Recent Developments in the Empirical Analysis of Auction Markets

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Introduction

Auctions have been the subject of a lot of good theory *and* good empirical work.

- Game is relatively simple, with well-specified rules.
- Actions are observed directly.
- Payoffs can sometimes be inferred.
- Data sets are readily available.

Why use an auction, instead of posting or negotiating a price?

- Buyers' willingness to pay is private information; auctions can be efficient price discovery process.
- Identity of highest value buyer is unknown; an auction can be an efficient allocation mechanism.
- Auctions can also be good at generating revenue.

Information asymmetries are fundamental.

There are many possible auction mechanisms.

- open outcry vs. sealed bid
- highest bid vs. second-highest bid
- reserve price, announced or secret
- entry fees or subsidies

In practice, most auctions are either first-price sealed bid (FPSB) or open, ascending price (English).

Goals of Theory

- Positive: describe how to bid rationally – Bayesian Nash equilibrium
- Normative: characterize optimal (e.g., revenue maximizing or efficient) selling mechanism

Goals of Empirical Studies

- Positive: what are the bid markups? Are buyers' valuations correlated and if so, what is the source of the correlation? Is observed bidding consistent with Bayesian Nash Equilibrium (BNE)? Is there evidence of buyer risk aversion? Do agents collude?
- Normative: recover value distribution, identify the revenue maximizing or efficient auction, simulate the effects of design changes.

There are many structural empirical papers which posit equilibrium bidding in the auction of a single item.

Recent surveys:

Athey & Haile (Handbook of Econometrics, Vol. 6, 2007)

Hendricks & Porter (Handbook of IO, Vol. 3, 2007)

Paarsch & Hong (MIT Press, 2006)

There has been considerable progress, but there remain important open issues.

In this talk, I discuss some recent developments that extend the basic empirical model of a one shot, single item auction.

I describe some research directions that might be of interest.

Outline of Talk

1. Standard Model and Notation
2. The Structural Program
3. Seller Incentives
4. Bidder Entry and Information Acquisition
5. Dynamics
6. Multi-Unit Auctions
7. Conclusion

1. Standard Model and Notation

n = number of (potential) bidders

m = number of bids ("active" bidders)

X_i = private signal of bidder *i*

$X = (X_1, \ldots, X_n)$

V = common payoff component

$U_i = u(X_i, V)$ bidder *i* utility if obtain one unit

F = joint distribution function of (X,V)

$Y_i = \max\{X_j, j \neq i\}$

W = winning bid

$\beta_i(x)$ bidder *i*'s (monotone) bid strategy

$\eta_i(b)$ inverse bid function of bidder *i*

Main Assumptions

- Each bidder wants only one unit.
- Utility u is non-negative, continuous, and increasing in each argument, and common across bidders.
- Bidders are risk neutral.
- F(X,V) is symmetric in the signals X.
- (X,V) are affiliated.
- X_i is real-valued.
- F, n and u are common knowledge.
- The losing bidders don't care who wins.

Special Cases

Private Values (PV): $u(X_i,V) = X_i$

Can normalize the signal X_i to be an unbiased estimator of expected valuation.

- IPV: X_i's are iid, F_x is marginal distribution of X_i
- APV: X_i's are affiliated.

If not PV, then say have Common Values (CV).

- Pure Common Value: $u(X_i,V) = V$
- CICV: X_i's are independent conditional on V.

If CV, then $E[U_i | X_i = x, Y_i < x] < E[U_i | X_i = x]$.

This is the winner's curse.

2. The Structural Program

Objective: Estimate F (and u) from bid data.

Basic idea: Bayesian Nash equilibrium (BNE) maps private signals into bids given F. Can we recover the primitives of the model from bid data?

Focus on symmetric BNE with increasing bid functions.

In open ascending auctions, problem of interpretation of losing bids.

Haile & Tamer (JPE 2003) make two assumptions about bidding behavior in an IPV environment, if b_i is *i*'s highest bid:

1. Winner is willing to pay more than the final bid, and losing bidders do not submit bids greater than their values, so $x_i \geq b_i$ for all *i*.

2. Losing bidders are not willing to raise the winning bid by the minimum bid increment Δ, so $x_i \leq w + \Delta$ for all *i* but the winning bidder.

These two assumptions provide upper and lower bounds on F_X, without fully specifying equilibrium play.

First Price Sealed Bid Auctions

Expected profits from bidding b, given a signal x:

$\pi(b,x)=\int^{\eta(b)} [w(x,y) - b]\, dF_{Y|X}(y|x)$

where $w(x,y) = E[u(V,X)|\, X=x, Y=y]$.

Differentiating with respect to b and imposing symmetry:

$[w(x,x) - \beta(x)]\, f_{Y|X}(x|x) = \beta'(x)F_{Y|X}(x|x)$

Laffont & Vuong idea: Let $M = \beta(Y)$, the highest rival bid.

Let $G_{M|B}$ denote the distribution function, conditional on one's own bid, and $g_{M|B}$ its p.d.f.

Then $F_{Y|X}(y|x) = G_{M|B}(\beta(y)|\beta(x))$ and $f_{Y|X}(y|x) = g_{M|B}(\beta(y)|\beta(x))\, \beta'(y)$.

Substitute into the FOC and evaluate at $b = \beta(x)$, to obtain the inverse bid function:

$w(\eta(b),\eta(b)) = b + (G_{M|B}(b|b)/g_{M|B}(b|b))$

Extensions of the Standard Model

The inverse bid equation has been adapted to estimate several variations on the standard model.

- Unobserved heterogeneity
  - Non-parametric (Krasnokutskaya (2004))
  - Parametric (Athey, Levin & Seira (2004), Krasnokutskaya & Seim (2007))
- Asymmetric bidders
  - Collusion (Bajari & Ye (REStat 2003))
  - Observable types (Athey, Levin & Seira)
- Risk averse bidders (Bajari & Hortacsu (JPE 2005))
- Identification of the CV model using ex post payoff data (Hendricks, Pinkse & Porter (RES 2003))
- Tests of PV vs. CV
  - Variation in number of bidders (Haile, Hong & Shum (2003))
  - Binding reserve price (Hendricks, Pinkse & Porter)

3. The Incentives of the Seller

Basic Question: What does the auction design reveal about the economic environment?

In most structural empirical analyses of the bidders' problem, the mechanism choice, or the reserve price policy, is treated as exogenous.

But the optimal reserve price is a monotonic function of the seller's valuation, which may be correlated with the buyers' values, and it is also a function of the distribution of buyers' values.

More generally, the mechanism choice may depend on the distribution of bidders' valuations, or on their behavior.

Examples:

In an IPV setting, if bidders are risk averse, the FPSB auction yields higher revenues than SPSB.

A seller may prefer SPSB or oral ascending if CV (Milgrom & Weber's linkage principle).

FPSB is less vulnerable to collusion.

Laffont, Ossard & Vuong (Ecma 1995): Marmande Eggplants

Model bidding in eggplant auctions (descending price, or Dutch) as BNE of IPV model, treating the reservation price as exogenous.

There is a strong correlation between the reserve price and the winning bid (see Figure 3 in LOV).

If the variation in the reserve price r is exogenous, so that F_X does not vary, the winning bid covaries with r in the BNE of the IPV model.

Here $\beta(x) = E[\max\{Y, r\} | X = x, Y \leq x]$, as in FPSB.

But it is also possible that both the reserve price r and the location and/or scale of the distribution of bidder values F_X are correlated with some factor (or factors) that are not observed by the econometrician.

In the variation in r is exogenous, should see more instances of no sale when r is high.

Under an optimal reserve price policy, there is a positive probability of no sale in many environments.

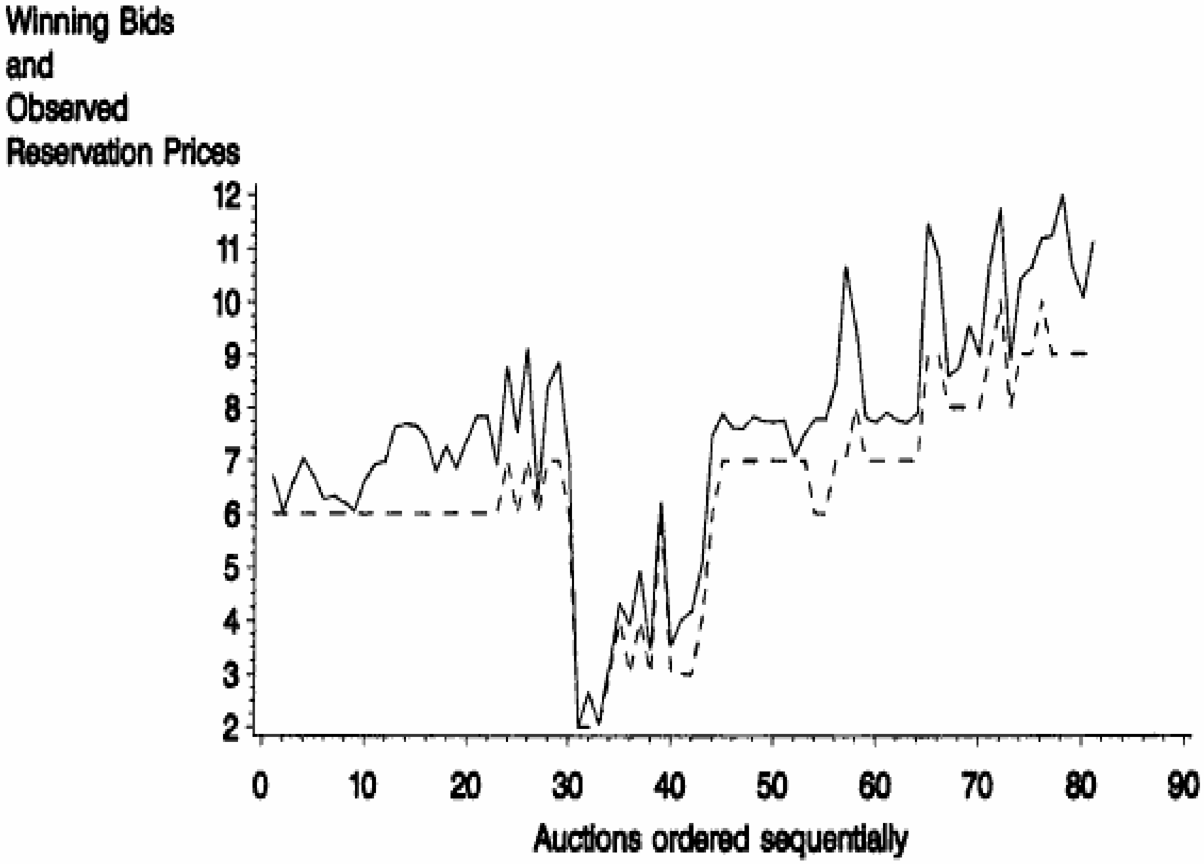

FIGURE 3.—Winning bids (continuous line) and reservation prices (dotted line).

Should be cautious in imposing full seller rationality.

The seller may have an objective other than static revenue maximization.

- If a government agency is the seller.
- If the seller can re-offer unsold items.
- If buyers can also go to competing sellers.

Nevertheless, if the reserve price is not exogenous, it may be informative about unobserved heterogeneity.

E.g., some authors deflate bids by the reserve price, to correct for proportional shifts in the mean valuation.

But need to be careful about higher order moments.

E.g., is the dispersion in bids proportional to the value of the item?

4. Bidder Entry and Information Acquisition

Auctions can be an important testing ground for studying entry.

- Auctions are held repeatedly, firms have to make frequent entry decisions.
- A rich variety of settings for studying entry decisions.

Much of the literature assumes that the number of bidders is fixed. But if participation is costly, the number should be determined as part of the equilibrium to the game.

- Who chooses to be a potential bidder?
- Which potential bidders choose to be active?
- Which active bidders submit a bid?
- In each instance, what do agents observe?
- Do auctions attract too many or too few bidders? This issue particularly important when bidders are asymmetric since, in this case, Revenue Equivalence does not hold and auction design matters (e.g., Athey, Levin & Seira), or if there are common values.

Empirical problem: multiplicity of entry equilibria $\Rightarrow$ likelihood function is not well-defined. Strategies for dealing with this issue include:

- Restrict payoffs so that number of entrants in set of pure strategy equilibria is unique and define likelihood in terms of this event. (Bresnahan & Reiss (RES 1990), Berry (Ecma 1992))
- Change game form: sequential entry, or private entry cost information (Seim (RAND 2006))
- Bound the probabilities of the outcomes (Tamer (RES 2003), Ciliberto & Tamer (2004))
- Append a set of selection rules and estimate joint distribution over outcomes and selection rules (Bajari, Hong & Ryan (2004)).

Auctions provide a natural context for implementing these strategies.

Sealed bid auctions – simultaneous move.

Oral auctions – sequential move.

Entry Models

Standard model:

All potential bidders are active; they submit a bid in the FPSB or SPSB, or participate in the open outcry auction, if their signal is above a threshold.

PV: Bid if $x \geq r$.

CV: Bid if $x \geq x^*(r,n)$,

where $x^*(r,n) = \inf\{x | E[u(V,X) | X=x, Y<x] \geq r\}$

and $x^*(r,n) > r$ is increasing in r and n.

In the PV case, $x^*(r,n) = r$.

Athey, Levin & Seira (2004): Timber Sales

Fixed number of potential bidders (of two types).

Bidders are endowed with a private signal, their bid preparation cost.

Bidders (simultaneously) choose to become active if this cost is below some threshold.

ALS consider the type symmetric pure strategy equilibrium, where bidders take as given the (binomial) distribution of the number of active rivals of each type.

Bidders then observe their private value, independent of their bid preparation cost, and they observe the number of active bidders.

Bidders submit a bid if their value is above the reserve price, as in the standard model.

The first stage is analogous to Seim's (RAND 2006) entry model.

The bidding game is that of Maskin & Riley (RES 2000), with a preceding round of entry decisions.

Bajari & Hortacsu (RAND 2003): eBay Coins

Model is in the spirit of Levin & Smith (AER 1994).

Large number of potential bidders, with a common bid preparation cost.

They (simultaneously) choose to become active.

BH consider the symmetric mixed strategy equilibrium.

Active bidders then observe their private signal of the common value, but not the number of active rival bidders.

Bidders take as given the (Poisson) distribution of the number of active rivals.

Bidders submit a bid if their signal is above the CV threshold, where this is the zero profit signal, taking expectations over the number of active rivals.

The bidding game is SPSB with an unknown number of rivals.

The common entry probability is uniquely determined by a zero ex ante profit condition.

Krasnokutskaya & Seim (2007): California Highway Procurement

KS consider two entry models.

In the first variant, firms observe a private bid preparation cost.

This model is essentially that of Athey, Levin & Seira, also with two bidder types.

In California, qualified small bidders are favored. The lowest small bidder wins if their bid is not 5% higher than the lowest large firm bid.

KS are interested in the effect on entry and bid levels for each bidder type.

In the second model, firms have a common bid preparation cost.

Firms randomize in their entry decisions, with type specific entry probabilities.

They observe the number of rivals of each type, which are distributed binomial.

Because values are assumed to be private, active bidders submit a bid if their signal is above the reserve price.

Active bidders choose bid levels in the FPSB, given the numbers of active rivals of each type, according to asymmetric PV BNE.

Hendricks, Pinkse & Porter (RES 2003): Offshore Oil & Gas

Model similar to McAfee & Vincent (AER P&P 1992).

Fixed number of potential bidders, with private signal of common value.

They (simultaneously) choose whether to become active.

Consider the symmetric pure strategy equilibrium.

Active bidders then observe a better signal of the common value, but not the number of active rival bidders.

Bidders' initial signals are informative about the number of active rivals.

Active bidders submit a bid if their second (better) signal is above the CV threshold, where this is the zero profit signal taking expectations over the number of active rivals.

The bidding game is FPSB, with an unknown number of rivals.

The active entry threshold is uniquely determined by a zero profit condition.

Here the entry decision is not independent of the bid level.

Endogenous Information Precision?

Almost all papers take the precision of information as given.

Potential bidders may not only choose whether to acquire information, but also the accuracy of their information.

In offshore oil and gas auctions, firms choose how much to invest in analyzing seismic data.

Firms' entry and bidding strategies will depend on their perceptions of how many serious rival bidders they face.

Information acquisition will be influenced by the auction mechanism (e.g., Compte & Jehiel (RAND 2007)).

A related issue: Much of the literature compares mean revenues. But in some instances bid dispersion varies with the mean bid level (e.g., offshore oil lease bidding).

This variation may be driven by variation in the level of competition.

But the entry decision, and decisions about the precision of information acquisition, may vary with the expected value of the item.

5. Dynamics

Aguirregabiria & Mira (Ecma 2007), Bajari, Benkard & Levin (Ecma 2007), Pakes, Ostrovsky & Berry (RAND 2007), and Pesendorfer & Schmidt-Dengler (RES 2008) have developed feasible estimators for dynamic games, extending the Hotz & Miller (RES 1993) approach to estimating dynamic decision problems.

Restrictive conditions: repeated stage game played in a stationary environment in which unobservables are independent across time and players.

Private value auctions, e.g., highway procurement auctions, can come close to meeting these conditions.

Most of empirical literature assumes that bidders treat auctions as static games, choosing bids to maximize profits in that auction.

Plausible in environments with no learning or payoff linkages, if the time interval between auctions is sufficiently long.

But when time between auctions is short, the current auction outcome can affect the state of play in future auctions.

- Winner of current auction may not be able to participate in future auctions, or have stochastically lower valuations.
- Losers are then more likely to win future auctions.

As a result, bidding is less aggressive, which must be accounted for in estimating the distribution of valuations.

Jofre-Bonet & Pesendorfer (2003): California Highway Procurement

Extend the Laffont & Vuong approach to the dynamic setting.

The inverse bid equation in a PV setting has an additional term:

$\eta(b,c) = b + G_M(b|c)/g_M(b|c) + \beta\, \partial V/\partial b$

where c is the vector of firm costs, which are increasing in backlog,

V is the expected continuation value, and β the discount factor.

Here $\partial V/\partial b > 0$, as losing bidders have relatively lower future costs

Hence bidding is less aggressive than in the static case (higher markups).

One issue: The distribution of valuations $F(x|c)$ is identified only for a fixed discount rate β.

Thus one cannot distinguish between myopic and forward-looking behavior.

Conjecture: Use variation in time between contracts to identify β, and thereby distinguish bidder myopia from foresight.

6. Multi-Unit Auctions

In many instances, multiple units are sold or procured simultaneously, rather than sequentially.

Examples: treasury bills, wholesale electricity, spectrum licenses.

An important issue is optimal mechanism design. For identical items:

Discriminatory auctions, in which winning bidders pay their own bids, vs. Uniform price auctions, in which winning bidders all pay the same price (such as the lowest winning bid, or the highest losing bid).

Ausubel & Cramton (2002) show that there is no clear ranking of the two according to expected seller revenues.

The revenue maximizing choice is an empirical issue.

A recent literature proposes structural estimation methods to estimate the distribution of valuations, and so draw inferences about the consequences of changes in the selling procedure. Buyers may value more than one of the multiple units being sold.

For non-identical items, there is also an issue of whether and how to bundle items, or allow package bids, to reflect complementarities.

Recent Studies of Treasury Bill Auctions

Hortacsu (2002) studies discriminatory Turkish auctions.

Model based on Wilson's (QJE 1979) analysis of share auctions, in which bid schedules are continuous.

Assume private values, real valued signals.

(Euler-LaGrange) necessary condition to demand q units at bid price b:

$v_i(q,x) = b + G_i(b,q)/(\partial G_i(b,q)/\partial b)$,

where $v_i(q,x)$ is the marginal value at q given x, and $G_i(b,q)$ is the probability that the bid b is accepted.

Here $v_i(q,x)$ is smooth, decreasing in q and increasing in x.

Analogous to the inverse bid equation in the single item case.

Uses resampling methods to estimate G_i, and hence recover $v_i(q,x)$.

Hortacsu treats $v_i(q,x)$ as an upper bound on the bid schedule in a uniform price auction, to bound revenues from above, and compare with revenues in the discriminatory format.

Kastl (2006) studies uniform price Czech auctions.
Explicit recognition of bids as step functions, not continuous.
Bidders are limited to at most 10 steps.
- In his sample this constraint is never binding.
- Larger bidders average 2.5 steps, small bidders 1.1 steps.

Allows common values, assumes real valued signals, plus an independent (and private) cost per bid point.
Bidders choose the number of bid points, and (b,q) for each point.
With step bid functions, ties are a possibility.
Assume rationing is pro-rata on the margin (as is typical in practice).
If values are private, ties have zero probability in equilibrium.
PV bidders may bid $b(q) > v_i(q,x)$, but not above average valuation.
Thus marginal valuation $v_i(q,x)$ is not a valid upper bound on the bid schedule in a uniform price PV auction.
Kastl shows that in the Czech data this is a practical concern.

Chapman, McAdams & Paarsch (2006) study discriminatory Canadian cash reserve auctions.

Also consider bids as step functions.

Quantity increments are taken as fixed, as with a finite number of identical discrete units for sale.

Bids are price offers for a given number of units (and prices are chosen from a fixed grid).

Assume affiliated private values, but signals are vectors of length equal to the number of quantity grid points.

Marginal values are only required to be non-increasing.

Characterization of equilibrium is daunting.

Build on work of McAdams, to derive bounds on values implied by equilibrium; essentially focus on necessary conditions for bid prices.

Look for bid patterns that are inconsistent with equilibrium.

Bidders do not appear to deviate in an important way from equilibrium predictions.

Conclusion

Empirical analysis of auction markets continues to be a fertile research area.

My focus today has been on methods that extend the standard model for single item auctions.

In some instances, the extensions involve relatively straightforward adaptations of existing methods.

But some extensions are not so simple.

Institutional details and the available data play a large role in the choice of research method.