The final argument in the proof of Theorem 7 is that these weights can only converge to a vector of weights associated with a first-best allocation.

We start with two simple lemmas, which show that each first-best vector of utilities corresponds to a unique vector of Pareto weights $(\alpha, 1-\alpha)$ and that when a first-best allocation is sustainable, any vector of Pareto efficient values must give higher value to one of the groups than the first best. Even though the vector of Pareto weights can be represented by a single number $\alpha \in [0,1]$, we sometimes use the notation $(\alpha, 1-\alpha)$ to emphasize that this is a vector of weights, and sometimes abuse notation by writing $(\alpha, 1-\alpha) \in \Delta$.

Lemma 4 *Suppose Assumptions 1 and 2 hold. Then,for every $(V_1, V_2) \in \mathbf{V}$, there exists a unique vector $\mathbf{s} \equiv (\alpha, j)$ such that the solution to (17) with weights $(\alpha, 1-\alpha) \in \Delta$ and starting with group j in power gives ex ante values V_1 and V_2 to members of the two groups according to equation (23). Moreover, suppose that $(V_1^*, V_2^*) \in \mathbf{V}^F$, then there exists a unique vector of weights $(\alpha, 1-\alpha) \in \Delta$ such that the solution to the maximization (3) with weights $(\alpha, 1-\alpha)$ leads to an optimal allocation with values (V_1^*, V_2^*).*

Proof. The first part follows from Theorem 2 and the second part follows from the definition of a first-best allocation, in particular, Fact 1. ■

Lemma 5 *Suppose that Assumptions 1 and 2 hold, that $(V_1^*, V_2^*) \in \mathbf{V}^F$ and that $(V_1, V_2) \in \mathbf{V} \setminus \mathbf{V}^F$. Then either $V_1 > V_1^*$ and $V_2 < V_2^*$; or $V_1 < V_1^*$ and $V_2 > V_2^*$.*

Proof. Clearly $V_1 > V_1^*$ and $V_2 = V_2^*$ is impossible by the fact that $(V_1^*, V_2^*) \in \mathbf{V}^F$. To obtain a contradiction, suppose that $V_1 \le V_1^*$ and $V_2 < V_2^*$. Then immediately choosing $(V_1^*, V_2^*) \in \mathbf{V}^F$ would give higher value to group 2 and the same or higher value to group

1. This shows that (V_1, V_2) cannot be a constrained efficient allocation. The proof of the remaining cases are similar. ■

The next lemma provides the most important step in the proof of Theorem 7.

Lemma 6 *Suppose Assumptions 1 and 2 hold and $(V_1^*, V_2^*) \in \mathbf{V}^F$. There exists $\hat{\beta} \in [\bar{\beta}, 1)$ such that if $\beta \geq \hat{\beta}$, then whenever $\left(V_1\left(h^{t-1}\right), V_2\left(h^{t-1}\right)\right) \notin \mathbf{V}^F$ and $V_j\left(h^{t-1}\right) > V_j^*$ for $j = 1$ or 2 and $\left(V_1\left(\tilde{h}^t\right), V_2\left(\tilde{h}^t\right)\right) \notin \mathbf{V}^F$ for $\tilde{h}^t \in P\left(h^{t-1}\right)$, we have $V_j\left(h^t\right) \geq V_j^*$ for all $h^t \in P\left(h^{t-1}\right)$.*

Proof. To be written. ■

Lemma 7 *Suppose that Assumption 1 and 2 hold, that $\partial^2 u_j / \partial c \partial l \leq 0$ for $j = 1, 2$, that $(V_1^*, V_2^*) \in \mathbf{V}^F$ with corresponding weights $(\alpha^*, 1 - \alpha^*)$, and that $\mathbf{j}\left(h^{t-1}\right) = \mathbf{j}\left(h^t\right) = 1$. Let ex post values in history h^{t-1} be $V_1\left[h^{t-1}\right]$ and $V_2\left[h^{t-1}\right]$ with $V_1\left[h^{t-1}\right] > V_1^*$. Then we have that $\alpha_1\left(h^t\right) = \alpha_1\left(h^{t-1}\right) > \alpha^*$.*

Proof. First, note that from Lemma 4, $V_1\left[h^{t-1}\right] > V_1^*$ implies that $\alpha_1\left(h^{t-1}\right) > \alpha^*$. Next, suppose, to obtain a contradiction, that $\alpha_1\left(h^t\right) \neq \alpha_1\left(h^{t-1}\right)$. Lemma 2 implies that $\alpha_1\left(h^t\right) \geq \alpha_1\left(h^{t-1}\right)$ since $\mathbf{j}\left(h^{t-1}\right) = \mathbf{j}\left(h^t\right) = 1$. Therefore, we must have $\alpha_1\left(h^t\right) > \alpha_1\left(h^{t-1}\right)$. Suppose this is the case. Then, from Proposition 3, it follows that

$$
\begin{aligned}
V_1\left[h^{t-1}\right] &= u_1\left(c_1', l_1'\right) + \beta \mathbb{E} V_1\left[h^t\right] \\
V_2\left[h^{t-1}\right] &= u_2\left(c_2', l_2'\right) + \beta \mathbb{E} V_2\left[h^t\right],
\end{aligned}
$$

where the allocation (c_1', l_1', c_2', l_2') is not a solution to (24) for any weights $(\alpha, 1 - \alpha)$ (i.e., this allocation is distorted).

Let $(c_1^*, l_1^*, c_2^*, l_2^*)$ be the allocation associated with first-best values $(V_1^*, V_2^*) \in \mathbf{V}^F$. Consider the following maximization problem

$$
\begin{aligned}
(c_1^{**}, l_1^{**}, c_2^{**}, l_2^{**}) &= \arg \max_{(c_1, l_1, c_2, l_2)} u_1\left(c_1, l_1\right) \\
& \quad \text{subject to} \\
l_1 + l_2 &= c_1 + c_2. \\
u_2\left(c_2, l_2\right) &= V_2\left[h^t\right] - \beta V_2^*.
\end{aligned}
$$

Construct

$$
\begin{aligned}
\tilde{V}_1\left[h^t\right] &= u_1\left(c_1^{**}, l_1^{**}\right) + \beta V_1^* \\
\tilde{V}_2\left[h^t\right] &= u_2\left(c_2^{**}, l_2^{**}\right) + \beta V_2^*.
\end{aligned}
$$

Since $\alpha_1\left(h^t\right) > \alpha_1\left(h^{t-1}\right)$, $\left(V_1\left[h^t\right], V_2\left[h^t\right]\right) \notin \mathbf{V}^F$, and therefore $V_2\left[h^t\right] < V_2^*$ (since $V_1\left[h^t\right] > V_1^*$). Moreover, since (c_1', l_1', c_2', l_2') is not a solution to (24), we have $\tilde{V}_1\left[h^t\right] > V_1\left[h^t\right]$. Also by construction, $\tilde{V}_2\left[h^t\right] = V_2\left[h^t\right]$. Therefore, if this variation is feasible, i.e., it satisfies (25), it would give higher value to group 1 and the same value to group 2, which would imply that $\left(V_1\left[h^t\right], V_2\left[h^t\right]\right)$ cannot be part of a constrained efficient allocation.

To see that this variation is feasible, note that since $V_2\left[h^t\right] < V_2^*$, we have $u_2\left(c_2^{**}, l_2^{**}\right) < u_2\left(c_2^*, l_2^*\right)$. Then $\partial^2 u_1/\partial c \partial l \leq 0$ implies that $l_2^{**} > l_2^*$ and $c_2^{**} < c_2^*$, and also $l_1^{**} < l_1^*$ and $c_1^{**} > c_1^*$. Now note that by virtue of $(c_1^*, l_1^*, c_2^{**}, l_2^{**})$ being sustainable, we have

$$u_1\left(c_1^*, l_1^*\right) + \beta V_1^* \geq u_1\left(c_1^* + c_2^*, l_1^*\right).$$

Since in view of the fact that and the fact that $c_2^{**} < c_2^*$, $l_1^{**} < l_1^*$ and $c_1^{**} > c_1^*$,

$$\begin{aligned} u_1\left(c_1^{**}, l_1^{**}\right) - u_1\left(c_1^*, l_1^*\right) &\geq u_1\left(c_1^{**} + c_2^{**}, l_1^{**}\right) - u_1\left(c_1^* + c_2^{**}, l_1^*\right), \\ &\geq u_1\left(c_1^{**} + c_2^{**}, l_1^{**}\right) - u_1\left(c_1^* + c_2^*, l_1^*\right) \end{aligned}$$

we have

$$u_1\left(c_1^{**}, l_1^{**}\right) + \beta V_1^* \geq u_1\left(c_1^{**} + c_2^{**}, l_1^{**}\right),$$

establishing that the variation is in fact incentive compatible and feasible. This completes the proof. ■

Lemma 8 *Suppose that Assumptions 1 and 2 hold, that* $\partial^2 u_j/\partial c \partial l \leq 0$ *for* $j = 1, 2$, *and that* $(V_1^*, V_2^*) \in \mathbf{V}^F$, $\left(V_1\left(h^t\right), V_2\left(h^t\right)\right) \notin \mathbf{V}^F$, $V_2\left(h^t\right) < V_2^*$ *and* $\mathbf{j}\left(h^t\right) = 2$. *Then,* $\alpha_2\left(h^t\right) > \alpha_2\left(h^{t-1}\right)$.

Proof. Suppose not. Then combined with Lemma 7, this would imply that $\alpha_j\left(h^t\right) = \alpha_j\left(h^{t-1}\right)$ for all $h^t \in P\left(h^{t-1}\right)$, and therefore $\left(V_1\left(h^t\right), V_2\left(h^t\right)\right) \in \mathbf{V}^F$, yielding a contradiction. ■

Now, combining the previous results in the Lemmas, we proveTheorem 7.

Proof of Theorem 7: The proof follows from combining Lemmas 5, 6, 7 and 8. Suppose that $\left(V_1\left(h^{t-1}\right), V_2\left(h^{t-1}\right)\right) \notin \mathbf{V}^F$ and, without any loss of generality, that $V_1\left(h^{t-1}\right) > V_1^*$. Lemma 6 then implies that $V_1\left(h^t\right) > V_1^*$ as long as there exists $\tilde{h}^t \in P\left(h^{t-1}\right)$ such that $\left(V_1\left(\tilde{h}^t\right), V_2\left(\tilde{h}^t\right)\right) \notin \mathbf{V}^F$. Applying Lemma 6 successively, we conclude that for any $k > 0$,

$V_j\left(h^{t+k}\right) > V_j^*$ as long as there exists $\tilde{h}^{t+k} \in P\left(h^{t-1}\right)$ such that $\left(V_1\left(\tilde{h}^{t+k-1}\right), V_2\left(\tilde{h}^{t+k-1}\right)\right) \notin \mathbf{V}^F$. Next, Lemma 7 implies that $\alpha_1\left(h^{t+k-1}\right) = \alpha_1\left(h^{t-1}\right)$ for all $h^{t+k-1} \in P\left(h^{t-1}\right)$ such that $\mathbf{j}\left(h^{t+s-1}\right) = 1$ for all $s, k \in \mathbb{N}$ with $s \leq k$. Combining this observation with Lemma 8 in turn implies that for all $k \in \mathbb{N}$ and for all $h^{t+k-1}, h^{t+k} \in P\left(h^{t-1}\right)$ such that $\mathbf{j}\left(h^{t+k}\right) = 2$, we have that $\alpha_1\left(h^{t+k}\right) < \alpha_1\left(h^{t+k-1}\right)$. Moreover, from Lemma 6 we have $\alpha_1\left(h^t\right) \geq \alpha^*$. Therefore, $\left\{\alpha_1\left(h^{t+k-1}\right)\right\}_{k=1}^{\infty}$ is a nondecreasing stochastic sequence in a compact set and must almost surely converge to unique limit point, say $\alpha_1\left(h^{t+k-1}\right) \rightarrow \alpha^{**} \geq \alpha^*$ where α^{**} is such that $\alpha_1\left(h^{t+k-1}\right) = \alpha^{**}$ for all $h^{t+k-1} \in P\left(h^{t-\mathbf{1}}\right)$. Unless $\mathbf{j}\left(h^{t+k}\right) = 1$ for all $k = 1, 2, ..., \infty$, this is only possible if at weights α^{**} the corresponding value vector $(V_1^{**}, V_2^{**}) \in \mathbf{V}^F$. Since $\mathbf{j}\left(h^{t+k}\right) = 1$ for $k = 1, 2, ..., \infty$ has zero probability given the assumption that the Markov process is irreducible, this establishes that $V_1\left(h^t\right), V_2\left(h^t\right) \rightarrow (V_1, V_2) \in \mathbf{V}^F$ almost surely as claimed in the theorem.■

The following two observations are useful.

Remark 1 Note that α^{**} is typically not equal to α^*, i.e., Pareto weights will typically converge to some $\alpha^{**} > \alpha^*$, which nonetheless induces a first-best allocation.

Remark 2 It is also useful to note that a first-best allocation does not mean that there are no transfers from one group to another. In a first-best allocation, typically one of the groups will be transfer resources to the other group (unless we happen to end up in the first-best allocation where Pareto weights are such that there should be no transfers between the two groups). The distinguishing feature of the first-best allocations is that transfers do not distort labor supply of either group and are independent of the identity of the group in power (thus ensuring full consumption and labor supply smoothing).

An immediate corollary of Theorem 7 is also useful to note.

Corollary 4 *Suppose that Assumptions 1 and 2 hold and that $\beta \geq \hat{\beta}$. Then as $t \rightarrow \infty$,* $\left(V_1\left(h^t\right), V_2\left(h^t\right)\right) \rightarrow (V_1^*, V_2^*) \in \mathbf{V}^F$ *almost surely.*

To illustrate the results of Theorem 7, let us now consider the same hypothetical economy, which was depicted in Figure 1, but with a higher discount factor $\beta = 0.75$, so that we are in the region where $\beta \geq \bar{\beta}$. For this economy, shown in Figure 2, we see that even though the

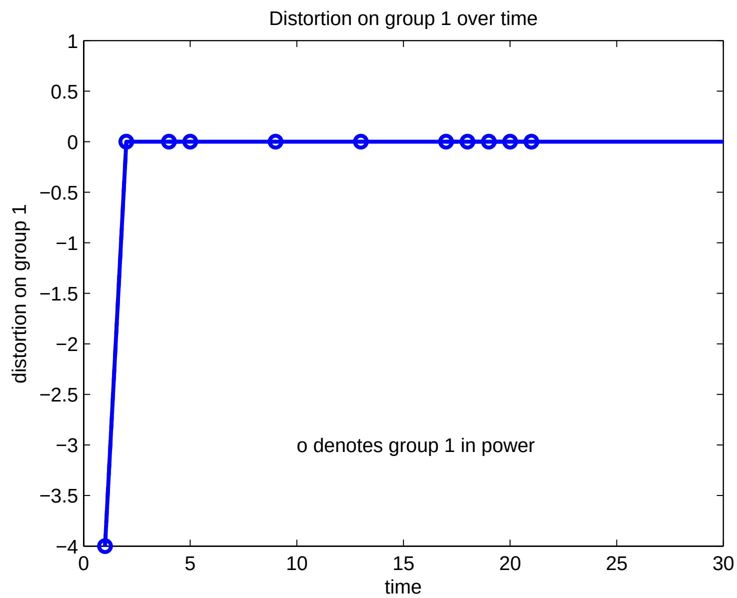

Figure 2: Distortions on the labor supply of members of group 1.

distortions start out as positive, the economy rapidly (in fact, in this case, in a single period) converges to a steady-state first-best allocation with no distortions.

5 Decentralization and the Structure of Taxation

The game (or the economic environment) we have considered so far has been both simple and abstract. We have only specified a rather simple rule for decision-making when a particular group comes to power. This has enabled us to characterize constrained Pareto efficient allocations without specifying the institutional structure regulating the interactions between or within groups. The drawback of this approach is that the Pareto efficient allocations specify equilibrium distortions and "wedges," but not taxes and transfers. However, as we emphasized in the title of the paper, our goal is to understand the structure of taxes (and how distortionary these taxes are) and how they evolve over time. Clearly there is a natural connection between taxes and the wedges between the marginal utility of consumption and disutility of labor.

In this section, we make this connection more explicit by considering a slightly modified version of the game form, in which the group in power explicitly sets taxes and transfers, which are allowed to be group specific but not individual specific. This game form will allow us to talk explicitly about taxes and transfers and also will show how the constrained Pareto efficient allocation considered so far can be decentralized as a competitive equilibrium. Our definition of competitive equilibrium here follows that of Chari and Kehoe (1990) and refers to allocations in which all individuals act as price (and policy) takers conditional on their beliefs regarding current and future policies that will be pursued by the group in power. Here in particular, the allocation will be competitive conditional on individuals' belief that the group in power will not expropriate all the output that has been produced in the economy.

This game form is as follows:

1. In each period t, we start with one group, j', in power.

2. Individuals make their labor supply decisions.

3. A representative agent from this group is chosen as the dictator and announces $\boldsymbol{\tau}_t = [\tau_{j,t}, T_{j,t}]_{j=1}^{N}$, where $\tau_{j,t}$ refers to a linear tax on the labor supply of all members of group j and $T_{j,t}$ is a lump-sum tax or transfer to all members of group j.[9]

4. The dictator representing the group in power decides whether to expropriate output, denoted by $\xi_t \in \{0, 1\}$. If $\xi_t = 1$, there are no factor payments and the group in power consumes the entire output.

5. The Markov process $m\,(j \mid j')$ determines who will be in power in the next period.

This game for makes it explicit that we can think of an equilibrium of an economy in which individuals supply their labor to a competitive labor market, where prices are determined competitively, but individuals face group-specific taxes and subsidies on their labor supply. Notice that somewhat differently from a standard competitive equilibrium, where all policies are given before individuals make their decisions, here the concept of competitive equilibrium requires individuals to make their decisions given their expectations of policies and government

[9] We could potentially allow more complex taxes, but linear and lump-sum taxes are sufficient to decentralize the constrained efficient allocation.

Also, in line with our restriction to group-specific, but not individual-specific allocation rules above, we cannot allow are tax rates that treat different members of a single group differentially. If we allow for such taxes, the individual chosen as the dictator would reallocate resources to himself at the expense of others in his own group as well.

actions (particular, over taxes, transfers and expropriation decisions). Though somewhat different than the standard competitive equilibrium notion, this feature is common in equilibrium models under endogenous policies without commitment (e.g., Chari and Kehoe, 1990).

Our first major result shows the equivalence between the allocations in this game and the Pareto efficient perfect equilibria considered above:

Theorem 8 *Suppose Assumptions 1 and 2 hold. Then, there exists a sequence of linear and lump-sum taxes* $\left\{\left[\tau_j\left(h^t\right), T_j\left(h^t\right)\right]_{j=1}^N\right\}_{t=0}^{\infty}$*, which constitutes a subgame perfect equilibrium in the game corresponding to political environment described by the game form above and implements the constrained efficient allocation characterized in Theorem 3. Let* $\lambda\left(h^t\right)$ *refer to the Lagrange multiplier on the sustainability constraint of the group in power in Theorem 3. Then, if* $\lambda\left(h^t\right) > 0$*, we have* $\tau_j\left(h^t\right) > 0$ *for all* $j \neq \mathbf{j}\left(h^t\right)$ *and* $\tau_{\mathbf{j}(h^t)}\left(h^t\right) < 0$*.*

Proof. Given competitive markets and taxes, each group will choose labor supply such that

$$\frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial c} = \left(1 - \tau_j\left(h^t\right)\right) \frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial l},$$

where from the budget constraint facing each individual, consumption is given by

$$c_j\left(h^t\right) = \left(1 - \tau_j\left(h^t\right)\right) l_j\left(h^t\right) + T_j\left(h^t\right). \tag{26}$$

The constrained Pareto efficient allocation must satisfy the first-order conditions in Lemma 9 and Theorem 3. Inspection of these conditions shows that for h^t such that $\lambda\left(h^t\right) > 0$, we must have $\tau_j\left(h^t\right) > 0$ for all $j \neq \mathbf{j}\left(h^t\right)$ and $\tau_{\mathbf{j}(h^t)}\left(h^t\right) < 0$, and then choose $\left\{T_j\left(h^t\right)\right\}_{j=1}^N$ so as to ensure the same levels of $\left\{c_j\left(h^t\right), l_j\left(h^t\right)\right\}_{j=1}^N$ in Lemma 9 and Theorem 3 according to (26) given $\left\{\tau_j\left(h^t\right)\right\}_{j=1}^N$. ■

An immediate implication of this theorem is the following.

Corollary 5 *For any* h^t *such that* $\lambda\left(h^t\right) > 0$*, the constrained efficient allocation cannot be decentralized with only lump-sum taxes.*

Theorem 8 and Corollary 5 imply that even though the political system (the group in power) has access to lump-sum taxes and transfers, it will tax the labor supply of other groups using linear, distortionary taxes. Moreover, it will subsidize the labor supply of the members of its own group, not simply using lump-sum transfers but again distortionary subsidies.

The intuition for this result is related to the fact that the constrained Pareto efficient allocation is being decentralized as a competitive equilibrium, whereby each individual takes market prices and policies as given. If all transfers and taxes were lump-sum, individuals would choose the level of labor supply equating the marginal utility of consumption to the marginal disutility of labor. However, as we saw above, this is not consistent with a constrained efficient allocation, because constrained efficiency requires the labor supply of the groups that are not in power to be below the unconstrained level so as to reduce the rents accruing to the ruling group. In a competitive equilibrium, where each individual does not take the impact of its own labor supply on the political economy constraints into account, this cannot be achieved using only lump-sum taxes and transfers. Put differently, there is an aggregate negative externality that an individual creates when he or she produces more output, by tightening the political economy constraints. In the constrained Pareto efficient allocation, the individual is made to internalize this negative externality by facing positive taxes on his or her labor supply. The role of pure political economy constraints in generating distortions and specific taxation patterns is further discussed in Acemoglu, Golosov and Tsyvinski (2006b).

6 Political Stability and Efficiency

As discussed in the Introduction, our simple environment enables us to investigate what types of Markov processes for power switches are conducive to sustainable first best allocations. Recall that the Markov process for power switches here captures the "stability of power allocations". For example, if the Markov process $m(j \mid j')$ makes it very likely that one of the groups, say group 1, will be in power all the time, we can think of this as a very "stable distribution of political power".

A common conjecture in the political economy literature is that such stable distributions of political power are conducive to better policies. For example Olson (1993) and McGuire and Olson (1996) reach this conclusion by contrasting an all-encompassing long-lived dictator to a "roving bandit". They argue that a dictator with stable political power is superior to a roving bandit and will generate better public policies. Similar insights emerge from the standard principal-agent models of political economy, such as Barro (1973), Ferejohn (1986), Persson, Roland and Tabellini (1997, 2000), because, in these models, it is easier to provide incentives to a politician who is more likely to remain in power. We next investigate whether a similar result applies in our context. In particular, we ask which types of Markov processes make it more likely that a large set of first-best allocations are sustainable.

To state the main result in this section, recall that $\mathbf{V}^F$ denotes the set of first-best sustainable allocations (Pareto efficient allocations that are first best). We write $\mathbf{V}^F[m]$ to emphasize that this set of first-best sustainable allocations may potentially depend on the Markov process for power switches, m. In addition, define $\mathcal{M}$ as the set of first-order Markov processes with N states, capturing the fluctuations of political power in our economy.

Our main result is that this common conjecture is not correct in our economy. On the contrary, the set of sustainable first-best allocations is independent of the Markov process for power switches.

Theorem 9 *Suppose Assumptions 1 and 2 hold. Fix β and the utility functions $\{u_j\}_{j=1}^N$. Then $(V_1^*, ..., V_N^*) \in \mathbf{V}^F[m]$ for some $m \in \mathcal{M}$ if and only if $(V_1^*, ..., V_N^*) \in \mathbf{V}^F[m']$ for all $m' \in \mathcal{M}$.*

Proof. By Fact 1, $(V_1^*, ..., V_N^*) \in \mathbf{V}^F[m]$ implies that

$$V_j^* = \frac{u_j\left(c_j^*, l_j^*\right)}{1-\beta} \text{ for } j = 1, ..., N,$$

such that

$$\begin{aligned}
\left\{c_j^*, l_j^*\right\}_{j=1}^N &= \arg \max_{\left\{c_j^*, l_j^*\right\}_{j=1}^N} \sum_{j=1}^N \alpha_j u_j\left(c_j, l_j\right) \\
& \quad \text{subject to} \\
\sum_{j=1}^N l_j &= \sum_{j=1}^N c_j.
\end{aligned}$$

for some $\boldsymbol{\alpha} \in \Delta^{N-1}$, where Δ^{N-1}, and moreover, this is the new constraints

$$\frac{u_j\left(c_j^*, l_j^*\right)}{1-\beta} \geq u_j\left(\sum_{j=1}^N l_j^*, l_j^*\right) \text{ for all } j = 1, ..., N.$$

Since neither left nor the right hand side of these constraints depend on the Markov process m, if $(V_1^*, ..., V_N^*) \in \mathbf{V}^F[m]$ for some $m \in \mathcal{M}$, then $(V_1^*, ..., V_N^*) \in \mathbf{V}^F[m']$ for all $m' \in \mathcal{M}$. ■

The following corollary is then immediate:

Corollary 6 *The set of first-best sustainable allocations and the cutoff discount factor $\bar{\beta}$ defined in Lemma 3 are independent of the Markov process for power switches m.*

This theorem and its corollary imply that the set of first-best allocations that are sustainable, i.e., that satisfy the sustainability constraints of all groups, is independent of the Markov process for power switches. This result contrasts starkly with the common conjecture discussed above. The reason why this conjecture does not apply in our model is quite informative. The intuition underlying the conjecture is that political instability reduces the "effective discount factor" of groups in power, because they anticipate that they will lose political power and thus the rents associated with power. In the economy studied here it is indeed the case that groups in power obtain rents (both in terms of greater current consumption and lower labor supply and also in terms of higher Pareto weights, which translate into higher consumption and lower labor supply in the future). However, the intuition about effective discount factors and the conjecture do *not* apply. This is because groups that hold political power can be rewarded (and punished) not only when they are in power, but also when they loose power. The conjecture mentioned above implicitly assumes that individuals or groups that hold power receive no consumption or a constant stream of consumption once out of power (e.g., they die or flee to the Cayman Islands). In contrast, the key distributional conflict in our model is between different social groups. The allocations that the social groups receive both when in power and when out of power are endogenous and determined as part of the Pareto efficient allocation. Consequently, the groups that hold power can be punished and rewarded when in power and also when out of power. This invalidates the simple intuition and the conjecture built on it.

In our baseline economy, the problem with the above-mentioned conjecture can be seen most easily. This is because possible deviations from the implicitly-agreed action profile are punished by switching to the worst subgame perfect equilibrium, which involves zero labor supply and zero consumption for all groups. Therefore, the punishment does not depend on the Markov process for power switches. Moreover, first-best allocations also do not depend on the Markov process, since they involve complete consumption and labor supply smoothing. Consequently, the Markov process for power switches affects neither the sustainability constraint nor the set of first-best sustainable allocations.

It is important to note, however, that this result does not imply that *the levels of utility* achieved by different groups starting from an arbitrary vector of Pareto weights are independent of the Markov process for power switches. The dynamics of utilities and distortions starting from such an arbitrary vector of weights will depend on the Markov process. Finally, this result refers to the "expected" stability of political power—i.e., whether power is expected to fluctuate between different social groups or whether it is supposed to remain the monopoly of one of them. Theorem 4 still applies and implies that for a given Markov process, a sample path in which the same group remains in power for a long time will induce a declining path of distortions.

7 Renegotiation-Proof Punishments

The analysis so far has followed the standard repeated games literature and focused on Pareto efficient allocations supported by the most severe punishments off-the-equilibrium path, which involve switching to the worst subgame perfect equilibrium characterized in Lemma 1. One disadvantage of these punishments is that they are not "renegotiation proof," meaning that the allocations during the punishment phase are Pareto dominated (e.g., Farrell and Maskin, 1989). The same reasoning which suggests that individuals will find ways of moving away from Pareto dominated allocations along-the-equilibrium path might apply off-the-equilibrium path as well. If so, we cannot support the allocations that rely on the worst subgame perfect equilibrium of Lemma 1.

In this section, we briefly discuss renegotiation-proof Pareto efficient allocations and show that most of our qualitative results continue to apply. Naturally, because punishments for deviating from the implicitly-agreed action profile are less severe, the set of Pareto efficient allocations will provide lower utilities for all players.

The main idea we use in this section is to use punishments that are still along the constrained Pareto efficient frontier. Let us return to Proposition 1. The best deviation for any group has not changed—it continues to be to allocate all production to itself. However, differently from before, the deviation continuation payoff for each group is no longer $V_j^d = 0$ (because this would involve the use of Pareto dominated punishments). What is the most severe Pareto efficient punishment on group j that is currently in power? The answer is clearly to move along the Pareto frontier to the point where this group has Pareto weight equal to 0. In fact, we can do better than this. Let $V_j(\boldsymbol{\alpha})$ be the expected value of group j given Pareto weight vector $\boldsymbol{\alpha} \in \Delta^{N-1}$. Since this function is continuous in $\boldsymbol{\alpha}$ and Δ^{N-1} is compact, the following expression is well defined:

$$\hat{V}_j^d = \min_{\boldsymbol{\alpha} \in \Delta^{N-1}} V_j(\boldsymbol{\alpha}). \tag{27}$$

Then, it is straightforward to see that in Proposition 1, the sustainability constraint for group j (when it is in power) becomes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_j\left(c_j\left(\omega^{t+s}\right), l_j\left(\omega^{t+s}\right)\right) \geq u_j\left(\sum_{j'=1}^{N} l_{j'}\left(\omega^t\right), l_j\left(\omega^t\right)\right) + \beta \hat{V}_j^d. \tag{28}$$

In other words, following a deviation by group j, we switch to a point along the Pareto frontier, which gives the lowest continuation utility to this group. Given the Pareto frontier, $\beta \hat{V}_j^d$ is just a number, thus all the analysis so far applies with $\beta \hat{V}_j^d$ replacing 0 as the deviation continuation payoff. The only complication is that at the end $\beta \hat{V}_j^d$ has to be determined as a function of the location of the Pareto frontier.

Despite this complication, the major results of our analysis so far apply. This is stated in the following result:

Theorem 10 *Suppose that Assumptions 1 and 2 hold and that we start with a Pareto weight vector* $\boldsymbol{\alpha} \in \Delta^{N-1}$ *such that* $\alpha_j > 0$ *for all* $j \in \mathcal{N}$*. Then the results in Theorems 2-8 generalize to the environment with renegotiation-proof punishments instead of the punishments given in Lemma 1.*

Proof. (sketch) The argument above shows that the set of constrained Pareto efficient allocations with renegotiation-proof punishments is a solution to a similar maximization problem to that in Proposition 3. To establish that the results in Theorems 2-8 apply, we need to prove two things. First, we need to ensure that the constraint set defined in (28) is nonempty. Second, we need to prove that the values given in (27) are well defined. Given these two features, the proofs of Theorems 2-8 generalize immediately.

First suppose that Theorem 2 applies. To show that the constraint set is nonempty, note that the argument in the proof of Theorem 4 establishes that along an equilibrium path supporting a constrained efficient allocation, Pareto weights are bounded away from 0 and 1. This implies that after any sequence of realizations, for each $j \in \mathcal{N}$, $\hat{V}_j^d$ is strictly less than continuation values, establishing that the constraint set given by (28) is nonempty.

To show that $\hat{V}_j^d$'s are well defined, let $\hat{V}^d \equiv \left(\hat{V}_1^d, ..., \hat{V}_N^d\right)$. Fix an arbitrary vector $\hat{V}^d$. Given this vector, the equivalent of the maximization problem (17) defines the vector of continuation values V for every vector of Pareto weights $\boldsymbol{\alpha} \in \boldsymbol{\Delta}^{N-1}$ and identity of the group in power given by $j \in \mathcal{N}$. Since these values are the solution to a maximization problem with continuous objective function and constraints given by compact-valued upper-hemi-continuous correspondences, by Berge's Maximum Theorem, V is continuous in the parameters, in particular, in the vector $\hat{V}^d$ and in the Pareto weights $\boldsymbol{\alpha}$. Then Brouwer's Fixed Point Theorem implies that $\hat{V}_j^d$ as defined in (27) exists for each $j \in \mathcal{N}$. Given the vector $\hat{V}^d$, then the sequence maximization problem (similar to that in Proposition 3) can be written recursively following the same steps as in the proof of Theorem 2, so 2 applies as hypothesized. The remaining theorems follow with essentially identical proofs. ■

Notice that Theorem 10 only generalizes Theorems 2-8 to the environment with renegotiation-proof punishments. The result that does not generalize is the one in Section 6, which concerns the relationship between first-best sustainable allocations and power switches is somewhat different. Now the right hand side of the sustainability constraint is no longer independent of the Markov process, since it involves $\beta\hat{V}_j^d$. Therefore, to maximize the set of first-best sustainable allocations, we need to minimize $\beta\hat{V}_j^d$ for each group. This implies that for the case of two-groups with identical utilities, we can establish the following result, which is the opposite of the common conjecture discussed above.

Proposition 4 *Consider a two-group economy where both groups have the same utility function $u(c,l)$ satisfying Assumption 1. Suppose also that there exists $\kappa > 0$ such that* $\min\{m(1 \mid 1), m(2 \mid 2)\} \geq \kappa$. *Then $\bar{\beta}$ as defined in Lemma 3 is minimized by setting*

$$m(1 \mid 2) = m(2 \mid 1) = 1 - \kappa,$$

i.e., by maximizing the probability of power switches between the two groups.

Proof. In a first best allocation, both groups receive constant levels of consumption and supply constant levels of labor. Therefore, the Markov process for power switches is irrelevant for the utility in the first best allocation. Recall that $\bar{\beta} = \beta(\alpha)$ for some α (see proof of Lemma 3). Focus on the first-best allocation for this α, say $(l_1^*, l_2^*, c_1^*, c_2^*)$, and we need to make sure that the right hand side of the sustainability constraint (28) is minimized for both groups.

Namely, when group j is in power, we need to satisfy

$$\frac{u\left(c_j^* l_j^*\right)}{1-\beta} \geq u\left(l_1^* + l_2^*, l_j^*\right) + \beta \hat{V}_j^d.$$

where $\hat{V}_j^d$ is the deviation continuation value for group j, which, as described above, involves setting their Pareto weight equal to 0. Given symmetry, to minimize $\bar{\beta}$, we need to minimize both $\hat{V}_1^d$ and $\hat{V}_2^d$. Moreover, for all $\beta < 1$, both $\hat{V}_1^d$ and $\hat{V}_2^d$ are minimized by maximizing the probability that the group that is in power today will loose power tomorrow, and thus the conclusion of the proposition follows. ■

Therefore in the environment with renegotiation-proof punishments, we obtain the converse of the conjecture discussed above. Namely, the set of sustainable first-best allocations is maximized when there are *frequent power switches* between different groups. Intuitively, the reason is that when a particular group is likely to stay in power in subsequent periods, then it perceives deviation as a more attractive option because after deviation, it can still obtain relatively high returns as it is likely to remain in power. This implies that a Markov process that makes it very likely that the group in power today will not be in power tomorrow implies more severe punishments against deviations and makes it more likely that first-best allocations are sustainable.

8 Conclusion

In this paper, we have studied the constrained Pareto efficient allocations in a dynamic production economy in which the group in power decides the allocation of resources. The environment is a canonical and simple model of political economy, in which different groups have conflicting preferences and at any given point in time, one of the groups has the political power to decide (or to influence) the allocation of resources. We have made relatively few assumptions on the interactions between the groups, except for taking the Markov process for power switches as exogenous. This has enabled us to look at a relatively general setup. In the spirit of focusing on the allocations that can be achieved given the distribution and fluctuations of political power in this society, we looked for the constrained Pareto efficient equilibria in this environment. Even in the constrained Pareto efficient equilibria, there are well-defined political economy distortions, which take an interesting form and also follow dynamics that we can characterize depending on the extent of discounting in the economy.

The distortions in constrained Pareto efficient equilibria are a direct consequence of the sustainability constraints, which reflect the political economy interactions in this economy. If these sustainability constraints are not satisfied, the group in power would allocate all production to itself. Clearly, if, instead of Pareto efficient allocations, we focused on Pareto dominated allocations, there would be further distortions. However, these distortions would be the result of possible coordination failures, of specific institutions preventing the emergence of constraint Pareto efficient allocations or of restrictions on policy spaces. Instead, the results here are driven by the location and shape of the Pareto frontier and by the "power" of different groups, which corresponds to what point the society is located along the Pareto frontier.

The analysis in the paper is simplified by the fact that these Pareto efficient allocations take a quasi-Markovian structure and can be represented recursively as a function of the identity of the group in power and updated Pareto weights. This recursive formulation allows us to provide a characterization of the level and dynamics of taxes and transfers in the economy.

We show that for high discount factors, the economy ultimately converges to a first-best allocation in which there may be transfers between groups, but labor supply decisions are not distorted and the levels of labor supply and consumption do not fluctuate over time. When discount factors are low, the economy converges to an invariant stochastic distribution in which distortions do not disappear and labor supply and consumption levels fluctuate over time, even asymptotically. In these allocations with distortions, the labor supply of individuals from groups that are not in power are taxed, while the labor supply of those in power is subsidized. The subsidies are useful to relax the political economy/sustainability constraints.

We also showed that the set of sustainable first-best allocations are independent of the Markov process for power change. This result contradicts a common conjecture that there will be fewer distortions when the political system creates a stable ruling group (see, e.g., Olson, 1993, or McGuire and Olson, 1996, as well as the standard principle-agent models of political economy such as Barro, 1973, Ferejohn, 1986, Persson, Roland and Tabellini, 1997, 2000). The reason why this conjecture is incorrect illustrates an important insight of our approach. In an economy where the key distributional conflict is between different social groups, these groups can be rewarded not only when they hold power, but also when they are out of power (and they engage in consumption and production). Consequently, the probability of power switches does not directly affect "effective discount factors" and potentially invalidating the insight on which this conjecture is based. In fact, in our baseline model the Markov process for power switches does not influence the punishments that can be imposed on social groups that deviate from the implicitly-agreed action profile and therefore the set of first-best allocations that are sustainable is independent of this Markov process. Nevertheless, it remains true that distortions decrease along sample paths where a particular group remains in power for a longer span of time.

We also discussed an alternative game in which the group in power sets taxes and transfers rather than directly deciding allocations. This game form also enables us to discuss the decentralization of the constrained efficient allocation. The interesting result we obtain here is that the constrained Pareto efficient allocation can be decentralized as a competitive equilibrium, but this involves the use of distortionary taxes. This result implies that the fluctuations of distortions, consumption and labor supply levels derived as part of the Pareto efficient allocations correspond to fluctuations in taxes—not simply to the presence of and fluctuations in "wedges" between the marginal utility of consumption and the marginal disutility of labor. The result that distortionary taxes must be used to decentralize the Pareto efficient allocation may be surprising, since the political system (the group in power) is assumed has access to lump-sum taxes then transfers. The intuition for this result is that in a competitive equilibrium individuals do not take the impact of their labor supply on the political economy constraints into account, and so each individual needs to face a distortionary tax so as to regulate his or her labor supply. The role of pure political economy constraints in generating distortions and specific taxation patterns is further discussed in Acemoglu, Golosov and Tsyvinski (2006b).

Finally, we showed that similar results apply when we consider renegotiation-proof Pareto efficient equilibrium. In this case, the Pareto efficient allocations provide lower utilities to all groups, because deviations from the implicitly-agreed behavior can be punished less severely and thus the sustainability constraints are tighter. Nevertheless, the general qualitative features of the Pareto efficient allocation are the same as in our baseline model. One interesting difference, however, is that now the set of first-best sustainable allocations is no longer independent of the structure of the Markov process for power switches. In particular, we show that, in contrast to the Olson-type conjecture, the set of sustainable first-that allocations is maximized when there are frequent power switches. This is because first-best allocations, by definition, do not depend on the Markov process for power switches (since they involve allocations that are independent of the identity of the group in power), whereas deviation continuation values payoffs are now minimized when there are frequent changes in the identity of the group that is in power.

We believe that the framework analyzed here is attractive both because we can analyze the effect of political economy distortions while specifying relatively few details about how groups with conflicting preferences interact and because the nature of the political economy distortions that arise do not rely on any type of coordination failures or on the absence of specific institutions or restrictions on policy spaces that lead to Pareto-dominated outcomes. A natural next step is then to investigate what types of institutional structures can support the constrained Pareto efficient allocations that arise in our economy. In other words, we would like to understand what types of specific institutions can ensure an allocation of political power and also impose the necessary constraints on the policies chosen by different groups so as to implement the constrained Pareto efficient allocations characterized here. Our analysis of decentralization with linear taxes and subsidies went one step in this direction, but remained silent on the political institutions necessary for such an implementation. An analysis of specific institutional structures that can implement constrained Pareto efficient allocations in economies with power switches between different social groups is an obvious area for future research. Another important area for future research is to endogenize the Markov process for power switches. In modern societies, fluctuations of political power between different groups arise because of electoral competition, possible political coalitions between different groups lending their support to a specific party or group, or in extreme circumstances, because different groups can use their *de facto* power, such as in revolutions or in civil wars, to gain *de jure* power (see, e.g., Acemoglu and Robinson, 2006a).

9 Appendix

In this Appendix, we provide some of the technical details and results omitted from the text. We start with the first-order conditions that characterize the Pareto efficient allocation. We then discuss how randomizations can be allowed without changing the conclusions in the text.

9.1 First-Order Conditions

Form a Lagrangean by assigning multipliers $\zeta\left(h^t\right)$ to (9), $\eta_j\left(h^t\right)$ to (14) and $\lambda\left(h^t\right)$ to (15) for history $h^t \in H_j^t$:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{t=0}^{\infty} \beta^t M\left(h^t \mid h^0\right)\left[\sum_{j=1}^{N} \alpha_j u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)\right] \qquad (29)\\
& + \sum_{h^t \in H^t, t} \zeta\left(h^t\right)\left[\sum_{j=1}^{N} l_j\left(h^t\right) - \sum_{j=1}^{N} c_j\left(h^t\right)\right] \\
& + \sum_{h^t \in H^t, t} \eta_j\left(h^t\right)\left[u\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)\right] \\
& + \sum_{h^t \in H^t, t} \lambda\left(h^t\right) \mathbf{I}_{j=\mathbf{j}(h^t)} \times \\
& \quad \left[\sum_{s=0}^{\infty} \beta^s M\left(h^{t+s} \mid h^t\right) u_j\left(c_j\left(h^{t+s}\right), l_j\left(h^{t+s}\right)\right) - u_j\left(\sum_{j=i}^{N} l_i\left(h^t\right), l_j\left(h^t\right)\right)\right],
\end{aligned}
$$

where $\mathbf{I}_{j=\mathbf{j}(h^t)}$ is the indicator function for the event $h^t \in H_j^t$. We then have:

Lemma 9 *Suppose that Assumptions 1 and 2 hold, then for any h^{t-1}, $h^t \in P\left(h^t\right)$ and $h_t \in H_{j'}^t$, the following first-order conditions are necessary:*

$$
\begin{aligned}
& \left[\alpha_j \beta^t M\left(h^t \mid h^0\right) + \eta_j\left(h^t\right) + \sum_{h^{t-s} \in H^{t-s}} \lambda\left(h^{t-s}\right) \mathbf{I}_{j=\mathbf{j}(h^t)} \sum_{s=0}^{t} \beta^{t-s} M\left(h^{t-s} \mid h^0\right)\right] \times \\
& \frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial c_j} = \zeta\left(h^t\right) \;\; \text{for } j \neq j', \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
- \quad & \left[\alpha_j \beta^t M\left(h^t \mid h^0\right) - \eta_j\left(h^t\right) + \sum_{h^{t-s} \in H^{t-s}} \lambda\left(h^{t-s}\right) \mathbf{I}_{j=\mathbf{j}(h^t)} \sum_{s=0}^{t} \beta^{t-s} M\left(h^{t-s} \mid h^0\right)\right] \times (31) \\
& \frac{\partial u_j\left(c_j\left(h^t\right), l_j\left(h^t\right)\right)}{\partial l} = \zeta\left(h^t\right) - \lambda\left(h^t\right) \frac{\partial u_{j'}\left(\sum_{j=i}^{N} l_i\left(h^t\right), l_{j'}\left(h^t\right)\right)}{\partial c} \;\; \text{for } j \neq j',
\end{aligned}
$$

$$\left[\alpha_{j'}\beta^t M\left(h^t \mid h^0\right)+\eta_j\left(h^t\right)+\sum_{h^{t-s}\in H^{t-s}}\lambda\left(h^{t-s}\right)\mathbf{I}_{j'=\mathbf{j}(h^t)}\sum_{s=0}^{t}\beta^{t-s}M\left(h^{t-s}\mid h^0\right)+\lambda\left(h^t\right)\right]\times$$
$$\frac{\partial u_{j'}\left(c_{j'}\left(h^t\right),l_{j'}\left(h^t\right)\right)}{\partial c}=\zeta\left(h^t\right), \tag{32}$$

and

$$-\left[\alpha_{j'}\beta^t M\left(h^t \mid h^0\right)-\eta_j\left(h^t\right)+\sum_{h^{t-s}\in H^{t-s}}\lambda\left(h^{t-s}\right)\mathbf{I}_{j=\mathbf{j}(h^t)}\sum_{s=0}^{t}\beta^{t-s}M\left(h^{t-s}\mid h^0\right)+\lambda\left(h^t\right)\right] \quad (33)\times$$
$$\frac{\partial u_{j'}\left(c_{j'}\left(h^t\right),l_{j'}\left(h^t\right)\right)}{\partial l}=\zeta\left(h^t\right)-\lambda\left(h^t\right)\frac{\partial u_{j'}\left(\sum_{j=i}^{N}l_i\left(h^t\right),l_{j'}\left(h^t\right)\right)}{\partial c}$$
$$-\lambda\left(h^t\right)\frac{\partial u_{j'}\left(\sum_{j=i}^{N}l_i\left(h^t\right),l_{j'}\left(h^t\right)\right)}{\partial l},$$

Proof. Given Assumption 1, first-order conditions of the Lagrangean (29) are necessary, and take the form given in the lemma. ■

9.2 Randomizations and Theorem 11

As discussed in the text, the maximization of (12) subject to (13), (14) and (15) is a potentially non-convex one. If non-convexities are important, allowing for public randomizations would improve the achievable value. In this part of the Appendix, we allow for such randomizations. We show that randomizations do not affect any of our major results, mainly because any randomization will be over a finite number of (in fact two) allocations, and each allocation in the support of the stochastic distribution induced by the randomizations will satisfy first-order conditions similar to those analyzed in the text. To establish this result, we will first state a generalized version of Theorem 2, which shows that the characterization of constrained Pareto efficient allocation takes a recursive form even when randomizations are allowed.

Formally, randomizations could be introduced by either considering correlated equilibria or by explicitly introducing a publicly-observed randomization device. We pursue the second strategy, since it allows for a tractable formulation in the context of the problem here.

Let us first formulate a version of Proposition 3 with randomizations. In particular, let $\mathcal{C}=\{\{c_j,l_j\}_{j=1}^N\in\mathbb{R}^{2N}:\sum_{j=1}^N c_j\le\sum_{j=1}^N l_j \text{ and } 0\le l_j\le\bar{l}\}$ be the set of possible consumption-labor allocations for different groups, and let $\mathcal{P}^\infty$ be the set of probability measures defined over the set $\mathcal{C}^\infty$. Moreover, for each t and $h^t\in H^t$, let $\zeta\left(\cdot\mid h^t\right)\in\mathcal{P}^\infty$ be a probability measure over consumption-labor allocations for different groups given history of power holdings h^t. Then, this randomized-version Proposition 3 can be written as

Problem A :

$$\max_{\zeta(\cdot|h^t)\in\mathcal{P}^\infty} \sum_{t=0}^{\infty} \beta^t \left[\int \sum_{j=1}^{N} \alpha_j u_j(c_j, l_j) \zeta(d(c_j, l_j) \mid h^t) \right]$$

subject to

$$\sum_{j=1}^{N} c_j \le \sum_{j=1}^{N} l_j \quad \zeta\left(\cdot \mid h^t\right)\text{-almost everywhere, for all } h^t \in H^t \text{ and all } t = 0, 1, ...,$$

$$\begin{aligned} u_j\left(c_j, l_j\right) &\ge 0\ \zeta\left(\cdot \mid h^t\right)\text{-almost everywhere,} \\ \text{for all } h^t &\in H^t, \text{ all } j = 1, ..., N, \text{ and all } t = 0, 1, ..., \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_j\left(c_j, l_j\right) &\ge u_j\left(\sum_{j'=1}^{N} l_{j'}\ , l_j\right)\ \zeta\left(\cdot \mid h^t\right)\text{-almost everywhere,} \\ \text{for all } h^t &\in H_j^t \text{ and all } t = 0, 1, ... \end{aligned}$$

We next assume that there exists an independently distributed uniform random variable υ_t publicly observed at each t before actions are taken. Consequently, actions can be conditioned on υ_t. This implies that formally, the full history ω^t should now include $\upsilon^t = (\upsilon_0, ..., \upsilon_t)$, and in terms of Proposition 3, we should now condition on $z^t = \left(h^t, \upsilon^t\right)$. Denote the set of z^t's by Z^t and partition this into $Z_1^t, ..., Z_N^t$, depending on which group is in power at time t. Once conditioning on this publicly-observed random variable is allowed, the maximization problem that characterizes the constrained Pareto efficient allocations can be written as

Problem B :

$$\max_{\{c_j(z), l_j(z)\}_{j=1}^N} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{N} \alpha_j u_j(c_j\left(z^t\right), l_j\left(z^t\right)) \right]$$

subject to

$$\sum_{j=1}^{N} c_j\left(z^t\right) \le \sum_{j=1}^{N} l_j\left(z^t\right)) \text{ for all } z^t \in Z^t \text{ and all } t = 0, 1, ..., \tag{34}$$

$$u_j\left(c_j\left(z^{t+s}\right), l_j\left(z^{t+s}\right)\right) \geq 0 \text{ for all } z^{t+s} \in Z^{t+s} \text{ and all } j = 1, ..., N. \tag{35}$$

and

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u_j\left(c_j\left(z^{t+s}\right), l_j\left(z^{t+s}\right)\right) \geq u_j\left(\sum_{j'=1}^{N} l_{j'}\left(z^t\right) \;, l_j\left(z^t\right)\right) \text{ for all } z^t \in Z_j^t \text{ and all } t = 0, 1, ...$$

Problem B allows for randomizations in the same way as Problem A, since allocations can be conditioned on the realization of the random variable today and in all past dates. Any solution to Problem A is a solution to Problem B and vice versa. Given this, our first result is a generalization of Theorem 2.

Theorem 11 *The constrained Pareto efficient allocation with randomizations has a quasi-Markovian structure whereby* $\{c_j(z), l_j(z)\}_{j=1}^N$ *depend only on* $\mathbf{s} \equiv \left(\{\alpha_j(z)\}_{j=1}^N, \mathbf{j}(z)\right)$*, i.e., only on updated weights and the identity of the group in power.*

Proof. The proof follows similar lines to that of Theorem 2. In particular, with a similar manipulation, we can write the maximization problem following history z^{t-1} as:

Problem C :

$$\max_{\{c_j(z), l_j(z)\}_{j=1}^N} \quad \mathcal{L} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \left[\sum_{j=1}^{N} \left(\alpha_j + \mu_j\left(z^{t-1}\right)\right) u_j\left(c_j\left(z^s\right), l_j\left(z^s\right)\right)\right]$$
$$-\mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \sum_{j=1}^{N} \lambda_j\left(z^s\right) \left[\mathbb{E}_s \sum_{s'=s}^{\infty} \beta^{s'-s} u_j\left(c_j\left(z^{s'}\right), l_j\left(z^{s'}\right)\right) - u_j\left(\sum_{i=1}^{N} l_i\left(z^s\right), l_j\left(z^s\right)\right)\right]$$

subject to (34) and (35). By construction, solutions to Problem B and Problem C coincide. The same argument as in the text establishes that Problem C has a recursive structure, where only the updated weights

$$\alpha_j\left(z^{t-1}\right) \equiv \frac{\alpha_j + \mu_j\left(z^{t-1}\right)}{\sum_{i=1}^{N}\left(\alpha_i + \mu_i\left(z^{t-1}\right)\right)}$$

and the identity of the group in power matter for future allocations. ■

Our next result states that the solution to Problem C will involve randomization using at most two values.

Theorem 12 *To characterize the Pareto efficient allocations, it is sufficient to restrict* $\zeta\left(\cdot \mid h^t\right)$ *for all* $h^t \in H^t$ *and for all* t *to have its support over two vectors of consumption, labor supply and updated Pareto weights.*

Proof. This theorem follows from Lemmas 7 and 8 in Appendix C of Acemoglu, Golosov and Tsyvinski (2006a). ■

In light of Theorem 12, we can consider the following simplified maximization problem, where following history h^{t-1}, the constrained Pareto efficient allocation is a solution to the following Lagrangian:

$$\max_{\left\{c_j^r(h), l_j^r(h), \xi^1(h)\right\}_{j=1}^N} \mathcal{L} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s M\left(h^s\right) \left[\sum_{r=1}^{2} \xi^r\left(h^s\right) \sum_{j=1}^{N} \left(\alpha_j + \mu_j^r\left(h^{t-1}\right)\right) u_j\left(c_j^r\left(h^s\right), l_j^r\left(h^s\right)\right)\right]$$

$$-\mathbb{E}_t \sum_{s=t}^{\infty} \beta^s M\left(h^{t+s}\right) \sum_{r=1}^{2} \xi^r\left(h^{t+s}\right) \sum_{j=1}^{N} \lambda_j^r\left(h^{t+s}\right) \left[\mathbb{E}_s \sum_{s'=s}^{\infty} \beta^{s'-s} u_j\left(c_j^r\left(h^{s'}\right), l_j^r\left(h^{s'}\right)\right) - u_j\left(\sum_{i=1}^{N} l_i^r\left(h^{t+s}\right), l_j^r\left(h\right.\right.\right.$$

subject (13) and (14). In this problem, $r = 1$ and 2 correspond to possible randomizations over two values given any history, so, for example, $c_j^r\left(h^t\right)$ and $l_j^r\left(h^t\right)$ are the consumption and labor supply levels for group j following history h^t into two possible events $r = 1$ and $r = 2$. Here the $\xi^r\left(h^t\right)$ correspond to the probabilities of these two possible events, (naturally with $\xi^1\left(h^t\right) + \xi^2\left(h^t\right) = 1$). Consequently, the first-order conditions in the text apply to $c_j^r\left(h^t\right)$ and $l_j^r\left(h^t\right)$ for $r = 1$ and $r = 2$, and all the necessary conditions and the resulting upward and downward distortions apply for each case separately.

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