

Incomplete:  
Conclusions and References missing

CREDIT CHAINS

by

Nobuhiro Kiyotaki<sup>†</sup> and John Moore<sup>††</sup>

January 1997

<sup>†</sup>University of Minnesota and Federal Reserve Bank of Minneapolis

<sup>††</sup>London School of Economics and Heriot-Watt University

\* This is a major revision of our July 1995 mimeo, "Credit Chains". We have benefitted from feedback from a number of colleagues and seminar audiences. We would particularly like to thank Martin Hellwig and Bengt Holmstrom for their help, and we are grateful to the University of Basel for hospitality. Financial assistance is acknowledged from the US National Science Foundation, the UK Economic and Social Research Council, and the Financial Markets Group at the London School of Economics.

## 1. Introduction

This paper is a theoretical study into how shocks propagate through a network of firms who borrow from, and lend to, each other. In particular, we investigate how a small, temporary shock to the liquidity of some firms may cause a chain reaction in which other firms get into financial difficulties, thus generating a large, persistent fall in aggregate activity. Also, we use our model to look at the aggregate effects of creditors' postponing the debts of delinquent debtors, rather than liquidating their assets. We show that, although it may be bilaterally efficient for a creditor and debtor in one link of a credit chain to reschedule debt, postponement can have serious social consequences because no new liquidity is injected into the system. Finally, we demonstrate that firms may privately decide not to insure against the risk of default by their debtors, even though such insurance may be available.

The economy we consider has many small firms owned and run by entrepreneurs who are unable to raise finance from outside investors, because they cannot precommit not to default, and because they do not have any collateral to offer potential investors by way of security. However, an entrepreneur can borrow from his suppliers, for the reason that they can withhold their supply if he defaults. That is, suppliers have additional leverage over him because the specific input that they supply to him is necessary for his production.

In our model we will demonstrate that supply contracts have to involve an element of lending, otherwise a supplier will be unable to compete against other suppliers. Since an entrepreneur sells to other entrepreneurs, he is thus forced to lend, even though he is short of funds. In other words, he simultaneously lends to his customers (other entrepreneurs) and borrows from his suppliers (who might be other entrepreneurs or unconstrained firms). Because his balance sheet has financial assets (accounts receivable from his customers), as well as liabilities (accounts payable to his suppliers), he is exposed to the risk of default by his debtors/customers.

Crucially, the entrepreneur cannot net out these gross positions in order to shed the default risk. In particular, he cannot raise money against

the accounts receivable, because only he is in a position to supply his customers with their specific input. If unconstrained third parties -- deep pockets -- were to buy the debt owed to him by his customers they wouldn't have the leverage necessary to get the customers to pay up. As a result, the entrepreneur cannot securitize the debt owed to him. If his customers experience a negative liquidity shock and default, he himself may run into financial difficulties, and he may have to default against his suppliers, thus causing further difficulties along the credit chain. This can be thought of as systemic risk.

To fix ideas, consider the following simple example. Suppose entrepreneur A has ordered 100 units of specific input from entrepreneur B at \$1 a unit. That is, A owes \$100 to B at the date the input is due for delivery, by which time she expects to have \$100 in cash with which to pay him. At the same time, B has ordered 100 units of specific input from entrepreneur C at \$1 a unit: B owes \$100 to C on delivery. B has no cash of his own, but expects to use the \$100 that A owes him to pay his debt to C. These may be two links in a longer credit/supply chain, which in turn might be part of a larger network.

Now suppose that A finds her cash holding is only \$60, rather than the \$100 she expected. She is unable to meet her obligations to B. If, following A's default, B continues to charge \$1 for each unit he delivers, then A can only afford to take delivery of 60 units. B can liquidate the remaining 40 undelivered units, but, since they are specific to A, they fetch less than \$1 a unit, say just \$0.5. Thus B gets \$80 in total: \$60 in cash from A plus \$20 in liquidation receipts.

This is less than the \$100 B had expected to receive from A, and so he is unable to meet his obligations to C. If C charges \$1 for each unit delivered, then B can only afford to take delivery of 80 units. C can liquidate the remaining 20 undelivered units, again say for \$0.5 each. Thus C gets \$90 in total: \$80 in cash from B plus \$10 in liquidation receipts. And if C is a debtor, there may be further default along the credit chain.

Notice that the secondary defaults by B, C, ... would be avoided if A's debt contract were with a deep pocket, who is not credit-constrained, rather

than with B. That is, the wrong people are being exposed to A's risk. The example suggests that the economy with credit chains reacts to shocks more than an economy where gross financial positions can be netted out (i.e. where the market for loans is anonymous on the supply side).

This is confirmed in Section 2 of the paper. Our formal model is of a network of credit rather than of a line of credit. Each person is owed money by several people; and each person owes money to several people, not all of whom are credit constrained. We exhibit an aggregate multiplier effect: everyone could in principle be made better off if people forgave each others' debts. We show that the greater are people's gross positions (the longer the credit chains), the stronger is the multiplier.

In practice, liquidation appears to be the exception not the rule, even though creditors usually have the right to liquidate the assets of a delinquent debtor. Typically, unpaid debts are postponed until such time as the debtor has enough cash to pay. How would the rescheduling of debt affect our analysis? This is the subject of Section 3 of the paper.

The reason why unpaid debts are postponed is that it may be in the joint interests of a creditor and debtor not to liquidate. That is, it may be privately efficient for them to reschedule the debt. Socially, however, postponement may be more damaging than liquidation if the two parties are a link in a credit chain, because postponement injects less liquidity into the system than does liquidation.

To see this, consider again the A, B, C, ... chain. When B liquidates 40 units of A's product, he injects \$20 cash into the system. (He defaults less than A: he pays \$80 of his debt, whereas A pays only \$60.) And when C liquidates 20 units of B's product, a further \$10 cash is injected into the system. As we move along the chain, the initial \$40 shock (to A's cash holding) converges to zero.

In fact, however, B may choose to postpone the unpaid portion of A's debt. Rather than liquidate the undelivered stock of 40 units today, B may be better off storing it until tomorrow, by which time A will have more cash (viz., the cash return from the 60 units of input that she can afford to buy

today). Liquidation generates \$20 today, which B can use to buy 20 more units of his own input from C. But if tomorrow's return on B's investment is less than \$2 per unit of input, this strategy yields him less than the \$40 he gets from simply postponing delivery of the 40 units until tomorrow. Moreover, A is also better off if B decides to postpone the debt, because she enjoys the gains from trade on those 40 units tomorrow, which she doesn't enjoy if B liquidates. Hence, provided the return on B's investment is less than \$2 per unit of input, postponement is bilaterally efficient.<sup>1</sup>

Postponement imposes a cost on C, however. Now C is in the same position as B: namely, of being owed \$100 by a debtor who only has \$60 cash. By the same argument, then, provided the return on C's investment is less than \$2 per unit of input, C will choose to postpone \$40 of B's debt rather than liquidate 40 units. And so on. Notice that there is no diminution of the shock along the credit chain if debts are postponed, because there is no liquidation to inject new cash into the system. In fact the social loss can be arbitrarily large, given a long enough chain! This extremely stark conclusion is tempered if, as in our formal model, each entrepreneur has many suppliers/creditors, not all of whom are credit constrained. Nevertheless, it is clear that postponement can be socially worse than liquidation, even though it is privately optimal. The root cause of the inefficiency is that a decision by a creditor to postpone debt imposes a negative externality on the creditor's creditors, and in turn on their creditors.

Thus far in the paper, the analysis assumes that the liquidity shocks are unexpected. Section 4 places the model of Section 2 in a fully-specified stochastic framework. Given rational expectations, shocks are anticipated, which has implications for the form of contract. In particular, we ask the question: If default risk has aggregate consequences, and so is publicly observable, why don't entrepreneurs insure against it? That is, why don't entrepreneurs insure themselves against fluctuations in their accounts receivable?

---

<sup>1</sup>More generally, if the delivery price is  $p$  per unit, if liquidation yields  $l$  per unit, and if the return on investment is  $\alpha$  per unit, where  $l < p < \alpha$ , then B will postpone A's debt if  $l(\alpha/p) < p$ .

An entrepreneur faces two kinds of uncertainty in his income: uncertainty in the returns from his own investment; and uncertainty in his accounts receivable arising from fluctuations in his customers' incomes. If the entrepreneur has many customers, fluctuations in his accounts receivable will be highly correlated with the aggregate state of the economy, and so can be insured. (We assume that, because individual returns are unverifiable, the entrepreneur cannot insure them.) The benefit of insurance is that the entrepreneur would obtain better terms from his own suppliers if he could demonstrate that he is less likely to default. Moreover, if all entrepreneurs were to take out such insurance, then the aggregate multiplier we have identified would go away: default risk would be absorbed by deep pockets via insurance companies. We find, however, that the private costs of insurance may outweigh the benefit.

There are two reasons why insurance is unattractive. First, given the limited enforcement of contracts, the entrepreneur cannot commit to pay the insurance premium out of the returns from his investment. That is, his "ideal" policy -- in which he pays out if and only if the economy booms (in return for being subsidized in a recession) -- is infeasible.<sup>2</sup> Instead, he has to pay the premium upfront, which eats into the funds he has available for undertaking his own investment. And, given that he is credit constrained, his internal rate of return will dominate that of the insurance company, so that investing in an insurance policy yields a poor return.<sup>3</sup> Second, the fluctuations in accounts receivable may be relatively small and not strongly correlated with the fluctuations in individual returns. So in a recession, when more of his customers default, the entrepreneur's own return may well be high enough to more than compensate for the shortfall in accounts received. In which case, he doesn't need any insurance in order to meet his own debt obligations. In other words, the payout from an insurance policy conditioned on the aggregate state may not be sufficiently correlated with

---

<sup>2</sup>We assume that there are no enforcement problems relating to the deep pockets: they can commit to pay out.

<sup>3</sup>A similar argument shows that self-insurance, in the form of investing enough in liquid assets to avoid default, also has too high an opportunity cost.

his own needs for cash to make the policy worth buying.

The first conclusion of Section 4, then, is that the aggregate multiplier may not be insured away in a full rational expectations model.

Another conclusion is that positive shocks cause an economy to respond less, in percentage terms, than negative shocks. The reason is that default or postponement is essentially asymmetric: a shortage of cash leads to the multiplier we have identified, whereas there is no counterpart if entrepreneurs have surplus cash. An implication of this asymmetry is that increased variance across entrepreneurs -- some defaulting, others with surplus cash -- leads to lower levels of activity in aggregate.

In the A-B-C chain, B is in effect playing the role of a financial intermediary. One is tempted to say that B stands for "bank". However we are far from modelling banks. As will become clear in the next section, ours is a primitive model of small entrepreneurial firms. We have found it easier to model the kinds of basic decision that small firms face -- such as how much to buy, produce, and sell -- rather than to model complex financial institutions like banks. In fact, though, most small firms do simultaneously hold gross credit and debt positions. And, arguably, systemic risk is just as important for inter-firm lending as it is for inter-bank lending. Trade credit is an important part of commercial life, and constitutes a significant fraction of total borrowing and lending. However, this paper should not be seen as an attempt to model trade credit as such, but rather as a framework for thinking about the aggregate behavior of an economy where agents simultaneously hold gross financial assets and liabilities.

In Section 5, we relate our paper to the literature, and make some final remarks.

## 2. The basic model: systemic risk

The economy has three dates, 0, 1 and 2; and there are two populations, entrepreneurs and deep pockets, which are both large. All agents are risk neutral and only consume a general commodity at date 2. We refer to this general commodity simply as "goods", and we take it as the numeraire. Goods can be stored without depreciation. Initially, the entrepreneurs and deep pockets are respectively endowed with  $M$  and  $\bar{M}$  goods in aggregate, where  $\bar{M}$  is large. Between dates 0 and 1, the entrepreneurs and the deep pockets respectively have an aggregate endowment of  $N$  and  $\bar{N}$  labor, where  $\bar{N}$  is large.

Deep pockets have access to a constant-returns, one-period technology for making goods at date 1 directly from their own labor: one unit of labor makes one unit of goods.

A typical entrepreneur,  $E$ , has access to two technologies for producing goods, one short-term and the other long-term. Both exhibit constant returns, and are specific to him. The short-term technology is effectively a superior storage technology:  $E$  can use one unit of goods at either date  $t = 0$  or 1 to make  $\sigma > 1$  units of goods at date  $t+1$ .

$E$ 's long-term technology has two stages, each stage taking one period to complete. In the first stage, between dates 0 and 1, labor is used to make an intermediate product, which is specific to  $E$  and has to be custom-made. We assume that  $E$  cannot use his own labor for this purpose. Instead, at date 0 he places orders with suppliers -- other entrepreneurs and deep pockets -- for intermediate product to be delivered at date 1. A supplier works between dates 0 and 1 with a blueprint that  $E$  gives her at the time of placing the order: one unit of her labor makes one unit of intermediate product. In the second stage, between dates 1 and 2,  $E$  uses his specific technology to make goods from the intermediate product: one unit of intermediate product at date 1 yields  $\alpha > 1$  units of goods at date 2.

At date 1, since the intermediate product is specific, it is of less value to a supplier than it is to  $E$ . We assume that the only use for the supplier is to liquidate it. Liquidation is instantaneous and exhibits constant returns: each unit of intermediate product can be liquidated for



$l < 1$  goods at date 1.

The model is symmetric: E uses his labor to make intermediate products for other entrepreneurs. At date 0 there is a competitive market: any entrepreneur is free to place an order with any other entrepreneur or any deep pocket to supply intermediate product at date 1.

To avoid counter-trade, however, we also assume that if E makes intermediate product for some entrepreneur  $E^*$ , then this precludes  $E^*$  producing for E.

### Restrictions on contracting

In Section 3 we will appeal to some auxiliary assumptions about an entrepreneur E's technology to justify a number of restrictions on contracting.<sup>4</sup> The first restriction prevents E from obtaining outside funds:

Restriction 1: E cannot commit to hand over any of the return from either his short-term or his long-term technology.

That is, since E has no collateral to offer, a deep pocket would be unwilling to put up funds to finance E's investment: E has to use his own resources.

The second restriction concerns the relationship between E and his suppliers:

Restriction 2: E and his suppliers cannot contract over the terms of delivery of the intermediate product.

---

<sup>4</sup>Note that these additional assumptions do not affect the analysis in any way other than to restrict contracting.

This means that at date 1 the terms of delivery will be a matter of negotiation. We suppose the outcome of the bargain is that at date 1 E pays a fixed amount,  $p$  goods, for each unit of intermediate product delivered. The parameter  $p$  is taken to satisfy  $\ell < p \leq 1$ ; i.e., there is an interior division of surplus.<sup>5</sup> In particular, then, the negotiated delivery price  $p$  is assumed to be strictly greater than the suppliers' valuation  $\ell$ .

It is important to notice that not all the units of intermediate product ordered at date 0 will be delivered at date 1 if E does not have sufficient funds to pay  $p$  for all of them. In this event, E "defaults", and the suppliers liquidate the undelivered units, losing  $p - \ell$  on each. Although the liquidated intermediate product isn't being put to best use, the outcome of the bargain is nevertheless constrained efficient, taking into account the fact that E has limited funds with which to pay his suppliers.

The third restriction is that suppliers have equal bargaining power in the event of default:

Restriction 3: If E defaults against his suppliers, then he does so on a pro-rata basis.

In particular, the bargaining outcome does not depend on the identities of E's suppliers: entrepreneurial suppliers and deep pocket suppliers are treated alike.

The final restriction deals with E as a supplier:

---

<sup>5</sup>Strictly speaking, since  $\alpha > 1$ , an interior division of surplus doesn't require that  $p \leq 1$ ; but this tighter upper bound on  $p$  helps with the interpretation of the model.

Generally,  $p$  could depend on the funds held by E and his suppliers, and could vary from supplier to supplier. Here we treat  $p$  as a fixed parameter in order to simplify the analysis.

Restriction 4: E cannot raise funds at date 0 against the future receipts from any intermediate product that he makes for other entrepreneurs.

In other words, E's earnings as a supplier cannot be securitized.

#### Implicit debt/supply contracts

In the competitive market at date 0, an entrepreneur agrees an implicit contract with his suppliers. Inter alia, the implicit contract deals with what fraction,  $\lambda$ , of the intermediate product that the entrepreneur orders at date 0 is expected to be liquidated at date 1. (In principle, each entrepreneur could have a different  $\lambda$ ; although, as we shall see, in the absence of any shocks, all  $\lambda$ 's will equal zero.)

The market clears by means of transfers of goods at date 0. Specifically, an entrepreneur makes a downpayment of  $q$  goods for each unit that he orders. ( $q$  is personal to the entrepreneur, insofar as  $\lambda$  is too.)

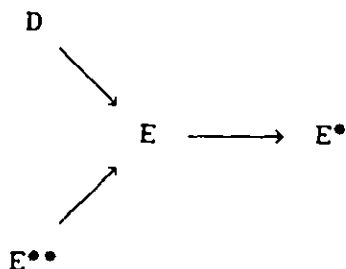
Thus the implicit contract between the entrepreneur and one of his suppliers takes the following form. At date 0, the entrepreneur gives the supplier his blueprint and orders a quantity  $x$  of intermediate product. At the same time, the entrepreneur makes a downpayment of  $qx$  goods. Then at date 1, the entrepreneur pays  $p(1 - \lambda)x$  goods for the delivery of  $(1 - \lambda)x$  units of intermediate product. The supplier liquidates  $\lambda x$  units. In total, the supplier gets a return of  $qx$  goods at date 0 and  $px - (p - \ell)\lambda x$  goods at date 1.

The implicit contract can be thought of as a debt contract and a supply contract bundled together. The deferment of part of the payment until date 1 can be construed as the entrepreneur borrowing from the supplier. (But this can just as well be turned round the other way: because the supplier is paid something in advance by the entrepreneur, she borrows from him. It is moot as to who is the borrower and who is the lender in a context where the product takes time to build.)

To recap: the reason why the deal between the entrepreneur and his supplier takes the above form is that formal contracts are difficult to enforce. By staggering the payments -- a downpayment of  $q$  goods per unit at the time of ordering, and a final payment of  $p$  goods per unit delivered -- the parties protect their respective interests. If the balance between  $q$  and  $p$  were shifted towards  $q$  (more downpayment), then the entrepreneur would be at risk from the supplier demanding more on delivery than was implicitly agreed. And if the balance were shifted towards  $p$  (more payment on delivery), then the supplier would be at risk from the entrepreneur refusing to pay. Unlike in a conventional spot transaction, the payments do not all coincide with delivery. And unlike in an Arrow-Debreu forward contract, the payments do not all occur at the time of ordering. Crucially, the fact that the implicit contract involves some form of debt contract is inescapable. Entrepreneurs who are credit-constrained are forced to offer credit to their customers, otherwise they will be unable to compete with other suppliers.

### Credit chains

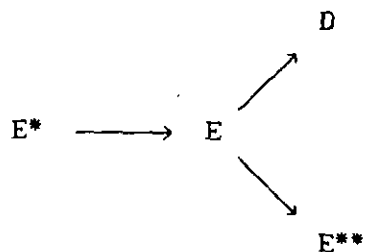
This opens the door to a model of credit/supply chains. Consider an entrepreneur  $E$  who supplies entrepreneur  $E^*$ .  $E$  purchases intermediate product both from another entrepreneur  $E^{**}$  ( $\neq E^*$ ) and from a deep pocket  $D$ . The chain of supply of intermediate product is thus



a supply chain:  $E^{**}$  and  $D$  supply  $E$

$E$  supplies  $E^*$

and the corresponding credit chain is



a credit chain: E\* is in debt to E

E is in debt to E\*\* and D

Here, the goods that E\* owes to E are part of E's accounts receivable; and the goods that E owes to E\*\* and D are part of E's accounts payable.

It is important to bear in mind that these links are part of an intricate network of such credit/supply relationships: E has many customers like E\*, and he has many suppliers like E\*\* and D. For future reference, it helps to draw the "canonical" credit network, viz. the triangle given in Figure 1. (Note that the figure may be misleading insofar as the actual credit/supply network comprises a jumble of bilateral trades.)

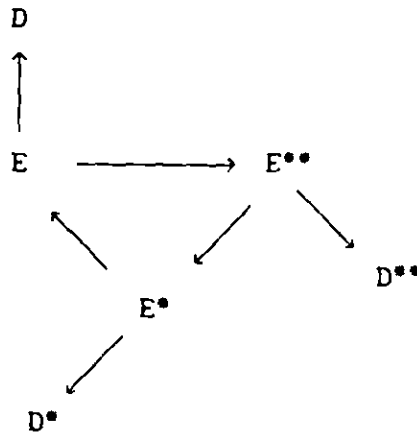


Figure 1: The canonical credit network

E is in debt to suppliers E\*\* and D

E\* is in debt to suppliers E and D\*

E\*\* is in debt to suppliers E\* and D\*\*

### Equilibrium

We now find the equilibrium value of  $q$ , for a given  $\lambda$ . Since the deep pockets' endowment of goods,  $\bar{M}$ , is large, in equilibrium some will be stored until date 2. That is, the deep pockets value goods equally at all three dates: their rate of interest is zero. Also, since the deep pockets' endowment of labor,  $\bar{N}$ , is large, in equilibrium some will be used to make goods using their own technology. For a deep pocket D to be indifferent between using her own technology and making intermediate product for some entrepreneur E, the  $q$  for that entrepreneur must satisfy

$$(2.1) \quad q + p - (p - \ell)\lambda = 1,$$

where  $\lambda$  is the fraction of E's intermediate product that is expected to be liquidated at date 1. The left hand side of (2.1) is the expected present value of the return to D of producing one unit of intermediate product for E,

and the right hand side is D's opportunity cost of withholding one unit of labor from her own technology. Our earlier assumption that  $p \leq 1$  ensures that  $q \geq 0$ .

Note that in equilibrium the aggregate demand for intermediate product by the entrepreneurs will exceed their aggregate supply, and the deep pockets supply the difference. This is why another entrepreneur  $E^{**}$  who supplies E with intermediate product charges E the same downpayment  $q$  that D charges -- namely, the  $q$  given by (2.1).<sup>6</sup>

It is clear from (2.1) that if E doesn't anticipate any shocks to his flow-of-funds at date 1, then he has an incentive to avoid default, since  $q$  rises with  $\lambda$ . The point is that liquidation is wasteful: the deep pocket D is more productively employed on her own technology (yielding one unit of goods per unit of labor) than on making intermediate product which is liquidated (effectively yielding only  $\ell$  goods per unit of labor).

Since E's  $\lambda$  equals zero, he has to pay a downpayment of  $q = 1 - p$  goods for each unit of intermediate product that he orders at date 0. And by symmetry, each of E's purchasers,  $E^*$ , will pay E a downpayment price of  $1 - p$ .

To avoid defaulting at date 1, E must arrange to have at least as many funds flowing in as he plans to spend paying for the delivery of his own intermediate product. His planned expenditure is  $pX$ , where  $X$ , say, is the quantity of intermediate product that he orders at date 0. ( $pX$  corresponds to his accounts payable.) His planned income comes from two sources. First, he expects a return  $\sigma$  on the short-term investment  $Y$ , say, that he undertakes at date 0.<sup>7</sup> Second, he expects to be paid by his customers (other entrepreneurs) for the delivery of intermediate product that they order from him at date 0. As he is endowed with  $N$  units of labor between dates 0 and 1, he can make  $N$  units of intermediate product. Moreover, none of his customers

---

<sup>6</sup>Because E defaults equally against all his suppliers,  $E^{**}$  cannot offer a supply contract different from D's.

<sup>7</sup>Since the short-term technology strictly dominates storage, E does not store any goods at date 0.

is planning to default, and so E expects to be paid  $pN$  goods at date 1. ( $pN$  corresponds to his accounts receivable.) Hence, in order to avoid defaulting at date 1, E must choose  $X$  and  $Y$  at date 0 so that

$$(2.2) \quad pX \leq \sigma Y + pN. \quad (\text{date 1 flow-of-funds})$$

$\uparrow$   
 accounts payable

$\uparrow$   
 revenue from short-term investment

$\uparrow$   
 accounts receivable

If E chooses  $X$  and  $Y$  so that this inequality is strict, then he will have surplus goods at date 1 to reinvest in his short-term technology, the return from which will augment the date 2 return  $\alpha X$  from his long-term technology. On the other hand, if E chooses  $X$  and  $Y$  so that (2.2) is an equality, E has just enough funds at date 1 to meet his debt obligations. This latter case, which we call a balanced investment strategy, yields E a date 2 consumption of  $\alpha X$ .

It is straightforward to show that the balanced investment strategy is optimal if the long-term return  $\alpha$  is sufficiently large relative to the short-term return  $\sigma$ . A sufficient condition is that

$$(A.1) \quad \alpha > \sigma^2.$$

Assumption (A.1) says that using the long-term technology yields more than using the short-term technology twice.

We can solve for E's balanced investments  $X$  and  $Y$  from (2.2) as an equality, together with his budget constraint at date 0:

$$(2.3) \quad (1 - p)X + Y = M + (1 - p)N \quad (\text{date 0 flow-of-funds})$$



Here, the left-hand side comprises E's downpayment  $(1 - p)X$  goods on the  $X$  units of intermediate product that he orders from his suppliers for delivery at date 1, plus the  $Y$  goods he invests in his short-term technology. The right-hand side comprises his initial goods endowment  $M$ , plus the  $(1 - p)N$  goods that he receives as downpayment for the delivery of  $N$  units of intermediate product to his customers at date 1.

Solving for  $X$  and  $Y$ , we obtain

$$(2.4a) \quad X = N + \sigma M / [p + \sigma(1-p)]$$

$$(2.4b) \quad Y = pM / [p + \sigma(1-p)].$$

This leads to E having a date 2 consumption  $C$  equal to  $\alpha X$ ; i.e.,

$$(2.5) \quad C = \alpha N + \alpha \sigma M / [p + \sigma(1-p)].$$

In sum, the equilibrium is characterised by all entrepreneurs choosing  $X$  and  $Y$  given by (2.4), and consuming  $C$  given by (2.5). The excess demand for intermediate product,  $X - N$  per entrepreneur, is supplied by the deep pockets.

#### The effects of an unanticipated shock at date 1

Now suppose that, at date 1,  $\sigma$  unexpectedly turns out to be smaller for all entrepreneurs, say equal to  $\hat{\sigma} < \sigma$ .<sup>8</sup>

---

<sup>8</sup>In Section 4 we look at the model where the shock is rationally expected at date 0.

Now E no longer has enough funds at date 1 to be able to take delivery of all  $X$  units of intermediate product: he must default. But then so too must all entrepreneurs, since they also experience the shock. Let  $\hat{X}$  and  $\hat{X}^*$  denote the respective quantities of intermediate product that E and a typical other entrepreneur  $E^*$  (who buys from E) can afford to pay for at date 1. (By symmetry,  $\hat{X}^* = \hat{X}$ . But it clarifies the discussion if we use an asterisk to denote other entrepreneurs' activities.) Both  $\hat{X}$  and  $\hat{X}^*$  are less than the amounts that E and  $E^*$  ordered at date 0,  $X$  and  $X^*$  respectively.

Recall that  $E^*$  defaults against his suppliers (including E) on a pro-rata basis. That is,  $E^*$  defaults on a fraction  $\lambda^* = (X^* - \hat{X}^*)/X^*$  of the order he placed with E at date 0. Since this is true of all the entrepreneurs that E supplies, E's sales revenues are reduced by  $(p - \ell)\lambda^*N$  at date 1. This indirect shock is over and above the direct productivity shock  $(\sigma - \hat{\sigma})Y$  to E's flow-of-funds:

$$(2.6) \quad p\hat{X} = \hat{\sigma}Y + pN - (p - \ell) \left( \frac{X^* - \hat{X}^*}{X^*} \right) N.$$

Equation (2.6) lies at the heart of the paper. A multiplier process is at work: E defaults on his suppliers, some of whom are entrepreneurs. As a result, these entrepreneurs default more -- that is, even more than the direct productivity shock dictates -- on their suppliers. And so on, until we find that the entrepreneurs that E is supplying default more on him; etc. The credit chains amplify the shock.

This can be seen most easily in terms of Figure 1. E defaults on  $E^{**}$  and D on a pro-rata basis. This means that  $E^{**}$  defaults more on E and D than he otherwise would.  $E^*$  thus defaults more on E and D, which worsens E's flow-of-funds, etc.

There is an easy way for the economy to effect a Pareto improvement. If each entrepreneur were to forgive some of the "debt" of the entrepreneurs he supplies -- that is, if he were to charge them less than  $p$  per unit of intermediate product that he delivers -- and if his suppliers who are

entrepreneurs were to forgive some of his debt, then all the entrepreneurs would be strictly better off (and none of the deep pockets would be affected). In terms of Figure 1, E can charge E\* less than p per unit, given that E\*\* is charging E less than p per unit (and E\* is charging E\*\* less than p per unit). Another way of saying this is that instead of E, E\* and E\*\* liquidating intermediate product that they have respectively custom-made for E\*, E\*\* and E, the three entrepreneurs could pass it around the triangle for free, and they would all be better off.

The root cause of the inefficiency is the lack of liquidity in the system, and the difficulty of coordination. Coordination is difficult because each entrepreneur can gain by continuing to insist that he be paid p goods per unit of intermediate product that he supplies, even though other entrepreneurs are forgiving his debt to them.

The longer the credit chains, the greater the multiplier. Think of a parallel economy in which entrepreneurs are able to make their intermediate product from their own labor. In such an economy, there would be no chains. In terms of Figure 1, the only links would be between E and D, E\* and D\*, and E\*\* and D\*\* (the deep pockets satisfy the three entrepreneurs' residual demand). Hence there would be no multiplier. Following the shock, the entrepreneurs would still default, but the deep pockets would absorb the loss without passing it on to others, and without having to cut back on any profitable investment of their own.

Appealing to symmetry (dropping the asterisks), we can solve (2.6) for  $\hat{X}$  in terms of its proportional deviation from X:

$$(2.7) \quad \frac{X - \hat{X}}{X} = \frac{\sigma - \hat{\sigma}}{\sigma} \left( \frac{\sigma p M}{\sigma p M + \ell N [p + \sigma(1-p)]} \right).$$

This is also the proportional fall in the entrepreneurs' consumption at date 2.

### A numerical example

Suppose the returns on the short-term and long-term technologies,  $\sigma$  and  $\alpha$ , are 1.2 and 1.8 respectively; and the liquidation return  $\ell$  is 0.5. (Note that assumption (A.1) is satisfied.) Let E's initial endowments of goods and labor, M and N, be 5 and 24. And let  $p = 1$ .

A 5% fall in the short-term productivity at date 1 -- which corresponds to a 1% fall in the entrepreneurs' net worth -- leads to a  $\frac{5}{3}\%$  fall in the entrepreneurs' consumption at date 2. Had the entrepreneurs been able to make their own intermediate product -- so that there were no credit chains -- then the fall in their consumption would have been 1%; i.e. the same as the drop in their net worth. The extra  $\frac{2}{3}\%$  fall is due to the multiplier.

### 3. A three period model: postponement

In practice, liquidation is the exception not the rule. Typically, unpaid debts are rescheduled. In this section, we extend our basic model by a period, so as to investigate the aggregate implications of postponement.

There are now four dates, 0, 1, 2, and 3. Consumption only occurs at date 3. The entrepreneurs' aggregate endowments are unchanged: they initially have  $M$  goods, and are endowed with  $N$  units of labor between dates 0 and 1. An entrepreneur can use his short-term technology at each of dates 0, 1 and 2. And he has the opportunity to use his long-term technology twice: between dates 0 and 2, and between dates 1 and 3.

The deep pockets have an aggregate endowment of  $\bar{M}$  goods initially, and  $\bar{N}_0$  units of labor between dates 0 and 1. In addition, they have  $\bar{N}_1$  units of labor between dates 1 and 2.  $\bar{M}$ ,  $\bar{N}_0$  and  $\bar{N}_1$  are all large. A deep pocket can use her own technology for producing goods directly from her own labor. Or she can use her labor to make intermediate product for entrepreneurs. Since only the deep pockets are endowed with labor between dates 1 and 2, only they are in a position to supply intermediate product to entrepreneurs at date 2.

We continue to assume that implicit contracts for the supply of intermediate product take the form given in Section 2: entrepreneurs pay  $q$  goods for each unit of intermediate product that they order, and pay  $p$  goods per unit delivered. A supplier can liquidate any undelivered intermediate product for  $\ell$  goods per unit.

We assume that intermediate product not delivered at date 1 need not be liquidated, however. Instead, a supplier can store it without depreciation until date 2, when it can be delivered to the entrepreneur for whom it was made. We also assume that the entrepreneur will pay a price of  $p$ . That is, if delivery is postponed, the terms of delivery are negotiated afresh at date 2, and  $p$  is the outcome of the bargain between the entrepreneur and his supplier.

Using arguments similar to those in Section 2, one can show that the entrepreneurs will choose balanced investment plans. Specifically, an

entrepreneur E draws up a plan at date 0 under which he invests  $Y$  in his short-term technology at date 0, orders  $X_1$  intermediate product to be delivered at date 1, and then at date 1 orders  $X_2$  intermediate product to be delivered at date 2 -- where  $Y$ ,  $X_1$  and  $X_2$  satisfy

$$(3.1) \quad (1 - p)X_1 + Y = M + (1 - p)N \quad (\text{date } 0)$$

$$(3.2) \quad (1 - p)X_2 + pX_1 = \sigma Y + pN \quad (\text{date } 1)$$

$$(3.3) \quad pX_2 = \alpha X_1 \quad (\text{date } 2)$$

The first expenditure terms on the left-hand sides of (3.1) and (3.2) reflect the fact that, since E plans not to default at either dates 1 or 2, he pays price  $q = 1 - p$  for each of the units of intermediate product that he orders at dates 0 and 1. The final income terms on the right-hand sides (3.1) and (3.2) reflect the fact that the other entrepreneurs also do not plan to default at date 1: E receives price  $1 - p$  for each of the  $N$  units of intermediate product that is ordered from him at date 0, and he expects to deliver all  $N$  units for price  $p$  at date 1.

The idea behind (3.1)-(3.3) is that E invests as much as he can long-term -- using the returns  $\alpha X_1$  at date 2 from the first round of long-term investment to pay for the delivery of intermediate product  $X_2$  in the second round. However, at date 1 he has to pay for the delivery of  $X_1$  and the downpayment for  $X_2$  partly out of the return from short-term investment  $Y$  at date 0.

E's planned consumption at date 2 equals

$$(3.4) \quad C = \alpha X_2.$$

The effects of an unanticipated shock at date 1

Now suppose that, for all the entrepreneurs, the return from the short-term investment unexpectedly turns out to be  $\hat{\sigma} < \sigma$  at date 1.<sup>9</sup> As a result, entrepreneurs are unable to execute the plans they drew up at date 0, because they are short of funds.

On the income side, an entrepreneur E finds that his customers (other entrepreneurs) take delivery of only  $(\hat{X}_1^*/X_1^*)N$  units of the intermediate product he made for them -- where  $\hat{X}_1^*$  is the total amount that they take delivery of, out of the  $X_1^*$  units they ordered at date 0. E has a choice. Either he can liquidate the  $N - (\hat{X}_1^*/X_1^*)N$  undelivered units for  $\ell$  goods each. Or he can postpone delivery of these units until date 2, by which time his customers will have more goods with which to pay him.

E's choice boils down to this. Is it more profitable to liquidate and invest in his own long-term technology -- i.e. to use the liquidation receipts  $\ell$  (per unit) to take delivery of an additional  $\ell/p$  units of his own intermediate product, yielding  $a\ell/p$  goods at date 2? Or is it more profitable to "invest in his customers" -- i.e. to wait until they can pay  $p$  at date 2? If

$$(A.2) \quad \ell(a/p) < p$$

then the latter strategy will be preferred. Note that E's customers are only too happy to go along with this, given that, if E postpones, they get intermediate product delivered at date 2 without their having to make any more downpayment to a new supplier at date 1. (They get nothing if E liquidates.) Hence, under assumption (A.2), postponement is bilaterally efficient between E and each of his customers.

---

<sup>9</sup>This is only a temporary shock. If an entrepreneur invests in his short-term technology at date  $t = 1$  or  $2$  then he earns  $\sigma$  at date  $t+1$ .

Notice that E's own suppliers face the same choice as he does: whether to liquidate the intermediate product they have made for him, or whether to postpone delivery until date 2. By a symmetric argument, the entrepreneurs who are supplying E will choose to postpone, given assumption (A.2). And, a fortiori, the deep pocket suppliers will postpone too, since postponement yields a price  $p$  at date 2, whereas liquidation yields only  $\ell$  at date 1 (and they have a zero rate of interest).

On the expenditure side, at date 1 E has to decide how to allocate his limited resources between taking delivery of  $\hat{X}_1$  intermediate product and placing orders for delivery of  $\hat{X}_2 - (X_1 - \hat{X}_1)$  additional intermediate product at date 2. (If  $\hat{X}_2$  is E's total input of intermediate product at date 2, and his date 1 suppliers are to postpone delivery of  $X_1 - \hat{X}_1$  units until that date, then at date 1 he only needs to make the downpayment  $1 - p$  on the difference,  $\hat{X}_2 - (X_1 - \hat{X}_1)$ .) It can be shown that E will adopt a balanced investment plan:

$$(3.5) \quad (1 - p) \left( \hat{X}_2 - (X_1 - \hat{X}_1) \right) + p \hat{X}_1 = \hat{\sigma} Y + p \left( \hat{X}_1^* / X_1^* \right) N \quad (\text{date 1})$$

$$(3.6) \quad p \hat{X}_2 = \alpha \hat{X}_1 + p \left( N - \left( \hat{X}_1^* / X_1^* \right) N \right). \quad (\text{date 2})$$

This new plan entails ordering just enough additional intermediate product  $\hat{X}_2 - (X_1 - \hat{X}_1)$  for delivery at date 2 that he can afford to take delivery of  $\hat{X}_2$  at price  $p$ , using the returns  $\alpha \hat{X}_1$  plus the receipts  $p \left( N - \left( \hat{X}_1^* / X_1^* \right) N \right)$  from the postponed delivery to his date 1 customers.

E's new consumption at date 3 equals

$$(3.7) \quad \hat{C} = \alpha \hat{X}_2.$$

$\hat{C}$  can be found by appealing to symmetry (dropping the asterisks), and solving (3.1), (3.5), (3.6) and (3.7).



### Comparison with liquidation

The point of this analysis is to compare  $\hat{C}$  with what an entrepreneur would consume if everyone liquidated rather than postponed. (This is what would occur if (A.2) did not hold.) Let a tilde denote this possibility. (3.5)-(3.7) would read:

$$(3.8) \quad (1 - p)\tilde{X}_2 + p\tilde{X}_1 = \hat{\sigma}Y + pN - (p - \ell) \left( \frac{X^* - \tilde{X}^*}{X^*} \right) N \quad (\text{date 1})$$

$$(3.9) \quad p\tilde{X}_2 = \alpha\tilde{X}_1 \quad (\text{date 2})$$

$$(3.10) \quad \tilde{C} = \alpha\tilde{X}_2.$$

The final income term in (3.8) reflects the lost revenue  $p - \ell$  on each unit that E liquidates at date 1.

The formulae for  $\hat{C}$  and  $\tilde{C}$  are messy, so it helps to consider the simple case  $p = 1$ ; and to look at the proportional deviation of consumption from planned consumption, given postponement and liquidation:

$$(3.11) \quad \frac{C - \hat{C}}{C} = \frac{\sigma - \hat{\sigma}}{\sigma} \frac{\alpha\sigma M + (\alpha - 1)N}{\alpha\sigma M + \alpha N} \quad (\text{postponement})$$

$$(3.12) \quad \frac{C - \tilde{C}}{C} = \frac{\sigma - \hat{\sigma}}{\sigma} \frac{\sigma M}{\sigma M + \ell N} \quad (\text{liquidation})$$

An inspection of (3.11) and (3.12) reveals that the proportional drop in  $\hat{C}$  may be more than the drop in  $\tilde{C}$ .

Consider again the numerical example from the end of Section 2:  
 $\sigma = 1.2$ ,  $\alpha = 1.8$ ,  $\ell = 0.5$ ,  $M = 5$ ,  $N = 24$  and  $\hat{\sigma} = 0.95\sigma$ . In this example, assumption (A.2) is satisfied, and so debts are postponed. However,  $\hat{C}$  is (25/9)% lower than  $C$ , whereas  $\tilde{C}$  is only (5/3)% lower. In other words, postponement may be socially inferior to liquidation, even though it is bilaterally efficient.

The reason is that each bilateral credit/supply relationship is part of a credit chain. When an entrepreneur  $E$  privately decides to postpone the debt (i.e. chooses not to liquidate the intermediate product) of one of his customers, he does not take into account the fact that it imposes a negative externality on his creditors (suppliers). From  $E$ 's perspective, more return at date 2 ( $p$  per unit postponed) is better than less at date 1 ( $\ell$  per unit liquidated) -- given assumption (A.2). But  $E$ 's suppliers are better off if  $E$  has more goods today with which to pay them.

In short, there is too little liquidity in the system at date 1, and postponement does nothing to relieve the situation, whereas liquidation helps. Were the agents able to participate in a grand Coasian bargain, then the inefficiency could be avoided, but we are considering an economy where only bilateral negotiations are feasible.

#### 4. A stochastic model: insurance

Thus far in the paper, the date 1 shocks have been unanticipated. In this section, we place the two-period model of Section 2 in a fully-specified stochastic framework, where agents have rational expectations.

There are two aggregate states at date 1. In the boom, which occurs with probability  $1 - \pi$ , all the entrepreneurs receive a deterministic return  $\sigma > 1$  from their short-term technologies. In the recession, which occurs with probability  $\pi$ , a fraction  $\theta$  of the entrepreneurs receive a return of only  $\sigma^- < \sigma$ ; and a fraction  $1 - \theta$  receive  $\sigma^+ > \sigma$ . We take  $\sigma^+ - \sigma$  and  $\sigma - \sigma^-$  to be small. It is not known at date 0 who will be the (relatively) unproductive entrepreneurs. There are no further shocks; in particular, the return on all the entrepreneurs' short-term technologies between dates 1 and 2 equals  $\sigma$ .

Notice that we allow for the case where  $\theta\sigma^- + (1 - \theta)\sigma^+ = \sigma$ . That is, if there is a mean-preserving spread of the entrepreneurs' short-term returns, then default by a fraction of the entrepreneurs is enough to bring about a recession. Moreover, as we shall see, the size of the downturn is exacerbated by a chain reaction of default.

We assume that at date 1 it cannot be publicly verified if an entrepreneur E has productivity  $\sigma^+$  or  $\sigma^-$ . For this reason, E cannot insure against fluctuations in his individual return in a recession. However, E can insure against the aggregate state, because this is publicly verifiable at date 1.

E's ideal insurance policy would be of the form: in return for agreeing to pay out at date 1 if one particular aggregate states occurs, he receives payment in the other state. However, the restrictions on contracting discussed in Section 2 (in particular Restriction 1) mean that E cannot commit to hand anything over at date 1. Hence the only feasible insurance policy is one in which he pays a premium in advance, at date 0, and he is paid according to the aggregate state at date 1. This is less than ideal for E because he spends valuable funds on the premium at date 0 which could otherwise be used for investment.

Deep pockets can set aside funds at date 0 as security. Thus in a competitive equilibrium, they can provide insurance to the entrepreneurs. The rate of return on insurance equals zero, the deep pockets' rate of interest.

If an entrepreneur E pays a premium of  $Z > 0$  goods at date 0, he can buy a policy that pays out  $Z/\pi$  goods in the recession. Equally, he can buy a different policy that pays out  $Z/(1 - \pi)$  goods in the boom.

At date 0, E has to decide what amount,  $X$ , of intermediate product to order at date 0; what amount,  $Y$ , to invest in his short-term technology; and what amount,  $Z$ , to invest in insurance. Let  $(X^*, Y^*, Z^*)$  denote the other entrepreneurs' choices. In a symmetric equilibrium,  $(X^*, Y^*, Z^*) = (X, Y, Z)$ .

We are interested in an equilibrium in which no entrepreneur buys insurance ( $Z = Z^* = 0$ ), and each arranges to have just enough funds in the boom not to default:

$$(4.1) \quad pX = \sigma Y + pN.$$

(This corresponds to the anticipated date 1 flow-of-funds constraint (2.2) from section 2, as an equality.) In the boom, then, there is no fall in accounts received: the final term in (4.1) is  $pN$ , the revenue E gets from delivering all his output of intermediate product to his customers at price  $p$ .

In the recession, however, the unproductive entrepreneurs will default, which means that everyone suffers a loss in accounts received. First, take the case where E himself is unproductive. Let  $\hat{X}$  denote the amount of intermediate product that he can afford to buy in this case; and let  $\hat{X}^*$  denote the equivalent for other unproductive entrepreneurs. ( $\hat{X}^*$  equals  $\hat{X}$  in a symmetric equilibrium.) Then, from E's flow-of-funds at date 1,

$$(4.2) \quad p\hat{X} = \sigma^- Y + pN - (p - \ell)\theta \left( \frac{X^* - \hat{X}^*}{X^*} \right) N.$$

(This corresponds to the realised date 1 flow-of-funds constraint (2.6) from Section 2.) The final term in (4.2) is the loss in accounts received arising from liquidation: the unproductive fraction,  $\theta$ , of E's customers default on a fraction  $(X^* - \hat{X}^*)/X^*$  of their order, incurring E a loss of  $(p - \ell)$  per unit.

Next, take the case where E is productive in the recession. In this case, we want the equilibrium to be such that E has spare funds available at date 1, in that the revenue  $\sigma^+ Y$  from his short-term investment more than makes up for the loss in accounts received, thereby enabling him to pay for all  $X$  units of intermediate product that he ordered at date 0 from his suppliers:

$$(4.3) \quad \sigma^+ Y + pN - (p - \ell)\theta \left( \frac{X^* - \hat{X}^*}{X^*} \right) N > pX.$$

E reinvests these surplus funds into his short-term technology between dates 1 and 2.

Turn now to date 0. There is a probability  $\pi\theta$  that E will default at date 1 and only be able to take delivery of a fraction  $(\hat{X}/X)$  of the  $X$  units of intermediate product he orders. Hence E has to make a downpayment of  $q$  goods on each unit, where  $q$  solves

$$(4.4) \quad q + p - (p - \ell)\pi\theta \left( \frac{X - \hat{X}}{X} \right) = 1.$$

(This corresponds to the equilibrium condition (2.1) from section 2.) By symmetry, all other entrepreneurs have to make the same downpayment  $q$  on the intermediate product that they order.

Given that he chooses  $Z = 0$ , E's date 0 flow-of-funds constraint

reduces to

$$(4.5) \quad qX + Y = M + qN.$$

(This corresponds to (2.3) from Section 2.)

E's expected consumption at date 2 equals

$$(4.6) \quad C = (1 - \pi)\alpha X + \pi\theta\hat{\alpha}\hat{X} + \pi(1 - \theta)\left[\alpha X + \sigma\left(\sigma^+ Y + pN - (p - \ell)\theta\left(\frac{X^* - \hat{X}^*}{X^*}\right)N - pX\right)\right].$$

where the first term corresponds to E's consumption following the boom, which happens with probability  $1 - \pi$ ; the second term corresponds to E's consumption following the recession if he is unproductive, which happens with probability  $\pi\theta$ ; and the third term corresponds to E's consumption following the recession if he is productive, which happens with probability  $\pi(1 - \theta)$ . Notice that the third term includes the return from investing surplus funds in his short-term technology between dates 1 and 2.

To sum up: we are interested in a symmetric equilibrium (dropping the asterisks) where  $q$ ,  $X$ ,  $Y$ ,  $\hat{X}$  and  $C$  solve (4.1)-(4.6). One can show that this is the unique equilibrium if the following condition holds:

$$(A.3) \quad \omega\sigma > \theta\alpha\sigma(1 - \ell) + \theta\alpha + (1 - \theta)\sigma.^{10}$$

---

<sup>10</sup>An additional condition on  $\sigma^+$  is needed, to ensure that (4.3) holds. Details are available from the authors.

Note that for  $\theta = \frac{1}{2}$ , (A.3) is satisfied by the numerical example at the end of Section 2.

The left-hand side of (A.3) reflects the marginal opportunity cost of insurance: the fact that an entrepreneur E has to pay the premium out of his date 0 investment funds.

The right-hand side of (A.3) gives the marginal benefits to E of taking out insurance against the recession. The first two terms reflect the fact that, with insurance, he will not default as much: the first term reflects the concomitant drop in his downpayment  $q$  at date 0, and the second term reflects his date 2 return from being able to pay for more intermediate product to be delivered. The final term on the right-hand side reflects the fact that if he is productive in the recession then he will spend the insurance payout on additional short-term investment between dates 1 and 2.

It is useful to look at the comparative statics of (A.3). The condition is more likely to hold as  $\alpha$ ,  $\sigma$  or  $\ell$  rise, or as  $\theta$  falls. The dominant effect of a rise in either the long-term return  $\alpha$  or the short-term return  $\sigma$  is to push up the opportunity cost of paying the insurance premium at date 0. If the liquidation value  $\ell$  is high (near to  $p$ ), then insurance doesn't cause the downpayment  $q$  to drop by much. Finally, if the probability  $\theta$  of being unproductive in the recession (and hence of defaulting) is low, then insurance doesn't bring much benefit either.

To sum up this section: there are circumstances where agents will choose not to insure against fluctuations in their accounts receivable, even when such insurance is available. As a result, there can be a chain reaction of default in a recession, which serves to exaggerate the extent to which the economy responds to negative shocks.

## Appendix: Justifying the restrictions on contracting

In this Appendix, we make some additional assumptions about an entrepreneur E's technologies to justify the four restrictions on contracting made in Section 2.

First, we assume that the final output of both E's short-term and his long-term technologies accrues directly to him. Moreover, E can divert, or steal, this output: there is no sanction against theft. In consequence, he is unable to mortgage any of his investment returns. (Restriction 1.)

We make a number of assumptions about E's long-term technology. These are perhaps easiest to understand if we start with the second stage, between dates 1 and 2, where the technology is Leontief: if, at date 1, E inputs an equal amount,  $X/k$  say, of  $k$  intermediate products then at date 2 he makes  $\alpha X$  goods, where  $\alpha > 1$ .

The  $k$  intermediate products are made at the first stage, between dates 0 and 1. Each requires the labor of someone other than E to make. To this end, at date 0, E provides  $k$  other people -- his suppliers -- each with a key, which is necessary to make intermediate product. Aside from the key itself, production exhibits constant returns: one unit of labor produces one unit of intermediate input at date 1. E is only endowed with  $k$  keys. No-one can work with more than one key, and no key can be shared. In effect, E's  $k$  intermediate products can only be supplied by  $k$  different people, and none by E himself. We assume that  $k$  is large enough for divisibility not to be an issue, but small compared with the sizes of the two populations.

We make one further assumption. Instead of delivering the intermediate product at date 1, a supplier S can costlessly make and deliver a fake product, which is of no value to E, but which outsiders (such as the courts) cannot distinguish from the real thing. As a result, there is no point in E signing a supply contract with S at date 0 specifying an amount that he must pay S on delivery at date 1 -- because S could always deliver the fake product and appeal to the courts to collect the contractually-agreed payment from E. Thus explicit contracts are useless. (Restriction 2.)



This does not mean that E and S cannot do business, however. At date 1, S and the other  $k-1$  suppliers to E each has some amount of intermediate product,  $X/k$ , which, collectively, is worth  $\alpha X$  to E, but only  $\ell X$  to them. We assume that the parties bargain over the quantity and terms of trade. Specifically, they negotiate a price  $p$  per unit, and E takes delivery of as much intermediate product as he can afford at that price (but of course no more than the supply  $X$  that has been made between dates 0 and 1). If E cannot afford to buy all of  $X$  at this price, then the sellers liquidate the remainder. We are assuming equal treatment across the sellers: given E's Leontief technology, all  $k$  the sellers are in same bargaining position. (Restriction 3.) We make no attempt to specify the details of the bargaining other than to suppose that  $p$  is a fixed parameter  $p$  satisfying  $\ell < p \leq 1$ ; that is, we assume an interior bargaining solution. At date 0, anticipating the bargain that will be struck at date 1, E and S agree on some additional amount,  $qX/k$  goods, that E must pay S, where the price  $q$  reflects market conditions.

E and S have an implicit contract. At date 0, E gives S a key and orders a quantity  $X/k$  of one of his  $k$  intermediate products. At the same time, E pays S an amount  $qX/k$  goods. At date 1, E buys up to  $X/k$  units of the intermediate product at a price  $p$ . Any remaining intermediate product can be liquidated by S for  $\ell$  per unit.

Finally, consider the position of E as a supplier of intermediate product (to some other entrepreneur). Any attempt by E to raise funds at date 0 from a third party (deep pocket) in return for handing over the intermediate product at date 1 is thwarted by the fact that E can deliver a fake product. (Restriction 4.)

# GENERAL EQUILIBRIUM MODELS OF FINANCIAL SYSTEMS

## THEORY AND MEASUREMENT IN VILLAGE ECONOMIES

Addendum for the Miami Conference on Exchange Structures, Jan 31 1997  
Robert M. Townsend

co-author is Youngjae Lim - Korean Development Institute

research assistants - Carlos Perez-Verdia and Edward Seiler -thank you!

- I. Picture each ICRISAT southern India village as a general equilibrium economy with much risk, much diversity—Facts
  - 1.1 yields across soils for a given crop are not highly correlated
  - 1.2 yields across crops are not highly correlated
  - 1.3 annual incomes across occupations are not highly correlated
  - 1.4 Youngjae Lim's factor model of these incomes shows factors have distinct weights across households with idiosyncratic shocks contributing much to variance
  - 1.5 graphs of annual incomes across households and over time show high-relief landscape
  - 1.6 Chauduri and Paxon show monthly incomes have large seasonal components for farmers
  - 1.7 seasonal components can vary by land class
  
- II. Outcomes -
  - 2.1 annual consumption is smoothed across household and over time- graph shows much less relief
  - 2.2 monthly consumption has a large common component across households- both across landclass and across occupations -Chauduri/Paxon

Models are an attempt to explain - taking stylized general equilibrium models to the "simple" village economies

### II. Full risk-sharing/ Complete Markets

3.1 description- each household and the village as a whole has the capacity to store grain, buy and sell livestock/durable goods and physical assets, to accumulate and decumulate currency, and to work inside and outside village. Inside the village there are complete contingent claim markets or an equivalent set of markets or institutions. Outside the village financial markets may be limited or nonexistent

#### 3.2 analysis—

3.2.1 apart from corners, weighted marginal utilities are equated across households to a common Lagrange multiplier. Thus consumption should comove with aggregate consumption and not with household income.

3.2.2 grain storage, money accumulation, physical assets movements, and possibly interaction in outside financial markets are determined by familiar Euler equations-- with two exceptions—"aggregate" marginal utility of consumption is tracked by the Lagrange multiplier, and the village may hit corners. Thus if grain, money, and savings are all positive, then it must be that no asset is dominated in rate of return. With external savings and currency both positive, the sure-return rate of interest on savings must be zero; otherwise there is an anomaly. Without the possibility of external savings this anomaly disappears; currency is held as in the turnpike models monetary exchange of Townsend, Manuelli/Sargent. The village as a whole could anticipate shortfalls in the future, and would plan to run buffers of money or grain down to zero. In that sense the aggregated

village could appear constrained in one or several objects. With complete markets in the village, and full household access to storage technologies in the village, it would seem Euler equations apply at the household level too. But since individual consumption tracks the aggregate, all households with access to storage technologies would evaluate storage decisions and contemplate marginal variations based on virtually identical Euler equations. If money is held by anyone internally, the internal interest rate on savings must be zero. In periods when this buffer stock is driven zero, there can be relatively high internal interest rates on savings. More generally, if we imagine that a full allocation of risk bearing were achieved by all means other than with a particular asset in question, as in Udry, then individual incomes would have no influence on saving or dissaving though that asset, controlling for aggregate consumption.

### 3.3 empirical results -

Econometrica - annual consumption appear to co move somewhat and more saliently, household incomes have positive, significant but low influence on household consumption. The exception is that the landless seem less well insured in the village of Aurepalle.

## VI Permanent Income/ Buffer stocks ( Hall, Deaton Zeldes, Paxon, Heaton and Lucas, etc)

4.1 description- within the village all households have access to the above-mentioned means of storage and possibly to a perfect internal credit market. Otherwise, contingent claims and cross household risk sharing are shut down exogenously. The external across-village financial market may allow savings only, if that.

4.2 analysis --Euler equations apply at the individual level. The Euler equation in credit must be satisfied as an equality if there is a perfect internal (or external) credit market - this would drive the internal sure-return interest rate to zero if money is held by anyone and otherwise allow a positive rate of interest, if money is held by no-one. Euler equations associated with grain, livestock, and other buffer stocks apply at an inequality at the individual level only when the asset is depleted, and this would tend to be associated with relative high internal interest rates on credit. But suppose the internal credit market is nonexistent. Then when income is low, when one or several of the buffer stocks hit zero, and when a household is unable to borrow, current consumption would drop, giving rise on average, subsequently, to high consumption growth. Village level asset movements are determined by summing over individuals households

As a special case of multiple buffers, specialized diverse preferences, and limited internal markets, suppose consumption is not continuous but takes place at the end of trading, that some buffers yield more disutility than others, and that exchange takes place through bilateral pairings, not through multiagent markets. As in Koyotaki and Wright, this can give rise to commodity money, and also give zero-cost fiat money a special advantage.

### 4.3 empirical results .

4.3.1 Murdoch has shown in annual data that the landless and small landholders are liquidity /credit constrained in the village of Aurepalle and Shirpur, while no landclass is constrained in Kanzara

4.3.2 Rosenzweig and Wolpin have argued in annual data that livestock and pumps are bought in good times and sold bad times, as buffer stocks, with adverse consequence on production.

4.3.3 Chauduri and Paxon have argued in monthly data that credit has few time specific fixed effects and plays little role in consumption smoothing, that (unobserved) buffers like money and gain must be presumed to play the dominate roles.

V. Private Information Models (Rogerson, Phelan -Townsend, Townsend, Atekeson- Lucas, Green, Spear and Srivastiva, etc)

5.1 description—agents in the village de facto solve an information-constrained insurance problem, trying to get resources to households suffering idiosyncratic hard times but knowing that too much insurance can have an adverse impact on the incentive to work hard, e.g., make effort to get high crop yields. The “planner”, or village as a whole, has access to a sure-return storage technology and can carry resources forward to the future, or, depending on parameter values, borrow from the future as if in a perfect external credit market. But individual access to this outside market is restricted deliberately.

5.2 analysis- if likelihood ratio are “regular”, household consumption should move with household income somewhat even after insurance premia are paid and indemnities are received. (In an extension household consumption would move also with aggregate shocks). This introduces so-called intertemporal links, that is, history matters in the contemporary consumption allocation, as in Townsend, Rogerson. In an information-constrained optimum, consumption will be controlled in such a way that the marginal utility consumption today is greater than the (expected) marginal utility tomorrow. That is, households would like to save more than they are allowed to do.

5.3 empirical results - Ligon has nested all three of the above models -- full insurance, permanent income, and private information -- though the full insurance model has little power. Using GMM estimation, he has shown that about half of the households in Aurepalle are permanent income households as opposed to private information households; far fewer households are permanent income households in Kanzara, and only one is in Shrapur.

#### VI. Open questions

6.1 How is the large (non perfect) degree risk sharing achieved- by what institutions, markets, and objects?

6.2 How thick are credit/insurance markets. That is, how large is the role of credit/ gifts versus the role of currency, grain stocks, and livestock? Can we distinguish distinct role?

6.3 What is the role currency - who uses it, how long is it held, what is its role as a store of value and as a medium of exchange?

6.4 Is control over household assets plausible?

#### VII. Measurement -Searching for Salient Facts—

-see current draft “Currency: Theory and Measurement in Village Economies” with Youngjae Lim revised April 1996

7.1 how to measure currency and other objects from the transactions, production files

7.1.1 distinguishing currency as entering transaction file only once, barter as twice

7.1.2 using the plot file and the transactions file to distinguish the date of production from the date of sale allowing us to create the use of crop inventory

7.1.3 every transaction enters a transaction balance equation twice, giving a complete decomposition of the deficit or surplus

7.1.4 justifying a mean square error criterion as a measure of tracking, goodness of fit of the asset change to the deficit

7.2 on the use of objects to fill the gap between expenditures and revenue, monthly data

7.2.1 livestock and capital assets are not bought and sold in good years and bad respectively - au contraire

7.2.2 crop inventory plays a big role in all three villages

7.2.3 Aurepalle is a relatively big user of credit cum gifts, Kanzara of currency

7.2.4 the landless are more prone to use currency, the landed to use crop inventory - use of credit is large but with no obvious pattern across landclass

7.3 on use of objects to fill the gap, annual data-

7.3.1 most of the annual results are unchanged from the monthly

7.3.2 the role currency is enhanced in all three villages, apparently as a long term store of value

7.4 exchange patterns

7.4.1 much exchange within (and outside) the villages is monetized, up to 80% of the value of all transactions, but credit and commodity barter are present

7.4.2 Aurepalle is less monetized in that sense than the other two villages

7.4.3 credit/gifts account for 7-14 % of the value all transactions, but less than 5% of the number of transactions, suggesting credit market "transactions costs" of some kind

7.4.4 labor, jewelry and other objects appear in barter exchange, especially in Aurepalle

7.5 village as a whole relative to the outside economy

7.5.1 crop inventory plays the dominate role in covering deficits, especially in Shirapur, in both monthly and annual data

7.5.2 outside credit is not nearly zero in Kanzara and is especially large in Aurepalle (but possible bias there)

7.5.3 currency movements are non negligible in Kanzara and Aurepalle, but the movement seems less related to the deficit itself -- sometimes credit and currency are an offset to one another

VIII. From Data back to the Theory. - New Empirical Results (highly tentative)

8.1 Tests of Risk Sharing and the Role of the Distinct Assets

8.1.1 nature of the test - regress household saving (total and one asset at a time) onto monthly time dummies, household specific effects and household income (both current, and one lead, one lag) - The maintained hypothesis, following Udry, is that there is achieved somehow an optimal allocation of risk bearing, i.e., full insurance against idiosyncratic shocks, and what is tested is that the asset in question plays no role in buffering idiosyncratic shocks. We might imagine for example that credit/gifts play the role of insuring idiosyncratic shocks, and the other assets movements are dictated by the optimal intertemporal smoothing conditions for the village as a whole. But other assets could play the role of buffering idiosyncratic shocks, e.g., gifts from money holding, and a rejection among one or several assets could also mean that the maintained hypothesis of full risk sharing is incorrect.

8.1.2 results are classified as a salient rejection of the hypothesis that the asset plays no role in buffering idiosyncratic shocks when the income term (or terms) are positive, increasing savings, and the time dummies capturing aggregate fixed effects for the village as a whole are not significant; as a salient acceptance of the maintained hypothesis jointly with the limited buffering role when incomes are not significant and the fixed time effects are; and as mixed when incomes are significant but so also are the fixed time effects Occasionally signs are noted and may be "perverse".

8.1.2.1 results by village- Aurepalle comes close to complete rejection with salient rejection for three of the four objects—money, credit, and crop inventory-- physical assets always has "wrong sign". Kanzara presents a mixed picture for every asset.

Shirapur has a salient acceptance, a salient rejection, and two mixed cases.

8.1.2.2 results by asset- crop inventory does the best with one acceptance in Shirapur and mixed, "wrong" sign in the other two villages. Money and physical assets come next presenting one salient rejection, in Aurepalle, and

two mixed results, in the other two villages (again physical assets always has a "wrong" sign). Credit does the worst with two salient rejections in Aurepalle and Shirapur -- only Kanzara is mixed. But we cannot not disentangle in the data simple intertemporal borrowing/lending from gifts, insurance, so the significance of the income term in the credit regression may only indicate that credit markets play a salient role in de facto insurance.-

8.1.3 quick summary- - under one interpretation, credit plays the dominate role of insuring idiosyncratic shocks, while grain satisfies the intertemporal Euler condition in smoothing shocks for the village over time, at least in Shirapur.--the role of money is less clear and this picture is blurred in the other two villages

## 8.2 Permanent Income/ Private Information relative to the Use of Assets

8.2.1 nature the procedure -sort households according to the Ligon criteria of whether they are private information households or permanent income households

8.2.2 results--compare the use of an asset in Tukey diagrams of the mean-square error criterion with the Ligon sort

### 8.2.2.1- Permanent Income households- by use an asset

borrowing.- In Aurepalle and Shirapur (one data point) those that satisfy the permanent income hypothesis in the sense of Ligon are heavier uses of credit. But in Kanzara this goes the other way!

money and crop inventory - In Aurepalle and Kanzara those that pass the permanent income model are more likely to use money (Shirapur with one data point goes the other way). Yet in all three villages there is a slight tendency for permanent income households to use crop inventory less.

physical assets- destabilizing and not much information

### 8.2.2.2 Private Information Households- by Village

Aurepalle -private information households are the heaviest users of crop inventory, then credit, finally money

Shirapur - crop inventory is used heavily

Kanzara - crop inventory is used heavily, currency is tied with it

8.2.2.3 - quick summary- credit seems to help achieve permanent income, while crop inventory, and also money, are more associated with private information households

8.3 Concerns that classification, accept/reject using income, consumption data, may be vulnerable to specification error and so patterns relative to use of assets may be misleading

8.3.1 Morduch versus Ligon- procedure- the first step is to find which households are liquidity constrained and which not, following Morduch. Real growth in food consumption is regressed on time dummies, age of the head, a demographic adjustment term, [with and without] log interest rate, and income -the latter is constrained to take on a coefficient which is at most different across different land classes

8.3.2 results - Shirapur show some consistency across Ligon and Morduch in the sense that by Ligon most household are not permanent income households but are

private information households ,and by Morduch-style most are not permanent income households in the sense being liquidity constrained

Aurepalle -inconsistency in that one half the sample is permanent income under Ligon's standard but are liquidity constrained under Morduch-style (the other half is like Shirapur)

Kanzara- consistency is mixed- by Ligon's standard almost all are private information households but virtually all pass the liquidity test (still the interest rate coefficient has the wrong sign suggesting risk loving – in Ligon's test this is equivalent with rejecting permanent income over private information)

8.3.3 - if original, not revised , land class is used it changes the significance of the income coefficient, and the classification relative to Ligon- however these regressions are less reliable because we can no longer control for household specific fixed effects

8.3.4 monthly data-- test of liquidity constraints – now under either the original or the revised land class specifications, virtually all household are constrained all the time regardless of land class, leaving no inference possible about use asset relative to whether or not constrained

8.3.5 aggregating across households in all land classes to deliver something at the aggregate, village level-aggregate income is sometimes significant as if the village as a whole were experience liquidity constraints- this depends much on the specification however

#### IX. Overall Summary and Assessment

Credit is not negligible in all three villages, and it plays a role in at least two of them to push households toward the permanent income standard . Indeed, credit cum gifts seem to play a big role in achieving better risk sharing than buffer stocks alone could do, though in the end complete risk sharing is not achieved. Internal credit markets appear limited in the sense that transactions in credit are large in value they occur and that many households, despite non negligible use of credit, appear liquidity constrained , among the poor, annually, and among virtually everyone, monthly. Grain inventory plays a big internal role in all three villages, particularly in intertemporal consumption smoothing, and particularly among the relatively rich . Currency also plays a big internal store-of-value role, even in annual data, despite what we would imagine to be rate of return anomalies. The poor in particular hold currency over time. Currency also plays a role in buffering idiosyncratic shocks. Finally, currency plays a large role in executing transactions, yet barter exchange and credit coexist with it.

These economies need to be approximated by "imperfect" markets and institutions of some kind. Private information seem a good candidate for an imperfection, and one private information model seems to fit the intertemporal consumption data reasonable well. But to tell this story we have to believe that households holding substantial crop inventory lose control over this asset and similarly for currency in Kanzara, . This seems particularly problematic. The reason for role of money as a store of value dominated perhaps in rate of return , as covering idiosyncratic shocks, and as a medium of exchange remains requires more scrutiny.. More theory and empirical work are needed.

The relationship of the village economies to the larger outside regional, national economy is less well understood. Outside credit markets appear limited, though non negligible. Crop inventory plays a big role, but currency is held as well. Salient patterns are less evident but the number of sample villages is low.

## CREDIT CHAINS

Nobuhiro Kiyotaki

John Moore

January 1997

### Questions

How do shocks propagate through a network of firms who borrow from, and lend to, each other?

What are the aggregate effects of creditors postponing the debts of delinquent debtors, rather than liquidating their assets?

Why don't firms insure against aggregate shocks to their accounts receivable?

### Overview of Model

Many small firms owned and run by entrepreneurs.

Entrepreneurs unable to raise outside finance.

But an entrepreneur can borrow from his suppliers, because they can withhold their supply if he defaults.

Supply contracts have to involve an element of lending.

Entrepreneurs buy and sell from each other

⇒ an entrepreneur simultaneously holds gross financial positions:

{ assets (accounts receivable)  
liabilities (accounts payable)

An entrepreneur cannot net out his gross financial positions (he cannot securitize the debt owed to him)

⇒ he is exposed to default risk

⇒ SYSTEMIC RISK



Example

A has ordered 100 units of specific input from B at \$1 a unit.

B has ordered 100 units of specific input from C at \$1 a unit.



A expects to have \$100 in cash at due date of delivery.

B has no cash, but expects to receive \$100 cash from A.

\*\*\*\*\*

Suppose A unexpectedly has only \$60 in cash at due date.

⇒ A defaults against B.

If B continues to charge \$1 for each unit delivered

⇒ A takes delivery of only 60 units.

B can liquidate remaining 40 undelivered units, say for \$0.5 each

⇒ B gets total of \$80:

{ \$60 in cash from A  
\$20 in liquidation receipts.

This is less than the \$100 B was expecting to receive

⇒ B defaults against C

If C continues to charge \$1 for each unit delivered

⇒ B takes delivery of only 80 units

C can liquidate remaining 20 undelivered units, say for \$0.5 each

⇒ C gets total of \$90:

{ \$80 in cash from B  
£10 in liquidation receipts.

And so on, along credit chain.

**Secondary defaults by B, C would be avoided if A's debt were with an unconstrained deep pocket, rather than with B.**

**Wrong people are being exposed to A's risk.**

**Hypothesis: Economy where gross positions can be netted out would react less to shocks?**

**Formal model is of a network of credit:**

- **each entrepreneur is owed money by several other entrepreneurs**
- **each entrepreneur owes money to several people, both to other entrepreneurs and to deep pockets**

**RESULT 1 (Systemic Risk)**

**Negative shock to liquidity**

**⇒ aggregate multiplier**

**everyone could in principle be made better off if people forgave each others' debts.**

**Larger gross financial positions ( $\equiv$  longer credit chains)**

**⇒ stronger multiplier**

**Postponement of unpaid debts**

Following A's default, B has choice:

either liquidate 40 undelivered units  
⇒ B has \$20 more cash  
⇒ B can buy 20 more units of input from C

or postpone A's outstanding debt of \$40

If B's return on investment < \$2 a unit

⇒ B is better off postponing  
A is also better off  
⇒ postponement is bilaterally efficient

Postponement imposes cost on C, however:

Now C is in same position as B:

viz. owed \$100 by a debtor who only has \$60.

Hence, if C's return on investment < \$2 a unit

⇒ C is better off postponing B's outstanding debt of \$40

And so on, along credit chain.

Note: No diminution of shock along the credit chain if debts are postponed.

Why? Because no liquidation ⇒ no new cash injected into system.

## RESULT 2

**Postponement of debt can be socially worse than liquidation, even though it is privately optimal.**

**Explanation:** A decision by a creditor to postpone debt imposes a negative externality on the creditor's creditors, and in turn on their creditors.

## Insurance

**In a stochastic environment, an entrepreneur faces:**

- (a) uncertainty in individual return from his own investment**
- (b) uncertainty in accounts receivable**  
**(arising from fluctuations in his customers' incomes)**

**Impossible to insure against (a).**

**But (b) is correlated with aggregate state**

**⇒ entrepreneur can insure against (b)**

**Ideally, entrepreneur would buy policy that pays in bad state, and he pays (the premium) in good state.**

**But entrepreneur cannot commit to pay in good state**

**⇒ he must pay premium upfront**

### Benefit of insurance

Entrepreneur can obtain better terms from his own suppliers if he is less likely to default

### Costs of insurance

- paying premium upfront eats into entrepreneur's own investment  
(inefficient to invest in an insurance policy)
- fluctuations in accounts receivable (b) may be relatively small and not strongly correlated with fluctuations in individual return (a)
  - ⇒ entrepreneur sometimes will not need any insurance to meet his own debt obligations in a recession.

On balance, costs may exceed benefit. Hence:

### RESULT 3

Entrepreneurs may not insure against default risk

⇒ systemic risk, even when shocks are anticipated.

Basic Model

Dates 0, 1, 2

Entrepreneurs - measure 1

Deep pockets

Consume general commodity ("goods") at date 2

- numeraire

Risk neutral

Goods can be stored without depreciation

Initial aggregate goods endowment

{	entrepreneurs $M$
	deep pockets $\bar{M}$ (large)

Aggregate labor endowment  
between dates 0 and 1

{	entrepreneurs $N$
	deep pockets $\bar{N}$ (large)

Deep pockets' technology  
(constant returns)

1 unit of own labor between dates 0 and 1 → 1 unit of goods at date 1

Entrepreneurs' short-term technology  
(constant returns)

1 unit of goods at date  $t$  →  $\sigma$  units of goods at date  $t + 1$

$t = 0$  or  $1$

$\sigma > 1$

In effect, entrepreneurs have a superior storage technology.

**Entrepreneurs' long-term technology**  
(constant returns)

**First stage:**

1 unit of labor between dates 0 and 1 → 1 unit of intermediate product (specific to entrepreneur) at date 1

Entrepreneur cannot use his own labor.

Instead, at date 0 he places orders with suppliers for intermediate product to be delivered at date 1. A supplier works between dates 0 and 1 with a blueprint that E gives her at date 0. Suppliers may be either other entrepreneurs, or deep pockets.

**Second stage:**

1 unit of intermediate product at date 1 →  $\alpha$  units of goods at date 2

$\alpha > 1$

Intermediate product is specific to entrepreneur.

Only use for the supplier is to liquidate it:

1 unit of intermediate product can be liquidated for  $\ell < 1$  units of goods at date 1

(liquidation has constant returns, and is instantaneous)

**Competitive market at date 0:**

**Any entrepreneur is free to place an order with any other entrepreneur or deep pocket for intermediate product to be supplied at date 1.**

**No counter-trade: if E makes intermediate product for E\*, then this precludes E\* producing for E.**

**Restrictions on contracting**

- (1) Limited enforcement of contracts**  
⇒ **entrepreneurs cannot raise outside investment funds against future returns**
  
- (2) Supply contracts incomplete**  
⇒ **terms of delivery renegotiated at time of delivery**
  
- (3) Equal treatment in default: an entrepreneur defaults against his suppliers on a pro-rata basis**
  
- (4) An entrepreneur cannot borrow against a promise to supply.**



**Implicit debt/supply contract**

**Date 0** Entrepreneur orders  $x$  units of his specific intermediate product, for delivery at date 1

Entrepreneur gives supplier blueprint

Entrepreneur gives supplier a downpayment  $qx$

**Date 1** Entrepreneur takes delivery of  $(1 - \lambda)x$  units by paying supplier  $p(1 - \lambda)x$  goods, where  $\lambda \geq 0$

Supplier liquidates  $\lambda x$  units

In total supplier gets  $\left\{ \begin{array}{l} qx \text{ goods at date 0} \\ px - (p - \ell)\lambda x \text{ goods at date 1} \end{array} \right.$

$p$  is a fixed parameter, determined through bargaining at date 1

Assume  $\ell < p \leq 1$  (interior bargaining solution)

$\lambda$  is determined by entrepreneur's goods holding at date 1.

$q$  clears market at date 0.

$q$  is a function of anticipated  $\lambda$  (i.e.  $q$  is specific to entrepreneur)

For deep pockets to be indifferent between using their own technology and making intermediate product for some entrepreneur:

$$q + p - (p - \ell)\lambda = 1$$

In the absence of shocks, each entrepreneur will plan to have just enough goods at date 1 to ensure  $\lambda = 0$

$$\Rightarrow q = 1 - p.$$

**Interpretation:** Implicit contract is a debt contract bundled with a supply contract.

**Balanced investment**

At date 0, entrepreneur:

- orders X units of his own intermediate product
- takes orders for N units of intermediate product from other entrepreneurs
- invests Y in short-term technology

Date 0 flow-of-funds:

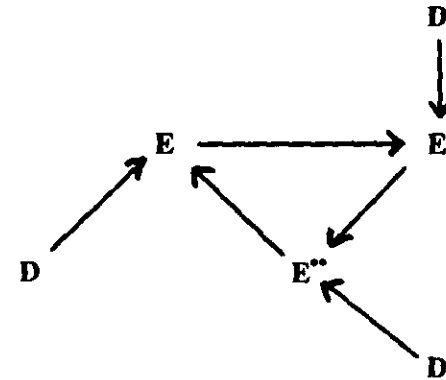
$$(1 - p)X + Y = M + (1 - p)N$$

Date 1 flow-of-funds:

$pX$	=	$\sigma Y$	+	$pN$
↑		↑		↑
accounts payable		return on short-term investment		accounts receivable

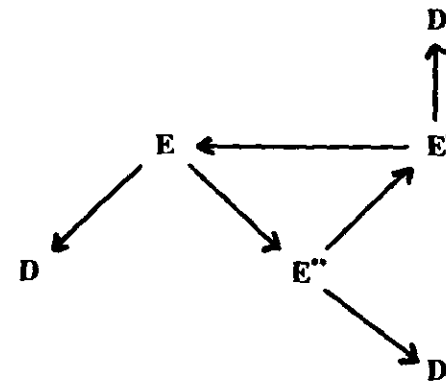
Balanced investment is best for entrepreneur if  $\alpha > \sigma^2$ .

**CANONICAL NETWORK**



Supply network:

E supplies E\*  
E is supplied by E\*\* and D



Credit network:

E\* is in debt to E  
E is in debt to E\*\* and D

Effects of an unanticipated shock at date 1

For all entrepreneurs, suppose that the date 1 return from short-term technology is unexpectedly  $\hat{\sigma} < \sigma$ .

⇒ an entrepreneur can only afford to take delivery of  $\hat{X} < X$ .

New flow-of-funds constraint at date 1:

$$\begin{array}{cccc}
 \hat{p}\hat{X} & = & \hat{\sigma}Y & + & p\left(\frac{\hat{X}^*}{X^*}\right)N & + & \ell\left(1 - \frac{\hat{X}^*}{X^*}\right)N \\
 \uparrow & & \uparrow & & \uparrow & & \uparrow \\
 \text{accounts} & \text{return from} & \text{accounts} & & \text{liquidation} \\
 \text{paid} & \text{short-term} & \text{received} & & \text{receipts} \\
 & \text{investment} & & & 
 \end{array}$$

Asterisks denote  $E^*$ 's behavior.

Multiplier.

All entrepreneurs would be better off if they forgive each other's debts

(≡ charge each other less than  $p$ )

Numerical example

Long-term return	$\alpha = 1.8$
Short-term return	$\sigma = 1.2$
Liquidation value	$\ell = 0.5$
Delivery price	$p = 1$
Entrepreneurs' initial goods endowment	$M = 5$
Entrepreneurs' labor endowment	$N = 24$

Suppose  $\hat{\sigma}$  is 5% less than  $\sigma$

(equivalent to a 1% shock to the entrepreneurs' wealth at date 1)

The entrepreneurs' consumption is 1.7% less than expected

(without credit chains, their consumption is only 1% less)

## Postponement

Changes to model: Add date 3

Consumption at date 3

Entrepreneurs can use short-term technology at  $t = 0, 1$  and 2

Entrepreneurs can use long-term technology twice:

between dates 0 and 2

between dates 1 and 3

Deep pockets have additional labor endowment between 1 and 2

Undelivered intermediate product can be stored from date 1 to 2

Let  $p = 1$ .

A supplier will postpone rather than liquidate if  $\alpha t < 1$

For numerical example, postponement

⇒ entrepreneurs' consumption is 2.8 % less than expected

(with liquidation, their consumption is only 1.7 % less)

## Stochastic Model

At date 0, the date 1 return on entrepreneurs' short-term investment is stochastic:

### Boom

All entrepreneurs receive  $\sigma$

(probability  $1 - \pi$ )

### Recession

Fraction  $1 - \theta$  of entrepreneurs receive  $\sigma^+ > \sigma$

(probability  $\pi$ )

Fraction  $\theta$  of entrepreneurs receive  $\sigma^- < \sigma$

(not verifiable)

Recession may be induced by a mean-preserving spread:

$$\theta\sigma^- + (1 - \theta)\sigma^+ = \sigma$$

## Insurance

Entrepreneurs can only insure against boom/recession

Premium must be paid at date 0

Typical policy:  $\left\{ \begin{array}{l} \text{Pay } Z \text{ goods at date 0 in premium} \\ \text{Receive } (Z/\pi) \text{ goods at date 1 if recession occurs.} \end{array} \right.$

At date 0, each entrepreneur chooses

$X$  = units of intermediate product ordered for delivery at date 1

$Y$  = investment in short-term technology

$Z$  = insurance premium

## Proposition

Assume  $(\sigma - \sigma')$  and  $(\sigma' - \sigma)$  are small, and that

$$(*) \quad \alpha\sigma > \theta\alpha + (1 - \theta)\sigma + \theta\alpha\sigma(1 - \theta)$$

Then the symmetric equilibrium has  $Z = 0$  (no insurance)

$(X, Y)$  chosen so that each entrepreneur just has enough funds in the boom not to default.

In recession, a fraction  $\theta$  of entrepreneurs default; but all entrepreneurs experience fall in accounts received.

⇒ multiplier

Note: (\*) is satisfied by numerical example.