Investment in Child Quality Over Marital States[1]

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September 2005

[1]This research is supported in part by a grants from the National Science Foundation and the C.V. Starr Center for Applied Economics at NYU. John Kennan, Ariel Pakes, Ken Wolpin and seminar participants at the Institute for Research on Poverty, NYU, Virginia, Wisconsin, Torino, the Society for Economic Dynamics 2002, ESPE 2002, and the American Economic Association 2002 Meetings provided valuable comments.

Abstract

Policies governing divorce and parenting, such as child support orders and enforcement, child custody regulations, and marital dissolution requirements, can have a large impact on the welfare of parents and children. Recent research has produced evidence on the responses of divorce rates to unilateral divorce laws and child support enforcement. We argue that in order to assess the child welfare impact of family policies, one must consider their influence on parents' investments in their children as well as the stability of the marginal marriage. Further, we expect that changes in the regulatory environment induce changes in the distribution of resources within both intact and divided families. We develop a continuous time model of parents' marital status choices and investments in children, with the main goal being the determination of how policies toward divorce influence outcomes for children. Preliminary estimates are derived for model parameters of interest, and simulations based on the model explore the effects of changes in custody allocations and child support standards on outcomes for children of married and divorced parents. We find that divorce laws have large impacts on investments in child quality improvements both within intact and nonintact households. Large changes in divorce regulations regarding custody arrangements and child support transfers have noticeable but small impacts on average child quality. However, they do have large effects on the distribution of parental welfare in the married and divorced states.

JEL codes: J12, J13, J18

1 Introduction

Divorced parenting in the U.S. is regulated through a combination of laws controlling marital dissolution, child custody and placement, and the assignment and enforcement of child support obligations. The primary objective of these activities is to increase the well-being of children and parents, and the divorce rate is often regarded as a first order measure of the success of family law. The rationale for this focus is the plethora of empirical evidence that suggests that children living in households without both biological parents are more likely to suffer from behavioral problems and have lower levels of a broad range of achievement indicators measured at various points in the life cycle (see, e.g., Haveman and Wolfe 1995). Recent empirical studies of unilateral divorce laws and child support enforcement have had some success in isolating the effects of changes in such legal structures on divorce rates (e.g., Friedberg 1998 and Gruber 2000). We suggest that in developing a complete picture of the influence of divorce regulations on the welfare of family members, particularly that of children, it would be productive to identify the manner in which changes in the legal climate affect child outcomes and the distribution of resources within the family. For example, how might a change in projected child custody allocations influence the probability of divorce? How might it affect each parent's interest in the quality of child outcomes? Taking each of these relationships into account, what is the net effect of the custody change on child welfare? Finally, if marriages vary in quality, what is the child welfare benefit of a policy that succeeds in stabilizing the marginal marriage?

Following the framework developed in Weiss and Willis (1985), Del Boca and Flinn (1995) and Flinn (2000) take as their starting point the problem of expenditure on children faced by divorced parents. The latter two papers model the role of institutions and the agents representing them in determining the welfare of divorced parents and children and take the models to data, but condition on the divorce event. In this research we remedy this potentially important omission by formulating and estimating a dynamic model of divorce and investment decisions. This extends the contribution of Weiss and Willis by looking at the joint evolution of children's human capital and parents' marital status.

We draw on two recent strands of the literature on marriage and childrearing. Analysis of family structure dynamics by Aiyagari, Greenwod and Guner (2000), Brien, Lillard and Stern (forthcoming), Chiappori, Fortin and Lacroix (2002) and others emphasizes the repeated interaction of a husband and wife over marital status and the allocation of household resources. The dynamic individual decision problem of a mother, or married parents with a common objective, is the focus of such child investment studies as Bernal (2003), Bernal and Keane (2005), and Liu, Mroz and van der Klaauw (2003). Our aim is to understand the endogenous growth of children's human capital where family structure and investments result from the distinct choices of mothers and fathers. Such an approach permits the study of divorce regulations with differing effects on the welfare and family attachment of mothers and fathers.

We develop a continuous time model of parents' marital status and child investment decisions. The value of marriage to parents is drawn from a population distribution of match values and evolves stochastically over time. Parents enjoy utility gains from marriage that result from the exogenously determined match value of the marriage and the output of their child investment decisions. The structure bears similarities to models of turnover and firm-specific human capital investment (e.g., Jovanovic 1979) in that parents invest in a project that produces greater returns while they remain attached to the family, and they have imperfect information on the future values of family attachment to each party. It differs from models of turnover in that parents' returns to investment in children may outlast the marriage match. The theoretical structure allows us to consider the influence of a change in the cost of divorce or each parent's access to the child in the divorce state on married parents' investment in children and their decision to continue in the marriage. Marital dissolution occurs as a result of exogenous changes in match quality, but whether a match quality shock is sufficiently negative to bring about divorce depends on earlier child quality investments and their results. Previous investment activities contribute to each parent's current benefit from remaining married and enjoying full access to the child. Thus the full history of marriage values and child investments determines current marital status and child investment. If the history of child investments and marriage values is poorer for the marginal marriage than it is for the representative marriage, then, all else equal, the child welfare gain associated with the continuation of the marginal marriage is smaller than that associated with the continuation of the representative marriage.

Regression analysis using our estimation sample of National Longitudinal Survey of Youth (NLSY) only-child families, described in more detail below, demonstrates a negative association between divorce and young children's academic achievement that is consistent with the catalog of such findings in Haveman and Wolfe. Table 1 contains estimates from the ordinary least squares regression of (roughly) six year old children's age-normed math test scores on child and parent characteristics. We find that divorce is associated with a decrease of 10.2 percentile points in children's test scores, and the coefficient on divorce differs significantly from zero. Measures of parents' abilities including the mother's Armed Forces Qualifying Test (AFQT) score and the father's education at the child's birth covary positively with children's test scores, as expected. Observable parent characteristics that we find to be associated with greater marriage stability are also positively associated with children's test scores. A year increase in the father's age at the child's birth is associated with a 0.8 percentile point increase in the child's test score. Mothers who report that their marriages are "very happy", as opposed to "fairly happy" or "not too happy," have children whose test scores are 5.5 percentile points higher, on average.[1] Finally, mothers earning greater shares of family income have children with substantially higher test scores: an increase of 0.25 in the mother's share of family income is associated with a 5.0 percentile point increase in her child's test score, on average.

It is difficult to interpret coefficient estimates obtained using this simple approach, and in general there are clear problems with this type of treatment of the evolution of family structure and children's academic performance. We estimate the dynamic model of marital status and child investment decisions described above in order to understand the processes that underlie the relationships evident in Table 1. We use the Method of Simulated Moments (MSM) and our sample of NLSY families to estimate the parameters of the model. The estimates indicate a diminishing return to child investments under our specification of the stochastic child quality production process. Child quality and parents' consumption contribute similar amounts to parents' welfare at the sample average income, child quality and investments and the estimated vector of parameters. Where parents remain married at the test date, the covariances of children's test scores with fathers' and mothers' incomes are 40.71 and 52.14, respectively. Among divorced parents they are 16.84 and 70.93. One question posed by the estimation is whether the model can fit such wide disparities in the covariances between parents' individual incomes and children's test scores without relying

[1] Sample restrictions that allow us to include marriage quality measures in all of the estimation discussed in this paper also lead to a smaller sample size, costing us some precision. The large negative coefficient on the unhappy marriage indicator reported in table 1 has a t-statistic of 1.3.

on differences in mothers' and fathers' tastes. We find that the model is in fact able to fit the patterns observed in the data based on the existing family law's treatment of mothers and fathers, while maintaining the requirement that mothers and fathers place equal preference weight on child quality.

The repeated interaction between independent decision makers modeled here allows us to perform otherwise impossible policy experiments that address the redistributive aspects of family law changes and their distinct influences on mothers' and fathers' family attachments. We use the NLSY data and the point estimates of model parameters to predict the child quality and welfare effects first of joint custody with zero child support and then of 90 percent maternal custody and a quarter of the father's income going to child support when divorced. Changes in average child quality across the three policy environments are relatively small, with the greatest difference being 4 percent of average child quality between the current policy and the shared custody environments. When compared with the existing policy, the joint custody experiment represents a substantial redistribution of welfare from mothers to fathers. Both the highest average child quality and the highest divorce rate are achieved under the joint custody regime. While these experiments demonstrate some of the tensions among the welfare of mothers, fathers and children confronted by policymakers, they must be interpreted with caution given the simplifying assumptions and sample restrictions we have imposed. Future research will address the fertility decision that we have omitted in this study, and we hope that this will lead to more comprehensive policy analysis.

The paper proceeds as follows. In Section 2 we present a model of the dynamic marriage and child investment decisions of two parents involved in rearing a child. Our intent is that the model be suited to address questions of the influence of the regulation of divorced parenting on both intact and divided households. Section 3 describes methods used in estimating the model parameters given data on parents' incomes, marital status and marriage quality and children's ages and attainments. Section 4 introduces data on parents and children from the National Longitudinal Survey of Youth (NLSY), and the results of the empirical analysis are reported in section 5. In Section 6 we present the output of simulations based on the model which are intended to illustrate the effects of policy changes resembling recent child custody and support reforms in the U.S. and western Europe. Section 7 concludes.

2 A Model of Child Investment and Divorce Decisions

There exist three agents in our model, two parents and one child. The welfare of the child at a point in time is summarized by its "quality," which is a scalar nonnegative index denoted by k. The model is set in continuous time, and the instantaneous utility function of parent p is given by

$$u_p(c_p, k, \theta, d) = \alpha_p \ln(c_p) + (1 - \alpha_p)\tau_p(d) \ln(k) + (1 - d)(r(z_p) + \theta),\ p = 1, 2; \tag{1}$$

where c_p is the consumption of a private good by parent p, d is an indicator variable that takes the value 1 if the parents are divorced, θ is a marriage-specific match value, $\tau_p(d)$ is the amount of contact that the parent has with the child, $r(z)$ is a deterministic function of observable characteristics z_p representing the "direct" utility of remaining married, and $\alpha_p \in (0, 1)$ is the preference weight on private consumption. We assume throughout that the price of private consumption is fixed at 1 for both parents.

The utility derived from current child quality by each parent is modified according to the amount of contact the parent has with the child in each marriage state. Time with the child is represented by $\tau_p(d)$ for $p = 1, 2$ and $d = 0, 1$. We assume that when married the parents enjoy complete and concurrent access to the child's time; without loss of generality, $\tau_1(0) = \tau_2(0) = 1$. Though their intrinsic valuation of the child remains the same in the divorce state, the fact that the child becomes an "excludable" good after divorce reduces the utility flows that parents receive from any given level of child quality. We assume that parents share time with the child in divorce, implying $\tau_1(1) + \tau_2(1) = 1$ and $\tau_p(1) \geq 0$, $p = 1, 2$, and that physical custody and visitation allocations are fully anticipated and set exogenously with respect to parental behaviors.

Parents receive incomes of $y_p(d)$, $p = 1, 2$ and $d = 0, 1$. Note that we assume that parents' incomes may differ from the marriage to the divorce state. We assume that this difference is generated by a child support transfer of $\pi y_1(0)$ from parent 1 to parent 2. We will consider equilibrium investment and divorce decisions under two divorce law regimes, represented by state variable $l \in \{0, 1\}$ and discussed below. Thus the set of exogenous policy parameters in the model is $(\tau_1(1), \pi, l)$.

The dynamics of the model are as follows. Parents begin life with a child of quality level k and with a marriage-specific match value of θ. At any moment in time, at most one of five possible events may occur. First, the child quality may change value. Child quality assumes one of a finite number of values, $k \in K = \{k_1, ..., k_T\}$, where $k_1 < k_2 < \ldots < k_T$. The current child quality will be interpreted in the analysis that follows as a measure of the child's achievements relative to her or his age cohort. The empirical analog to k that we consider is an age-normed measure of academic performance or behavioral traits. Gains and losses in child quality follow a process that we divide into exogenous and endogenous components. Costly investments in child quality made by the parents increase the rate at which improvements in child quality arrive. The child quality improvement rate is described by the function $\delta(i_1, i_2)$, where i_p denotes the child quality investment of parent p and $\frac{\partial \delta(i_1,i_2)}{\partial i_p} > 0$ for $p = 1, 2$. We assume a transition function for child quality in which arriving improvements increase current quality from k_t to k_{t+1} with certainty whenever $1 \leq t < T$.

Second, the child may experience a setback. Setbacks occur at exogenous rate $\widetilde{\sigma}$, and lead to a decline in child quality from k_t to k_{t-1} whenever $1 < t \leq T$. For notational convenience, we define

$$\sigma(k_t) = \begin{cases} \widetilde{\sigma} \text{ where } 1 < t \leq T \\ 0 \text{ where } t = 1. \end{cases}$$

The third and fourth possible events are an increase and a decrease in match value θ. The value of the marriage match is included to permit parents' investments to respond to current information on the stability of the marriage. Like child quality, match quality assumes one of a finite number of values, $\theta \in \Theta = \{\theta_1, ..., \theta_M\}$, where $\theta_1 < \theta_2 < \ldots < \theta_M$. Match quality increases arrive at rate $\widetilde{\gamma}^+$. Given a current match quality of θ_m, the arrival of a match quality increase leads with certainty to a new match quality of θ_{m+1} whenever $1 \leq m < M$. Symmetrically, match quality decreases arrive at rate $\widetilde{\gamma}^-$, and an arriving decrease in match quality leads to a drop from θ_m to θ_{m-1} whenever $1 < m \leq M$. As with the child quality setback rates, for convenience of notation we define

$$\gamma^+(\theta_m) = \begin{cases} \widetilde{\gamma}^+ \text{ where } 1 \leq m < M \\ 0 \text{ otherwise} \end{cases} \quad \text{and} \quad \gamma^-(\theta_m) = \begin{cases} \widetilde{\gamma}^- \text{ where } 1 < m \leq M \\ 0 \text{ otherwise.} \end{cases}$$

Thus the values of $\gamma^+(\theta_m)$ and $\gamma^-(\theta_m)$ determine the degree of persistence in marriage quality.

Finally, the child may attain functional independence at the current age-normed child quality, in which case the child quality improvement process ends. The parents enjoy a terminal value that increases with the current child quality level and continues to depend on the parents' marital status.[2] Termination of the investment process occurs at exogenous rate η; state variable $g \in \{0, 1\}$ indicates the current investment condition, and equals 1 when the investment process has been terminated.

Each parent is assumed to have a fixed income flow y_p. While conceptually it is straightforward to augment the model with an exogenous income process for both parents, the computational cost of doing so is prohibitive. Moreover, we face the standard problem of limited data on the income trajectories of divorced fathers. To keep the model tractable, we abstract from the phenomenon of remarriage by assuming that once parents exit the marriage state they never reenter it.

In modeling the behavior of married and divorced parents an important specification choice is the manner in which spouses interact. One may assume that spouses interact cooperatively or noncooperatively. Under the cooperative specification, spouses make decisions that place the welfare of family members on the Pareto frontier, and some sharing rule is chosen for the division of the surplus from cooperation.[3] In the noncooperative case, spouses make decisions representing the equilibrium of a Nash or Stackelberg game, and the family may not achieve the Pareto frontier.[4] We look to the empirical literature on divorce regulation for guidance in constructing our model. A testable implication of the hypothesis that married spouses behave cooperatively is that only divorces that are efficient for the family occur. Since laws governing the consent to divorce do not change total family resources, but rather shift property rights within the marriage, a change from bilateral to unilateral divorce laws should have no effect on the decision to divorce when married partners behave cooperatively. As discussed above, in recent studies on the subject both Friedberg and Gruber find significant effects of unilateral divorce laws on rates of marital dissolution in the U.S., indicating noncooperative interaction in married households. Below we assume that parents behave noncooperatively no matter what their marital state, and that investment strategies

[2] An alternative approach to finalizing the child investment process would be to impose a fixed time horizon of 18 or 21 years, after which children achieve independence. The drawback to this approach is that it generates strategic manipulations by parents approaching the date of independence that we find unrealistic.

[3] See, for example, Browning and Chiappori (1998).

[4] See, for example, Lundberg and Pollak (1994) and Udry (1996).

constitute a Markov Perfect Equilibrium.[5] In our discussion of the theoretical results we dedicate some attention to the effects of this modeling choice.

2.1 Divorced Parents

Under our assumptions, divorce is an absorbing state in each parent's marital "life cycle." When the parents are divorced and the child quality improvement process is terminated, a parent p whose child is of quality k_t enjoys terminal value

$$V_p(k_t, d = 1, g = 1) = \frac{\alpha_p \ln(y_p(1)) + (1 - \alpha_p)\tau_p(1)\ln(k_t)}{\rho},$$

where ρ is the instantaneous discount rate. In the case of divorce with an ongoing child quality improvement process, each parent's decision is how much to invest in the child. We therefore look for an equilibrium in parental investments, which is determined by the state of child quality and the parental income distribution. To find the equilibrium, we first solve for the reaction function of parent p; this is the decision rule used by parent p in determining his or her investment level conditional on the investment level of the other parent. The conditional value of the future to divorced parent p is given by

$$\begin{aligned} V_p(k_t, d = 1, g = 0|i_{p'}) = \max_{i_p}(\rho + \delta(i_p, i_{p'}) + \sigma(k_t) + \eta)^{-1}\{\alpha_p \ln(y_p(1) - i_p) + (1 - \alpha_p)\tau_p(1)\ln(k_t) \\ +\delta(i_p, i_{p'})V_p(k_{t+1}, d = 1, g = 0) \\ +\sigma(k_t)V_p(k_{t-1}, d = 1, g = 0) + \eta V_p(k_t, d = 1, g = 1)\}. \qquad (2) \end{aligned}$$

To find the equilibrium investment levels we solve the dynamic reaction functions. Let the function $i_p^*(i_{p'}, k_t, d = 1)$ denote the optimal level of investment by divorced parent p given current child quality level k_t and investment by the other parent of $i_{p'}$. This function is the argument i_p that maximizes the right hand side of [2]. Given the reaction functions $i_1^*(i_2, k_t, d = 1)$ and

[5] See, for example, Pakes and McGuire (2000).

$i_2^*(i_1; k_t, d = 1)$, an equilibrium is a pair of investment values $(\hat{\imath}_1, \hat{\imath}_2)(k_t, d = 1)$ such that

$$\begin{aligned} \hat{\imath}_1 &= i_1^*(\hat{\imath}_2, k_t, d = 1) \\ \hat{\imath}_2 &= i_2^*(\hat{\imath}_1, k_t, d = 1). \end{aligned} \tag{3}$$

The properties of this reaction function depend critically on the properties of the improvement rate function δ. Along with $\frac{\partial \delta(i_1, i_2)}{\partial i_p} > 0,\ p = 1, 2$, we assume that δ is twice continuously differentiable and concave, and add to these the restriction that i_1 and i_2 behave as (weak) substitutes. Under these assumptions, $\frac{d i_p^*(i_{p'}, k_t, d=1, g=0)}{d i_{p'}} < 0$ and the reaction function is negatively sloped for each parent p and for all values of $k_t < k_T$.

The expressions in [3] do not fully characterize the equilibrium of the model, since the reaction functions themselves depend upon the equilibrium values $V_p(k_{t'}, d = 1, g = 0)$, $\forall t' \neq t$. Equilibrium in the divorce state for a family with an active child investment process is therefore determined over the $2T$ parent- and child quality-specific values as well as the $2T$ parent- and child quality-specific investments. The solution is obtained numerically, and the numerical technique employed is simplified by restrictions on the relationships among equilibrium values arising from the theory and the use of the $2T$ values of terminal child qualities. Given the ordering of child qualities and the possibility of setbacks when the investment process is active, we know that $V_p(k_T, d = 1, g = 1)$ dominates the divorce-state values of (a) all terminal child qualities k_t such that $t < T$ and (b) *all* non-terminal child qualities. Additionally, $V_p(k_t, d = 1, g)$ increases monotonically with k_t for both $g = 0$ and 1. The numerical solution produces equilibrium investment levels $\{\hat{\imath}_1(k_t, d = 1, g = 0), \hat{\imath}_2(k_t, d = 1, g = 0)\}\}_{t=1}^T$ and value functions $\{V_1(k_t, d = 1, g = 0), V_2(k_t, d = 1, g = 0)\}_{t=1}^T$.

2.2 Married Parents

The experiences they will have if they enter the divorce state can meaningfully affect the investment decisions of forward-looking married parents. In particular, currently married parents who believe that divorce is likely in the near future will make investment decisions that look more like those made by divorced parents than will couples who believe that divorce is a remote possibility. In our model, the likelihood of divorce is partially endogenous and partially exogenous. We posit the existence of a match value of the marriage θ that evolves according to an exogenous stochastic process. We structure the problem so that when this value becomes sufficiently low, parents divorce. Given the existence of a critical match value property for divorce decisions, we will show that the critical value depends both on exogenous characteristics, such as the parental income distribution, and on the endogenous history of investments in child quality.

We must also specify the manner in which divorce decisions are made. Under our assumption of noncooperative behavior, these decisions are not, in general, efficient. The nature of the decisions depends critically on legal statutes. In our empirical work, we will look at two different cases, one in which it is enough for one of the parents to ask for a divorce for the couple to enter the divorce state and the second in which both parents must agree to the divorce for it to occur. These cases are commonly called unilateral and bilateral divorce regimes, and they will be represented in the model by $l = 0$ and 1, respectively. Side payments between parents are permitted in the married state. Given the irreversibility of the divorce decision in our model, and the consistent availability of the divorce option to each spouse under unilateral divorce law, we infer that side payments from a marriage-preferring spouse to a divorce-preferring spouse may credibly sustain a marriage. However, a promise of side-payments following divorce from a divorce-preferring spouse to a marriage-preferring spouse is not credible, and therefore no series of side payments can buy a marriage-preferrer into divorce under bilateral divorce law. With knowledge of the process by which divorce decisions are made, we define $Q_p(k_t, \theta, g, l)$ as the value to parent p of the marital status chosen in equilibrium by both parents in state (k_t, θ, g, l).

The derivation of the marriage state equilibrium is similar to that of the divorce state equilibrium, with one major difference being the search for an equilibrium in divorce decisions and side payments, as well as investments and values. As before, we begin with the value of a terminated child investment process at k_t for parent p:

$$\begin{aligned} V_p(k_t, \theta_m, d = 0, g = 1, l) = (\rho + \gamma^+(\theta_m) + \gamma^-(\theta_m))^{-1} \{ & \alpha_p \ln(y_p(0)) \\ & + (1 - \alpha_p) \ln(k_t) + r(z_p) + \theta_m + \gamma^+(\theta_m) Q_p(k_t, \theta_{m+1}, g = 1, l) \\ & + \gamma^-(\theta_m) Q_p(k_t, \theta_{m-1}, g = 1, l) \} \end{aligned} \tag{4}$$

Next, given the current child quality level and match value, we solve for the equilibrium investment levels and associated values for each parent conditional on the continuation of the marriage. As in the divorce case, using the reaction functions we can define a pair of equilibrium investment levels and parent-specific state values associated with marriage that are given by

$$(\hat{\imath}_p, \hat{\imath}_{p'})(k, \theta, d = 0, g = 0);\ (V_p, V_{p'})(k, \theta, d = 0, g = 0). \tag{5}$$

The investment equilibrium depends on the current marriage quality both through its direct influence on the productivity of child investment and through its effect on the anticipated duration of the parents' marriage, which contributes to the value gain parents experience with an increase in child quality.

With the parents' equilibrium investments in the child found as in [5], the value to parent p of marriage, an ongoing child improvement process, and child quality k_t is

$$\begin{aligned} V_p(k_t, \theta_m, 0, 0, l) \quad = \quad & (\rho + \gamma^+(\theta_m) + \gamma^-(\theta_m) + \delta(\hat{\imath}_p, \hat{\imath}_{p'}) + \sigma(k_t) + \eta)^{-1}\{\alpha_p \ln(y_p(0) - \hat{\imath}_p) \\ & +(1 - \alpha_p)\tau_p(0)\ln(k_t) + r(z_p) + \theta_m + \gamma^+(\theta_m)Q_p(k_t, \theta_{m+1}, 0, l) \\ & +\gamma^-(\theta_m)Q_p(k_t, \theta_{m-1}, 0, l) + \delta(\hat{\imath}_p, \hat{\imath}_{p'})V_p(k_{t+1}, \theta_m, 0, 0, l) \\ & +\sigma(k_t)Q_p(k_{t-1}, \theta_m, 0, l) + \eta Q_p(k_t, \theta_m, 1, l)\}, \end{aligned}$$

where we have ignored the possibility of side-payments for expositional convenience. To find equilibrium investments, values, and divorce decisions over the marriage quality distribution and for all child quality levels, we again make use of the restrictions on the relative values of the possible child and marriage quality states implied by the theory. Again the solution is obtained numerically, but in this case equilibrium occurs over all $2T$ parent- and child quality-specific values and investments across all M possible values of θ. Computation of the equilibrium is simplified by the presence of the terminal values represented in [4]. Having followed the above steps, we have the complete solution for the marriage state,

$$\{(\hat{\imath}_1, \hat{\imath}_2)(k_t, \theta, 0, g, l), (V_1, V_2)(k_t, \theta, 0, g, l)\}_{t=1}^T, s = 0, 1, l = 0, 1,$$

along with divorce decisions and side payments for every value of θ in the set Θ.

Figures 1.a-c represent equilibrium parental investments and divorce decisions for a specific parameterization of the model. They are included in order to build some intuition regarding the roles of custody, income and marriage quality in determining the investment equilibria in marriage and divorce and the parents' divorce decision. We assume that the child quality takes one of five values. Figures 1a and b show the total investment and father's investment share over the range of child quality levels assuming 80% maternal custody in divorce. The model solutions presume that the mother's income is \$20,000 and the father's income is \$30,000, scaled to units of \$5000 in the solution. The set of possible marriage quality values is discrete, indexed by $m = 1, ..., 10$, and ordered such that $\theta_1 < \theta_2 < ... < \theta_{10}$. The father's share of investments in the child in the divorce state is 0 under 80% maternal custody. Interestingly, the father's share of child investment increases with child quality in marriage where the father expects less than majority custody in the event of divorce. Given our assumption of bilateral divorce laws in the solutions represented, it appears that the parent who least prefers divorce makes a growing share of the investments as child quality approaches its ceiling. This must have the effect of increasing the value of continued marriage to the parent who most prefers divorce as the welfare loss with divorce to the marriage-preferring parent increases.

Figure 1.c shows the sets of (k, θ) for which parents will choose to remain married, and those for which parents will choose to divorce given bilateral divorce standards. It is evident that marriage will be sustained at lower θ for a given k when one parent obtains majority custody. When custody is shared, parents' values of marriage and divorce are similar, and as a result the θ at which the parent least preferring divorce chooses to sustain the marriage must be greater. We observe that as k increases the θ required to sustain the marriage declines, indicating that the value of marriage to the parents does not dominate the value of child quality, or vice versa, in the determination of marital status outcomes for this specification of the model.

3 Estimation Method

We estimate the model using a simulated method of moments estimator. In this section we provide the details of the estimation procedure. Some preliminary model estimates are presented in the following section.

While the general estimation strategy we outline below can be used with any number of functional form assumptions on the investment process that satisfy our conditions for uniqueness of the Nash equilibrium investment choices, in the results reported below we assume that

$$\delta(i_1, i_2) = \delta_0[i_1 + i_2]^{\nu},$$

where $\nu \in (0, 1)$ and δ_0 is a positive constant. This form of the δ function satisfies the requirement that $\frac{\partial^2 \delta((i_p^*, i_{p'})}{\partial i_p \partial i_{p'}} \leq 0$. To economize on parameters, we have also assumed that $\gamma^-(\theta_m) = \gamma^+(\theta_m) = 0$ for all θ_m. Thus we view the match quality draw as permanent; some rationalization of this assumption is provided below when we discuss the empirical results.

The endogenous variables utilized in the estimation procedure consist of a youth's score on a mathematics examination administered as part of the NLSY Child survey (some details on the nature of this examination are provided in the following section) and whether the child's parents are divorced at the time the test was taken. We denote child $j's$ score on the test by $Z(j)$, and to assist in identification we will assume that there is a deterministic mapping that exists between this score and the child's human capital level. In particular, we assume that

$$k(j) = k_t \Leftrightarrow z_{t-1} < Z(j) \leq z_t, \ t = 1, ..., T,$$

where $z_0 = 0$, $z_T = 100$, and $z_0 < z_1 < ... < z_T$. We denote child $j's$ age at the time the test was taken by $a(j)$, and the binary variable that indicates that the parents were divorced at this time is given by $d(j)$. The parents' incomes are assumed constant for purposes of this analysis and are denoted by $y_p(j)$, $j = 1, 2$.

Conditional on the parental income observations and the age of the child at the survey, the endogenous variables are $k(j)$ and $d(j)$. As the model clearly demonstrates, these variables are functions of realizations of exogenous and endogenous stochastic processes. The exogenous stochastic processes are those that describe the termination date of the "window" for child quality improvement and the realization of the permanent marriage quality characteristic θ. The endogenous stochastic process is the one determining the timing of improvements in child quality; this process is endogenous since parental behavior determines the (average) rate of change.

For each sampling unit, the dependent variables (jointly) take one of $T \times 2$ possible values. Because the stochastic process generating these outcomes is rather complicated due to the endogeneity of the investment in child quality improvements, we utilize the method of simulated moments to estimate the model. To implement this procedure requires access to a large number of simulated sample paths for each sample household j, all of which terminate at age $a(j)$ with a realization of $(k^r(j), d^r(j))$. For the moment, condition on the states of marriage and child quality at the time of the birth of the child (θ and k^0). Given $\theta, k^0, y_1(j), y_2(j)$, and that the parents are married at the time of the birth, we first solve for the equilibrium investment rate in child quality, $(\hat{\imath}_1, \hat{\imath}_2)(k^0, \theta, d = 0, g = 0)$.[6]

The rate of child quality improvement immediately following the birth of the child is given by $\delta(1) = \delta_0[\hat{\imath}_1(k^0, \theta, d = 0, g = 0) + \hat{\imath}_2(k^0, \theta, d = 0, g = 0)]^{\nu}$. The rate of arrival of a negative shock to the child's quality level, one that results in a decrease of one level, is given by $\widetilde{\sigma} > 0$, for $t = 2, ..., T$. The rate of arrival of the (exogenous) termination of the child quality process is fixed once and for all at η; recall that this is a one time event.

Let there be N sample observations. For each observation we perform R replications for each possible set of initial conditions (k^0, θ). The "base draws" for the random number generation are kept constant across iterations of the estimation algorithm to facilitate the convergence process. For any given individual, we draw a total of $R \times S$ values from a uniform pseudo-random number generator for use in generating the timing of changes in the child quality improvement process (these values are denoted $u^{(1)}$), another $R \times S$ uniform random numbers for use in the generation of the timing of decreases in the level of child quality ($u^{(2)}$), and an $R \times 1$ vector for generating the duration of the "window" for child quality improvement ($u^{(3)}$). S is chosen as an upper bound on the number of spells experienced by any one family prior to reaching the child's age at the time the test is taken.

Consider the generation of the first event for a sample member with endogenous arrival rate parameter $\delta(1)$ and exogenous rate parameters $\sigma(k^0)$ and η in replication r. We define the length

[6] We have suppressed the parental income arguments in the equilibrium investment functions to simplify the notation.

of time until the improvement in child quality by

$$\hat{a}_1(r,1) = -\frac{\ln(1-u^{(1)}(r,1))}{\delta(1)}.$$

The length of time until a decrease in the quality level of the child is given by

$$\hat{a}_2(r,1) = -\frac{\ln(1-u^{(2)}(r,1))}{\sigma(k^0)}.$$

Note that if $k^0 = k_1$ then $\sigma(k^0) = 0$, so that $\hat{a}_2(r,1)$ is indefinitely large and the first event cannot be a decrease in child quality. Finally, the length of time until the exogenous termination of the child quality process is given by

$$\hat{a}_3(r,1) = -\frac{\ln(1-u^{(3)}(r))}{\eta}.$$

Which event is actually observed is determined using the competing risks framework, namely, cause j is observed if

$$\hat{a}_j(r,1) = \min[\hat{a}_1(r,1), \hat{a}_2(r,1), \hat{a}_3(r,1)].$$

Before the second event is generated the state variables are updated as follows. If there is an improvement in child quality, then the child quality level is increased by one level. At this new quality level, the parents resolve their investment problems and a new equilibrium rate of arrival of quality changes is found,

$$\delta(2) = \delta_0[\hat{\imath}_1(k_{\mathrm{level}(k^0)+1}, \theta^0, d=0, g=0) + \hat{\imath}_2(k_{\mathrm{level}(k^0)+1}, \theta, d=0, g=0)]^{\nu}.$$

Recall that the value of marriage is enhanced by a higher level of child quality, so that by definition the arrival of a positive shock to k cannot bring about divorce.

If the first event to occur is a decrease in child quality, then the parents will change their quality investment decisions, and, more importantly, may choose to divorce. If they choose to divorce, then the state variable d switches from 0 to 1 and the value of θ is set to 0 in the instantaneous payoff functions for the spouses. Thus the improvement in child quality parameter will either become

$$\delta(2) = \delta_0[\hat{\imath}_1(k_{\text{level}(k^0)-1}, \theta, d = 0, g = 0) + \hat{\imath}_2(k_{\text{level}(k^0)-1}, \theta, d = 0, g = 0)]^\nu$$

or

$$\delta(2) = \delta_0[\hat{\imath}_1(k_{\text{level}(k^0)-1}, 0, d = 1, g = 0) + \hat{\imath}_2(k_{\text{level}(k^0)-1}, 0, d = 1, g = 0)]^\nu.$$

Other parameter changes may occur as well. For example, if the child quality level was initially the second, i.e., $k^0 = k_2$, then a reduction in the child quality level to k_1 implies that the rate of decrease in child quality now becomes 0; if $k^0 > k_2$, then the rate of decrease remains at $\tilde{\sigma}$.

Finally, if the first event is an exogenous termination of the child quality improvement process, then the state variable g is reset from 0 to 1. In this case, parental investment will remain at the value of 0 for the remainder of this sample path. In addition, when this event occurs, it is possible that the parents could find that the divorce option dominates the marriage option. Thus, we also have to evaluate these values in this case. If the divorce state dominates the marriage state, then the state variable d is set to 1.

The second event that takes place is then determined as follows. Using the new value of δ, $\delta(2)$, the current value of k (i.e., the level of child quality after the first event has occurred), and the new values of d and g, define the latent time to the next increase in child quality by

$$\hat{a}_1(r, 2) = -\frac{\ln(1 - u^{(1)}(r, 2))}{\delta(2)},$$

the latent time until the next decrease in child quality by

$$\hat{a}_2(r, 2) = -\frac{\ln(1 - u^{(2)}(r, 2))}{\sigma(k)},$$

and the remaining time until the end of the possibilities of changes in child quality by

$$\hat{a}_3(r, 2) = \hat{a}_3(r, 1) - t_1(r),$$

where $t_1(r)$ is the age of the child when the first event occurred in replication r. Then the time of the second event is given by

$$t_2(r) = \min[\hat{a}_1(r,2), \hat{a}_2(r,2), \hat{a}_3(r,2)].$$

Further events are generated in a similar manner.

Formally, let us define the sample path associated with the r^{th} replication given parameter values ψ, income values y_1 and y_2, and initial conditions k^0 and θ by $\varsigma_r(y_1, y_2, k^0, \theta; \psi)$. Let the state of the sample path defined in terms of whether the parents are divorced and the child quality level at age a be denoted

$$(k^r, d^r)(k^0, \theta) = \Gamma_r(y_1, y_2, a, k^0, \theta; \psi) \equiv \Gamma(\varsigma_r(y_1, y_2, k^0, \theta; \psi), a).$$

Given $a(j)$, let $\{(k^r(j), d^r(j))\}_{i=1}^R$ denote the R simulation draws, where each simulation draw r itself consists of $T \times M$ replications, one for each of the T values of k and M values of θ.

As mentioned above, we utilize simulated method of moments estimation to obtain estimates of the primitive parameters. The simulated moments are computed as follows. Let the exogenous, time-invariant characteristics of household j be given by X_j, and let this vector be of dimension $1 \times Q$. This vector includes the parental incomes y_1 and y_2 for household j. Then for a given initial condition, we can define the conditional expectation

$$E(x_{(1)}^{c_1} x_{(2)}^{c_2} \cdots x_{(Q)}^{c_Q} k^{c_{Q+1}} d^{c_{Q+2}} | x, a; \psi), \tag{6}$$

where $c_k \in \mathbb{N}$, the set of all nonnegative integers, a is the age of the child at the time of the observation, and ψ is the vector of primitive parameters that characterize the model. A vector $C_l \equiv (c_1^l, c_2^l, ..., c_{Q+2}^l)$ defines a particular moment. A set of L (raw) moments is denoted $\mathbb{C} = \{C_1, ..., C_L\}$.

Given the complexity of the model there exists no closed form expression for (6); we approximate the value of a particular moment (defined by C_l) through the use of simulation. Given the R sample paths associated with a given value of the initial conditions, for the moment defined by C_l we have

$$\begin{aligned}
&\frac{1}{R}\sum_{i=1}^{R} x_{(1)}^{c_1^l} x_{(2)}^{c_2^l} \cdots x_{(Q)}^{c_Q^l} k_r^{c_{Q+1}}(x, a, k^0, \theta; \psi) d_r^{c_{Q+2}}(x, a, k^0, \theta; \psi) \\
&= x_{(1)}^{c_1^l} x_{(2)}^{c_2^l} \cdots x_{(Q)}^{c_Q^l} R^{-1} \sum_{i=1}^{R} k_r^{c_{Q+1}}(x, a, k^0, \theta; \psi) d_r^{c_{Q+2}}(x, a, k^0, \theta; \psi) \qquad (7) \\
&\equiv A_l(x, a, k^0, \theta; \psi). \qquad (8)
\end{aligned}$$

Now let the probability distribution over the initial values for household x be defined by

$$\omega^0(k^0, \theta | x, \psi). \tag{9}$$

Then we have

$$A_l(x, a; \psi) = \sum_{m=1}^{M} \sum_{t=1}^{T} A_l(x, a, k_t, \theta_m; \psi) \omega^0(k_t, \theta_m | x, \psi). \tag{10}$$

Finally, our approximation to the population moment defined by C_l is given by

$$A_l(\psi) = N^{-1} \sum_{n=1}^{N} A_l(x_n, a_n; \psi). \tag{11}$$

An important component of this specification of the unconditional moments is the distribution over the initial conditions. We assume that k^0 and θ are independently distributed, and in keeping with our assumption that the state space is finite, assume that both random variables are discrete. We assume that the set of child quality values is $\{1, 2, ..., T\}$. Let $X_k \subset x$ be a set of exogenous household-specific variables that influence the distribution of k^0. Then let

$$\omega^0(k^0 = t | x) = \begin{cases} .5[\Phi(\frac{1 - X_k \beta_k}{\sigma_k}) + \Phi(\frac{2 - X_k \beta_k}{\sigma_k})] & t = 1 \\ .5[\Phi(\frac{(t+1) - X_k \beta_k}{\sigma_k}) - \Phi(\frac{(t-1) - X_k \beta_k}{\sigma_k})] & t = 2, ..., T-1 \\ 1 - .5[\Phi(\frac{(T-1) - X_k \beta_k}{\sigma_k}) + \Phi(\frac{T - X_k \beta_k}{\sigma_k})] & t = T \end{cases}, \tag{12}$$

where Φ is the standard normal c.d.f. Then the probability distribution of the initial condition is parametric, and determined by β_k and σ_k.

The distribution of match values is similarly determined. Each spousal pair draws a match value from a common support of $\{\theta_1, ..., \theta_M\}$, with $\theta_1 < 0$ and $\theta_M > 0$. Define $X_\theta \subset x$ as a set of household characteristics that affect the match value distribution, and define

$$\omega(\theta=\theta_m|x)=\begin{cases} .5[\Phi(\frac{\theta_1-X_\theta\beta_\theta}{\sigma_\theta})+\Phi(\frac{\theta_2-X_\theta\beta_\theta}{\sigma_\theta})] & m=1 \\ .5[\Phi(\frac{\theta_{m+1}-X_k\beta_k}{\sigma_\theta})-\Phi(\frac{\theta_{m-1}-X_k\beta_k}{\sigma_\theta})] & m=2,...,M-1 \\ 1-.5[\Phi(\frac{\theta_{M-1}-X_\theta\beta_\theta}{\sigma_\theta})+\Phi(\frac{\theta_M-X_\theta\beta_\theta}{\sigma_\theta})] & m=M \end{cases} \quad (13)$$

Then the probability distribution of the marriage quality value is completely determined by $\{\theta_1,...,\theta_M\}$, β_θ, and σ_θ.

3.1 Estimator

Calculation of the decision rules used by agents with current state variables $s\in S$ is an extremely time-intensive task, and to compute the moments from the simulated histories requires access to these rules. We have adopted a strategy to speed the convergence process which was inspired by the insightful work of Jain, Imai and Ching (2003). They recognized the wastefulness of recomputing decision rules "from scratch" at each new set of trial parameter values as we work through the iterative process to find the parameter estimates. The idea, as implemented here, is to compute some "exact" solutions to the household's investment and divorce problem at a fixed set of parameter values, and to approximate the household investment rule as a convex combination of these parameter values, where the weights attached to the rules are a function of the relative distance between the current parameter guesses and the reference parameter vectors. Using the approximate investment rules and the current guesses of the parameters $\tilde{\psi}$, we generate simulated moments. We iterate over $\tilde{\psi}$ until we adequately approximate the observed sample moments, and call this estimator $\tilde{\psi}_1^*$. We find investments over all states s at this value of the parameter vector, and compare these with the investments predicted from the approximation. If the divergence is sufficiently great for any $s\in S$, we add $\tilde{\psi}_1^*$ to our collection of parameter vectors with exact investment solutions, and restart the iteration process using as starting value $\tilde{\psi}_1^*$. We repeat the process until the exact and approximation investment rules are sufficiently close over all $s\in S$.

More formally, let the number of parameter vectors at which exact solutions are computed be given by H, and let the collection of these parameter vectors be given by $\Lambda=\{\psi_1,\psi_2,...,\psi_H\}$, where each $\psi_h\in\Omega_\psi$, the parameter space associated with ψ. The Nash equilibrium investment rules for the household are given by $i^*(s;\psi_h) = i_1^*(s;\psi_h) + i_2^*(s;\psi_h)$ at the parameter vector ψ_h. Let the true value of the parameter vector be given by ψ_0. Both ψ_0 and $\tilde{\psi}$ are interior points in the K-dimensional parameter space Ω_ψ. Estimation proceeds as follows.

1. Begin by selecting H distinct points in the parameter space Ω_ψ, which we denote by ψ_h, $h = 1, ..., H$, with the collection of these points define as Λ. For these H values of the parameter vector we solve for the investment rules for all values s in the finite state space S.

2. Given any current guess of the values of the parameters $\tilde{\psi}$, compute the weights

$$w_{\tilde{\psi}}(m) = \frac{[D(\tilde{\psi}, \psi_m)]^{-1}}{\sum_{j=1}^{M}[D(\tilde{\psi}, \psi_j)]^{-1}}, \tag{14}$$

   where $D(x, y)$ is a distance function so that $D(x, y) = D(y, x)$, $D(x, y) > 0$ for all $x \neq y$, and $D(x, x) = 0$. As a result, $w_{\tilde{\psi}}(h) \in [0, 1]$, $\forall h$, and

$$\sum_{h=1}^{H} w_{\tilde{\psi}}(h) = 1, \ \forall \tilde{\psi} \in \Omega_\psi. \tag{15}$$

3. Form the approximate decision rules for every value of s,

$$\hat{\imath}^*(s; \tilde{\psi}) = \sum_{h=1}^{H} w_{\tilde{\psi}}(h) i^*(s; \psi_h). \tag{16}$$

4. Generate the simulated moments at the parameter vector $\tilde{\psi}$ using the approximate decision rules $\hat{\imath}^*(s; \tilde{\psi})$.

5. Define the distance function

$$L_1(\tilde{\psi}; C^N) = (C^N - \hat{C}(\tilde{\psi}))' W (C^N - \hat{C}(\tilde{\psi})), \tag{17}$$

   where C^N are the sample moments, $\hat{C}(\tilde{\psi})$ are the analogous moments computed from the simulated sample at the parameter vector $\tilde{\psi}$, and W is a positive-definite weighting matrix.

6. Using the Nelder-Mead simplex algorithm, repeat steps (2)-(5) until

$$L_1(\tilde{\psi}; C^N) < \varepsilon_N, \tag{18}$$

   where ε_N is a small positive number.

7. Denote the value of $\tilde{\psi}$ that satisfies (18) by $\tilde{\psi}_1^*$, where the subscript '1' suggests that this is an estimator that has passed the first convergence criterion.

8. Compute the optimal investments at $\tilde{\psi}_1^*$ for each $s \in S$. Define

$$L_2(\tilde{\psi}_1^*) = \max_{s \in S} \{|i^*(s; \tilde{\psi}_1^*) - \hat{\imath}^*(s; \tilde{\psi}_1^*)|\}_{s=1}^S. \tag{19}$$

9. If $L_2(\tilde{\psi}_1^*) < \zeta_N$, where ζ_N is a small positive number, then we say that the final estimator of ψ is

$$\tilde{\psi}_2^* = \tilde{\psi}_1^*. \tag{20}$$

   If not, then add the point $\tilde{\psi}_1^*$ to the set Λ (or $\Lambda' = \Lambda \cup \tilde{\psi}_1^*$) so that the cardinality of this set increases to $H + 1$. Then repeat all steps beginning with (2), keeping the current guess of the parameter vector fixed at $\tilde{\psi}_1^*$.

In practice we have had good success with this estimation method. At this point we cannot supply a formal proof of consistency of this estimator, but we turn to a sketch its elements.

First consider the approximation of the investment rule as a function of the parameter vector ψ. Given our H element set Λ, for a given value of $\psi \in \Omega_\psi$, we compute

$$\hat{\imath}^*(s; \psi) = \sum_{m=1}^{M} w_\psi(m) i^*(s; \psi_m).$$

If

$$\max |\hat{\imath}^*(s; \psi) - i^*(s; \psi)| \geq \zeta_N,$$

then we add the point ψ to the set Λ as element $H + 1$ of the set. If not, we say that we have adequately approximated the decision rule.

If the convergence criterion is not satisfied, we return to recompute the weights attached to the "exact" investment rules associated with the new set of points $\Lambda' = \Lambda \cup \psi$. Since the weight attached to any arbitrary evaluation point h can be expressed as

$$\begin{aligned} w_{\psi}(h) &= \frac{[D(\psi,\psi_h)]^{-1}}{\sum_{j=1}^{H+1}[D(\psi,\psi_j)]^{-1}}, \;\; h = 1, ..., H+1 \\ &= \frac{\frac{1}{D_h(\psi)}}{\frac{1}{D_1(\psi)} + \frac{1}{D_2(\psi)} + ... + \frac{1}{D_{H+1}(\psi)}} \\ &= \frac{\frac{1}{D_h(\psi)}}{\frac{D_{-1}(\psi)+D_{-2}(\psi)+...+D_{-(H+1)}(\psi)}{D_1(\psi)D_2(\psi)\cdots D_{H+1}(\psi)}} \\ &= \frac{D_{-h}(\psi)}{D_{-1}(\psi)+D_{-2}(\psi)+...+D_{-(H+1)}(\psi)}, \end{aligned}$$

where $D_j(\psi)$ is shorthand for $D(\psi, \psi_j)$ and

$$D_{-j}(\psi) = D_1(\psi)\cdots D_{j-1}(\psi)D_{j+1}(\psi)\cdots D_{H+1}(\psi).$$

But note that in this case $\psi = \psi_{H+1}$, so $D_{H+1}(\psi) = 0$ and $D_j(\psi) > 0$, $\forall j \neq H+1$, since all points of evaluation are distinct. Then $D_{-(H+1)}(\psi) > 0$, while $D_{-j}(\psi) = 0$, $\forall j \neq H+1$. Thus $w_{\psi}(H+1) = 1$, and the new "approximate" decision rule is the "exact" one computed at the point ψ, or

$$\hat{\imath}^*(s;\psi) = i^*(s;\psi), \;\; \forall s.$$

This completes the discussion of the ability of the investment rule approximation method to fit the actual investment rule solution for every value of s. While it is always capable of providing a perfect fit, we will not want to enforce this in practice since this would imply an indefinite number of iterations over steps (2)-(5). For consistency of the entire estimator, we will only require that critical value used for convergence to get arbitrarily small as sample size grows.

Now we need to consider the convergence of the stage one estimator, $\tilde{\psi}_1^*$, which is computed on the basis of a fixed collection of decision rules. Since the weights attached to the exact investment rules used in forming the approximation are functions of the current parameter guess $\tilde{\psi}$, the approximation is as well. As long as the distance function is a continuous function of $\tilde{\psi}$, then the weights are as well, which implies that the approximation is continuous in $\tilde{\psi}$.

Given that certain events involved in the moment computation are discrete (such as divorce), it is not possible to claim that the functions $\hat{C}(\tilde{\psi})$ are continuous. However, continuity is not required for consistency, as is made clear in Pakes and Pollard (1989). We have not explicitly noted dependence of $\hat{C}$ on R, but for now write $\hat{C}_R(\tilde{\psi})$. Then we need uniform convergence of $\hat{C}_R(\tilde{\psi})$, so that there exists a value $\overline{R}$ and $\kappa > 0$ such that

$$|\hat{C}_R(\tilde{\psi}) - C(\tilde{\psi})| < \kappa \tag{21}$$

for all $R \geq \overline{R}$ and $\tilde{\psi} \in \Omega_\psi$. Standard Law of Large Numbers results yield $\text{plim}_{N\to\infty} C^N = C$. Then the key elements required for $\text{plim}(\tilde{\psi}_2) = \psi_0$ are:

1. $\varepsilon_N \to 0$ as $N \to \infty$
2. $\zeta_N \to 0$ as $N \to \infty$
3. $R \to \infty$ as $N \to \infty$
4. $C(\psi)$ continuous function of ψ
5. Uniform convergence of $\hat{C}_R(\psi)$.

We do not attempt to characterize the requirements for deriving a well-defined limiting distribution for the estimator $\tilde{\psi}_2$. Although computation of the estimator is demanding, it is still feasible to construct bootstrap estimates of it sampling distribution. We will do this once we have arrived at a final specification of the econometric model.

4 Data and Descriptive Statistics

The estimation employs the Child and Young Adult Data associated with the 1979 cohort of the National Longitudinal Survey of Youth. Our sample consists of families with only children in which the parents were married in the first interview after the date of the child's birth. The use of single-child families allows us to abstract from issues of investment allocation across children and from problems relating to remarriage and step-siblings.

The child outcome measure employed in the empirical analysis is based on the child's score on the Peabody Individual Achievement Test (PIAT) in mathematics. The PIAT is administered to all children aged five and older in the NLSY Child sample, and is ceased when children exit the Child sample and enter the Young Adult sample at the age of 14. In order to include children who reach the age of five during the sampling widow and are born to mothers with as broad a range of ages as possible, we measure the child's age and test scores and the parents' marital status in the first year in which the child undergoes the PIAT mathematics assessment. Thus, our outcome measures for sample children are collected when the children are between the ages of 5 and 14, but in practice almost all of our sample children are tested between the ages of 5 and 7. This step adds a third sample selection criterion to the requirements that sample children have no siblings and sample parents were married at the children's births. We require that sample children undergo the PIAT mathematics assessment in at least one interview. The test score measure employed in the estimation and policy experiments is the child's age-specific PIAT mathematics percentile.[7]

Parental incomes are measured at the date of birth of the child. A common difficulty faced by empirical studies of married and divorced parents' interactions with their children is tracking divorced fathers who no longer reside with their children. The appeal of the NLSY in this regard is that it allows us to observe families from the date of birth of the child, and therefore we have some information on each sample father no matter how quickly the family dissolves after the birth of the child. We measure each father's income at his child's date of birth in order to avoid admitting error in the measure of the father's income whose variance depends on the parents' marital status. Further, we restrict our measure of the mother's and father's incomes to those observed at the date of birth of the child, and we assume that incomes are constant from that date. This step is also taken to avoid relying on income variation in the data that is only observable for families that remain intact. Each parent's income is determined as the sum of reported incomes in the NLSY that are attributable to the individual parent and not to her or his spouse. Attributable income sources are wage and salary, farm and business income, military income, and unemployment income. Regardless of the date of birth of the child, parents' incomes are inflated to 1998 dollars for the purposes of reporting and estimation.

[7] The age norming performed in the 1979 NLSY Child Data uses an age-based norming sample from 1968. Therefore, NLSY sample children show average scores that exceed those of the norming sample. A useful reference on this point is Dunn and Markwardt (1970).

The divorce outcome measure used in the estimation is zero if parents remain married and not separated from the first interview after the birth of the child through the interview in which the child's first PIAT assessments take place. Otherwise, it is one. As discussed in the section on estimation, the probability distribution of initial child quality k^0 is permitted to rely on a vector of characteristics of the parents and child observed at (or, in one case, before) the date of birth of the child. Characteristics entering X_k are a constant, the mother and father's ages at the date of birth of the child, the mother and father's years of schooling at the date of birth of the child and the mother's score on the Armed Forces Qualifications Test (AFQT) administered to NLSY respondents in 1980.

Like the initial child quality, the probability distribution of the quality of the parents' marriage at the child's birth relies on a vector of characteristics of the parents. Characteristics entering X_θ pertain to the likely stability of the parents' marriage. The first is the mother's response when asked whether the marriage is "very happy, fairly happy or not too happy" at the interview following the child's birth. An indicator for whether her answer is "very happy" covaries positively with the child's PIAT test score in our sample, and mothers who respond with "very happy" are more often married at the child's first PIAT test. As a second measure of marriage quality, we include the number out of a list of ten topics on which the mother states that she and the father sometimes or often argue. The topics are chores and responsibilities, children, money, showing affection, religion, what to do with leisure time, drinking, other women, her relatives and his relatives. This measure shows a substantial negative covariance with children's test scores, and parents who divorce by the test date argue over more of the ten points on average. Since we find that the divorce rate among Roman Catholics in the sample is 8.4 percentage points lower than among non-Catholics in our sample, we also include an indicator for whether the mother reports her religion as Roman Catholic at the start of the NLSY79 panel. Though a positive coefficient on the Catholic indicator would explain the lower divorce rate among Catholics in the sample as the result of a higher overall welfare while in marriage, it is also possible that this distinction results from a greater cost of divorce among Catholics. Without an explicit decision to marry, our model cannot distinguish between these two possibilities. It does, however, allow information on the apparent difference in the stability of marriage for Catholics and non-Catholics to influence parents' child investments.

The marriage quality questions described here, other than the mother's religion, are fielded with the main surveys of the NLSY79 cohort starting in 1988. This poses a further difficulty in our sample construction. In order to include measures of marriage quality, we must observe families between 1988 and 2002. If we retain children born before 1988 in our sample, then we can only observe the quality of their parents' marriages if the marriages survive until the 1988 interview. As with fathers' incomes, this leads to measurement error in an independent variable whose distribution depends on a dependent variable. In order to avoid this problem, we consider only families in which the child was born between the 1987 and 1988 interviews or later, and we measure marriage quality at the date of the child's birth. The choice to include marriage quality measures in the only manner we determine to be valid has two important costs. First, the mothers we consider are older on average than the set of all NLSY mothers when their children are born. Since respondents were aged 14 to 21 at the time of the 1979 wave, between 1988 and 1998 the first time mothers we study are all aged 23 to 40. Second, we forfeit some sample size in order to include marriage quality measures from the time of the child's birth. While we have complete information excepting marriage quality for 427 only-child families, our final estimation sample including marriage quality consists of 202 families. Finally, because what marriage quality responses we observe across waves for surviving marriage show only small changes, we have fit families' child quality and marital status paths assuming that marriage qualities are permanent characteristics; i.e. $\gamma^+ = \gamma^- = 0$. This forces the child quality update, setback and termination processes to generate both divorces and child attainments on their own, without exogenous shocks to the value of marriage that might help us to fit these observed outcomes. We are interested in whether the child quality production processes are sufficient to fit actual divorce and test score realizations for these families, and if they are not we will turn to the limited variation displayed by the repeated marriage quality measures among surviving marriages to attempt to explain observed divorces and child attainments.

We have imposed a number of stringent selection conditions in defining our sample, primarily regarding family composition. The benefit of these restrictions is that they allow us to abstract from concerns involving the allocation of parental investments across groups of siblings and step-siblings, and from the complex marital status choices facing unwed parents. The cost is that the use of only-child families in which parents were married at their children's births certainly limits the generalizability of our findings to families with more complex structures. In future research, we plan to investigate the role of evolving family structures in parents' ongoing child investment decisions.

Table 2 contains the means of the variables used in estimation for the estimation sample. At the time of the test, 17.3 percent of parents are divorced. The average child in the sample is 5.6 years old at her or his first PIAT assessment, and 95 percent of children complete at least one PIAT test by the age of 7. The average age of mothers at the child's birth is 29.5, and the average age of fathers is 32.8. Mothers' average education is 13.9 years of schooling, and fathers' is 13.8. All of this reflects the use of later NLSY births in the construction of our sample. Children's average PIAT score percentile is 58.26. Fathers' incomes are substantially higher than mothers' incomes on average, at $38,130 as compared with $22,103 1998 dollars. More than three quarters of mothers report that their marriages are "very happy", and couples argue over an average of 2.56 of the ten points listed above. The argument measure does show meaningful variation, with a standard deviation of 2.03. A third of the sample is Roman Catholic.

5 Empirical Results

The simulated outcomes described above are obtained under the following conditions. We fix policy parameters $\{\pi, \tau_1(1)\}$ at values intended to reflect the state of existing family law. The father is assumed to make a child support payment of 17 percent of his income to the mother in the divorce state, which represents a fairly moderate child support order for one child in a state that maintains numerical guidelines for child support. We assume that custody standards allocate 80 percent of the custody of the child to the mother in the divorce state. Custody averages over the period in which we observe our NLSY sample have been studied for eight states. All but California maintain approximately 80-20 custody division averages; California's custody decisions favor fathers substantially more than those of other states, with as much as 40 percent custody going to fathers on average.[8] Durations are measured in years. The instantaneous discount rate ρ is fixed at 0.05. We assume $\eta = 0.06$, implying an average age for the termination of investment productivity of between 16 and 17. The number of discrete child quality levels, T, is set to 10 and chosen to reflect deciles of the age-specific PIAT math score distribution. We assume that there are $M = 5$ marriage quality levels, and that the probability mass at each level is determined in

[8] See, for example, Cancian (1998) on states' custody averages.

part by family characteristics and approximates a discrete normal distribution as described above by $\omega(\theta = \theta_m|x)$. Parents' incomes are measured in units of \$2500 1998 dollars. Simulated moments are based on $R = 100$ replications per family per (k^0, θ) pair, or 5000 replications per family.

The model relates exogenous household characteristics $X = \{y_1, y_2, a, \theta, k^0\}$ to outcomes k and d for a given family. Therefore the moments we choose pertain to the relationship between parents' incomes, children's test ages, determinants X_θ of marriage quality or determinants X_k of child quality and the child's test score; the relationship between parents' incomes, children's test ages, determinants X_θ of marriage quality or determinants X_k of child quality and the parents' marital status at the test; the test and marital status outcome averages for the full sample; and higher-order interactions among the two outcome measures and elements of X as described by expression (6). The 21 moments that we fit are described in table 4. Note that we choose to measure and simulate unconditional moments in almost every instance, due to the complication associated with simulating and evaluating conditional moments across family-initial condition combinations that are each associated with unique weights. However, the moments chosen contain information equivalent to conditional moments where, for example, we compare moments that condition on marital status to unconditional moments involving products of d and k or elements of X.

The vector of parameters estimated using our MSM procedure govern the parents' preferences, the production of child quality, the relationship of observed marriage quality measures to true marriage quality and the relationship of characteristics of the parents observed at or before the child's birth to the child's initial quality. The complete vector of parameters we estimate is $\psi = \{\alpha_1 = \alpha_2, \delta_0, \nu, \sigma, \beta_\theta, \sigma_\theta, \beta_k, \sigma_k\}$, which contains a total of 15 free parameters. The estimated values of β_k reflect the standardization of each element of X_k to maintain a zero mean and unit variance within the sample.

The parameter estimates are reported in Table 3. Estimated preference parameters $\alpha_1 = \alpha_2 = 0.3526$ reflect the preference weight placed on own consumption by both the mother and father. Each parent appears to be relatively "altruistic" toward the child, but of course these weights are not independent of the manner in which consumption and child quality have been measured. Production coefficients are estimated to be $\delta_0 = 0.2708$, $\nu = 0.4853$ and $\sigma = 0.8859$. These point estimates indicate that there are significantly decreasing returns to child investments. Setbacks occur for children on average about every thirteen months, and parents' investments in their children are close to offsetting the setbacks on average. However, we predict substantial variation in family-specific rates of child progress.

Perhaps the initial condition parameters are most easily interpreted at this point, while a more complete understanding of the implications of the estimated production parameters requires the simulation of child quality and parental value distributions. We estimate large effects of each of the measures of marriage quality on parents' welfare. The average welfare levels we simulate for mothers and fathers based on the parameter estimates are 45.31 and 42.88, respectively. The estimated coefficient on the number of points of argument between the parents implies that the marginal point of argument decreases the permanently married parent's overall welfare by a fifth of average total lifetime welfare or more. The estimated coefficient on the indicator for whether the mother claims that her marriage is fairly or not too happy implies that a permanent marriage that is very happy contributes an additional 38.53 units of welfare. Remaining married to the horizon is estimated to imply a welfare for Roman Catholic parents that is 29.17 units higher on average than the value of remaining married to non-Catholics. Of course, many of the observed marriages would dissolve if parents made marital status decisions until the termination of the child improvement process, implying that these permanent marriage value differences overstate the estimated effects of the marriage quality measures on parents' welfare.

Parents' ages, educations and measured abilities are each estimated to influence children's initial quality levels positively, and some of these effects are substantial. The estimated values of the elements of β_k imply that an increase of one standard deviation from the sample average in the mother's AFQT score leads to an increase of 5.07 percentile points in the child's age-normed math score. An increase of a standard deviation in the mother or father's education, or the mother's age, leads to a similar change in the child's PIAT score percentile. A one standard deviation increase in the father's age from the sample average has less than half of this effect on the test score percentile.

The parameter estimates are reported without standard errors for now. Given the difficulty of characterizing the limiting distribution of our estimator, we are currently in the process of constructing bootstrap estimates of its sampling distribution. The resulting standard errors will be added in future versions.

The data and simulated moments listed in Table 4 give an idea of the fit of the model. Overall, the simulated moments match the patterns in the data reasonably well. The data and simulation values of the first, third and fourth moments listed in the table, the divorce rate by the date of the child's test, the average of kd and the average of $k(1-d)$ at the test date, together indicate that we have fit the two most obvious targets of the estimation reasonably well. The divorce rate in the sample is 17.32 percent, while the simulated divorce rate is 16.15 percent. The average test score in the sample is 58.26; we simulate an overall average test score, implied by moments three and four, of 58.67. Further, the simulated moments replicate the differences in test scores between children with married and divorced parents quite closely. Thus the model is able to fit observed divorces along with test scores even where marriage quality is assumed to be a permanent feature of the household. Divorces here result from the interaction of the permanent marriage quality and realizations of the child quality processes, and do not arise from exogenous preference shocks. Based on this finding we do not attempt to include time series data on the relatively stable marriage quality responses from mothers in surviving marriages in the estimation.

Recall also that the estimates are generated under the restriction that $\alpha_1 = \alpha_2$. One concern is whether the model is able to generate the observed relationships between the incomes of mothers and fathers and children's test scores across marriage and divorce states without relying on a difference in the tastes of mothers and fathers for child quality. Beyond preferences, mothers and fathers differ in the model in their individual incomes and in their treatment by existing policy in the event of divorce. Moments that particularly speak to our success in fitting the described relationship given the preference restriction in question include $E($ test score $\times$ father's income $\times$ married $)$, $E($ test score $\times$ mother's income $\times$ married $)$, $E($ test score $\times$ father's income $\times$ divorced $)$ and $E($ test score $\times$ mother's income $\times$ divorced $)$. Their respective values in the data are 820.89, 501.33, 117.98 and 84.72. Their simulated values are 856.57, 486.75, 126.48 and 80.45. We take from this that the model has done a fair job of fitting the observed relationships between parents' incomes and children's test scores across the marital states. Clearly the difference between parents in the interactions is overpredicted in both marriage and divorce. It appears that a common preference parameter may not quite capture the value of the child to the mother, and that, were α_2 permitted to deviate from α_1, a slightly better fit might be achieved with a slightly higher preference weight on child quality for the mother (even though we know that identification of different parental preference parameters is extremely precarious in practice). We are, nonetheless, satisfied with the ability of the differences in parents' incomes and legal treatment, absent different preferences, to fit the patterns in the data fairly well based on these moments.

6 Custody and Child Support Experiments

Recent law changes and social movements in U.S. states and in western Europe have advocated shared custody and placement or lesser increases in fathers' access to their children in divorce. Over the past several years, fathers' groups in the UK and US such as Fathers 4 Justice, the American Coalition for Fathers and Children, Dads Against Discrimination and the Alliance for Noncustodial Parents' Rights have agitated for shared physical custody.[9] Major law changes from 2000 to 2004 in Iowa, Maine, Wisconsin and Austria, among others, encourage judges to grant joint physical custody, or to divide the child's time between the two parents as close to equally as possible. In Wisconsin court record data, Brown and Cook (2005) show that the 2000 Wisconsin law change was followed by a continuing upward trend in the rate of joint placement. Early results from the new and ongoing custody research of Atteneder, Boheim, Buchegger and Halla (2005) suggest that, along with increasing the number of joint custody arrangements, the 2001 Austrian custody law reform has had an effect in practical terms on the time that children spend with their non-resident parents. We look forward to the availability of data on children's academic or behavioral performance following relevant custody law changes.[10]

The trend toward shared placement suggests the first of our two policy experiments. We simulate child quality and parent welfare for our estimation sample using the vector of estimated parameters and assuming a 50-50 allocation of the child's time between the mother and father in divorce, so that $\tau_1(1) = 0.5$. In addition, many states use unequal shared placement thresholds of 25, 30 and 35 percent to determine whether child support should be based on non-shared or shared child support formulas.[11] Generally speaking, state policies have established a negative relationship between a parent's share of placement and child support obligation. We therefore assume zero child support in our joint placement experiment.

For the purpose of comparison, we simulate outcomes given a change in placement and child

[9] A description of their activities can be found in Dominus (2005), among other places.

[10] The NLSY's 2002 wave follows the Maine and Wisconsin custody law reforms by a year and two years, respectively. However, NLSY79 cohort children are fairly old by the 2002 wave on average, and in order to study the effects of the custody law change directly we would require a longer (or larger) post-reform panel in order to observe a sufficient number of post-reform divorces.

[11] See Melli and Brown (2004).

support that moves in the opposite direction from the baseline policy of $\{\pi, \tau_1(1)\} = \{0.17, 0.2\}$. In this second policy experiment, we grant mothers 90 percent of the child's time and impose a 25 percent child support requirement on fathers. Studying outcomes for each family member under the existing, joint and 90 percent maternal custody regimes allows us to consider at once the direction of the influence of parents' relative family attachments on child outcomes and the redistributive pressures inherent in custody and support arrangements.

Like the estimation, the baseline simulations involve 100 replications for each marriage quality and initial child quality pair for each family. As before, this implies 5000 total replications per family. We calculate the divorce rate, the average simulated math test percentile, the average mother's welfare and the average father's welfare at the test date, weighting simulated outcomes by the estimated probability that each family begins in the (k^0, θ) pair on which they are based. We generate divorce rates, average test percentiles and parental welfare for the first policy experiment, in which $\{\pi, \tau_1(1)\} = \{0, 0.5\}$, and the second policy experiment, in which $\{\pi, \tau_1(1)\} = \{0.25, 0.1\}$, by similar means. Table 5 reports the sample averages and standard deviations of the four outcome measures under each of the three family law regimes.

There is limited variation in the average child qualities predicted for the three regimes. Average child quality is 58.66 in the baseline simulation, 61.04 in the joint placement experiment and 59.08 in the 90 percent maternal placement experiment. Interestingly, while the standard policy objective of decreasing the divorce rate presumes that a low divorce rate benefits children, in this instance the policy regime that generates the largest average child quality also leads to the highest divorce rate. The divorce rate predicted under joint placement is 20.78 percent, as opposed to 16.15 (15.81) percent under 80 (90) percent maternal custody. The variation in families' child quality averages is nearly fixed across the three custody regimes.

Panels a-c of Figure 2 show histograms of the sample average k, V_1 and V_2 in the baseline simulations. Figure 2 panels d-f graph the proportionate changes in k, V_1 and V_2 with a change in the custody and support parameters from $\{\pi, \tau_1(1)\} = \{0.17, 0.2\}$ to $\{\pi, \tau_1(1)\} = \{0, 0.5\}$. The distribution of child quality changes from the standard to the shared custody regime is centered over +3 to 4 percent. There is a fair amount of dispersion in the family-specific changes in children's outcomes, with changes ranging from -8 to +15 percent. The graphs of the changes in parents' welfare with the policy change demonstrate that the simulated shift from 80 percent maternal to

50-50 custody is largely redistributive for these parents. Despite the fact that greater attachment to the child in divorce leads to greater paternal investments in children on average, no sample mother benefits on net from the simultaneous decreases in her custody share and the child support that she receives. Despite the negative effect of lower divorce-state custody on mothers' average child investments, no father experiences a net decrease in welfare following the increase in fathers' custody shares and the decrease in their child support obligations.

Panels g-i of Figure 2 present the analogous proportionate changes for the second policy experiment. The histogram of proportionate changes in k from $\{\pi, \tau_1(1)\} = \{0.17, 0.2\}$ to $\{\pi, \tau_1(1)\} = \{0.25, 0.1\}$ is centered more closely around zero, with a range of roughly -11 to +11 percent changes in families' average simulated child qualities. Again the displayed proportionate changes in mothers' and fathers' values describe a largely redistributive policy change. Here every mother benefits from the simultaneous increase in maternal custody and child support, and every father's welfare is decreased. In sum, the policy experiments generate a 4 percent average increase in child quality with the adoption of 50-50 placement and a substantial redistribution from mothers to fathers as paternal custody shares increase and child support obligations decrease. The magnitudes of the proportional changes in child quality and parental welfare suggest that we consider effects on the distribution of resources between parents to be a key concern of divorce policy. Further, the experiments demonstrate that under the best-fitting parameterization of the model children's attainments are not necessarily greatest where the divorce rate is minimized.

7 Conclusion

We have developed and estimated a continuous time model that allows for strategic behavior between parents in making child quality investment choices and divorce decisions. An important component of the behavioral model is the family law environment, which has a large impact on the rewards attached to the marital states and, in turn, the returns to investment in child quality. We use data from the Mother-Child subsample of the NLSY to estimate model parameters using a relatively involved Method of Simulated Moments estimation procedure. We find that the parameter estimates are roughly in accord with our priors, and that the correspondence between simulated and sample moments is adequate to good.

The most important contribution of our work, still in its preliminary stages, is to the understanding of the dynamic relationship between divorce decisions and the evolution of child quality, and the dependence of this process on family law "parameters." We have conducted some initial investigations of how substantial changes in these parameters - those reflecting contact time between divorced parents and the child and the child support transfers between parents - impact the parental welfare distribution and the child quality outcomes. To date, our experiments suggest relatively small, but 'noticeable,' impacts of changing the family law environment on the average value of child quality in the population. Instead, the concurrent impact on the welfare distribution of parents is substantially greater. Such a result may suggest a rationale for why changes in family law tend to occur very gradually over time. While "better" family law environments may favorably impact the child outcome distribution, the gains are slight compared to the shifts in the parental welfare distribution. It follows that it may be difficult to attain the wide-spread support from both mothers and fathers that radical changes in family law require.

Though complex, the model is very stylized and it seems important to generalize it along several dimensions before taking the results of our policy experiments too seriously. As we have mentioned in passing, it would be highly desirable to allow for endogenous fertility decisions, and this is one of our current research directions. We are less convinced that allowing for cooperative behavior on the part of parents will have substantive impacts on our results, but for theoretical reasons believe that it is probably the correct modeling choice. Perhaps the most daunting task we face is to develop a reasonable set of measures of child quality stretching over the period from birth to young adulthood. The test scores we use are clearly a ridiculously simplistic measure of child 'quality.' We must be able to splice together various measures of child performance over the development period if we are to adequately characterize the long run relationships between the household environment and the growth process.

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Table 1: Ordinary Least Squares Regression of PIAT Math Percentile Scores on Family Characteristics

Independent Variable	Coefficient (Standard Error)	Independent Variable	Coefficient (Standard Error)
Constant	-34.806* (18.252)	Mother's AFQT score	0.256** (0.106)
Total income x $10,000^{-1}$	0.1339 (0.3608)	Father's age at child's DOB	0.7714** (0.3198)
Mother's income share	20.130** (7.765)	Father's education at child's DOB	2.528† (0.6736)
Parents divorced	-10.216** (4.735)	Marriage "fairly" or "not too" happy	-5.454 (4.263)
Age of child at test	1.589 (1.719)		

$N = 202$; $R^2 = 0.2275$. Income is measured at the child's date of birth (DOB) and is reported in 1998 dollars. * represents significance at the ten percent, ** at the five percent and † at the one percent level.

Table 2: Estimation Sample Descriptive Statistics

Variable	Mean	Standard Deviation	Minimum	Maximum
PIAT percentile	58.26	27.94	1.00	99.00
Marital status ($d = 1$)	0.1733	0.3794	0.0000	1.0000
Child's age at test	5.629	1.077	4.000	13.000
Mother's income	22,102.74	18,307.84	0.00	146,869.84
Father's income	38,129.54	49,009.40	0.00	595,689.67
I(marriage very happy)	0.7673	0.4236	0.00	1.00
Points of arguement	2.564	2.029	0.000	10.000
Mother's AFQT score	72.57	19.13	0.00	102.00
Mother's education	13.86	2.45	4.00	20.00
Mother's age	29.53	3.81	19.00	39.00
Father's education	13.77	2.91	0.00	20.00
Father's age	32.79	5.67	22.00	55.00
I(Roman Catholic)	0.3366	0.4737	0.0000	1.0000

$N = 202$. The mother and father's age, education and income are each measured at the first interview following the child's birth. Income is reported in 1998 dollars.

Table 3: Parameter Estimates

Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)
δ_0	0.2708	β_{k_4} on father's age	0.1875
ν	0.4853	β_{k_5} on father's education	0.5955
σ	0.8859	β_{θ_0} on constant	-0.7688
$\alpha_1 = \alpha_2$	0.3526	β_{θ_1} on marriage fairly/not too happy	-1.9266
β_{k_0} on constant	7.3026	β_{θ_2} on # argument points	-0.4378
β_{k_1} on AFQT	0.5066	β_{θ_3} on Roman Catholic	1.4586
β_{k_2} on mother's age	0.3743	σ_θ	3.2253
β_{k_3} on mother's education	0.4261	σ_k	0.8950

$N = 202$. All elements of X are standardized to have a zero mean and unit variance in the sample. Parents' incomes are scaled to units of 2500 1998 dollars.

Table 4: Data and Simulated Moments

Moment	Data	Simulation
[1] Proportion divorced by test	0.1732	0.1615
[2] E(child's age at test $\times$ I(divorced at test))	1.0248	0.9057
[3] E(test score $\times$ I(married at test))	49.713	49.867
[4] E(test score $\times$ I(divorced at test))	8.545	8.798
[5] E(test score $\times$ I(marriage very happy))	45.124	44.704
[6] E(test score $\times$ # argument points)	142.67	147.39
[7] E(test score $\times$ mother's education)	826.01	833.49
[8] E(test score $\times$ mother's AFQT score)	4405.19	4435.42
[9] E(test score $\times$ mother's age)	1745.79	1755.69
[10] E(test score $\times$ father's education)	829.36	831.44
[11] E(test score $\times$ father's age)	1941.16	1947.85
[12] E(test score $\times$ father's income $\times$ married)	820.89	856.57
[13] E(test score $\times$ mother's income $\times$ married)	501.33	486.75
[14] $E(\overline{y}_1 \vert d=0) - E(\overline{y}_1 \vert d=1)$	2.3638	2.5494
[15] $E(\overline{y}_2 \vert d=0) - E(\overline{y}_2 \vert d=1)$	0.7042	0.8461
[16] $E(\overline{d} \vert$ not Catholic$) - E(\overline{d} \vert$ Catholic$)$	0.0839	0.0794
[17] E(test score $\times$ father's income $\times$ divorced)	117.98	126.48
[18] E(test score $\times$ mother's income $\times$ divorced)	84.72	80.45
[19] E(test score^2)	4170.52	4121.49
[20] E(test score $\times$ father's income $\times$ mother's income)	9870.04	9900.65
[21] E(test score $\times$ child's age $\times$ divorced)	51.173	48.453

$N = 202$. The simulations are based on $R = 5000$ replications per family (100 per initial conditions pair).

Table 5: Custody and Child Support Policy Experiments

Custody:	80% maternal	Shared	90% maternal
Child support:	17% of $y_1(0)$	0% of $y_1(0)$	25% of $y_1(0)$
$\overline{d}$	0.1615	0.2078	0.1581
$\overline{k}$	58.66	61.04	59.08
$SD(\overline{k}_i)$	11.89	12.30	12.23
$\overline{V}_1$	42.88	45.80	41.76
$SD(\overline{V}_{1_i})$	14.71	14.20	14.94
$\overline{V}_2$	45.31	43.00	46.71
$SD(\overline{V}_{2_i})$	13.71	14.11	13.46

$N = 202$. Simulated outcomes are based on $R = 5000$ replications for each family.

Figure 1: Illustration of Decision Rules

Figure 1.a
Total Child Quality Investment
τ = .2

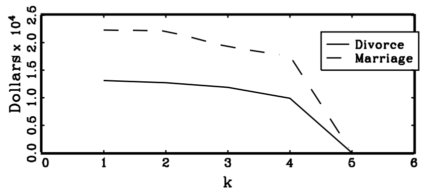

Figure 1.b
Father's Investment Share
τ = .2

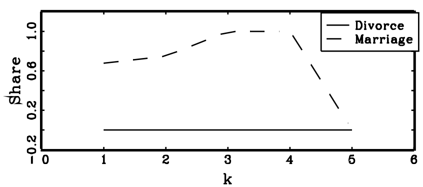

Figure 1.c
Divorce Sets
Largest ϑ Resulting in Divorce

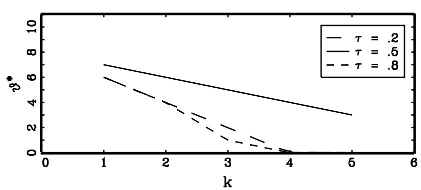

Figure 2.a
Histogram of k
pi=.17, $\text{tau}_1(1)$=.2

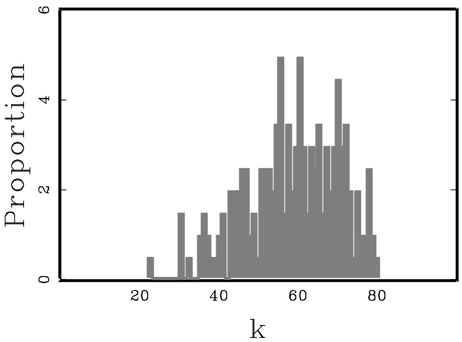

Figure 2.b
Histogram of V_1
pi=.17, $\text{tau}_1(1)$=.2

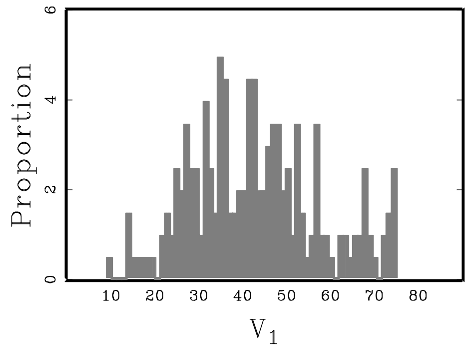

Figure 2.c
Histogram of V_2
pi=.17, $\text{tau}_1(1)$=.2

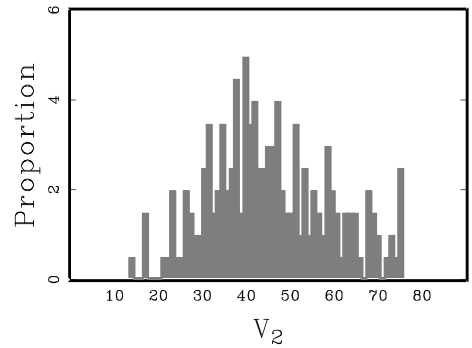

Figure 2.d
Proportionate k Gain
pi=0, $\text{tau}_1(1)$=.5

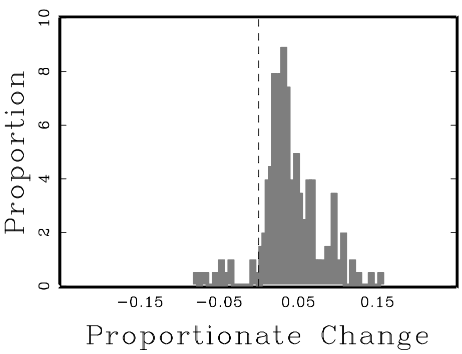

Figure 2.e
Husbands' V_1 Gain
pi=0, $\text{tau}_1(1)$=.5

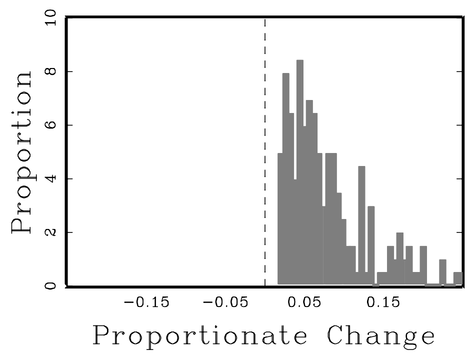

Figure 2.f
Wives' V_2 Gain
pi=0, $\text{tau}_1(1)$=.5

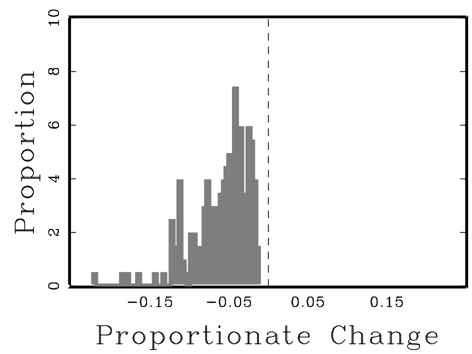

Figure 2.g
Proportionate k Gain
pi=.25, $\text{tau}_1(1)$=.1

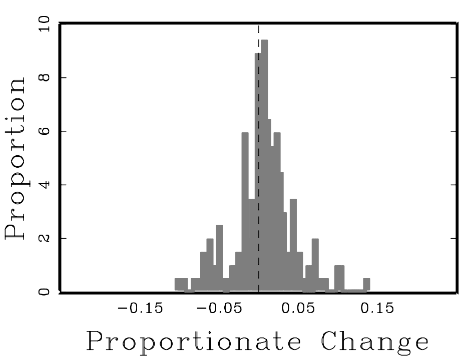

Figure 2.h
Husbands' V_1 Gain
pi=.25, $\text{tau}_1(1)$=.1

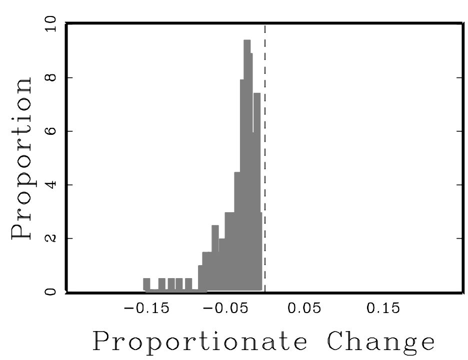

Figure 2.i
Wives' V_2 Gain
pi=.25, $\text{tau}_1(1)$=.1