The Diffusion of Wal-Mart and Economies of Density

by

Thomas J. Holmes¹
University of Minnesota

Federal Reserve Bank of Minneapolis

and National Bureau of Economic Research

July, 2005

Note: This is a very preliminary and incomplete draft for a conference presentation.

¹ The views expressed herein are solely those of the author and do not represent the views of the Federal Reserve Banks of Minneapolis or the Federal Reserve System. The research presented here was funded by NSF grant SES 0136842. I thank Junichi Suzuki for excellent research assistance for this project. I thank Emek Basker for sharing data with me and I thank Ernest Berkas for help with the data.
1. Introduction

A retailer can often achieve cost savings by locating its stores close together. A dense networks of nearby stores facilities the logistics of deliveries and facilitates the sharing of infrastructure such as distribution centers. When stores are close together they are easier to manage and it is easier to reshuffle employees between stores. Stores located near each other can potentially save money on advertising. All such cost savings are *economies of density.*

Understanding these benefits is of interest because they matter for determining policies towards mergers. To the extent that merger of nearby facilities into one company confers cost savings, these benefits potentially offset concerns about increased market power. Study of these benefits is also of interest for understanding firm behavior. To the extent these benefits matter, firms may have an incentive to preemptively build a large network of stores to grab a first-mover advantage. Finally, in recent years there has been a general interest in the industrial organization literature in network benefits of all kinds. The network benefits of a dense network of stores is a potentially important efficiency and there has been little work on this topic.

Wal-Mart is the world’s largest corporation in terms of sales. It is regarded as a company that excels in logistics. The goal of this paper is assess the importance of economies of density to Wal-Mart. Preliminary results suggest the benefits are significant.

Wal-Mart is notorious for being secretive about internal data—I am not going to get access to confidential data on its logistics costs, managerial costs, advertising, or any of the other cost components that depend upon economies of density. Instead, I draw inferences about the cost structure that Wal-Mart faces by examining its revealed preferences in its site-selection decisions. I study the time path of Wal-Mart’s store openings, the diffusion of Wal-Mart. The idea underlying my approach is that alternative sites vary in quality. If economies of density were not important, Wal-Mart would go to the highest quality sites first and work its way down over time. The highest quality sites wouldn’t necessarily be bunched together, so initial Wal-Mart stores would be scattered in different places. But

---

2 There is a larger literature on *economies of density* in electricity markets (e.g. Roberts (1986)) and transportation markets (e.g....)

3 Gowrisankaran and Stavins, etc.
when economies of density matter, Wal-Mart might chose lower quality sites that are closer to its existing network, keeping the stores bunched together, putting off the higher quality sites until later when it can expand out to them.

This latter is what happened. Wal-Mart started with its first store near Bentonville, Arkansas, in 1962. The diffusion of store openings radiating out from this point was very gradual. It is very helpful to view a movie of the entire year-by-year diffusion path that will be posted on the web. Figure 1 shows the process over the years 1970-1980. Wal-Mart did not first grab the “low hanging fruit” in the most desirable location throughout the county and then come back for the “high hanging fruit,” with fill-in stores. Desirable locations far from Bentonville had to wait to get their Wal-Marts.

I bring to the analysis a number of pieces of information about Wal-Mart’s problem. I use store-level data from ACNeilsen and demographic data from the Census to estimate a model of demand for Wal-Mart at a rich level of geographic detail. I use this to estimate Wal-Mart’s sales from alternative location configurations. I also incorporate information about other aspects of costs that can be measured, store-level labor costs, land costs, etc. The underlying principle I use here is to plug into the model the things that I can estimate, and back out the economies of density as a residual. Of course this leaves open the possibility that I have left something else out.

Given the enormous number of different possible combinations of stores that can be opened, it is difficult to solve Wal-Mart’s optimization problem. This makes conventional approaches used in the industrial organization literature infeasible. Instead, I follow a perturbation approach. I consider a set of selected deviations from what Wal-Mart actually did and determine the set of parameters consistent with this decision. The deviations take the form of a resequencing the date of store openings. I interchange the opening dates of “small market” stores (e.g. under 10,000 in population) and “medium market stores (e.g. 10,000 to 40,000). Medium market stores are more advantageous than small market stores because of the larger demand. But going to the medium market stores earlier stretches Wal-Marts market area and reduces economies of density.

I leave the timing of openings in large market areas alone, because there are a variety of difficulties in incorporating the Wal-Mart model in large cities and I want to avoid this
complication here. The Wal-Mart model is a big box with a large parking lot near an interstate exit. It is not surprising that there still is no Wal-Mart in New York City. By medium size cities, I am including what others would view as fairly small cities (on the order of Mason City, Iowa) for which the Wal-Mart model is obviously applicable and which are most definitely nothing like New York City.

There are interesting issues here in the dynamic interaction between Wal-Mart and other retailers. I have already brought these up when I alluded to the issue of first-mover advantages. As a first cut, I abstract from this here. I use a single-agent decision theoretic approach. I treat Wal-Mart as a new innovation and study the diffusion of this innovation. I would be particularly worried about taking this approach if it looked like Wal-Mart had been preempted from a large number of markets. And it may be that Wal-Mart has been preempted by the likes of Target in some markets. But the overall impression one gets from looking at the diffusion path of Wal-Mart is that of a continually-moving force that wasn’t stopping for nobody. So I look here at this force in isolation.

In addition to contributing to the literature on economies of density, the paper also contributes to a new and growing literature about Wal-Mart itself (e.g., Basker (forthcoming), Stone (1995)). Wal-Mart has had a huge impact on the economy. It has been argued that this one company contributed a non-negligible portion of the aggregate productivity growth in recent years. Wal-Mart is responsible for major changes in the structure of industry, of production, and of labor markets. One good question is: what exactly is a Wal-Mart, why is it different from a K-Mart or a Sears? One thing that distinguishes Wal-Mart is its emphasis on logistics and distribution. (See, for example, Holmes (2001)). It is plausible that Wal-Mart’s recognition of economies of density and its knowledge of how to exploit these economies distinguished it from K-Mart and Sears and is part of the secret of Wal-Mart’s success.

2. Model

Consider a model of a retailer that I will call “Wal-Mart.” At a particular point in time, Wal-Mart has as set of stores and consumers make buying decisions based on the location of the stores. I first describe consumer demand holding the set of Wal-Mart store locations
as fixed. Next I describe the cost structure and the process through which Wal-Mart opens new stores.

2.1 Demand

We expect that consumers will tend to shop at the closest Wal-Mart to their home. Nonetheless, in some cases, a consumer might prefer a further Wal-Mart. For example, for a particular consumer, a further Wal-Mart might be more convenient for stopping on the way home from work. Since a consumer at a given location might potentially shop at several different Wal-Marts, we need a model of product differentiation across different Wal-Marts. To this end, I follow the common practice in the literature of taking a discrete choice approach to product differentiation. I specify a nested logit model and put the various Wal-Marts in a consumer’s vicinity in one nest and put the outside good in a second nest.

Now for some notation. Consumers are located across $L$ discrete locations indexed by $\ell$. Suppose at a point in time Wal-Mart has $J$ stores indexed by $j$, with each store in a unique location. For a given location $\ell$, let $y_{\ell j}$ denote the distance in miles between location $\ell$ and store $j$. Let $n_{\ell}$ denote the population of location $\ell$ and let $m_{\ell}$ be the population density at $\ell$.

Consider a particular consumer $i$ at a particular location $\ell$. Let $B_\ell$ denote the set of Wal-Marts in the vicinity of the consumer’s home. (In the empirical work, this will be defined as the set of Wal-Mart’s within 25 miles of the consumer’s home.). The consumer has a dollar amount of spending $\lambda$ that he or she allocates between the following discrete choices: the outside good (good 0) or one of the nearby Wal-Marts in $B_\ell$ (if $B_\ell$ is non-empty). The utility of the outside good 0 is

$$u_{i0} = f(m_\ell) + z_i\omega + \zeta_{i0} + (1 - \sigma)\varepsilon_{i0}. \quad (1)$$

The first term is a function $f(\cdot)$ that depends upon the population density $m_\ell$ at consumer $i$’s location. Assume $f'(m) > 0$; i.e., the outside option is better with more people around. This is a sensible assumption as we would expect there to be more substitutes for Wal-Mart in larger markets for the usual reasons. A richer model of demand would explicitly specify the alternative shopping options available to the consumer. In my empirical analysis this
isn’t feasible for me since I don’t have detailed data on all various shopping options besides Wal-Mart a particular consumer might have. Instead I specify the reduced form relationship between \( f(m_{\ell}) \) and population density.

The second term allows demand for the outside good to depend upon a vector of characteristics \( z_i \) of consumer \( i \) (demographic characteristics and income) times a parameter vector \( \omega \). The final two terms, \( \zeta_{i0} \) and \( \varepsilon_{i0} \), are random taste parameters for the outside good that are specific to consumer \( i \). The distributions for these draws are explained momentarily.

The utility of a given Wal-Mart store \( j \in B_{\ell} \) is

\[
  u_{itj} = -\tau (m_{\ell}) y_{tj} - x_j \gamma + \zeta_1 + (1 - \sigma) \varepsilon_{ij}.
\]

The first term is the utility decrease from travelling to the Wal-Mart \( j \) that is a distance \( y_{tj} \) from the consumer’s home. The weight \( \tau (m_{\ell}) \) the consumer places on distance depends upon population density. This is another reduced form relationship; because of differences in the availability of substitutes induced by differences in population density, consumers in areas with high population density may respond differently with distance than consumers in low density areas. The second term allows utility to depend upon other characteristics \( x_j \) of Wal-Mart store \( j \). In the empirical analysis, the store-specific characteristic that I will focus on is store age. In this way, it will be possible in the demand model for a new store to have less sales, everything else the same. This captures in a crude way that it takes a while for new store to ramp up sales. The final two terms are random utility components specific to store \( j \).

As discussed in Wooldrige (2002), McFadden (1984) showed that under certain assumptions about the distribution of \( (\zeta_0, \zeta_1, \varepsilon_{i0}, \varepsilon_{i1}, \ldots, \varepsilon_{iJ}) \) that I impose here, the probability consumer \( i \) purchases from some Wal-Mart is

\[
  p_{it}^{W} = \frac{\left[ \sum_{j \in B_{\ell}} \exp \left( (1 - \sigma) \delta_{tij} \right) \right]^\frac{1}{1-\sigma}} {\left[ \exp \left( \delta_{i0} \right) \right] + \left[ \sum_{j \in B_{\ell}} \exp \left( (1 - \sigma) \delta_{tij} \right) \right]^\frac{1}{1-\sigma}} \tag{2}
\]

for

\[
  \delta_{i0} \equiv f(m_{\ell}) + z_i \omega,
\]

\[
  \delta_{tij} \equiv -\tau (m_{\ell}) y_{tj} - x_j \gamma,
\]
and the probability of purchasing at a particular store \( j \in B \), conditional on purchasing from some Wal-Mart is

\[
p_{ij}^{W} = \frac{\exp((1 - \sigma) \delta_{ij})}{\sum_{k \in B} \exp((1 - \sigma) \delta_{ik})}.
\]

(3)

In the empirical work, I won’t have consumer characteristic data at the level of the individual. Instead, I will have average characteristics at the level of the location (here a Census Block Group). So I assume that all consumers at a location \( \ell \) have the characteristics of the average consumer at the location; i.e., \( z_{i} \) is a constant \( z_{\ell} \) for all individuals at location \( \ell \). So we can drop the subscript \( i \) in (2) and (3). The probability a consumer at \( \ell \) shops at Wal-Mart \( j \) is

\[
p_{\ell}^{j} = p_{ij}^{W} \times p_{W}^{j}.
\]

Total revenue of store \( j \) is

\[
R_{j} = \sum_{\{\ell \in B\}} \lambda \times p_{\ell}^{j} \times n_{\ell}.
\]

(4)

This equals the spending \( \lambda \) of a consumer times the probability a consumer at \( \ell \) shops at \( j \) times the population \( n_{\ell} \) at \( \ell \), aggregated over all locations in the vicinity of store \( j \).

## 2.2 Cost Structure and Openings of New Stores

This subsection describes the cost structure. It first specifies input requirements for merchandise, labor, land, miscellaneous inputs. It next specifies an urbanization cost. Finally, it specifies the form of the density economies, which will be the main target of the estimation.

### 2.2.1 CGS, Labor, Land, and Miscellaneous Costs

Suppose the gross margin is \( \mu \), so that \( \mu R \) equals sales minus cost of goods sold.

Assume that the labor requirements \( Labor \) of store in a period depend upon the sales \( R \) at the store in a log linear fashion,

\[
Labor = \nu_{Labor} R^{\nu_{0}},
\]

for parameters \( \nu_{Land} \) and \( \nu_{0} \).
Suppose the wage for retail labor at location \( \ell \) is \( W_\ell \) so that the wage bill is \( W_\ell L \). Assume that wage at a location depends on population density

\[
W_\ell = g_{\text{Labor}}(m_\ell).
\]

Assume for now that land and building requirements are proportional to sales,

\[
\begin{align*}
\text{Land} &= v_{\text{Land}} R \\
\text{Bldg} &= v_{\text{Bldg}} R
\end{align*}
\]

(5)

(In later work I plan to allow for scale economies and a richer structure). Let \( P_{\text{Land}} \) and \( P_{\text{Bldg}} \) be the rental prices. Assume the land prices depend upon population density,

\[
P_{\text{Land}} = g_{\text{Land}}(m_\ell).
\]

Assume that building prices are the same everywhere; i.e. \( P_{\text{Bldg}} \) is a constant. I discuss this further below.

Finally, there are miscellaneous costs. They have two parts, a fixed cost and a marginal cost. Assume the fixed miscellaneous is constant across stores. This means I can ignore it in the analysis since it will be independent of where stores are located. (And then number of stores in a period is fixed, just their location is endogenous.). The second part is proportionate to \( R \) and is denominated in dollars,

\[
C_{\text{Misc}} = v_{\text{Misc}} R.
\]

Importantly, the cost \( v_{\text{Misc}} \) is assumed to be constant across locations.

2.2.2 Urbanization Costs

The Wal-Mart store has a distinct format, a big box one-floor store with huge parking lot on a convenient interstate exit. This approach has obvious limitations in a big city. To capture this in the model, assume an urban fixed cost \( C_{\text{Urban}}(m) \) that depends upon the population density \( m \) of a location. If Wal-Mart were to locate in an highly urbanized area, they would have to do things, like make a multi-store structure, that is not necessary in a less urbanized area. For example, there are reports that best Buy Buy expects to pay $200
per square foot in construction costs to enter the Los Angeles market which is four times
their normal building cost of $50 per square foot.

Assume there is range of \( m \), \( m \leq \overline{m} \), where \( C_{Urban}(m) = 0 \) and it is only above the
threshold \( \overline{m} \) where the urbanization cost is positive. The idea here is that Dubuque, Iowa,
the eight largest city in Iowa with a population of 62,220, is relatively similar to the small
towns in Iowa, in terms of the applicability of the Wal-Mart model while Dubuque is very
different from New York City. In other words, \( m_{Dubuque} < \overline{m} \).

### 2.2.3 The Density Cost Component

I now specify the main target of this inquiry, *density economies*. There is a store-level fixed
cost that is made lower by a higher density of stores. This component is intended to capture
a broad set of costs, including managerial costs. Certainly a significant component of these
costs is logistics and distribution cost. A delivery struck may cost the same to operate
whether full or half full. If two stores are near each other, the stores can be replenished on
the same delivery run. Also included here are savings in marketing cost (advertising) by
locating stores near each other.

Rather than develop a micro-model of distribution economies and route structures or
micro-model of economies of management, I follow the literature on productivity spillovers
and take a reduced-form approach. I assume a parametric form whereby a cost saving
“spills over” from one store to another. These spillovers won’t give rise to any externalities,
of course, since central headquarters will be making location decisions that internalize these
benefits. The functional form for the spillover collected by store \( j \) is

\[
s_j = \sum_{k \neq j} \exp(-\alpha y_{jk}),
\]

where \( y_{jk} \) is the distance in miles from store \( j \) to store \( k \). If store \( j \) is right next to another
store \( j \) so that \( y_{jk} \) is approximately zero, the spillover collected by \( j \) from \( k \) is approximately
one spillover unit. As the distance to store \( k \) is increased, the spillover decays at a rate
\( \alpha > 0 \).

The fixed cost depends upon the level of spillover \( s \) it collects according to the following
exponential form:

\[ C_{Density}(s) = \phi \frac{2e^{-\xi s}}{1 + e^{-\xi s}}, \]

where \( \phi \geq 0 \), \( \xi > 0 \). The function is scaled so that with no spillovers the density cost component equals the parameter \( \phi \). Since \( \xi > 0 \), the density cost decreases with \( s \), unless \( \phi = 0 \), in which case it is always zero.

2.2.4 Store Openings

Everything that has been discussed so far considers quantities for a particular time period, i.e., revenues or fixed cost. I now explain the dynamic aspects of the model.

Let \( B_t \) be the store of Wal-Mart stores that consumers can shop at in period \( t \). This consists of stores \( B_{t-1} \) operating in the previous period as well as a set of \( A_t \) new stores opened in the current period, so \( B_t = B_{t-1} + A_t \). This is a good assumption for Wal-Mart. It virtually never exits a location once it opens a store.

Let \( J_t \) be the number of stores operating at \( t \), the cardinality of \( B_t \). Let \( N_t \) be the number of stores opened at \( t \), i.e., the cardinality of \( A_t \). I take \( N_t \) as exogenous in my analysis. Wal-Mart in its first years added only one or two stores a year. The number of new store openings has grown substantially over time and now they sometimes several stores in one week. I am not going to make any attempt to model the growth rate at which Wal-Mart adds stores. Presumably capital market considerations played an important role here. Rather I will take as given that Wal-Mart gets to add a certain number of new stores in each period and the question of interest is where Wal-Mart puts them. Formally, the number of new openings \( \{N_1, N_2, ..., N_T\} \) in periods \( t = 1 \) through the terminal period \( t = T \) is taken as given.

I allow for exogenous productivity growth of Wal-Mart at a rate of \( \rho_t \) per period. What I mean by this is that if Wal-Mart where to hold fixed the set of stores, from period \( t - 1 \) to period \( t \), then revenue at store and all components of level costs would grow at a constant amount \( \rho_t \), i.e.

\[
R_{j,t} = (1 + \rho_t)R_{j,t-1}
\]
\[
C_{j,t} = (1 + \rho_t)C_{j,t-1}.
\]
This means the profit grows at a rate \( \rho_t \), holding fixed the set of Wal-Mart’s stores. As will be discussed later, the growth of sales per store of Wal-Mart has been remarkable. Part of this growth is due the gradual expansion of its product line, from hardware and variety items to eye glasses and tires later, to groceries today, and perhaps banking tomorrow. Rather than model this expansion of product variety directly, I take the process as occurring exogenously.

Let \( \beta \) be the discount factor. Let \( B_0 \) be set of stores open in period 0. Let \( a = (A_1, A_2, ..., A_T) \) be a vector specifying the new stores opened in each period \( t \). Require this vector to be feasible so that the number of new openings in a given period is \( N_t \). Wal-Mart’s problem at time \( t \) is

\[
\max_{\alpha} \sum_{t=1}^{T} \beta^{t-1} \sum_{j \in B_t} [R_{jt} - C_{jt}].
\]  

(8)

for

\[
C_{jt} = (1 - \mu) R_{jt} + W_{jt} \text{Labor}_{jt} + P_{\text{Land},jt} \text{Land}_{jt} + P_{\text{Bldg},jt} \text{Bldg}_{jt} + C_{\text{Misc},jt} + C_{\text{Urban},jt} + C_{\text{Density},jt}
\]

### 3. Data and Some Facts

This section begins by explaining the basic data sources. It then discusses some facts about Wal-Marts expansion process.

#### 3.1 Data

There are five main data elements used in the analysis. The first element is store-level data on sales and other store characteristics that I have obtained from a commercial source. The second element is information about the timing of store openings that has been cobbled together from various sources. The third element is demographic information from the Census. The fourth is land price data for Wal-Mart stores obtained from tax records. The fifth element is data on how retail wages vary with population density from the Census.

Data element one, store-level data variables such as sales, was obtained from *TradeDimensions*, a unit of ACNeilsen. This data provides estimates of average weekly store level
sales for all Wal-Marts open at Feb. 2004, as well as the following additional store characteristics: employment, square footage of the store building, store location exact geographic coordinates and whether or not the store is a supercenter. (Supercenters sell perishable groceries like meat and vegetables in addition to the products carried by regular stores.) The sale estimates are obtained on the basis of a variety of sources including the actual values of direct store deliveries by manufacturers such as Coke and Pepsi, consumer diary information collected by ACNielsen, and information directly provided to TradeDimensions by Wal-Mart. TradeDimensions is a partner of Wal-Mart and the company has several employees that work at Wal-Mart’s central headquarters. This data is the best available and is the primary source of market share data used in the retail industry. Ellickson (2004) is a recent user of this data for the supermarket industry.

Table 1 presents summary statistics of the TradeDimensions data for the 2,936 Wal-Marts in existence in the contiguous part of the United States as the end of 2003.4 5 (Alaska and Hawaii are excluded in all of the analysis.) As of the end of 2003, slightly over half of Wal-Mart’s stores are supercenters. The average Wal-Mart racks up annual sales of $60 million. The breakdown is $42 million per regular store and $76 million per supercenter. The average employment is 223 and the average square feet is approximately 150,000.

As part of its expansion process, Wal-Mart routinely tears down old stores and builds larger ones either on the same property or just down the road. However, it is an extremely rare event for Wal-Mart to shut down a store and exit a location. I estimate this has happened on the order of 30 times over a 42 year period in which Wal-Mart has opened 3,000 stores. Since it is negligible, I am going to ignore exit in the analysis and focus only on openings.

Every Wal-Mart store has a store number. Wal-Mart stores retain this number even when they are upgraded and relocated down the street, which makes it very convenient for keeping track of the stores. The first Wal-Mart store opened in Rogers, AR in 1962. This is Wal-Mart store #1. The next store opened in 1964 in Harrison, AR, store #2. Since

---

4 I will refer to the TradeDimensions data as from 2003, even though it is for Feb 2004. I will think of this as the beginning of 2004, so the data is for 2003.
5 The Wal-Mart Corporation has other types of stores that I exclude in the analysis. In particular, I am excluding Sam’s Club (a wholesale club) and Neighborhood Market stores, Wal-Marts recent entry into the pure grocery store segment.
numbers are assigned in sequential order, store number provides very good information about relative rankings of store opening dates. In the first version of this paper, I just pulled the information about store numbers and addresses from Wal-Mart’s web site, and combined this with counts of stores by year from annual reports to come up with estimates of store age. This is a reasonably accurate dating system but it isn’t perfect. A potential store site picks up a number in the planning process, but it might not be built right away, so store openings aren’t perfectly sequenced with store number. Also, the original store #3 was closed, but this number was reused for a store that opened in 1989. This sort of thing is rare, but it does happen.

Fortunately, additional information is available to give a more precise dating method. Emek Basker (see Basker (forthcoming)) has assembled data on store openings from store #1 in 1962 up to about the year 2001. and I use her data. In its annual reports up to 1978, Wal-Mart published a complete list of all its stores. Basker uses this information as well as analogous information from directories for later years up to the year 2001. I combined the Basker data with information from TradeDimensions and information from Wal-Mart’s web site about openings since 2001 to determine the year that each store opened. This is data element two. Table 2 reports the frequency distribution of opening year categories. In the 1970s, Wal-Mart added about 30 stores a year. Since that time it has averaged over 100 new stores a year.

The third data element, demographic information, comes from the three decennial censuses, 1980, 1990, 2000. The data is at the level of the block group, a geographic unit finer than the Census tract. Summary statistics are provided by Table 3. In 2000, there were 206,960 block groups with an average population of 1,350. The Census provides information about the geographic coordinates of the block group which I use extensively in the analysis. For each block group I determine all the block groups within a five mile radius and add up the population of these neighboring areas. This population within a five mile radius is the population density measure \( m \) I use in the analysis. With this measure, the average block group in 2000 had a population density of 219,000 people per five mile radius. The table also reports mean levels of per capita income, share old (65 or older), share young (21 or younger), and share black. The per capita income figure is in 2000 dollars for all the Census
years using the CPI as the deflator.\textsuperscript{6}

The fourth data element, data on land values for Wal-Mart stores, was obtained from county tax records. At this point, only data for stores in Minnesota and Iowa have been collected (more to follow). The data was obtained from the internet for those counties posting records. Through this method, I was able to obtain the assessed valuations for half of the stores in these states (50 stores in total). Counties in rural areas are less likely to post valuations on the internet for obvious fixed cost reasons. But this selection is not an issue in my analysis since I control for population density.

The fifth data element is average retail wage by county for the year 2000 from County Business Patterns. The variable is total payroll divided by number of retail employees. This wage information is cruder than some other possibilities in terms of its wage information, e.g. the PUMS data. However, its availability at the county level affords a richer geography than other sources.

3.2 Facts about the Diffusion of Wal-Mart

Any discussion of the diffusion of Wal-Mart is best started by viewing on a map the year-by-year expansion of stores. Figure 1 shows the expansion process for years 1971-1980. The expansion process over the entire period 1962-2004 will be posted on the Web.

From inspection of this process it is clear that Wal-Mart diffusion path was from the inside out. Starting from Bentonville AR as the center, it gradually expanded its radius over time. There is one case of a jump where between 1980 and 1981 it filled in South Carolina, skipping most of Georgia. (But coming back to fill it in soon enough.) This is due so an external expansion when it bought Kuhn’s Big K and added a large number of stores. The rest of the expansion process is smooth. External expansion such as what happened in 1981 is rare. (My comment refers to domestic expansion. Foreign expansion has frequently taken place though acquisition.)

Along its expansion path, Wal-Mart made choices along the way about priority locations. It is well known that it avoided very large cities, at least initially. Some evidence of Wal-Mart’s priorities can be obtained by looking at where they are at now. Table 4 presents\textsuperscript{6} Per capita income is truncated from below at $5,000 in year 2000 dollars.
information on the average distances to the nearest Wal-Mart across blockgroups. Consider those blockgroups in the highest density category, 500 thousand or more within a 5 mile radius. Average distance to the nearest Walmart, weighted by population, is 6.7 miles. If we look at the next lower density category, distance falls to 4.2 miles and then again it falls to 3.7. Thereafter distance increase as density falls. If we go all the way to extremely sparse locations, the average distance is 24 miles. Wal-Mart is known for preferring small towns. But as Table 4 makes clear, it is actually medium-sized towns that are the “sweet spot” for Wal-Mart.

The next column conditions on blockgroups that are within 25 miles of a Wal-Mart to start. This decreases average distance, of course, but the pattern remains the same. I condition in this way to make comparisons with earlier years. By conditioning in this way, we are restricting attention to Wal-Mart’s market area and then we can look where it is putting its stores in its market area. The basic pattern is the same if we go back to 1990 or 1980, a U-shaped relationship. Interestingly, the “sweet spot” is changing. In 1980, blockgroups in the 10-20 density range used to be the closest to a Wal-Mart. In 1990 the sweet spot was 20-40. Now the broad range of 40-250 is the sweet spot.

4. First Stage Parameter Estimation

As I first stage I estimate in pieces various parameters of the model. I take the pieces to the second stage analysis of the dynamic problem of Wal-Mart.

Part 1 of this section estimates the demand parameters. Part 2 estimates various cost parameters. I only have data from one year to estimate demand. So Part 3 explains how I extrapolate to other years.

In the analysis of costs, particular attention will be placed on how costs vary across cities with under 10,000 in population density (people per 5 mile radius circle) and cities in the 10,000-40,000 density range. As discussed in the introduction, my second stage procedure will interchange the timing of store openings of stores in this city size groupings. So cost differences here are of particular interest.
4.1 Demand Estimation

With a given vector $\theta$ of parameters from the demand model, we can plug in the demographic data and obtain predicted values of revenues $\hat{R}_j(\theta)$ for each store $j$ from equation (4). Let $\varepsilon_j$ be the difference between log actual sales and log predicted sales,

$$\varepsilon_j = \ln(R_j) - \ln(\hat{R}_j(\theta)).$$

I assume the discrepancy is normally distributed measurement error. There most certainly is measurement error in the sales data. My sales figure is an estimate TradeDimensions comes up with based on things like direct store shipments of Coke and Pepsi and other proxies of sales. I estimate the parameters with maximum likelihood.

Before going to the estimates I have to take care of two unresolved issues. The first is about specification. I need to specify the forms of the reduced form functions $f(m)$ and $\tau(m)$. Assume

$$f(m) = \omega_0 + \omega_1 \ln(m) + \omega_2 (\ln(m))^2$$
$$\tau(m) = \tau_0 + \tau_1 \ln(m)$$

for $m = \max\{1, m\}$, for population density in thousands. (Thus the minimum value of $\ln(m)$ is zero.)

The second issue is what to do about supercenters. As can be seen in Table 1, supercenter sales are almost twice as large as regular store sales. What is going on here is clear: supercenters have a broader product line, so everything else the same we would expect supercenters to have larger sales. But this is not something that fits easily into the model just outlined. Even if I were able to put supercenters cleanly in the demand model, in my later analysis I would have the problem that I don’t know the dates when a given supercenter was converted from a regular store, I only know store openings. (A large percentage of supercenter were once regular stores.) My product would be a lot simpler if Wal-Mart had never got into the supercenter format.

I finesse the supercenter issue in the following way. I imagine that for the consumer, shopping for groceries and shopping items found at a regular Wal-Mart are two separate
things and the activities take place at separate shopping trips. (Of course this goes against one of the basic premises of the supercenter format.) A supercenter is then two distinct stores: a regular Wal-Mart combined with a grocery store. The demand model described above just applies for the regular Wal-Mart component of a supercenter. The predicted sales $\hat{R}_j$ for a store $j$ that is a supercenter is only the predicted sales of the items in a regular store. If I observed a breakdown of sales for each supercenter into those items carried at regular-store items and those not carried, then my sales figure I would use in the estimation would just be the regular items component. However, this is unobserved for supercenters. My strategy then is to exclude the unobserved data in my likelihood function. But importantly, the supercenters remain in the choice set of consumers. So if a regular store is near a supercenter, it’s sales will be lower, everything else the same.

Table 5 reports the demand estimates for three specifications. The specifications differ in the extent to which store age is used as a store characteristics. Specification 1 uses no store-age information. It fits the data reasonably well, with an $R^2$ of .674. Specification 2 adds a dummy variables for stores 2 years and older from brand new stores. The effect age is substantial, a mature store increases log sales by .25. Specification 3 breaks the mature category into four different groups. There is some effect of further increases in age. The effect increase from .24 for 3-5 to .319 for 6-10. But the differences are relatively small compared to the effect of just being 2 or above. And there is not much improvement in goodness of fit. I will use specification 2 for my baseline model of demand. An advantage of this specification for later use relative to specification 3 is that the impacts of a change in store location will not have a lagged effect 20 years down the line as is the case for Specification 3.

The parameters in Table 5 are difficult to interpret directly so I will look at how fitted values vary with the underlying determinants of demand. Table 6 examines how demand varies with distance to the closest Wal-Mart and population density. For the analysis, the demographic variables are set to their mean level from Table 3. There is assumed to be only one store within the vicinity of the consumer (i.e. within 25 miles) and the distance of this single Wal-Mart is varied in the table. Consider the first row, where distance is set to zero (the consumer is right-next door to a Wal-Mart) and population density is varied. As
expected, there is a substantial negative effect of population density on demand. A rural consumer right next to a Wal-Mart shops there with a probability that is essentially one. With a population density of 40 this falls to .77 and up to 250 it falls to less than .25. In a large market there are many substitutes. Even a customer right next to a Wal-Mart is not likely to shop there. While per capita demand falls, overall demand overwhelmingly increases. A market that is 250 times as large as an isolated market may have a per capita demand that is only a fourth as large, but overall demand is over 50 times as large.

Next consider the effect of distance holding fixed population density. In a very rural area, increasing distance from 0 to 5 miles has only a small effect on demand. This is exactly what we would expect. Now raising the distance from 5 to 10 miles does have an appreciable effect, .971 to .596. In thinking about the reasonableness of this effect, it is worth noting the miles here are “as the crow flies,” not driving distance. An increase of 5 to 10 could be the equivalent of a 10 to 20 mile increase in driving time. In that light, the change in demand from .971 to .596 seems highly plausible. Demand taper out at 15 miles and goes to zero at 20 miles.

Next consider the effect of distance in larger markets. The negative effect of distance begins much earlier in larger markets. For a market of size 250, an increase in distance from 0 to 5 miles reduces demand by on the order of 80 percent while the effect of distance in rural markets is miniscule. This is what we would expect.

Other demand characteristics are of note. It is possible to calculate consumer demand when there are multiple Wal-Marts in his or her area. At the mean characteristics, if a consumer is zero miles to one Wal-Mart and 2 miles to another, (and no others are in the area), the consumer goes to the one next door with probability .75 and the other with probability .25, conditioned upon shopping at one. So allowing for product differentiation among Wal-Mart, instead of just assuming consumers shop at the closest one, is important. But if the distance disadvantage of the further store is increased, demand for the further store drops off sharply.

Demand varies by demographic characteristics in interesting ways. Wal-Mart is an inferior good in that demand decreases in income. Demand is higher among whites and lower among younger people and older people.
4.2 Labor Costs

Regressing log of employment on log of sales for 2004, I obtain the labor requirements function,

\[
\ln \text{Labor} = 2.29 + 0.74 \ln R
\]

(.06) (.02)

Next I obtain an estimate of the function \( W(m) \) which specifies how the retail wage varies with population density. I project average county wage on a quartic equation in population density (the coefficients not reported here.) Table 7 shows how average actual wage and the fitted wage varies with population density. As is typical, measured wages increase in density. For the under 10 category, the wage is $17,150 which increases to $18,520 for the 10-40 category and even higher thereafter.

There are obvious measurement difficulties here. Pay divided by total employment is a crude measure of the wage since hours worked varies substantially across individuals, particularly in retail. However, since my labor input level is in employment, not hours, even if I could come upon hourly wage information I would have to get data on hours from Wal-Mart to use it and such data is not available.

Of course I am not taking into account differences in labor quality across locations either here. There is evidence in the urban economics literature that workers in larger cities are better quality (see Glaeser and Mare). Later I show that Wal-Mart could have earned substantially more revenues if it reordered its opening sequence and went to larger cities first as compared to smaller cities. To the extent Wal-Mart could have obtained higher quality workers from this perturbation, it means my results understate the density cost savings it achieved by doing what it did.

4.3 Land Costs

Wal-Marts typically use relatively large plots of land, on the order of 10, 15, to even 20 acres. To open a Wal-Mart with this size of a plot of land in Manhattan would cost a fortune. So to open a Wal-Mart in a very urban area would results in substantial increases in land rents compares to a less urban area. Nevertheless, \textit{a priori}, it is not obvious that rents in medium
size cities will be more than rents in small cities or rural areas. Wal-Mart tends to open its stores on the outskirts of town. In the standard urban theory, rents on the outskirts of town equal the agricultural land rent.

To examine this hypothesis, I use the land value data on the 50 Wal-Marts in Iowa and Minnesota that I have collected. I don’t know acreage, so I make use of the fixed coefficient assumptions made in (5) and assume acreage is proportionate to building size. I then regress the log of land prices (assessed value divided by building square footage) on dummy variables by population density class. I also include state fixed effects as well as age of the store. The results are reported in Table 8. Comparing the “Under 10” density class with the 40-80 and 80 and above density classes I find significant differences in land prices. Plugging in the coefficient estimates, the predicted prices differ by factors of 2.6 and 3.4, respectively, from the “Under 10” group. But the differences between “Under 10” and “10-40” are negligible. In the analysis I will treat the land prices for these groups the same.

4.4 Other Costs

In the analysis I set gross margin less nonlabor variable costs equal to

\[ \mu - P_{Land}v_{Land} - P_{Bldg}v_{Bldg} - \nu_{Misc} = .17. \]

The price of land applies for locations with density \( \leq 40 \). (Since locations with density \( \geq 40 \) are not altered, pricing for such parcels is not needed). Note land and buildings are variable costs here because larger sales require more space.

Wal-Mart’s gross margin over the years has ranged from .22 to .26 (from Wal-Marts annual reports.), so \( \mu = .24 \) is a sensible value. The mean ratio of assessed value of land and building to annual sales in my sample is .14. Converting this to rental values results in a figure on the order of .01 to .02 for the quantity \( P_{Land}v_{Land} + P_{Bldg}v_{Bldg} \). Setting \( \nu_{Misc} \) to be on the order of .05 to .06 is on the high side. There is much cost that takes place outside of the store. I have already discussed how I am taking store-level labor costs out of this. An there are is also that large profit margin to consider. Here I am being conservative and erring in the direction of understating variable profit. This works against the incentive to increase revenues by going to medium cities instead of small cities.
4.5 Extrapolation to Other Years

So far I have constructed a model of Wal-Marts demand and costs circa 2003, the year of the TradeDimensions data. I will need a demand and cost model for all the years that Wal-Mart was in business to study its diffusion path.

Growth in Wal-Mart on a per store basis is remarkable. We see from Table 1 that in 2003, average store sales (regular stores) was $42.4 million. In 1972, average sales (in 2003 dollars) was only $11.1. How can I take this into account.

I applied the following procedure. First, I took the exact demand model from 2003 and evaluated average sales per store in the prior years, given the configuration of stores for each of these prior years. The 2003 demand model evaluated at the store configuration for 1972 predicted an average store sales (in 2003 dollars) of $31.4 million. So one third of the difference in average in average store size of 11.1 in 1972 and 42.4 in 2003 is due to the change in the average market size from the two periods. The rest of the difference is unexplained. I attribute this to productivity growth. I determine the average growth $r_{1972}$ from 1972 to 2003 that would generate the sales difference of 11.1 to $31.4$. The annual growth in this case is approximately .04. Proceeding this way, I determined that the following simple series fit well. Growth before 1980 at $r = .04$, growth after 2000 at $r = .02$ and linearly interpolating for the 20 years in between.

This growth factor was applied to all the cost functions as well. The impact of this assumption is that if Wal-Mart keeps the same set of stores over a given time period, and demographics were held fixed, then revenue and costs increase by a proportionate amount, so profit increases by a proportionate amount.

The growth factor applies holding demographics fixed. But demographics changed over time and I take this into account as well. I use data from the 1980, 1990, and 2000, decennial censuses. For years before 1980, I use 1980, for years after 2000 I use 2000. For years in between I use a convex combination of the appropriate censuses as follows. For example, for 1984 I convexify by placing .6 weight on 1980 and .4 weight on 1990. I so this by assuming that only 60 percent of the people in the people from the given 1980 blockgroup are still there and that 40 percent of the people form the 1990 block group are already there as of 1984. This procedure is clean, since I avoid the issue of having to link the block groups
longitudinally over time, which would be very difficult to do. Given my continuous approach to the geography, there is no need to link block groups over time.

5. Stage Two: The Density Cost Component

This section attempts to quantify the importance of economies of density. The procedure developed delivers a lower-bound on its importance. It then uses the information to determine the effect on costs of splitting up Wal-Marts operations into two parallel operations.

5.1 The Methodology

We know what Wal-Mart did. I consider perturbations of its actual strategy, re-orderings of the stores it actually selected.

Recall the problem (8) faced by the firm. Let $\Pi(a, \theta)$ be discounted profit from taking action $a$ given parameter vector $\theta$. Suppose $a_0$ is the path the Wal-Mart actually selected. Let $a_k$ be a deviation, $k \in \{1, 2, ..., K\}$ and let $A$ be the set of deviations. Define a parameter vector $\theta$ to admissible if

$$\Pi(a_0, \theta) \geq \Pi(a_k, \theta), a_k \in A. \quad (9)$$

Let $\Theta(A)$ be the set of admissible parameter vectors, given the set of deviations $A$.

I consider two classes of deviations. A type 1 deviation resequences store openings over a given time interval so that store openings in medium sized markets occur before openings in small sized markets, leaving openings in large sized markets the same. This kind of deviation raises discounted revenue, but potentially reduces spillovers increasing the density cost component. The purpose of this deviation is to put a lower bound on the density cost component. A type 2 deviation sequences store openings for the same set of stores in descending order of spillovers at time of openings. The purpose of this deviation it to put an upper bound on the density cost component.

To define the deviations formally, let $t_{begin}$ and $t_{end}$ be the first and last periods of a time interval. This is the time interval over which store openings will be reordered. Openings before $t_{begin}$ and after $t_{end}$ are left unaltered. Recall the parameter $\overline{m}$. This is the threshold below which the urbanization cost is zero, $C_{Urban}(m) = 0, m < \overline{m}$. Define two cutoffs $m^1$ and $m^2$, for $m^1 < m^2 \leq \overline{m}$. For a given store $j$, let $m_j$ be defined as the population density.
for store $j$ at the time the store actually opened. In the deviations considered, all stores with $m_j \geq m^2$ are left alone while stores with $m_j < m^2$, are potentially resequenced. Since all of the resequenced stores are below the cut-off $\overline{m}$, and since the urban cost is specified as a fixed cost and independent of sales, I need not know the parameters of the urban-cost function to determine the change in profits from the deviations. (I do need to know $\overline{m}$. As already noted, $\overline{m} > m_{Dubuque} = 65.8$).

For a type 1 deviation, let the store openings that opened between $t_{\text{begin}}$ and $t_{\text{end}}$ with $m_j < m^1$ be defined as small-market openings for that interval. Let the openings with $m_j \in [m^1, m^2]$ be medium-market openings. The deviation is constructed as follows. The timing of opening of all stores with $m_j \geq m^2$ is left unchanged. The small and medium market stores are resequenced so that all medium-market stores are opened ahead of small-market stores. Within medium-market stores, the stores are opened in the same order that actually took place. The total number of store openings in a given year is kept the same as what actually took place.

The Type 2 deviation is defined for a given value of the spillover decay parameter $\alpha$. For a given value of $t_{\text{begin}}$, $t_{\text{end}}$, $m^1$, $m^2$, the deviation is constructed as follows. Take the set of all stores opening in the interval with $m_j < m^2$. Call this the pool of candidates for the next opening. Call the stores actually opened in period $t_{\text{begin}}-1$ or before plus the stores with $m_j \geq \overline{m}$ opening in $t_{\text{begin}}$ the already-opened set. For each store $j$ in the pool, calculate the spillover $s_j$ coming from the stores in the already-opened set using formula (6) and the value of spillover decay $\alpha$ taken as given. Find the store in the pool with the highest spillover. Let this be the first store opened. Remove it from the pool and add it to the already-opened set. Repeat this process until the number of openings in the period is the same as the actual number of periods. Then go to the next period and do the same thing. With this path, stores with the highest amount of spillover at time of opening are opened first.

### 5.2 Admissible Parameters

I set $m^1 = 10$ and $m^2 = 40$. I consider three different deviation time intervals. (1) 1971-1980, there are 91 store openings with $m_j < m^1$ in this interval and 149 with $m_j \in (m^1, m^2)$. (2) 1982-1990, with 210 and 459 stores affected. (3) 1991-2002, with 102 and 345 stores
affected.  

In Table 9 I report discounted values of revenues, wages, and operating profits for the actual policy, the type 1 deviation, and the type 2 deviation, for the three different time intervals. The values are discounted to the first period of the deviation. The present values include flows up to the period \( t_{end} + 1 \). I go one period after to take account of the lag in demand. At period \( t_{end} + 2 \) and beyond, revenues are the same regardless of the deviation selected up to period \( t_{end} \), since the configuration of stores is the same beginning \( t_{end} \) and since the lag in demand is two periods.

As expected, the type 1 deviation raises revenues, wages, and operating profits. Operating profits subtract out all costs except for the urbanization cost and the fixed cost that varies with density. The deviations have no effect on urbanization costs since \( m^2 < \bar{m} \). So if a particular deviation raises discounted operating profits by \( x \), it must be that the loss in density economies more than outweigh the gain in operating profit.

The change in operating profits from the type 1 deviation are 87, 553, and 370 in millions of dollars. The biggest impact is in the second interval since the highest number of stores are reshuffled around for this case. As a percent of operating profit, the numbers are 6.2, 3.6, and .5. The impact for time interval (3) since the absolute number of stores that are reshuffled is smaller than in (2) and a larger portion of Wal-Mart’s business was in big cities in the later time period and this is left alone in the deviation.

The increase in operating profits from deviation 1 comes at a cost. The last column in table 9 reports average spillover collected in the period \( t_{begin} + 2 \) of the deviation interval. I use a decay parameter of \( \alpha = .04 \) to calculate the spillover. In for the firm time interval deviation, average spillover collected at \( t_{begin} + 2 \) is 1.94. With the type 1 deviation, the average falls to 1.52, the equivalent of half of a store located very nearby. The effect of the spillovers enters in a nonlinear way, so the effect on the density costs depends upon more than just the mean. But the difference in mean provides some information about the tradeoff facing the firm. Analogously, average spillover falls from 3.81 to 3.72 in time interval 2. The effect is smaller here because by this point there is a larger stock of existing

---

7 I leave out 1981 because that is the year of the Kuhn’s K acquisition.
8 This includes land and building cost as these are assumed to be variable inputs for the exercise.
9 I look two periods in to allow more time for the policy change to take effect. But I don’t go all the way to \( t_{end} \) because by that point everything is the same.
firms that aren’t begin moved around and also the number of firms under 40 in population density is smaller as a percent of Wal-Mart’s total. But the qualitative effect is the same. For time interval 3, the mean is actually larger with the deviation, though the difference is negligible. For deviation 1 not to be prefered, and to have higher density costs, will depend upon the nonlinear relationship (7) between spillover and cost.

Type 2 deviations are defined for a given value of \( \alpha \). This is needed to select on the amount of spillover collected. The type 2 deviation which selects openings based on the highest spillover reduces operating profit. The deviation for all three cases leads to losses in operating profits, albeit a small loss for interval (2). As expected, mean spillover increases with the deviation.

Let \( \Theta_1, \Theta_2, \Theta_3 \), be the set of admissible parameters that correspond to each time interval. Each set is large, and takes some work to describe. The sets are qualitatively similar, and roughly similar quantitatively. The intersection is non-empty.

Fix \( \alpha = .04 \). Consider first only type 1 deviations. Recall that \( \phi \) is a multiplicative parameter while \( \xi \) affects curvature. Figure 2 plots the level of \( \phi \) as a function of \( \xi \) that makes Wal-Mart indifferent to the type 1 deviation for each \( \xi \), for each time interval \( j \). Let \( \hat{\phi}_j^1(\xi) \) denote this critical level. (It is convenient to plot the function on a log scale for \( \xi \))

A remarkable feature of Figure 2 is the extent to which the critical levels for intervals 1 and 2 overlap each other. It indicates a degree of continuity in the store location behavior between the 1970s and 1980s. The pattern for interval 3 is different. For low levels of \( \xi \), density costs are actually lower for the deviation than for the actual policy, so there is no feasible \( \phi \). For \( \ln(\xi) \) in the range of 2 to 6, there is a feasible range of \( \phi \). For \( \ln(\xi) \geq 3 \), the cutoff is actually close in magnitude to the cutoffs for time intervals 1 and 2.

Turning next at type 2 deviations, it turns out that the region with \( \ln(\xi) \leq 3 \), for time intervals 1 and 2, there exist no \( \phi \) that simultaneously satisfies inequality 1 and inequality 2. So this is inadmissible. So the admissible region of \( \xi \) for intervals 1 and 2 approximately overlaps the admissible region of \( \xi \) for interval 3. Moreover, for most of this region, constraint 2 that Wal-Mart not prefer a type 2 deviation is automatically satisfied. (Over this region density fixed cost is actually lower for the actual policy than for deviation 2 so the actual policy dominates it in revenues and costs.) Thus there is no upper-bound for \( \phi \) in this
region. The bottom line is that in the region of admissible $\xi$, the lower bound for $\phi$ is roughly of the same order of magnitude for all three time interval, as illustrated in figure 2.

The pattern discussed also holds for other values of $\alpha$. Intervals 1 and 2 are remarkably similar with respect to the type 1 deviation. As $\alpha$ is increased beyond .04, the admissible region of $\xi$ decreases.

This is a very preliminary discussion. In the next draft of this paper, clearly more works has to be done in characterizing the extent to which the admissible regions from the three time intervals are similar.

A second issue is what the parameters mean, what is the magnitude of the density economies? Some insite about magnitude can be obtained in the next subsection where a policy experiment is discussed.

5.3 Policy Experiment

Consider a policy that in each period $t$, breaks up Wal-Mart into two separate companies. Suppose the company is broken up so that the stores overlap. So store density falls in half, as opposed to what would have with a different breakup on a regional basis that would leave store density in a particular region in tact. Suppose things remains the same on the demand size, so total revenue of the two companies combined remain the same. Suppose, however that spillovers do not cross company boundaries. Hence, economies of density are reduced after the breakup.

I determine the effect of the change in density fixed cost as a percent of total operating profits, which are unaffected. For a given parameter vector $\theta$, let $\Delta^1(\theta)$ be the discounted percentage change in fixed costs as a percent of operating profit over the time interval 1 discussed above. Analogously, define $\Delta^2(\theta)$ and $\Delta^3(\theta)$. Let $\Delta^{avg}(\theta) = \frac{\Delta^1(\theta) + \Delta^2(\theta) + \Delta^3(\theta)}{3}$.

Consider the problem

$$\min_{\theta \in \Theta} \Delta^{i}(\theta) = \Delta^{*i}$$

Table provides the results of this exercise. The results from the different time intervals are similar. The minimum welfare cost averaged over the three time periods ranges from .10 from the interval 3 admissible set to .27 for the interval 1 admissible set. Either way, this is a substantial effect. Density issues affect Wal-Mart’s profits in a significant way.
One oddity in the table is that the minimum effect is zero for welfare in time period 1, using the admissible set constructed from time-period 1 deviations. This minimum is obtained in the limit at $\xi$ goes to infinity and $\phi$ goes to infinity. This is odd, and needs further scrutiny.
6. References

REFERENCES


Table 1
Summary Statistics: TradeDimensions Data

<table>
<thead>
<tr>
<th>Store Type</th>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>Sales ($millions/year)</td>
<td>2,936</td>
<td>59.6</td>
<td>29.2</td>
<td>5.2</td>
<td>170.3</td>
</tr>
<tr>
<td>Regular Store</td>
<td>Sales ($millions/year)</td>
<td>1,457</td>
<td>42.4</td>
<td>19.3</td>
<td>5.2</td>
<td>122.2</td>
</tr>
<tr>
<td>SuperCenter</td>
<td>Sales ($millions/year)</td>
<td>1,479</td>
<td>76.5</td>
<td>27.4</td>
<td>13.0</td>
<td>170.3</td>
</tr>
<tr>
<td>All</td>
<td>Employment</td>
<td>2,936</td>
<td>223.4</td>
<td>132.6</td>
<td>31.0</td>
<td>801.0</td>
</tr>
<tr>
<td>Regular Store</td>
<td>Employment</td>
<td>1,457</td>
<td>112.2</td>
<td>38.2</td>
<td>47.0</td>
<td>410.0</td>
</tr>
<tr>
<td>SuperCenter</td>
<td>Employment</td>
<td>1,479</td>
<td>332.8</td>
<td>96.5</td>
<td>31.0</td>
<td>801.0</td>
</tr>
<tr>
<td>All</td>
<td>Building (1,000 sq feet)</td>
<td>2,936</td>
<td>143.1</td>
<td>54.7</td>
<td>30.0</td>
<td>286.0</td>
</tr>
<tr>
<td>Regular Store</td>
<td>Building (1,000 sq feet)</td>
<td>1,457</td>
<td>98.6</td>
<td>33.3</td>
<td>30.0</td>
<td>219.0</td>
</tr>
<tr>
<td>SuperCenter</td>
<td>Building (1,000 sq feet)</td>
<td>1,479</td>
<td>186.9</td>
<td>31.1</td>
<td>76.0</td>
<td>286.0</td>
</tr>
</tbody>
</table>

Table 2
Distribution of Wal-Mart Stores by Year Open

<table>
<thead>
<tr>
<th>Period Open</th>
<th>Frequency</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971-1980</td>
<td>277</td>
<td>302</td>
</tr>
<tr>
<td>1981-1990</td>
<td>1,236</td>
<td>1,538</td>
</tr>
<tr>
<td>1991-2000</td>
<td>1,080</td>
<td>2,618</td>
</tr>
<tr>
<td>2001-2003</td>
<td>318</td>
<td>2,936</td>
</tr>
</tbody>
</table>

Table 3
Summary Statistics for Census Block Groups

<table>
<thead>
<tr>
<th></th>
<th>1980</th>
<th>1990</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>269,738</td>
<td>222,764</td>
<td>206,960</td>
</tr>
<tr>
<td>Mean population (1,000)</td>
<td>0.83</td>
<td>1.11</td>
<td>1.35</td>
</tr>
<tr>
<td>Mean Density (1,000 in 5 mile radius)</td>
<td>165.3</td>
<td>198.44</td>
<td>219.48</td>
</tr>
<tr>
<td>Mean Per Capita Income (Thousands of 2000 dollars)</td>
<td>14.73</td>
<td>18.56</td>
<td>21.27</td>
</tr>
<tr>
<td>Share old (65 and up)</td>
<td>0.12</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Share young (21 and below)</td>
<td>0.35</td>
<td>0.31</td>
<td>0.31</td>
</tr>
<tr>
<td>Share Black</td>
<td>0.1</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>----------------------------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>Under 1</td>
<td>1.3</td>
<td>24.2</td>
<td>15.1</td>
</tr>
<tr>
<td>1-5</td>
<td>9.6</td>
<td>16.7</td>
<td>13.2</td>
</tr>
<tr>
<td>5-10</td>
<td>16.1</td>
<td>11.3</td>
<td>9.9</td>
</tr>
<tr>
<td>10-20</td>
<td>24.0</td>
<td>7.2</td>
<td>6.6</td>
</tr>
<tr>
<td>20-40</td>
<td>33.2</td>
<td>5.1</td>
<td>4.8</td>
</tr>
<tr>
<td>40-100</td>
<td>50.4</td>
<td>4.0</td>
<td>3.9</td>
</tr>
<tr>
<td>100-250</td>
<td>76.2</td>
<td>3.7</td>
<td>3.6</td>
</tr>
<tr>
<td>250-500</td>
<td>90.2</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>500 and above</td>
<td>100.0</td>
<td>6.9</td>
<td>6.9</td>
</tr>
</tbody>
</table>
Table 5
Demand Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>scaling parameter</td>
<td>29.742</td>
<td>29.057</td>
<td>18.702</td>
</tr>
<tr>
<td>( \rho = 1/(1-\sigma) )</td>
<td>correlation parameter</td>
<td>.781</td>
<td>.767</td>
<td>.959</td>
</tr>
<tr>
<td>( \tau_0 )</td>
<td>constant</td>
<td>.616</td>
<td>.621</td>
<td>.464</td>
</tr>
<tr>
<td>( \tau_1 )</td>
<td>population density within 5 miles</td>
<td>-.046</td>
<td>-.049</td>
<td>-.001</td>
</tr>
<tr>
<td>( \omega )</td>
<td>constant</td>
<td>-7.769</td>
<td>-7.586</td>
<td>-10.517</td>
</tr>
<tr>
<td>( \ln \max_c(\text{neig5}) )</td>
<td></td>
<td>1.503</td>
<td>1.605</td>
<td>2.596</td>
</tr>
<tr>
<td>( \ln \max_c(\text{neig5})^2 )</td>
<td></td>
<td>-.027</td>
<td>-.037</td>
<td>-.140</td>
</tr>
<tr>
<td>pcitrun</td>
<td></td>
<td>.023</td>
<td>.021</td>
<td>.018</td>
</tr>
<tr>
<td>blackshr</td>
<td></td>
<td>.928</td>
<td>.909</td>
<td>.841</td>
</tr>
<tr>
<td>youngshr</td>
<td></td>
<td>1.241</td>
<td>.881</td>
<td>.633</td>
</tr>
<tr>
<td>oldshr</td>
<td></td>
<td>1.369</td>
<td>1.158</td>
<td>1.288</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>store age 3- dummy</td>
<td>.246</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>store age 3-5 dummy</td>
<td></td>
<td>.240</td>
<td></td>
</tr>
<tr>
<td></td>
<td>store age 6-10 dummy</td>
<td></td>
<td>.319</td>
<td></td>
</tr>
<tr>
<td></td>
<td>store age 11-20 dummy</td>
<td></td>
<td>.340</td>
<td></td>
</tr>
<tr>
<td></td>
<td>store age 20- dummy</td>
<td></td>
<td>.225</td>
<td></td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>measurement error</td>
<td>.092</td>
<td>.090</td>
<td>.090</td>
</tr>
<tr>
<td>( N )</td>
<td></td>
<td>1457</td>
<td>1457</td>
<td>1457</td>
</tr>
<tr>
<td>SSE</td>
<td></td>
<td>134.746</td>
<td>131.039</td>
<td>130.554</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td>.674</td>
<td>.683</td>
<td>.684</td>
</tr>
<tr>
<td>( \ln (L) )</td>
<td></td>
<td>-333.020</td>
<td>-312.701</td>
<td>-309.963</td>
</tr>
<tr>
<td>Distance (miles)</td>
<td>Neig5 (thousands)</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>.999</td>
<td>.984</td>
<td>.957</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>.997</td>
<td>.973</td>
<td>.930</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>.995</td>
<td>.954</td>
<td>.890</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>.991</td>
<td>.923</td>
<td>.829</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>.984</td>
<td>.875</td>
<td>.745</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>.971</td>
<td>.803</td>
<td>.637</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>.596</td>
<td>.213</td>
<td>.122</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td>.062</td>
<td>.018</td>
<td>.011</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>.003</td>
<td>.001</td>
<td>.001</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
</tbody>
</table>

Notes: These values are evaluated when black ratio, young ratio, old ratio and PCI are set to be at their
Table 7
Average Retail Wages and Population Density

<table>
<thead>
<tr>
<th>Density Category (1,000 in 5 mile radius)</th>
<th>Actual Wage</th>
<th>Fitted Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 10</td>
<td>17.15</td>
<td>17.06</td>
</tr>
<tr>
<td>10-40</td>
<td>18.52</td>
<td>18.55</td>
</tr>
<tr>
<td>40-100</td>
<td>19.70</td>
<td>20.06</td>
</tr>
<tr>
<td>100-250</td>
<td>21.61</td>
<td>21.32</td>
</tr>
<tr>
<td>250-and up</td>
<td>22.76</td>
<td>22.88</td>
</tr>
</tbody>
</table>

Source: County Business Patterns 2000 and author’s calculations.
Table 8  
Land Price Regression  
Dependent Variable: Log of Estimated Land Price  
(Excluded density group is 0-10)  

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>2.09</td>
<td>.29</td>
</tr>
<tr>
<td>Population Density 10-40</td>
<td>- .04</td>
<td>(.27)</td>
</tr>
<tr>
<td>Population Density 40-80</td>
<td>.96</td>
<td>(.28)</td>
</tr>
<tr>
<td>Population Density 80 and above</td>
<td>1.23</td>
<td>(.27)</td>
</tr>
<tr>
<td>Store Age</td>
<td>.02</td>
<td>(.02)</td>
</tr>
<tr>
<td>Iowa Dummy</td>
<td>-.64</td>
<td>(.23)</td>
</tr>
</tbody>
</table>

N = 43  
R² = .63
<table>
<thead>
<tr>
<th>Interval: 1971-1980</th>
<th>Revenue (Millions)</th>
<th>Wages (Millions)</th>
<th>Operating Profit (Millions)</th>
<th>Mean Spillover t_{begin+2} (\alpha = .04)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Policy</td>
<td>14,965</td>
<td>1,131</td>
<td>1,413</td>
<td>1.94</td>
</tr>
<tr>
<td>Type 1 Deviation</td>
<td>15,797</td>
<td>1,185</td>
<td>1,500</td>
<td>1.52</td>
</tr>
<tr>
<td>Type 2 Deviation</td>
<td>14,602</td>
<td>1,109</td>
<td>1,374</td>
<td>3.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interval: 1982-1990</th>
<th>Revenue (Millions)</th>
<th>Wages (Millions)</th>
<th>Operating Profit (Millions)</th>
<th>Mean Spillover t_{begin+2} (\alpha = .04)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Policy</td>
<td>159,992</td>
<td>12,006</td>
<td>15,192</td>
<td>3.81</td>
</tr>
<tr>
<td>Type 1 Deviation</td>
<td>165,231</td>
<td>12,345</td>
<td>15,745</td>
<td>3.72</td>
</tr>
<tr>
<td>Type 2 Deviation</td>
<td>159,757</td>
<td>11,988</td>
<td>15,169</td>
<td>4.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interval: 1991-2002</th>
<th>Revenue (Millions)</th>
<th>Wages (Millions)</th>
<th>Operating Profit (Millions)</th>
<th>Mean Spillover t_{begin+2} (\alpha = .04)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual Policy</td>
<td>749,676</td>
<td>55,498</td>
<td>71,947</td>
<td>5.16</td>
</tr>
<tr>
<td>Type 1 Deviation</td>
<td>753,275</td>
<td>55,740</td>
<td>72,317</td>
<td>5.18</td>
</tr>
<tr>
<td>Type 2 Deviation</td>
<td>747163</td>
<td>55,367</td>
<td>71,651</td>
<td>5.68</td>
</tr>
</tbody>
</table>
Table 10
Effects of Policy Experiment
Minimum Effect over Admissible Set

<table>
<thead>
<tr>
<th>Percentage Effect on Density Costs of Policy</th>
<th>Admissible Set for Time Interval 1</th>
<th>Admissible Set for Time Interval 2</th>
<th>Admissible Set for Time Interval 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^*_{1}$</td>
<td>.22</td>
<td>0</td>
<td>.15</td>
</tr>
<tr>
<td>$\Delta^*_{2}$</td>
<td>.14</td>
<td>.24</td>
<td>.08</td>
</tr>
<tr>
<td>$\Delta^*_{3}$</td>
<td>.11</td>
<td>.19</td>
<td>.07</td>
</tr>
<tr>
<td>$\Delta^*_{\text{avg}}$</td>
<td>.17</td>
<td>.27</td>
<td>.10</td>
</tr>
</tbody>
</table>
Fig 1: Dispersion of Wal-Mart Stores
Value of phi parameter that Leaves Wal-Mart Indifferent to Deviation 1 For All Three Intervals as a function of ln(xzi)