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A Theory of Sticky Rents: Search and Bargaining with Incomplete Information

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The housing rental market offers a unique laboratory for studying price stickiness. This paper is motivated by two facts: 1. Tenants' rents are remarkably sticky even though regular and expected recontracting would, by itself, suggest substantial rent flexibility. 2. Rent stickiness varies significantly across structure type; for example, detached unit rents are far stickier than large apartment unit rents. We offer the first theoretical explanation of rent stickiness that is consistent with these facts. In this theory, search and bargaining with incomplete information generates stickiness in the absence of menu costs or other commonly used modeling assumptions. Tenants' valuations of their units, and whether they are considering other units, are both private information. At lease end, the behavior of risk-averse landlords differs according to the number of units managed. Multi-unit landlords, aided by the law of large numbers, exploit tenant moving costs. When renegotiating rent contracts, they set rent increases that exceed the inflation rate; while the majority of tenants stay, those who place low value on the unit search elsewhere and leave. Landlords with one unit loathe vacancy and offer tenants the identical contract to pre-empt search; only those who really hate the unit leave.

JEL Codes: C7, C78, D4, D83, D9, E3, R31.

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1. Introduction

Although price stickiness is a central issue in macroeconomics (for example, see Ball and Mankiw, 1994, or Boivon, Giannone, and Mihov, 2009) there is little consensus about its deep sources. Most New Keynesian models—which are the workhorse models for central banks around the world—use tractable simplifications like Calvo pricing or state-dependent pricing, which posit arbitrary "costs" or constraints to changing prices. But the modeling details matter: The appropriate monetary policy response to shocks depends upon the precise source of price stickiness (see, e.g., Caballero and Engel 1993 or Huang and Meng 2014). Unsurprisingly then, explaining price stickiness remains an area of active theoretical and empirical research (Golosov and Lucas, 2007; Nakamura and Steinsson, 2008, 2011; Gopinath and Itskhoki 2011; Knotek, 2011; Head et al., 2012; Midrigan, 2011; Kehoe and Midrigan, 2012; L'Huillier and Zame, 2015).[1] But rent stickiness has garnered almost no theoretical attention.

There are three main reasons for our interest in rent stickiness. First, as documented below, rental markets exhibit a large and puzzling degree of rent stickiness, but the degree of stickiness varies greatly by structure type; neither fact is adequately explained by extant theories. Second, rent stickiness is quite important for aggregate inflation because rent changes drive 30 percent of the US Consumer Price Index, and play an even more important role in "core" inflation measures: Rent changes drive 42 percent of the core CPI and 19 percent of the core PCE price index (Verbrugge and Garciga, 2015). Third, rent stickiness is intriguing because it suggests that stickiness can emerge even in markets that are characterized by a lack of "menu costs" and a prevalence of repeated bargaining over contracts.

For our purposes, several features of the rental housing market are salient. First, contract rents in the U.S. have exhibited a large degree of stickiness during the past 40 years, despite an overall increase in rents over this period. Using the micro data that form the basis for the Consumer Price Index in the US, Gallin and Verbrugge (2017) show that from 1999 to 2008, about half of all rents were unchanged after 12 months and about one-third of all rents were unchanged after two years. Moreover, Gallin and Verbrugge show that about two-thirds of the

[1] Wolman (2007) and Nakamura and Steinsson (2013) provide excellent surveys.

single-family units had no rent change over a year while only about one third of units in a large buildings saw no rent change over a year.

Extant theories that explain price stickiness are not appropriate for the housing rental market because rent setting is substantively different from price setting for things like cabbage, clothing, computers, catering, and even cars. Most transactions in these other goods and services involve search, but little or no bargaining. Moreover, even transactions that often involve bargaining—such as cars and other durables—are sold repeatedly to different customers, on different days, in different locations. Perhaps as a result, while there is a large literature of price setting involving search,[2] most models of price setting abstract from bargaining (see Camera and Sulcuk 2009).[3]

Search and bargaining over match-specific surpluses with incomplete information play a central role in the rental housing market. Fixed costs such as search and vacancy, in conjunction with idiosyncratic valuations of particular rental units, produce a match-specific surplus that is potentially bargained over, particularly given repeat transactions over the same product (when tenants re-sign leases and renegotiation may occur). Landlords and tenants almost always enter into long-term contracts rather than negotiating terms every hour or day. But contracts expire; at the end of twelve months (a typical length in the U.S.), we often see landlords and tenants agree to *the identical contract*, despite inflation in various costs such as maintenance and taxes, and inflation in the outside options of both parties.

Our model of rent setting relies upon information frictions and risk aversion on the part of landlords. Initially, landlords post prices, but unlike the directed search literature (see, e.g., Eeckhout and Kircher 2010), prospective tenants and units are ex ante identical and there is no benefit to posting a low price, so a "law of one price" prevails for vacant units (cf. Curtis and Wright 2004). However upon the expiration of the initial rental contract, when the contract is potentially renegotiated, there is asymmetric information: a landlord does not know the tenant's

[2] Stigler (1961), Butters (1977) and Reinganum (1979) are early entries, and Menzio and Trachter (2015) and Camera and Kim (2015) are recent contributions.

[3] Post-match bargaining is more common in the wage-setting literature; generally this is Nash bargaining (such bargaining is a typical feature, for example, of the standard Diamond-Mortenson-Pissarides model of the labor market.) A simple form of "bargaining" or renegotiation is a feature of some models in which list prices differ from realized prices depending upon the number of agents matched (as in Albrecht, Gautier and Vroman 2006 or Camera and Selcuk 2009). Post-match bargaining under asymmetric information appears in Kennan (2010)'s model of the labor market.

valuation of the unit, or whether the tenant has located another suitable unit. When a contract is about to expire, a landlord must balance the benefit of higher nominal rent revenue against the possibility that the tenant will be provoked to move. All landlords dislike leaving units vacant, but vacancies are more costly for the numerous landlords who own relatively few units.[4] In our theory, we make the stark assumption that landlords are of two types: "small" landlords, who own a single unit, and "large" landlords, who own an infinite number of units. With an appeal to the Law of Large Numbers, we assume that a large landlord knows with certainty her per-unit (average) return for any given offer or renegotiated rent, while a small landlord, whose unit is either vacant or not, faces a potentially large standard deviation of returns.[5]

Tenants in our model dislike moving, but may find the present unit a poor match. For example, they may have found the unit at the last minute, or may have obnoxious neighbors, or may now understand that the unit does not meet their needs. Toward the end of the initial contract period, tenants may engage in costless passive search or costly active search to find another unit, but their incentive to search will be reduced if they expect an attractive rent offer from the landlord. In equilibrium, "large" landlords always play tough and enter the risky renegotiation process. They are usually able to extract a higher rent from tenants who decide not to move, but sometimes experience lost rent revenue on the unit.. On the other hand, in equilibrium "small" landlords play it safe and always offer the tenant the same contract in order to pre-empt active search, and this strategy is usually successful. Though not a part of our model, informally we think that our framework also captures another idea: a tenant may have imperfect information about her outside options (owing to uncertainty about the state of the rental market, how much she will like the new neighborhood after all, etc.), but she knows for sure that a zero rent increase is a good deal.

Our theory is a dynamic model of search and bargaining with incomplete information. Such models are notoriously difficult to solve analytically. As a result, we show that an

[4] In 2005, about one-third of the rental units in the US were single-family detached homes or townhouses. About half of all rental units in the US are in large apartment complexes, though most landlords in the US own 4 or fewer units; Chambers, Garriga and Schlagenhauf (2009) and DiPasquale (2011) review the evidence.

[5] While we wish to tie our hands by focusing only upon the risk aversion channel, we do not deny the presence of other advantages that owners of multi-unit complexes enjoy. Among these advantages: large complexes are easy to find; fixed costs like advertising, contract writing, and maintaining a demo unit can be spread over a larger number of units; and large complexes have better information about the state of the rental market owing to monthly turnover and the law of large numbers.

equilibrium exists by examining a "case" in the sense of Gilboa et al. 2011.[6] That is, because the model is so complex, we do not provide an analytical proof of the existence of an equilibrium for arbitrary parameter values. Rather, as is fairly standard for dynamic asymmetric information games, we prove that an equilibrium exists for a specific set of parameter values (which we do not attempt to calibrate). See, for example, Milgrom and Roberts (1982), Cho and Kreps (1987) and Geanokoplos (2010).

One popular alternative explanation of rent stickiness is the "good tenant" theory (Barker, 2003) in which landlords who find good tenants attempt to retain them by not changing the rent. Another is the "depreciation equals inflation" theory in which landlords can raise effective rent without changing actual rent by allowing the quality of the housing unit to deteriorate. [7] Although both these theories may explain why rents may not fully adjust, neither can explain why the rent change so often is *zero* rather than some small number. In an otherwise standard Calvo model of the rental market which includes a "good tenant" discount and allows for depreciation, the first rent reset is often *negative* (Verbrugge and Krsinich, 2011), and Barker (2003) establishes that new tenants often receive a discount, rather than a markup (and provides a theory to rationalize this pricing). Moreover, good tenants and depreciation cannot explain why rent stickiness differs so significantly across property type. Our model provides a better explanation of the data.

The rest of the paper is organized as follows. Section (2) briefly presents a summary of some key empirical features of rent stickiness, section (3) provides a description of the model, section (4) defines the equilibrium and prove the existence of an equilibrium for specific parameter values, and section (5) presents our discussion and conclusions.

[6] Gilboa, Postlewaite, Samuelson and Schmeidler (2014) point to Akerlof's (1970) celebrated "lemons market" paper as a classic example of a case: it is quite stylized, uses particular numerical values, yet provides helpful insights into the functioning of markets with asymmetric information. One role of economic reasoning and theoretical modeling, these authors argue, is to enrich the set of cases.

[7] While direct evidence on the depreciation rate experienced by rental units is difficult to locate, Sommers, Sullivan and Verbrugge (2013) estimate this rate at roughly 3.7%. The Bureau of Labor Statistics adjusts rent inflation for aging using estimates from cross-sectional data (Randolph 1988a,b, Gallin and Verbrugge 2007); the size of the (average) inflation adjustment varies from year to year, from +0.2% to +0.4%. Gallin and Verbrugge (2007) emphasize that aging-related depreciation is highly variable across units; unit age is especially important.

2. Data and Key Facts

In Gallin and Verbrugge (2017), we document key facts about rent-setting behavior in U.S. housing markets from 1998 to 2008. The micro-data in that paper are from the rent-inflation measurement program at the Bureau of Labor Statistics (BLS). As part of this program, the BLS collected and followed a nationally representative sample of rental housing units, where each unit in the sample was surveyed every six months.

Table 1 (excerpted from Gallin and Verbrugge, 2017) provides evidence on the degree of nominal rent rigidity.[8] About half of rent observations are unchanged during a 6-month period. A fair amount of nominal rent rigidity is to be expected in rental markets because of the prevalence of one-year leases.[9] However, about half of rents are still unchanged after one year, about 40 percent of rents remain the same after 18 months, and one third of rents are still unchanged after two years.[10]

Table 1
Distribution of rent changes

	Percent decrease	No change	Percent increase
6-month	9	67	24
12-month	11	49	39
18-month	12	39	49
24-month	12	32	55

Source: BLS micro-data for rent. January 1998 to October 2008.

Of the rents that change, the majority show increases. While this should not be surprising given the tendency for rents to increase on average during this time period, a nontrivial fraction of housing units display rent decreases in our sample. As a comparison, Klenow and Kryvtsov

[8] This manuscript provides additional evidence on rent stickiness.
[9] Crone, Nakamura and Voith (2010) note: “The annual lease is the predominant form for rentals. Data from the U.S. Census Bureau’s Property Owners and Managers Survey in 1995 (single-family and multifamily units, excluding data not reported or for rent-free units) showed that 44.4 percent of all units had annual leases, 4.0 percent had leases longer than one year, 36.1 percent had leases less than one year, and 15.5 percent had no leases.”
[10] Verbrugge et al. (2016) studies dimensions along which rent markets are stratified, motivated by a desire to ensure that the BLS undertakes representative sampling and can adequately address sample shortfall; it does not study rent stickiness per se. However, it does show that rent changes over three-year periods exhibit a large spike at zero.

(2008) investigated the dynamics of retail prices of goods and non-shelter services in the CPI microdata, and found that on average over the 1988-2005 period, about 12 percent of regular (non-sale) prices fell each month and about 15 percent of regular prices rose every month.

Table 2 presents some cross-tabulations of nominal rent rigidity at 12-month horizons by several observable characteristics of the housing unit.

Table 2
Nominal wage rigidity by characteristic

	Twelve-month price change Percent		
	Negative	Zero	Positive
Full Sample	11	49	39
Structure type			
Mobile home	10	57	33
Single-family detached	9	57	33
Single-family attached	11	49	40
Multi-unit without elevator	13	40	46
Multi-unit with elevator	13	33	55

Source: BLS micro-data for rent. January 1998 to October 2008.

Note that while just over half of all rents were unchanged after 12 months, the degree of rent stickiness differs starkly across structure types. In particular, almost 60 percent of rents for single-family detached homes remain unchanged over a 12-month period while only a third of rents for multi-unit housing structures with elevators remain unchanged over a 12-month period.

Very few other studies of rent stickiness exist. Genesove (2003) used data from the Annual Housing Survey (AHS) from 1976 to 1981 to examine rent stickiness and found broadly similar results. Looking outside the U.S., Aysoy, Aysoy, and Tumen (2014) find that about one third of nominal contract rents are unchanged from year to year in Turkey from 2006 to 2011. However, unlike Genesove (2006) and Gallin and Verbrugge (2017), they find no evidence for a differential rent stickiness across structure property type. Aysoy et al. attribute the absence of differential stickiness to the fact that the Turkish rental market is dominated by individual landlords rather than professional management companies, which implies that the type of structure in Turkey is likely a poor proxy for whether a housing unit is owned by a professional landlord who owns many units. Shimizu, Nishimura, and Watanabe (2010) use data on the housing rents in Japan from 1986 to 2008 show that over 90 percent of rents remain unchanged from one year to the next, but they did not look at differential rent stickiness across property type.

3. Model Description

The economy exists for two periods. As depicted in Figure 1, each period is preceded by a subperiod during which activities such as entry, search, matching, contracting, and renegotiation take place. Entry of units precedes the first period, followed by a three-stage search, matching and contracting subperiod. During period one, tenants occupy rental units and pay rent. Play at the end of the period one involves tenant and landlord decisions, followed by a three-stage search, matching and contracting subperiod. Some tenants relocate at the end of period one. During period two, tenants occupy rental units and pay rent.

There is a measure 2 of tenants, and a continuum of landlords whose total measure L is determined in equilibrium. Landlords are of two types: Large landlords, each of whom owns a continuum of rental units of type A, and small landlords, each of whom own a single rental unit of type D. Landlords are risk averse; their return function (as a function of income in both periods) is given below. The cost of creating a D unit is $\delta_D P_1$, while cost of creating an A unit is $\delta_A P_1$, where δ_D and δ_A are constants and P_1 denotes the overall price level in the first period. (In this paper, prices or costs in lowercase letters denote *real* prices, while prices or costs in uppercase letters denote nominal prices. Furthermore, for notational simplicity we suppress time subscripts if the period in question is clear from the context.) Given perfect competition in conjunction with these fixed costs of entry, then in equilibrium the number of extant units of a given type is such that the expected profit from entry, taking into account the possibility of vacancy, is zero. Occupied units incur maintenance and repair costs, totaling $k_m P_t$. The landlord of each unit that is available to a new leasee in period t must post an offer rent, $R_t^{offer} = P_t r_t^{offer}$, extended to any new tenant who wishes to sign the 1-period lease. To occupy a unit, a tenant must sign a lease. The lease may not be terminated early by either party, but is renewable at the end of period 1, at a potentially renegotiated rent level, as described below. The price level P equals 1 in the first period, and $(1+\iota)$ in the second, where $\iota > 0$.

Tenant utility

Tenants potentially obtains utility from shelter services s and nondurable consumption c, and maximize

$$EU(s_1,c_1)+bEU(s_2,c_2)=E\{u(s_1)+c_1\}+bE\{u(s_2)+c_1\}$$

Utility from consuming the shelter services of a rental unit is equal to a random variable Z, which is a tenant- and unit-specific quality or amenity attribute.[11] Utility from shelter services for "tenants" that do not occupy a rental unit are determined/defined by an outside option $\underline{s}$. Tenant income equals Y in both periods and there is no saving; income not spent on shelter is spent on c. Hence the per-period indirect utility function is

$$V\left(R_t,P_t,k_n,k_r,k_s,Y;Z\right)=\begin{cases} Z+\dfrac{Y-R_t-P_t\left(1_n k_n+1_r k_r+1_s k_s\right)}{P_t} & s_t=1 \\ \underline{s}+\dfrac{Y}{P_t} & s_t=0 \end{cases} \tag{0}$$

R_t is nominal rent. 1_n 1_r, and 1_s are indicator functions for the events "renegotiation," "relocation," and "active search," each of which implies the incurrence of costs: k_r is real relocation costs (incurred if the tenant switched units between periods),[12] k_n is real negotiation costs (incurred if the tenant faced renegotiation between periods, with various interpretations given below), and k_s is real search costs (incurred if the tenant actively searched this period). The value of the outside option, $\underline{s}$, can be interpreted as the value of the best inferior option that is available to an agent for whom a large home purchase is ruled out, such as living with one's parents, purchasing an inexpensive mobile home, etc.

[11] While preferences appear to be risk-neutral, this is a matter of interpretation. Consider the preferences $U(s,c)$ = $\ln(zs) + (y - r)$.For a given distribution of z, any decision rules derived from these preferences can be replicated using (1) by suitably altering the distribution of z.

[12] Moving costs include the physical costs of transporting durables and semi-durables, but also time costs associated with packing and unpacking, transport, making phone calls, notifying firms of one's change of address, and so on.

Units of a given structure type are ex ante identical. When a tenant first enters a unit, whether during search or after signing a lease on the unit sight-unseen, Z is drawn from a discrete distribution with probability density function $P(z_i)$ defined over $i = 1, \ldots, 4$. This quality attribute is specific to the tenant/unit match. Its value is private information of the tenant, and cannot be credibly communicated.

There are two types of tenants, *A* and *D*, each of measure 1.Type *A* tenants prefer (receive a higher amenity value from) units of structure type *A*, while type *D* tenants prefer units of structure type *D*. Tenant type is also private information. When a tenant of type j ($j = A,D$) faces a unit of same type j, Z takes on values in the set $\{z_1, z_2, z_3, z_4\}$ with respective probabilities $\{\mu_1,\mu_2,\mu_3,\mu_4\}$. The distribution of Z for a tenant of type j facing the structure type k ($k \neq j$) is identical, except that each Z outcome is reduced by $\underline{z}$. Nominal income Y is assumed to be adequate to cover the maximum rent in any period.

Search and matching

Before each period, there is a three-stage search, matching and contracting subperiod. The search and matching process captures a number of salient ideas, including: tenants decide how intensively to search for available units; search may be unsuccessful, and units may remain vacant; and tenants may opt to keep searching if the first unit they locate is unsuitable. The process begins when landlords submit "searchable" units to the matching process (along with an associated binding offer rent), and tenants actively or passively search for a unit. If a landlord submits multiple units to the matching process, the offer rent applies to every unit submitted. There are two types of searchable units: units currently vacant, and – at the end of the first period – units occupied by tenants who have announced their intention to depart.[13] Given our discrete time environment, we posit a simple matching process M which randomly matches searching tenants to searchable units.[14] Searching tenants inform the matching mechanism of any restrictions on their search, but since units are ex ante identical except for structure type and possibly offer rent, the only restrictions possible are structure type and offer rent.

[13] These matching restrictions are intended to capture the notion that it is costly for a landlord to bring prospective renters into occupied units.
[14] Note that details of the search and matching process can matter; see Chade, Eeckhout and Smith (2017) for a discussion.

The first stage is termed “orderly search.” Tenants who are matched to a unit during this stage are able to preview it and determine its suitability prior to signing a lease – that is, they are able to observe the match-specific realization of Z. If the tenant so decides, the lease is signed in stage 2, the “contracting” stage; if not, the tenant keeps looking. Units without a contract remain available to the matching process for the final stage, “desperate search and signing”.

During the orderly search stage, a tenant may search actively, passively, or not at all. Passive search, which is costless, places the tenant into the search/matching mechanism with probability $q < 1$, while active search, which is costly, places a tenant into the search/matching mechanism with probability one. Let h denote the measure of tenants in the search process for a given structure type, and let v denote the measure of searchable units of that type. The matching process M is such that searchers locate a searchable unit with probability $\min\left(1, \frac{v}{h}\right)$, and a searchable unit is visited with probability $\min\left(1, \frac{h}{v}\right)$. Thus, if there are more searchable units than searchers, a searcher locates a unit with probability one, while some units are never visited. In the equilibrium we characterize below, $h < v$ during all search phases. Hence, passive search allows a tenant to visit one searchable unit with probability q; active search allows a tenant to visit one searchable unit with probability 1.

In the contracting stage of the search and matching process, searchers who located a unit are given the option of signing a lease on that unit at the rent offered by the landlord. Stage two involves an additional activity at the end of period 1, in that it is the stage during which renegotiation over a current lease may take place. Such renegotiation imposes renegotiation costs, as specified below.

All searchable units that do not obtain a signing tenant, and all tenants or prospective tenants who did not sign or renegotiate a lease during stage two, enter stage three, the desperate search and signing stage. During this “secondary market” stage, search is costless, all unmatched “desperate” tenants search actively, and the matching process M matches these searchers to the remaining available units. A successful match in this stage simply means that a tenant located an available unit (a failed search, which is possible only if there are not enough vacant units, would imply $s = \underline{s}$). Tenants who are matched to a unit must sign a lease without previewing it. Moving into rental units occurs at the end of stage three.

We assume for simplicity that landlords do not know if their unit was matched in stage one or stage three; this captures the notion that a landlord knows very little, at least initially, about how intensely any given leasee has searched, or how much the leasee values the unit.

Bargaining and Renegotiation

Due to search frictions, matches can generate quasi-rents, i.e. a surplus that depends upon the realization of *Z* (and landlord costs) and that is divided between the two agents according to the rent level in the lease. When match surplus is not common knowledge, the Nash Bargaining Solution is not applicable. Myerson (1984) developed a generalization of the Nash bargaining solution for games with incomplete information. The Myerson "neutral bargaining" solution involves agreement over a mechanism to divide the surplus equitably, often a random dictatorship mechanism (see also Kennan 2010). In the random dictatorship mechanism, one of the players is randomly selected to make an offer, and if the offer is rejected then the match dissolves. We posit the (weighted) random dictatorship mechanism as a simple and tractable way to incorporate into our model bargaining under incomplete information and unequal bargaining weights, both of which are prominent features of the tenant/landlord setting. However, we abstract from bargaining over mechanisms.[15]

Bargaining (and renegotiation) only pertains to existing relationships. As noted above, initial rent contracts are all associated with an observable non-negotiable initial rent offer. We fold initial rent price discovery into the search process, since our goal is not to explain rent dispersion per se, but rather the striking rent stickiness (and differential rent stickiness) observed in the face of obvious rent reset opportunities. In the equilibrium we describe, all landlords end up posting the same offer rent. Bargaining is thus only applicable at the end of the first period, when an existing lease is ending and may be renewed, possibly after renegotiation. But as discussed below, landlords always have the option of pre-empting the uncertain renegotiation process, by foregoing an expected profit opportunity and offering tenants a good deal.

Play at the end of the first period is depicted in Figure 2. Tenants move first, their only option being to walk away (and search actively or passively), or to signal a desire to maintain the relationship (and a concomitant willingness to incur renegotiation costs). If a tenant announces

[15] Note that the Kalai and Kalai (2013) "coco value" solution is not applicable in this strategic setting, as there are no side payments and the tenant's utility level is not observable, even ex post.

her decision to walk away, the landlord is then free to place the unit into the search process; otherwise an occupied unit cannot be placed into the search process. If the tenant chooses to continue to play, landlords move next. At this stage, landlords may either offer tenants the identical lease as a take-it-or-leave-it offer, or signal a desire to negotiate a new lease. (A landlord might offer the same lease in order to minimize expected vacancy costs. In our theory, to emphasize that rent rigidity does not derive from menu costs, we make the stark assumption that landlords face no explicit renegotiation costs; however imposing modest renegotiation costs would not substantively alter the equilibrium we describe.) If the landlord offers the same lease, and the tenant rejects the offer, the match will dissolve, and the tenant will enter the first search stage, actively or passively. If the landlord signals for negotiation, the tenant will enter the orderly search stage, actively or passively. Both the intensity and the outcome of the search process are private information. Since renegotiation will occur, the tenant incurs renegotiation costs $k_n P$, which may be interpreted as information-gathering costs, contract preparation costs, contract reading costs (as in, e.g., Rasmusen 2001), or "hassle" or other psychological costs (as in, e.g., Rotemberg 2008, 2011). Renegotiations occur independently of one another. During renegotiation, Myerson's dictator chooses a party, either the landlord (with probability χ) or the tenant (with probability $1-\chi$), to make the take-it-or-leave-it offer. If the offer is rejected, the match dissolves, the unit will remain vacant the next period, and the tenant must find somewhere else to live. She will either sign a lease on a unit that she located during orderly search (if she found one), or enter the desperate search and signing stage.

Landlord return function

Landlords are risk-averse, and the landlord return function is specified as

$$E\ln\left(\pi_1\right)+\beta E\ln\left(\pi_2\right)$$

where π_t (t = 1, 2) is profit, defined as rental (or vacancy) revenue minus costs. The real interest rate is zero. A landlord contemplating entry of a unit of type k compares this return to its expected opportunity cost, which is the utility return achievable from optimal consumption with an initial asset position of δ_k. In equilibrium, the net return to entry for a unit of either type is 0.

For simplicity, we assume that a unit in period t which is vacant generates nominal revenue (or utility) equal to εP_t.[16] Landlord costs consist of maintenance and repair on occupied units.

While in almost all respects the problem facing a small landlord is identical to that facing a large landlord, in one key respect these problems are quite different, owing to the law of large numbers. In particular, the large landlord's per-unit return is the average over an infinite number of units; thus, large landlords know with certainty, for any given offer rent and renegotiated rent, the fraction of units which will be vacant. Conversely, for a small landlord, the unit is either vacant or not, and the standard deviation of returns is therefore much greater. Risk aversion will imply that small landlords are more willing to make concessions in order to avoid vacancy.

4. Equilibrium

4.1 Definition and discussion

The equilibrium concept is a **perfect Bayesian equilibrium** (Kreps and Wilson, 1982):[17] a strategy profile Ξ and a set of beliefs α such that at each node of the game:

(1) All players know the probabilities with which Nature chooses.
(2) The strategies at each node of the game are Nash, given the beliefs and strategies of the other players.
(3) The beliefs at each information set are rational, given the evidence appearing thus far in the game (meaning that they are based, if possible, on priors updated by Bayes's Rule, given the observed actions of the other players under the hypothesis that they are in equilibrium).

Given the presence of asymmetric information, an equilibrium is a strategy profile *and* a set of beliefs such that the strategies are best responses. The deductions used to update beliefs are based on the actions specified by the equilibrium; players update their beliefs assuming that the

[16] The positive vacancy return ε plays two roles in our theory, both driven by the fact that we are using a two-period model to represent a more complicated reality. First, a return of zero – a real possibility for a small landlord – yields infinite disutility. Second, ε represents the outside option of a landlord, and will thus pin down the lowest rent a hard-bargaining tenant could ever obtain. In reality, a vacant unit is not worthless; but in a two-period model, a vacancy in the second period would ordinarily be worth nothing. Since we desire a stationarity property for the equilibrium, we posit the existence of an explicit return ε to units which are vacant in either period.
[17] See Rasmusen (2005).

other players are following the equilibrium strategies. Since best responses depend on the beliefs, an equilibrium cannot be defined based upon strategies alone. Players use Bayes's Rule (and their priors) to update beliefs on the equilibrium path; but *off* the equilibrium path, beliefs are specified by the modeler. A perfect Bayesian equilibrium puts no restrictions on off-path beliefs. These beliefs must be specified carefully, otherwise players might have an incentive to undertake out-of-equilibrium actions in order to set in motion other players' out-of-equilibrium beliefs and strategies.[18] As Rasmusen (2005) notes, finding a perfect Bayesian equilibrium of a game is not necessarily straightforward.

Table 3: Parameters

Parameters		
(z_1, z_2, z_3, z_4)	quality attribute levels	(200, 500, 700, 900)
$(\mu_1, \mu_2, \mu_3, \mu_4)$	Parameters in F_z	(0.1, 0.2, 0.6, 0.1)
$\underline{z}$	utility penalty to tenant who lives in non-preferred shelter type	100
$\underline{s}$	value of outside shelter option to tenant	-500
k_r	relocation cost to tenant	150
k_s	search cost to tenant	25
k_n	negotiation cost to tenant	25
k_m	maintenance costs for an occupied unit	80
ε	return to vacant unit	250
χ	probability that Myerson's dictator chooses landlord	0.9
q	probability passive search results in a match	0.2
β	discount rate	0.95
P_1 (P_2)	price level in period 1 (period 2)	1 (1+ι)
ι	growth rate of price level	0.03
δ_D, δ_A	cost to creating unit (D, A)	1800, 1800
Endogenous prices and quantities (measures)		
r_t^{offer}	real offer rent for a new tenant	
r_2'	rent that landlord offers upon contract renewal in period 2	
h	measure of searching tenants	
v	measure of searchable units	

[18] There are many refinements of perfect Bayesian equilibrium, which agree on the equilibrium path but place various restrictions on off-equilibrium beliefs and behavior. Given our purpose in this paper, these refinements are tangential.

We seek to find a symmetric pure strategy equilibrium, which is a set of strategies at each node and a set of beliefs that satisfy Bayes's rule on all nodes reached by equilibrium play, in which all landlords of a given type and all tenants of a given type play the same pure strategy, and which has the "stationarity" property that the real offer rent is the same in period 2 as it is in period 1 (i.e., so that offer rents go up at the rate of inflation).

Our parameter choices are given in Table 1, and we remind the reader that our goal is to provide a model whose equilibrium conveys the key ideas, rather than to present a calibrated model. That said, our chosen numbers strike us as reasonable. Taking the average rent in Cleveland, $1000 per month, as an example, total moving costs would be rescaled to $1600 and search cost or the utility penalty to facing renegotiation would be $265. A household would be willing to pay $1000 more per year to live in the preferred structure type, and—as we shall see—an aggressive landlord will offer a rent increase of about $130 more per month.

Throughout, we adopt some tie-breaking rules. First, any tenant receiving a renegotiated rent offer who is indifferent between the choices will choose to accept the offer, and any tenant facing an offer rent who is indifferent between that offer rent and the outside option will choose to accept the offer rent. Second, any landlord receiving a renegotiated rent offer who is indifferent between accepting the offer and rejecting the offer will accept it. Third, any tenant who believes that the prevailing real offer rent is r^* will inform the matching mechanism that the search should be restricted to units with real offer rents less than or equal to r^*. Fourth, tenants who expect to gain nothing from playing wait and see will announce that they are vacating the unit. Finally, if the expected benefit to searching is zero, a tenant will not search.

With these parameter choices, a symmetric pure strategy equilibrium exists.

4.2 Demonstration of the equilibrium

This subsection provides all the propositions; longer proofs are relegated to the appendix. Outside options play a key role in games of asymmetric information. In this paper, these options determine equilibrium offer rents, and the proposals made by either party when renegotiation ensues. For example, it is trivial to prove that if negotiation ensues and Myerson's dictator selects the tenant to make the offer, the tenant will propose a contract of $P(\varepsilon + k_m)$, the landlord's outside option. The tenant's outside options likewise pin down the landlord's proposal, although proving this is more involved. We provisionally assume that tenants stick with their preferred structure type, and that there are always more searchable units than searching tenants at every stage of the game (i.e., $h < v$ at all times). Later we prove that these assumptions hold in equilibrium.

Lemma 1 (Rejection of inferior units in stage 1) Assume $h < v$. In either period, tenants who search a unit and obtain a valuation $z < 700$ will not sign a lease on the unit, but will instead enter the secondary market.

Proof: For a tenant entering the secondary market who restricts his or her search to the preferred structure type, the expected value of z is 630, and (by our symmetry restriction) all real offer rents in equilibrium are the same, r_t^{offer}. Thus, the secondary market delivers a higher return in expectation. (A unit of the non-preferred structure type has an expected z value of 530, which also dominates the current z.) This refers to the static return; however, dynamic considerations do not overturn it. If signing a lease on the searched unit in the first period dominates playing the outside option, any such benefits could equally be obtained, in expectation, from signing a lease on the secondary market; there is no inherent advantage to signing a lease in the first stage rather than the second stage of contracting, since landlords cannot distinguish between first and second stage matches.■

Proposition 1 (The value of r^{offer}) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type. Then for either structure type, and either time period, a real offer rent r_t^{offer} of 1130 is a symmetric Nash equilibrium: given that all other landlords set their real offer rent at 1130, and given that tenants believe the prevailing real offer rent is 1130, it is the best response of each landlord to also set his or her real offer rent at 1130. The beliefs of tenants at this node are specified to be that the prevailing real offer rent is 1130, consistent with this equilibrium. With a prevailing real offer rent of 1130, tenants who search a unit and draw $z \geq 700$ will sign a contract on the unit, and the secondary market is operative.

Proof: See Appendix.

The real offer rent of 1130 is the unique rent such that the expected return from entering the secondary market matches the outside option; higher equilibrium rents would rule out the secondary market, and can be ruled out as potential equilibria, at least if the total measure of units exceeds 1.1. The beliefs restriction mainly serves to ensure that a *unique* pure strategy equilibrium exists.

Establishing the value of the renegotiated rent offer of the landlord is straightforward but considerably more involved. Lemma 2 conjectures a particular value *r'* for this rent, and establishes the equilibrium tenant response (rejection or acceptance) to this conjectured offer. Given the real rent offer *r'* of 1280, tenants who value the current unit at 900 will stay, tenants who value the current unit at 700 will stay unless they locate another unit valued at 900, and others will move. Lemma 3 focuses on equilibrium search behavior and some immediate implications. With no alternative unit in hand, active search yields E(Z) = 699 (implying a *net* benefit of 699-25 = 674), while passive search yields E(Z) = 644; thus all tenants actively search to begin with. However, one's options matter. Passive search is optimal for a tenant residing in a unit valued at 700 who expects a renegotiated rent offer of $1280P_2$. Finally, Proposition 2 verifies the conjectured renegotiated rent offer. An offer of $1280P_2$ is profit-maximizing, balancing additional revenue against the loss of some tenants.

Lemma 2 (Accept or reject landlord offer) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type. Consider a tenant who has entered the negotiation process, and suppose that the landlord has chosen to negotiate rather than offer the same contract, and suppose that Myerson's dictator has selected the landlord to make the offer. If the landlord makes an r_2' offer of 1280 (in real terms), then:

a) tenants who value the current unit at 900 will accept the offer and stay.
b) tenants who value the current unit at 700 will accept the offer and stay, unless they have located a unit with valuation 900.
c) all other tenants will reject the offer.

Proof: see Appendix.

Lemma 3 (Active versus passive search) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type.

Lemma 3a: Consider a tenant who has entered the negotiation process and who values the current unit at 700. If this tenant expects the landlord to make an r_2' offer of $1350P_2$ or greater, this tenant will actively search. If this tenant expects the landlord to make an r_2' offer of $1280P_2$, this tenant will passively search.

Lemma 3b: If all agents actively search in the initial pre-period, then in equilibrium in the first period, 13% reside in units valued at 900, 78% reside in units valued at 700, 6% reside in units valued at 500, and 3% reside in units valued at 200.

Lemma 3c: Let E(Z|active,ncuo) denote the expected value of Z, given that the tenant searches actively and has no current unit option (either is not in a unit presently, or will be vacating said unit), and define E(Z|passive,ncuo) in like manner. Then E(Z|active,ncuo)=699 and E(Z|passive,ncuo)=644.

Proof: see Appendix.

Corollary 1 (Active search initially) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type. Then all tenants actively search in the initial pre-period.
Proof: At worst, a matched tenant must move and enter the secondary market at the end of the first period. If we can establish that even in this pessimistic scenario, a tenant would actively search in the initial pre-period (even though the benefits of search would dissipate after one period), we are done.

In this scenario, tenants would pay the offer rent in the first period, then pay relocation costs plus the offer rent in the second period (and possibly, incur search costs again, if that is optimal). Ignoring costs paid with certainty, the benefit to active search equals 699-25 = 674. Passive search conversely yields 644.■

Proposition 2 (The value of *r'*) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type. Then the landlord's renegotiation rent offer r_2' equals $1280P_2$.
Proof: see Appendix.

We next establish some properties of the equilibrium behavior of the unlucky first-period tenants, those who ended up in a unit that they value at 500 or 200. If they expect renegotiation to ensue, both types will agree to enter the process, hoping to be selected by Myerson's dictator – though if not so selected, both types will reject the renegotiated rent offer of the landlord. However, if they expect the landlord to offer them the same contract, behavior diverges. Tenants who value the current unit at 200 will not stick around. But a tenant who values the current unit at 500 will accept the offer and stay. Even though such a tenant is likely to find a more suitable unit if she searches, the real rent reduction and the avoidance of search and moving costs prompts her to accept the landlord's offer. This accords with our intuition: retention will be enhanced if the landlord offers the same contract, at the cost of lower revenue per tenant. The proof consists of checking numerous inequalities.

Lemma 4 (Wait and see, or vacate I) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type. Then tenants who value the current unit at 500 will choose to wait, rather than to announce that they will vacate the unit. If such a tenant receives an offer of the same contract from his landlord, he will accept the offer. If not, he will actively search in preparation for the negotiation process. Tenants who value the current unit at 200 and anticipate that the landlord will renegotiate the contract will also choose to wait, rather than to announce that they will vacate the unit, and will actively search. However, if such a tenant anticipates that the landlord will offer the same contract, she will announce that she intends to vacate the unit.
Proof: see Appendix.

Corollary 2 (Wait and see, or vacate II) Assume that $h < v$. Suppose that there is a tenant who prefers the structure type *A*, but is nonetheless in a structure of type *D*, and values the current unit at 400. If this tenant anticipates receiving an offer of the same contract, she would immediately vacate the unit, rather than waiting.
Proof: The net benefit to accepting the same contract is 400–(1130/1.03) = <u>–697</u>, which is smaller than the benefit to immediately vacating the unit and actively searching in the preferred market. ■

Proposition 3 (Sticky rents) Assume that a) $h < v$, and b) tenants restrict themselves to their preferred structure type. Then when given the opportunity to make this decision, small landlords play it safe and offer their tenants the same contract; large landlords instead renegotiate the contract.

Proof: Landlords compare the expected returns to negotiating, versus the expected returns to offering the same contract. The return function differs across landlord types, however: Given the law of large numbers, large landlords maximize the logarithm of expected outcomes, while small landlords maximize the expected return across various log outcomes. By Lemma 4, a tenant who values the unit at 500 will accept an offer of the same contract; but by Lemma 2, such tenants will refuse a renegotiated offer of $1280P_2$.

At the end of period 1, and normalizing revenue terms by P_2, the small landlord compares the following two options:

Negotiate:

$$p_{900}\{\chi\ln(r'-k_m)+(1-\chi)\ln\varepsilon\} + p_{700}\{\chi(1-q\mu_4)\ln(r'-k_m)+[1-\chi(1-q\mu_4)]\ln\varepsilon\} + (p_{500}+p_{200})\ln\varepsilon = \underline{6.78}.$$

Play it safe:

$$(p_{900}+ p_{700}+ p_{500})\ln\{(r^{offer}/[1+\iota]) - k_m) + p_{200}\ln\varepsilon = \underline{6.88}$$

where p_{900} refers to the probability that the tenant values the unit at 900 (i.e., 13%), etc. Landlords who negotiate retain all tenants who value the unit at 900, but Lemma 3, tenants who value the unit at 700 and face renegotiation passively search, and vacate if they find a superior unit, which occurs with probability $q\mu_4$. Thus the probability of obtaining second period revenue of $(r' - k_m)$ from such tenants is given by $\chi\left(1-q\mu_4\right)$.

Similarly, but owing to the law of large numbers, the large landlord compares the following two options:

Negotiate:

$$\ln(p_{900}\{\chi(r'-k_m)+(1-\chi)\varepsilon\} + p_{700}\{\chi(1-q\mu_4)(r'-k_m)+[1-\chi(1-q\mu_4)]\varepsilon\} + (p_{500}+p_{200})\,\varepsilon) = \underline{6.92}.$$

Play it safe:

$$\ln[(p_{900}+ p_{700}+ p_{500})\{(r^{offer}/[1+\iota]) - k_m) + p_{200}\varepsilon] = \underline{6.90} \quad \blacksquare$$

It is trivial to verify that given a sufficiently large inflation rate, playing it safe would no longer be the small landlord's preferred option.

We finally return to establishing two properties which, up until now, have merely been conjectures: tenants stick to their preferred structure type, and there are always more searchable units than searching tenants. Per-unit revenue is higher in the A market, hence the equilibrium measure of vacant units is larger in the A market.

Proposition 4 (Tenants stick to their preferred structure type) If $h < v$, all tenants stick to their preferred structure type in equilibrium. The beliefs of landlords at this node are consistent with this equilibrium.
Proof: see Appendix.

Proposition 5 ($h < v$) There exists an equilibrium in which the measure of type A units is 1.26, and the measure of type D units is 1.11, so that $h < v$.
Proof: see Appendix.

This completes the characterization of the equilibrium. The results are now collected and summarized.

4.3 Summary description of the equilibrium

In the initial pre-period, all tenants search actively, since a well-suited unit is so valuable. In either period, offer rents equal 1130 in real terms, consistent with tenant expectations. Tenants only search for units of their preferred structure type. In the pre-period, those who find a unit with valuation 700 or 900 will sign a lease; all others recognize that any valuation below 700 falls beneath the expected value of a "sight unseen" unit, prompting those tenants to enter into the secondary market. Most tenants (91%) end up in a unit that they value at 700 or 900; 6% end up in units that they value at 500. At the end of period one, in the small landlord "D" market, all landlords play "offer same contract," and this behavior is correctly anticipated by all tenants. In this market, even tenants who value the current unit at 500 will plan on staying, although those who value it at 200 announce that they will vacate the unit, and search actively. Conversely, in the large landlord "A" market, everyone knows that all landlords will play "negotiate." Tenants recognize that landlords chosen by Myerson's dictator will offer a rent of $1280P_2$, and this prompts tenants who value their unit at 700 to search passively, while tenants who value the unit at 500 or 200 are prompted to search actively. Still, *all* tenants play "wait" in hopes of getting a lucky Myerson draw – since if selected by Myerson, they offer the landlord his outside option, a mere 330 (the vacancy return plus maintenance). If Myerson selects the landlord to make the offer, the following tenants accept the offer: those who value the current unit at 900, and those who value the current unit at 700 *and* did not locate another unit valued at 900. Thus, the majority of tenants stay, but turnover is higher in this market than in the D market. In equilibrium, the higher per-tenant profit enjoyed by large landlords prompts more entry, so the excess supply of units is of measure 0.27 in the A market, while it is only 0.11 in the D market. (We once again remind the reader that no attempt was made to calibrate parameters.)

5. Discussion and Conclusions

We have here produced the first explanation of rent stickiness grounded in theory, which builds upon key aspects of the rental market: while turnover is costly for both the tenant and the landlord, the landlord has no way of knowing how much the tenant likes the unit. Our theory is intended to explain two key facts in the data: rents for housing units are remarkably sticky despite the fact that contracts must be renewed, whether rent changes or not. Furthermore, rents for units in larger buildings, which are more likely to be managed by large landlords, are much more flexible than those in single-family structures. The intuition of the theory is straightforward. Landlords who are very concerned about tenant retention – namely, landlords with few rental units, for whom the loss of a single tenant means proportionally large vacancy and search costs – will be much more willing to offer the same rent as they did in their previous contract, in hopes of pre-empting tenants' search and enhancing retention. In times of inflation, a fixed nominal rent means that real rents drop. Landlords for whom the loss of a single tenant is not a big deal—in that all such losses merely represent a fraction of the total potential revenue from the entire complex—will instead raise the rent in excess of the inflation rate, counting on the moving costs of tenants to keep most tenants in place.[19] But occasionally they have to reduce the rent, consistent with the empirical evidence presented in Table 2.

Our theory is necessarily stark and leaves out aspects of rental markets that might be important for generating rent stickiness. The leading contender is the so-called "good tenant" explanation, in which landlords prefer to retain good tenants who do not impose high maintenance and repair costs and who pay their rent on time. Importantly, this is particularly true of small landlords. But the "good tenant" explanation cannot, by itself, explain the rent change

[19] Indeed, the incentive for large tenants to raise rents upon contract renewal helps explain why, upon occasion, some large landlords advertise and commit themselves to maintaining the rent unchanged for a certain number of years. For example, in 2012, the Privacy World at Glenmont Metrocentre apartment complex in Wheaton, MD advertised "Same Rent till 2016, Utilities Included."

patterns we document. This type of information is revealed relatively quickly. Consider a landlord now realizes that she has a bad tenant. When the rental contract expires, this landlord will insist upon a large rent increase, reflecting the higher costs. Under these circumstances, many tenants would not renew the lease, so we would rarely observe such large rent increases in equilibrium. Any good tenant, meanwhile, will receive a discount; and this discount would be larger for small landlords. Hence, the "good tenant" explanation can explain rent changes that are smaller than the inflation rate. But it fails to explain the extreme rigidity of rents, it fails to explain the small-versus-large landlord rigidity distinction observed in the data, and it fails to explain (at least on its own) the fact that rent discounts are less likely in detached units (see Gallin and Verbrugge 2017)

One might also argue that the key difference between rents offered by small and large landlords is due to differences in renters. Specifically, moving costs are arguably higher for people who rent single-family homes rather than apartments in large complexes, perhaps because they have more material belongings or are more likely to have children. However, this fact actually deepens rather than explains the differential rent stickiness puzzle, since smaller landlords—who are more likely to be renting out their starter home—would find it accordingly *easier* to raise rents. Something powerful must be at play to keep so many rents *exactly* unchanged.

However, our goal here was not to produce a model of the rental market that includes all plausible determinants of rent setting. Instead, we have focused on how landlord risk aversion, the size of landlords' rental stock, and moving costs interact in a search model with incomplete information and bargaining to generate the pronounced rent rigidity in rental markets in general and among smaller landlords in particular. One can consider our model a proof of concept. Future work should focus on generalizing our result and bringing a model to grips with the data in a more complete fashion.

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Appendix

Proof of Proposition 1:

We first establish that the secondary market is operative, given a prevailing real offer rent of 1130. Tenants entering the secondary market receive an expected value of *z* equaling 630 (yielding expected utility of $630+\frac{Y-R_t^{offer}}{P_t}$), while the outside option yields $\frac{Y}{P}-500$; hence, the tenant ultimately compares –500 to $630-R_t^{offer}/P_t$. Hence the condition for tenants to accept an offer in the secondary market is $R_t^{offer}/P_t \leq 1130$.

If $r_t^{offer}=R_t^{offer}/P_t=1130$, all tenants who search a unit and draw $z \geq 700$ will sign a contract – since entering the secondary market yields no advantage to these tenants. Landlords do not have an incentive to deviate from setting r^{offer} = 1130. Why? First, since a landlord cannot discriminate between primary and secondary market leases, but must post one offer rent, if the prevailing real rent is 1130, there is no advantage to an individual landlord setting his real offer rent *below* 1130, since this does not increase the probability of a match, but does decrease revenue from a match: Since tenants expects a real offer rent of 1130, none will specify $r_t^{offer} \leq r*$ for any $r*$ < 1130 as a matching criteria (since this is expected to result in no matches). Second, there is no advantage to setting a real offer rent above 1130 either, since tenants each specify $r_t^{offer} \leq 1130$ as a matching criteria. ■

Proof of Lemma 2 The net benefit received by a tenant who relocates to another unit (the preferred outside option) is given by

$$E(z'')-k_r-R_2^{offer}/P_2=E(z'')-1280$$

where $E(z'')$ denotes the expected valuation on the alternative unit. The net benefit to a tenant who remains in the unit and accepts an offered rent R_t^{offer} of $1280P_2$ is given by

$$z^{\text{current unit}}-1280$$

Thus, with a postulated real rent offer of 1280, tenants merely compare the value of the current unit to the (known or expected) value of the best alternative unit. Of course, any searched unit yielding 200 or 500 will be rejected by the tenant, since the secondary market yields an expected valuation of 630. It is trivial to establish that, given a real rent offer of 1280:

a) A tenant who values the current unit at 900 and has located a new unit with valuation 900 is indifferent between moving and staying; all other tenants who value the current unit at 900 are better off staying.
b) A tenant who values the current unit at 700 and has located a new unit with valuation 900 prefers to move. Conversely, a tenant who values the current unit at 700 and has located a new unit with valuation 700 is indifferent between moving and staying, and a tenant who

values the current unit at 700 but has *not* found an alternative unit with valuation at least 700, prefers to stay.

c) All other tenants are better off leaving, even if their best option is the secondary market. ■

Proof of Lemma 3: In the comparison of active versus passive search, the negotiation cost is a sunk cost. For the moment, consider real rent offers that satisfy $1280 \le r' \le 1350$. Facing a real rent offer $1280 < r' \le 1350$, a tenant who values the current unit at 700 will vacate the unit if she finds an alternative unit valued at 900 or 700, but will not vacate it otherwise, given that remaining weakly dominates the expected net return to the secondary market (which equals 630–1280). Hence under these conditions, the expected return to **active** search for an agent of this type, assuming $h < v$, is given by

$$-Pk_s/P+(1-\chi)(700-P(\varepsilon+k_m)/P)+\chi\{\mu_4(900-P(r^{offer}+k_r)/P)+\mu_3(700-P(r^{offer}+k_r)/P) +(\mu_2+\mu_1)(700-Pr'/P)\}$$

For $r' = 1350$, this expression equals

$$-25+0.1(700-330)+0.9(0.1(900-1130-150)+0.6(700-1130-150)+0.3(700-1350)) = \underline{-510.9}$$

The expected return to **passive** search, if $h < v$ and the real r' offer is 1350, is given by

$$0 + (1-\chi)(700-P(\varepsilon+k_m)/P)+\chi\{q\mu_4(900-P(r^{offer}+k_r)/P)+q\mu_3(700-P(r^{offer}+k_r)/P) +((1-q(\mu_4+\mu_3))(700-Pr'/P)\}$$

For $r' = 1350$, this expression equals

$$0.1(700-330)+0.9(0.1(0.2)(900-1280)+0.6(0.2)(700-1280)+(1-(0.2)0.7)(700-1350))= \underline{-535.6}$$

Thus active search is preferred facing an expected real rent offer of 1350.

When the tenant expects the landlord to make an r_2' offer of $1280P$, and $h < v$, then a tenant who values the current unit at 700 would remain in the unit unless he found a unit with a valuation of 900, and thus the tenant compares:

Active search:

$$-25+0.1(700-330)+0.9(0.1(900-1130-150)+0.6(700-1280)+0.3(700-1280)) = \underline{-492}$$

Passive search:

$$0.1(700-330)+0.9(-1280+0.1(0.2)(900)+0.6(0.2)(700)+(1-(0.2)0.7)(700)) = \underline{-481.4}$$

Thus passive search is preferred facing an expected real rent offer of 1280.

For which expected real rent offer r' does active search become preferable? Once r' reaches approximately 1301.03, active search becomes preferable.

Expected rent offers in excess of 1350 would prompt *all* "700" agents to vacate the unit if they were not selected by Myerson's dictator (see proof of Proposition 2). All search actively in this case. Intuitively, all tenants know that they will be vacating the unit anyway, unless they get lucky during the renegotiation. More formally, if we let E(Z|active,ncuo) denote the expected value of Z, given that the tenant searches actively and has no current unit option (either is not in a unit presently, or will be vacating said unit), and define E(Z|passive,ncuo) in like manner, then the relevant comparison in this case is

$$-25+0.1(700-330)+0.9(-1280+E(Z|\text{active,ncuo})) \le 0.1(700-330)+0.9(-1280+E(Z|\text{passive,ncuo})).$$

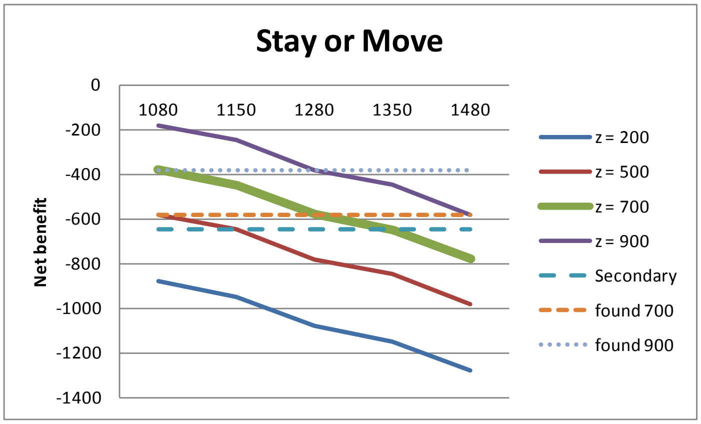
We now compute E(Z|search actively,ncuo) for $h < v$.

By Lemma 1, a tenant who searches a unit and draws an amenity value of 700 or 900 will sign a lease on that unit, and if her valuation is lower, she will enter the secondary market. Thus under active search in the initial matching, 10% of tenants find units with a valuation of 900, 60% find units with a valuation of 700, and 30% – who are initially matched to units of quality 500 or 200 – enter the secondary market. In the secondary market, of the 30% of agents who are in the

market, 10% are matched to units with valuation 900, 60% are matched to units with valuation 700, 20% are matched to units with valuation 500, and 10% are matched to units with valuation 200. Now just make the computations: 10% + 10%(30%) = 13%, 60% + 60%(30%) = 78%, 20%(30%) = 6%, and 10%(30%) = 3%. Similarly, one can show that under passive search, 10.6% locate units valued at 900, 63.6% locate units valued at 700, 17.2% locate units valued at 500, and 8.6% locate units valued at 200. Hence,
E(Z|search actively,ncuo) = .13(900) + .78(700) + .06(500) + .03(200) = 699, while
E(Z|search passively,ncuo) = .106(900) + .636(700) + .172(500) + .086(200) = 644.
As E(Z|search actively,ncuo) – E(Z|search passively,ncuo) > 25, active search is preferred.■

Proof of Proposition 2: From proof of Lemma 2, it is clear that tenants receiving a renegotiated rent offer of r' compare (Ez''-1280=-650) to (z-r'), where z is the valuation on the current unit and Ez'' is the expected valuation on the alternative unit. Ez'' takes on the values 900, 700, or 630, since a tenant may have located another unit with valuation 900 or 700, and otherwise obtains the expected valuation of 630 on the secondary market. The landlord seeks the profit-maximizing rent offer. Denote tenants with z = 200 by "200s", and so on.

The figure below plots (z-r') highlighting five alternative z's, along with the benefits associated with moving under different circumstances: when the individual will be using the secondary market (net benefit of 630–1280 = –650), when the individual has located an alternative unit with z = 700, and when the individual has located an alternative unit with z = 900. Tenants choose the highest-valued option; thus, for example, a "700" who has located a unit valued at 900 will move unless r' is 1080 or less. The landlord choice variable is r'. Only r' between 1080 and 1480 are considered, since (as will become clear) other rent offers are clearly dominated.



Higher r' not only directly provoke some types of tenants to leave, they also – see Lemma 3 – provoke more types of tenants to search actively, increasing the probability of tenant departure. As can be seen, r' = 1280P_2 has the property that this rent offer is the highest rent offer that will be accepted by all "700s" who did not locate a unit valued at 900, and by all "900s"; meanwhile "500s" and "200s" reject this offer. Furthermore, by Lemma 3, if this rent offer is expected, all "700s" search passively rather than actively.

The "700s" comprise 78% of the tenant population, and their behavior is crucial to landlord profits. Landlord expected profits from units under negotiation equal (r'-k_m)(% who stay)+(ε)(% who leave). Given the discontinuities in profits induced by discrete unit valuations (and hence discrete tenant "types"), it suffices to consider six real offer rent possibilities: 1080, 1150, 1280, 1301, 1350, and 1480.

- Facing a real rent offer of 1080, all "900s" and "700s" accept the offer, as well as all "500s" who did not locate another unit with $z = 900$. However, counterfactually assume that all "500s" stay, which will overstate profits; in this case, 97% of all tenants accept this offer, yielding expected landlord profits of (1080-80)*(.97) + (250)(.03) = 977.5. (Even if all renters had been retained, expected profits would only have been 1000.)
- Facing a real rent offer of 1150, all "900s" stay, all "700s" who did not locate a $z = 900$ unit stay, and all "500s" who did not locate a $z \geq 700$ unit stay. Expecting this offer rent, "700s" passively search; under the assumption that all "500s" passively search (which again possibly overstates landlord profits), then those who reject are: all "200s" (3%); "500s" who find units such that $z \geq 700$ ($(0.06)q(\mu_4+\mu_3) = (0.06)0.2(0.7) = 0.84\%$); and all "700s" who locate $z = 900$ units ($(0.78)q(\mu_4) = (0.78)0.2(0.1) = 1.56\%$). Thus 1-0.03-0.0084-0.0156 = 94.6% stay, yielding expected landlord profits of (1150-80)(0.946) + 250*(0.054)=1025.7.
- Facing a real rent offer of 1280, all "900s" stay, all "700s" who did not locate a $z = 900$ unit stay, and all "500s" leave. Expecting this offer rent, "700s" passively search. Those who reject are: all "200s" and "500s" (9%); and all "700s" who locate $z = 900$ units ($(0.78)q(\mu_4) = (0.78)0.2(0.1) = 1.56\%$). Thus 1-0.09-0.0156 = 89.44% stay, yielding expected landlord profits of (1280-80)(0.8944)+250*(0.1056)=1099.7.
- Facing a real rent offer of 1301, counterfactually assume that all "900s" stay, but now all "700s" who did not locate a unit such that $z \geq 700$ stay, and all "500s" leave. Expecting this offer rent, "700s" passively search. Those who reject are: all "200s" and "500s" (9%); and all "700s" who locate $z \geq 700$ units ($(0.78)q(\mu_4+\mu_3) = (0.78)0.2(0.7) = 10.92\%$). Thus 1-0.09-0.1092 = 80.08% stay, yielding expected landlord profits of (1301-80)(0.8008))+250*(0.1992)=1027.6.
- Facing a real rent offer of 1350, counterfactually assume that all "900s" stay; as in the r'=1301 case, all "700s" who did not locate a unit such that $z \geq 700$ stay, and all "500s" leave. Expecting this offer rent, "700s" actively search. Those who reject are: all "200s" and "500s" (9%); and all "700s" who locate $z \geq 700$ units ($(0.78)(\mu_4+\mu_3) = (0.78)(0.7) = 54.6\%$). Thus 1-0.09-0.546 = 36.4% stay, yielding expected landlord profits of (1350-80)(0.364)+250*(0.636)=621.3.

Triggering active search by "700s" is extremely costly to landlords, and losing "700s" to alternative units with z=700 is also extremely costly to landlords. Setting $r' = 1280P_2$ is the highest proposed rent level that avoids both of these consequences. For proposed real rents above 1350, no "700s" stay, leaving landlords with only some of the "900s" – only 13% of the population. Even counterfactually retaining all such tenants at a real rent of 1480 only yields expected revenues from renters of (1480-80)*(0.13) = 182, with total profits of 399.5. All tenants reject rent offers above $1550P_2$; even if all "900s" stayed at that rent, total profits would only be 191.1+217.5=408.6.

Clearly, then, landlords offer = $1280P_2$. ■

Proof of Lemma 4 A tenant who announces that she will vacate the unit will either actively or passively search. Lemma 3 has established that E(Z|search actively,ncuo) = 699, and E(Z|search passively,ncuo) = 644. This implies that any tenant who intends to vacate the unit will actively search.

Conversely, a tenant who does not announce that he is vacating the unit, but instead waits, faces two possibilities: either the landlord will offer the same contract, or the landlord will renegotiate. As in the proof of Proposition 2, define a "900" to be a tenant who values the current unit at 900, and so on. A "500" who receives an offer of the same contract will obtain a benefit of 500–(1130/1.03) = –597. Rejection and active search would yield a net benefit of E(Z|search actively,ncuo) – k_r – k_s – r^{offer} = 699–150–25–1130 = –606, while rejection and passive search would conversely yield 644–150–1130 = –636. Hence, even though a "500" is likely to find a more suitable unit if she searches, the real rent reduction and the avoidance of search and moving costs prompts her to accept this offer. Conversely, a "200" who receives an offer of the same contract will obtain a benefit of 200–(1130/1.03) = –897, which is even lower than the expected return from entering the secondary market. This implies that a "200" who believes his landlord will offer the same contract has no incentive to wait and see.

A tenant who does not receive an offer of the same contract must incur a renegotiation cost of 25, and then decides whether to actively or passively search, in the hope that Myerson's dictator chooses him to offer the contract. If Myerson's dictator selects the tenant to make the offer, an event with probability 0.1, a "500" ("200") will stay and obtain a net benefit of 500-330 (200-330). If Myerson selects the landlord to make the offer, then by Lemma 2 and Proposition 2, the tenant is going to reject the offer and move (either to another unit he has located, or to the secondary market). As has been noted above, active search is preferred when there is no current unit option.

Hence, regardless of beliefs, a "500" will play wait and see. A "200" will vacate the unit if she believes she will be offered the same contract; but if she believes the landlord will renegotiate, she will play wait and see, prepare for negotiation and hope that she is selected by Myerson's dictator.■

Proof of Proposition 4 Let $V_{900,D,D}$ denote the value function of a tenant who during period one is matched with a unit of type D with an amenity value of 900, and who will remain within this structure type during period 2, with $V_{900,A,A}$, $V_{900,D,A}$, $V_{900,A,D}$, $V_{800,D,A}$, … defined analogously. Given the landlord and tenant behavior specified in the preceding Propositions and Lemmata, and putting all expressions in real terms, the following hold:

$$V_{900,A,A} = (900-1130) + \beta\left(.1(900-330) + .9(900-1280) - 25\right) = -525$$

$$V_{700,A,A} = (700-1130) + \beta\left(.1(700-330) + .9\left[.02(900-1280) + .98(700-1280)\right] - 25\right) = -911$$

$$V_{500,A,A} = (500-1130) + \beta\left(.1(500-330) + .9\left[\Sigma-1280\right] - 25 - 25\right) = -1158$$

$$V_{200,A,A} = (200-1130) + \beta\left(.1(200-330) + .9\left[\Sigma-1280\right] - 25 - 25\right) = -1487$$

$$V_{900,D,D} = (900-1130) + \beta\left(900 - \frac{1130}{1.03}\right) = -417$$

$$V_{700,D,D} = (700-1130) + \beta\left(700 - \frac{1130}{1.03}\right) = -807$$

$$V_{500,D,D} = (500-1130) + \beta\left(500 - \frac{1130}{1.03}\right) = -1197$$

$$V_{200,D,D} = (200-1130) + \beta\left(\Sigma - 1130 - 150 - 25\right) = -1505$$

where Σ= E(Z|search actively,ncuo) = 699.

The second period rental costs facing a tenant dwelling in a structure of type *D* in the first period are lower than those of a tenant dwelling in structure type *A*. As long as $h < v$, a tenant who prefers structure type *D* has no incentive to enter the *A* market in either period. Furthermore, a tenant dwelling in structure type *A* in the first period has no incentive to enter the type *D* market in the second period. Thus, we need only consider the possibility that tenants who prefer *A* nonetheless enter the *D* market in period 1, in order to capitalize on the expected rent stickiness. By Lemma 2 and its corollary, tenants who inhabit a structure with amenity value 400 or less will re-enter the *A* market in period two. The value functions of type-*A* tenants who enter the *D* market in period 1 are as follows:

$$V_{800,D,D} = (800-1130)+\beta\left(800-\frac{1130}{1.03}\right) \qquad = -612$$

$$V_{600,D,D} = (600-1130)+\beta\left(600-\frac{1130}{1.03}\right) \qquad = -1002$$

$$V_{400,D,A} = (400-1130)+\beta(\Sigma-1130-150-25) \qquad = -1306$$

$$V_{100,D,A} = (100-1130)+\beta(\Sigma-1130-150-25) \qquad = -1606$$

The second-period rent benefits don't outweigh the utility cost. Thus if $h < v$, tenants stick to their preferred structure type. ■

Proof of Proposition 5 The structure of the proof is as follows. First, we first derive expressions for equilibrium profit under the assumption that $h < v$. Next we compute the value of a potential landlord's outside option, given the cost of entry. Next we verify that the zero-net-benefit-to-entry condition implies that sufficient entry occurs so that the total number of units is sufficiently large so that $h < v$ at every stage of the game.

We introduce some notation for the proof. Let $\#_{i,j}$ denote the *number* of units in the matching process in stage $j \in \{1,2\}$ of period $i \in \{1,2\}$. (Thus, for example, $\#_{1,1}$ is the total number of units in existence.) Ultimately we must determine $\#_{1,1}$ for each structure type.

1. We start with the **market for type A units** (offered by large landlords). Since tenants who value the current unit at 900 do not search, and tenants who value the current unit at 700 search passively, we can deduce the following relationships:

$\#_{1,2} = \#_{1,1} - 0.7$ (70% of households find a unit valued at 900 or 700, and do not enter the secondary market.)

$\#_{2,1} = \#_{1,1} - 1$ (given equilibrium behavior when $h<v$, every unit in this market which is occupied in period 1 (totaling mass 1) is not searchable and not on the market.)

$\#_{2,2} = \#_{2,1} - 0.7(0.09) - 0.1(0.156)$ (all tenants who value their period-1 unit at 200 or 500, who comprise 9%of the households, search actively and thus locate units with probability one; of these units, 70% are valued at 700 or 900 and thus these households do not enter the secondary market. Similarly, all tenants who value their period-1 unit at 700 search passively, so that $0.78q = 0.156$ tenants of this type are searching. 10% of these find a unit valued at 900.)

Landlords receive ε from vacant units, and from units occupied by tenants who were selected by Myerson's dictator, which are a fraction $(1-\chi)$ of all occupied units. The return to a large landlord in equilibrium is given by the expression

$$\ln E\pi_1 + \beta \ln E\pi_2 = \ln\left[Y\varepsilon + (1-Y)\left(r^{off} - k_m\right)\right]$$
$$+\beta \ln\left[A\chi\left(r' - k_m\right) + \left((1-Y)(1-\chi)+Y\right)\varepsilon + (1-A-Y)\chi\left(r^{off} - k_m\right)\right]$$

where Y = vacancy rate (probability that a unit is vacant in the first period), given by $(\#_{1,1}-1)/\#_{1,1}$, and A (which represents the probability that a unit is occupied either by someone who values the current unit at 900, or by someone who values the current unit at 700 and did not find another unit valued at 900) is given by the expression

$$A = \left(\frac{1}{\#_{1,1}}\right)\left(0.1 + 0.6(1-0.1q)\right) + \left(\frac{\#_{1,2}}{\#_{1,1}}\right)\left(\frac{0.3}{\#_{1,2}}\right)\left(0.1+0.6(1-0.1q)\right)$$
$$= \left(\frac{1}{\#_{1,1}}\right)\left(0.13 + 0.78(1-0.1q)\right)$$

where the second part of this expression indicates refers to tenants who were matched to this unit on the secondary market. These are compared to the alternative (where optimal consumption behavior and a zero real interest rate are assumed):

$$\text{(Return to cash):}\quad \ln c_1 + \beta \ln c_2 = \ln\left(\frac{1}{1+\beta}\delta_A\right) + \beta \ln\left(\frac{\beta}{1+\beta}\delta_A\right),$$

which for d_A = 1800 is given by 13.27. At this level of d_A, then if $\#_{1,1}$ = 1.26, equilibrium profits total 13.27; the marginal landlord would be indifferent to creating another unit. When $\#_{1,1}$ = 1.26, $h < v$ for every stage of the matching process in this market, as postulated.

2. We now consider the **market for type *D* units** (offered by small landlords). The return is given by

$$E\ln\pi_1 + \beta E\ln\pi_2 = Y\ln\varepsilon + (1-Y)\ln\left(r^{off} - K_m\right)$$
$$+\beta\left[A'\ln\left(\frac{r^{off} - K_m}{1+\iota}\right) + Y\ln\varepsilon + (1-A'-Y)\ln\left(r^{off} - K_m\right)\right]$$

where $A' = \left(1/\#_{1,1}\right)0.91$. In this market, in equilibrium tenants who draw an amenity value of 200 in period 1 will announce that they will vacate the unit in period 2; all others will remain in the unit. The return to cash is no different for small landlords: 13.27. If $\#_{1,1}$ = 1.109, equilibrium profits total 13.27; the marginal landlord would be indifferent to creating a unit and entering. When $\#_{1,1}$ = 1.109, $h < v$ for every stage of the matching process in this market, as postulated. ■

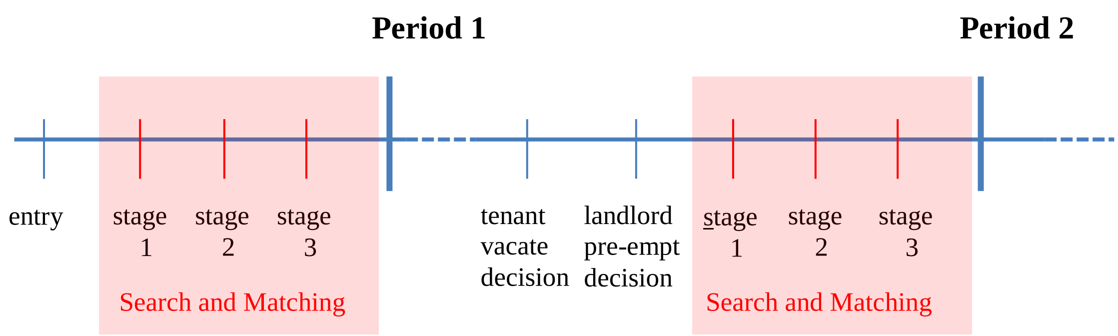


Figure 1: The Timing of Play

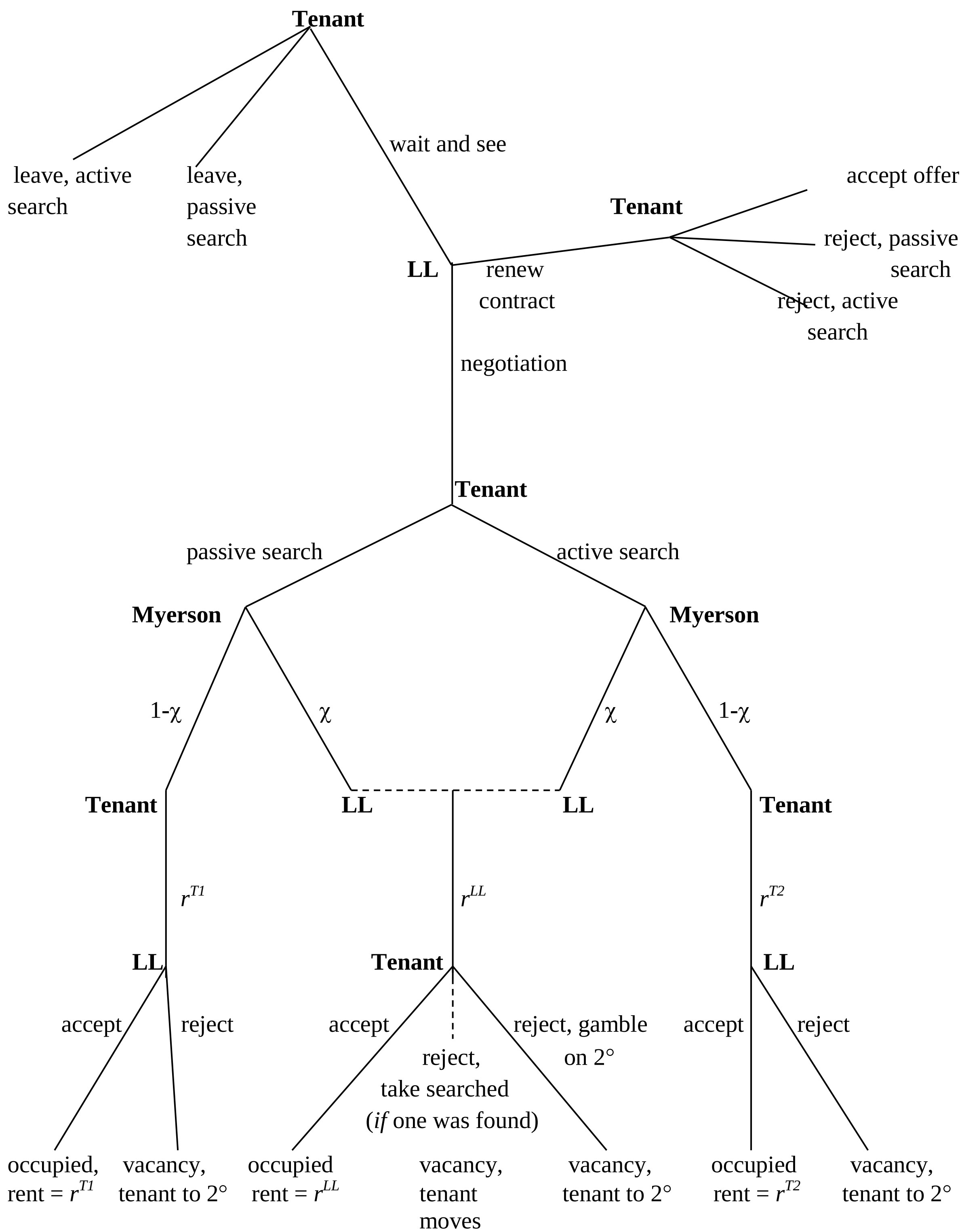


Figure 2: The Rent Renegotiation Process