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SEASONALITY AND EQUILIBRIUM BUSINESS CYCLE THEORIES

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ABSTRACT

Barksy-Miron [1989] find that the postwar U.S. economy exhibits a regular seasonal cycle, as well as the business cycle phenomenon. Are these findings consistent with current equilibrium business cycle theories as surveyed by Prescott [1986]? We consider a dynamic, stochastic equilibrium business cycle model which includes deterministic seasonals and nontime-separable preferences. We show how to compute a perfect foresight seasonal equilibrium path for this economy. An approximation to the stochastic equilibrium is calculated. Using postwar U.S. data, GMM estimates of the structural parameters are employed in the perfect foresight and simulation analyses. As in Constantinides and Ferson [1990], the estimates of consumption preferences exhibit habit-persistence, but a local optimum also exists which exhibits local durability.

The nontime-separable model predicts most of the seasonal patterns found in aggregate quantity time series; notable exceptions are the seasonal patterns in investment and the fourth quarter seasonal in labor hours. An evaluation of the model's predictions for deseasonalized second moments finds strong support for the parameterization with local durability in consumption. This model broadly displays a seasonal cycle as well as the business cycle phenomenon.

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1. Introduction

The postwar U.S. economy exhibits a regular seasonal cycle, as well as the business cycle phenomenon: this is the principal finding by Barsky and Miron [1989]. These researchers analyze aggregate data which has not been adjusted for seasonality and find that deterministic seasonals account for between 50 - 95% of the variation in the growth rates of aggregate quantity variables such as GNP, consumption, and investment. Are these findings consistent with current equilibrium business cycle theories as surveyed by Prescott [1986]? Prescott concludes that variations in the rate of technological change are an important source of economic fluctuations, accounting for about 70% of cyclical fluctuations (Kydland and Prescott [1989]). Theory predicts cyclical fluctuations, but does it also predict seasonal cycles? Answering this question is a potentially important step in assessing the validity of equilibrium theories. The similarities between the seasonal cycle and the business cycle suggest that the economic mechanism generating business cycle fluctuations also generates seasonal cycles. Consequently, a proper theory should predict seasonal cycles as well as business cycles.

We consider a dynamic, stochastic equilibrium business cycle model which includes deterministic seasonals. Our seasonal specification is parsimonius: we include only a technology seasonal, a preference seasonal, and a government spending seasonal. As in Kydland and Prescott [1982], Eichenbaum, Hansen, and Singleton [1989], and Braun [1990], preferences exhibit nontime-separability in consumption goods and leisure. This model is tractable. In particular, we show how to compute a perfect foresight seasonal equilibrium path for this economy. An approximation to the stochastic equilibrium is also calculated around this equilibrium path. Using a Generalized Method of Moments (GMM) estimator, the model's structural parameters and the seasonals are estimated using postwar U.S. data. The overidentifying restrictions of the model are not rejected at conventional levels; and the technology seasonal estimates indicate a strong seasonal pattern, sufficient to drive an equilibrium seasonal cycle. As in Constantinides and Ferson [1990], our GMM estimation strategy finds evidence of habit-persistent preferences for consumption goods.

Given the parameter estimates, the seasonal patterns and business cycle properties of the equilibrium model accord well with the data. First, the model replicates many of the seasonal patterns in the aggregate data, particularly for output, consumption, capital, average labor productivity, and the real interest rate. The principal shortcomings are with respect to third and fourth quarter investment, and fourth quarter labor hours. The model captures the large contribution of deterministic seasonals for the total variation in most aggregate variables; the principal shortcomings are the overstated contributions for the real interest rate and labor hours. Second, in this model seasonal variation in technology is essential to produce seasonal patterns in output. Seasonal variation in preferences and government purchases alone generate output seasonals no larger than 0.2% per quarter. Third, without nontime-separabilities in preferences, seasonal variation in output, hours, and investment is much too large. Habit-persistent preferences for leisure are an important element in the model's ability to match the magnitude of seasonal variation in aggregate hours. Interestingly, habit-persistent preferences for consumption are not important for matching the seasonal moments, and actually detract from the model's ability to match the cyclical second moment properties of the data. The model's fit is greatly enhanced when local durability in consumption is considered. The evidence presented here complements research by Heaton [1988], Singleton [1990], Gallant-Tauchen [1990], and Gallant-Hansen-Tauchen [1990], who find empirical support for local durability when considering the asset pricing implications of representative agent models.

Does theory predict seasonal fluctuations as well as cyclical fluctuations? The equilibrium model with nontime-separable preferences displays the seasonal patterns emphasized by Barsky and Miron [1989] as well as the business cycle phenomenon. The fact that the model successfully offers predictions across both business and seasonal cycle frequencies is perhaps its greatest strength.

The paper is organized as follows. Section 2 presents the model economy which includes seasonality and exogenous growth, and the perfect foresight seasonal equilibrium path is defined. Section 3 presents the GMM estimation strategy, describes the data, and discusses the structural parameter estimates. Section 4 analyzes the perfect foresight seasonal equilibrium paths implied by the parameter estimates. Section 5 presents and analyzes the simulation results for the stochastic economy with seasonality. Section 6 offers conclusions.

2. An Equilibrium Business Cycle Economy with Seasonality

This section presents a one-sector, real business cycle economy which is subjected to seasonal variation in the technology, preferences, and government purchases. The model is similar to the models considered by Christiano-Eichenbaum [1990] and Braun [1990].

2.1 The economy with growth and seasonality

Consider an economy composed of a large number of identical, infinitely-lived households each of which seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tau_t \log c_t^* + \gamma_2 \log l_t^* \right\}, \qquad \gamma_2>0 \qquad [2.1]$$

where c^* and l^* represent consumption and leisure services, respectively. Consumption services are related to private consumption (cp) and public consumption (g) as follows:

$$c_t^* = cp_t + \gamma_1 g_t + a\,(cp_{t-1} + \gamma_1 g_{t-1}), \quad 0 \le \gamma_1 < 1, \ |a|<1 \qquad [2.2]$$

where γ_1 governs the substitutability of public goods for private consumption goods. The parameter a governs the character and degree of nonseparability: if a is negative (positive), consumption goods are complements (substitutes) across adjacent time periods. The complementarity case can also be interpreted as habit-persistence in preferences. The variable τ_t is a deterministic preference seasonal which follows:

$$\tau_t = \tau_1 Q_{1t} + \tau_2 Q_{2t} + \tau_3 Q_{3t} + \tau_4 Q_{4t}, \quad \tau_j>0 \text{ for all } j \qquad [2.3]$$

and the variable Q_{jt} is a dummy variable taking on the value of 1 when period t corresponds to season j, and zero otherwise; consequently, τ_j is the preference seasonal in season j. Leisure (l) is time not devoted to labor (n), leading to the time allocation constraint that $n_t + l_t = T$, where T is the maximum number of hours available per period. Preferences are defined over the leisure service l^*:

$$l_t^* = l_t + b\, l_{t-1} \qquad |b|<1. \qquad [2.4]$$

The parameter b governs the character and degree of nonseparability: if b is negative (positive), then leisure choices are complements (substitutes) across adjacent time periods. Finally, the operator E_t is the mathematical expectations operator conditional on all information known at time t.

Each household has access to a production function of the form:

$$y_t = (k_t^d)^{\theta} (z_t n_t^d)^{1-\theta} \qquad [2.5]$$

where y is output and k^d and n^d are the quantities of capital and labor services demanded by the entrepreneur-household. The household's output can be consumed (privately or publicly) or stored in the form of additional capital next period. Each period, the existing capital stock depreciates at the geometric rate δ. The variable z_t is a labor-augmenting technology shock which includes deterministic seasonal components:

$$z_t = z_{t-1} \exp (\lambda_t) \qquad [2.6]$$

$$\lambda_t = \lambda_1 Q_{1t} + \lambda_2 Q_{2t} + \lambda_3 Q_{3t} + \lambda_4 Q_{4t} + \epsilon_t$$

where ϵ_t is a purely indeterministic, white noise random variable. Notice that log z_t is a random walk with seasonal drift: when the seasonal growth rates λ_j do not sum to zero, this economy experiences growth.

The economy possesses competitive markets in labor and capital services: suppliers of labor services receive a wage w_t, suppliers of capital services receive the rental rate r_t. Finally, the government taxes each household in a lump-sum fashion, TL_t. This leads to the household's period budget constraint:

$$cp_t + k_{t+1} = y_t + (1-\delta)k_t - w_t(n_t^d - n_t) - r_t(k_t^d - k_t) - TL_t \qquad [2.7]$$

where k and n represent the supply of capital and labor services by the household.

The government chooses a stochastic process for g_t which is uncontrollable from the household's perspective. Government purchases are assumed to contain a permanent and a transitory component. The permanent component is related to the technology shock z_t; the transitory component is an autoregressive process of order 1 with a seasonal mean. The stochastic process for g_t is:

$$\log \frac{g_t}{z_t} - \log \tilde{g}_{jt} = \rho \left(\log \frac{g_{t-1}}{z_{t-1}} - \log \tilde{g}_{j-1,t-1}\right) + u_t \ , \ 0<\rho<1 \quad [2.8]$$

where $\tilde{g}_{jt}$ is the seasonal mean of transitory government purchases when period t corresponds to season j; and u_t is an indeterministic, white noise random variable. A specification such as this one, but without seasonality, was adopted in Christiano-Eichenbaum [1990] and Braun [1990].

In this Ricardian environment, we assume without loss of generality that the government's budget constraint is $g_t = TL_t$. This leads to the economy-wide resource constraint (in per capita terms):

$$cp_t + k_{t+1} + g_t = y_t + (1-\delta)k_t - w_t(n_t^d - n_t) - r_t(k_t^d - k_t) \qquad [2.9]$$

When the supply of labor and capital equals the demand for labor and capital (respectively), equation [2.9] is the familiar per capita national income accounting identity for this closed economy. Since all individuals are identical in this economy, each individual's net supply of either factor will be zero, in equilibrium.

As in King-Plosser-Rebelo [1988], an empirical analysis of this economy is facilitated by rescaling the economy in a way which induces a stationary environment. To this end, define the following scaled variables:

$$\tilde{k}_{t+1} = \frac{k_{t+1}}{z_t} \ , \quad \tilde{y}_t = \frac{y_t}{z_t} \ , \quad \tilde{g}_t = \frac{g_t}{z_t} \ , \quad \tilde{i}_t = \frac{i_t}{z_t} \ , \quad \tilde{c}_t^* = \frac{c_t^*}{z_t} \ ,$$

$$\tilde{cp}_t = \frac{cp_t}{z_t} \ , \quad \tilde{w}_t = \frac{w_t}{z_t} \ , \quad \tilde{TL}_t = \frac{TL_t}{z_t}$$

where i_t is gross investment. Under the assumption that the unscaled economy exhibits balanced growth, the scaled variables are stationary; the remaining unscaled variables are leisure services (l^*), labor (n) and the rental rate (r), which are stationary without any rescaling. The household's problem in the scaled economy becomes:

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tau_t \log \tilde{c}_t^* + \gamma_2 \log l_t^* + \tau_t \log z_t \right\} \qquad [2.1']$$

subject to the constraints

$$\tilde{c}_t^* = \tilde{c}p_t + \gamma_1 \tilde{g}_t + a\,(\tilde{c}p_{t-1} + \gamma_1 \tilde{g}_{t-1})\, e^{-\lambda_t} \qquad [2.2']$$

$$l_t^* = T - n_t + b\,(T - n_{t-1}) \qquad [2.4']$$

$$\tilde{y}_t = (\tilde{k}_t^d)^{\theta} (n_t^d)^{1-\theta} e^{-\theta\lambda_t} \qquad [2.5']$$

$$\tilde{c}p_t + \tilde{k}_{t+1} = \tilde{y}_t + (1-\delta)\tilde{k}_t e^{-\lambda_t} - \tilde{w}_t(n_t^d - n_t) - r_t(\tilde{k}_t^d - \tilde{k}_t)e^{-\lambda_t} - \tilde{g}_t \qquad [2.7']$$

where the uncontrollable $\tilde{g}_t$ has replaced $\tilde{T}L_t$, and the presence of log z_t poses no analytical difficulties since it is uncontrollable and stochastically dominated.

Given initial values for the capital stock, leisure, government purchases and private consumption, as well as a law of motion for $\tilde{g}_t$, the equilibrium in this economy is essentially a sequence of contingency plans $\{ \tilde{c}p_t, \tilde{k}_{t+1}, n_t; t>0 \}$ which satisfies: (1) the household's first-order necessary conditions for a constrained maximum of [2.1'], (2) a transversality condition on capital, (3) the economy-wide per capita resource constraint, and (4) market-clearing in the labor and capital markets. The system of equations which characterize the equilibrium of this scaled economy are:

$$\frac{\gamma_2}{l_t^*} + b\,\beta\,E_t \frac{\gamma_2}{l_{t+1}^*} = \tilde{\mu}_t\,(1-\theta)\,\tilde{k}_t^{\theta}\,n_t^{-\theta}\,e^{-\theta\lambda_t} \qquad [2.10]$$

$$\beta\,E_t \left\{ \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \left(\theta\,\tilde{k}_{t+1}^{\theta-1}\,n_{t+1}^{1-\theta}\,e^{-\theta\lambda_{t+1}} + (1-\delta)\,e^{-\lambda_{t+1}} \right) \right\} = 1 \qquad [2.11]$$

$$\tilde{\mu}_t = \frac{\tau_t}{\tilde{c}_t^*} + a\,\beta\,E_t \left\{ e^{-\lambda_{t+1}} \frac{\tau_{t+1}}{\tilde{c}_{t+1}^*} \right\} \qquad [2.12]$$

$$\tilde{cp}_t + \tilde{k}_{t+1} + \tilde{g}_t = \tilde{k}_t^{\theta}\,n_t^{1-\theta}\,e^{-\theta\lambda_t} + (1-\delta)\,\tilde{k}_t\,e^{-\lambda_t} \qquad [2.13]$$

$$\tilde{y}_t = \tilde{k}_t^{\theta}\,\tilde{n}_t^{1-\theta}\,e^{-\theta\lambda_t} \qquad [2.14]$$

$$\tilde{c}_t^* = \tilde{cp}_t + \gamma_1\,\tilde{g}_t + a\,(\tilde{cp}_{t-1} + \gamma_1\,\tilde{g}_{t-1})\,e^{-\lambda_t} \qquad [2.15]$$

$$l_t^* = T - n_t + b\,(T - n_{t-1}) \qquad [2.16]$$

$$\tilde{w}_t = (1-\theta)\,\tilde{k}_t^{\theta}\,n_t^{-\theta}\,e^{-\theta\lambda_t} = (1-\theta)\,\tilde{y}_t/n_t \qquad [2.17]$$

$$r_t = \theta\,\tilde{k}_t^{\theta-1}\,n_t^{1-\theta}\,e^{-\theta\lambda_t}\,e^{\lambda_t} = (\theta\,\tilde{y}_t/k_t)\,e^{\lambda_t} \qquad [2.18]$$

$$\lim_{t\to\infty} \beta^t\,\tilde{\mu}_t\,\tilde{k}_{t+1} = 0 \qquad [2.19]$$

The variable $\tilde{\mu}_t$ is the Lagrange multiplier associated with the resource constraint. These conditions determine the equilibrium of the economy.

2.2 Characterizing the Seasonal Equilibrium

In general, a perfect foresight path would solve the system of equations [2.10] - [2.19], with the uncertainty removed. We restrict attention to a particular perfect foresight path. This path has the characteristic that the value of a variable x in quarter j is always equal to its realization four quarters ago. For our economy with (quarterly) seasonals, equations [2.10] - [2.16] can be reduced to twelve restrictions which $\{ \tilde{cp}_j, \tilde{k}_j, \tilde{n}_j ; j = 1,2,3,4 \}$ must satisfy. These seasonal restrictions (without uncertainty) can be written as:

$$\frac{\gamma_2}{l^*_j} + b\,\beta\,\frac{\gamma_2}{l^*_{j+1}} = \tilde{\mu}_j\,(1-\theta)\,\tilde{k}^{\theta}_j\,n^{-\theta}_j\,e^{-\theta\lambda_j} \qquad [2.20]$$

$$\tilde{\mu}_j = \beta \left\{ \tilde{\mu}_{j+1} \left(\theta\,\tilde{k}^{\theta-1}_{j+1}\,n^{1-\theta}_{j+1}\,e^{-\theta\lambda_{j+1}} + (1-\delta)\,e^{-\lambda_{j+1}} \right) \right\} \qquad [2.21]$$

$$\tilde{cp}_j + \tilde{k}_{j+1} + \tilde{g}_j = \tilde{k}^{\theta}_j\,n^{1-\theta}_j\,e^{-\theta\lambda_j} + (1-\delta)\,\tilde{k}_j\,e^{-\lambda_j} \qquad [2.22]$$

where the seasonal index j runs from 1 to 4. We adopt a wrap-around dating convention that when j=5, this represents the first quarter (j=1). The variables $\tilde{\mu}_j$, $\tilde{c}^*_j$, and l^*_j are functions of the essential variables $\tilde{cp}$ and n, as well as the exogenous variables $\tilde{g}$, τ, and λ:

$$\tilde{c}^*_j = \tilde{cp}_j + \gamma_1\,\tilde{g}_j + a\,(\tilde{cp}_{j-1} + \gamma_1\,\tilde{g}_{j-1})\,e^{-\lambda_j} \qquad [2.23]$$

$$l^*_j = T - n_j + b\,(T - n_{j-1}) \qquad [2.24]$$

$$\tilde{\mu}_j = \frac{\tau_j}{\tilde{c}^*_j} + a\,\beta \left\{ e^{-\lambda_{j+1}}\,\frac{\tau_{j+1}}{\tilde{c}^*_{j+1}} \right\} \qquad [2.25]$$

This leads to the following definition:

Definition A sequence { $\tilde{cp}_i$, $\tilde{k}_i$, $\tilde{n}_i$; i = 1,2,3,4 } *which satisfies* equations [2.20] - [2.22] is a perfect foresight seasonal equilibrium.

For all parameterizations of the model which we consider below, we were able to numerically calculate a unique seasonal equilibrium: no evidence of multiple equilibria was found. Furthermore, in the course of solving for the stochastic equilibrium (in Section 5), we calculate roots for the log-linear system which exhibit the proper characteristics to ensure local stability of the perfect foresight seasonal equilibrium. That is, in the state space representation, the fundamental matrix had equal numbers of roots inside and outside the unit circle. Also, Chatterjee and Ravikumar [1989] characterize existence and uniqueness in a model similar to ours.

Given the perfect foresight path for { $\tilde{cp}_i$, $\tilde{k}_i$, $\tilde{n}_i$; i = 1,2,3,4 } and the seasonals { λ_j, τ_j, g_j ; j = 1,2,3,4 }, seasonal paths for { $\tilde{y}_j$, $\tilde{i}_j$, $\tilde{w}_j$, r_j } can be computed using seasonal counterparts to equations [2.14], [2.13], [2.17], and [2.18]:

$$\tilde{y}_j = \tilde{k}_j^{\theta} n_j^{1-\theta} e^{-\theta\lambda_j} \qquad [2.26]$$

$$\tilde{i}_j = \tilde{k}_{j+1} - (1-\delta)\, \tilde{k}_j\, e^{-\lambda_j} \qquad [2.27]$$

$$\tilde{w}_j = (1-\theta)\, \tilde{y}_j / n_j \qquad [2.28]$$

$$r_j = \theta\, \tilde{y}_j \,/\, (\tilde{k}_j e^{-\lambda_j}) \qquad [2.29]$$

To compare these seasonal means with the Barsky-Miron seasonal results, the data set must be precisely defined, the seasonals in { λ_j, τ_j, g_j ; j = 1,2,3,4 } must be estimated, and the model's other parameters set.

3. Estimation of the Structural Parameters

Given seasonally unadjusted time series data for the U.S economy, the Euler equation methods of Hansen-Singleton [1982] can be used to estimate the model's structural parameters and test the overidentifying restrictions implied by the model and choice of instruments. The parameter vector to be estimated is:

$$\Psi = (\theta, a, b, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \tau_1, \tau_2, \tau_3, \tau_4, d_1, d_2, d_3, d_4, \rho, \sigma_u, \sigma_\epsilon),$$

where the d_i seasonals are related to the log $\tilde{g}_i$ seasonals by the relationship $d_i = \log \tilde{g}_i - \rho \log \tilde{g}_{i-1}$. The parameters β, γ_1, γ_2, δ, and T are set a priori, in accordance with previous studies. The discount factor β was chosen to be $1.03^{-.25}$, as in Christiano-Eichenbaum [1990] and Braun [1990]. The depreciation rate δ was chosen to be 2.5% per quarter, as in Kydland-Prescott [1982] and King-Plosser-Rebelo [1988]. The utility parameter γ_1 governing the substitutability of government purchases for private consumption was chosen to be 0.4, as found by Aschauer [1985]. The utility weight γ_2 on leisure was normalized to be 1. The total time endowment for the household is 1369 hours per quarter.

The moment equations chosen for the estimation consist of two stochastic Euler equations, the production function, the transitory government spending autoregression, and two variance estimates. Specifically, the household's time t decision for n_t and k_{t+1} yield the conditions:

$$\beta E_t \left\{ \left(\frac{\tau_{t+1}}{c^*_{t+1}} + \beta a \frac{\tau_{t+2}}{c^*_{t+2}} \right) \left(\theta \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) - \left(\frac{\tau_t}{c^*_t} + \beta a \frac{\tau_{t+1}}{c^*_{t+1}} \right) \right\} = 0 \qquad [3.1]$$

$$E_t \left\{ \left(\frac{\tau_t}{c_t^*} + \beta\, a \frac{\tau_{t+1}}{c_{t+1}^*} \right) (1-\theta) \frac{y_t}{n_t} - \frac{\gamma_2}{l_t^*} - \beta\, b \frac{\gamma_2}{l_{t+1}^*} \right\} = 0 \qquad [3.2]$$

where c^* and l^* can be constructed from equations [2.2] and [2.4] given values of γ_1, a, and b. The technology specification [2.5] - [2.6] yields the stochastic equation:

$$\frac{1}{1-\theta}\Big(\Delta \log y_t - \theta\, \Delta \log k_t - (1-\theta)\, \Delta \log n_t$$
$$- \lambda_1 Q_{1t} - \lambda_2 Q_{2t} - \lambda_3 Q_{3t} - \lambda_4 Q_{4t} \Big) = \epsilon_t \qquad [3.3]$$

where ϵ_t is an unconditionally mean zero random variable which is not observed by the econometrician. The law of motion for transitory government spending [2.8] yields the stochastic equation:

$$\log \frac{g_t}{z_t} - \rho \log \frac{g_{t-1}}{z_{t-1}} - d_1 Q_{1t} - d_2 Q_{2t} - d_3 Q_{3t} - d_4 Q_{4t} = u_t \qquad [3.4]$$

where u_t is an unconditionally mean zero random variable which is not observed by the econometrician. The d_i seasonals are related to the log $\tilde{g}_i$ seasonals by the relationship $d_i = \log \tilde{g}_i - \rho \log \tilde{g}_{i-1}$. Finally, the residual errors ϵ_t and u_t are used to estimate the standard deviations σ_ϵ and σ_u:

$$E\left(\epsilon_t^2 - \sigma_\epsilon^2 \right) = 0 \qquad [3.5]$$

$$E\left(u_t^2 - \sigma_u^2 \right) = 0 \qquad [3.6]$$

Equations [3.1] - [3.6] are estimated simultaneously.

The instruments for equations [3.1] - [3.6] were selected as follows: in equation [3.1], four seasonal dummies, and the time t growth rates of private consumption, leisure, output-capital ratio and output-labor ratio; in equation [3.2], four seasonal dummies and the time t and t-1 growth rates of private consumption, leisure, output-capital ratio and output-labor ratio; in equation [3.3], four seasonal dummies; in equation [3.4], four seasonal dummies and the logarithm of g_{t-1}/z_{t-1}; and in equations [3.5] and [3.6], only unity. A total of 31 instruments are used to estimate the 18 parameters of the nontime-separable specification, yielding 13 overidentifying restrictions. For a time-separable specification in which the parameters a and b are set to zero a priori, only 16 parameters are estimated, yielding 15 overidentifying restrictions.

The original data set employed in this study is the Barsky-Miron [1989] data: U.S. quarterly data which has not been adjusted for seasonality. For the empirical analysis to conform to the theoretical constructs of our model, however, we redefine some of the variables as follows (and convert to per capita values). Output (y) is Gross National Product per capital. Private consumption (cp) is nondurables plus services consumption expenditures per capita. Investment (i) is the sum of Business Fixed Investment plus Durable consumption expenditures, per capita. Government (g) is Federal, State, and Local government purchases, per capita. The capital stock is computed using the flow investment expenditures, a quarterly depreciation rate of 2.5%, and an initial capital stock value for 1950. Labor hours are computed as the product of total nonagricultural employment times average hours per week of nonagricultural production workers times 13 weeks per quarter (per capita). All other variables are the same as reported by Barsky-Miron: the real wage, average labor productivity, and the real interest rate. The data is converted to per capita values by using the civilian population, 16 years and older.

Table 3.1 presents three sets of parameter estimates of Ψ. The time-separable (TS) estimates set the parameters a and b equal to zero a priori; the nontime-separable estimates (NTS #1 and NTS #2) allow a and b to be nonzero. The NTS #1 estimates are the GMM estimates of Ψ since these estimates globally minimize the criterion function; however, the NTS #2 parameterization is an interesting local optimum discovered during the estimation procedure. The NTS #1 estimates display habit-persistence (or complementarity) in preferences for consumption goods and leisure hours; that is, a and b are estimated to be negative. Habit-persistence in leisure is consistent with previous empirical analyses using these preference specifications. In seasonally adjusted quarterly data, Braun [1990] has found evidence of habit-persistence in consumption goods and leisure hours. In seasonally adjusted monthly data, Eichenbaum-Hansen-Singleton [1989] have found evidence of habit-persistence in leisure hours, but not in consumption goods.

The NTS #2 parameterization is of interest since it produces a positive value of a, indicating local durability of consumption, not habit-persistence. A number of researchers have estimated consumption preferences which are consistent with a>0. For example, using monthly data on consumption and returns, Eichenbaum-Hansen-Singleton [1989] and Gallant-Tauchen [1990] find evidence of local durability in consumption. Constantinides [1990], on the other hand, has shown that negative values of a can help explain the equity premium puzzle. Constantinides and Ferson [1990] find evidence in favor of habit persistence when using quarterly data and argue that other researchers may have settled on a local optimum or used poor instruments. Our results are consistent with Constantinides and Ferson. We find a local optimum with local durability of consumption (NTS #2) and a global optimum with habit persistence (NTS #1). In Section 5, we consider the implication of these alternative parameterizations for the first and second moment properties of the data at seasonal and business cycle frequencies.

In Table 3.1, many of the parameter estimates are similar across each set of estimates. The capital and labor shares are tightly estimated to be around .29 and .71, respectively. The seasonal pattern in transitory government spending is similar: fourth quarter spending is low, while first quarter spending is high. The government spending autoregressive coefficient ρ is estimated to be approximately .85. The estimated standard deviations of u_t and ϵ_t are roughly similar.

Each parameter set displays a large degree of seasonal variation in technology. The fourth quarter growth in total factor productivity is estimated to be 6% (or 24% on an annualized basis). The first quarter experiences technical regress (on average), growing at a rate of -7.4% in one quarter. If the technological specification [2.4] is correct, and the factor inputs and output are properly measured, then these seasonals represent true seasonal variation in aggregate technology. The plausibility of seasonal variation in technology cannot be rejected outright. As Barro [1990] suggests, weather conditions and seasonality in construction probably account for some of the seasonal patterns in aggregate technological growth. Under this interpretation of the estimates, the equilibrium model may display the seasonal patterns uncovered by Barsky-Miron.

The estimates which differ substantially across the three parameter sets are the preference seasonal estimates. For time-separable preferences, the quarterly growth rates for τ (beginning in the first quarter) are -5.64%, +2.45%, +0.37%, and +3.06%. For the nontime-separable preferences with habit persistence in consumption (NTS #1), the same growth rates are -19.13%, +17.25%, -2.69%, and +8.38%. With local durability in consumption (NTS #2), these growth rates are -3.84%, +0.15%, +0.75%, and +3.06%. Since the estimated standard errors for the τ's are small, these differences are due to the preference specifications. In the time-separable case, the implied consumption expenditure patterns are variable (relative to the nontime-separable case with habit persistence). Only small changes in the preference seasonals are required to match the fourth quarter rise and first quarter fall in actual consumption expenditures. In the nontime-separable case with habit-persistence in consumption (NTS #1), the implied consumption expenditures are smooth; now relatively large changes in preferences are required to match the actual seasonal pattern in consumption expenditures. With local durability in consumption (NTS #2), the implied consumption expenditures are more variable than the other cases, due to the substitutability of consumption across adjacent periods; now only small changes in the preference seasonals are necessary.

Finally, Hansen's [1982] J-statistic indicates that neither model's overidentifying restrictions can be rejected at conventional significance levels. Having passed this initial specification test, the next section examines the seasonal implications of these models under these parameter estimates.

4. The Perfect Foresight Seasonal Equilibrium Analysis

Table 4.1 reports the growth rates of aggregate variables along the PFSE growth path. Table 4.1A reports the results for the time-separable estimates; Table 4.1B, for the nontime-separable estimates. Four cases are presented: 1) technology seasonals only, 2) preference seasonals only, 3) transitory government seasonals only, and 4) all seasonals. As the discussion below indicates, this decomposition analysis highlights the important role of technology seasonals in generating seasonals in output and hours.

<u>Case</u> <u>1</u>: Technology Seasonals

Panel one of Table 4.1A contains results from solving the perfect foresight TS model for the scenario in which the only source of seasonality is in technology. The quarterly growth rates in the technology are expressed in terms of deviations from the average quarterly growth rate of 0.25%. Although the seasonals in the transitory government purchases process ($\tilde{g}_t$) have been set to their mean value, panel one still shows *seasonal variation in the growth rate of* g_t. *This variation is inherited* from the seasonal variation in the permanent component of government purchases.

Along the seasonal equilibrium path, seasonal variation in technology has important effects on hours, output, investment and the real wage. All of these variables are proseasonal, exhibiting positive growth rates in seasons where the growth rate of technology (λ) is positive. The seasonal variations in quantity variables display several of the gross characteristics of business cycle variations. Investment is four times as variable as output. Fluctuations in hours are about as large as those in output (85%), and private consumption moves smoothly over the seasons relative to output. Some seasonal variations differ importantly from the typical business cycle features. The variation in the growth rate of labor productivity (real wage) is about one-fifth of the size of hours, and private consumption moves counterseasonally, opposing the seasonal movements in output. This latter feature is due to the substitutability of public consumption for private consumption: when γ_1 is set equal to zero (unreported), seasonal variation in private consumption is proseasonal, but virtually flat. The former observation is sensitive to the specification of preferences. Table 4.1B displays the PFSE path for preferences which exhibit habit persistence in leisure: fluctuations in hours are only half as large as in output, and labor productivity is more variable than hours.

A comparison of Tables 4.1A and 4.1B reveals the role of nontime-separable preferences in the PFSE paths. The pattern of seasonal variation in Case 1 (and the others) is the same for both sets of preferences; these preferences do not alter the determination that a variable is pro- or counterseasonal. Instead, the seasonal variability in output, investment, and hours is lower under habit-persistence preferences. The real wage (labor productivity) is more variable, and private consumption is slightly more variable. In response to a positive fourth-quarter technology seasonal, habit persistence in leisure reduces the positive substitution effect for labor hours relative to the TS economy. In the NTS economy, the labor market clears with a smaller increase in hours and at a higher wage. The smaller response of labor hours reduces the increase in output. The equilibrium interest rate falls less, investment rises less, and private consumption falls more. This pattern is repeated across the seasons. Given our estimates of the stuctural parameters, therefore, the nontime-separable preferences play an important role in an attempt to match the magnitude of a seasonal cycle, but not the pattern.

Both tables demonstrate that seasonal variation in technology can lead to seasonal fluctuations in output. For the TS economy, output fluctuations are about 70% larger than the technology fluctuations; for the NTS economy, output fluctuations are about as large as the technology fluctuations.

Case 2: Preference Seasonals

Panel two of Table 4.1A contains results from solving the perfect foresight TS model for the scenario in which the only source of seasonality is in preferences. Technology and government purchases do not contain any seasonal effects: the quarterly growth rates of technology and government purchases are set at the baseline average of 0.25%.

In our economy with only preference seasonals, the equilibrium effects essentially involve only consumption and investment. Movements in consumption growth closely parallel the seasonal movements in preferences. Seasonal changes in equilibrium investment essentially offset the changes in consumption. Output, labor hours, the real wage, and the interest rate are largely unchanged over this seasonal equilibrium path. These essential findings also hold for the NTS economy, Table 4.1B, panel two: output, labor hours, and productivity display no perceptible seasonal variation due to preferences. Apparently, any attempt to match the seasonal variation in GNP will not be aided by selecting among either of these preferences or preference seasonals. This reinforces the important role of seasonal fluctuations in technology for generating seasonal variation in output.

Case 3: Transitory government spending seasonals

Panel three of Table 4.1A contains results from solving the perfect foresight TS model for the scenario in which the only source of seasonality is in transitory government purchases. Technology and preferences have no seasonal variation in this case. Along the seasonal growth path, consumption and investment move together; these movements run counter to the seasonal pattern in government purchases. The NTS results are about the same as for the TS economy. The seasonal equilibrium responses to government seasonals are similar to the Case 2 analysis: changes in output, labor hours, the real wage, and the interest rate are virtually imperceptible. Again, this reinforces the important role of seasonal variations in technology.

Case 4: Technology, Preference, and Government Seasonals

Collecting the results from Cases 1 - 3 suggests that seasonal output, labor hours, wages, and interest rate movements will be determined largely by the technology seasonal; seasonal consumption and investment patterns will depend on the joint configuration of technology, preference, and government spending seasonals. Case 4 bears this out. The seasonal movements in output, labor hours, wages, and the interest rate are virtually identical between cases 1 and 4. Despite the disparities in the estimated preference seasonals for the TS and NTS economies, the seasonal patterns in consumption are remarkably similar for both economies. In the TS economy, the consumption seasonals roughly mimic the preference seasonals. In the NTS economy, the preference seasonals and nontime-separabilities interact to produce essentially the same pattern as in the TS economy. The investment seasonal is in fact determined by the joint contribution of technology, preference, and government seasonals. While the investment pattern is similar for both economies, the seasonal variability of investment in the NTS economy is about one-half of the variability in the TS economy.

5. Evaluation of the Stochastic Model

This section presents results from solving a stochastic version of the seasonal business cycle model. In the course of describing the empirical characteristics of this model two issues will be addressed. First, the models' predicted seasonal patterns will be compared with the data. Second, (deseasonalized) cyclical properties of the seasonal business cycle models will be examined and compared with the data.

5.1 Solving the Stochastic Model

Before discussing the results we will briefly outline the methodology used in solving for the stochastic equilibrium. In the last section the perfect foresight seasonal equilibrium was calculated and analyzed. In this section we approximate the true stochastic equilibrium. In solving the model we modify the standard method (see Kydland and Prescott [1982], King, Plosser and Rebelo [1988] and Christiano and Eichenbaum [1990]) of approximating the true stochastic equilibrium with a Taylor expansion about the steady-state. The modifications arise because the perfect foresight equilibrium we consider exhibits seasonal cycles. The nonlinear model has the characteristic that technology and preferences change with the season. Varying the location of the Taylor approximation with the season allows the linearized system to inherit this characteristic of the nonlinear model. We consider this approach to be the appropriate generalization of the strategy adopted by Hansen and Sargent [1990], Ghysels [1989], and Todd [1990] who analyze seasonality in explicitly linear-quadratic frameworks.

The first step in solving the model is to linearize the equations [2.10]-[2.16] about the perfect foresight seasonal equilibrium path calculated in section 2. The linearized system can be reduced to twelve stochastic difference equations governing the evolution of the capital stock, hours, and private consumption in each season. These twelve equations are linearized, stochastic counterparts to equations [2.20]-[2.22]. In a technical appendix we display the linearized system and describe how these twelve difference equations are mapped into a state space representation which can be solved using methods described in King, Plosser, and Rebelo [1990]. The state space representation essentially has the same structure as Todd's time-invariant linear-quadratic representation (TILQ) or Hansen and Sargent's time-varying strictly periodic equilibrium. The model's solution is a series of twelve equations that describe the optimal decision rule for capital, hours, and private consumption, one equation for each season:

$$K_{t+1} = A\ S_t \qquad [4.1]$$

where

$$K_{t+1} = [\ k^1_{t+1}\ k^2_{t+1}\ k^3_{t+1}\ k^4_{t+1}\ cp^1_t\ cp^2_t\ cp^3_t\ cp^4_t\ n^1_t\ n^2_t\ n^3_t\ n^4_t\ ]'$$

and

$$S_t = [\ K'_t\ \ g^1_t\ \ g^2_t\ \ g^3_t\ \ g^4_t\ \ \lambda^1_t\ \ \lambda^2_t\ \ \lambda^3_t\ \ \lambda^4_t\ ]'.$$

where the superscripts denote the seasons.

Given these log-linear decision rules for capital, private consumption and hours, it is straightforward to generate time series for the model economy. First, a sequence of normal variables is drawn to mimic the empirical covariance structure of the forcing processes u_t and ϵ_t. Once u_t and ϵ_t have been constructed it is straightforward to calculate λ_t and g_t. Then given an initial K_o we can construct a sequence of realizations for the capital stock, hours, and private consumption using the following method. If this is the jth quarter then use the jth, j+4th and j+8th row of matrix A along with the current states: k_t^j, cp_{t-1}^{j-1}, n_{t-1}^{j-1}, λ_t^j, and g_t^j to determine the current decisions for next period's capital, and today's consumption and hours. Given the values of next period's stock of capital, today's consumption and today's work effort, it is straightforward to determine the current choices of output, investment, real wages, and the real interest rate using equations [2.14], [2.13], [2.17], and [2.18]. In practice we choose an effective sample length of 88 and provide results based on 500 draws.

In order to facilitate comparison with other research in this field we report results based on data that has been filtered in two distinct manners to induce stationarity. These are the log first difference filter, and the Hodrick-Prescott filter. A third filter (log fourth differences) was investigated, and the results were very similar to the Hodrick-Prescott filter results (and so are unreported).

The equilibrium model developed in this paper imposes restrictions across the entire spectrum. To focus attention on a specific set of moments, researchers often decompose time series. Examples of such decompositions include first differencing to remove low frequency moments and seasonal adjustment to remove particular high frequency moments. Our objective is to investigate the model's ability to match both the seasonal patterns uncovered by Barsky and Miron, as well as the data's cyclical moments which Prescott defines to be the business cycle phenomena. To facilitate these comparisons, we adopt Barsky and Miron's decomposition of the stationary, stochastic processes into "deterministic seasonal" and "indeterministic" components. Specifically, after filtering the data to induce stationarity, we regress (the logarithm of) each series on four seasonal dummies: the estimates on the dummy variables define the seasonal patterns emphasized by Barsky and Miron. We also adopt the convention of referring to moments calculated using the indeterministic residuals from these regressions as relating to cyclical or business cycle phenomena. Obviously, the definition of the seasonal cycle versus the business cycle will depend on the particular decomposition used. Another approach is to simply avoid decompositions. Hansen and Sargent [1990] observe that seasonally unadjusted data can be *stationary, conditional on a starting* season. In light of this result, we also report moments for seasonally unadjusted growth rates; this approach ignores the entire distinction between business cycles and seasonal cycles.

5.2 Seasonal Predictions of the Stochastic Model

First we report results on the seasonal properties of the model. In summarizing the results particular attention is paid to the following question: can a parsimoniously parameterized real business cycle model capture the central features of the data at seasonal frequencies? In order to facilitate comparision with Barsky and Miron's work we address this question by considering the same set of moments reported in their paper. Table 5.1 presents the seasonal patterns for the data set using the log first difference filter and the Hodrick-Prescott filter. The seasonal means are reported in terms of percentage deviations from average growth rates for the sample period 1964:I to 1985:IV. The real interest rate, which is the exception to this rule, is reported in terms of annualized rates of return. The table also includes R-square statistics for each variable which describe the percentage of the total variation in the particular time series that is attributable to the determinstic seasonal.

Finally, we report standard errors for each estimate that are based on the Newey-West [1987] weighting matrix with 12 autocorrelations.

Table 5.2 contains simulation results for the time-separable preference specification. Results are reported for the first difference filter in the top panel and the Hodrick-Prescott filter in the lower panel. For each variable, columns 1 through 4 label the average seasonal means for 500 draws. The fifth column contains the average R-square of the regressions.

Comparisons of the results in table 5.2 with those in table 5.1 reveal several significant shortcomings of the model. First, under the first difference filter the model sharply overstates the seasonal means in output, hours and investment. The predicted R-squares for these variables also exceeds the respective number in the data in each instance. Comparisons on the basis of Hodrick-Prescott filtered data produce a similar picture. The model does capture, however, the seasonal movements in consumption, mimicing both the sign and the magnitude of the seasonal means. Overall, the time-separable specification does not capture the seasonal properties of the data.

One way to interpret the time-separable model's shortcomings is to focus on hours. In the fourth quarter, for instance, the model predicts a large positive seasonal in hours under the first difference filter while the data indicates that hours are only slightly above their four quarter mean value. The model's surge in fourth quarter hours induces a large increase in output which is high already due to a positive technology seasonal. Since these seasonals are anticipated and temporary, their wealth effects on consumption are small. Instead, optimizing households choose to increase desired savings which shows up as increased equilibrium investment. Based on this analysis, it is conceivable that the excess variability in hours across the seasons is the cause for the excess variability in investment and output. Alternatively, generalizations of the model that act to smooth hours across the seasons (such as habit-persistence in leisure preferences) will also smooth output and investment.

This proposition is explored in the context of the nontime-separable preferences' results in Table 5.3. Recall from Section 3 that estimates of the nontime-separabilities produced evidence of habit persistence in consumption and leisure. Habit persistence in leisure acts to smooth desired labor supply between adjacent periods. Examination of table 5.3 reveals that the NTS #1 specification captures many of the seasonal movements in the data. The model successfully mimics the seasonal patterns in output, consumption, government purchases, average productivity and capital. For these variables the model reproduces the nature of seasonal movements in the data and in most cases the magnitudes. The main failures of the model lie in its inability to capture the magnitude of the movements in investment and hours: for both of these variables, the largest deviations of the theory from the data are in the fourth quarter. Finally, these properties hold for both filters.

Comparing R-squares indicates also a generally good fit for most of the variables. The model correctly predicts the fraction of total variation due to deterministic seasonals. The worst performance using this metric is the R-square for the real interest rate, which the model overstates. Again, these results are robust to the method of inducing stationarity in the data.

In table 5.4 we report results from solving the model using the NTS #2 parameterization. The key observation to note is the striking similarity between the results in table 5.3 and 5.4. The NTS #2 parameterization captures the same seasonal features of the data that the NTS #1 parameterization captures. This is a striking result since the parameterization differs from the previous one in important respects. Here, consumption in adjacent periods are substitutes (local durability), whereas for the previous parameterization they were complements (habit-persistence). As we observed above our specification has the characteristic that seasonal movements in equilibrium consumption are determined primarily by preferences. The results here make this point quite sharply. The two parameterizations are similar in all respects except for the pattern of seasonal preference shocks and nature of complementarities in consumption. We see in tables 5.3 and 5.4 that there are two ways to capture the seasonal patterns in consumption. The NTS #2 parameterization exploits the negative first order autocorrelation that a positive value of **a** implies for consumption, leaving little seasonal variation for the seasonals to explain. The alternative (NTS #1) is to have a negative value of **a** which acts to smooth consumption in adjacent time periods and leaves the entire burden of capturing seasonal fluctuations to the preference shocks. In the next section we exploit information at other frequencies that allows us to make a sharper distinction between the predictions of the two parameterizations.

5.3 Cyclical Predictions of the Stochastic Model

This subsection examines the cyclical properties of the stochastic model. Three distinct parameterizations are considered: the time separable GMM optimum (TS), the nontime-separable global optimum (NTS #1) and a local nontime-separable optimum (NTS #2).

Table 5.5 contains results relating to relative variability and cross-correlations with output for the three parameterizations and the data under three different filters. The heading "one-quarter growth rate" corresponds to moments calculated using data that has been first differenced and regressed on four dummies. The heading "HP filter" corresponds to data that has been Hodrick-Prescott filtered and then regressed on four seasonal dummies. The heading "Unadjusted one-quarter rates" corresponds to moments calculated using data that has been first differenced only.

For each filter we report moments for U.S data running from 1964:1-1985:4 in the first column. The second column contains standard errors for the data's moments reported in column one. The standard errors were calculated using a Newey-West weighting matrix with 12 lags. The third through fifth columns contain results from simulating the model using the three parameterizations. In each case the reported statistics are sample averages based on 500 draws of length 88.

Looking first at the properties of the data, observe that the two filters which incorporate deseasonalization (one-quarter growth and HP filter in table 5.5) produce the same general patterns. With respect to relative variability, investment is about twice as variable as output, government purchases are about as variable as output, and hours and consumption are less variable than output. We do observe some differences in moving from the deseasonalized one-quarter growth rates to the deseasonalized HP filtered data. In particular, there is a significant increase in the relative variability of hours. With respect to correlations we also observe similar general patterns accross the two filters. Here the most significant differences occur in the instances of investment and hours.

More important differences are observed when comparing the first two filters with the third filter, first differenced seasonally unadjusted data. While many of the relative volatility statistics are similar across the three filters, output is considerably more variable prior to removal of seasonal means. Furthermore, some of the correlations are quite different. For instance, government purchases have a correlation with output of about .3 under the first two filters, but a correlation of .8 under the third filter. In general, the strongest cross-correlations with output occur in data which have not been seasonally adjusted, but have been first differenced in order to induce stationarity. This is another way of capturing Barsky and Miron's finding that aggregate variables exhibit strong comovement across seasonal frequencies as well as cyclical frequencies.

Turning to the theory, consider next a comparison of the cyclical properties of the NTS #1 parameterization with the TS parameterization. The NTS #1 parameterization is successful in matching many of the properties of unadjusted 1-quarter growth rates. It captures most of the relative variability statistics and cross-correlation patterns found in the data. The major shortcomings lie in the failure of the model to capture precisely the correlation of hours with output and the relative variability of consumption, investment, and average productivity. The TS parameterization has considerably more difficulty matching the unadjusted moments, missing the relative variability of consumption, investment, hours and average productivity by wider margins and overstating the correlation of capital with output. On the basis of these seasonally unadjusted moments, the NTS #1 parameterization captures more features of the data.

If we compare the predictions of the NTS #1 and TS parameterizations under the two seasonally adjusted filters, we get a different picture. In many cases the two models' predictions lie outside a two standard deviation band around the data. The largest differences appear to occur under the 1-quarter growth filter. Both model's fail to capture the relative variability of consumption, and most of the cross correlations with output. Under the HP-filter, the two parameterizations look only slightly better. For the NTS #1 parameterization this represents a striking deterioration in performance relative to the seasonally unadjusted moments we considered above.

How do these results compare with the performance of standard business cycle models that ignore seasonality? Many of the model's predictions are similar to those of standard real business cycle models. These models typically predict that consumption is about half as variable as output, understate the relative variabilities of hours and productivity, and produce a counterfactually strong correlation between output and hours (see for example, Christiano-Eichenbaum [1990] and Braun [1990]).

Next we turn to a comparison of the NTS #1 and NTS #2 parameterizations. It was noted in the previous subsection that the main distinction between these two parameterizations lies in the preference parameterization of consumption. The NTS #1 parameterization exhibits large seasonal shifts in preferences and habit persistence ($a<0$). The NTS #2 parameterization has significantly less seasonal variation in preferences and local durability ($a>0$).

The value of a has been found to have important implications for asset pricing. Using quarterly data Constantinides [1989] finds that preference specifications with habit persistence in consumption offer a potential solution to the equity premium puzzle. Nonparametric tests of marginal rates of substitution by Gallant, Hansen and Tauchen [1990] confirm this possibility. However, many researchers, including Eichenbaum and Hansen [1990], Heaton [1988], Singleton [1990], Dunn and Singleton [1986], and Gallant and Tauchen [1989] find evidence of local durability in consumption preferences using monthly data, which implies that $a>0$. For quarterly data, Braun [1990] estimates a to be negative but close to zero using seasonally adjusted data. Constantinides and Ferson [1990] find a local optimum with $a>0$, but the global optimum is with $a<0$. Constantinides and Ferson's finding is consistent with the results we report in Section 3. One way to distinguish between positive and negative values of a is to consider the implications of the two specifications for business cycle observations.

In table 5.5 we report results for the NTS #2 parameterization. On the basis of the seasonal moments reported in Table 5.4, the NTS #2 parameterization captures the seasonal features of the data as well as the NTS #1 parameterization. When we consider the cyclical properties of this parameterization, however, the two most significant failures of the NTS #1 parameterization disappear. The NTS #2 parameterization mimics the relative variability of investment and captures the correlation of consumption with output under the 1-quarter growth and HP-filters! With respect to other moments reported in Table 5.5, both seasonally adjusted and unadjusted, it performs about as well as the NTS #1 parameterization.

This raises two questions. First, why does GMM favor the NTS #1 parameterization; and second, is there a priori information that we can use to choose in favor of one or the other parameterization? To answer the first question we have explored a larger set of moments than we report in table 5.5. The mostly likely set of moments to consider in looking for a distinction between the two parameterizations are the autocorrelations of consumption. In the absence of seasonal shocks, negative values of a will produce a positive first order autocorrelation in equilibrium consumption, while positive values will produce a negative first order autocorrelation in consumption. Proceeding along these lines, we first considered seasonally unadjusted autocorrelations of first differenced consumption. The top panel of Table 5.6 reports these statistics for the data and the two parameterizations. The main distinction between the two parameterizations lies in the fourth order autocorrelation. Here the prediction for the NTS #1 parameterization lies within one standard deviation of the data, while the NTS #2 parameterization lies outside the two standard deviation band of the data.

The second panel of Table 5.6 contains the same autocorrelations after removing seasonal dummies from consumption. Consumption still has a lot of power at seasonal frequencies, in contrast to most of the other time series we consider. The fourth order autocorrelation for consumption is .57 with a tight standard error. Examination of the lower panel of Table 5.6 reveals that the NTS #1 parameterization is more successful at capturing this aspect of the data. The cost, however, is a failure of this parameterization to capture the first two autocorrelations of consumption, which the NTS #2 parameterization successfully captures. On the basis of this analysis, we believe that it is the ability of the NTS #1 specification to capture more of the seasonal movements in consumption that is responsible for its lower rejection level.

This inference leads to our second question. Is it possible on an a priori basis to evaluate the plausibility of the two parameterizations. It is clear that the NTS #2 parameterization does a better job of mimicing the cyclical properties of the data, while capturing the seasonal moments as well as the NTS #1 parameterization. Furthermore, the seasonal pattern of the estimated NTS #2 preference shocks are more consistent with a priori reasoning. Notice that in table 3.1 the estimates of the τ's for the NTS #2 parameterization indicate that there is one large preference shock in the fourth quarter, with no evidence of seasonality in the first, second or third quarters. The preference seasonals τ_1, τ_2, and τ_3 are all of about the same magnitude. On the other hand, the NTS #1 parameterization produces estimates of the τ's that imply large positive preference shocks in the second and fourth quarters. It is easy to associate the fourth quarter increase in demand for consumption services with Christmas, but much more difficult to produce an economic explanation for an increased desire for consumption services in the second quarter.

We conclude that the NTS #2 parameterization displays the seasonal cycle and the business cycle phenomenon. The choice of NTS #2 over NTS #1 is based on three informal arguments. First, while our GMM estimator selects NTS #1 as the global optimum, a different set of instruments might select the NTS #2 parameterization. In fact, Constantinides and Ferson [1990] find that different instrument selections lead to different estimates of a in seasonally adjusted data. Second, NTS #2 better displays the business cycle phenomenon, while equally displaying the seasonal patterns in the data. Third, the consumption preference seasonals estimated in the NTS #2 parameterization seem more plausible than the NTS #1 preference seasonals. Consequently, the preferred parameterization of the equilibrium model is the one in which consumption exhibits local durability, not habit-persistence.

6. Conclusions

In this paper we introduce deterministic seasonals into an equilibrium model of the business cycle. The model is tractable: the perfect foresight seasonal equilibrium is computable without any approximations, and an approximate linear solution of the stochastic model can be found using methods analogous to those of King-Plosser-Rebelo [1988], Hansen-Sargent [1990], and Todd [1990]. The structural parameters and seasonals were estimated using GMM with seasonally unadjusted, postwar U.S. data. The overidentifying restrictions implied by the model cannot be rejected at conventional significance levels.

Are Barsky and Miron's findings consistent with current equilibrium business cycle theories as surveyed by Prescott [1986]? Conditional on our parameterization, the nontime-separable model predicts most of the seasonal patterns found in aggregate quantity time series; notable exceptions are the seasonal patterns in investment and the fourth quarter seasonal in labor hours. The model also predicts many of the deseasonalized second moment properties of the data. Our answer to this question is yes: this equilibrium model generally displays the seasonal patterns discovered by Barsky and Miron [1989] as well as the business cycle phenomenon.

Finally, this model is a benchmark in the tradition of Kydland and Prescott [1982] and Long and Plosser [1983]. The predictions of the theory match the data's properties fairly well; the assumptions of theory, however, require further investigation. In particular, for this class of equilibrium models, seasonal variation in technology is crucial for delivering seasonal fluctuations in output. Does the aggregate technology vary exogenously as much as our estimates suggest, or is this seasonal variation due to some misspecification of the technology? Future research should assess the plausibility of seasonality in aggregate Solow residuals by examining alternative general equilibrium economies. We conjecture that this seasonal investigation will provide new macroeconomic insights into the importance of labor hoarding (as in Summers [1986]), increasing returns due to endogenous growth (Romer [1986]), increasing returns due to market externalities (Diamond [1982], Murphy-Shleifer-Vishny [1989]), countercyclical markups of price over cost (as suggested by Hall's [1989] evidence), and propagation mechanisms in general. In principle, each of the above phenomenon could produce endogenous seasonal movements in measured Solow residuals, even if true technological advances are nonseasonal. Whether or not any of these theories can encompass the results of this benchmark model and the seasonality in measured Solow residuals is an important topic for future research.

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Table 3.1 GMM Estimates of the Structural Parameters

	Time - separable		Nontime-separable #1		Nontime-separable #2	
	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
θ	.2945	.0053	.2872	.0229	.2911	.0049
a	0	--	-.5671	.0224	.2892	.0736
b	0	--	-.4560	.0332	-.5145	.0148
λ_1	-.0722	.0029	-.0742	.0046	-.0754	.0029
λ_2	.0390	.0033	.0403	.0042	.0418	.0030
λ_3	-.0172	.0018	-.0168	.0022	-.0172	.0018
λ_4	.0604	.0018	.0584	.0032	.0621	.0026
τ_1	.1304	.0005	.1171	.0017	.1327	.0006
τ_2	.1336	.0004	.1373	.0007	.1329	.0005
τ_3	.1341	.0004	.1336	.0006	.1339	.0005
τ_4	.1382	.0005	.1448	.0009	.1380	.0005
d_1	.7802	.2447	.7327	.2367	.5322	.1788
d_2	.7427	.2452	.6926	.2384	.4909	.1794
d_3	.7800	.2446	.7321	.2373	.5300	.1791
d_4	.7109	.2463	.6628	.2395	.4566	.1792
ρ	.8434	.0508	.8525	.0505	.8949	.0375
σ_u	.0190	.0012	.0191	.0016	.0173	.0012
σ_ϵ	.0198	.0010	.0179	.0015	.0172	.0010
J-statistics	19.71		16.71		19.05	
distribution	$\chi^2(15)$		$\chi^2(13)$		$\chi^2(13)$	
P-value	.1834		.2132		.1214	

Table 4.1A Perfect Foresight Seasonal Equilibrium Growth Rates*
Time-Separable Preferences

Case 1: Technology Seasonals Only

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	0.00	0.00	0.00	0.00	0.00
Technology	(Δ log λ)	-7.47	3.65	-1.97	5.79	0.25
Government	(Δ log g)	-7.47	3.65	-1.97	5.79	0.25
Output	(Δ log y)	-12.62	6.17	-3.38	9.83	0.25
Consumption	(Δ log cp)	1.39	-0.75	0.31	-0.95	0.25
Investment	(Δ log i)	-48.31	25.91	-13.36	35.76	0.25
Capital	(Δ log k)	0.79	-0.58	0.08	-0.28	0.25
Labor Hours	(Δ log n)	-10.73	5.33	-2.85	8.24	0.00
Real Wage	(Δ log w)	-1.89	0.84	-0.53	1.58	0.25
Real Rate	(r)	4.15	3.65	5.15	3.20	4.04

Case 2: Preference Seasonals Only

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	-5.97	3.03	-0.00	2.94	0.00
Technology	(Δ log λ)	0.00	0.00	0.00	0.00	0.25
Government	(Δ log g)	0.00	0.00	0.00	0.00	0.25
Output	(Δ log y)	-0.16	0.16	0.00	0.00	0.25
Consumption	(Δ log cp)	-7.27	3.70	-0.00	3.57	0.25
Investment	(Δ log i)	14.99	-6.89	0.01	-8.11	0.25
Capital	(Δ log k)	-0.21	0.20	0.00	0.00	0.25
Labor Hours	(Δ log n)	-0.14	0.13	0.00	0.00	0.00
Real Wage	(Δ log w)	-0.02	0.02	0.00	0.00	0.25
Real Rate	(r)	4.03	4.03	4.03	4.04	4.04

*These figures are quarterly percentage rates, expressed as deviations from the Baseline average, except for the interest rate. For example, in Case 1 fourth quarter output grew at a quarterly rate of +10.08%, or 9.83% above the average quarterly rate of +0.25%. The figures for the interest rate report the annualized level of the seasonal real rate.

Table 4.1A (continued)

Case 3: Transitory Government Seasonals Only

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	0.00	0.00	0.00	0.00	0.00
Technology	(Δ log λ)	0.00	0.00	0.00	0.00	0.25
Government	(Δ log g)	3.02	-1.15	2.73	-4.60	0.25
Output	(Δ log y)	0.03	-0.01	0.01	-0.03	0.25
Consumption	(Δ log cp)	-0.65	0.25	-0.60	1.00	0.25
Investment	(Δ log i)	-1.95	0.77	-1.91	3.08	0.25
Capital	(Δ log k)	0.04	-0.01	0.01	-0.04	0.25
Labor Hours	(Δ log n)	0.03	-0.01	0.01	-0.03	0.00
Real Wage	(Δ log w)	0.00	-0.00	0.00	-0.00	0.25
Real Rate	(r)	4.04	4.03	4.04	4.03	4.04

Case 4: Technology, Preference and Transitory Government Seasonals

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	-5.97	3.03	0.00	2.94	0.00
Technology	(Δ log λ)	-7.47	3.65	-1.97	5.79	0.25
Government	(Δ log g)	-4.44	2.50	0.76	1.19	0.25
Output	(Δ log y)	-12.74	6.32	-3.37	9.80	0.25
Consumption	(Δ log cp)	-6.55	3.18	-0.28	3.64	0.25
Investment	(Δ log i)	-35.13	18.04	-15.38	32.48	0.25
Capital	(Δ log k)	0.62	-0.39	0.09	-0.32	0.25
Labor Hours	(Δ log n)	-10.84	5.46	-2.84	8.22	0.00
Real Wage	(Δ log w)	-1.90	0.86	-0.53	1.58	0.25
Real Rate	(r)	4.14	3.65	5.15	3.21	4.04

Table 4.1B Perfect Foresight Seasonal Equilibrium Growth Rates*
Nontime-separable Preferences

Case 1: Technology Seasonals Only

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	0.00	0.00	0.00	0.00	0.00
Technology	(Δ log λ)	−7.61	3.83	−1.86	5.65	0.19
Government	(Δ log g)	−7.61	3.83	−1.86	5.65	0.19
Output	(Δ log y)	−8.43	3.81	−1.47	6.09	0.19
Consumption	(Δ log cp)	1.65	−0.83	0.41	−1.23	0.19
Investment	(Δ log i)	−31.55	14.90	−5.25	21.90	0.19
Capital	(Δ log k)	0.50	−0.38	0.01	−0.13	0.19
Labor Hours	(Δ log n)	−4.41	1.66	−0.20	2.95	0.00
Real Wage	(Δ log w)	−4.01	2.15	−1.27	3.14	0.19
Real Rate	(r)	3.82	3.62	4.51	3.25	3.80

Case 2: Preference Seasonals Only

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	−21.21	15.91	−2.72	8.02	0.00
Technology	(Δ log λ)	0.00	0.00	0.00	0.00	0.19
Government	(Δ log g)	0.00	0.00	0.00	0.00	0.19
Output	(Δ log y)	−0.13	0.11	0.03	−0.00	0.19
Consumption	(Δ log cp)	−8.32	3.36	0.40	4.56	0.19
Investment	(Δ log i)	18.22	−6.55	−0.77	−10.90	0.19
Capital	(Δ log k)	−0.27	0.22	0.03	0.01	0.19
Labor Hours	(Δ log n)	−0.07	0.06	0.02	−0.01	0.00
Real Wage	(Δ log w)	−0.06	0.05	0.00	0.01	0.19
Real Rate	(r)	3.80	3.79	3.79	3.81	3.80

*These figures are quarterly percentage rates, expressed as deviations from the Baseline average, except for the interest rate. For example, in Case 1 fourth quarter output grew at a quarterly rate of +6.28%, or 6.09% above the average quarterly rate of +0.19%. The figures for the interest rate report the annualized level of the seasonal real rate.

Table 4.1B (continued)

Case 3: Transitory Government Seasonals Only

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	0.00	0.00	0.00	0.00	0.00
Technology	(Δ log λ)	0.00	0.00	0.00	0.00	0.19
Government	(Δ log g)	3.11	−1.36	2.79	−4.54	0.19
Output	(Δ log y)	0.02	−0.01	0.00	−0.02	0.19
Consumption	(Δ log cp)	−0.67	0.30	−0.62	0.99	0.19
Investment	(Δ log i)	−2.15	0.97	−2.04	3.22	0.19
Capital	(Δ log k)	0.04	−0.01	0.01	−0.04	0.19
Labor Hours	(Δ log n)	0.01	−0.00	0.00	−0.01	0.00
Real Wage	(Δ log w)	0.01	−0.00	0.00	−0.01	0.19
Real Rate	(r)	3.80	3.80	3.80	3.80	3.80

Case 4: Technology, Preference and Transitory Government Seasonals

		Q1	Q2	Q3	Q4	Baseline Avg
Preferences	(Δ log τ)	−21.21	15.91	−2.72	8.02	0.00
Technology	(Δ log λ)	−7.61	3.83	−1.87	5.65	0.19
Government	(Δ log g)	−4.50	2.47	0.92	1.10	0.19
Output	(Δ log y)	−8.53	3.91	−1.44	6.07	0.19
Consumption	(Δ log cp)	−7.36	2.80	0.20	4.35	0.19
Investment	(Δ log i)	−15.92	8.13	−8.09	15.89	0.19
Capital	(Δ log k)	0.28	−0.17	0.05	−0.16	0.19
Labor Hours	(Δ log n)	−4.48	1.72	−0.17	2.93	0.00
Real Wage	(Δ log w)	−4.06	2.19	−1.27	3.14	0.19
Real Rate	(r)	3.82	3.61	4.51	3.26	3.80

Table 5.1 Seasonal Patterns, 1964-1985*

A. Seasonal Deviations, Log Growth Rate Filter

	Q1	Q2	Q3	Q4	R^2
Output	-7.72	4.26	-1.04	4.16	.905
	(.59)	(.42)	(.22)	(.35)	
Consumption	-7.30	2.59	0.04	4.34	.936
	(.48)	(.40)	(.09)	(.26)	
Investment	-14.64	11.79	-2.15	4.33	.903
	(.68)	(1.08)	(.34)	(.58)	
Government	-4.89	2.86	0.49	1.31	.696
	(.37)	(.79)	(.46)	(.32)	
Capital	0.16	-0.26	0.09	0.02	.214
	(.10)	(.08)	(.10)	(.10)	
Labor Hours	-3.26	2.31	0.73	0.07	.842
	(.16)	(.21)	(.20)	(.17)	
Real Wage	-0.06	-0.27	0.03	0.29	.076
	(.21)	(.24)	(.16)	(.11)	
Avg. Productivity	-4.47	1.95	-1.78	4.09	.846
	(.52)	(.40)	(.30)	(.40)	
Capital Rental Rate (MPK)	4.76	4.64	5.25	4.11	.084
	(.69)	(.63)	(.71)	(.60)	

B. Seasonal Deviations, Hodrick-Prescott Filter

	Q1	Q2	Q3	Q4	R^2
Output	-3.73	0.37	-0.33	3.71	.655
	(.58)	(.55)	(.56)	(.67)	
Consumption	-3.10	-0.44	-0.34	3.90	.845
	(.42)	(.32)	(.35)	(.47)	
Investment	-8.81	2.70	0.87	5.34	.552
	(1.38)	(1.39)	(1.37)	(1.34)	
Government	-2.71	0.18	0.61	1.95	.436
	(.51)	(.61)	(.52)	(.48)	
Capital	-0.01	-0.07	-0.08	0.14	.056
	(.30)	(.15)	(.19)	(.16)	
Labor Hours	-2.20	0.11	0.89	1.26	.394
	(.45)	(.37)	(.40)	(.44)	
Real Wage	-.34	0.01	-0.03	0.37	.017
	(.59)	(.33)	(.33)	(.35)	
Avg. Productivity	-1.76	0.35	-1.32	2.73	.766
	(.28)	(.34)	(.26)	(.36)	

**Standard errors are in parentheses.* 12 autocorrelations were used in The Newey-West procedure.

Table 5.2 Seasonal Patterns, Time-Separable Economy
500 Simulations of Stochastic Model

A. Seasonal Deviations, Log Growth Rates*

	Q1	Q2	Q3	Q4	AVG	R^2**
Output	-12.72	6.26	-3.34	9.79	0.25	0.950
Consumption	-6.35	2.43	0.18	3.74	0.25	0.944
Investment	-35.79	19.65	-16.33	32.47	0.25	0.964
Government	-4.41	2.45	0.76	1.20	0.25	0.484
Capital Stock	0.11	-0.32	0.61	-0.40	0.25	0.694
Labor Hours	-10.82	5.42	-2.81	8.21	0.00	0.983
Wage	-1.90	0.84	-0.53	1.58	0.25	0.592

B. Seasonal Deviations, Hodrick-Prescott Filter+

	Q1	Q2	Q3	Q4	AVG	R^2**
Output	-5.47	0.79	-2.55	7.23	0.00	0.786
Consumption	-2.85	-0.41	-0.24	3.50	0.00	0.751
Investment	-14.62	4.97	-11.34	21.03	0.00	0.865
Government	-2.51	-0.07	0.68	1.89	0.00	0.189
Capital Stock	0.03	-0.29	0.33	-0.08	0.00	0.184
Labor Hours	-4.71	0.71	-2.10	6.10	0.00	0.941
Wage	-0.76	0.08	-0.45	1.13	0.00	0.189

C. Seasonal Real Rate of Return, Annalized Percentage

	Q1	Q2	Q3	Q4	AVG	R^2**
Real Rate	4.15	3.66	5.16	3.22	4.05	0.551

+These figures are quarterly percentage rates, expressed as deviations from trend.

*These figures are quarterly percentage rates, expressed as deviations from the average growth rate in the sample period. For example, fourth quarter output grew at a quarterly rate of 10.04%, or 9.79% above the average quarterly rate of 0.25%.

**The R^2 from a regression on quarterly seasonal dummies.

Table 5.3 Seasonal Patterns, NonTime-Separable Economy (#1)
500 Simulations of Stochastic Model

A. Seasonal Deviations, Log Growth Rates*

	Q1	Q2	Q3	Q4	AVG	R^2**
Output	-8.52	3.90	-1.45	6.06	0.19	0.923
Consumption	-7.33	2.78	0.20	4.34	0.19	0.974
Investment	-15.85	8.03	-8.14	15.96	0.19	0.866
Government	-4.52	2.50	0.90	1.12	0.19	0.518
Capital Stock	0.05	-0.16	0.28	-0.17	0.19	0.308
Labor Hours	-4.47	1.72	-0.18	2.93	0.00	0.961
Wage	-4.05	2.19	-1.28	3.13	0.19	0.872

B. Seasonal Deviations, Hodrick-Prescott Filter+

	Q1	Q2	Q3	Q4	AVG	R^2**
Output	-3.71	0.18	-1.27	4.80	0.00	0.655
Consumption	-3.28	0.49	-0.29	4.06	0.00	0.841
Investment	-5.91	2.05	-6.07	9.93	0.00	0.532
Government	-2.60	-0.10	0.80	1.89	0.00	0.215
Capital Stock	0.02	-0.14	0.14	-0.03	0.00	0.036
Labor Hours	-1.93	-0.22	-0.39	2.54	0.00	0.761
Wage	-1.78	0.39	-0.88	2.26	0.00	0.540

C. Seasonal Real Rate of Return, Annalized Percentage

	Q1	Q2	Q3	Q4	AVG	R^2**
Real Rate	3.86	3.65	4.55	3.29	3.79	0.403

+These figures are quarterly percentage rates, expressed as deviations from trend.

*These figures are quarterly percentage rates, expressed as deviations from the average growth rate in the sample period. For example, fourth quarter output grew at a quarterly rate of 6.25%, or 6.06% above the average quarterly rate of 0.19%.

**The R^2 from a regression on quarterly seasonal dummies.

Table 5.4 Seasonal Patterns, NonTime-Separable Economy (#2)
500 Simulations of Stochastic Model

A. Seasonal Deviations, Log Growth Rates*

	Q1	Q2	Q3	Q4	AVG	R^2**
Output	-8.06	3.65	-1.47	5.89	0.28	0.921
Consumption	-6.90	2.06	0.01	4.83	0.28	0.938
Investment	-14.68	8.28	-7.54	13.93	0.28	0.928
Government	-4.61	2.71	0.81	1.09	0.28	0.569
Capital Stock	0.06	-0.14	0.25	-0.17	0.28	0.322
Labor Hours	-3.67	1.31	-0.08	2.44	0.00	0.942
Wage	-4.40	2.34	-1.39	3.45	0.28	0.899

B. Seasonal Deviations, Hodrick-Prescott Filter+

	Q1	Q2	Q3	Q4	AVG	R^2**
Output	-3.47	0.16	-1.30	4.61	0.00	0.648
Consumption	-2.76	-0.71	-0.68	4.15	0.00	0.817
Investment	-5.91	2.32	-5.19	-8.79	0.00	0.592
Government	-2.68	0.01	0.80	1.90	0.00	0.244
Capital Stock	0.03	-0.12	0.13	-0.04	0.00	0.037
Labor Hours	-1.54	-0.25	-0.33	2.12	0.00	0.675
Wage	-1.92	0.41	0.97	2.48	0.00	0.613

C. Seasonal Real Rate of Return, Annalized Percentage

	Q1	Q2	Q3	Q4	AVG	R^2**
Real Rate	4.23	4.01	4.91	3.69	4.21	0.382

+These figures are quarterly percentage rates, expressed as deviations from trend.

*These figures are quarterly percentage rates, expressed as deviations from the average growth rate in the sample period. For example, fourth quarter output grew at a quarterly rate of 6.17%, or 5.89% above the average quarterly rate of 0.28%.

**The R^2 from a regression on quarterly seasonal dummies.

Table 5.5 Second Moment Properties, Various Detrending Filters*

1. Relative Volatility: σ_x/σ_y

	1-Quarter Growth					HP Filter					Unadjusted 1-Quarter Rates				
x-variables	Data	Std Err	NTS #1	NTS #2	TS	Data	Std Err	NTS #1	NTS #2	TS	Data	Std Err	NTS #1	NTS #2	TS
Output[a]	.016	.002	.016	.016	.020	.026	.002	.023	.022	.025	.051	.003	.059	.056	.091
Consumption	0.73	.054	0.45	0.71	0.47	0.60	.036	0.50	0.55	0.53	0.88	.033	0.78	0.80	0.44
Investment	1.94	.259	3.07	2.05	2.65	2.42	.116	2.77	2.30	2.27	1.96	.069	2.32	2.15	3.09
Government	1.21	.193	1.61	1.54	1.37	0.86	.243	1.44	1.40	1.40	0.66	.063	0.64	0.66	0.42
Capital[b]	0.18	.026	0.18	0.17	0.14	0.32	.060	0.25	0.23	0.20	0.07	.009	0.06	0.06	0.05
Labor Hours	0.52	.065	0.35	0.36	0.49	0.73	.033	0.40	0.42	0.40	0.42	.025	0.49	0.42	0.83
Avg. Prod.	0.89	.050	0.67	0.66	0.56	0.75	.110	0.63	0.61	0.63	0.70	.024	0.52	0.59	0.19
Real Wage	0.42	.064	na	na	na	0.73	.114	na	na	na	0.14	.020	na	na	na
Real Rate[c]	0.36	.073	0.36	0.38	0.34	na	na	na	na	na	0.28	.086	0.13	0.13	0.11

2. *Contemporaneous Correlation:* x with Output

x-variables	Data	Std Err	NTS #1	NTS #2	TS	Data	Std Err	NTS #1	NTS #2	TS	Data	Std Err	NTS #1	NTS #2	TS
Output	1.00	--	1.00	1.00	1.00	1.00	--	1.00	1.00	1.00	1.00	--	1.00	1.00	1.00
Consumption	0.72	.075	0.18	0.76	0.73	0.87	.030	0.49	0.75	0.81	0.96	.010	0.94	0.97	0.96
Investment	0.72	.059	0.94	0.90	0.93	0.93	.024	0.94	0.96	0.95	0.94	.010	0.96	0.97	0.99
Government	0.38	.081	0.71	0.73	0.74	0.30	.110	0.73	0.75	0.76	0.83	.030	0.75	0.77	0.71
Capital[b]	0.26	.108	0.42	0.27	0.40	0.28	.070	0.44	0.30	0.36	0.50	.060	0.55	0.54	0.78
Labor Hours	0.46	.085	0.95	0.95	0.95	0.67	.100	0.96	0.96	0.95	0.81	.020	0.99	0.99	0.99
Avg. Prod.	0.82	.040	0.99	0.99	0.96	0.59	.080	0.98	0.98	0.98	0.93	.010	0.99	0.99	0.88
Real Wage	0.47	.103	na	na	na	0.45	.250	na	na	na	0.17	.080	na	na	na
Real Rate[c]	0.25	.098	-0.08	-0.09	-0.08	na	na	na	na	na	-0.09	.095	-0.32	-0.32	-0.45

*The sample period of the data is 1964:II - 1985:IV. The standard errors are for the data's moments; and 12 lags are employed in Newey-west estimator. The stochastic models were simulated 500 times.

[a]The "output" row reports the Standard Deviation of output.
[b]The capital stock refers to K_{t+1}, whereas the other variables are X_t.
[c]The real rate is not filtered, for comparability with Barsky-Miron [1989].
na--not applicable. In the case of the real wage, it is proportional to Avg. Prod.

Table 5.6
Autocorrelations of Consumption*

Unadjusted Growth Rates

Lag	Data	NTS #1	NTS #2
1	-.602 (.022)	-.603	-.616
2	.272 (.032)	.252	.265
3	-.604 (.018)	-.606	-.590
4	.972 (.009)	.977	.931

Adjusted Growth Rates

Lag	Data	NTS #1	NTS #2
1	-.174 (.096)	.085	-.301
2	.088 (.244)	.256	.055
3	-.043 (.093)	-.026	.077
4	.569 (.039)	.277	-.039

*Standard errors are in parentheses

Technical Appendix

to

Seasonality and Equilibrium Business Cycle Theory

This technical appendix describes how the competitive equilibrium of the stationary, stochastic economy in Braun and Evans[1990] can be linearized and mapped into the state space framework of King, Plosser, and Rebelo[1988].

The following statement of the household's problem is equivalent to the scaled economy given by equations [2.1'], [2,2'], [2.4'], [2.5'] and [2.7']. To conserve on notation in what follows, all variables are stationary. That is the tilda's which appear in [2.1']–[2.7'] are omitted below. The household's objective is given by:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t\{\tau_t \ln(c_t^*) + \gamma_2 \ln(l_t^*)\} +$$

$$\beta^t \mu_t^1\{w_t n_t + r_t k_t e^{-\lambda_t} + tr_t + (1-\delta)k_t\, e^{-\lambda_t} - cp_t - k_{t+1}\} +$$

$$\beta^t \mu_t^2\{-c_t^* + cp_t + \gamma_1 g_t + a(cp_{t-1} + \gamma_1 g_{t-1})e^{-\lambda_t}\} +$$

$$\beta^t \mu_t^3\{-l_t^* + (T-n_t) + b(T-n_{t-1})\}$$

First Order Necessary Conditions

$$(1) \qquad \frac{\tau_t}{c_t^*} - \mu_t^2 = 0$$

(2) $$\frac{\gamma_2}{l^*_t} - \mu^3_t = 0$$

(3) $$-\mu^1_t + \mu^2_t + E_t\{\mu^2_{t+1} a\beta e^{-\lambda_{t+1}}\} = 0$$

(4) $$\mu^1_t w_t - \mu^3_t - E_t\{\beta b \mu^3_{t+1}\} = 0$$

(5) $$-\mu^1_t + E_t\{\beta \mu^1_{t+1}(r_{t+1} e^{-\lambda_{t+1}} + (1-\delta)e^{-\lambda_{t+1}})\} = 0$$

Substituting in the marginal product pricing relationships yields:

(4') $$\mu^1_t(1-\theta)k^{\theta}_t n_t^{-\theta} e^{-\theta\lambda_t} - \mu^3_t - E_t\{\beta b \mu^3_{t+1}\} = 0$$

(5') $$-\mu^1_t + E_t\{ \beta\mu^1_{t+1}(\theta k^{\theta-1}_{t+1} n^{1-\theta}_{t+1} e^{-\theta\lambda_{t+1}} + (1-\delta)e^{-\lambda_{t+1}})\} = 0$$

Next we derive the log–linearization of the first order conditions (1)–(3),(4'),(5'), the aggregate resource constraint and the μ^2 and μ^3 constraints. The linearization is taken about the perfect foresight seasonal equilibrium path. Consequently, the point where the linearization is centered shifts with the season. We will use the following additional notation:

$\tilde{x}_t$ – the natural logarithm of the variable x_t,

x – the current period's steadystate value of the variable x,

x' – next period's steadystate value of the variable x,

x^- – last period's steadystate value of the variable x.

Thus, if today is Spring then y is the Spring steadystate value of output, y^- is the Winter steadystate value of output and y' is the Summer steadystate value of output.

(1L) $$\frac{\tau}{c^*}(\tilde{c}^*_t - \tilde{c}^*) + \mu_2(\tilde{\mu}^2_t - \tilde{\mu}^2) = 0$$

(2L) $$\frac{\gamma_2}{l^*}(\tilde{l}^*_t - \tilde{l}^*) + \mu_3(\tilde{\mu}^3_t - \tilde{\mu}^3) = 0$$

(3L) $$-\mu^1(\tilde{\mu}^1_t - \tilde{\mu}_1) + \mu_2(\tilde{\mu}^2_t - \tilde{\mu}^2) + a\beta e^{-\lambda'}\mu_2'(\tilde{\mu}^2_{t+1} - \tilde{\mu}^{2\prime}) - a\beta e^{-\lambda'}\mu_2'(\lambda_{t+1} - \lambda') = 0$$

(4'L) $$\mu^1(1-\theta)y/n(\tilde{\mu}^1_t - \tilde{\mu}^1)$$

$$+\mu^1(1-\theta)\theta y/n(\tilde{k}_t - \tilde{k}) - \mu^1(1-\theta)\theta y/n(\tilde{n}_t - \tilde{n})$$

$$-\mu^1(1-\theta)\theta y/n(\lambda_t - \lambda)$$

$$-\mu_3(\tilde{\mu}^3_t - \tilde{\mu}^3) - \beta b\mu_3'(\tilde{\mu}^3_{t+1} - \tilde{\mu}^{3\prime}) = 0$$

(5'L) $$-\mu^1(\tilde{\mu}^1_t - \tilde{\mu}^1) + \mu^{1\prime}\beta\{\theta y'/k' + (1-\delta)e^{-\lambda'}\}(\tilde{\mu}^1_{t+1} - \tilde{\mu}^{1\prime})$$

$$+\mu^{1\prime}\beta\theta(\theta-1)y'/k'(\tilde{k}_{t+1} - \tilde{k}')$$

$$-\frac{\mu^{1\prime}\beta\theta(1-\theta)y' l^{*\prime}}{n'k'}(\tilde{l}^*_{t+1} - \tilde{l}^{*\prime})$$

$$-\frac{\mu^{1\prime}\beta\theta(1-\theta)y' bn}{n'k'}(\tilde{n}_t - \tilde{n})$$

$$+ \mu^{1\prime}\beta\{-\theta^2 y'/k' -(1-\delta)e^{-\lambda'}\}(\lambda_{t+1} - \lambda') = 0$$

(6L) Aggregate Resource Constraint:

$$-k'(\tilde{k}_{t+1} - \tilde{k}') + (\theta y + (1-\delta)ke^{-\lambda})\{(\tilde{k}_t - \tilde{k}) - (\lambda_t - \lambda)\}$$

$$+ (1-\theta)y(\tilde{n}_t - \tilde{n}) - cp(\tilde{cp}_t - \tilde{cp}) - \rho g(\tilde{g}_{t-1} - \tilde{g}^-) - g(u_t - u) = 0$$

(7L) μ_2 constraint.

$$- c^*(\tilde{c}_t^* - \tilde{c}^*) + cp(\tilde{cp}_t - \tilde{cp}) + \gamma_1 g(u_t - u)$$

$$+ ae^{-\lambda}cp^-(\tilde{cp}_{t-1} - \tilde{cp}^-) + (a\gamma_1 e^{-\lambda}g^- + \rho\gamma_1 g)(\tilde{g}_{t-1} - \tilde{g}^-)$$

$$-a(\tilde{cp}^- + gama1*g^-)e^{-\lambda}(\lambda_t - \lambda) = 0$$

(8L) μ_3 constraint

$$-l^*(\tilde{l}_t^* - \tilde{l}^*) - n(\tilde{n}_t - \tilde{n}) - bn^-(\tilde{n}_{t-1} - \tilde{n}^-) = 0.$$

It is important to recognize that equations (1L)–(8L) hold on a season by season basis. Each expression implicitly defines four equations, one for each season. Following King, Plosser, and Rebelo, the linearized system of equations (1L)–(8L) can be expressed in a state space systems representation using the following two matrix equations:

(9) $$Mcc\begin{bmatrix} \vec{c}^{\,*}_t \\ \vec{i}^{\,*}_t \end{bmatrix} = Mcs \begin{bmatrix} \vec{k}_t \\ \vec{c}_{t-1} \\ \vec{n}_{t-1} \\ \vec{\mu}^{1}_t \\ \vec{\mu}^{2}_t \\ \vec{\mu}^{3}_t \end{bmatrix} + Mce \begin{bmatrix} \vec{\boldsymbol{\lambda}}_t \\ \vec{\mathbf{u}}_t \\ \vec{g}_{t-1} \end{bmatrix}$$

and,

(10) $$Mss(L) \begin{bmatrix} \vec{k}_{t+1} \\ \vec{c}_t \\ \vec{n}_t \\ \vec{\mu}^{1}_{t+1} \\ \vec{\mu}^{2}_{t+1} \\ \vec{\mu}^{3}_{t+1} \end{bmatrix} = Msc(L) \begin{bmatrix} \vec{c}^{\,*}_{t+1} \\ \vec{i}^{\,*}_{t+1} \end{bmatrix} + Mse(L) \begin{bmatrix} \vec{\boldsymbol{\lambda}}_{t+1} \\ \vec{\mathbf{u}}_{t+1} \\ \vec{g}_t \end{bmatrix}$$

for L=0,1. The vector notation $\vec{x}_t$ corresponds to:

$$\vec{x}_t \equiv \begin{bmatrix} \tilde{x}_{wt} - \tilde{x}_w \\ \tilde{x}_{st} - \tilde{x}_s \\ \tilde{x}_{sut} - \tilde{x}_{su} \\ \tilde{x}_{ft} - \tilde{x}_f \end{bmatrix}$$

and $\vec{\mathbf{x}}_t = \exp(\vec{x}_t)$. In addition the lag operator L has the following definition:

$$L\vec{x}_t = \vec{x}_{t-1} \equiv \begin{bmatrix} \tilde{x}_{wt-1} - \tilde{x}_w \\ \tilde{x}_{st-1} - \tilde{x}_s \\ \tilde{x}_{sut-1} - \tilde{x}_{su} \\ \tilde{x}_{ft-1} - \tilde{x}_f \end{bmatrix}.$$

The letter w denotes winter, s denotes spring, su denotes summer and f denotes fall. In addition, L denotes the backshift operator. Implicit in this sytems representation is the treatment of consumption and leisure services as controls, capital, consumption and leisure as endogenous state variables, with the La–Grange multipliers the costate variables. The two innovations and last periods government purchases are exogenous state variables. In order to fill the elements of each of these matrices we use equations (1L) and (2L) in equation (9), and equations (3L)–(8L) in equation (10). Since each of these equations holds on a season by season basis we have a total of eight equations defining (9) and 24 equations defining equation (10). The dimensions of the various matrices are as follows:
Mcc: (8x8), Mcs: (8x24), Mce: (8x12), Mss(0) and Mss(1): (24x24), Msc(0) and Msc(1): (24x8), Mse(0) and Mse(1): (24x12). Note further that the transition of the state variables has the same structure used by Todd [1990] and Hansen and Sargent [1990]. If today is spring then the current stock of capital is k_{spt}, last period's stock was k_{wt-1} and tomorrows will be k_{sut+1}. Given the matrices of the systems representation it is now straightforward to solve for the loglinear decision rules using programs developed by King, Plosser and Rebelo.