BUSINESS CYCLES AND THE ASSET STRUCTURE OF FOREIGN TRADE

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ABSTRACT

Since the primary role of international financial linkages is to facilitate consumption smoothing in the face of country-specific shocks, the degree of international financial integration should play an important role in the international transmission of business cycles. This paper therefore studies the business cycle implications of restricting international trade in financial assets. The key restriction is that domestic residents must hold all risky claims to domestic output, trading only noncontingent bonds on the international asset markets. We find that restricting asset trade may or may not change the business cycle implications of the model relative to complete markets, depending on the parameterization of the stochastic process for productivity. When there are important differences, these stem largely from differential wealth effects. We also find that restricting asset trade can resolve the chief problem inherent in complete markets models, which is their predictions of too-high consumption correlations and too-low output correlations. When technology follows a random walk process, the restricted asset markets model predicts that cross-country output correlations are positive, and cross-country consumption correlations are smaller than the output correlations, as is typically observed in the data.

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1. Introduction

The primary economic function of financial markets is to permit individuals to smooth consumption in the face of fluctuations in income. On a national level, domestic financial markets provide opportunities for (i) intertemporal trade between citizens who face different expected income profiles, and (ii) risk-sharing between those who are subject to idiosyncratic income risks. On an international level, financial markets provide a potentially important avenue for citizens of a particular country to smooth consumption in the face of country-specific fluctuations (e.g., shifts in national taxes or government expenditure). The extent of financial linkages among countries may therefore be a central determinant of the amplitude, persistence, and international transmission of business cycles.

This paper explores the implications for open economy business cycles of restricting international trade in financial assets. The key restriction that we impose is that domestic residents must hold all risky claims to domestic output, trading only noncontingent bonds on the international asset markets. We build a quantitative general equilibrium model of interacting economies subject to random shocks to productivity; interest rates and asset prices are thus determined endogenously. As a benchmark for evaluating the implications of restricted asset trade, we compare the predictions of this model to those of the complete markets model in our earlier study (Baxter and Crucini [1991]). We find that, when there are important differences between the two models, these differences can be traced primarily to differential wealth effects of shocks under alternative asset structures.

This paper is related to several recent contributions to the literature on open economy business cycles. This literature is divided into partial equilibrium analyses of small open economies (e.g., Cardia [1991], Finn
[1989], and Mendoza [1991]), and general equilibrium analyses of a world comprising two national economies (Backus, Kehoe and Kydland [1992], Baxter and Crucini [1991], and Stockman and Tesar [1991]). In the partial equilibrium analyses, asset markets are highly restricted; at most, individuals are assumed to be able to trade noncontingent debt with the rest of the world. In the general equilibrium analyses, asset markets are assumed to be complete in the sense that the dynamic equilibria of these models display complete risk-pooling.

Both branches of the literature have serious shortcomings. The partial equilibrium approach postulates an exogenous interest rate process, shutting down any possibility of discussing the determinants of the world interest rate, and making the analysis very sensitive to the stochastic process specified for this key variable. The small open economy approach also prohibits the study of business cycle linkages among non-infinitesimal economies: the countries that comprise the OECD, for example.

The existing general equilibrium analyses, on the other hand, have been conducted under the assumption that all risks are fully pooled internationally—including risk deriving from fluctuations in labor income, and shocks to government expenditure and tax rates. While questionable on empirical grounds, this assumption was primarily justified on the basis of conceptual and computational ease: with complete risk-pooling, the equilibrium can be computed using straightforward extensions of methods developed in the closed economy real business cycle literature (e.g., Kydland and Prescott [1982] or King, Plosser, and Rebelo [1988]).

Skepticism concerning the validity of the complete markets assumption arises for at least two reasons. First, there are no internationally-traded assets currently traded which are explicitly contingent on realizations of
many types of uncertainty (variations in national tax rates, for example). Whether existing assets effectively act to hedge this type of risk is a more subtle question, and whether the risks are empirically important is also open to debate (see the recent contribution by Cole and Obstfeld [1991]).

However, some of the implications of these complete markets models are strongly at variance with the stylized facts of international business cycles; many of these implications plausibly stem from the extreme assumptions concerning risk-pooling. Specifically, one-sector complete-markets models generically predict international consumption correlations that are too high, relative to the data, and cross-country correlations of investment, labor input, and output that are too low. Notably, the one-sector models of Backus, Kehoe, and Kydland [1992] and Baxter and Crucini [1991] predict near-perfect correlation of consumption movements across countries. In the data, we find that cross-country consumption correlations are typically very weak, and are not even always positive (see the Appendix to Baxter and Crucini [1991]).

Further, these models have trouble generating positive comovement of investment, labor, and output because of two reinforcing factors. First, these one-sector equilibrium models all possess a version of the neoclassical "accelerator" mechanism by which investment responds rapidly and strongly to changes in the return to capital. In a multi-country setting with shocks that are partially country-specific in nature, this translates into a strong tendency for negative international comovement of investment. There is a simple economic reason for this: with one final good in the world economy, capital owners' primary concern is to locate their capital in the most productive location. Second, complete risk pooling implies that the equilibrium quantities of consumption and labor input in each country are
those that would be observed under optimal labor income insurance. Although these quantities may be supported as equilibria under a variety of financial market structures (i.e., there are many ways to "decentralize" the equilibrium) it is useful to think about the equilibrium as if this insurance were explicitly utilized. The optimal risk-sharing arrangements involve the following state-contingent responses to productivity shocks. First, those individuals who receive a favorable productivity shock work harder. Second, the optimal insurance character of the equilibrium requires that they transfer part of the proceeds to individuals living in the less productive country. Individuals who live in the less productive location work less hard, but their consumption increases because of the international transfer of goods (i.e., the insurance payments). Because of these two reinforcing factors, complete-markets models must be driven by shocks that are highly correlated across countries if they are to be able to replicate the observed tendency for national outputs and investments to move together. However, even in this case the complete markets model predicts that labor inputs are negatively correlated across countries.

Finally, is hard to imagine that optimal labor insurance would be sustainable (i.e., enforceable) in an international context: the country receiving the favorable shock would, ex post, not wish to pay the "insurance benefits" to the other country. These considerations motivated us to study the link between the international character of business cycles and the structure of international asset markets.¹

¹ Cole [1988] studied the implications of financial structure for business cycles in a two-period model with production. Many of our findings are qualitatively similar to his results. More recently, the relationship between asset markets and real activity has been studied by Conze, Lasry, and Sheinkman [1990], and Kollman [1990], and Backus [1991]. Their models differ from the one developed here; Kollman's model is the closest to ours. Kollman
The paper is structured as follows. Section 2 describes the model economy and discusses aspects of the solution procedure that differ from the prior, complete markets analyses. Section 3 begins with a review of previous work on estimating the stochastic process for productivity, and presents some new results. Taken together, these results suggest that productivity shocks are highly persistent, are correlated across countries, and may contain unit roots. Therefore, we compared the predictions of the complete markets model to the predictions of the model with restricted asset markets under two alternative parameterizations of the productivity process: (i) a trend-stationary process with innovations that are correlated across countries and with international transmission of shocks; and (ii) a difference-stationary process with correlated innovations. We find that the empirical implications of the models are very sensitive to the specification of the stochastic process for productivity. If productivity follows a trend-stationary process with highly persistent shocks and international transmission, the business cycle implications of the incomplete markets economy are very similar to those of the complete markets economy. If productivity follows a random walk, however, the implications of the alternative models are quite different. Section 4 explores the reasons behind the differential response under alternative asset structures by studying the dynamic response to a productivity shock originating in one country. Using King's [1990] method for decomposing consumption and labor responses into wealth and substitution effects, we find that the primary differences across asset structures can be

finds, as we do, that restrictions on asset markets leads to lower international correlations of consumption (Conze, et al. obtain this prediction as well). Kollman also explores the implications of additive productivity shocks as well as the traditional multiplicative shocks. He finds that the problem of negative international comovement is less severe with additive shocks.
traced to differential wealth effects. Section 5 briefly summarizes the paper's main results and discusses avenues for future research.

2. The Model

The basic structure of this model, in terms of preferences and technology, is identical to the structure in Baxter and Crucini [1991]. The main difference arising from restrictions on asset trade appears in the flow constraints (budget constraints), which differ across the two models. Foreign country variables are denoted by stars, and all variables are in national per capita terms.

Preferences: Individuals consume two goods: a produced consumption good, C, and leisure, L. They maximize expected lifetime utility, given by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [C_t^\theta L_t^{1-\theta}]^{1-\sigma} \quad \text{home country;}
\]

\[
E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} [C_t^\theta L_t^*^{1-\theta}]^{1-\sigma} \quad \text{foreign country.}
\]

In each country, individuals are subject to the constraint that hours worked in the marketplace plus hours of leisure cannot exceed the time endowment, normalized to one unit:

\[
1 - L_t - N_t \geq 0 \quad \text{home country;}
\]

\[
1 - L_t^* - N_t^* \geq 0 \quad \text{foreign country.}
\]

Technology: Production functions exhibit constant returns to scale. Production of the single final good requires as input labor and capital. Capital used in production in a specific country is not necessarily owned by residents of that country. Thus \( K_t \) represents capital in place in the home
country, not capital owned by residents of the home country. Labor is internationally immobile. Letting $N_t$ denote labor employed in the home country, these production functions are given by:

$$Y_t = A_t K_t^{1-\alpha} (X_t N_t)^\alpha$$  
home country;  \hspace{1cm} (5)

$$Y^*_t = A^*_t K^*_t^{1-\alpha*} (X^*_t N^*_t)^{\alpha*}$$  
foreign country  \hspace{1cm} (6)

In these production functions, the variables $X_t$ and $X^*_t$ represent the level of purely labor-augmenting technical change in the home and foreign countries, respectively, and each grows at a common, constant gross rate: $\tau = X_{t+1} / X_t = X^*_{t+1} / X^*_t$. The variables $A_t$ and $A^*_t$ represent the stochastic component of the productivity variable, and are assumed to follow a vector Markov process.

New capital goods are internationally mobile, subject to costs of adjustment governed by the function $\phi(I/K)$, with $\phi>0$, $\phi'>0$, $\phi''<0$. Capital accumulates over time according to:

$$K_{t+1} = (1-\delta)K_t + \phi(I_t/K_t)K_t$$  
home country;  \hspace{1cm} (7)

$$K^*_{t+1} = (1-\delta)K^*_t + \phi(I^*_t/K^*_t)K^*_t$$  
foreign country.  \hspace{1cm} (8)

2.1 A two country general equilibrium model with complete markets.

The first model that we shall study is also, in many ways, the simplest. There are two countries in the world, and individuals in the two countries are free to trade any state contingent asset they wish. Thus, in equilibrium, individuals will bear no idiosyncratic risk. The assumption of complete markets means that we can compute the competitive equilibrium for this world economy as the solution to the Euler equations from a standard Lagrangian problem.

Resource constraints: Since the consumption/investment good is internationally mobile, there is a single world resource constraint for this good. Letting $\pi$ denote the fraction of the world population residing in the
home country, the world resource constraint is:

$$\pi [Y_t - C_t - I_t] + (1-\pi) [Y^*_t - C^*_t - I^*_t] \geq 0.$$  

(9)

**Model solution:** The equilibrium of this economy consists of a set of functions describing the behavior of endogenous variables such as consumption, saving, investment, etc., as functions of the exogenous shocks to the model (i.e., the productivity shocks). Before solving our model, we transform it to remove deterministic trend components; this is accomplished by dividing all home country variables by $X_t$, and all foreign country variables by $X^*_t$. Lowercase letters are used below to denote transformed variables. Note that labor and leisure cannot have deterministic trends; otherwise, the "time constraints" (3) and (4) would eventually be violated. These variables continue to be represented by uppercase letters. Finally, the rate of time preference for the transformed (world) economy is

$$\beta = \beta(\gamma)^{\theta(1-\sigma)}.$$  

It can easily be shown that the second welfare theorem applies in our economy, i.e., competitive equilibrium and Pareto optimum will coincide. Thus a straightforward way to compute the equilibrium for the economy is to solve the following Lagrangian problem:

$$\mathcal{J} = \sum_{t=0}^{\infty} \beta^t \left\{ \pi u(c_t, L_t) + (1-\pi)u(c^*_t, L^*_t) \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right.
In programming this model, we found it convenient to distinguish between the capital stock in a particular location (\(k\) and \(k^*\)) and capital services used in production (\(ks\) and \(ks^*\)). The notation above reflects this distinction. The multipliers on the constraints in (10) have natural interpretations as (utility-denominated) shadow prices, as follows:

- \(w_t, w_t^*\): wage rate
- \(\lambda_t, \lambda_t^*\): price of existing capital
- \(\zeta_t, \zeta_t^*\): value marginal product of capital
- \(p_t\): price of the final good (price of new capital).

Letting \(D\) to denote the total derivative of a function of a single variable, and letting \(D_j\) denote the partial derivative of a function with respect to its \(j\)th argument, the first-order necessary conditions for this Lagrangian problem are:

\[
\begin{align*}
(c_t) & \quad D_1 u(c_t, L_t) - p_t = 0 & (11) \\
(L_t) & \quad D_2 u(c_t, L_t) - w_t = 0 & (12) \\
(ks_t) & \quad p_t A_t D_1 F(ks_t, N_t) - \zeta_t = 0 & (13) \\
(N_t) & \quad p_t A_t D_2 F(ks_t, N_t) - w_t = 0 & (14) \\
(i_t) & \quad \lambda_t D\phi(i_t/k_t) - p_t = 0 & (15) \\
(\zeta_t) & \quad k_t - ks_t = 0 & (16) \\
(w_t) & \quad 1 - L_t - N_t = 0 & (17) \\
(\lambda_t) & \quad \gamma k_{t+1} - (1-\delta)k_t - \phi(i_t/k_t)k_t = 0 & (18) \\
(k_{t+1}) & \quad E_t \mu(i_{t+1}/k_{t+1}) - \beta\lambda_{t+1} + \beta E_t \zeta_{t+1} - \gamma\lambda_t = 0 & (19) \\
p_t & \quad \pi[A_t F(ks_t, N_t) - c_t - i_t] + (1-\pi)[A_t^* F(ks_t^*, N_t^*) - c_t^* - i_t^*] = 0 & (20) \\
& \quad E_0 \lim_{t \to \infty} \frac{\beta^t}{\lambda_t k_{t+1}} = 0 & (21)
\end{align*}
\]

for all \(t \geq 0\), where \(\mu(z) \equiv [\phi(z) - zD\phi(z) + (1-\delta)]\). In addition, there are foreign-country analogs to equations (11)-(19) and (21). These Euler
equations, derived from an optimum problem as specified in the Lagrangian (10), also describe the equilibrium of a decentralized economy in which atomistic consumers interact with atomistic, competitive firms.

It is well-known that the system of equations that implicitly defines the equilibrium of the one-sector closed-economy model does not have an analytic solution, except in a small number of special cases. A variety of numerical methods have recently been developed for obtaining approximate solutions to a particular nonlinear equilibrium problem: see the summary paper by Taylor and Uhlig (1990) and the papers cited therein. One method which has been shown to work well for the closed-economy neoclassical model is the method by which one obtains log-linear approximations to the equilibrium decision rules that solve the Euler equations. The point around which the approximation is taken is the model's deterministic steady state. The resulting linear system is solved by application of standard linear systems theory. This method is described in detail in King, Plosser, and Rebelo [1987], and is the method we use in this paper.

2.2 A Partial Equilibrium Model of a Small Open Economy

This sub-section describes a model of an open economy that is assumed to be too small to affect the world interest rate. This is a partial equilibrium model because the small open economy optimizes in the face of an exogenous process for the world interest rate. Solving the small open economy model is a useful step toward constructing a general equilibrium model with restricted asset trade.

Flow budget constraint: In this model, the bonds take the form of pure discount bonds. We let \( r_t \) denote the exogenously-given world rate of return
on risk-free securities, and let $R_t = (1 + r_t)^{-1}$ denote the price per unit of one-period discount bonds purchased in period $t$. $B_{t+1}$ denotes the quantity of bonds purchased in period $t$ (maturing in $t+1$). Following our earlier convention of letting lowercase letters refer to the transformed economy, we let $b_t = B_t / X_t$ denote the value of bonds in the transformed economy. Then the flow budget constraint for the small open economy in period $t$ is:

$$\gamma R_t b_{t+1} + c_t + i_t \leq y_t + b_t. \quad (22)$$

The Lagrangian for the partial equilibrium small open economy problem is:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, L_t) + \right.$$  
$$+ \omega_t (1 - N_t - L_t)$$  
$$+ \lambda_t \left[ \gamma k_{t+1} - \phi(i_t/k_t)k_t + (1-\delta)k_t \right]$$  
$$+ \zeta_t [k_t - k_{t-1}]$$  
$$+ p_t \left[ b_t + A_t F(k_{t-1}, N_t) - c_t - i_t - R_t \gamma b_{t+1} \right] \left. \right\}.$$  

The interpretation of the Lagrange multipliers for this problem is similar to the equilibrium problem discussed above: $\omega_t$ is the (utility-denominated) wage rate; $\lambda_t$ is the value of a unit of installed capital, $\zeta_t$ is the value marginal product of capital, and $p_t$ is the value of an additional unit of the consumption good.

Many of the first-order necessary conditions for this problem are exactly the same as the corresponding efficiency conditions for the complete-markets economy described above. Specifically, the first-order conditions for $c_t$, $L_t$, $k_{t-1}$, $N_t$, $i_t$, $\zeta_t$, $\omega_t$, $\lambda_t$, and $k_{t+1}$ are given by equations (11) to (19) above. There are two first-order conditions that are different: these are the efficiency conditions for $b_{t+1}$ and for $p_t$, which are as follows:
\[ (b_{t+1}) - \beta P_{t+1} - \gamma P_t R_t = 0 \]  
\[ (p_t) \quad b_t + A_t F(k_s_t, N_t) - c_t - i_t - R_t \gamma b_{t+1} = 0 \]

Thus the steady state real interest rate is determined by equation (23): $R \gamma = \beta$. The conditions (11)-(17) determine the vector of controls $[c_t, L_t, k_s_t, N_t, i_t, c_t, q_t, \omega_t]$ as functions of the controlled states $(b_t, k_t)$; the corresponding costates $(p_t, \lambda_t)$; and the exogenous variables $(A_t, R_t)$. The four equations (18), (19), (23), and (24) define the fundamental state–costate difference equation.

**Model Parameterization:** By permitting asset accumulation, we have introduced a new parameter: the steady state level of assets relative to output. In closed economies and in multi–country models with fully–pooled equilibria, each country’s holdings of assets which are in zero net supply must be constant along any equilibrium path. In general, however, whenever there are multiple countries and incomplete risk pooling, the steady state level of asset holdings will not be invariant to shocks to the world economy. This introduces an additional degree of freedom into the parameterization.

In our applications, we use this degree of freedom to specify $\theta_b = b/y$: the initial steady state bond–to–output ratio of the home country.

### 2.3 General Equilibrium in a Two Country World Trading Goods and Bonds

The model differs from the general equilibrium economy described in section 2.1 above in that this world economy is restricted to trade only goods and non–contiguous debt. In world general equilibrium, each of the countries faces the problem described in Section 2.2, but in general equilibrium the interest rate process ($R_t$) is endogenously determined. As before, let $\pi$ be the share of home country in the world economy. Then, bond
market clearing requires that:

\[ \pi b_t + (1-\pi)b^*_t = 0 \]  

(25)
since the bonds are in zero net supply in the world economy. Combining equation (25) with aggregate financial asset accumulation equation

\[ \pi R_t b_{t+1} + (1-\pi)R_t b^*_t \leq \pi \left\{ b_t + A_t F(ks^*_t, N_t) - c_t - i_t \right\} + (1-\pi) \left\{ b^*_t + A^* F(ks^*_t, N^*_t) - c^*_t - i^*_t \right\}. \]

implies goods-market clearing (due to Walras' Law):

\[ \pi (A_t F(ks^*_t, N_t) - i_t - c_t) + (1-\pi)(A^* F(ks^*_t, N^*_t) - c^*_t - i^*_t) > 0. \]  

(26)

**Modifications of efficiency conditions:** In this incomplete markets setting, we nevertheless have the same equilibrium condition for \( p_t \) as in the complete-markets model of section 2.1:

\[ (p_t) \quad \pi [A_t F(ks^*_t, N_t) - c_t - i_t] + (1-\pi) [A^* F(ks^*_t, N^*_t) - c^*_t - i^*_t] = 0 \]  

(27)

In addition, we have one additional pair of state and co-state equations which are the efficiency conditions for \( b^*_{t+1} \) and \( p^*_{t+1} \). These are given by (28) and (29) below:

\[ (b^*_{t+1}) \quad E_t (p^*_{t+1}/p^*_t) = E_t (p_{t+1}/p_t) \]  

(28)

\[ (p^*) \quad b^*_t + A^*_t F(ks^*_t, N^*_t) - c^*_t - i^*_t - \gamma R_t b^*_{t+1} = 0 \]  

(29)

where (28) uses the fact that \( R_t = E_t (p_{t+1}/p_t) \).

**Model solution:** The key issue is how to use the information contained in (25) and (26) to compute the world general equilibrium. The procedure we use is as follows. First, we drop one of the asset accumulation equations since (25) implies that, in a two-country world, only one of the asset stocks are
independent. In this two-country setting, we assume that this is the foreign country's asset accumulation equation. Second, we treat the home country's shadow price \((p)\) as an additional control variable. That is, we add the efficiency condition \((26)\) to the system of equations \({((11)-(19), (23)-(24)})\) for both countries. This augmented system determines the world control vector as a function of the world state vector \(\left[ k_t, k_t^*, b_t^* \right]\); the world costate vector \(\left[ \lambda_t, \lambda_t^*, p_t^* \right]\); and the exogenous variables \(\left[ A_t, A_t^* \right]\).

Third, we impose the equilibrium condition that \(R_t = \beta E_t(p_{t+1}/\gamma p_t)\). That is, we replace \(R_t\) with the expression \(\beta E_t(p_{t+1}/p_t)\) in the accumulation equations for \(b_{t+1}^*\). This three-step procedure yields a dynamic system with that can be linearized and solved in the standard manner.

3. Implications for Business Cycles

In this section we examine the implications for the character of international business cycles of restricting the portfolio of internationally tradable assets. We compare two asset structures: (i) the complete-markets structure employed in the equilibrium business cycle research program; and (ii) a restricted structure in which the only traded asset is noncontingent bonds, as assumed in traditional small open economy models and in Friedman's [1957] and subsequent partial equilibrium models embodying the permanent income hypothesis.

Previous research in the international real business cycle literature has found that aspects of the models' implications for business cycles are sensitive to specification of the exogenous process driving the model, and we find that this is the case here as well. Therefore we briefly review the findings of this prior literature and present some new empirical evidence on international productivity.
3.1 Measuring productivity

Following the work of Solow [1957], it has become commonplace to measure disembodied productivity ($A_t$ and $A^*_t$, in our notation) as a residual from a Cobb–Douglas production function. In the notation of our model, the "Solow residuals" would be measured (using (5) and (6)) as:

$$
\log(A_t) \equiv \log(y_t) - (1-\alpha)\log(k_t) - \alpha \log(N_t)
$$

$$
\log(A^*_t) \equiv \log(y^*_t) - (1-\alpha^*)\log(k^*_t) - \alpha^* \log(N^*_t)
$$

Measurement of the Solow residual therefore requires measures of output; capital input; labor input; and factor shares. For the United States, measures of all these variables are available, although there naturally is substantial disagreement concerning the accuracy of these measures. For other countries, in many cases, the necessary data are not readily available.

Backus, Kehoe, and Kydland (BKK) [1992] used output data and employment data to construct estimates of Solow residuals for the U.S., Canada, and an aggregate of six European countries. These measures omit the term involving capital input. However, Costello [1990] has shown that the stochastic properties of Solow residuals is sensitive to the measure of capital used to construct the residuals. Further, the mismeasurement of labor input by using employment in place of total hours worked is a potentially serious problem; Burdett and Wright [1989] show that for many European countries, more of the variance in total labor input is explained by hours variation than by employment variation. Despite the measurement problems, however, the BKK estimates provide a valuable starting point, and we briefly review their findings here.
Backus, Kehoe, and Kydland [1992] modeled the productivity shock process as the following vector-autoregressive process:

\[
\begin{bmatrix}
\log A_t \\
\log A^*_t
\end{bmatrix}
= \begin{bmatrix}
\rho & \nu^* \\
\nu & \rho^*
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A^*_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_t \\
\epsilon^*_t
\end{bmatrix},
\]

with \(E(\epsilon) = E(\epsilon^*) = 0\) and \(E(\epsilon^2) = \sigma^2_\epsilon, E((\epsilon^*)^2) = \sigma^2_{\epsilon^*}\), and \(E(\epsilon_t, \epsilon^*_t) = \psi\) for all \(t\). Backus, Kehoe, and Kydland [1992] estimate (30) for (i) the U.S. versus Canada, and (ii) the U.S. versus an aggregate of six European countries. Their estimates are given below; standard errors are in parentheses.

**U.S.:**
\[
\begin{bmatrix}
\log A_t \\
\log A^*_t
\end{bmatrix}
= \begin{bmatrix}
0.989 & 0.000 \\
0.060 & 0.093
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A^*_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_t \\
\epsilon^*_t
\end{bmatrix}
\]

\(\rho(\epsilon_t, \epsilon^*_t) = 0.434\).

**Canada:**
\[
\begin{bmatrix}
\log A_t \\
\log A^*_t
\end{bmatrix}
= \begin{bmatrix}
0.131 & 0.796 \\
0.052 & 0.079
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A^*_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_t \\
\epsilon^*_t
\end{bmatrix}
\]

**U.S.:**
\[
\begin{bmatrix}
\log A_t \\
\log A^*_t
\end{bmatrix}
= \begin{bmatrix}
0.904 & 0.052 \\
0.073 & 0.041
\end{bmatrix}
\begin{bmatrix}
\log A_{t-1} \\
\log A^*_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\epsilon_t \\
\epsilon^*_t
\end{bmatrix}
\]

\(\rho(\epsilon_t, \epsilon^*_t) = 0.258\).

We see from these estimates that shocks to productivity are highly persistent, and that there is some evidence of transmission of shocks from one country to another (\(\nu, \nu^* > 0\)). Further, the innovations to productivity are positively correlated across countries.

Because the BKK estimates indicate that shocks to the productivity process are highly persistent, we investigate the hypothesis that the Solow
residuals contain a unit root. Table 1, panel A reports the results of the J(p,q) test for a unit root in each of the three Solow residual series generated by Backus, Kehoe, and Kydland (Canada, Europe, and the U.S.). In each case, we fail to reject at conventional significance levels the null hypothesis that the series contains a unit root. A natural next question is whether there is a cointegrating relationship between the Solow residual time series. Table 1, Panel B reports the results of tests for cointegration: there is evidence of cointegration between the U.S. and Canada, but the evidence for cointegration is weaker for the U.S. versus Europe. Based on these results, we estimated a vector error-correction model for the U.S. versus Canada, and a standard VAR in first-differences for the U.S. versus Europe; the results are in Table 1, panel C. There appears to be no significant international transmission of shocks, with the possible exception of transmission from the U.S. to Canada. Based on these estimates, we cannot reject the hypothesis that productivity in each country follows a random walk with drift, with innovations that are positively correlated across countries.

In the remainder of the paper, therefore, we examine the business cycle implications of alternative asset structures under two assumptions concerning the stochastic process for productivity: (i) BKK's "symmetrized parameterization" characterized by trend-stationary shocks with correlated innovations and substantial international transmission ("spillovers"); and (ii) a random walk process for productivity with no spillovers, but with correlated innovations.

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2 All test statistics used in this analysis are discussed in Park [1990].
3.2 Trend stationary productivity with spillovers

We begin by comparing the cyclic behavior of the complete markets economy to that of the bond economy when the stochastic process for productivity is given by the BKK "symmetrized parameterization" of the relationship between the U.S. and Europe under which $\rho = \rho^* = .906$, $\nu = \nu^* = .088$. We set $\psi = .40$. Under this parameterization, innovations to productivity are positively correlated across countries ($\psi > 0$) and shocks that originate in one country "spill over" to the other country at the rate of 8.8% per quarter ($\nu = .088$). We set the innovation variances equal to one.\(^3\) The world is assumed to comprise two equally-sized countries, and there are small costs of adjustment in investment. The parameters of preferences and technology are the same as in Baxter and Crucini [1991]: $\sigma = 2$; $\alpha = .58$; $\beta = .9875$; $\gamma = 1.004$; $\delta = .025$. The parameters of the adjustment cost process are set as follows: (i) $\phi'$ is set so that the steady state value of Tobin's "q" ($1/\phi$) is one (i.e., the model with adjustment costs has the same deterministic steady state as the model without adjustment costs); and (ii) the elasticity of the investment-capital ratio with respect to Tobin's "q," is $\eta = -(\phi'/\phi) \cdot (i/k) = 15$. (See Baxter and Crucini [1991] for additional discussion regarding parameter choice, and sensitivity analyses for some critical parameters).

Table 2 presents summary business cycle statistics for eight OECD countries; the data has been filtered with the Hodrick and Prescott [1980] filter. The central stylized facts of business cycles are similar across countries: there is a tendency for consumption to be less volatile than

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\(^3\) Since our log-linear solution algorithm generates decision rules that display certainty equivalence, only the scale of volatility changes as we change the innovation variances. Relative volatilities, such as the standard deviation of consumption divided by the standard deviation of output, are invariant in this setup to the size of the shock variance.
output, while investment is more volatile; (ii) output movements are highly persistent; and (iii) consumption and investment are both highly correlated with fluctuations in output. The lower panel of Table 2 shows that both outputs and consumptions have a tendency to covary positively across countries, but international consumption correlations tend to be lower than international output correlations. Finally, labor market data for the U.S. shows that (i) labor input is less volatile than output, as is average labor productivity; (ii) labor input is highly correlated with output, as is the average product of labor, and (iii) average labor productivity and the level of labor input are roughly uncorrelated.\footnote{Labor market statistics for other countries are omitted since accurate measures of total labor input are not readily available, as discussed in Section 3.1 above.}

Table 3 compares the response of the complete markets economy to the economy which is restricted to financial trade in bonds alone when both economies are driven by the trend-stationary productivity shock with spillovers. The statistics reported in this table are the model's population moments for Hodrick-Prescott [1980] filtered time series.\footnote{The population moments for the filtered time series are computed using the rational polynomial version of the Hodrick-Prescott filter applicable to an infinite sample of data, as discussed in King and Rebelo [1992]. Even though the incomplete markets model implies that there is a unit root component to each country's real quantity variables, the HP filter contains four differences in the numerator of the rational polynomial, so that population moments for the filtered series are well-defined.} Surprisingly, the differences between the business cycle implications of these two (apparently) very dissimilar models are really quite minor. Compared with the complete markets economy, the bond economy displays similar volatility of output, consumption, investment, labor input, and the wage rate.\footnote{In our model, the wage rate equals the average product of labor.} However, the volatility of the net-export-to-output ratio is substantially higher in the
bond economy and bond holdings (as a fraction of output) are about three
times as volatile as output. (Bond holdings have zero variance in the
complete markets economy; a well-known characteristic of fully-pooled
equilibria is that asset holdings need not fluctuate.) In terms of
persistence, the two models are essentially indistinguishable.

Turning to the contemporaneous correlation of macro aggregates with
output, we see that the bond economy predicts uniformly higher correlations
of all variables with output (in absolute value), although the numerical
differences are very small. The bond economy predicts higher international
correlations of output, investment, and labor input, and smaller
international correlations between consumption and wage rates. The
within-country correlation between saving and investment is slightly lower in
the bond economy compared with the complete markets economy. This might seem
surprising, since one's intuition is that closing asset markets, thus forcing
individuals to bear more country-specific risk, would act to increase
within-country saving-investment correlations. However, this "basic saving
measure" (defined as output minus consumption) need not be a good measure of
true saving in an open economy, as discussed by Obstfeld [1986], Stockman and
Svensson [1987], and Baxter and Crucini [1991].

Finally, the asset structure is important for the predicted correlation
between the wage rate and the level of labor input. The complete markets
economy predicts a correlation of .66, which is about the same as the
predictions of the closed-economy real business cycle model calibrated to
U.S. data (e.g., the model in Baxter and King [1991] predicts a correlation
of .65). Restricting asset trade to bonds alone has the effect of greatly
reducing the predicted correlation, to a level of .23. (In the U.S. data,
this correlation is −0.04.)
How do these models do overall in terms of generating empirically accurate predictions? As discussed by Backus, Kehoe, and Kydland [1992] and Baxter and Crucini [1991], the complete markets economy does reasonably well in matching the stylized facts concerning volatility and persistence of macro aggregates. Much more problematic are the complete markets model’s implications for cross-country correlations of output, consumption, investment, and labor input. Specifically, this model has difficulty generating positive output comovement (and correspondingly positive comovement of investments and labor inputs across countries). Further, the model predicts a level of cross-country consumption correlation that is much too high relative to the data.

Because individuals are subject to idiosyncratic (nation-specific) risk in the bond economy, in equilibrium this economy will display nation-specific fluctuations in consumption. Thus we expect that the international correlation between consumptions should be lower in the bond economy, and it is—but not much lower. The complete markets economy predicts an international consumption correlation of .95, while the bond economy predicts a correlation of .92. (Table 2 shows that the empirical correlations range from 0.11 to 0.65.)

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7 These results are not surprising in light of the results obtained by Backus, Kehoe, and Kydland [1992] and Baxter and Crucini [1991]. Both of these paper explore, in a primitive way, the importance of asset structure for consumption correlatedness by computing international consumption correlations under the assumption that (i) shocks are trend-stationary, and (ii) both countries operate in complete autarky. Even under these assumptions, international consumption correlations exceed .90. The reason is that, within each country, individuals act as permanent income consumers. With highly correlated outputs (stemming from the specification of the productivity process), consumption-smoothing leads individuals to choose consumption paths that are highly correlated across countries.
Similarly, the absence of insurance against labor income risk in the bond economy is important for the cross-country correlation of labor inputs. In the complete markets economy, the response to a positive productivity shock in one country generates an increase in labor input in the productive country, and a tendency for a decline in labor input in the relatively unproductive country. Because of the optimal insurance character of the complete markets equilibrium, workers in the productive country agree to "share" some of the additional output generated by the increase in productivity and labor input, in exchange for similar "sharing" when the other country receives a positive productivity shock. In the bond economy, individuals can only smooth consumption across time (by buying or selling bonds); they cannot smooth consumption across different "states of nature" because of the absence of contingent securities. This reduces the tendency for labor input to decline in the temporarily unproductive location (we will discuss the details of these mechanisms further in Section 4). But again, while we see this effect in somewhat higher international correlations between output, investment, and labor input, the effect is not strong enough to make the bond economy a good description of the international data along these dimensions.

In summary, with the BKK parameterization of the productivity process, we find that restricting international trade in financial assets to noncontingent bonds alone has very minor effects on the model's predictions for the business cycle behavior of the key macroeconomic aggregates. In particular, restricting asset markets helps only slightly in remedying the chief empirical failings of the one-sector international equilibrium business
cycle model, which are (i) predicted international correlations of consumptions that are too high, relative to the data, (ii) a tendency to predict very low international correlations of output, labor input, and investment, and (iii) too-high predicted correlations between labor input and wage rates.

3.3 A unit root in productivity

As shown in Section 3.1, we cannot reject the statistical hypothesis that the logs of total factor productivity (log $A_t$ and log $A^*_t$) follow random walk processes with correlated innovations. This section therefore examines the implications of a unit root for the behavior of the complete markets economy and the bond economy. Table 4 presents the two models' predictions for the central business cycle statistics, under the assumption that $\rho=\rho^*=1$, $\nu=\nu^*=0$ (as in Table 3, these are HP filtered population moments). All other parameters, including the contemporaneous correlation of the shocks ($\psi=.40$) and the unit innovation variances, are exactly the same as in Table 3.

It is immediately evident from Table 4 that there are important differences between the complete markets economy and the bond economy. In contrast to the results for the BKK parameterization, reported in Table 3, market structure matters a great deal when shocks are purely permanent. First, the levels of output volatility, investment volatility, and labor input are substantially higher in the complete markets economy, compared with the bond economy; in fact investment and labor input are about twice as volatile in the complete markets economy. Recall that one effect of the complete risk-pooling in the complete markets economy is a strong increase in labor input in response to positive productivity shocks; some of the additional product generated is sent to citizens of the nonproductive
location. Because of the complementarity of labor input and capital input, the stronger labor response in the complete markets setting is accompanied by a stronger investment response (and, consequently, a stronger output response).

The most striking differences between the models appear when we look at the international correlations of output and consumption. As noted above, a widely-discussed failing of the complete markets model is its robust prediction of too high an international consumption correlation, combined with a too-low prediction for the international correlation of outputs. In section 3.1, we saw that the bond economy shares this flaw when shocks to total factor productivity are trend-stationary and subject to spillovers. When the shocks are purely permanent, as in Table 4, the complete markets economy continues to exhibit this counterfactual pair of predictions; in fact, the predictions for output correlations are even worse (i.e., even more strongly negative). But when the bond economy is subject to purely permanent shocks, this model predicts a substantial, positive international output correlation (0.55) and a negative consumption correlation (-0.28)! (While this configuration of correlations is unusual in the data, Baxter and Crucini [1991] did find this pattern of positive output correlations and negative consumption correlations for four country pairs.) Evidently, in the context of the bond economy, allowing the shocks to be purely permanent has more than fixed the problem of overpredicting consumption correlations and underpredicting output correlations. However, it is likely that productivity contains both temporary and permanent components. Since the bond economy with purely temporary shocks overpredicts consumption correlations, and the model with purely permanent shocks underpredicts consumptions, the combination of both types of shocks may do quite well along this dimension.
With random walk productivity, the two asset structures also differ importantly in their implications for the cyclic behavior of labor input. The bond economy predicts a weak (.17) contemporaneous correlation of labor with output compared to the prediction of .93 for the complete markets economy. However, despite the fact that asset market restrictions have increased output correlations to an empirically reasonable level, the bond economy continues to underpredict international comovement of labor input and investment. The contemporaneous correlation between the net-export-to-output ratio and output is another dimension along which the models differ strongly: the complete markets economy predicts a strong negative correlation (−0.81), while the bond economy predicts a strong positive correlation (0.93).

Under the random walk parameterization, the complete markets model continues to predict high saving–investment correlations but the bond economy does not. In fact, the predicted correlation of 0.06 in the bond economy is much lower than saving–investment correlations typically found in the data for this measure of saving. (As noted earlier, however, this "basic saving" measure may not be an accurate measure of true saving in the economy.)

Finally, the complete markets model predicts a substantial positive correlation between productivity and labor input, while the bond economy generates a strongly negative correlation. In the data, these variables are roughly uncorrelated, so this is another case in which the combination of random walk productivity and restricted asset markets has moved the model predictions in the right direction, but by too much. And, as in the case of the consumption correlations, the combination of permanent and temporary shocks may lead the model to generate empirically reasonable predictions.
4. Dynamic Response to a Productivity Shock

The preceding section explored the implications for the summary statistics of business cycles of alternative assumptions concerning (i) the stochastic process for productivity shocks and (ii) market structure. The chief findings of that section were that restricting financial trade to noncontingent bonds alone had minor effects on the business cycle statistics when productivity was assumed to follow a trend stationary process exhibiting high persistence with substantial international transmission of shocks. However, when productivity contained a unit root, the restrictions on asset trade had important effects.

In order to explore the economic mechanisms behind these differential responses, this section studies the impulse responses of the alternative models when driven by the trend-stationary (BKK) process of section 3.2 versus random walk shocks, as in section 3.3. Throughout, we study the response of the world economy to a 1% increase in total factor productivity which originates in the home country: \( \Delta_t = 0.01 \).

4.1 Random walk productivity

In many ways the responses to purely permanent shocks are easier to understand, so we start with this case. Figure 1 plots the responses of aggregate quantity variables in the two countries, and Figure 2 plots the responses of real wages, real interest rates, and bond holdings. In both figures, stars denote the response of the bond economy, and open circles denote the response of the complete markets economy.

Figure 1 shows that, under both asset structures, home country output, consumption, and investment increase in response to the shock, while foreign country investment falls. However, labor market behavior in both countries
is sensitive to the asset structure, as is the comparative behavior (across countries) of consumption and output. First, under complete markets, labor increases in the home country and falls in the foreign country; the reverse is true in the bond economy. Second, under complete markets, consumptions move together across countries while outputs move in opposite directions. In the bond economy, by contrast, consumptions move in opposite directions while output rises in both countries (at least for the first few periods).

Figure 2 shows that the real interest rate implications of the shock are identical under the two asset structures; and that the positive productivity shock causes the home country wage rate to rise under both structures, reflecting the positive effect of the shock on labor productivity. However, the foreign country wage rate rises on impact in the complete markets economy but falls in the bond economy, mirroring the labor responses. Finally, with asset trade restricted to bonds alone, the foreign country accumulates bonds in response to the productivity shock in the home country, (there is no change in asset holdings in the complete markets economy). We have already seen that, in the bond economy, the foreign country responds by decreasing consumption and increasing labor input; thus they must be accumulating bonds over time. When adjustment to the shock is complete, the foreign country will work less and consume less than in the pre-shock steady state; however, a higher share of this consumption will be financed by the interest generated by the increased stock of debt accumulated over the transition path.

Figures 1 and 2 illustrated that the the within-country and cross-country responses of consumption and labor are sensitive to the asset structure. In order to understand the equilibrium responses of consumption and labor input, we employ King's [1990] "Hicksian" method for decomposing the consumption and labor supply responses into (i) a wealth effect, (ii) a real interest rate
effect, and (iii) a wage effect, we gain additional insight into the reasons why consumption and labor responses differ across asset structures. The wealth effect is computed as follows. First, compute the discounted present value of the change in utility caused by the altered time path of consumption and leisure (in response to the shock). Next, compute the constant consumption and leisure profiles that yield the same change in utility, using initial steady-state wages and interest rates. The real interest rate effect is that part of the response due to alterations in the interest rate alone, holding fixed wealth and wage rates at their initial steady state levels; the wage effect is computed in a similar fashion. These effects are plotted in Figure 3.

Beginning with the wealth effect on home country consumption, we find that the positive productivity shock has a positive wealth effect in the bond economy, but has a negative wealth effect under complete markets! The positive wealth shock in the bond economy is easy to understand — the positive productivity shock means that more output can be obtained using the same level of inputs. In the bond economy, these inputs are completely domestically-owned. Because individuals value both consumption and leisure, the natural response to a positive wealth shock (holding fixed all prices) is to consume more and work less; we see that the wealth effect on labor input is in fact negative in the bond economy (Figure 3-B).

Why is the home country wealth effect on consumption negative under complete markets? Recall that, under complete markets, the response to a location-specific positive productivity shock is for individuals living in the productive location to increase labor supply, taking advantage of the increase in productivity, while transferring some of the proceeds of the increased labor input to individuals living in less productive locations.
Although home country consumption rises in response to the shock, home country leisure falls so much that home country discounted utility actually falls in response to the shock. Thus, in order to produce the post-shock level of home country discounted utility at initial steady-state wages and interest rates, the home country must suffer a negative wealth effect.

In addition to the wealth effect, the productivity shock also induces substitution effects associated with (i) alterations in the time profile of real interest rates (the intertemporal price of consumption and leisure), and (ii) alterations in the time profile for the real wage rate. Since the real interest rate response is identical under the two asset structures (see Figure 2), the substitution effect stemming from this channel is identical in both cases. The substitution effect on consumption arising from the increase in the wage profile is positive in both cases (although the difference is minor), reflecting the fact that wages rise in response to the shock under both asset structures. Thus in the home country, the differential consumption response under the alternative asset structures is almost entirely due to differences in the size of the wealth effect.

Similar arguments explain the responses of home country labor supply: under complete markets, the wealth effect on labor is positive (i.e., the negative wealth effect induces an increase in labor input). With financial trade restricted to bonds alone, the productivity shock implies a positive wealth shock, thus labor input falls. As with home country consumption, the discount rate effects are identical across the two market structures: the increase in current real interest rates (an increase in the current price of leisure, relative to future leisure) leads to an increase in labor supply from this channel. The wage effect on labor input is positive in the bond economy, but negative in the complete markets economy. As with the
consumption response, the biggest difference between the labor response across asset structures lies in the wealth effects. Because the wealth effects are of different sign under the alternative structures, we find that labor input rises in the complete markets setting, but falls in the bonds-only economy.

In the foreign country it is also the case that the dominant differences across the asset structures lie in the wealth effects. Because foreign country residents do not own productive factors located in the home country, and because there is no international transmission of the productivity shock, there is a zero wealth effect of the shock on consumption and on labor supply. Under complete markets, however, there is a positive wealth effect on consumption and a negative wealth effect on labor supply. With optimal labor insurance, the efficient arrangement for the less-productive country to "take a paid vacation," working less and consuming more. Under complete markets, the strength of the wealth effect in depressing labor input is sufficient to counteract positive substitution effects from the increase in the real interest rate and the increase in the real wage rate. Thus foreign labor input rises in response to the shock in the bond economy, but falls in the complete markets economy.

4.2 Trend-stationary shocks with spillovers

Figures 4–6 plot the dynamic response to an innovation in home country productivity when productivity follows the trend-stationary process with spillovers specified in Section 3. We have already seen, in Table 3, that the summary statistics of business cycles are largely invariant to the asset structure under this parameterization of the productivity process. The dynamic responses detailed in Figures 4–6 give a similar impression: the
responses of the quantity variables (Figure 4) and prices and interest rates (Figure 5) show very similar responses under the alternative asset structures. The only significant difference is that assets are decumulated in the foreign country in response to the home country productivity shock in the bond economy, whereas there is no change in asset holdings in the complete markets economy.

The Hicksian decompositions of the consumption and labor responses plotted in Figure 6 confirm the general impression that there is little practical difference between the two asset structures. Recall that, with random walk productivity shocks, the primary difference across asset structures was due to differential wealth effects. In Figure 6, we see that the wealth effects on the two countries of the productivity shock are (i) small, and (ii) are virtually indistinguishable across the two asset structures. The wealth effect of a temporary shock will always be smaller than the wealth effect of a permanent shock, so that this in itself is not surprising. The fact that the wealth effects are almost identical across asset structures is more surprising, and this is due to the fact that the productivity shock is transmitted across countries over time via the "spillover" parameter, \( \nu \). In fact, 8.8% of the shock is transmitted each quarter, and apparently this is rapid enough so that the wealth effects of the shock are nearly identical across countries even when asset trade is restricted to bonds alone (the wealth effect is identical across countries under complete markets). As before, the real interest rate effects are identical across countries and across asset structures.

Figure 6-B is useful in understanding the forces behind negative comovement of labor input. Although the wealth effects and real interest rate effects of the shocks are approximately the same across countries, the
wage effects are quite different. The wage effect on home country labor is positive, reflecting the higher productivity due to the shock itself, combined with a rapid run-up in investment (see Figure 4). In the foreign country, there is no immediate effect on productivity, although individuals in that country realize that productivity will increase in the future due to the "spillovers." That is, labor productivity in the foreign country is low, on impact, compared with its expected future value. Intertemporal substitution considerations mean that foreign country residents are induced to increase current leisure with the expectation of lower future leisure when the spillover effect brings increased productivity to the foreign country.

Once we understand the importance of the international transmission of the shock (the "spillover") for the wealth effects on labor and consumption, it is easy to understand why foreign country residents decumulate bonds in response to the shock. As noted above, they know that the favorable shock will be coming to their country in a few quarters—rapidly enough that the positive wealth effect is nearly as large as in the originating economy. But on impact, productivity in the foreign country is low relative to its expected future level. Thus individuals respond by decreasing current labor supply and increasing current consumption, financing part of current consumption from the proceeds of bond sales.

In summary, we find that with trend-stationary shocks and fairly large spillovers, the absence of risk-sharing arrangements stemming from asset market restrictions is not important for the character of international business cycles. With this parameterization of the productivity shock process, nearly all of the fluctuations in productivity are common across countries, i.e., there is little scope for risk-sharing in the first place, although there is a role for intertemporal trade since productivity arrives
in the non-originating country with a lag. Thus closing markets for
risk-pooling by restricting asset trade to noncontingent bonds has little
effect on equilibrium outcomes.

5. Conclusions

This paper explored the importance of financial market linkages for the
character of international business cycles. The technical innovation of this
paper is the development of methods for studying dynamic general equilibrium
models with restrictions on international trade in financial assets. Our
main findings are as follows.

First, we found that restricting asset to noncontingent bonds alone does
not necessarily alter in an important way the predictions of the standard,
complete markets model. If the international productivity process is trend
stationary with substantial international "spillovers" of productivity
shocks, the two models are essentially indistinguishable. If, however,
productivity in each country follows a random walk, restricting asset trade
alters the predictions of the model in several important dimensions. In
particular, under this parameterization, the complete markets model predicts
low cross-country output correlations and near-perfect consumption
correlations; the bond economy conversely predicts high output correlations
and low consumption correlations. This finding is important, since the
complete markets model has been heavily criticized for its counterfactual
prediction of near-perfect international consumption correlations for a wide
range of parameterizations. With random walk shocks, restricting asset
markets brings the consumption correlation down substantially.

Second, we found that the major differences in the macroeconomic response
to shocks under the alternative asset structures are due almost entirely to
differential wealth effects. In particular, we found strong differences across asset structures when productivity follows a random walk. In this case, under complete markets, individuals receiving a favorable productivity shock experience a negative wealth effect, because the optimal insurance character of equilibrium requires them to increase labor supply while transferring a large proportion of the additional output to residents of the other country. In the bond economy, however, individuals own all the risky claims to their country's output. Thus individuals receiving a favorable productivity shock experience a positive wealth effect, which induces them to increase consumption by more than in the complete markets economy and, more importantly, causes them to decrease labor input (at least on impact).

Although the dominant source of differences across asset structures was found to lie in wealth effects, consumption and leisure are strongly affected in both asset structures by interest rate effects and wage effects. Thus the general equilibrium structure, in which interest rates and asset prices are determined endogenously, is important for understanding the way in which economies respond to exogenous shocks. This consideration is important even if the economy in question is "small" in the sense of having a small share of world product. As shown in Baxter and Crucini [1991], and as confirmed by the empirical analyses of Backus, Kehoe, and Kydland [1992] and the present paper, for this type of model to have sensible business cycle implications, it is necessary that productivity have a substantial component that is common across countries. Even though shocks to the small open economy may not have any direct effect on world interest rates, the commonness of shocks means that movements in the world interest rate are correlated with shocks in the small economy. This implies that the traditional assumption that the small open economy faces a fixed interest rate is not empirically defensible.
Further, as shown in section 4 of this paper, the interest rate effects on consumption and labor input are quantitatively important, even when asset trade is restricted to noncontingent bonds. Thus, even with incomplete financial markets, problems involving small open economies must nevertheless be studied in a general equilibrium framework, as stressed in Baxter and Crucini [1991] and Crucini [1991].

In summary, we have found that the asset structure of foreign trade can be important for the character of international business cycles, but that many model predictions are very sensitive to the parameterization of the productivity process. An important avenue for future research, therefore, is the continuation of the line of research begun by Costello [1990] and Backus, Kehoe, and Kydland [1992] on measuring international productivity. While this paper has studied the polar cases of (i) purely temporary shocks and (ii) purely permanent shocks, it is likely that fluctuations in productivity contains both temporary and permanent components. Since the bond economy with purely temporary shocks overpredicts consumption correlations, and the model with purely permanent shocks underpredict consumption, the combination of both types of shocks may do quite well along this dimension. Further, the complete markets model predicts a substantial positive correlation between productivity and labor input, while the bond economy generates a strongly negative correlation. (In the data, these variables are roughly uncorrelated). Thus combining temporary and permanent shocks may improve the empirical predictions of the restricted-asset-markets model along many dimensions. However, we leave this question for future research.
References


TABLE 1

Statistical Properties of International Solow Residuals

Panel A: Park and Choi J(p,q) Test for Unit Root

(The null hypothesis is a unit root: the hypothesis is rejected if the test statistic is smaller than the critical value)

<table>
<thead>
<tr>
<th>Measure of Solow Residual (time period)</th>
<th>Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>J(1,2)</td>
</tr>
<tr>
<td>U.S. (1965:3–1988:3)</td>
<td>0.124</td>
</tr>
<tr>
<td>Canada (1965:3–1988:3)</td>
<td>0.343</td>
</tr>
<tr>
<td>U.S. (1970:2–1986:4)</td>
<td>0.010</td>
</tr>
<tr>
<td>Europe (1970:2–1986:4)</td>
<td>0.740</td>
</tr>
</tbody>
</table>

critical values: 1% 8.6e-5 0.011 0.055 0.123 5% 0.002 0.055 0.160 0.295 10% 0.009 0.120 0.290 0.452

Panel B: Tests for Cointegration

We used Park's canonical cointegrating regression to estimate \( \alpha_1 \) such that \( A_t - \alpha A^*_t = \epsilon_t \), a stationary random variable. Next, we used Park's \( H(p,q) \) test for stochastic cointegration; p-values are given in the table below. In each case, the U.S. is the unstarred variable (i.e., \( \alpha \) is the coefficient on Canada or Europe).

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\alpha} )</th>
<th>se(( \hat{\alpha} ))</th>
<th>( H(1,2) )</th>
<th>( H(1,3) )</th>
<th>( H(1,4) )</th>
<th>( H(1,5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. – Canada</td>
<td>0.580</td>
<td>0.061</td>
<td>0.313</td>
<td>0.523</td>
<td>0.707</td>
<td>0.462</td>
</tr>
<tr>
<td>U.S. – Europe</td>
<td>0.603</td>
<td>0.041</td>
<td>0.046</td>
<td>0.039</td>
<td>0.082</td>
<td>0.145</td>
</tr>
</tbody>
</table>
Panel C: Estimates of stochastic processes for Solow residuals

$\Delta$ denotes the first difference of a variable, i.e., $\Delta A_t = A_t - A_{t-1}$; as before the U.S. is the unstarred country. Standard errors are in parentheses.

U.S.–Canada:

$\Delta A_t = 0.003 + 0.113 \Delta A_{t-1} + 0.048 \Delta A^*_{t-1} - 0.074 (A_{t-1} - A^*_{t-1}) + u_t$

\( (0.001) \quad (0.117) \quad (0.101) \quad (0.052) \)

$\Delta A^*_t = 0.005 + 0.283 \Delta A_{t-1} + 0.035 \Delta A^*_{t-1} + 0.021 (A_{t-1} - A^*_{t-1}) + u^*_t$

\( (0.001) \quad (0.131) \quad (0.112) \quad (0.058) \)

$\hat{\sigma}^2_u = 8.38e-3; \quad \hat{\sigma}^2_{u*} = 9.34e-3; \quad \hat{\rho}(u,u*) = 0.392.$

U.S.–Europe: (error-correction term omitted due to lack of cointegration)

$\Delta A_t = 0.002 + 0.003 \Delta A_{t-1} + 0.193 \Delta A^*_{t-1} + u_t$

\( (0.001) \quad (0.126) \quad (0.134) \)

$\Delta A^*_t = 0.005 + 0.196 \Delta A_{t-1} - 0.076 \Delta A^*_{t-1} + u^*_t$

\( (0.001) \quad (0.110) \quad (0.117) \)

$\hat{\sigma}^2_u = 9.07e-3; \quad \hat{\sigma}^2_{u*} = 7.95e-3; \quad \hat{\rho}(u,u*) = 0.228.$

Notes:

1. All the tests reported in this table are discussed in Park [1990].
### Table 2

Business Cycle Statistics for 8 OECD Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>$\sigma_c/\sigma_y$</th>
<th>$\sigma_i/\sigma_y$</th>
<th>$\sigma_{nx}/\sigma_y$</th>
<th>$\rho_y$</th>
<th>$\rho(c,y)$</th>
<th>$\rho(i,y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.69</td>
<td>2.17</td>
<td>1.46</td>
<td>0.67</td>
<td>0.62</td>
<td>0.55</td>
</tr>
<tr>
<td>Canada</td>
<td>0.88</td>
<td>2.83</td>
<td>0.83</td>
<td>0.79</td>
<td>0.72</td>
<td>0.62</td>
</tr>
<tr>
<td>France</td>
<td>0.89</td>
<td>1.92</td>
<td>0.81</td>
<td>0.79</td>
<td>0.58</td>
<td>0.45</td>
</tr>
<tr>
<td>Germany</td>
<td>0.70</td>
<td>3.40</td>
<td>0.88</td>
<td>0.71</td>
<td>0.64</td>
<td>0.80</td>
</tr>
<tr>
<td>Italy</td>
<td>0.82</td>
<td>2.49</td>
<td>1.76</td>
<td>0.78</td>
<td>0.70</td>
<td>0.80</td>
</tr>
<tr>
<td>Japan</td>
<td>1.12</td>
<td>2.31</td>
<td>0.93</td>
<td>0.74</td>
<td>0.47</td>
<td>0.60</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.77</td>
<td>2.88</td>
<td>1.50</td>
<td>0.70</td>
<td>0.74</td>
<td>0.73</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>0.67</td>
<td>3.00</td>
<td>0.41</td>
<td>0.84</td>
<td>0.88</td>
<td>0.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>correlation with same U.S. variable</th>
<th>Additional labor market statistics for the USA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>output</td>
<td>cons.h.</td>
</tr>
<tr>
<td>Australia</td>
<td>0.24</td>
<td>0.11</td>
</tr>
<tr>
<td>Canada</td>
<td>0.77</td>
<td>0.65</td>
</tr>
<tr>
<td>France</td>
<td>0.50</td>
<td>0.28</td>
</tr>
<tr>
<td>Germany</td>
<td>0.44</td>
<td>0.45</td>
</tr>
<tr>
<td>Italy</td>
<td>0.47</td>
<td>0.23</td>
</tr>
<tr>
<td>Japan</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Notes to Table 2:

With the exception of the U.S. labor market statistics, all statistics are taken from Baxter and Crucini [1991]. The data is from the International Financial Statistics, and is quarterly postwar data, with coverage varying by country. This is the same database used by Backus, Kehoe, and Kydland [1992] who graciously provided us with their data.

Statistics for U.S. labor markets were taken from Baxter and King [1991]; the original data source was Citibase. The data is quarterly data from 1955:1–1990:3. In this table, "H" denotes labor input (hours worked), and "prod" denotes productivity computed as output per manhour.

All data has been detrended using the Hodrick-Prescott [1980] filter.
Table 3
Trend Stationary Shocks

(1): results for complete markets economy
(2): results for economy trading noncontingent bonds and goods only

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Std. Dev.</th>
<th>Persistence</th>
<th>corr w/y, lag 0</th>
<th>Other Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2)</td>
<td>(1) (2)</td>
<td>(1) (2)</td>
<td>(1) (2)</td>
<td>(1) (2)</td>
</tr>
<tr>
<td>y</td>
<td>2.00 1.98</td>
<td>1.00 1.00</td>
<td>0.75 0.75</td>
<td>1.00 1.00</td>
<td>y,y* 0.04 0.06</td>
</tr>
<tr>
<td>c</td>
<td>0.98 0.98</td>
<td>0.49 0.50</td>
<td>0.81 0.81</td>
<td>0.81 0.84</td>
<td>c,c* 0.95 0.92</td>
</tr>
<tr>
<td>i</td>
<td>3.52 3.37</td>
<td>1.76 1.70</td>
<td>0.74 0.74</td>
<td>0.98 0.98</td>
<td>i,i* 0.11 0.21</td>
</tr>
<tr>
<td>N</td>
<td>1.06 1.01</td>
<td>0.53 0.51</td>
<td>0.71 0.72</td>
<td>0.90 0.91</td>
<td>N,N* -0.71 -0.68</td>
</tr>
<tr>
<td>w</td>
<td>1.13 1.14</td>
<td>0.57 0.58</td>
<td>0.80 0.80</td>
<td>0.92 0.93</td>
<td>w,w* 0.75 0.72</td>
</tr>
<tr>
<td>nx/y</td>
<td>1.65 3.57</td>
<td>-- --</td>
<td>0.77 0.76</td>
<td>-0.96 -0.99</td>
<td>s,i 0.94 0.93</td>
</tr>
<tr>
<td>b</td>
<td>0.00 3.26</td>
<td>0.00 1.65</td>
<td>0.00 0.98</td>
<td>0.00 -0.21</td>
<td>w,N 0.66 0.23</td>
</tr>
</tbody>
</table>
Table 4

Unit Root in Productivity

(1): results for complete markets economy

(2): results for economy trading noncontingent bonds and goods only

<table>
<thead>
<tr>
<th></th>
<th>Standard Deviation</th>
<th>Relative Std. Dev.</th>
<th>Persistence</th>
<th>corr w/y, lag 0</th>
<th>Other Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
<td>(1)</td>
</tr>
<tr>
<td>y</td>
<td>2.53</td>
<td>1.58</td>
<td>1.00</td>
<td>1.00</td>
<td>0.86</td>
</tr>
<tr>
<td>c</td>
<td>1.03</td>
<td>1.67</td>
<td>0.41</td>
<td>1.06</td>
<td>0.81</td>
</tr>
<tr>
<td>i</td>
<td>11.19</td>
<td>4.54</td>
<td>4.42</td>
<td>2.87</td>
<td>0.77</td>
</tr>
<tr>
<td>N</td>
<td>1.54</td>
<td>0.71</td>
<td>0.61</td>
<td>0.45</td>
<td>0.89</td>
</tr>
<tr>
<td>w</td>
<td>1.24</td>
<td>1.62</td>
<td>0.49</td>
<td>1.02</td>
<td>0.83</td>
</tr>
<tr>
<td>nx/y</td>
<td>3.63</td>
<td>3.89</td>
<td>--</td>
<td>--</td>
<td>0.78</td>
</tr>
<tr>
<td>b</td>
<td>0.00</td>
<td>7.92</td>
<td>0.00</td>
<td>5.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Random walk productivity: Quantity responses to innovation in home country
Random walk productivity: Price and interest rate responses to innovation in HC
Random walk productivity: Hicksian decomposition of consumption
FIGURE 3-B

Random walk productivity: Hicksian decomposition of labor

HC WEALTH EFFECT
* bond economy
o complete markets

HC REAL INTEREST RATE EFFECT

HC WAGE EFFECT

HC TOTAL LABOR EFFECT

PC WEALTH EFFECT

PC REAL INTEREST RATE EFFECT

PC WAGE EFFECT

PC TOTAL LABOR EFFECT
FIGURE 4
Trend-stationary productivity: Quantity responses to innovation in home country

HC output: $= bond, \omega = CM$

HC consumption: $= bond, \omega = CM$

HC investment: $= bond, \omega = CM$

HC labor: $= bond, \omega = CM$

FC output: $= bond, \omega = CM$

FC consumption: $= bond, \omega = CM$

FC investment: $= bond, \omega = CM$

FC labor: $= bond, \omega = CM$
FIGURE 5

Trend-stationary productivity: Price and interest rate responses to innov. in HC

HC wage rate: * = bond, o = CM

FC wage rate: * = bond, o = CM

Real interest rate: * = bond, o = CM

FC bond holdings: * = bond, o = CM
Trend-stationary productivity: Hicksian decomposition of consumption

* bond economy
○ complete markets
FIGURE 6-B

Trend-stationary productivity: Hickian decomposition of labor

**HC WEALTH EFFECT**

* bond economy
  o complete markets

**HC REAL INTEREST RATE EFFECT**

**HC WAGE EFFECT**

**HC TOTAL LABOR EFFECT**

**FC WEALTH EFFECT**

**FC REAL INTEREST RATE EFFECT**

**FC WAGE EFFECT**

**FC TOTAL LABOR EFFECT**