Early Development

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Abstract

We explain how a long period of slow pre-industrial development triggers an Industrial Revolution that leads to modern balanced growth. Development in the pre-industrial period is driven by increasing returns to specialization made possible by a growing population. Increasing access to specialized intermediate goods eventually makes fundamental technological innovation possible. Innovation initiates the Industrial Revolution, after which productivity grows endogenously regardless of population growth. Industrialization reconciles the crucial role of population early on with its weak relation to per capita product in developed economies. Faster population growth speeds early development, though if it results from a highly productive primitive technology, the consequences for development are ambiguous.

Introduction

Economic growth since the Industrial Revolution has been truly remarkable. Maddison (1982) estimated, for a sample of sixteen industrialized countries, that total product increased sixty-fold and per capita product rose thirteen-fold since 1820.[1] This extraordinary performance is even better appreciated by contrasting it with prior historical experience. The table below, also from Maddison, presents population and per capita product growth rates since 500 A.D. Although the numbers are highly aggregated both across countries and over time, they tell a dramatic story.

Performance Characteristics of Four Eras

	Average Annual Compound Growth Rates	
	Population	Per Capita GDP
500-1500	0.1	0.0
1500-1700	0.2	0.1
1700-1820	0.4	0.2
1820-1980	0.9	1.6

Source: Maddison (1982), Table 1.2, p.6. The sample includes sixteen countries, twelve from Europe, plus Australia, Canada, Japan, and the U.S.A.

For the thousand years following the fall of Rome, there was little net progress in population and none in per capita product. From 1500 to 1700, progress was also very poor, although population growth doubled and per capita product began to grow slightly.[2] The period from 1700 to 1820 was marked by a doubling of both population and per capita product growth compared to the preceding two hundred years, although per capita growth

[1] Maddison (1982), p. 4.

[2] Maddison notes that his estimates are below those of Kuznets and Landes, but above those of Le Roy Ladurie and Abel (Maddison, 1982, p. 7).

was still very poor by today's standards.[3]

Improvement in transportation during the latter period helped break down the isolation of self-sufficient villages and greatly increased the scope for economies of scale and specialization. Such progress prompted Adam Smith (1776) to observe that productivity gains were made possible by the division of labor, which in turn was limited by the extent of the market. In his day, markets grew not only because of rising population, but also because improved transportation greatly enlarged regional markets for many goods and services.

From 1820 to 1980 population growth again more than doubled compared to the previous hundred and twenty years. This time, however, growth in per capita product increased eight-fold, reflecting the startling jump in technological improvement and productivity growth associated with industrialization.

The rise of urban population in Europe indicates the extent to which specialization associated with the division of labor accompanied economic development. For towns of five thousand or more, Bairoch (1988) estimates urban population in 1000 A.D. at about 10% of the total, rising only slightly to 12% by 1700. Urbanization quickened over the next hundred and fifty years with urban population rising to 19% of the total by 1850. But the pace exploded after that, bringing urban population to 67% of the total by 1980.[4] The trend in city dwelling thus mirrors the growth in population and per capita product over the same long history.

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[3] The flat population profile before 1700 was marked by two major declines due to epidemic disease. The first occurred in the sixth and seventh centuries, the second in the fourteenth century. Such devastating epidemics had tapered off by 1700, helping to make possible the sustained growth in population that was to follow. See McNeill (1976).

[4] Bairoch (1988), Table 13.3, p. 219.

In the late 1700s and early 1800s there occurred the great break with the past that we call the Industrial Revolution. The Industrial Revolution was marked by a widespread systematic application of empirical knowledge and methods to the production of goods and services. Attempting to understand the onset of industrialization, Landes (1969) pointed out that many technical improvements became feasible only after advances in associated fields. The steam engine is the classic example of this. In the 1800s, the know-how associated with the division of labor had progressed sufficiently that the search for technological improvements became routine and productive. Widespread innovation, in turn, created more specialized knowledge which further raised the productivity of time devoted to innovation. In this way, the Industrial Revolution initiated the cumulative, self-sustaining, technical improvements that give rise to modern economic growth.

Our paper presents a model motivated by the broad picture of early development described above. The model generates a long pre-industrial period in which growth in per capita product is tied to population growth. It also generates an Industrial Revolution after which productivity grows endogenously regardless of the growth of population. In the model, per capita product grows as production shifts from primitive processes to modern specialized techniques. The model economy is slow to specialize in the pre-industrial period if the population grows slowly, reflecting the relatively slow pace of urbanization prior to the Industrial Revolution. But thereafter, the pace of modernization quickens, reflecting the rapid urbanization that accompanied industrialization.

Each household in our model is endowed with a *primitive*, diminishing-returns technology allowing it to produce its own consumption goods. A purely primitive economy is one in which all goods production is carried out by each household independently using this technology. There is also a *modern* technology that exhibits increasing returns due to specialization along the lines of Romer (1987, 1990).[5]

Population drives early development in our model. Initially, population is too small to sustain a specialized modern sector, but if population continues to grow, then the modern sector eventually opens. Continuing population growth enables the economy to benefit more fully from increasing returns. The growing population raises modern sector wages, lowers the primitive sector marginal product, and thereby causes the modern sector to expand at the expense of the primitive sector. Our model thus reproduces the stylized fact emphasized by Locay (1990) that production shifts from households to firms as the economy develops. It also embodies Smith's (1776) idea that the division of labor is limited by the extent of the market. Population is the scale factor governing the size of the market in the pre-industrial period, although we recognize that other factors, such as transportation costs, also help determine market size.

Productivity gains during the pre-industrial period arise primarily from an ever - finer division of labor without much improvement in fundamental productive techniques themselves. The pre-industrial period in our model economy is one in which households choose to remain in a "no-learning corner" in the sense that they devote no time to devising fundamental improvements in technology. They choose not to innovate because learning productivity depends on the economy-wide degree of specialization, and initially the specialized modern sector is too small to make learning productive. The idea is that innovation involves problem solving which, in turn, is facilitated by access to specialized tools and techniques.

An Industrial Revolution occurs in our model when the growing population expands the specialized modern sector to the point where households finally choose to

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[5] Notable discussions of specialization and the division of labor also appear in Becker and Murphy (1990), Stigler (1951), and Young (1928).

leave the no-learning corner. Thus, we associate industrialization with learning that yields fundamental improvements in technology.[6] We do not have physical capital in our model. Instead we index the state of technological know-how by the stock of human capital. The Industrial Revolution is marked by the initiation of human capital accumulation by households. The accumulation of human capital enables endogenous per capita productivity growth to take place independently of the population growth rate, and greatly speeds the transition from primitive to modern production techniques.

When the transition is complete, and all goods are produced in the modern sector, the economy attains a balanced growth path along which time allocations are constant and per capita product grows at a constant rate. This is more familiar territory, given the work of Uzawa (1965), Romer (1986), and Lucas (1988) among others, though the engine of endogenous balanced growth in our model more closely resembles that in Romer (1987).

Conditions for industrialization have recently been studied in three important papers. Murphy, Shleifer, and Vishny (1989) show how a backward economy could raise per capita product by coordinating investment in a big push. Becker, Murphy, and Tamura (1990) produce a model with two kinds of equilibria: a "Malthusian" one with large families, little human capital, and zero growth; and a "Development" equilibrium with small families and rising living standards. A development trap is also present in the paper by

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[6] Sokoloff's (1988) analysis of patent records in early industrial America (1790 - 1846) provides evidence that early industrialization was characterized by a surge in the commitment of resources to the search for fundamental advances in technology.

Of course, technological progress did not begin with industrialization. Countless advances in the pre-industrial era compare favorably with any the industrial world has produced. See, for example, Cipolla (1980). Past progress was, however, sporadic and slow, whereas modern invention and innovation is routine and widespread.

Azariadis and Drazen (1990). All three papers are primarily concerned with the possibility of multiple equilibria and how it might account for observed differences across countries. Our paper, however, focuses on explaining how a long period of slow pre-industrial development eventually triggers an Industrial Revolution that leads to modern balanced growth.

We recognize that a population-driven model of development such as ours must confront a puzzle. How can a model that relies on increasing returns to labor be consistent with the lack of evidence that per capita product rises with population in modern economies? And how can such a model explain why some of the world's most populous countries have such low per capita income?

Two features of our model allow us to address this population puzzle. First, the occurrence of an Industrial Revolution enables us to reconcile the importance of population growth for early development with its minimal influence on modern economies. Second, our dual-technology model allows per capita product to differ enormously among countries depending on the relative sizes of the primitive and modern sectors. Moreover, the transition to a modern economy can be quite slow without industrialization. Since the timing of the Industrial Revolution does not depend entirely on population, but also involves the three technologies -- primitive, modern, and learning -- more heavily populated countries need not pass through the transition most quickly.

The plan of the paper is as follows. The modern reduced-form production function is derived in Section 1. The technology for accumulating human capital is motivated and described in Section 2, and Section 3 presents the primitive technology. The household's optimization problem is stated and characterized in Section 4. Modern balanced growth is analyzed in Section 5. We then study early development, first without human capital accumulation in Section 6, and then with learning in Section 7. In the latter we consider the timing of the Industrial Revolution in some detail. The population puzzle is dealt with in

Section 8, and a brief summary concludes the paper.

1. Production With Specialized Inputs

Modern final good production uses labor and intermediate goods in the following way:

$$Y = (e_Y \, h \, N)^{1-\alpha} \int_0^M [x(i)]^\alpha \, di \;, \text{ with } 0 < \alpha < 1. \tag{1.1}$$

There are N workers in the economy each of whom possesses human capital h and devotes a fraction of time e_Y to the production of final goods. Total effective work effort engaged in final goods production is thus $e_Y\, h\, N$. The x(i) are the quantities of distinct intermediate inputs indexed by i and arranged on a continuum of measure M. Thus, M denotes the range of intermediate inputs. Our final-good technology is a straightforward extension of that in Ethier (1982) and Romer (1987) to incorporate human capital.

We model the production of intermediate goods as in Romer (1987), except that effective labor is the primary factor instead of capital.[7] The cost of producing the quantity x of any intermediate good, in units of effective labor, is given by:

$$g(x, h) \equiv b(x) / h, \quad \text{with } b'(x) > 0 \text{ and } b''(x) \geq 0, \tag{1.2}$$

where b(x) is the cost in terms of workers with the minimum (h = 1) human capital. Although inaction at zero cost is feasible [so b(0) = 0], b(x) includes a fixed cost, $\bar{b}$, for positive production levels.

The modern sector must satisfy two labor-market clearing conditions. First, total work effort involved in producing intermediate goods must equal its supply:

$$M\, g(x, h) = e_I\, N\, . \tag{1.3}$$

Here e_I is the fraction of time an individual devotes to intermediate-good production.

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[7] Our concept of effective labor is the same as in Lucas (1988), pp. 17 - 18.

Second, individual effort utilized by both final and intermediate-good firms must equal total work effort, e_M, supplied to the modern sector :

(1.4) $$e_Y + e_I = e_M \ .$$

At this stage, we take e_M as given, and let firm demands allocate the total between intermediate and final-good firms.

Each final-good firm chooses labor hours and intermediate inputs to maximize profit, taking the final-good wage, w, and the final-good prices of the intermediate inputs, p_i, as given. Setting marginal products equal to relative prices and exploiting the symmetry from (1.2) yields:

(1.5) $$w = (1 - \alpha)\,(e_Y\, h\, N)^{-\alpha}\, h\, M\, x^{\alpha}$$

(1.6) $$p = (e_Y\, h\, N)^{1-\alpha}\, \alpha\, x^{\alpha - 1} \ .$$

The wage, w, is the competitively determined compensation per hour in units of the final good paid to a worker with human capital of h. Symmetry implies that the price of each intermediate good will be the same in equilibrium.

Each intermediate-good firm behaves like a monopolist, since it is aware that the demand for its product is given implicitly from (1.6). Taking the wage, w, as given, each firm maximizes profit,

(1.7) $$\Pi = p\, x - w\, g(x, h) \ ,$$

by choosing x, substituting from (1.6) for p before undertaking the maximization.

Although each intermediate firm is a monopolist, each must earn zero profit in equilibrium. If intermediate-firm profits were positive, new firms would arise to produce additional specialized inputs: M would increase, eliminating the original profit. Setting (1.7) to zero, and using conditions (1.3) - (1.6), results in:

(1.8) $$e_Y = (1 - \alpha)\, e_M$$

(1.9) $$e_I = \alpha\, e_M \ .$$

Profit maximization, together with (1.3), (1.8) and (1.9), yields the following:

$$x\, g_x(x, h) / g(x, h) = x\, b'(x) / b(x) = \alpha . \tag{1.10}$$

This implicitly defines the constant equilibrium output level, $\tilde{x}$, for each intermediate firm.

Condition (1.3), combined with (1.2) and (1.9), yields the following expression for the degree of specialization:

$$M = \alpha\, e_M\, h\, N / b(\tilde{x}) . \tag{1.11}$$

Aggregate effective labor ($e_M\, h\, N$) is the primary determinant of M. As the modern sector grows, in terms of the fraction of time worked (e_M), the labor force (N), or human capital per person (h), it becomes more and more specialized.

Our reduced-form production function, obtained by substituting (1.11) and (1.8) into (1.1), is:

$$Y = A\, (e_M\, h\, N)^{2-\alpha}, \tag{1.12}$$

where the productivity coefficient $A \equiv \alpha\, (1-\alpha)^{1-\alpha}\, \tilde{x}^{\alpha} / b(\tilde{x})$. The function exhibits *increasing returns* to e_M, h, and N separately.[8]

Substituting (1.8) and (1.11) into (1.5) yields a reduced-form expression for the wage:[9]

$$w = A\, h^{2-\alpha}\, (e_M\, N)^{1-\alpha} . \tag{1.13}$$

Notice that there is one factor of production, labor, augmented with human capital. All output is exhausted in compensating that one factor, since $w\, e_M = Y/N$. The fact that w rises with the population plays an important role in our model.

The wage that an individual commands in the labor market can be written as the

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[8] The source of increasing returns to effective labor is easily seen by rewriting (1.1) as $Y = [\,(1-\alpha)\, e_M\, h\, N\,]^{1-\alpha}\, M\, \tilde{x}^{\alpha}$. By (1.11), $e_M\, h\, N$ has a unit elastic effect on Y through M. Holding M constant, the elasticity of Y with respect to $e_M\, h\, N$ is $1-\alpha$; so the total elasticity is $2-\alpha$.

[9] The reduced-form wage cannot be found by differentiating (1.12). The wage is determined by firms who do not see the external effect of labor in increasing the degree of specialization.

marginal product of effective labor in final-goods production given M, $\partial Y / \partial(e_Y h N)$, times the units of effective labor that each hour of his time is worth, h. This is another way of arriving at (1.5). An individual with only the minimum human capital (h = 1) just earns $\partial Y / \partial(e_Y h N)$, which we refer to as the "base wage", w_b. Therefore, dividing (1.13) by h yields the base wage:

$$w_b = A\,(e_M\, h\, N)^{1-\alpha}\,. \tag{1.14}$$

Since w_b depends on aggregate effective labor, the individual must take it as given in making his decisions. The base wage plays an important role when we discuss the household's problem in Section 4.

2. *Human Capital Accumulation*

Individuals can accumulate human capital by allocating current time to learning. As discussed below in Section 4, time allocations are chosen by households. Family production of human capital depends, in part, on effective learning time, $e_L hn$, where e_L is time spent learning, h is human capital per person, and n is the number of family members. The productivity of learning time is enhanced by the economy-wide degree of specialization, M. Learning time does not use up intermediate inputs, but it is made more productive by access to knowledge associated with the production and use of specialized inputs.[10] For this reason, we assume that the quantity of each intermediate input does not matter, but the range of such inputs does.

Household acquisition of human capital is governed by the following:

$$\dot{H} = e_L\, H^{1-\gamma} M^{\gamma}\ , \tag{2.1}$$

where $H \equiv hn$ is the household's stock of human capital and $0 < \gamma < 1$. The technology

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[10] Our intermediate goods, to use Romer's (1990) terminology, are both excludable and rivalrous in final good production, but they are neither in the learning technology.

exhibits diminishing returns to the family's stock of human capital and the range of intermediate inputs separately.[11] Note that the family's stock of human capital is constant unless time is devoted to learning; that is, unless $e_L > 0$.

Equation (2.1) captures the idea that holding the economy-wide degree of specialization, M, constant a household's learning time is more productive the greater its human capital. Conversely, holding household human capital constant, learning time is more productive the greater the economy-wide degree of specialization. We assume decreasing returns to the household's stock of human capital, H, because we recognize that limited human capabilities make it increasingly difficult for a household to increase its knowledge. Development, however, raises the degree of specialization and offsets the decreasing returns to accumulation, making balanced growth with human capital accumulation feasible.

Using (1.11) to eliminate M, we can write (2.1) as:

$$\dot{H} = e_L\,(h\,n)^{1-\gamma}\,[h_a\,\alpha\,n\,f\,/\,b(\tilde{x})]^{\gamma}\,e_{Ma}{}^{\gamma}, \tag{2.2}$$

where f is the constant number of households in the economy, and h_a and e_{Ma} are, respectively, *economy-wide per capita averages* of human capital and hours worked in the modern sector. An individual household takes h_a and e_{Ma} as given when choosing its own time allocations.

New family members are born with the average stock of household human capital, without a family's having to incur any (time) costs of education.[12] This allows us to write a per capita accumulation equation of the following form:

$$\dot{h} = \delta\,e_L\,h^{1-\gamma}\,h_a{}^{\gamma}, \tag{2.3}$$

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[11] We follow Uzawa (1965), Lucas (1988) and Becker, Murphy and Tamura (1990) in assuming constant returns to e_L because it simplifies the solution for time allocations in balanced growth.

[12] This assumption corresponds to adding the term ηh to (2.1).

where

$$\delta \equiv [\alpha f / b(\tilde{x})]^{\gamma} e_{Ma}^{\gamma} \quad . \tag{2.4}$$

Our accumulation technology is similar to Lucas's (1988, eq. 13), but differs from his in two important respects. Ours recognizes an external effect of the economy-wide average per capita human capital, h_a, on the productivity of an individual's learning time. And our δ depends on the economy-wide average time devoted to modern production, e_{Ma}. Both externalities enter through the degree of specialization, M.

Industrial development in our model relies on the notion that per capita human capital can grow without bound. Thus, our human capital represents scientific knowledge that exists outside of any individual. In effect, ours is a generalized sort of human capital that indexes the economy-wide state of technological know-how.

3. Primitive Production

Each household has a primitive technology that allows it independently to produce a final good. We assume that this output is a perfect substitute for the good produced with specialized inputs in the modern sector. A household's primitive production function is:

$$Y_p = \beta \ln (1 + e_p n) \quad , \tag{3.1}$$

where Y_p is total household primitive production of final goods, e_p is the fraction of time that each household member works in the primitive sector, and n is a household's size. Thus, $e_p n$ is the total household time allocated to primitive production. β is the productivity coefficient. We choose this technology because it exhibits diminishing returns to the labor input, and because the marginal product reaches an upper bound at $e_p = 0$. The latter property is not necessary for what follows, but is convenient since it means that the primitive sector will be abandoned in finite time.

We conceive of primitive production as applying to virtually all household activities since, at least until recently, such production utilized relatively simple and traditional techniques. For those goods produced both at home and in the market, such as agricultural products, we regard as primitive only that portion produced at home for domestic use with traditional techniques.

Three characteristics distinguish the primitive technology from the modern production process. First, each household operates its own primitive process independently, whereas modern processes employ workers from all households. Second, individual households are too small to use specialized inputs economically with the primitive technology. Third, human capital does not enhance labor productivity in the primitive production of goods. Only innate human capital ($h = 1$) is relevant in the primitive sector.

The above three points notwithstanding, our primitive technology has much in common with pre-industrial ($h = 1$) modern production processes. As production moves from the household to the market in the pre-industrial period, productivity gains arise primarily from an ever-finer division of labor applied to the familiar primitive technology, without much improvement in fundamental productive methods themselves.

4. The Household's Problem

In our model economy there are f families, each of which has n members. A representative family grows at the exogenous rate η, as does the total population, $N = n f$, since the number of families stays fixed. Each household head maximizes the present value of the utility of his infinitely-lived family dynasty, where instantaneous utility of each member depends only on his own consumption, c(t). This means the household head maximizes:

$$U(t) = \int_0^\infty n(t)\, u[c(t)]\, e^{-\rho t}\, dt \ .$$

In addition to choosing per capita consumption, the family head decides how much of each member's time to allocate to primitive production, $e_p(t)$, production in the modern sector, $e_M(t)$, and learning, $e_L(t)$. We have the following time constraint:

$$1 = e_P + e_M + e_L . \tag{4.1}$$

There is also a budget constraint,

$$c = y_P + w_b\, h\, e_M . \tag{4.2}$$

Output from a household's primitive production process (3.1) is split equally among family members, so per capita primitive sector output is $y_P = Y_P / n$. Modern sector earnings of each member are $w_b\, h\, e_M$. Each household takes as given the path of the modern-sector base wage, w_b, which should now be written as $w_b = A\,(e_{Ma}\, h_a\, N)^{1-\alpha}$, in light of our distinction between individual values and economy-wide averages. But the wage that its members earn in the modern sector, $w = w_b\, h$, depends on its own human capital.

Firms in the modern sector are owned by households. There is no physical capital so, as we noted above, all factor income in the modern sector is paid as wages. This is consistent with the zero-profit conditions for both intermediate firms (because of free entry) and final-good firms (because of competition and constant returns to scale).

A household saves by allocating time to learning in order to accumulate human capital. We ignore the credit market because in this representative-agent model there can be no borrowing or lending in equilibrium. Human capital accumulation is governed by the learning technology (2.3). The productivity of learning time depends on the economy-wide averages h_a and e_{Ma}. An individual household takes the paths of h_a and e_{Ma} as given when making its own decisions.

We solve for a *perfect foresight equilibrium* where the paths $\{w_b(t), h_a(t), e_{Ma}(t)\}$ are forecast correctly for $t \geq 0$. To see what this means, consider arbitrary paths for w_b, h_a, and e_{Ma}. These lead each household to choose paths for consumption, work effort, and learning. These choices, in turn, imply paths for w_b, h_a, and e_{Ma}. The economy-wide equilibrium paths for w_b, h_a, and e_{Ma} are those that reproduce themselves given optimal decisions by households and firms.

The maximization problem is stated formally in Appendix A. There we show that optimal household choice of e_P requires:

(4.3) $$\lambda_2 = u_c\,(\partial y_P/\partial e_P) + \Omega_1 \ ,$$

where λ_2 has the interpretation of the marginal utility of time. This condition says that hours per person allocated to the primitive production process must equate the utility-value of the marginal per capita product of e_P to the marginal utility of time, unless the non-negativity constraint on e_P is binding, in which case the multiplier $\Omega_1 > 0$.

Analogously, the choice of e_M requires:

(4.4) $$\lambda_2 = u_c\, w_b\, h + \Omega_2 \ .$$

Work effort in the modern sector earns a market wage of $w_b h$. Such effort, if positive, will also have to confer a marginal utility benefit equal to that of time, λ_2, unless the constraint on e_M is binding, in which case $\Omega_2 > 0$.

The choice of e_L requires:

(4.5) $$\lambda_2 = q\,\delta\, h^{1-\gamma} h_a^{\gamma} + \Omega_3 \ ,$$

where q is the marginal utility value of h. This condition says that learning must also yield a marginal benefit equal to that of time although, again, e_L may be constrained, in which case $\Omega_3 > 0$.

The utility price of human capital, q, must be a continuous function of time and satisfy the pricing equation:

(4.6) $$\rho - \eta - \dot{q}/q \ = \ (1-\gamma)\,\delta\, e_L\, h^{-\gamma} h_a^{\gamma} + (u_c\, w_b / q)\, e_M \ .$$

Over time, the utility value attached to h must change so that the net percentage capital loss on h just equals the percentage marginal productive value of h. This value has two sources: the marginal product of h in human capital production, and its marginal value in enhancing wages. In other, words, q must change so that there is zero net marginal benefit to h along an optimal path.

There is also a transversality condition, which takes the usual form:

$$\lim_{t \to \infty} e^{-\rho t} q(t)\, h(t)\, n(t) = 0 \, . \tag{4.7}$$

This says that the utility value of the family's stock of human capital must become valueless as time goes to infinity but not before. Finally, there are two initial conditions: $n(0) = n_0$ and $h(0) = 1$.

Aggregate Consistency

In moving from the household optimum to the economy equilibrium, we must impose the two aggregate consistency conditions:

$$h_a = h \tag{4.8}$$

$$e_{Ma} = e_M \, . \tag{4.9}$$

Imposing these onto equations (2.3) and (2.4), and conditions (4.4) - (4.6), and using the definition $w \equiv w_b\, h$, yields:

$$\lambda_2 = u_c\, w + \Omega_2 \tag{4.10}$$

$$\lambda_2 = q\, \delta\, h + \Omega_3 \tag{4.11}$$

$$\dot{h} / h = \delta\, e_L \tag{4.12}$$

$$\dot{q} / q = \rho - \eta - (1 - \gamma)\, \delta\, e_L - [u_c\, w / q\, h]\, e_M \tag{4.13}$$

$$\delta = [\alpha\, f / b(\tilde{x})]^{\gamma}\, e_M^{\ \gamma} \, . \tag{4.14}$$

To study development, we use (4.10) - (4.14), together with (4.1) - (4.3), the primitive technology (3.1), the modern sector wage (1.13), and the transversality condition (4.7). For convenience, we work with logarithmic utility, letting $u[c(t)] = \ln c(t)$.

5. Modern Balanced Growth

In this section, we analyze an economy that has developed to the point where it is no longer efficient for households to operate the primitive technology. An economy that is fully modern, in the sense that $e_p = 0$, need not, however, accumulate human capital. Time will be allocated to learning only if it is *efficient* to do so. It is efficient to learn, in our fully modern economy, if and only if:

$$\delta(1) > \rho - \eta \,, \tag{5.1}$$

where $\delta(1)$ refers to the value of δ in (4.14) when $e_M = 1$.

This critical condition is derived as follows. Learning is desirable if, at a position of zero learning (i. e., $e_L = 0$ and $e_M = 1$), the rate at which a household can transform per capita consumption intertemporally by accumulating human capital exceeds $\rho - \eta$, its utility rate of time discount.[13] A household can transform one unit of per capita consumption over time by cutting current work effort and increasing learning time. At the margin, reducing per capita income and consumption by one unit allows a household to raise per capita learning time by $1/w_b h$ hours. Each additional hour of learning, in turn, raises human capital by $\delta(1)\, h$ units. And the added human capital raises future per capita consumption by $w_b e_M$. Multiplying these terms together, and noting that $e_M = 1$ when e_L and e_p are zero, yields the *instantaneous marginal rate of transformation*, $\delta(1)$. This rate must exceed the family discount rate, $\rho - \eta$, for households to find accumulation of human capital worthwhile.[14]

Balanced Growth With Human Capital Accumulation

Consider, first, the case where condition (5.1) is satisfied so that human capital accumulation is efficient. When time is devoted to learning, the multipliers Ω_2 and Ω_3 are zero, so (4.10), (4.11), and (4.14) imply:

$$u_c\, w = v\, e_M^{\ \gamma} q\, h \,, \tag{5.2}$$

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[13] Existence of an optimal plan, with or without growth, requires that $\rho - \eta > 0$: this is equivalent to assuming positive discounting in models without population growth.

[14] Our continuous-time learning condition (5.1) may be derived formally by letting the period length become arbitrarily short in a discrete-time version of the model.

where $v \equiv \delta(1) \equiv [\alpha f / b(\bar{x})]^{\gamma}$. This expression, together with (4.2) and log utility, allows us to express e_M and e_L as functions of q and h:

$$e_M = (v\,q\,h)^{-1/(1+\gamma)} . \tag{5.3}$$

$$e_L = 1 - (v\,q\,h)^{-1/(1+\gamma)} . \tag{5.4}$$

Substituting these into (4.12) and (4.13), yields the following dynamic system:

$$\dot{h} / h = v\,(v\,q\,h)^{-1/(1+\gamma)} - (q\,h)^{-1} \;; \tag{5.5}$$

$$\dot{q} / q = \rho - \eta - (1-\gamma)\,v\,(v\,q\,h)^{-1/(1+\gamma)} - \gamma\,(q\,h)^{-1} . \tag{5.6}$$

The modern economy inherits a per capita stock of human capital, called h_M, from the early phase of development. The household must choose an initial shadow price for human capital, called q_M, to satisfy the transversality condition (4.7) given that the economy evolves according to (5.5) and (5.6). Once q_M is chosen, the two differential equations govern the future paths of q and h, and by extension, the allocations, e_L, e_M, and c over time. Here we consider h_M to be given and q_M to be the object of choice, but in Section 7 this pair will constitute the endpoint of the early development path.

The phase diagram in Figure 1 indicates the motion of q and h implied by the household's first-order conditions when the economy is fully modern. The locus labelled $\dot{h} = 0$, is the *boundary* between the regions where $e_L > 0$ and $e_L = 0$. It is a rectangular hyperbola given from (5.5) as:

$$\dot{h} = 0 \quad \text{boundary} \quad \Rightarrow \quad q\,h = 1 / v . \tag{5.7}$$

From (5.5) we see that above this boundary $e_L > 0$ and human capital is rising. On or below it, $e_L = 0$ and h is constant.

The shadow price of human capital, q, is constant along the $\dot{q} = 0$ locus, which is found by setting equation (5.6) to zero:

$$\dot{q} = 0 \quad \text{locus} \quad \Rightarrow \quad q\,h = \gamma\left[\rho - \eta - (1-\gamma)\,v\,(v\,q\,h)^{-\gamma/(1+\gamma)}\right]^{-1} . \tag{5.8}$$

This locus is also a rectangular hyperbola, although one cannot explicitly find the value of qh that defines it. From (5.6), q is rising above the $\dot{q} = 0$ locus and falling below it . As

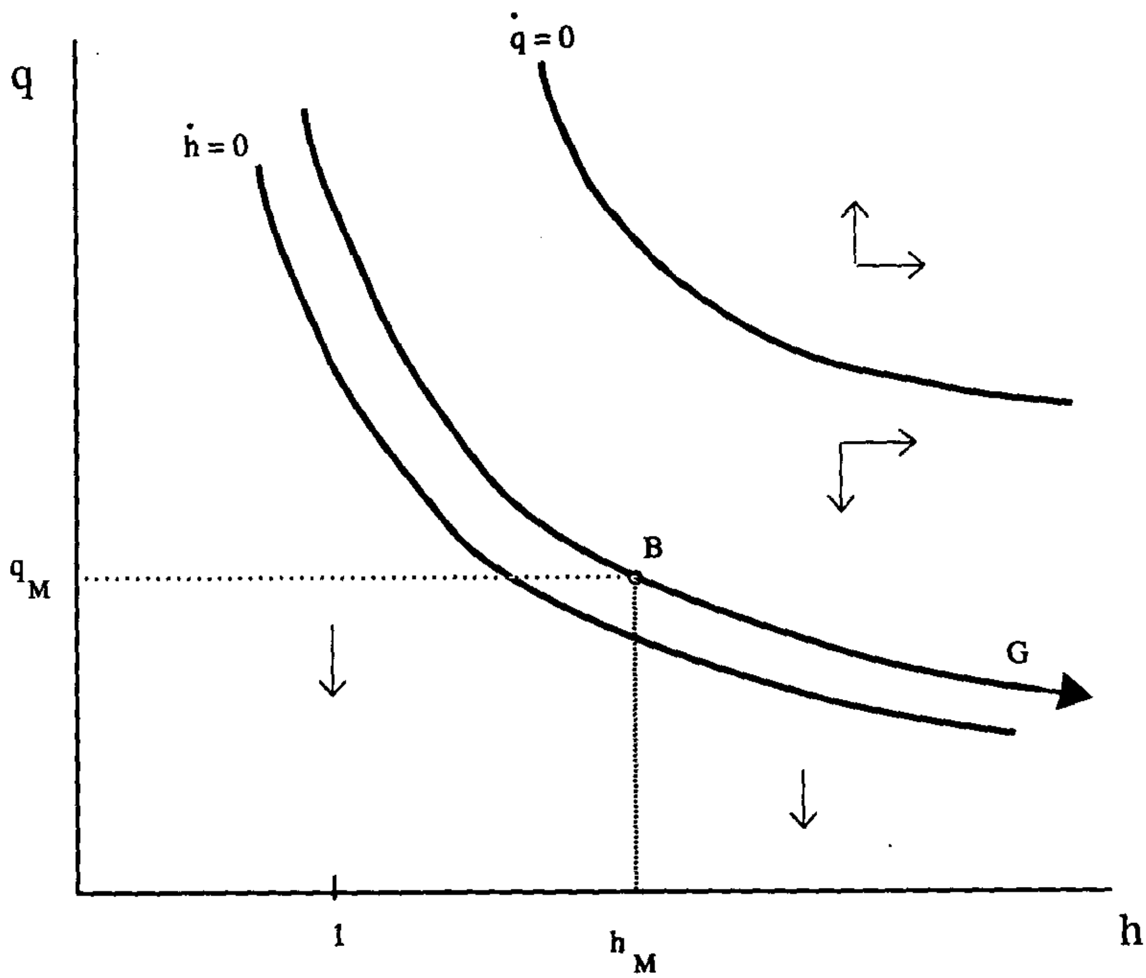

Figure 1:
Balanced Growth

drawn, $\dot{h} = 0$ lies below $\dot{q} = 0$. This is the case if and only if (5.1) is satisfied.

The dynamic system can be expressed compactly in terms of the variable $\xi \equiv qh$. From (5.5) and (5.6), we get:

$$\dot{\xi} = -(1+\gamma) + \left[\rho - \eta + \gamma v (v\xi)^{-\gamma/(1+\gamma)}\right] \xi \quad (5.9)$$

This has a unique stationary value, ξ^*. The optimal plan is to choose the initial shadow price, q_M, so that $\xi(t) = \xi^*$, since doing so satisfies transversality (4.7) as well as the necessary motion equations, (5.5) and (5.6). No other choice of q_M does so.

The optimal q_M puts the economy on the rectangular hyperbola labelled BG in Figure 1 along which $q\,h = \xi^*$. If (5.1) is satisfied, BG lies above the $\dot{h} = 0$ boundary and below the $\dot{q} = 0$ locus. Hence, the equilibrium path proceeds down the BG locus over time. The fact that qh is constant at ξ^* gives us constant time allocations, $e_M{}^*$ and $e_L{}^*$, and *balanced growth* in the modern economy as a direct implication of (5.3) and (5.4).[15]

Since $y_P = 0$ in balanced growth, consumption of each individual equals $w\, e_M$ by (4.2). Moreover, since e_M is constant, consumption and the wage grow at the same rate. By (1.13) the wage rises continually by force of both population growth and human capital accumulation. Since e_L is constant, human capital grows at the constant rate $\delta(e_M{}^*)\, e_L{}^*$ along the balanced path. Recall that population grows at η. Taking logs and differentiating (1.13) with respect to time yields the growth in wages and, by (4.2), the growth rate of per capita consumption along the balanced path:

$$(\dot{c}/c)^* = (2-\alpha)\,\delta(e_M^*)\,e_L^* + (1-\alpha)\,\eta \quad (5.10)$$

Balanced Growth Without Human Capital Accumulation

If condition (5.1) fails to hold, there will be no learning and slower growth along

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[15] Since qh is constant along the balanced growth path, the transversality condition is satisfied without any restrictions in addition to (5.1). This also means that utility is necessarily bounded.

the balanced growth path. If $\delta(1) \leq \rho - \eta$, then the $\dot{q} = 0$ locus would lie on or below the $\dot{h} = 0$ boundary. The only value of q_M satisfying transversality would lie on the $\dot{q} = 0$ locus above h_M: q would be constant for all time, and e_L would be zero. Since qh would be constant, the transversality condition would be satisfied. The modern economy would be fully viable, but households would not find it worthwhile to accumulate human capital.

In this case, the intertemporal rate of transformation is too low to make learning worthwhile. Therefore, per capita consumption growth is given by (5.10) with e_L set to zero. This is plainly below the growth rate with learning. Accumulation is absent, but wages and per capita consumption rise continually due to increasing returns from population growth. Growth in living standards may be quite slow, however, if the parameter α is near unity, so that increasing returns to labor are small.

6. *Early Development*

In this section we analyze how an economy using the purely primitive technology of Section 3 transforms itself into a modern economy using specialized intermediate inputs as described in Section 1. Exogenous population growth drives the development process. Initially, population is too small for the efficient use of specialized intermediate inputs. Eventually, however, rising population pushes the scale of operation past the critical point at which the modern sector becomes viable, and modern specialiazed processes come into use alongside the primitive sector.[16] If population continues to grow, eventually it becomes

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[16] One should think of the opening of the modern sector in our model as the birth of urbanization. Roughly speaking, urbanization followed the Neolithic or Agricultural Revolution (cerca 8,500 B.C.) when society progressed from an economy based on hunting, gathering, and fishing to one based on farming and livestock.

The first region to the see the emergence of genuine cities with craft activities and inter-city trade efficient to shut down the primitive sector altogether, completing the transition to modern balanced growth.

To best illustrate the role of population growth in early development, we assume for now that conditions are such that there is no human capital accumulation ($e_L = 0$). We analyze early development with accumulation in Section 7.

Once the modern sector begins to operate ($e_M > 0$), household choices for c, e_P, and e_M are determined by the time constraint (4.1), the period budget constraint (4.2), the two first-order conditions (4.3) and (4.10), the wage equation (1.13), and the primitive technology (3.1).

The bottom curve in Figure 2 shows how, according to (1.13), the modern sector wage depends on per capita hours worked there, e_M. The wage rises with e_M because the modern sector is characterized by increasing returns to effective labor input. Differentiating (1.13) shows the modern wage curve to be concave to the horizontal axis. As population,

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in specialized goods was the area between the Tigris and Euphrates rivers in Iraq. That region, known as Sumer, had about 10 to 15 cities around 2800 B.C. The Sumerian city of Ur, for example, had perhaps 24,000 inhabitants. The urban population of Sumer is thought to have been as high as 5% of the total. By about 1700 B.C. Babylon had become the first city with a population in excess of 300,000.

Urbanization in Greece was even more extensive, representing about 20% of the total. At its height in the second century A.D., the city of Rome had over 800,000 people. Bairoch (1988, p. 86) estimates that the Roman Empire had at least 350 cities and towns with over 5,000 inhabitants, although estimates of the urban population of the empire are notoriously imprecise.

As mentioned in the introduction, by 1,000 A.D. the urban population of Europe was roughly 10% of the total. Thus, if one wishes to use our model to study the development of pre-industrial Europe, one should assume that the modern sector had been open for quite awhile, though it was still relatively small as of 1,000 A.D.

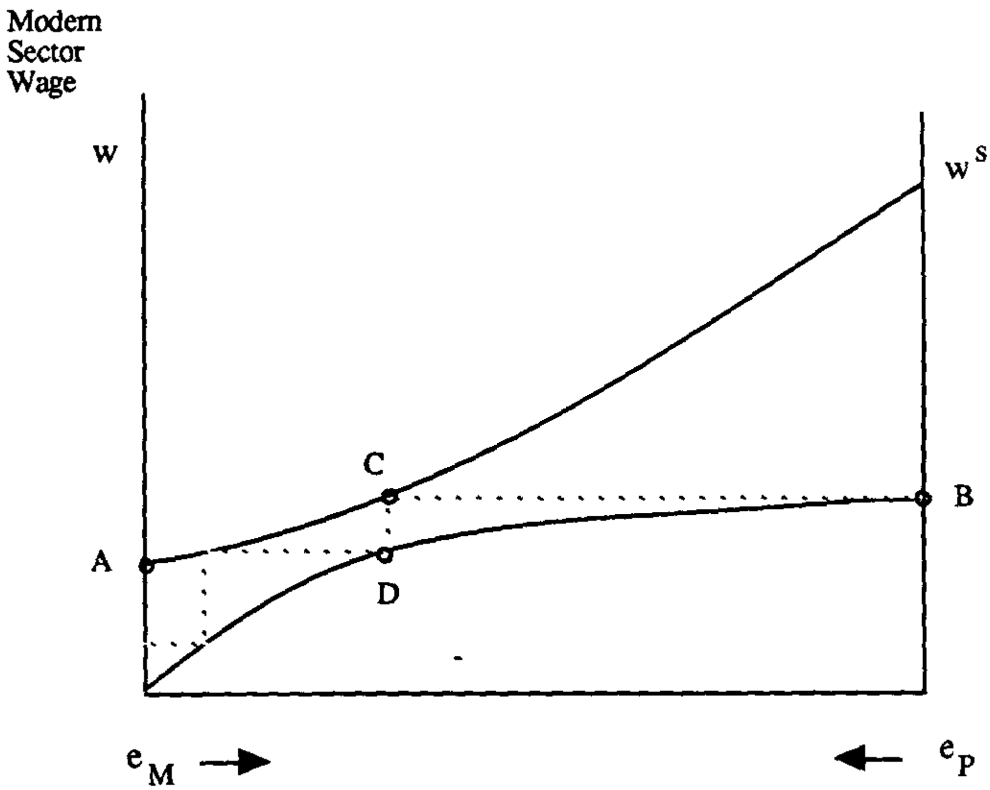

Figure 2:
The Primitive Economy

N, grows, this curve rotates upwards around the lefthand origin.

The top curve shows how, according to (3.1), the primitive-technology marginal product per man-hour of labor, $\partial y_P / \partial e_P = \beta / (e_P n + 1)$, depends on per capita hours allocated to that sector, e_P (measured from the right-hand origin). The maximum marginal product per man/hour, β, is shown as w^s in Figure 2. If the modern sector wage rises to, or above, w^s it is optimal to shut down the primitive technology (set $e_P = 0$). Differentiating $\partial y_P / \partial e_P$, it is straightforward to verify that the top curve is convex to the horizontal axis, as drawn. The shut-down wage, $w^s = \beta$, is invariant with respect to household size, n, so the curve is anchored at w^s. But, because of diminishing returns to labor, the curve rotates down over time as the population grows.

The width of the box is $e_P + e_M \equiv e$, total work effort, which is equal to unity by (4.1) under the assumption that $e_L = 0$.

Equilibrium in Figure 2 requires that two conditions be satisfied. First, household necessary conditions (4.3) and (4.10) imply that the modern sector wage must equal the primitive sector marginal product if both e_P and e_M are positive. Second, an (e_P, e_M) allocation is an equilibrium only if an individual household has no incentive to deviate from it when taking other households' allocations as given.

Figure 2 illustrates the case where population is small enough that the primitive-sector marginal product curve lies everywhere above the modern sector wage curve. When this is the case, no equilibrium exists with e_P and e_M both positive, since there is no way for the modern wage to equal the primitive marginal product while e_P and e_M sum to unity.

This leaves either corner, point A or B, as a possible equilibrium. The second condition, however, is not satisfied at point B, because if all households chose $e_M = 1$ and $e_P = 0$, then the marginal product of work in the primitive sector would exceed the modern-sector wage at B. Hence, an individual household, taking everyone else's allocation (and, therefore, the modern wage) as given would be better off shifting work effort to its own primitive technology up to point C. If everyone did so, however, the modern-sector wage would fall to point D. Repeating the argument, it is clear that point A is the only stable equilibrium in Figure 2. So $e_P = 1$ is the only equilibrium and the economy remains purely primitive for a sufficiently small population.

Now let population grow. This drives the upper curve down and the lower curve up until, eventually, a critical size is reached at which the two touch at a point like Z in Figure 3. This tangency point satisfies both conditions mentioned above, so it represents a *second equilibrium* in addition to the one at the $e_P = 1$ corner. Importantly, the new equilibrium at point Z can support a modern sector.

The economy is better off at an interior equilibrium like Z than at the corner, in the sense that per capita consumption is higher in the former. To see this, note that modern-sector product per person is $w\, e_M$, while primitive-sector per capita product is the area under the upper curve, measured from the right to the equilibrium e_M. Therefore, an economy-wide move from $e_M = 0$ to $e_M = e_M^z$ results in a net increase in product per person.

When the tangency appears at Z, however, the economy is at the $e_P = 1$ corner. The equilibrium will remain there if individual households believe that everyone else will not move. On the other hand, if each believes that everyone else will switch to e_M^z, then the equilibrium will jump to Z. Which equilibrium will be selected is indeterminate. But, given the potential gains in current and future consumption, there is reason to believe that society would jump to Z. We shall, at any rate, assume that the jump to Z is made, so the modern economy springs into existence as soon as possible.

The dotted lines in Figure 3 locate the primitive marginal product and the modern wage curves for a population that has grown beyond the critical level. As one can see, the Z allocation is no longer an equilibrium with the larger population. An individual household, taking other households' allocations as given, now has an incentive to move to point U.

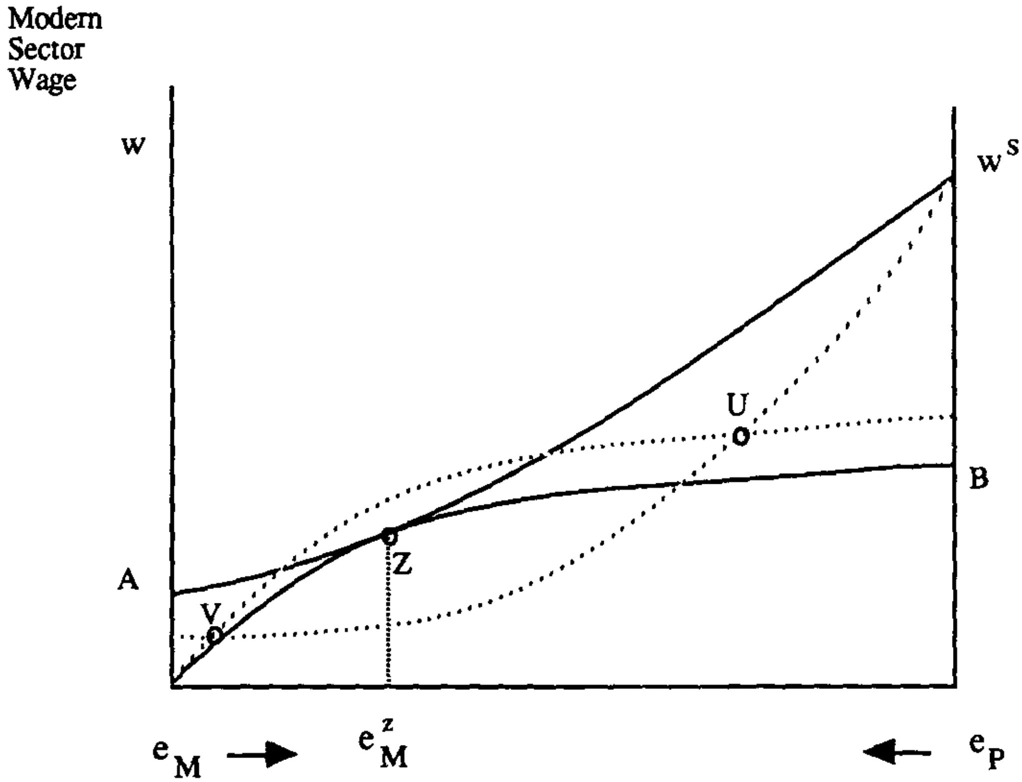

Figure 3:
Transition Equilibria

Hence, all households do so, and U is the new equilibrium.[17] Moreover, as population continues to grow, the interior equilibrium follows U until it eventually reaches w^s, at which time the primitive technology is completely abandoned. Early development -- the transition from a primitive economy to a specialized, modern economy -- is then complete, and the economy attains balanced growth without human capital accumulation.

7. The Industrial Revolution

In the previous section, development proceeded solely on the strength of increasing returns to specialization made possible by a growing population. This reflects pre-industrial growth, in which productivity gains arise from an ever-finer division of labor without much improvement in fundamental productive techniques themselves. We associate industrialization with learning that yields fundamental improvements in technology. The state of technological know-how is indexed by the stock of human capital, and the Industrial Revolution is marked by the initiation of human capital accumulation by households. In Section 5, we saw that if condition (5.1) were satisfied, the economy would accumulate human capital continually in modern balanced growth. Here in Section 7 and in Appendix C we see that when (5.1) is satisfied, accumulation through learning begins before the economy becomes fully modern. Indeed, we show at the end of Appendix C that (5.1) is both necessary and sufficient for the economy to industrialize eventually, provided only that the population grows in the pre-industrial era .

An Industrial Revolution is not an accident of history in our model; rather, it is a deliberate choice made by forward-looking, decentralized household decision makers. Households in our economy choose to devote time to learning to acquire human capital

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[17] For a given population size, a point like V is also an equilibrium. We ignore such equilibria, however, since they are not stable with respect to population growth.

when it becomes efficient to do so. The timing turns on the interaction of preferences, the primitive technology, modern production processes, and the learning technology. To the extent that primitive technologies differ across regions, our model implies diverse development histories, including the timing of the Industrial Revolution.

The Phase Diagram

It is most convenient to analyze the Industrial Revolution with a phase diagram. To do so, we generalize the phase portrait introduced in Section 5 to include, besides the fully modern region, a region that is relevant when the economy is purely primitive, and a "transition" region that is relevant when both the primitive and modern technologies are in use. The expanded phase diagram is shown in Figure 4.

In Section 6 we saw that rising population leads the economy to shift work effort from the primitive to the modern sector. A larger population favors the modern sector over the primitive because the former is more efficient at larger scales of operation. An analogous scale effect occurs with respect to increases in per capita human capital, h, since these raise the modern-wage curve in Figures 2 and 3. Thus, we can partition the expanded phase plane into the three regions described above.

The phase diagram is partitioned by the two kinked loci rising above the human capital stocks labelled $h^T(N_t)$ and $h^M(N_t)$, where N_t is the population at time t. To the left of h^T the stock of human capital is below the critical level at which the modern sector is viable, given N_t. All production takes place in the primitive sector there. Points on the h^T border correspond to tangency points like Z in Figure 3. Both primitive and modern sectors are in use in the transition region, while the modern sector alone operates at and to the right of h^M. This rightmost region, where human capital is high, is identical to that depicted in Figure 1.

Both the h^T and h^M loci move to the left over time as population grows, since a larger population requires a smaller stock of human capital to make the modern sector

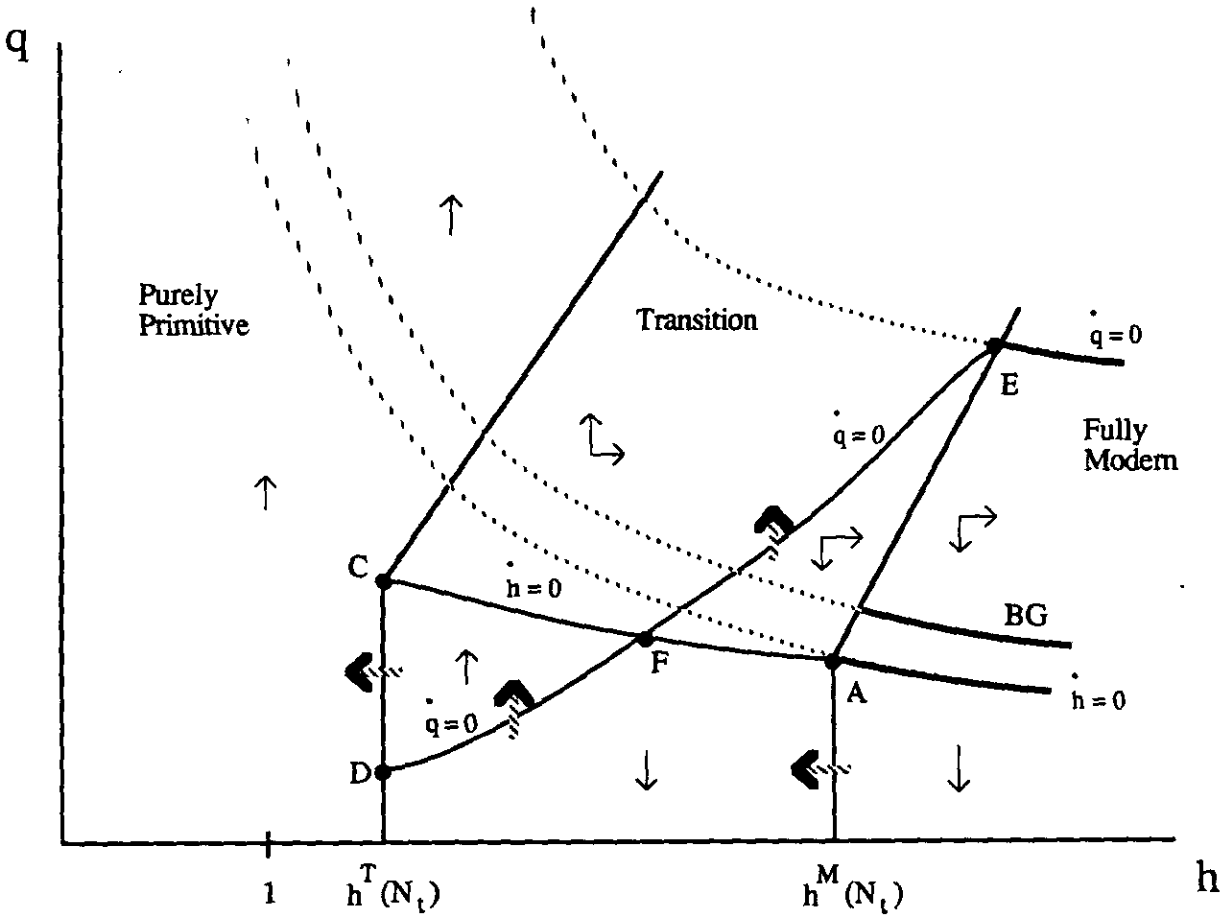

Figure 4:
The Complete Phase Diagram

viable or exclusive. This movement is indicated by the fat arrows.

The hyperbolas to the right of h^M labelled $\dot{q} = 0$, BG, and $\dot{h} = 0$ are the same as in Figure 1. They are dashed to the left of the h^M locus, since they are not immediately relevant there. They become relevant, however, as population grows and the modern region expands leftward.

The $\dot{h} = 0$ boundary in the transition region is given by the curve labelled CFA. We show in Appendix B that this boundary connects with its modern sector counterpart at point A along the h^M border. As in the completely modern region, h is rising above the $\dot{h} = 0$ boundary in the transition region and constant below it. When population grows and h^M shifts to the left, point A slides leftward up the hyperbola, bringing the endpoint of the $\dot{h} = 0$ boundary along with it. We assume that the boundary runs below the dashed hyperbola in the transition region, which is true for reasonable parameterizations of the model.

The $h^T(N_t)$ and $h^M(N_t)$ borders are kinked at points C and A, respectively, where they cross the $\dot{h} = 0$ boundary. The reason is as follows. Above the boundary learning time, e_L, is positively related to the shadow value of human capital, q, so that work effort, e, must fall when q rises. (See Appendix B.) From a point along the h^T border (which corresponds to a tangency like Z in Figure 3) a rise in h would move the economy towards greater modernization. It would take an increase in q to reduce e and re-establish the tangency in Figure 3 that defines the border. A similar argument implies that the h^M border slopes upward in the region of positive learning. The logic of Figure 3 guarantees that the h^T and h^M border loci never intersect.

We will assume that, at the time the modern sector opens, the $\dot{q} = 0$ locus starts at point D below point C on the h^T border. This is the case if $e_M(\tau)$, modern sector effort at the *moment* the modern sector opens, is small enough that $e_M(\tau)\,\delta\,(e_M(\tau)) < \rho - \eta$. If so, as shown in Appendix B, the $\dot{q} = 0$ locus must be continuous at the $\dot{h} = 0$ boundary (point F) and must connect with the fully modern $\dot{q} = 0$ locus at the h^M border (point E). Thus, the

$\dot{q} = 0$ locus is shown as DFE in Figure 4. The locus is repellent in the transition region, as in the fully modern region, and rises over time with population growth.

The thin arrows indicate the motion of q and h for a *given* population. In the primitive region, for example, q rises at the rate $\rho - \eta$, by (4.13) with e_L and e_M set to zero. In the transition region, on the other hand, motion depends upon where the (q,h) pair lies relative to the $\dot{q} = 0$ locus and the $\dot{h} = 0$ boundary. Motion in the fully modern region is identical to that in Figure 1.

The Timing of the Industrial Revolution

The timing of the Industrial Revolution is part of the overall development process driven by optimizing, decentralized households. The representative household's optimal plan is governed above all by the requirement that the (q, h) pair reach the fully modern balanced growth path the instant that the primitive technology is shut down. As discussed in Section 5, the fully modern economy requires such a starting point to satisfy transversality. If (5.1) is satisfied, human capital is accumulated along the BG path. Optimality, then, requires an Industrial Revolution: households must begin to devote time to learning sometime before, or just when, balanced growth is attained.

As noted above, we assume that the initial population level, N_0, is small enough that $h^T(N_0) > 1$: initially, society uses primitive techniques exclusively. The modern sector will open, at time τ, when population has grown enough to support such a move.

Households select their optimal development plan by choosing the initial condition for q when the modern sector first opens,[18] q_τ, so that the (q, h) pair follows the motion in the phase plane thereafter and reaches the balanced growth path at the instant the transition

[18] Actually, they choose q before this, when the economy is entirely primitive, so that q_τ will be reached just when the transition begins.

phase ends. The equilibrium development history is shown in Figure 5.[19] Panel (1) shows the situation just as the modern sector opens. Households choose q_τ to begin development at point P. Note that P must be above D, but cannot be too high. As drawn, P lies beneath C so that there is no learning for a time after the opening of the modern sector.

As time passes and population continues to grow, the h^T and h^M borders move leftward, and the $\dot{q} = 0$ locus rises in the transition region. As drawn, the optimal (q, h) path remains above the rising $\dot{q} = 0$ locus, eventually crossing the $\dot{h} = 0$ boundary at point P' in Panel (2). This marks the Industrial Revolution when households begin learning and accumulating human capital.

It is feasible for households to begin to accumulate human capital through learning the instant the modern sector opens, but they do not. Why? Human capital benefits individuals by raising the present discounted value of future income earned in the modern sector, $w_b h\, e_M$. A marginal increase in human capital raises an individual's modern sector earnings by the present discounted value of future increments, $w_b\, e_M$, where the base wage is given by (1.14). In the early transition, per capita human capital is unity, population is small, and few hours are worked in the modern sector. Hence, the base wage is very low and the marginal consumption benefit to human capital is also low.

An individual's consumption opportunity cost of accumulating a unit of human capital is the time cost, $1 / h\, \delta\, (e_M)$, multiplied by the wage, $w_b\, h$; i. e., $w_b / \delta\, (e_M)$. Thus, both the benefit and the cost of learning depend positively on the base wage and, as a first approximation, we can ignore the marginal effect of w_b on the benefit net of cost. In the early transition, however, we are left with a benefit that is low relative to the cost because few hours are worked in the modern sector, and because the small size of the modern sector restricts specialization, keeping learning productivity $\delta\, (e_M)$ down.

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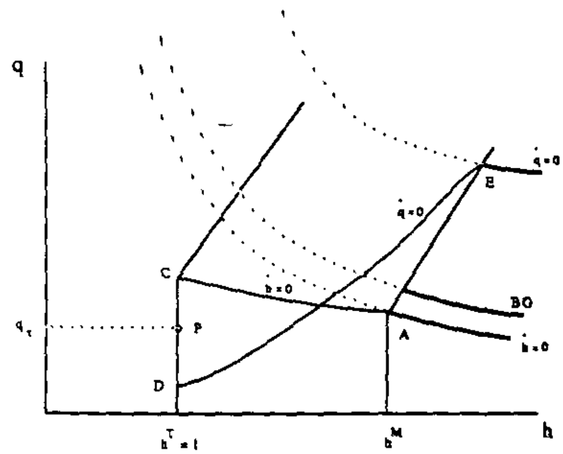
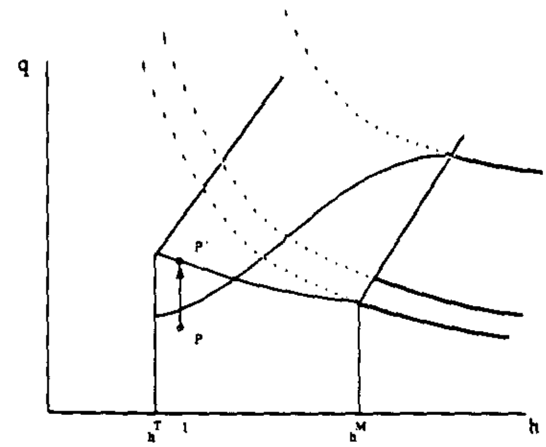
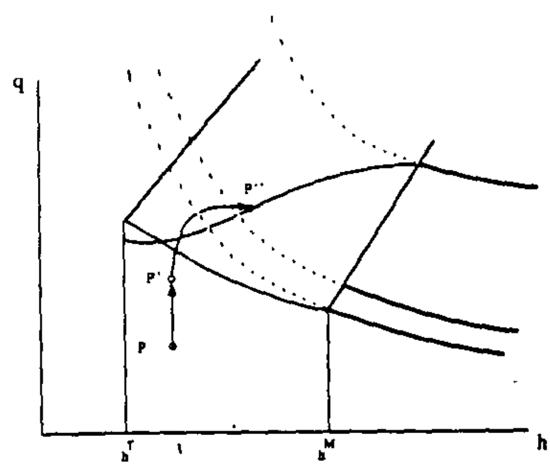
[19] Appendix C shows that such an equilibrium path exists and that it is unique.



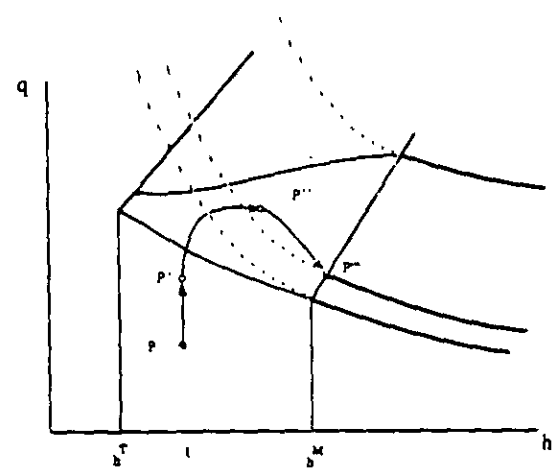
(1) The Modern Sector Opens



(2) The Industrial Revolution Begins



(3) Early Development with Accumulation



(4) Modern Balanced Growth Achieved

Figure 5:
The Industrial Revolution

A sufficiently small e_M in the early transition, therefore, makes it inefficient to accumulate from the start. Accumulation becomes efficient only when e_M becomes large enough that the present discounted consumption value of the increments to future earnings exceeds the consumption opportunity cost of the learning time needed to accumulate human capital. This is the condition reached at P' in Panel (2) when the Industrial Revolution occurs.

The rising population continues to move h^T and h^M leftward and to raise the learning boundary in the transition. Most importantly, however, the $\dot{q} = 0$ locus continues to rise and eventually overtakes the P path at P'', shown in Panel (3), at which time q must begin to fall. Since the P path is optimal by construction, the phase plane motion thereafter brings the (q, h) pair to P''' in Panel (4), where the transition ends and the economy attains fully modern balanced growth.

Several factors govern the timing of the Industrial Revolution and the speed with which a society makes the transition from a primitive to a modern economy. Using Figure 3 to analyze the pre-industrial period, we see that faster population growth causes the modern sector to open sooner and speeds development thereafter. It is also apparent that the modern sector opens later, the more productive the primitive technology (the higher is β) and the less productive the modern sector (the lower is A). Likewise, development proceeds more slowly when the primitive technology exhibits smaller diminishing returns and the modern sector is characterized by smaller increasing returns. The slower development of the modern sector, in turn, limits learning productivity $\delta(e_M)$ and delays the Industrial Revolution.

Nor are these factors necessarily independent. For example, rapid population growth early on may be the *result* of a highly productive primitive technology, a high β. If population growth depends on the technology in this manner, there is little reason to expect a heavily populated region to have developed rapidly, since its highly productive primitive technology would have discouraged development of a modern sector.

Our model suggests that different development experiences may be due to different geographical initial conditions. To interpret different experiences in terms of our model, we need to understand how geographical factors account for differences in population growth, primitive technologies, early modern techniques, and learning productivities. We believe that the primitive technology is critical in this, since it provides the foundation for population growth and the specialized goods that arise initially.

8. The Population Puzzle

Population growth plays a central role in our model of early development. It must grow to a threshold level before the economy is large enough to support a modern sector. Moreover, a rising population continues to be essential for pre-industrial development by enabling the economy to benefit more fully from increasing returns to specialization. A growing population raises modern sector wages, reduces the primitive sector marginal product, and so causes the modern sector to expand at the expense of the primitive sector. Most importantly, population must become sufficiently large before industrialization can begin. If population were to stop growing prior to triggering an Industrial Revolution, development would cease.

Yet empirically, the relationship between population and development appears weak. Consider, for example, Kuznets's (1973, pp. 41 - 48) post-War cross-country evidence from a sample of 63 developed and less-developed countries. He finds a statistically significant negative correlation between population growth and growth in per capita product. But this negative correlation is due entirely to the differences between the two groups of countries. Disaggregating, he finds that the correlation becomes insignificant both for the developed group by itself and the less-developed countries taken by themselves.

The negative correlation between groups arises because all high income countries have gone through a transition from high fertility and mortality rates to low rates, due to improved health care and the higher opportunity cost of having children.[20] Thus, the higher income growth appears to be the result of a high level of economic development, rather than a lower population growth rate. Controlling for the level of economic development, Kuznets finds little correlation between growth in population and growth in per capita product.

The problem for our model is this. Since the model relies on increasing returns to population in the modern sector, it would seem to imply that the most populous countries should have the highest per capita product. Yet the most heavily populated countries in the world are far from having the highest per capita product. Our population-driven model faces a population puzzle. How are we to reconcile the central role of population growth in our model with the evidence that finds little correlation between population growth and per capita output growth for either developed or less developed countries?

Developed Countries

The possibility of an Industrial Revolution provides a solution to the population puzzle. The accumulation of human capital means that population size and growth eventually become relatively unimportant for the level and growth of per capita product. To see this, use (4.2) and (1.13) to express per capita product in modern balanced growth as:

$$y = A\, h^{2-\alpha} e_M^{2-\alpha} N^{1-\alpha} \quad (8.1)$$

For population growth to drive development in its early phase, we require only that the modern sector exhibit *some* increasing returns. Technically, we need $1 - \alpha > 0$, no matter how small. In fact, a small value for $1 - \alpha$ may be very reasonable, since it implies a lengthy transition, as we in fact observe. If so, then (8.1) shows that the level of per capita

[20] On this point, see Barro and Becker (1988) and Becker, Murphy, and Tamura (1990).

product in modern balanced growth is not very sensitive to the population size.

In contrast, per capita output would be approximately *proportional* to both e_M and h if $1 - \alpha$ were near zero.The fraction of time spent working is constant in balanced growth and one would not expect it to be a major source of differences among national product levels. But the stock of human capital is unbounded, and can potentially account for large discrepancies across countries.

Hence, our model predicts that differences in human capital per capita, not population size, are most likely to account for diversity of per capita product across developed nations. What might account for such differences in human capital? According to the model, the answer lies in the fact that countries may have different transition histories, based on different initial conditions, giving rise to varying degrees of industrialization.

Taking logs and differentiating (8.1) yields the balanced growth rate of per capita product. If the coefficient on population growth is very small, then since e_M is constant, per capita output grows approximately at the rate of growth in human capital per capita. Thus, the model is consistent with a weak relationship between population and per capita output growth, and suggests that diverse growth rates among developed countries are due to factors governing learning productivity $\delta(e_M)$ and time devoted to learning, e_L.

Less-Developed Countries

In terms of our model, a less-developed country is one that has not yet completed the transition to fully modern balanced growth. This accords with reality, since a significant portion of LDC output is produced by relatively primitive techniques without much benefit of modern specialization.

While our model predicts a positive, though possibly very weak, effect of population size on per capita product in the fully modern economy, even this need not be the case for an economy in transition. As we show in Figure 6, *product per person may actually decline* during the transition. The dark curves there determine an equilibrium at

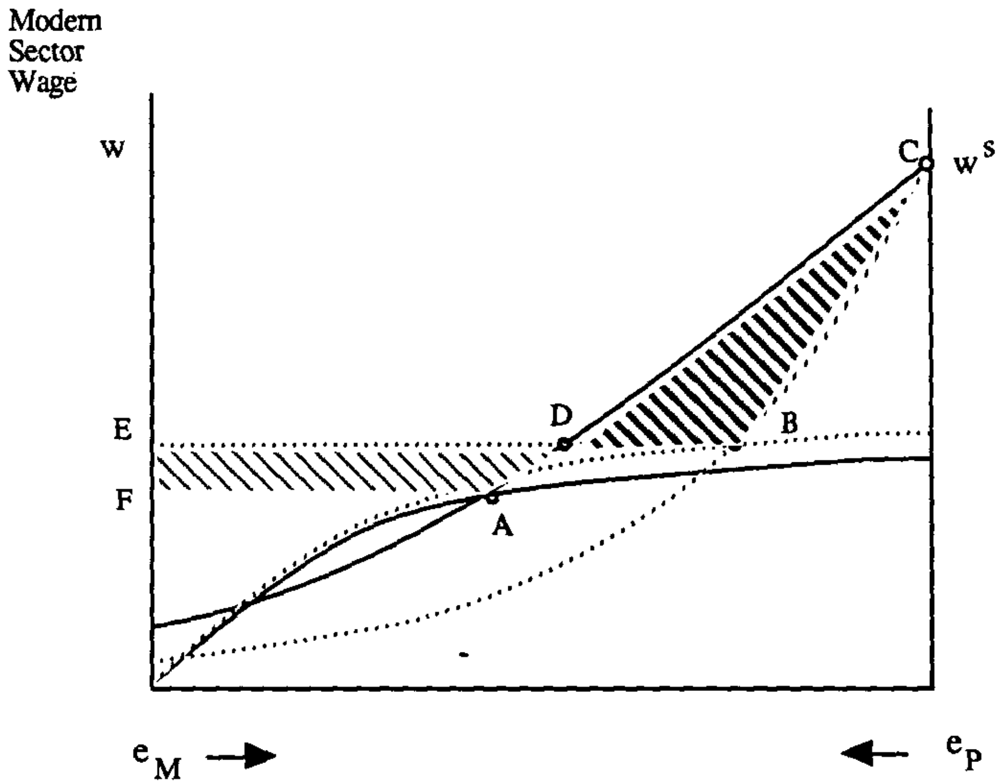


Figure 6:
Per Capita Consumption During
the Transition

point A, so that per capita output is the area under FAC. The dashed curves depict the situation after a relatively short period of population growth. The equilibrium has moved to point B and per capita product is now the area under EBC. The net change in per capita output is EDAF less DBC. As drawn, per capita product falls slightly.

The possibility that population growth reduces the standard of living arises because per capita product is an average of output from the primitive and modern sectors. Because of diminishing returns in the primitive sector, rising population forces down per capita product there. Although per capita product in the modern sector rises with population, the increase is slight if $1 - \alpha$ is near zero. Effort shifts to the modern sector continually in response to the higher population. But the increasing returns to e_M there may not be high enough to offset the diminishing returns in the primitive sector.

The effect of a rising population on per capita product is more likely to be negative: (1) earlier in the transition, when the primitive sector is relatively large; (2) the smaller the increasing returns in the modern sector, so the modern sector wage curve is less steep and shifts up less with population growth; (3) the larger the diminishing returns in the primitive sector, so that the primitive marginal product curve is steeper and shifts down more with population growth.

While population growth may reduce per capita product temporarily in our model, the cumulative effect of population during the entire transition *must* be positive. A related point is that per capita product may be considerably higher for countries at or near the end of the transition, than for countries whose primitive sectors are still relatively large. Countries that continue to produce a relatively large share of output with primitive techniques will have low per capita product regardless of population size.

Our model reconciles two apparently contradictory notions about the influence of population in LDC's. It emphasizes the essential role of population in driving the transition from a primitive society to a modern economy. But it also allows for the possibility that population growth could actually reduce per capita product during the transition. Thus, our model explains how population growth might appear detrimental to LDC development over some periods. A key idea, however, is that *without* the the specialized modern sector, per capita output would fall by considerably *more* under the weight of greater population. Our model, then, predicts a potentially loose, short-term relationship between population growth and per capita output growth, depending on where an LDC is in its transition. This, in turn, is consistent with the insignificant correlation across LDC's found by Kuznets.

Taking a long, historical perspective, the above argument implies a U-shaped curve for per capita product as a function of population in the pre-industrial era.[21] The effect on per capita product of an exogenous population shock due to war or plague, for example, would therefore depend on the society's position along this curve when the shock occurred. A plague in a relatively primitive society would raise output per person, but it would reduce it in a highly urbanized economy. The pre-industrial data summarized in the introduction suggests that population raised per capita product after 1500, but may have had little effect prior to that date.

9. Conclusion

We have modelled early development as the transformation of society from a collection of autonomous producing units into a modern economy based on firms, specialization, and the division of labor. To do so, we relied on the idea that the extent of the market limits society's ability to take advantage of increasing returns inherent in modern production processes. For us, population is the critical factor limiting the extent of the

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[21] The possibility thus exists that a growing population could reduce per capita product to a minimum subsistence level, precluding further increases in scale. Such a binding subsistence constraint in the pre-industrial period would leave the economy in a Malthusian trap.

market in the earliest phase of development. When population is too small, the economy remains in a primitive state, unable to take advantage of increasing returns to specialization. Once population becomes sufficiently large, a specialized modern sector arises side-by-side with the primitive one. We showed, however, that primitive techniques are not abandoned immediately. Rather, there is a long period of slow pre-industrial development in which ongoing population growth expands the modern sector by gradually increasing the size of the market.

Productivity gains during the pre-industrial period arise primarily from an ever-finer division of labor applied to traditional techniques. An Industrial Revolution occurs in our model when individuals begin to devise fundamental technological improvements by devoting time to learning. Since learning productivity depends critically on access to specialized tools and techniques, it is relatively low when the modern sector is small. Individuals only begin to learn after the specialized modern sector has expanded sufficiently to make learning worthwhile. In this way, we explained how a long period of slow pre-industrial development eventually triggers an Industrial Revolution that leads to modern balanced growth.

Population growth is no longer necessary for ongoing development once industrialization begins, and plays only a minor role in modern balanced growth. Industrialization thereby reconciles the crucial role of population in early development with its apparently weak relation to per capita product in developed economies. Less developed countries, in our model, are those with relatively large primitive sectors. For them, we showed that population growth could actually reduce per capita product. Moreover, per capita product would be considerably lower for countries with relatively large primitive sectors, regardless of population size.

Several factors govern the timing of the Industrial Revolution and the speed with which a society makes its transition from a primitive to a fully modern economy.

Development proceeds more slowly, the more productive the primitive technology and the smaller its diminishing returns. Faster population growth speeds development, though if it results from a highly productive primitive technology the combined consequences for development are ambiguous. Industrialization tends to occur sooner the faster the modern specialized sector develops. Our model suggests that different development experiences are due to different geographical initial conditions. To interpret different experiences in terms of our model, we need to understand how geographical factors account for differences in the primitive technology, since it provides the basis for the modern processes that follow.

Appendices

A. The Household's Problem

The household decision maker maximizes:

$$J = \int_0^{\infty} n(t)\, u[c(t)]\, e^{-\rho t}\, dt$$

by choice of $c(t)$, $e_P(t)$, $e_M(t)$, and $e_L(t)$, subject to:

(A.1) $$y_P(t)n(t) + w_b(t)h(t)e_M(t)n(t) - c(t)n(t) = 0$$

(A.2) $$e_P(t) + e_M(t) + e_L(t) = 1$$

(A.3) $$\dot{n}(t) = \eta\, n(t)$$

(A.4) $$\dot{h}(t) = \delta\, e_L(t)\, h^{1-\gamma}(t)\, h_a^{\gamma}(t)\,, \text{ where } \delta \equiv [\alpha\, f/b(\tilde{x})]^{\gamma}\, e_{Ma}(t)^{\gamma}$$

(A.5) $$y_P(t) = Y_P(t)/n(t)$$

(A.6) $$Y_P(t) = \beta \ln\,(e_P(t)\, n(t) + 1)$$

(A.7) $$e_P(t) \geq 0$$

(A.8) $$e_M(t) \geq 0$$

(A.9) $$e_L(t) \geq 0$$

(A.10) paths $\{w_b(t), h_a(t), e_{Ma}(t)\}$ for $t \geq 0$.

The current-valued Hamiltonian, with utility price θ used to value increments to human capital, is given by:

(A.11) $$\begin{aligned} \mathcal{H} &= nu(c) + \theta\delta\, e_L\, h^{1-\gamma} h_a^{\gamma} \\ &+ \lambda_1(y_P n + w_b h e_M n - cn) + \lambda_2(1 - e_P - e_M - e_L)n + \Omega_1 e_P n + \Omega_2 e_M n + \Omega_3 e_L n\,, \end{aligned}$$

where the λ_i and Ω_i are the multipliers for the two equality and three inequality constraints.

The first-order conditions for a maximum are found by differentiating $\mathcal{H}$ with respect to c, e_P, e_M, and e_L; the shadow price θ; and the multipliers λ_i. The first-order conditions are, respectively:

(A.12) $$u_c = \lambda_1$$

(A.13) $$\lambda_2 = \lambda_1\, (\partial y_P/\partial e_P) + \Omega_1$$

(A.14) $$\lambda_2 = \lambda_1 w_b h + \Omega_2$$

(A.15) $$n\lambda_2 = \theta \delta h^{1-\gamma} h_a^{\gamma} + \Omega_3 n$$

(A.16) $$\dot{h} = \delta e_L h^{1-\gamma} h_a^{\gamma}$$

(A.17) $$y_P + w_b h e_M = c$$

(A.18) $$e_P + e_M + e_M = 1.$$

The utility-price of human capital must be a *continuous* function of time, and it must change according to:

(A.19) $$\dot{\theta} = \theta\rho - \partial\mathcal{H}/\partial h = \theta\rho - \theta(1-\gamma)\delta e_L h^{-\gamma} h_a^{\gamma} - \lambda_1 w_b e_M n .$$

Together with the transversality condition,

(A.20) $$\lim_{t \to \infty} e^{-\rho t} \theta(t) h(t) = 0 ,$$

and the initial conditions

(A.21) $$n(0) = n_0$$

(A.22) $$h(0) = 1 ,$$

conditions (A.12) - (A.19) are necessary and sufficient for a household optimum.

Finally, in moving from this system to the one presented in the text, we define $q \equiv \theta / n$. This is done to eliminate n from the first-order conditions, except insofar as it influences wages and the marginal product of labor in the primitive sector.

B. Motion in the Transition

Here we characterize the motion of q and h in the transition region of the phase diagram of Figure 4. We do so by deriving the $\dot{h} = 0$ boundary and the $\dot{q} = 0$ locus, using the household's necessary conditions from Section 4, including the restriction that the modern sector wage equal the primitive marginal product of labor.

Wage equalization and the logic of Section 6 imply that e_M and e_P depend on h, n, and e as follows:

(B.1) $$e_M = e_M(h, n, e) \ ; \ e_{Mi} > 0 \ , e_{Me} > 1 \ ;$$

(B.2) $$e_P = e_P(h, n, e) \ ; \ e_{Pi} < 0 \ ; \ \text{for } i = h, n \ .$$

Since a rise in work effort, e, e_p, we know that e_M must rise by more than e itself; hence, $e_{Me} > 1$.

Work effort is determined by the state variables, h and n, and the shadow price, q. To see this, note that when e_L is unconstrained, the necessary conditions (4.10) and (4.11) together with log utility imply:

(B.3) $$\delta(e_M)\, q\, h \ = \ [e + \tfrac{s}{w}]^{-1} \ ,$$

where $s \equiv y_p - w\, e_p > 0$, and δ depends on e_M by virtue of (4.14). The variable s is output per capita in the primitive sector, less per capita earnings in the modern sector. The logic of Figure 2 -- wage equalization, diminishing returns to the primitive technology, and increasing returns in modern production -- implies that s depends negatively on h, n and e. The modern sector wage, w, depends positively on these same variables. Hence, the ratio s / w depends negatively on them, and we write:

(B.4) $$s / w \ = \ S\,(h, n, e) \ , \quad S_i < 0 \ .$$

Using, (B.1) and (B.4), (B.3) implicitly defines work effort, e, as a function of qh, h, and n. Express this as:

(B.5) $$(\, q\, h\,)^{-1} = \delta\,[e_M\,(h, n, e)]\ \{e \ + \ S\,(h, n, e)\ \} \quad \Rightarrow \quad e \ = \ E\,(qh, h, n) \ .$$

Assuming that $-S_e < 1$, we have that $E_{qh} < 0$, while E_h and E_n are ambiguous. This means that as qh falls, work effort rises to its upper bound of 1. For any (h, n) pair, there is a specific q just low enough to make $e_L = 0$ and $e = 1$.

Still assuming that e_L is unconstrained, equation (B.5) allows us to express all of the time allocations in terms of the shadow price and the state variables. Substituting (B.5) into (B.1) and (B.2), and using the time constraint, yields:

(B.6) $$e_M = E_M\,(qh, h, n)\ ; \ E_{Mqh} < E_{qh} < 0 \ ;$$

(B.7) $$e_P = E_P\,(qh, h, n)\ ; \ E_{Pqh} > 0 \ ;$$

(B.8) $\qquad e_L = E_L\,(qh, h, n) = 1 - E\,(qh, h, n)\,;\;\; E_{Lqh} = -\,E_{qh} > 0\,.$

Since e_P^- falls when e rises, $E_{Pqh} > 0$. This means that e_M must fall *more* than e does when qh rises, so $E_{Mqh} < E_{qh}$. We assume that the signs of the partials with respect to h and n in (B.6) and (B.7) are the same as those in (B.1) and (B.2): $E_{Mh}, E_{Mn} > 0$; and $E_{Ph}, E_{Pn} < 0$.

Now we derive the $\dot{h} = 0$ boundary. The motion equation for h, (4.12), can be written as:

(B.9) $\qquad \dot{h} = h\,\delta\,(e_M)\,e_L = h\,\delta\,[E_M\,(qh, h, n\,)\,]\,\{1 - E(qh, h, n)\,\}\,,$

from which it can be seen that the boundary where e_L and $\dot{h}$ are zero is also the set of points where E(qh, h, n) is unity. This corresponds to the smallest value of qh for which the interior condition (B.5) is valid. Setting e to unity in (B.5) yields:

(B.10) $\;\dot{h} = 0\;$ boundary $\;\Rightarrow\;\; q\,h \;=\; \left\{\,\delta\,[\,e_M\,(h, n, 1)]\,\{1 + S\,(h, n, 1)\,\}\,\right\}^{-1}.$

We see that $e_L > 0$ above the $\dot{h} = 0$ boundary, and is equal to zero below it, since, according to (B.9), $\partial\,\dot{h}\,/\,\partial\,q > 0$ when evaluated at e = 1.

Next, we derive the $\dot{q} = 0$ locus. This must be done in two parts. First, we find the $\dot{q} = 0$ locus below the $\dot{h} = 0$ boundary where e_L is constrained to zero. From (4.13) we can write:

(B.11) $\qquad \dot{q}\,/\,q = (\rho - \eta) - e_M\,(h, n, 1)\;\{[\,1 + S\,(h, n, 1)]\,q\,h\}^{-1}\,,$

where we have set $e_L = 0$, and used (B.1) and (B.4) with e = 1. Setting (B.11) to zero yields the $\dot{q} = 0$ locus:

(B.12) $\;\;\dot{q}\; = 0\;$ locus $\;\Rightarrow\;\; q\,h \;=\; e_M\,(h, n, 1)\;\{[\,1 + S\,(h, n, 1)]\,(\rho - \eta)\}^{-1}\;.$

This locus repels q itself, as is evident from (B.11) where q appears only in the second term.

Now we turn our attention to that portion of the $\dot{q}\; = 0$ curve that lies on or above the $\dot{h} = 0$ boundary, where e_L is not constrained. To do so, note that (4.13), (4.1), and (4.10) and (4.11) now yield the following:

(B.13) $\quad \dot{q}/q = (\rho - \eta) - \delta\,[E_M(qh, h, n)]\,\{(1-\gamma)\,E_L(q\,h, h, n) + E_M(q\,h, h, n)\}.$

Setting (B.13) to zero and using (4.1) yields:

$$\dot{q} = 0 \ \text{locus} \ \Rightarrow$$

$$\text{(B.14)} \qquad \rho - \eta = \delta\,[E_M(qh, h, n)]\,\Big\{1 - \gamma - (1-\gamma)\,E_P(qh, h, n) + \gamma\,E_M(qh, h, n)\Big\}.$$

By (B.6) and (B.8), we know that $E_{Lqh} < -E_{Mqh}$, which means, by (B.13), that the $\dot{q} = 0$ locus is repellent in the region where learning takes place, just as it is in the region below the boundary. Together, (B.12) and (B.14) imply that the $\dot{q} = 0$ locus rises with population.

In the text, we assume that at the moment the modern sector opens the $\dot{q} = 0$ locus starts at point D below C on the h^T border . If so, it must eventually cross into the learning region while still in the transition. To see why, suppose the contrary, so that $\dot{q} = 0$ hits the h^M border below A in Figure 4. But at h^M, below the $\dot{h} = 0$ boundary, we know that e_M = 1 and S = 0, so that $qh = 1/(\rho - \eta)$, according to (B.12). But at A, $qh = 1/v = 1/\delta(1)$ which is *less than* $[1/(\rho - \eta)]$ if (5.1) is satisfied. Hence, there is a contradiction.

The $\dot{q} = 0$ locus must also be continuous as it crosses the $\dot{h} = 0$ boundary. To see this, note that the value of h at which the $\dot{q} = 0$ locus hits the boundary from below is the one that solves the expression found by equating (B.10) and (B.12). Using (B.10) with (B.14), we see that the $\dot{q} = 0$ locus hits the $\dot{h} = 0$ boundary from above at the same h.

Finally, we can see that the $\dot{q} = 0$ locus connects with its fully modern counterpart exactly at the h^M border (point E) as follows. Use (4.10) and (4.11) to substitute $\delta(e_M)$ for $u_c w/qh$ in (4.13). Now set $\dot{q}$ equal to 0 in the result. Then write e_M as $1 - e_L - e_P$. Use (B.3) and $e = 1 - e_L$ to express $e_L = 1 + S - 1/[\delta(e_M)\,q\,h]$ and substitute into the expression for $\dot{q} = 0$. As $h \to h^M$, $e_P \to 0$ and $S \to 0$, implying that $qh \to \gamma/\{\rho - \eta - (1-\gamma)\,\delta(e_M)\}$, which is (5.8) by virtue of (5.3).

C: Existence and Uniqueness of an Equilibrium

Path When (5.1) Is Satisfied

Following the discussion in the text, we assume $e_M(\tau)$ is sufficiently small so that the left endpoint of the $\dot{q} = 0$ locus in the transition region, point D in panel (1) of Figure 5, is beneath point C when the modern sector opens at time τ. At $t = 0 < \tau$, the household can pick a value for q(0) such that $q(\tau)$, as given by the point $P(\tau)$ in panel (1) of Figure 5, lies anywhere between D and C along h = 1. This is true since the speed of the rising q(t) to the left of h^T is $\dot{q} = q(\rho - \eta)$, and h^T itself is moving leftwards at a rate governed by exogenous population growth, η.

Let time t_1 be the time it takes for the intersection point of $\dot{q} = 0$ and $\dot{h} = 0$ (labelled F in Figure 4) to reach h = 1. Note that t_1 is exogenous. Let t_2 be the time at which P(t) reaches the $\dot{h} = 0$ boundary. By selecting $P(\tau)$ appropriately, we can insure that $\tau \le t_2 \le t_1$. A t_2 equal to τ means that $P(\tau)$ coincides with C in panel 1 of Figure 5. When $t_2 = t_1$, on the other hand, P(t) is "overtaken" by the rising $\dot{q} = 0$ locus just as it reaches the $\dot{h} = 0$ boundary.

In fact, an equilibrium requires that t_2 must be somewhere between the two extremes. If $t_2 = \tau$, then the path would head into the purely primitive region. If, on the other hand, $t_2 = t_1$, the path would immediately get caught by the rising $\dot{q} = 0$ locus, and the economy would never accumulate human capital. In order to reach BG eventually, the system must enter into the region of positive learning ahead of the $\dot{q} = 0$ locus. This is the situation shown in panel (2) Figure 5.

By appropriate choice of $P(\tau)$ we can achieve any position we like for $P(t_2)$ along the lefthand portion of the $\dot{h} = 0$ boundary relative to the intersection point of the $\dot{q} = 0$ locus. In terms of Figure 4, we can effectively "place" the starting point anywhere along the segment FC of the $\dot{h} = 0$ boundary.[22]

Suppose for the moment that, after the path point reaches $P(t_2)$ somewhere along the $\dot{h} = 0$ boundary, population growth, η, becomes zero. The system would then be *autonomous*. Under these conditions, it is always possible to hit the terminus of BG -- which we now call point Z -- given the motion illustrated in Figure 4. To see this, consider Figure C.1 below. Transition region motion indicates that paths starting along FC near F must spiral either toward F or away from it.[23] Note, first, that paths cross the $\dot{q} = 0$ locus horizontally and then head to the southeast. Second, note that point Z must lie between points E and A; that is, Z itself must lie to the southeast of the $\dot{q} = 0$ locus. Because of continuity from variation in initial conditions, there must be a point beginning on the $\dot{q} = 0$ line that just hits Z. Working back from this point on the $\dot{q} = 0$ locus allows us to find the equilibrium point, $P(t_2)$. That point, moreover, must be unique.

If η were positive, but very small, the system would be *nearly autonomous* and the above arguments would go through with minor modification, since the path point would still move much faster than the loci defining the transition region.

For faster population growth (although η must be less than ρ) the transition region moves more rapidly to the left and we need, in particular, to account for: (i) the fact that the $\dot{q} = 0$ locus rises more quickly now relative to the path point and, (ii) the fact that the terminal point Z of BG moves more rapidly to the northwest along its rectangular hyperbola. Both move at exogenous speeds governed by η. The problem now is to intercept the rising Z. Figure 5 in the text shows how this is done. The question here is:

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[22] This statement is, of course, misleading, in the sense that it is the FC segment (appropriately distorted by the collapsing of the transition region) that moves over the value $h = 1$, and not the value of h that is being moved under the FC segment.

[23] This is true only of the region above the $\dot{h} = 0$ boundary, which is all we care about.

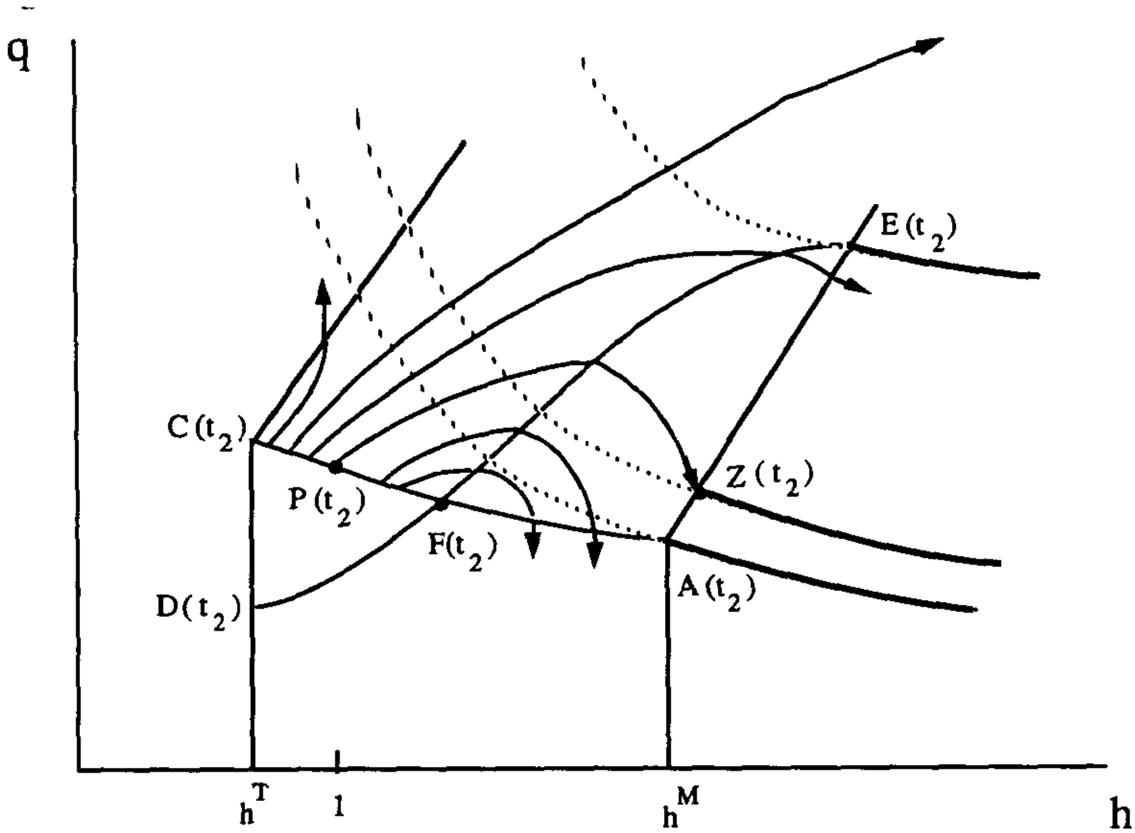

Figure C.1:
Existence and Uniqueness

must there always exist a path beginning along FC that intercepts Z?

By continuity, we can always select a point on the now-rising $\dot{q} = 0$ locus to intercept the rising Z. This only requires us to find paths that deviate to either side of Z. Using the same reasoning, we can always find a $P(t_2)$ that is guaranteed to hit the proper point along the $\dot{q} = 0$ locus. Thus, an equilibrium path always exists. Moreover, even though the P(t) path intersects the trajectory of Z twice, P(t) can only intercept Z when P(t) is falling. Therefore, the equilibrium path must be unique.

The above establishes that the economy industrializes eventually if (5.1) is satisfied. We can also show that (5.1) must be satisfied for the economy to industrialize. To begin, suppose that (5.1) is not satisfied so that $\delta(1) \leq \rho - \eta$.

Recall that (B.13) describes the motion of q above the $\dot{h} = 0$ boundary in the transition region. Divide (B.13) by $\delta[E_M(..)]$. Since $E_M < 1$ above the $\dot{h} = 0$ boundary, it follows that if (5.1) is not satisfied, then by (2.4) the first term on the RHS of (B.13) exceeds one. Recalling (4.1), the second term on the RHS is less than one. Hence, $\dot{q} > 0$ everywhere above the $\dot{h} = 0$ boundary in the transition region.

We have established already that the $\dot{h} = 0$ boundary is continuous across the transition and modern regions. We also know that the $\dot{q} = 0$ locus lies below the $\dot{h} = 0$ boundary in the fully modern region when (5.1) is not satisfied, and that $\dot{q} > 0$ above the $\dot{q} = 0$ locus in that case.

It follows directly that if learning ever took place during the transition, it would have to continue forever. But we know from Section 5 that if (5.1) fails to be satisfied households would not wish to learn when the transition is complete. Hence, we have a contradiction and we conclude that the economy will never learn, i. e. industrialize, unless (5.1) is satisfied.

We have thus shown that (5.1) is both necessary and sufficient for the economy to industrialize eventually, provided only that population continues to grow until the Industrial

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