Testing for Common Valuation in the Presence of Bidders with Informational Advantage*

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Abstract

We develop a test for common values in auctions in which some bidders possess information about rivals' bids. Information about rival's bids causes a bidder to bid differently when she has a private value than when there is a common valuation component or her value depends on rivals' information. In a divisible good setting, such as treasury bill auctions, bidders who obtain information about rivals' bids in the private values model use this information only to update their prior about the distribution of the residual supplies. In the model with a common value component, they also update their prior about the valuation itself. We use this different updating effect to construct our test. The proposed test displays quite a good performance in our Monte Carlo studies. We apply it to data from Canadian treasury bill market, where some bidders have to route their bids through dealers who also submit bids on their own. We cannot reject the null hypothesis of private values in our data.

Keywords: multiunit auctions, treasury auctions, structural estimation, nonparametric identification and estimation, test for common value

JEL Classification: D44

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1 Introduction

Is the private or common valuation component more important in treasury bill auctions? Can we use data to provide an answer? These are two major questions that we attempt to address in this paper. We exploit variation in observed bids by several bidders before the deadline for bid submission to develop an econometric test for presence and importance of the common valuation component in treasury bill markets. The basic idea underlying our test is that as new information about bidding behavior of her rivals becomes available a bidder should augment her bidding strategy in a different way when her valuation is private and when the valuation has an important common component.

Most governments sell their short term debt via auctions. The economic theory does not have a definitive answer as to what the optimal selling mechanism would be, and it is perhaps not surprising that the actual auction mechanisms differ substantially across countries. In previous empirical work discussed below, researchers tried to compare discriminatory and uniform price auction formats, yet most of the structural work restricts attention to the private values paradigm. While many economists agree that for the short term debt the private valuation component is probably more important, because most investors hold these papers in their portfolios until maturity so that there is almost no resale, there is still some controversy in modelling auctions of government debt using private valuation models. In particular, for example due to different expectations of some global risk, say of interest rate fluctuations, there might still be an important common valuation component involved. It therefore remains a matter of taste as to which model to apply. So far, there is relatively thin literature on testing for common value component, moreover, it deals solely with a setting where a single unit of a good is being auctioned. The auctions of government debt clearly do not fall into this category. In particular, in these multiunit auctions bidders submit whole demand curves as their bids rather than just a simple real-valued bid signalling their willingness to pay. It turns out that using the two-dimensionality of bidders demands will help us develop our test. The proposed test is quite different from those employed previously in the literature and is less susceptible to unobserved heterogeneity across auctions. In particular, we will make use of dynamics in bidding behavior within a particular auction, where the common and private value paradigm would predict different bidding patterns.

As an example, consider a situation in which bidder i is about to submit her bid (demand) function y_i, but before submitting y_i she observes a bid actually submitted by bidder j. With private valuations bidder i obtains better information about the location and shape of residual supply she will be facing in the upcoming auction. Using this additional information, she revises her initial bid y_i and submits an aternative bid y_i'. In an auction with a common value component, on top of the additional information about the location and shape of the residual supply curve, she also obtains new important information about the common value. Therefore she submits a new bid y_i'' taking into account both of these two pieces of new information. In general, the way she will revise her bid y_i will differ under the two scenarios and this distinction motivates our test.

The question of finding a way to distinguish between the common and private valuation paradigms is not new to economics literature. The theory of equilibrium bidding in different auction environments which was spelled out in the seminal paper of Milgrom and Weber (1982) motivated empirical researchers to develop formal techniques that would help them decide which theoretical model would seem more appropriate in a given setting. In a single unit setting, in which a single object is auctioned, researchers proposed a reduced form testing approach based on examining how bids vary with the number of participants (e.g., Gilley and Karels (1981)). For second-price sealed-bid and English aucions, Paarsch (1991) and Bajari and Hortaçsu (2003) suggest testing for CV using standard regression techniques. Pinkse and Tan (2002) establish, however, that such a reduced form test cannot distinguish unambiguously a CV from PV model in first price auctions. Therefore, structural modelling seems necessary in order to achieve the goal of distinguishing CV and PV. Paarsch's (1992) seminal paper was indeed motivated by this question. His method, however, relies on parametric assumptions about the distribution of bidder's private information, and hence it is hard to disentangle the influence of the parametric assumptions on the actual outcomes of the testing procedure. Our approach, instead, will be non-parametric. Haile, Hong and Shum (2003) (henceforth HHS) is the most closely related paper. They propose a non-parametric test for common value making use of variation in the number of bidders across auctions. They use non-parametric techniques developed in empirical auctions literature (e.g., Laffont and Vuong (1995), and Guerre, Perrigne and Vuong (2002)) to estimate the distribution of valuations given the observed bids. In particular, the theory predicts a certain ordering between the distribution of bids under common valuation paradigm as the number of bidders varies, while the expected value of the object conditional on winning should not vary with the number of participants under PV. An important problem they have to deal with though is the unobserved characteristics of the auctions, which in turn could influence the number of participating bidders. Our testing approach will not suffer from this potential difficulty as it is based on dynamics of submitted bids within an auction.

As mentioned above our analysis involves a multi-unit environment. In particular, we look at auctions of divisible good, i.e., auctions of very large number of homogeneous units of a good, so that the quantity can be treated as continuous choice variable. The theory of such auctions has been laid out in Wilson (1979) and these auctions have generated a lot of interest recently, as they seem to be a fitting model for auctions of securities, electricity or emission permits. Empirical literature on divisible good auctions can be classified into two groups.

The first group of papers is interested in modelling behavior in electricity auctions (e.g., Wolak (2003, 2005), Hortaçsu and Puller (2005)). The private value framework seems like an appropriate setting for these auctions, and hence we will not be talking about these in more detail.

The second group consists of papers that aim to compare the revenue and efficiency of alternative auction mechanisms so that to provide a recommendation for the auctioneer. These papers usually use data from auctions of government treasury bills (e.g., Armantier and Sbai (2002), Fevrier, Preget and Visser (2002), Hortaçsu (2002), Kastl (2006a)). The only paper from this list that employs a common value framework is Fevrier, Preget and Visser (2002). They, however, look at the other extreme - pure common value environment, and they are able to make progress only by assuming a particular functional form for the distribution of private information because it allows for closed form solutions of equilibrium strategies. Another problem with their approach is that the implied equilibrium strategies are continuous downward sloping demand schedules, which is not what is observed in practice. Bidders are usually required to characterize their demands only by using a finite (and low) number of price-quantity pairs, which specify how much quantity they demand at a given price. Kastl (2006a) points out that ignoring this feature of bidding can have important consequences on the estimated valuations. All other papers in the list above look at a private value setting, and each provides some intuition as to why the private setting seems to be appropriate. In our view a formal test for validity of this assumption conducted in a similar environment to provide supportive evidence for private values would be quite handy. On the other hand, should this test point towards an important common valuation component, then we should pay more attention to defending the private value paradigm in any given setting.

The remainder of the paper is organized as follows. In Section 2 we lay out the model of a discriminatory auction of a perfectly divisible unit good and characterize the necessary conditions for equilibrium bidding under private values. We use these necessary conditions to conduct structural estimation of bidders' marginal valuations under the null hypothesis. We describe the actual test for common values in Section 3. To evaluate the performance of the proposed test, we conduct a Monte Carlo simulation in Section 4. In Sections 5 and 6 we describe our dataset and present the results. Finally, Section 7 concludes.

2 The Model and Test Description

The basic model underlying our analysis is based on the share auction model of Wilson (1979) with private information, in which both quantity and price are assumed to be continuous. There are N bidders, who are bidding for a share of a perfectly divisible good. Each bidder receives a private (possibly multidimensional) signal, s_i, which is the only private information about the underlying value of the auctioned goods. The joint distribution of the signals will be denoted by $F(\mathbf{s})$.

Assumption 1 *Bidder i's signal s_i is drawn from a common support $[0,1]^M$ according to an atomless marginal d.f. $F_i(s_i)$ with strictly positive density $f_i(s_i)$.*

Winning q units of the security is valued according to a marginal valuation function $v_i(q, s_i, s_{-i})$. In the special case of independent private values (IPV), the s_i's are distributed independently across bidders, and bidders' valuations do not depend on private information of other bidders, i.e., the valuation has the form $v_i(q, s_i)$. At the estimation stage we will not impose full symmetry, since we will allow for different groups, within which the signal is distributed identically across bidders. We will impose the following assumptions on the marginal valuation function $v(\cdot, \cdot, \cdot)$:

Assumption 2 *$v_i(q, s_i, s_{-i})$ is measurable and bounded, strictly increasing in (each component of) s_i $\forall (q, s_{-i})$ and weakly decreasing in q $\forall (s_i, s_{-i})$.*

We will denote by $V_i(q, s_i, s_{-i})$ the gross utility: $V_i(q, s_i, s_{-i}) = \int_0^q v_i(u, s_i, s_{-i})\, du$. Throughout the paper we will distinguish between private values and other valuation structures, where bidders' valuations could be interdependent (for example could have a common value component). The following defition states what we understand under these terms using our notation.

Definition 1 *(i) Bidders have private values when $\forall i : v_i(q, s_i, s_{-i}) = v_i(q, s_i,)$.*
(ii) Bidders have interdependent values if $\forall i, j$ and a.e.s_i $\exists S'_j, S''_j : S'_j \cap S''_j = \emptyset$ such that $\Pr\left(S'_j\right) > 0, \Pr\left(S''_j\right) > 0$ and $\mathbb{E}_{s_{-i}}\left(v_i(q, s_i, s_{-i})\,|s_j \in S'_j, s_i\right) \neq \mathbb{E}_{s_{-i}}\left(v_i(q, s_i, s_{-i})\,|s_j \in S''_j, s_i\right)$.

Our definition of interdependent values simply states that each bidder posseses with positive probability some private information that is relevant for valuation of each of his rivals. In particular, in the context of our empirical application it implies that at least some customer information is valuable to the dealers.

Bidders' pure strategies are mappings from private signals to bid functions: $\sigma_i : S_i \to \mathcal{Y}$, where the set $\mathcal{Y}$ includes all possible functions $y : \mathbb{R}^+ \to [0, 1]$. A bid function for type s_i can thus be summarized by a function, $y_i(\cdot|s_i)$, which specifies for each price p, how big a share $y_i(p|s_i)$ of the securities offered in the auction (type s_i of) bidder i demands. Q will denote the amount of T-bills for sale, i.e., the good to be divided between the bidders. Q might itself be a random variable if it is not announced by the auctioneer ex ante, or if the auctioneer has the right to augment or restrict the supply after he collects the bids. We assume that the distribution of Q is common knowledge among the bidders. Furthermore, the number of bidders participating in an auction, denoted by N, is also commonly known. This assumption is reasonable in the context of our empirical application as all participants have to register with the auctioneer before the auction and the list of registered participants is publicly available. The natural solution concept to apply in this setting is Bayesian Nash Equilibrium. The expected utility of type s_i of bidder i who employs a strategy $y_i(\cdot|s_i)$ in a discriminatory auction given that other bidders are using $\{y_j(\cdot|\cdot)\}_{j\neq i}$ can be written as:

$$EU_i(s_i) = E_{Q,s_{-i}|s_i}\left[\begin{array}{c} \int_0^{q_i^c(Q,\mathbf{s},\mathbf{y}(\cdot|s))} v_i(u,s_i)\,du \\ -\sum_{k=1}^{K} \mathbf{1}\left(q_i^c(Q,\mathbf{s},\mathbf{y}(\cdot|s)) > q_k\right)(q_k - q_{k-1})\,b_k \\ -\sum_{k=1}^{K} \mathbf{1}\left(q_k \geq q_i^c(Q,\mathbf{s},\mathbf{y}(\cdot|s)) > q_{k-1}\right)\left(q_i^c(Q,\mathbf{s},\mathbf{y}(\cdot|s)) - q_{k-1}\right)b_k \end{array}\right]$$

where $q_i^c(Q,\mathbf{s},\mathbf{y}(\cdot|s))$ is the (market clearing) quantity bidder i obtains if the state (bidders' private information and the supply quantity) is $(\mathbf{s},Q)$ and bidders bid according to strategies specified in the vector $\mathbf{y}(\cdot|s) = [y_1(\cdot|s_1),...,y_N(\cdot|s_N)]$, and similarly $p^c(Q,\mathbf{s},\mathbf{y}(\cdot|s))$ is the market clearing price associated with state $(\mathbf{s},Q)$. A Bayesian Nash Equilibrium in this setting is thus a collection of functions such that almost every type s_i of bidder i is choosing his bid function so as to maximize his expected utility: $y_i(\cdot|s_i) \in \arg\max EU_i(s_i)$ for a.e. s_i and all bidders i.

2.1 Equilibrium strategy of a bidder in a private value auction

In this subsection we describe equilibrium behavior of a bidder in a private value setting. The discriminatory auction version of Wilson's model with private values has been previously studied in Hortaçsu (2001). Kastl (2006b) extends this model to empirically relevant setting, in which bidders are restricted to use step functions with limited number of steps as their bidding strategies. He proves the following result summarizing necessary conditions for an equilibrium:

Proposition 1 *Suppose values are private, rationing is pro-rata on-the-margin, and bidders can use at most K steps. Then in any Bayesian Nash Equilibrium of a Discriminatory Auction, for almost all s_i, with a bidder of type s_i submitting $\hat{K}(s_i) \leq K$ steps, every step k in the equilibrium bid function $y_i(\cdot|s_i)$ has to satisfy:*
(i) $\forall k < \hat{K}(s_i)$ such that $v(q,s_i)$ is continuous in a neighborhood of q_k for a.e. s_i:

$$v(q_k,s_i) = b_k + \frac{\Pr(b_{k+1} \geq p^c)}{\Pr(b_k > p^c > b_{k+1})}(b_k - b_{k+1}) \tag{1}$$

and if $v(q, s_i)$ *is continuous in a neighborhood of* $q_{\hat{K}}$ *for a.e.* s_i*, the demand at the last step* $\hat{K}(s_i)$ *has to satisfy:*

$$v\left(q_{\hat{K}}, s_i\right) = b_{\hat{K}} \tag{2}$$

(ii) if $v(q, s_i)$ *is a step function in* q *such that* $v(q, s_i) = v_k \ \forall q \in (q_{k-1}, q_k]$ *for a.e.* s_i*, then*

$$v_k = b_k + \frac{\Pr\left(b_k > p^c\right)}{\frac{\partial \Pr\left(b_k > p^c\right)}{\partial b_k}} \tag{3}$$

Using these necessary conditions and assuming either continuity of marginal valuation function in q or assuming $v(\cdot, s_i)$ is a step function, we can obtain point estimates of marginal valuations at submitted quantity-steps nonparametrically using (1) and (2) or (3) as described in Hortaçsu (2002) and Kastl (2006a). The resampling method that we employ in these papers is based on simulating different possible states of the world (realizations of the vector of private information) using the data available to the econometrician and thus obtaining an estimator of the distribution of the market clearing prices. It works as follows:

Suppose there is N_d potential dealers and N_c potential customers and both types of players are (ex ante) symmetric within their respective group. Fix a dealer's bid (or a customer). From the observed data, draw (with replacement) $N_d - 1$ actual bid functions submitted by dealers, and similarly draw N_c bid functions submitted by customers. This simulates one possible state of the world, a possible vector of private information, and thus results in one potential realization of the residual supply. Intersect this residual supply with the fixed dealer's bid to obtain the market clearing price. Repeat this procedure large number of times in order to obtain an estimate of the distribution of the market clearing price conditional on the fixed bid. Using this simulated distribution of market clearing price, we can obtain our estimates of valuation at each step submitted by the bidder whose bid we fixed using (1) and (2) or (3) depending on the assumption on the marginal valuation function we are willing to impose.

One additional caveat that we need to be careful about when dealing with bidders with informational advantage is the following. A dealer that observes demand from her customer of course is no longer symmetric to her counterpart who does not posses this information. Therefore, when resampling, we do not want to pool all dealer bids together and draw from such a pool. Since customers actually participate in every auction, to simulate the states of the world correctly, we would have to perform conditional drawing. This works as follows:

Start drawing N_c customer bids. Conditional on the bid drawn, draw a dealer's bid. If a zero customer bid is drawn, draw from the pool of dealers' bids, which have been submitted without observing any bid by the customers. If a non-zero customer bid is drawn, draw from the pool of dealers' bids, which have been submitted having observed the same customer bid. After drawing N_c customer bids, continue drawing from the pool of bids submitted by uninformed dealers until N_d dealer bids are drawn. Obtain the market clearing price and repeat.

Performing such a conditional drawing procedure does, unfortunately, greatly reduce the number of states that can be simulated. As a robustness check, we can perform an uncoditional simulation, where among the dealer bids, we first flip a coin whether this dealer has hypothetically seen a customer's bid or not, where the coin is biased such that it reflects the actual probability of a dealer observing a customer's bid. If the coin determines a bid has been seen, then we draw an "updated" bid, otherwise we draw from an original dealer bid. This is performed for each of N_d potential dealer draws, i.e., independently of the customer bids actually drawn in a given simulation round.

What would happen as additional information about a bid submitted by a rival becomes available to a bidder? In a private value setting, this bidder would simply update his belief about the distribution of the residual supply he will be facing in the auction. The following proposition states formally that using the conditional resampling procedure outlined above replicates a bidder's updating process.

Proposition 2 *Under private values the conditional resampling procedure where the known rival's bid is subtracted from the supply at each resampling draw and 1 less bids are drawn from the pool of potential bids leads to a consistent estimate of marginal valuation of a bidder with information about a rival's bid.*

Applying the conditional resampling procedure therefore results in two sets of marginal valuation estimates - before and after the information about rival's bid arrives, and our test will be based on comparing the two sets of marginal valuation estimates. One caveat involved in constructing this test is that the bids before and after the information about rival's bid arrives are not necessarily submitted for the same quantities(even though for a subset of bidders in our sample updated bids are submitted at the same quantities), and hence we will face an inference problem of how to compare the two sets of estimates. We will discuss these issues and the solutions in Section 3 of the paper which deals with the test specification.

2.2 Asymptotic Distribution of the Estimates

As suggested above our test will be based on comparing two sets of estimates. Therefore we have to be able to account for the sampling error when constructing our test statistic and deriving its asymptotic distribution. Let us first look at the asymptotic behavior of the estimates of marginal valuation. It is easy to see from equation (1) that these estimates are a non-linear function of the distribution of the market clearing price, which is estimated by the resampling method described above. Let us rewrite (1) as

$$v(q_k, s_i) = b_k + \frac{H(b_{k+1})}{G(b_{k+1}) - H(b_k)}(b_k - b_{k+1})$$

where $H(X)$ (resp. $G(X)$) is the probability that market clearing price is weakly (resp. strictly) lower than X. The following proposition establishes the asymptotic distribution of the resampling estimator $\hat{H}^R$.

Proposition 3 *Let $\hat{H}^R(X)$ denote the resampling estimator, N number of bidder in an auction, T number of auctions and let $\Phi(y_1, ..., y_{N-1}; X) = I\left(Q - \sum_{j=1}^{N-1} y_j(X|s_j) \geq y_i(X|s_i)\right)$, then*

$$\sqrt{T}\left(\hat{H}^R(X) - H(X)\right) \rightarrow N\left(0, \frac{(N-1)^2}{N}\zeta\right) \tag{4}$$

where $\zeta = E_{s_{-i}}\left[(\Phi(y_1, ..., y_{N-1}; X))^2\right] - \left(\binom{NT}{N-1}^{-1} \sum_{(1,1)\leq\alpha_1<\alpha_2<...<\alpha_{N-1}\leq(T,N-1)} \Phi\left(y_{\alpha_1}, ..., y_{\alpha_{N-1}}, X\right)\right)^2$

and where that last summation is taken over all combinations of $N-1$ *indices* $\alpha_i \in \{(1,1),(1,2),...,(1,N-1),...,(T,N-1)\}$ *such that* $\alpha_1 < \alpha_2 < ... < \alpha_{N-1}$

Proof. Consider the following statistic based on all subsamples of size $(N-1)$ from the full sample of NT datapoints:

$$\theta\left(\hat{F};c\right) = \binom{NT}{N-1}^{-1} \sum_{1\leq\alpha_1<\alpha_2<...<\alpha_{N-1}\leq NT} \Phi\left(y_{\alpha_1},...,y_{\alpha_{N-1}},c\right)$$

where $\hat{F}$ is the empirical distribution of bid functions. θ is a U-statistic and the result thus follows from applying Theorem 7.1 of Hoeffding (1948) which provides a useful version of a central limit theorem for this class. A sufficient condition for asymptotic normality is the existence of the second moment of the kernel of the functional θ: in our case $E\left[\Phi\left(\cdot\right)^2\right]$, which clearly holds. ■

Using the asymptotic variance of the distribution of the market clearing prices, $H(X)$ we can use the delta-method to derive the asymptotic variance of the estimates of the marginal valuations, i.e., $Var\left(v\right) = J_v^{'}\Sigma J_v$, where J_v is the matrix of partial derivatives with respect to $H\left(b_{k+1}\right), H\left(b_k\right)$ and $G\left(b_{k+1}\right)$ and Σ is the asymptotic variance/covariance matrix for those estimates. How do we obtain the asymptotic covariance matrix of $\{H\left(c_1\right), H\left(c_2\right), G\left(c_3\right)\}$ at particular three values of c? An advantage of Hoeffding's theorem is that it applies also to vector-valued random variables and the off-diagonal elements of the asymptotic variance/covariance matrix are the asymptotic covariances between two corresponding U-statistics.

In order for basing the hypothesis tests on confidence invervals using the asymptotic normal approximation, we may need to use $T > N$ auctions. Using this route we might, however, run into the problem of unobserved heterogeneity across auctions as if some unobserved characteristics of these auctions differ, the observed bids would be no longer be generated from the same distribution. Instead, we will use bootstrap confidence intervals to reduce the need to use many auctions for estimation. The following proposition establishes the validity of bootstrap in our setting.

Proposition 4 *Let* $\hat{F}$ *denote the empirical distribution of the bid functions and let* F^b *denote its* *bootstrap approximation. Then*

$$T^{\frac{1}{2}}\left(\theta\left(F^{b};c\right)-\theta\left(\hat{F};c\right)\right)\rightarrow N\left(0,\frac{(N-1)^{2}}{N}\zeta\right)$$

where ζ is as defined in Proposition 3.

Proof. The result follows from Theorem 3.1 of Bickel and Freedman (1981) using the fact that the variance and any covariances of our kernel $\Phi\left(y_1,...,y_{N-1}\right)$ in the U-statistic $\theta\left(F;c\right)$ are bounded. ■

2.3 Equilibrium strategy of a bidder in an auction with affiliated values

If the valuation of a bidder has a common value component, then we will not be able to replicate the updating process of this bidder as new information becomes available to him. While the updating part due to better information about the location and shape of the residual demand is still the same as in the private value setting, there is a second updating component due to the additional information about the signal of a rival and hence about the common value. In particular, the necessary condition for optimality at k^{th} step in an affiliated value environment is (if $v\left(q,s_i\right)$ is continuous in a neighborhood of q_k for a.e. s_i):

$$\begin{aligned}&\Pr\left(b_k>p^c>b_{k+1}\right)\left[E\left[v\left(q_k,s_i,s_{-i}\right)\mid\left\{b_k,q_k\right\}_{k=1}^{K}\right]-b_k\right]=\\&=\Pr\left(b_{k+1}\geq p^c\right)\left(b_k-b_{k+1}\right)+\frac{\partial E\left(p;b_k\geq p^c\geq b_{k+1}\right)}{\partial q_k}\int_0^{q_k}\frac{\partial E\left[v\left(u,s_i,s_{-i}\right)\mid\left\{b_k,q_k\right\}_{k=1}^{K}\right]}{\partial p}du\end{aligned}$$

In other words, we have the familiar trade-off in a discriminatory auction that occurs even with private values: marginally shading the quantity demanded at k^{th} step results on the one hand in loss of expected surplus of $E\left[v\left(q_k,s_i,s_{-i}\right)\mid\left\{b_k,q_k\right\}_{k=1}^{K}\right]-b_k$ in the states that exactly that quantity would be won in. On the other hand it results in saving of b_k-b_{k+1} whenever the market clearing price is lower than the bid at the next quantity step. But now, because of the presence of the affiliation of values, there is an additional effect: marginally shading the quantity at k^{th} step can lead to a different slope of expected market clearing price in the region where k^{th} quantity demand effects the market clearing price or allocation and thus it can effect the way inference is drawn from the market clearing price realization on the unknown valuation (through updated information about rival's signals).

Since we do not know enough about $E\left[v\left(q_k, s_i, s_{-i}\right) \mid \{b_k, q_k\}_{k=1}^{K}\right]$, we cannot identify $v\left(q_k, s_i, s_{-i}\right)$ non-parametrically without imposing more structure. Therefore the main purpose of this paper is to construct a test that would enable us to empirically test whether or not the data are consistent with private values just using our identification results from the private value setting.

3 Test Specification

The fact that the dealers (large bidders) submit bids on behalf of customers (smaller clients) and that these bids are visible to the econometrician provides a unique environment for testing for the presence of a common component in bidders' valuations which are not observed by the econometrician. In particular, dealers sometimes submit their own bids, but after fulfilling a request of one or more of their customers to submit a bid on their behalf, they decide to adjust their previously submitted bid. Since we observe the bid both before and after the additional information was made available to the dealer, we will now argue formally that we can potentially distinguish a setting in which common valuation component plays a key role from a private one. In a pure private valuation setting any such bid adjustment should be driven solely by more information about the residual supply that this bidder will be facing in the actual auction. In a setting with a common valuation component, the adjustment reflects both more information about the residual supply AND more information about the common valuation component, and hence these two adjustment results should be different. In Hortaçsu (2002) and Kastl (2006a) we proposed nonparametric methods based on simulating rivals' strategies for estimation of marginal valuations in private value divisible good auctions. We will utilize these methods to estimate the marginal valuation schedules implied by the initial bid, and by the updated bid, taking into account the new information about the residual supply. In other words, as suggested in Proposition 2 we are able to mimick the bid updating process under private values hypothesis, but we are not be able to mimick it under common values. Therefore, under the null hypothesis of private valuation setting the estimates before and after the additional information should coincide. Should we find that the two marginal valuation schedules are significantly different, we would have to reject the null and conclude that the common valuation component plays an important role in these auctions.

The test for common values in a single unit setting proposed in Haile, Hong and Shum (2003) relies crucially on the ability of the auctioneer to observe repetitions of the same experiment over time, where the number of bidders varies exogenously. The problem of some auction characteristics that are unobserved by the econometrician, but observed by (potential) bidders, would severly hamper their test. In our data, as we observe exact time of each bid submission, we can distinguish a change in bid due to more information coming from the bids by smaller bidders from a change in bid due to some new publicly available information. In the latter case, conditional on some small time window, all adjustments by large bidders should be positively correlated, whereas in the former case they should be independent. Therefore if we subject to our test only those changing bids that are not accompanied by similar changes in rival's bids, we can be more confident that no commonly observed (but unobserved by us) piece of information is biasing our test as we do not need to compare estimates (such as valuation distributions) across auctions.

One important caveat of our approach is that since the bids in multiunit auctions are two-dimensional, and since bidders usually characterize their demand functions using only few points, bids submitted before and after the additional information becomes available can be quite different. But because there is an estimation error in the estimates of marginal valuations, we can write the estimated marginal valuation function of bidder i as:

$$v_{ik} = f_k\left(q_{ik}\right) + \varepsilon_{ik} \quad \text{for} \quad k = B, W$$

where B and W stands for "before information" and "with information" respectively and ε_{ik} is the estimation error in marginal valuation estimates, i.e., $v_{ik}\left(q_{ik}, \bar{s}\right) = \hat{v}_{ik}\left(q_{ik}, \bar{s}\right) + \varepsilon_{ik}$ with $\hat{v}$ being the estimates of marginal valuation from our resampling procedure. Since our estimate of marginal value at quantity q_i is consistent, $E\left(\varepsilon|q\right) = 0$, and hence the level curve of the marginal valuation function at $\bar{s}$ $f_k\left(q_i\right)$ woud be nonparametrically identified whenever the number of observed bids at different quantities for this particular signal level would go to infinity. It is reasonable to believe that this assumption which is necessary for consistency of nonparametric regression might be violated in practice, since the quantity demands at submitted bids are not a random selection from the support of quantities. Therefore, in the subsequent section we discuss two alternative tests that can be performed. The first is based on testing for monotonicity of the estimated marginal valuation function and the second is based on comparing the two sets of estimates of marginal valuations, $k = B, W$.

3.1 Test for Equality of Nonparametric Regressions

Under the null hypothesis of private values, $f_{BI} = f_{WI}$ and hence we can simply test for equality of two nonparametric regressions. Few of such tests have been proposed in the statistics literature on treatment evaluations (e.g., Koul and Schick, 1997).

Consider the statistic

$$T = \sqrt{\frac{n_B n_W}{n_B + n_W}} \frac{1}{n_B n_W} \sum_{i=1}^{n_B} \sum_{j=1}^{n_W} \frac{1}{2} \left(\eta\left(q_{B,i}\right) + \eta\left(q_{W,j}\right)\right) \rho\left(v_{B,i} - v_{W,j}\right) w_a\left(q_{B,i} - q_{W,j}\right)$$

where a is a small positive number depending on the sample sizes. H_0 is rejected for large values of T. Koul and Schick call this test a covariate-matched test.

The test statistic considered above assumes that for any given level curve of the marginal valuation function $v(q, \bar{s})$ at an unobserved signal $\bar{s}$, the set of quantities at which the value is estimated grows asymptotically, so that $v(q, \bar{s})$ can be identified nonparametrically. The test then rejects H_0 if the two estimated regression curves are sufficiently different. As mentioned above, in practice, however, the number of steps in the observed bids is very low and there is no compelling reason to believe that it would vary much as the number of observed auctions increases (for a given unobserved signal realization $\bar{s}$). One possibility to obtain the asymptotic behavior consistent with the construction above is to assume private cost c per bidpoint as in Kastl (2006a), which is drawn independently of s. As $c \downarrow 0$, bidders would submit bids with more and more steps (a continuous function in the limit of zero cost) for any s and thus $v(q, \bar{s})$ would again be nonparametrically identified.

3.2 Non-parametric Test for Monotonicity

An alternative, and possibly more natural way to think about the asymptotics is to consider the number of steps and thus the number of quantities at which the marginal value can be estimated as fixed, and let just the number of auctions increase, which is necessary for these estimates to be consistent. Then we could test for monotonicity of the estimated marginal values at quantities submitted before and after the additional information. Order the quantities at which a bid was submitted by bidder i either before the additional information was revealed or after it was revealed in an increasing order: $q_{i1} < q_{i2} < ... < q_{iK}$ and let $\hat{v}_{i1}, ..., \hat{v}_{iK}$ denote the associated estimated marginal values. Consider the following test statistic:

$$S_i = \max_j \{\hat{v}_{ij+1} - \hat{v}_{ij}, 0\} \tag{5}$$

Clearly, when monotonicity is satisfied at all quantities, then $\hat{v}_{ij} \geq \hat{v}_{ij+1}$ and hence $S_i = 0$. On the other hand we could get violations of monotonicity due to the sampling error in a finite sample and hence $S_i > 0$ could be consistent with the null hypothesis. The major advantage of this approach is that it does not restrict the class of possible marginal valuation functions in any other way than that it be non-increasing in quantity.

Critical Values

We obtain the critical value for this test statistic using bootstrap. Using B bootstrap draws, the critical values are computed as follows:

$$\tilde{c}_{1-\alpha} = \inf \left\{ x : \frac{1}{B} \sum_{b=1}^{B} 1 \left\{ \tilde{S}_b \leq x \right\} \geq 1 - \alpha \right\} \tag{6}$$

For each bootstrap draw of the test statistic, the marginal valuation is reestimated by the resampling method described earlier, where a new sample of bid functions from which this resampling is performed is drawn. For each bootstrap sample of bid functions, we draw from the observed sample N_d dealer bids with replacement giving $\frac{1}{TN_d}$ probability to each dealer bid, and similarly we draw N_c customer bids with replacement giving $\frac{1}{TN_c}$ probability to each. In constructing these bootstrap samples we include also the 'zero' bids for those potential bidders that do not end up actually submitting a bid.

Discussion

A short discussion of the testing approach is now necessary. Since our testing for monotonicity via the test statistic S falls into the framework of partial identification, there could be situations in which the true model in fact has a common value component and thus the level curves of (expected) marginal valuation are different before and after the information is revealed, but our monotonicity test fails to reject the null hypothesis (possibly even asymptotically). In other words, the monotonicity test proposed above might not be consistent against all alternatives. However, its special case discussed below **is** consistent against all alternatives provided that there is some dependence of a bidder's payoff on rivals' information. More specifically, asymptotic consistency requires that the common valuation component depends nontrivially on the information of those bidders whose bid a dealer gets to observe.

3.2.1 Special Case

A special case of the above described monotonicity test can be performed if bids are submitted at the same quantities before and after the information becomes available. In that case, under private values the two estimates of marginal valuations should coincide asymptotically and thus could differ in a finite sample only due a sampling error. Consider the test statistic:

$$T_i(q) = \left|\hat{v}_i^{BI}(q, s_i) - \hat{v}_i^{AI}(q, s_i)\right|$$

where $\hat{v}_i^{BI}(q, s_i)$ is the estimated marginal valuation for share q before the information was revealed and similarly $\hat{v}_i^{AI}(q, s_i)$ is the estimated marginal valuation for share q after the additional information arrived. The following proposition reveals a nice feature of this test.

Proposition 5 *(Asymptotic consistency)*

Under H_1 of interdependent values, $\Pr\left(T_i(q) > T_i^B(q)\right) \to 1$ *as* $T \to \infty$ *where* T^B *is the critical value obtained by bootstrap.*

This special case of our test is thus consistent against all alternatives, since if under any alternative (affiliated values) $E_{s_{-i}}\left[\hat{v}_i^{BI}(q, s_i, s_{-i})\,|p, s_i\right] = E_{s_{-i}}\left[\hat{v}_i^{AI}(q, s_i, s_{-i})\,|p, s_i\right]$ with probability 1, then the additional knowledge of s_j would not contribute any additional information about the value at q, which is not consistent with the basic assumption of the interdependent value model that the value depends on rivals' signals.

4 Monte Carlo Study

Our ability to test the performance of the above described testing procedure in multiunit auctions is limited by the fact that in most general cases we do not have closed form solutions for equilibrium strategies, either in the private or in the affiliated values settings. We circumvent this problem by conducting two sets of Monte Carlo exercises. In the first set we look at a first price auction with independent private values, with interdependent values and pure common values. In all examples, we generate the data from an equilibrium model of bidding with 3 uninformed bidders and non-parametrically estimate the marginal values implied by the bids using Guerre, Perrigne and Vuong (2002) (henceforth GPV). Then we assume that bidder 1 observes bidder 2's bid and submits an updated bid instead. For the purposes of our Monte Carlo experiment, we focus on a single-agent problem, i.e., we keep strategies of bidder 2 and 3 fixed, and we use only the data on bids by bidder 1 in estimation. Finally, we again estimate the implied values of (informed) bidder 1 using GPV which assumes private values. We construct our test statistic and bootstrap the critical values.

In the second set of Monte Carlo exercises we turn to a special case of a discriminatory auction with 2 bidders and private values. Hortaçsu (2002a) constructs an example of a discriminatory auction with private values, two bidders and exponential distribution of signals that has a closed form solution. We will use an extension of this example which involves also supply uncertainty to conduct a Monte Carlo experiment for our test.

4.1 First Price Auction with Informed Bidders

4.1.1 Independent private values (IPV)

The first exercise we consider is a first price auction with 3 bidders, valuations $v(s_i) = s_i$ and signals distributed uniformly on [0,1]. The unique equilibrium in strictly increasing differentiable strategies when all bidders are uninformed is $b^U(s_i) = \frac{2}{3}s_i$. Now consider the case that bidder 1 would be able to observe bidder 2's bid and for the purposes of our exercise suppose that bidder's 2 and 3 continue to bid as if bidder 1 was uninformed. In that case the optimal bid by the informed bidder would be:

$$b^I(s_1, s_2) = \begin{cases} \frac{s_1}{2} \; if \; \frac{s_1}{2} > \frac{2s_2}{3} \\ \frac{2s_2}{3} \; if \; s_1 > \frac{2s_2}{3} > \frac{s_1}{2} \end{cases}$$

[UPDATED] Figure 1 compares the estimated valuations of a bidder before and after she is informed. The figure suggests that except at the boundaries of the support of the bid/valuation distribution, the valuation estimated with and without conditioning on observed information is likely to be very close.

[NEW PARA] Of course, this figure depicts what happens in one randomly chosen data set on bids. We then implement a test of the equality of the estimated valuations before and after information is received using 200 randomly drawn data sets. In particular, we calculate the mean and median differences between $\hat{v}_{informed}$ and $\hat{v}_{uninformed}$ in each Monte Carlo sample, and construct the 5th and 95th bootstrap percentiles (using 200 resamples of the Monte Carlo sample) of these differences. Equality is rejected if the null hypothesis of zero is not within this confidence interval.

The null rejection frequencies of this testing procedure across 200 Monte Carlo is displayed in Table 1. In order to understand how sampling error affects the rejection performance, we replicated this exercise for data sets of size 50, 100, 200 – which are data sets of similar size to the empirical exercise. For data sets of size 100 and 200, the test statistic we constructed tends to reject the null hypothesis slightly more frequently than we would like (8.5% as opposed to 5%), though for data sets of size 50, the rejection probability appears right no target.

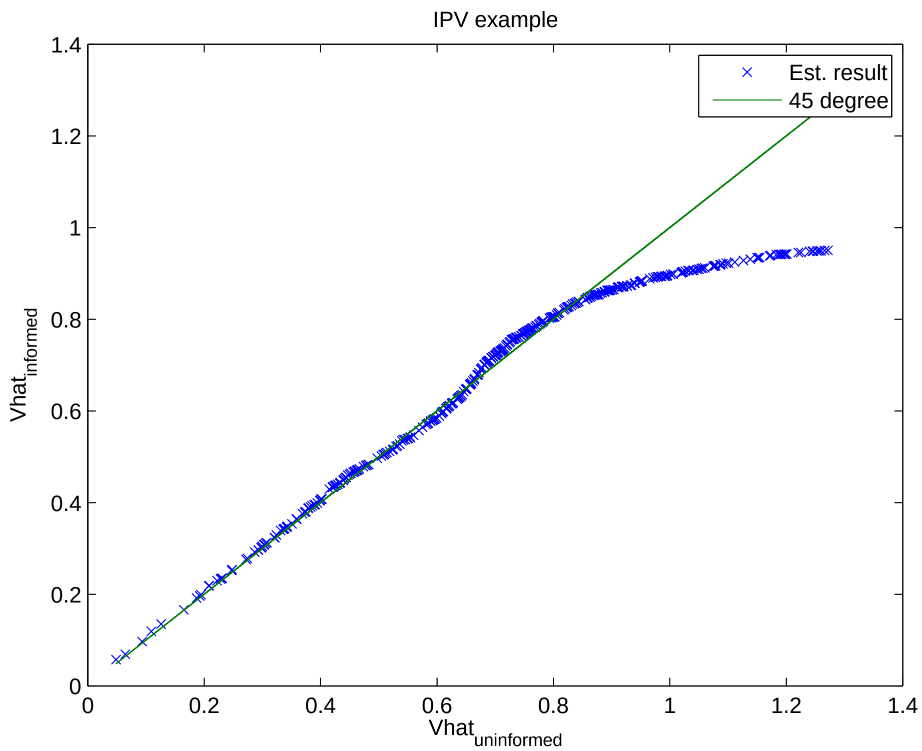

Figure 1: Estimated values for informed and uninformed bidders in a FPA with private values

4.1.2 First price auction with interdependent values and independent signals (IIV)

In the second exercise we look at a first price auction with interdependent values and independent signals (IIV). The valuation function is $v(s_i, s_{-i}) = \frac{s_i}{2} + \frac{\sum_{j \neq i} s_j}{2(n-1)}$ where $s_i \sim U[0,1]$. The unique symmetric equilibrium in strictly increasing differentiable strategies involved biddgin according to $b^U(s_i) = \frac{7}{12}s_i$. In the appendix we show that the equilibrium stragety of an informed bidder who observes a bid of his rival (and thus for practical purposes another signal S_2) is bidding according to:

$$b_1^I(s_1, s_2) = \begin{cases} \left(\frac{5}{16}s_1 + \frac{s_2}{8}\right) \, if \; s_1 > \frac{22}{15}s_2 \\ \frac{7s_2}{12} \, if \; \frac{22}{15}s_2 \geq s_1 \geq \frac{5}{12}s_2 \end{cases}$$

[NEW PARA] Figure 2 depicts the results of estimating the implied values using GPV for a randomly selected data set. The null rejection frequencies of the testing procedure utilized in the IPV example is displayed in Table 1. Observe that for data sets of size 100, the test appears to be right on target: we reject the null hypothesis 94% of the time. For data sets of size 50, we reject slightly less frequently at 87%. However, for data sets of size 200, we rejected the null hypothesis in every Monte Carlo sample.

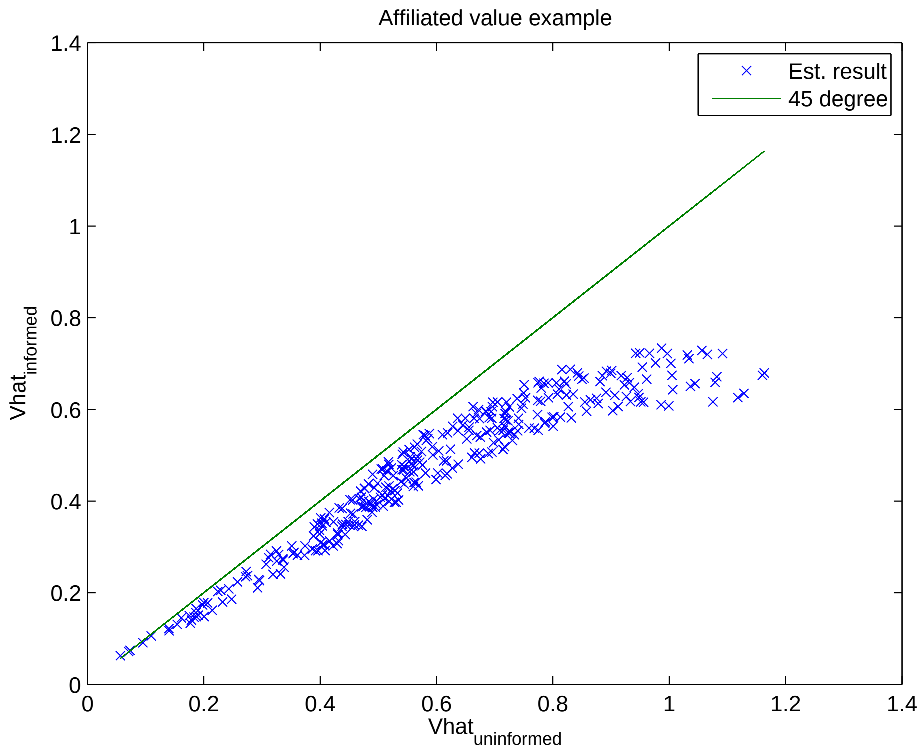

Figure 2: Estimated values for informed and uninformed bidders in a FPA with interdependent values

4.1.3 First price auction with pure common values

In our third exercise we examine at a first price auction with pure common values described in Matthews (1984). Let the utility be $u_i(s_i) = v$ where $v \sim \text{Pareto}(\alpha) : g(v) = \alpha v^{-(\alpha+1)}$ for $1 \leq v \leq \infty$ and $F(s|v) = \frac{s}{v}$.

In this case Matthews shows that there is a unique equilibrium in differentiable strictly increasing strategies of the form:

$$b(s) = \left(\frac{(N-1) + [\max\{1,s\}]^{-(N-1)-1}}{(N-1)+1} \right) \hat{v}(s,N)$$

where

$$\hat{v}(s,N) = \frac{N+\alpha}{N+\alpha-1} \max\{1,s\}$$

is the expected valuation conditional on winning. Notice that for $s \geq 1$ we have

$$b(s) = \frac{(N+\alpha)\, s \left[(N-1) + s^{-N}\right]}{N(N+\alpha-1)}$$

Now if bidder 1 were again to observe bidder 2's bid, two cases can occur: either he can infer s_2 or that $s_2 < 1$.

Suppose that $s_2 \leq s_1$. Then the optimal bid is as before since no such signal is informative about realized v conditional on winning ($s_{\max}$ is a sufficient statistic of the sample $(s_1, ..., s_N)$ for v). On the other hand, if $s_2 > s_1$, then the optimal bid becomes:

$$b(s_1, s_2) = \frac{(N+\alpha)\, s_2 \left[(N-1) + s_2^{-N}\right]}{N(N+\alpha-1)}$$

In other words, bidder uses just the highest signal he observes to base his bid upon and updates the prior on the distribution of v using the winning event.

[UPDATED] We once again generated data for an informed and an uninformed bidder using the above described bidding strategies and used GPV to estimate the implied valuations under the null hypothesis of private values. The results, for a randomly chosen data set, are displayed in Figure 3. Note that there is a large divergence between the (incorrect) null hypothesis of the equality between estimated valuations before and after conditioning for information. Not surprisingly, in Table 1 we report that we reject this null hypothesis in every Monte Carlo experiment.

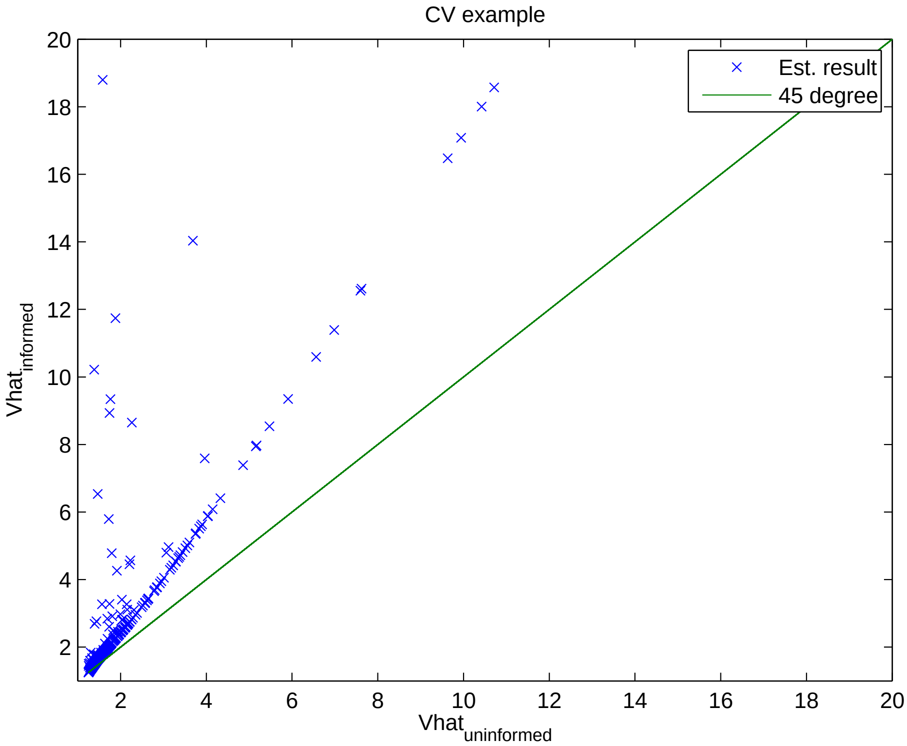

Figure 3: Estimated values for informed and uninformed bidders in a FPA with pure common values

4.2 Discriminatory auction with private values

Consider 2 bidders with true demands $D(p, s_i) = \frac{1}{\beta}[\alpha - p + \gamma s_i]$ and the corresponding valuations $v(q, s_i) = \alpha + \gamma s_i - \beta q$ where $\alpha, \beta, \gamma > 0$. The signals are independently and exponentially distributed: $F(s_i) = \exp[\theta s_i]$ with $\theta > 0$ and $s_i < 0$. In this setting Hortacsu (2002) shows that there exists an equilibrium in linear strategies of the form: $y(p, s_i) = \frac{1}{\beta}\left(\alpha + \gamma s_i - p - \frac{\gamma}{\theta}\right)$. We will also assume that the auctioneer does not commit to a supply $Q = 1$ before the auction, but the supply is rather a random variable from perspective of the bidders which is distributed normally with mean 1 and variance σ^2.

In the appendix we show that the equilibrium bidding strategy with a normal supply uncertainty remains unchanged and still takes the form $b(q, s_1) = \alpha + \gamma s_1 - \beta q - \frac{\gamma}{\theta}$.

On the other hand the equilibrium bidding strategy of a bidder who observes his rival's signal

Table 1: Monte Carlo Exercises

Dataset size	Rejection prob. IPV Mean Δ	IPV Median Δ	IIV Mean Δ	IIV Median Δ	CV Mean Δ	CV Median Δ
50	0.055	0.055	0.87	0.87	1	1
100	0.085	0.085	0.94	0.94	1	1
200	0.085	0.085	1	1	1	1

(fixing rival's linear strategy) becomes:

$$b\left(q,s_1,s_2\right)=\alpha+\gamma s_1-\beta q-\beta\frac{1-\Phi\left(\frac{q+\frac{1}{\beta}\left(\alpha+\gamma s_2-p-\frac{\gamma}{\theta}\right)-1}{\sigma}\right)}{\phi\left(\frac{q+\frac{1}{\beta}\left(\alpha+\gamma s_2-p-\frac{\gamma}{\theta}\right)-1}{\sigma}\right)}$$

Figure 4 depicts the results for a particular bidder with signal draw $s_i=-1.6$ and for parameter values $a=10$, $b=2$, $c=2$, and 20 signal draws for each bidder from exponential distribution with $\theta=1$, random supply is distributed as $N\left(1,0.04\right)$, each bid is discretized to 100 steps and there is 5000 resampling draws for the estimation.

With finely specified bids (100 steps) the two estimates of marginal valuation curve for the given signal are very similar for all bidders that have drawn signals for which they find it worthwhile to submit a bid.

5 Data and Background

Treasury bills and other Bank of Canada securities are issued in the primary market through sealed-bid discriminatory auctions. Bids are submitted electronically and can be revised at any point before the submission deadline. There are two major groups of potential bidders: primary dealers (PDs) and customers.

The major distinction between these two groups of potential bidders is that customers cannot bid on their own account in the auction, but have to route their bids through one of the dealers. The PDs are required to identify bids on behalf of the customers in the electronic bidding system.

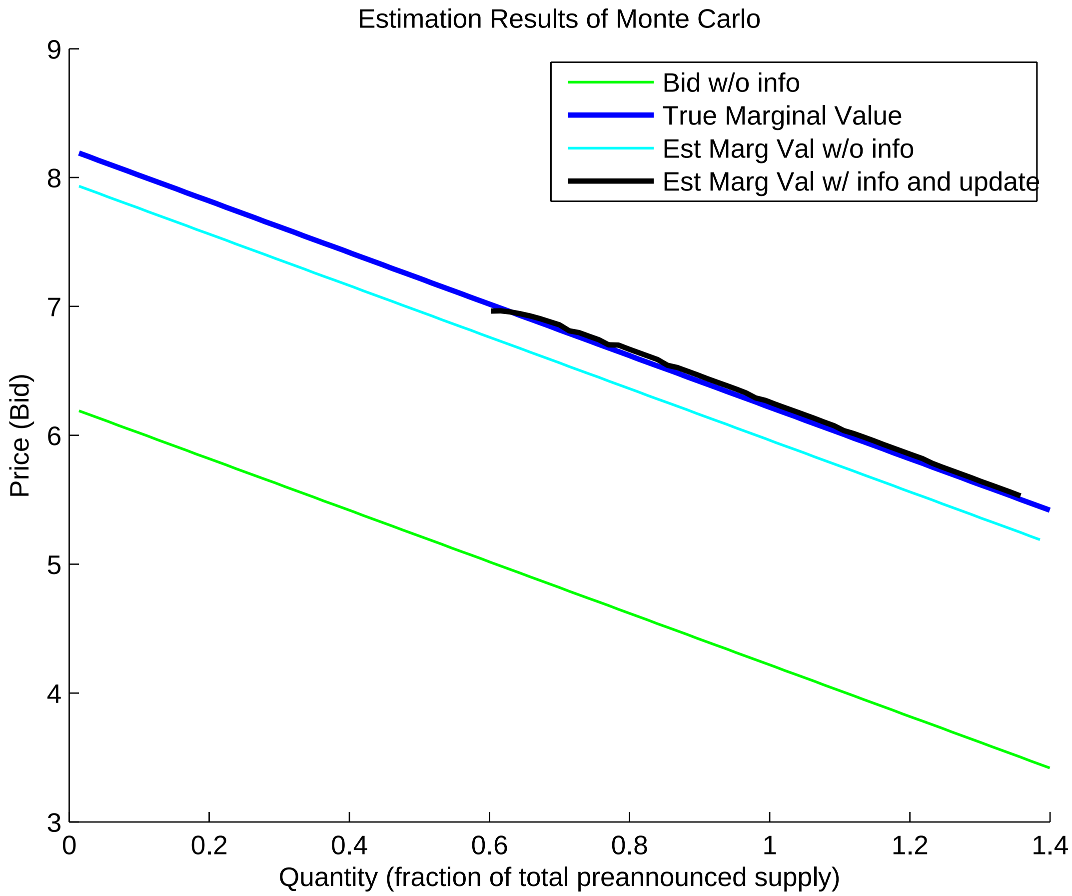

Figure 4: Estimated values in a DA with IPV

On average, there is about 2.5 primary dealers for 1 customer in an auction. In contrast, in all auctions of Bank of Canada's securities Hortaçsu and Sareen (2006) report that on average 1 dealer services 0.8 customers and that on average 8.6 customers participate. The auctions of treasury bills generate therefore less interest among the customers relative to the auctions of bonds and other securities.

In order to encourage liquidity provision and activity in the primary market, the rules of the auctions specify that a maximum amount a dealer can bid either for himself or his customers is based on his past primary market winning share and secondary market trading share, net of his current holdings of the auctioned security. However, there is also an institutionally set maximum of 25% of the issue amount for a bidder (dealer or customer individually) and 40% for a dealer (sum of all awarded bids submitted by the dealer including those on behalf of customers).

As usual in most government securities auctions, bids can be submitted both as competitive tenders and as noncompetitive tenders. Each participant is allowed to submit a single noncompetitive tender. A noncompetitive tender specifies a quantity that the bidder wishes to purchase at the price at which the auction clears. In our data, there are on average 3.6 noncompetitive tenders in an auction for on average 4.4% of the preannounced amount for sale.

Since there are no restrictions on how many times a primary dealer (or a customer) can revise her bid before the bid submission deadline, the information flow caused by customers' routing their bids through dealers causes the dealers to update their bids exactly in the spirit of the test that we propose in this paper.

[HORTACSU SAREEN] Hortaçsu and Sareen (2006) report various descriptive measures suggesting that obtaining customer information has a causal impact on dealers' bidding patterns. They find that the direction of changes in a dealer's (quantity-weighted price) bid typically follows the direction of discrepancy between the dealer's pre-customer information bid, and the customer's bid. They also report the phenomenon of "late bidding" in these auctions, where customer bids come in a very narrow window before the bid submission deadline, followed by changes in dealer bids that do not always make it in time to be considered by the Bank of Canada. Hortaçsu and Sareen report examples of such late bid changes by dealers that would have had an important impact on the dealer's profit from the auction, which again suggests that the information contained in customer bids is important for the dealer. Hortaçsu and Sareen point out that both common value and private value models are consistent with their descriptive patterns, however, and do not conduct tests to distinguish between these informational environments.

[PUBLIC INFO] An important potential caveat regarding our testing strategy is that privately observed customer bids *per se* are not the causal drivers of observed changes in dealer bids, and that customer bids are correlated with unobservable *public* information flows driving modifications to dealer bids. The presence of such unobservable public information flows would confound our testing strategy, since these information flows may affect the dealer's marginal valuation, and/or allow them to observe an extra piece of information regarding the auction environment that we are not able to account for in our marginal valuation estimation procedure.

To examine the plausibility of this confound, we examined the timing of changes in dealer bids in our data set. If information flows are publicly observed across dealers, we should observe some amount of clustering in the timing of bid modifications in our data set. We failed to find an important degree of clustering in this dimension – within any 5 minute window around a particular bid updating event, there was at most one other dealer changing his/her bid (and such a dealer was only found in 40 instances out of the total 213 updated bids in our sample). This suggests that it is unlikely that customer bids were driven by or accompanied with important public information releases that are unobservable to us. As a complement to this finding, Hortaçsu and Sareen (2006) report that unobservable public information releases by official sources are highly unlikely, as Bank of Canada and Treasury pay careful attention to avoid public disclosures during the bidding period.

Our sample consists of all submitted bids in 116 auctions of 3-months treasury bills of the Canadian government issued between 10/29/1998 and 3/27/2003. The following tables offer some summary statistics.

Table 2: Data Summary

Summary Statistics				
Auctions	116			
	Mean	St.Dev.	Min	Max
Dealers	12.34	1.64	9	15
Customers	4.66	2.3	0	12
Participants	17	2.83	11	23
Submitted steps	2.88	1.69	1	7
Price bid	989353	3244	984515	994969
Quantity bid	0.092	0.07	0.00023	0.2512
Issued Amount (billions C$)	3.881	0.552	2.8	5

6 Results

We observe 213 bids that have been updated after a customer bid arrived. Figure 5 depicts updating of a bid by one dealer. After observing a relatively low bid by one customer, the dealer submits a

Table 3: Summary of Noncompetitive Bids

Auctions with NC bid	116			
	Mean	St.Dev.	Min	Max
Number of NC bids	3.6	1.1	2	7
NC bid	0.044	0.08	0.00003	0.27

new bid which is below his original bid.

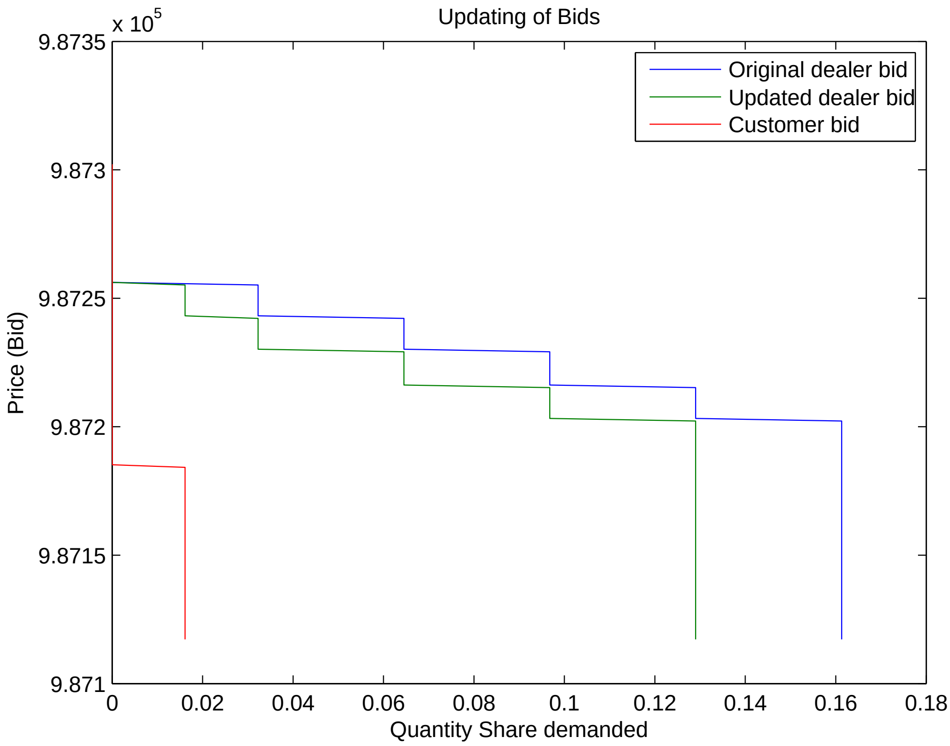

Figure 5: Updating of a dealer's bid

Before updating, these 213 bids consist of 792 bidsteps (price-quantity pairs) and after updating they consist of 848 bidsteps. We use these bids to conduct our tests. We construct a bootstrap sample of 200 replications of the test statistics (using always 5000 resampling draws for estimating each bidder's marginal valuation) for each of these bidders as defined by (5) and construct the corresponding critical values given by (6). Figures 6 and 7 depict the estimation results for two bidders. In figure 6 we depict a bidder for whom the two estimates of marginal valuations are statistically indinstinguishable, while in figure 7 we depict a bidder for whom the equality of estimated marginal valuations when information is taken into account can be rejected at two of his steps.

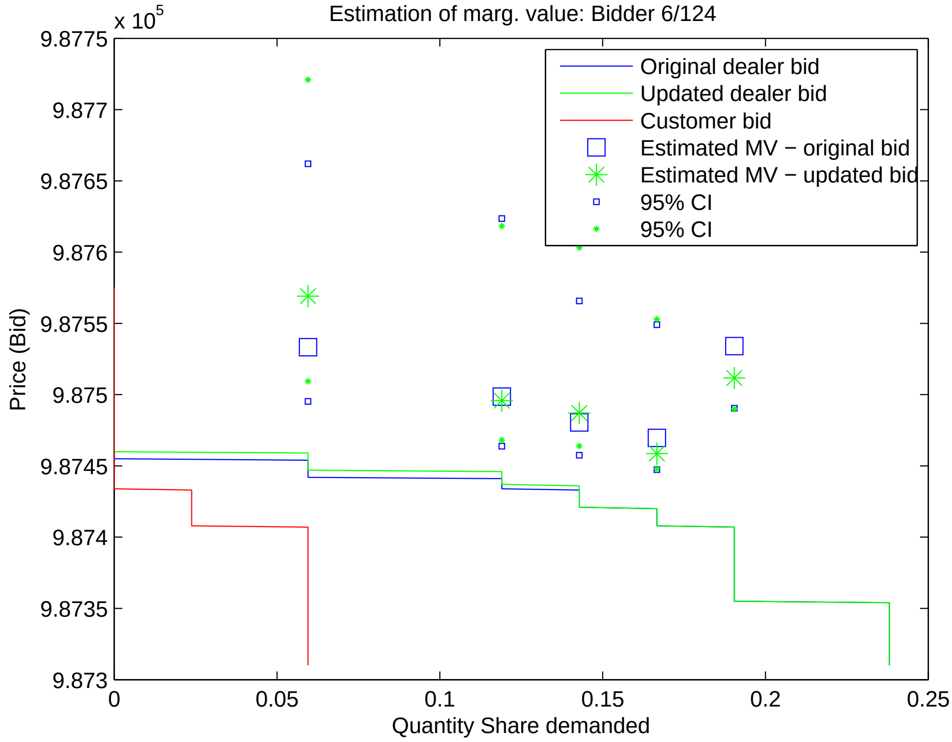

Figure 6: Estimated valuations - dealer 6

Overall bidders and steps, the median critical value for $\alpha = 0.05$ is 1525 and the mean critical value is infinite[1] which suggests that there are a few bidders (in fact there are 2 such bidders) whose marginal valuation cannot be estimated with satisfactory precision (recall that the value of the test statistic is the maximal violation of monotonicity between two adjacent steps of the estimated marginal valuation function for a given bidder). Evaluating the test statistics on the actual sample of estimated marginal valuations at updated bids results in a distribution with mean 1635 and median 82.5. On a bidder-by-bidder basis, the sample test statistic is lower than the

[1]This problem arises for a few bidders because the denominator on the RHS of (1) is on a few occasions virtually zero.

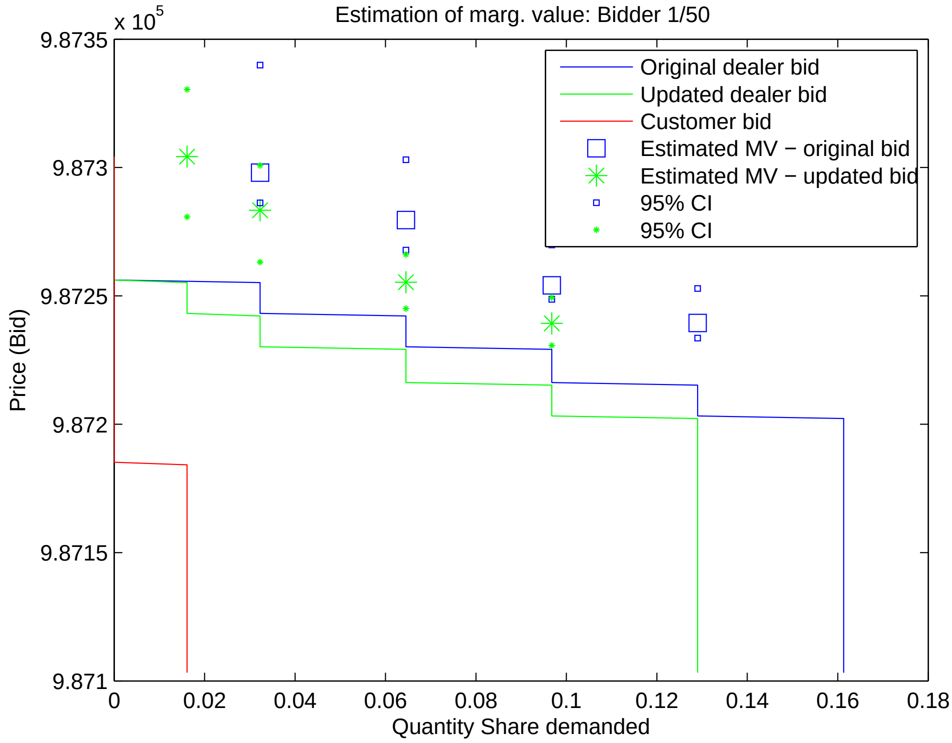

Figure 7: Estimated valuations - dealer 1

critical value (for $\alpha = 0.05$) for 208 out of the 213 bidders[2], which suggests that we cannot reject the null hypothesis of private values.

Out of those 213 bids that have been updated, we observe 575 bidsteps for which the quantity demanded at one of the bidsteps of the updated bid is the same as that of the bidsteps of the bid before updating (such as for bids displayed in figures 6 and 7) and therefore the estimated marginal valuation before and after incorporating the additional information should coincide under private values. To provide better illustration in terms of magnitudes, the mean difference in the submitted bids which differ before and after the updating is 44 (median is 16) and the standard deviation of the difference in bids is 89. The median critical value for difference in the estimated marginal valuations is 405 while the median of the corresponding statistic evaluated on the sample is 38. The hypothesis of equal marginal valuation is rejected for 9 bidsteps when the critical values are

[2]For $\alpha = 0.1$ the test statistic exceeds the critical value for 12 bidders.

constructed for $\alpha = 0.05$ and for 28 bidsteps when the critical values are constructed for $\alpha = 0.1$. In either case, the number of rejections amounts to less than 5% of observed bidsteps with the same quantity before and after the information was revealed. While we believe that using this last test provides evidence that the test we constructed points towards private values, we are currently working on more Monte Carlo experiments to verify the power of this test.

6.1 Value of Information

Given that our test failed to reject private values, in what follows we will use our estimates of marginal valuations generated by assuming the private values paradigm to estimate the value of information. In particular, we try to answer the question what is the effect of the additional information on a dealer's interim (expected) and ex post utility. Let $U_d^{EP}(I)$ denote the ex post utility of a dealer d, where $I = 1$ if additional information is incorporated, i.e., when the updated bid is used to compute the utility.

One measure of the value of information in terms of this notation would be:

$$VI^{EP} = U_d^{EP}(1) - U_d^{EP}(0)$$

An alternative measure of the value of information is in terms of a difference in interim payoffs:

$$VI^{I} = EU_d(1) - EU_d(0)$$

where we average over the distribution of the market clearing price which differs when $I = 1$ and $I = 0$.

Using our estimates we find that that the point estimate of VI^{EP} is on average about $1.65 per T-bill for sale, or about 15-20% of the payoff of the dealers. [IN PROGRESS]

7 Conclusion

In this paper we proposed a novel non-parametric test for common values. The test can be applied universally in both single-unit first-price auctions and multiunit auctions. On the other hand a necessary condition for the test to be applicable is the ability of the researcher to distinguish between more and less informed bidders, who are ex ante symmetric. The test is based on comparing two estimated distribution of valuations, which should conincide under the null hypothesis of private values. Our Monte Carlo experiments suggest that the test performs well. We also apply our test to data from Canadian treasury bill auctions and we cannot reject the null hypothesis of private values. Since we compare two estimates of valuations within an auction, our test is less susceptible to unobserved heterogeneity of individual auctions than the recent alternative test for common values proposed in Haile, Hong and Shum (2003).

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A Appendix

Here we present the derivation of the closed form solution for bidding used to generate data in our Mont Carlo studies with 3 bidders.

A.1 First price auction with independent private values

Let the utility function be:

$$u_i = x_i$$

In this case bidder 1 maximizes $\Pr(b_1 > max\{b_2, b_3\})(x_1 - b_1)$ which implies that the symmetric equilibrium bidding function is:

$$b(x) = \frac{2}{3}x$$

If he observed 2's bid, he would bid in 2 cases (assuming any tie is broken in 1's favor and bidders 2 and 3 continue using the strategies given above) using the rule:

$$b(x_1, x_2) = \begin{cases} \frac{x_1}{2}\, if\, \frac{x_1}{2} > \frac{2x_2}{3} \\ \frac{2x_2}{3}\, if\, x_1 > \frac{2x_2}{3} > \frac{x_1}{2} \end{cases}$$

where the second case occurs whenever bid of bidder 1 using the rule for the first case would be lower than 2's bid, but bidder 1 would prefer to win the object.

A.2 First price auction with interdependent values

Let the utility function be:

$$u_i = \frac{x_i}{2} + \frac{\sum_{j \neq i} x_j}{2(n-1)}$$

where

$$x_i \sim U[0, 1]$$

With 3 bidders there exists a unique symmetric equilibrium in differentiable strictly increasing strategies:

$$b(x) = \frac{7}{12}x$$

Now suppose that bidders 2 and 3 follow these strategies. Suppose bidder 1 can observe bidder 2's bid (we will denote its realization as B_2). It is easy to see that he can thus recover the signal x_2. In this case it can be shown that his expected value conditional on winning depends on two cases:

$$1)\, x_1 > x_2 :\; E[u|win] = \frac{5}{8}x_1 + \frac{x_2}{4}$$

$$2)\, x_2 > x_1 :\; E[u|win] = \frac{x_1}{2} + \frac{3}{8}x_2$$

Notice that obviously, if he were to bid this expected utility conditional on winning, he would win only in case 1) as in case 2) bidder 2's bid is always higher as his expected utility conditional on winning is higher.

Ad Case 1) Bidder 1 maximizes:

$$Pr\left(b_1 > \max\left\{b_2, b_3\right\} | b_2 = B_2\right)\left(\frac{5}{8}x_1 + \frac{x_2}{4} - b_1\right)$$

where

$$Pr\left(b_1 > \max\left\{b_2, b_3\right\} | b_2 = B_2\right) = b_1$$

(if $b_1 \geq B_2$) and 0 otherwise and hence the FOC give:

$$\left(\frac{5}{8}x_1 + \frac{x_2}{4}\right) = 2b_1$$

or

$$b_1 = \left(\frac{5}{16}x_1 + \frac{x_2}{8}\right)$$

hence he would win with such a bid in Case 1 only if it exceeds 2's bid $B_2 = \frac{7}{12}x_2$ and therefore

$$\left(\frac{5}{16}x_1 + \frac{x_2}{8}\right) > \frac{7}{12}x_2$$

$$\frac{15}{48}x_1 + \frac{6}{48}x_2 > \frac{28}{48}x_2$$
$$x_1 > \frac{22}{15}x_2$$

Otherwise, he would lose to $2's$ bid and hence as long as 1's expected value is higher than 2's bid, bidder 1 would prefer to raise his bid to 2's level and win. So now suppose that $x_1 < \frac{22}{15}x_2$ then

$$Pr\left(b_1 > \max\left\{b_2, b_3\right\} | b_2 = B_2\right) = B_2$$

and thus we get a corner solution: $b_1 = B_2$ if $B_2 \leq E[u_1 | win, x_1 \geq x_2] = \frac{5}{8}x_1 + \frac{x_2}{4}$ but since whenever $x_1 > x_2$ we have

$$\frac{5}{8}x_1 + \frac{x_2}{4} > \frac{7}{12}x_2 = B_2$$

bidder 1 would always prefer to bid B_2 whenever

$$x_2 \leq x_1 \leq \frac{22}{15}x_2$$

Ad Case 2: $x_1 < x_2$. In this case bidder 1 would like to beat bidder 2 if

$$E[u|win] = \frac{x_1}{2} + \frac{3}{8}x_2 > B_2 = \frac{7}{12}x_2$$

i.e., when $x_1 \geq \frac{5}{12}x_2$. Clearly, we are again in the corner as the bid $b_1^*(x_1)$ derived above would be below B_2.

Hence summarizing both cases:

$$b_1^*(x_1, x_2) = \begin{cases} \left(\frac{5}{16}x_1 + \frac{x_2}{8}\right) \; if \; x_1 > \frac{22}{15}x_2 \\ \frac{7x_2}{12} \; if \; \frac{22}{15}x_2 \geq x_1 \geq \frac{5}{12}x_2 \end{cases}$$

A.3 Discriminatory Auction with PV and informed bidders

Guess symmetric strategies $y\left(p, s_j\right) = a + bp + cs_j$

$$\begin{aligned} H\left(p, y\right) &= \Pr\left(p^c < p|y\right) = \Pr\left(Q > y + a + bp + cs_j\right) \\ &= \Pr\left(s_j < \frac{Q - y - a - bp}{c}\right) \\ &= \exp\left[\theta \frac{Q - y - a - bp}{c}\right] \end{aligned}$$

Hence $\frac{H}{H_p} = -\frac{1}{\theta}\frac{c}{b}$. Using the optimality equation: $v\left(y\left(p, s_i\right), s_i\right) = p + \frac{H}{H_p}$, a linear guess for the strategy $y\left(p, s_i\right)$ and equating coefficients we obtain: $y\left(p, s_i\right) = \frac{1}{\beta}\left(\alpha + \gamma s_i - p - \frac{\gamma}{\theta}\right)$. Notice that this equilibrium exhibits constant shading of $\frac{\gamma}{\theta}$ for every unit.

Now if the auctioneer does not commit to a supply $Q = 1$ before the auction, but the supply is rather a random variable from perspective of the bidders which is distributed normally with mean

1 and variance σ^2.

We will again guess the linear strategies $y\left(p, s_i\right) = a + bp + cs_i$, the distribution of the market clearing price becomes:

$$
\begin{aligned}
H\left(p, y\right) &= \Pr\left(p^c < p|y\right) = \Pr\left(Q > y + a + bp + cs_j\right) \\
&= \Pr\left(-\frac{Q}{c} + s_j < \frac{-y - a - bp}{c}\right) \\
&= \Pr\left(u + s_j < \frac{-y - a - bp}{c} + \frac{1}{c}\right)
\end{aligned}
$$

where $u = -\frac{Q}{c} + \frac{1}{c}$. The probability density of a sum of a normal random variable with $\mu = 0$ and variance φ^2 and an (negative) exponential r.v. with parameter θ is (approximately) exponential (for $x \ll -\varphi$) with a cdf:

$$
F\left(x\right) = \frac{e^{\theta x + \frac{\theta^2 \varphi^2}{4}}}{\varphi \sqrt{2\pi}}
$$

Since $Q \sim N\left(1, \sigma^2\right)$, $u \sim \left(0, \varphi\right)$, where $\varphi = \frac{\sigma^2}{c^2}$ and thus we obtain:

$$
H\left(p, y\right) = \frac{e^{\theta \frac{1 - y - a - bp}{c} + \frac{\theta^2 \varphi^2}{4}}}{\varphi \sqrt{2\pi}}
$$

Using (1), we get:

$$
v\left(y\left(p, s_i\right), s_i\right) = p - \frac{c}{\theta b}
$$

Hence equating coefficients we get exactly the same equilibrium bidding strategies as with no supply uncertainty. So the equilibrium demand function becomes: $y\left(p, s_i\right) = \frac{1}{\beta}\left(\alpha + \gamma s_i - p - \frac{\gamma}{\theta}\right)$. Since the true demand is: $D\left(p, s_i\right) = \frac{1}{\beta}\left(\alpha - p + \gamma s_i\right)$ bidders are shading their demand by a constant $\frac{\gamma}{\theta}$.

To incorporate the feature of updating the bids, suppose that after submitting the bid described above, bidder 1 observes the realization of bidder 2's signal s_2. In this case, the only remaining uncertainty in his bid is the supply uncertainty. Since the supply is normally distributed, his optimal bid function is defined implicitely:

$$
\begin{aligned}
v(q, s_i) &= p + \frac{1-\Phi\left(\frac{q+y_2(p,s_2)-1}{\sigma}\right)}{-\phi\left(\frac{q+y_2(p,s_2)-1}{\sigma}\right) y_2'(p, s_2)} \\
q &= \frac{1}{\beta}\left(\alpha+\gamma s_1 - p - \frac{1-\Phi\left(\frac{q+y_2(p,s_2)-1}{\sigma}\right)}{-\phi\left(\frac{q+y_2(p,s_2)-1}{\sigma}\right) y_2'(p, s_2)}\right)
\end{aligned} \tag{A-1}
$$

where $\Phi(\cdot)$ is a standard normal CDF, $\phi(\cdot)$ the corresponding PDF and σ is the standard deviation of the random supply. Fixing strategy of bidder 2, we have $y_2'(p, s_2) = -\frac{1}{\beta}$. We will generate data from the model described above. The optimal bid function of player 1 has to satisfy:

$$
q = \frac{1}{\beta}(\alpha+\gamma s_1 - p) - \frac{1-\Phi\left(\frac{q+\frac{1}{\beta}\left(\alpha+\gamma s_2-p-\frac{\gamma}{\theta}\right)-1}{\sigma}\right)}{\phi\left(\frac{q+\frac{1}{\beta}\left(\alpha+\gamma s_2-p-\frac{\gamma}{\theta}\right)-1}{\sigma}\right)}
$$

In other words, his bid for q solves:

$$
p = \alpha+\gamma s_1 - \beta q - \beta\frac{1-\Phi\left(\frac{q+\frac{1}{\beta}\left(\alpha+\gamma s_2-p-\frac{\gamma}{\theta}\right)-1}{\sigma}\right)}{\phi\left(\frac{q+\frac{1}{\beta}\left(\alpha+\gamma s_2-p-\frac{\gamma}{\theta}\right)-1}{\sigma}\right)}
$$