Measuring the Rate of Technological Progress in Structures

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Abstract

How much technological progress has there been in structures? An attempt is made to measure this using panel data on the age and rents for buildings. This data is interpreted through the eyes of a vintage capital model where buildings are replaced at some chosen periodicity. There appears to have been significant technological advance in structures that accounts for a major part of economic growth. *Journal of Economic Literature* Classification Numbers: O3 and 04.

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1. Introduction

So how much technological progress has there been in structures? Not much, might be the answer. Perhaps this answer should be reconsidered; because, as each decade passes new limits are reached in building airports, bridges, dams, highways, oil platforms, sea barriers, skyscrapers, stadiums, towers, and tunnels. Moreover, as new technology enables these advances, increased output per square foot of land is realized due to benefits such as faster and better communication and higher productivity resulting from more and better work space. Some examples may be in order to bolster this claim.

Skyscrapers: The Home Insurance Building is generally considered to be the world's first skyscraper. Built in Chicago in 1885 it was 10 stories tall. Compare this with Chicago's 110-story Sears towers completed in 1974. In less than 100 years the tallest building went from 10 to 100 floors. The 443 meter Sears Tower now plays second fiddle to the 452 meter Petronas Twin Towers in Kuala Lumpur built in 1997. The increase in building height reflects significant advances in engineering. While the Sears Towers are 200 feet taller than the Empire State Building (circa 1931) they weigh much less, 223,000 tons versus 365,000: a testimonial to better materials and design. Providing comfort to the occupants of a skyscraper is a major concern. For instance, the 29th floor is taken up by five chillers that cool the air in the building. Three of these weigh 5,000 tons a piece. Water that has been used in the chillers is pumped up 77 floors to four three-story high cooling towers located on levels 106 to 109. As the water cascades down the walls of the towers it is cooled by a huge fan. The tops of tall buildings are also subject to substantial movement from wind, causing motion sickness to the occupants.

To prevent this, two tuned dynamic dampers were installed in Boston's Hancock Tower (1969). Here, two three ton masses of lead are set on thin layers of oil on opposite ends of the 59th floor of the tower. They are connected to the structure with springs and shock absorbers. These dampers serve to mitigate the sway in the tower.

Suspension Bridges: The Brooklyn Bridge was a technological marvel when it opened in 1883. Its center span is 486 meters long. Contrast this to Japan's Akashi Kaikyo Bridge, opening in 1998, who's center span is 1990 meters long. The Messina Bridge planned for the year 2006 will connect mainland Italy with Sicily and will have a central span of 3300 meters. Long suspension bridges are very susceptible to the vicissitudes of nature, especially wind and water. Wind-excited vibrations at the natural frequency of the structure caused the collapse of the Tacoma Narrows suspension bridge in Puget Sound in 1940. To protect against oscillations, tuned mass dampers were added to the towers of Akashi Kaikyo Bridge, the first time in a bridge. These devices contain pendulums that rock in a direction opposite to the towers, thus dampening motion. This, together with other innovations, should allow the bridge to withstand winds up to 290 kilometers per hour.

Tunnels: The world's longest railway tunnel spans the Tsugaru Straits in Japan. Completed in 1988, it is 34 miles long and was dug through some of the most difficult rock ever encountered. The rock under the Tsugaru Straits is porous and unstable, permitting large water flows. So, before tunnelling, the rock had to be prepared. The fissures in the rock were sealed by pumping, under high pressure, a mixture of cement and a gelling agent into small holes that were drilled into the rock. Digging tunnels under water is dangerous. Once, during construction, water flooded in at a rate of 80 tons per minute forcing a rapid evacuation. Even today, after the tunnel has been lined, without the aid of four pumping stations it would flood within 78 hours. Tunnelling has come a long way from the world's first railway tunnel, the 12.3 mile Simplon Tunnel built through the Alps between France and Italy, that opened in 1871.

Oil Platforms: At 630 feet tall and 824,000 metric tons it was the heaviest man-made object ever moved when it was hauled out to sea in 1981. This is taller than the United Nations Building and three times heavier than the World Trade Center. The Statfjord B oil platform was a mammoth undertaking. One hundred miles from shore with 200 people aboard, it needs to be able to withstand the worst of weather. In waves of 100 feet and in wind of 100 miles per hour it is designed to shift less than one half of an inch. It can produce 150,000 barrels of oil a day.

The Analysis: So again, how much technological progress has there been in structures? The answer here is that the rate of technological progress in structures is about 1% per year, and accounts for 15% of economic growth. The method of estimating technological progress employed here differs significantly from current growth accounting practice. In particular, a vintage capital model is developed where technological progress is embodied in the form of new capital goods, namely equipment and structures. Production in the economy is undertaken at a fixed number of locations, each using equipment, structures, and labor. Investment in structures is assumed to be lumpy. Once a building is erected on a site it remains there until torn down and replaced with a new and improved one. The decision about when to replace a building is modeled in the analysis. In equilibrium some sites will have new, efficient buildings and others old, less efficient ones. Equipment and labor are mobile across sites. By using the structure of the developed model, in conjunction with some observations from the U.S. data, an estimate of the rate of technological progress in structures and its contribution to economic growth can be made.

A novel aspect of the analysis relative to traditional growth accounting is the use of price data to shed information on technological progress. Gordon's [2] data on the price for new producer durable equipment shows that there has been a substantial secular decline in the relative price of new equipment over the postwar period.[1] In constructing his price index, Gordon [2] attempts to control for the operating characteristics of equipment that are important for production. This suggests that there has been significant technological progress in the production of new equipment. There is no similar series available for new structures. To the extent that new office buildings have new and improved technology embodied in their structures, however, they should rent for more than old ones, ceteris paribus. A panel data set of office buildings is used in the current analysis to estimate the rent gradient for buildings (as a function of age). A key assumption in the analysis is that buildings are continually kept in good repair. This allows the decline in rents with age to be identified solely with technological advance, and not with wear and tear as well. To the extent that buildings must be kept in good repair either because of building codes or rental contracts this assumption may not be that stringent. The rent gradient obtained is then connected with the rate of technological progress in structures by using various equilbrium conditions

[1] Here the analysis follows the lead of Greenwood, Hercowitz and Krusell [3] who use Gordon's [2] prices to calculate how much of postwar economic growth was due to equipment-specific technological progress.

arising from the vintage capital model.

Contrast this with conventional growth accounting. There an aggregate production function and input measures are used to decompose growth into technological progress and changes in inputs. In a world where technological progress is embodied in the form of new equipment and structures the use of capital input measures becomes suspect. They are avoided here. Also, conventional growth accounting is incomplete because it does not allow for the growth in output due to capital accumulation to be broken down into its underlying sources of technological progress. The analysis here takes this factor into account by imposing balanced growth conditions on the developed model.

2. Theory

2.1. Environment

Production is undertaken at a fixed number of locations, distributed uniformly on the unit interval, and requires the use of three inputs: equipment, structures, and labor. Each location has associated with it a stock of structures of a certain age or vintage. The manager of a location must decide at each point in time whether to replace this stock of structures or not. Equipment and labor can be hired each period on a spot market. Let production at a location using structures of vintage j be given by

$$o(j) = zk_e(j)^{\alpha_e}k_s(j)^{\alpha_s}l(j)^{\beta}, \tag{2.1}$$

where z is the economy-wide level of total factor productivity and $k_e(j)$, $k_s(j)$, and $l(j)$ are the inputs of equipment, structures and labor. Denote the number of locations using structures of vintage j by $n(j)$ and let the oldest age of structures be T. Then $\int_0^T n(j)dj = 1$. Aggregate output is thus

$$y = \int_0^T n(j) z k_e(j)^{\alpha_e} k_s(j)^{\alpha_s} l(j)^{\beta} dj. \tag{2.2}$$

Output can be used for four purposes: consumption, c, investment in new equipment, i_e, investment in new structures, i_s, and for investment in repair and maintenance on old structures, i_m. Hence

$$c + i_e + i_s + i_m = y. \tag{2.3}$$

Equipment is mobile and can be freely rented on an economy-wide equipment market. The law of motion for equipment has the form[2]

$$\frac{dk_e}{dt} = -\delta_e k_e + q i_e. \tag{2.4}$$

The variable q represents equipment-specific technological progress. This occurs over time at rate γ_q. As q increases over time a unit of forgone consumption can purchase ever increasing quantities of equipment. Here $1/q$ can be thought of as the relative price of equipment. This price declines over time. The rate of physical depreciation on equipment is δ_e.

Imagine constructing a new building at some location. Suppose that a unit of forgone consumption can purchase v new units of structures. Then, building $k_s(0)$ units of new structures would cost $k_s(0)/v$ units of consumption. Let v grow at the fixed rate γ_v; this denotes structure-specific technological progress.[3]

[2] Greenwood, Hercowitz and Krusell (1997) use this formulation to study investment-specific technological progress. A more detailed discussion on the notion of investment-specific technological advance is contained there.

[3] The focus of the analysis is on balanced growth paths. So, some variables, such as aggregate output, will grow over time at constant rates while others, for instance the interest rate, will be constant.

Aggregate gross investment in structures will therefore read

$$i_s = n(0)k_s(0)/v. \tag{2.5}$$

Structures remain standing until they are replaced. While structures suffer no physical depreciation, they must be maintained. Let the initial maintenance cost be a fraction $\mu(0)$ of the building's purchase price. These costs grow exogenously at rate $\gamma_\mu + \gamma_y$ as the building ages, where γ_y is the economy's growth rate. Therefore, $\mu(j) = e^{(\gamma_\mu+\gamma_y)j}$. Aggregate investment in repair and maintenance is[4]

$$i_m = \int_0^T n(j)\mu(j)k_s(j)/(ve^{-\gamma_v j})dj.$$

The question of interest here is: When should a building be replaced?

2.2. The Location Manager's Decision

2.2.1. Static Profit Maximization

At a point in time the manager of a location should hire equipment and labor to maximize the location's profits, given his stock of structures. Consider the static profit-maximizing decision at a location using vintage-j structures:

$$\pi(j) = \max_{k_e(j),l(j)} \{zk_e(j)^{\alpha_e}k_s(j)^{\alpha_s}l(j)^{\beta} - r_ek_e(j) - wl(j)\}, \tag{P(1)}$$

where r_e is the economy-wide rental price for equipment and w is the wage rate. The first-order conditions are

$$\alpha_e zk_e(j)^{\alpha_e-1}k_s(j)^{\alpha_s}l(j)^{\beta} = r_e, \tag{2.6}$$

[4]Note that $1/(ve^{-\gamma_v j})$ is the price that a unit of structures cost j periods ago.

and

$$\beta z k_e(j)^{\alpha_e} k_s(j)^{\alpha_s} l(j)^{\beta-1} = w. \tag{2.7}$$

By multiplying (2.6) by $k_e(j)$ and (2.7) by $l(j)$ it is easy to establish from P(1) that

$$\pi(j) = (1-\alpha_e-\beta) z k_e(j)^{\alpha_e} k_s(j)^{\alpha_s} l(j)^{\beta}.$$

Next, observe that (2.6) and (2.7) imply that

$$k_e(j) = \frac{\alpha_e}{\beta}\frac{w}{r_e} l(j), \tag{2.8}$$

and

$$l(j) = [\frac{\beta^{1-\alpha_e}\alpha_e^{\alpha_e} z(1/r_e)^{\alpha_e} k_s(j)^{\alpha_s}}{w^{1-\alpha_e}}]^{\frac{1}{1-\alpha_e-\beta}}, \tag{2.9}$$

so that rents at a point in time (the return to the fixed factor, here land) can be expressed as

$$\pi(j) = (1-\alpha_e-\beta) z^{\frac{1}{1-\alpha_e-\beta}} \alpha_e^{\frac{\alpha_e}{1-\alpha_e-\beta}} \beta^{\frac{\beta}{1-\alpha_e-\beta}} r_e^{\frac{-\alpha_e}{1-\alpha_e-\beta}} w^{\frac{-\beta}{1-\alpha_e-\beta}} k_s(j)^{\frac{\alpha_s}{1-\alpha_e-\beta}}. \tag{2.10}$$

The profits from each location, net of any repair and maintenance costs and investment in structures, are rebated to consumers each period.

2.2.2. The Replacement Problem

When should the manager of a location replace the structures on his site? Suppose that at date 0 the manager has $k_{s,0}(0)$ units of new structures.[5] At what date T should he replace his building and how much should his investment in new

[5]Time is indicated by subscripts. So for example, $k_{s,t}(j)$ indicates the amount of age-j structures at time t.

structures, $k_{s,T}(0)$, be at that time? Clearly, he should choose these variables to maximize the value of the location as denoted by $V(k_{s,0}(0))$. The manager's date-0 problem can be written as

$$V(k_{s,0}(0)) = \max_{k_{s,T}(0),T} \{ \int_0^T [\pi_t(t) - \mu(t)k_{s,0}(0)/v_0]e^{-\iota t}dt \qquad \text{P(2)}$$
$$+e^{-\iota T}[V(k_{s,T}(0)) - k_{s,T}(0)/v_T]\},$$

where ι represents the time-invariant interest rate. The solution dictates that

$$[\pi_T(T) - \mu(T)k_{s,0}(0)/v_0] - \iota[V(k_{s,T}(0)) - k_{s,T}(0)/v_T] = 0, \qquad (2.11)$$

and

$$V_{k_s}(k_{s,T}(0)) = 1/v_T. \qquad (2.12)$$

2.3. Equipment Rentals

At each point in time the equipment manager has k_e units of equipment that he can rent out at r_e. He must decide how much to invest, i_e, in new equipment. This investment can be financed at the fixed interest rate ι. The optimal control problem governing the accumulation of equipment is summarized by the current-value Hamiltonian shown below and its associated efficiency conditions.

$$\mathcal{H} = r_e k_e - i_e + \lambda[i_e q - \delta_e k_e],$$

$$\mathcal{H}_{i_e} = -1 + \lambda q = 0, \qquad (2.13)$$

and

$$d\lambda/dt = \iota\lambda - \mathcal{H}_{k_e} = \iota\lambda - r_e + \lambda\delta_e. \qquad (2.14)$$

Observe that from (2.13) that $\gamma_\lambda = -\gamma_q$ so that (2.14) can be expressed as

$$r_e = (\iota + \delta_e + \gamma_q)/q. \qquad (2.15)$$

This gives the rental price for equipment. This formula has a simple interpretation. A unit of forgone consumption can purchase q units of equipment that will rent for $r_e q$. This rental income must cover the forgone interest, ι, physical depreciation, δ_e, and the capital loss, γ_q, induced by the fact the price of equipment (in terms of consumption) is falling across time.[6]

2.4. The Representative Consumer's Problem

Let a consumer's lifetime utility function be given by

$$\int_0^\infty \ln c_t e^{-\rho t} dt.$$

Now, the consumer is free to lend in terms of bonds, a, earning the return ι. In addition to the interest he realizes on his lending activity, the consumer earns labor income, w, and the profits from his locations (net of any repair and maintenance costs and investment in structures), $\int_0^T n(j)[\pi(j) - \mu(j)k_s(j)e^{\gamma_v j}/v]]dj - n(0)k_s(0)/v$. The law of motion governing his asset accumulation reads

$$da/dt = w + \iota a + \int_0^T n(j)[\pi(j) - \mu(j)k_s(j)e^{\gamma_v j}/v]dj - n(0)k_s(0)/v - c.$$

The efficiency condition governing asset accumulation is

$$\frac{1}{c}\frac{dc}{dt} = (\iota - \rho), \tag{2.16}$$

which states the familiar condition that consumption should grow at the rate at which the interest rate exceeds the rate of time preference.

[6]A unit of equipment sells for $1/q$ units of consumption, and this price is falling over time at rate γ_q.

2.5. Market Clearing Conditions

At each point in time the markets for labor and bonds must clear [in addition to the goods market as represented by (2.3)]. Consequently,

$$\int_0^T n(j)l(j)dj = 1, \tag{2.17}$$

and

$$\int_0^T n(j)k_e(j)dj = qa.$$

2.6. Balanced Growth

The balanced growth path will be uncovered using a guess and verify procedure. To this end, conjecture that along a balanced growth path consumption, investment in equipment and structures, aggregate output, and the stocks of equipment and structures at a location of any given age, will all be growing at constant rates. Likewise, it seems reasonable to believe that the age distribution of structures and the amount of labor allocated to an age-j location will be constant through time. If so, equation (2.2) then implies that along a balanced growth path output will grow at rate

$$\gamma_y = \gamma_z + \alpha_e\gamma_e + \alpha_s\gamma_s, \tag{2.18}$$

where $\gamma_y \equiv (1/y)dy/dt$, $\gamma_z \equiv (1/z)dz/dt$, $\gamma_e \equiv [1/k_e(j)]dk_e(j)/dt$, and $\gamma_s \equiv [1/k_s(j)]dk_s(j)/dt$. Additionally, from the resource constraint (2.3), consumption, equipment investment, structure investment, and maintenance investment will all need to grow at the same rate as output, or would disappear relative to output. Consumption growing at the fixed rate γ_y requires that the interest rate remains constant at

$$\iota = \rho + \gamma_y, \tag{2.19}$$

a fact evident from (2.16).

Next, note that the law of motion for equipment in balanced growth reads

$$\gamma_e = -\delta_e + \frac{qi_e}{k_e}.$$

Thus, γ_e can be constant if and only if qi_e/k_e is too. This can only be true when

$$\gamma_e = \gamma_q + \gamma_y, \tag{2.20}$$

Analogously, $i_s/y = [n(0)k_s(0)/v]/y$ can only remain fixed if

$$\gamma_s = \gamma_v + \gamma_y, \tag{2.21}$$

where, as must be obvious by now, $\gamma_q \equiv (1/q)dq/dt$ and $\gamma_v \equiv (1/v)dv/dt$.

The rate of growth in output, as a function of the underlying sources of technological progress, can now be uncovered by substituting (2.20) and (2.21) into (2.18) to obtain

$$\gamma_y = \frac{1}{1-\alpha_e-\alpha_s}\gamma_z + \frac{\alpha_e}{1-\alpha_e-\alpha_s}\gamma_q + \frac{\alpha_s}{1-\alpha_e-\alpha_s}\gamma_v. \tag{2.22}$$

In turn, using this in (2.20) and (2.21) gives

$$\gamma_e = \frac{1}{1-\alpha_e-\alpha_s}\gamma_z + \frac{1-\alpha_s}{1-\alpha_e-\alpha_s}\gamma_q + \frac{\alpha_s}{1-\alpha_e-\alpha_s}\gamma_v, \tag{2.23}$$

and

$$\gamma_s = \frac{1}{1-\alpha_e-\alpha_s}\gamma_z + \frac{\alpha_e}{1-\alpha_e-\alpha_s}\gamma_q + \frac{1-\alpha_e}{1-\alpha_e-\alpha_s}\gamma_v. \tag{2.24}$$

Equation (2.22) is the key for opening the door to growth accounting. Not surprisingly, the contribution of equipment-specific technological progress to economic growth will be larger the bigger is equipment's share of income, α_e, relative to that of the nonreproducable factors, $1-\alpha_e-\alpha_s$. The contribution of structure-specific technological progress to growth depends in a similar way on structure's share of income, α_s. Observe that stocks of equipment and structures grow at a faster rate than output, since $\alpha_e < 1-\alpha_s$ and $\alpha_s < 1-\alpha_e$.

Next, it is easy deduce from (2.15) and (2.7) that the factor prices r_e and w will grow at rates

$$\gamma_{r_e} = -\gamma_q,$$

and

$$\gamma_w = \gamma_y.$$

By using the above two conditions, in conjunction with (2.22), in (2.10), it is easy to show that, *when the stock of structures is held fixed*, profits on a building will rise over time at rate

$$\begin{aligned}\gamma_\pi &= \left(\frac{1-\alpha_e-\alpha_s-\beta}{1-\alpha_e-\beta}\right)\left(\frac{1}{1-\alpha_e-\alpha_s}\right)\gamma_z + \left(\frac{1-\alpha_e-\alpha_s-\beta}{1-\alpha_e-\beta}\right)\left(\frac{\alpha_e}{1-\alpha_e-\alpha_s}\right)\gamma_q \\ &\quad -\left(\frac{\beta}{1-\alpha_e-\beta}\right)\left(\frac{\alpha_s}{1-\alpha_e-\alpha_s}\right)\gamma_v \\ &< \gamma_y. \end{aligned} \tag{2.25}$$

Observe that profits grow at a rate less than output. This, together with rising maintenance costs, motivates the replacement of buildings. The location manager's replacement decision is driven by the lure of profits. For a given stock of structures, profits are forever being squeezed by rising labor costs. To increase these dwindling profits the manager must replace his old structure with a new and improved building.

Now, consider the economy's cross section of buildings at a point in time. It is easy to calculate from (2.10) that the percentage change in rents as a function of age, or the rent gradient δ_s, should be given by

$$\delta_s = -\frac{\alpha_s}{1-\alpha_e-\beta}\gamma_s, \tag{2.26}$$

since the stock of structures declines at rate γ_s as a function of age (while factor prices remain constant). This formula plays a starring role in the analysis. It is a measure of obsolescence in buildings. In the absence of depreciation, a new building rents for more than an old one only because it offers more efficiency units of structures. Figure 2.1 plots the rent gradient as derived from (2.10).[7] As can be seen, at a moment in time rents are a decreasing function of a building's age. The rent gradient shifts out over time due to growth in the economy.

Along a balanced growth path the profits of an age$-j$ building will grow at rate γ_y, a fact readily apparent from (2.10). Since T is constant it then follows that $V(k_{s,0}(0)) = e^{-\gamma_y T}V(k_{s,T}(0))$. Furthermore, note that $k_{s,0}(0)/v_0 = e^{-\gamma_y T}k_{s,T}(0)/v_T$. This allows the first-order condition (2.11) to be written as

$$[e^{\gamma_\pi T}\pi_0(0) - e^{(\gamma_\mu+\gamma_y)T}\mu(0)k_{s,0}(0)/v_0] - \iota e^{\gamma_y T}[V(k_{s,0}(0)) - k_{s,0}(0)/v_0] = 0, \tag{2.27}$$

where everything has now been expressed in terms of date-0 values. From P(2) it is easy to calculate that

$$\begin{aligned} V(k_{s,0}(0)) &= \frac{\pi_0(0)\int_0^T e^{-(\iota-\gamma_\pi)t}dt - \mu(0)[k_{s,0}(0)/v_0]\int_0^T e^{-(\iota-\gamma_\mu-\gamma_y)t}dt}{1-e^{-(\iota-\gamma_y)T}} \\ &\quad -\frac{e^{-(\iota-\gamma_y)T}k_{s,0}(0)/v_0}{1-e^{-(\iota-\gamma_y)T}} \\ &= \frac{\pi_0(0)[1-e^{-(\iota-\gamma_\pi)T}]/(\iota-\gamma_\pi)}{1-e^{-(\iota-\gamma_y)T}} - \frac{e^{-(\iota-\gamma_y)T}k_{s,0}(0)/v_0}{1-e^{-(\iota-\gamma_y)T}} \end{aligned} \tag{2.28}$$

[7] The figure uses the calibration discussed in Section 3.

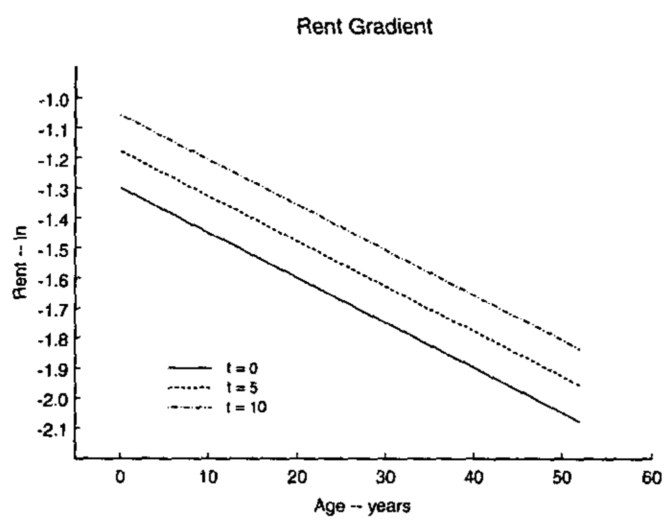
Rent Gradient
-1.0
-1.1
-1.2
-1.3
-1.4
-1.5
-1.6
-1.7
-1.8
-1.9
-2.0
-2.1
Rent -- ln
t = 0
t = 5
t = 10
0
10
20
30
40
50
60
Age -- years

Figure 2.1:

$$-\frac{\mu(0)[k_{s,0}(0)/v_0][1-e^{-(\iota-\gamma_\mu-\gamma_y)T}]/(\iota-\gamma_\mu-\gamma_y)}{1-e^{-(\iota-\gamma_y)T}}.$$

Likewise, from P(2) it is easy to see that in balanced growth

$$V_{k_s}(k_{s,0}(0)) = \frac{[1-e^{-(\iota-\gamma_\pi)T}]\pi_{k_s,0}(0)}{(\iota-\gamma_\pi)} - \frac{[1-e^{-(\iota-\gamma_\mu-\gamma_y)T}]\mu(0)}{(\iota-\gamma_\mu-\gamma_y)}, \tag{2.29}$$

where

$$\pi_{ks,0}(0) = \alpha_s z^{\frac{1}{1-\alpha_e-\beta}} \alpha_e^{\frac{\alpha_e}{1-\alpha_e-\beta}} \beta^{\frac{\beta}{1-\alpha_e-\beta}} r_{e,0}^{\frac{-\alpha_e}{1-\alpha_e-\beta}} w_0^{\frac{-\beta}{1-\alpha_e-\beta}} k_{s,0}(0)^{\frac{\alpha_e+\alpha_s+\beta-1}{1-\alpha_e-\beta}}, \tag{2.30}$$

so that the first-order condition will read

$$V_{k_s}(k_{s,0}(0)) = 1/v_0. \tag{2.31}$$

The model is almost complete except that the date-0 market clearing wage rate needs to be computed. The age distribution of structures over locations will be uniformly distributed on the interval $[0,T]$. The labor market clearing condition (2.17) can accordingly be rewritten as $(1/T)\int_0^T l(j)dj = 1$. Substituting (2.9) into this condition and using the fact that $k_{s,0}(j) = k_{s,0}(0)e^{-\gamma_s j}$ yields

$$\begin{aligned} w_0 &= [\beta^{1-\alpha_e}\alpha_e^{\alpha_e} z_0(\frac{1}{r_{e,0}})^{\alpha_e} k_{s,0}(0)^{\alpha_s}]^{1/(1-\alpha_e)} \\ &\quad \times [\frac{1-e^{-T\alpha_s\gamma_s/(1-\alpha_e-\beta)}}{T\alpha_s\gamma_s/(1-\alpha_e-\beta)}]^{(1-\alpha_e-\beta)/(1-\alpha_e)}. \end{aligned} \tag{2.32}$$

The solution to the model's balanced growth path is now completely characterized. To see this note that equations (2.10), (2.15), (2.19), (2.22), (2.24), and (2.27) to (2.32) represent a system of 11 equations in the 11 unknowns $\pi_0(0)$, ι, $r_{e,0}$, γ_y, γ_s, T, $V(k_{s,0}(0))$, $V_{k_s}(k_{s,0}(0))$, $\pi_{ks,0}(0)$, $k_{s,0}(0)$, and w_0.[8]

[8] Equations (2.10) and (2.15) at time zero read $\pi_0(j) = (1-\alpha_e-\beta)z_0^{\frac{1}{1-\alpha_e-\beta}} \alpha_e^{\frac{\alpha_e}{1-\alpha_e-\beta}} \beta^{\frac{\beta}{1-\alpha_e-\beta}} r_{e,0}^{\frac{-\alpha_e}{1-\alpha_e-\beta}} w_0^{\frac{-\beta}{1-\alpha_e-\beta}} k_{s,0}(j)^{\frac{\alpha_s}{1-\alpha_e-\beta}}$ and $r_{e,0} = (\iota+\delta_e+\gamma_q)/q_0$.

Last, it was stated that the replacement of structures was driven by the lure for profits. For a given stock of structures, profits are squeezed over time for two reasons: rising real wages and maintenance costs. To see the important role that profits play in replacement, assume that there are no profits, because production is governed by constant returns to scale, and that buildings can be maintained cost free. It is easy to deduce that in this situation structures will never be replaced.

Proposition 2.1. *(No Replacement): If* $\alpha_e + \alpha_s + \beta = 1$ *and* $\mu(0) = 0$ *then* $T = \infty$.

Proof. Observe that in this situation $\pi_{k_s,0}(0)k_{s,0}(0) = \pi_0(0)$, since $1-\alpha_e-\beta = \alpha_s$. Using (2.28) and (2.29) this then implies that $V_{k_s}(k_{s,0}(0))k_{s,0}(0) = V(k_{s,0}(0))$ so that $V(k_{s,0}(0)) = k_{s,0}(0)/v_0$. The right-hand side of (2.27) will therefore always be strictly positive so that there does not exist a finite T satisfying this equation. Hence, in balanced growth path it must transpire that $T = \infty$. ■

3. Measurement

3.1. Estimation

There are three parameters that need to be estimated — the rent gradient, δ_s, maintenance costs for newer buildings, ϕ, and the growth rate in maintenance costs, γ_μ. To do this, was obtained data from the Building Owners and Managers Association International (BOMA). The data used for the estimation is based on a panel covering approximately 200 office buildings across the United States from 1988 to 1996.[9] The data set includes information on age, location, size,

[9] This data was assembled by BOMA International with the names and addresses removed.

rent, and several categories of expenses. Clearly, office buildings are only part of the private sector's nonresidential stock of structures. So, hopefully the extent of technological progress in office buildings reflects the amount of technological advance in the broader aggregate. In any event, this is all of the data that could be found.

Summary statistics for the sample are given in Table 1. The average size building is 312,403 sq. ft.; the smallest being 15,683 sq. ft. and the largest 2,529,269 sq. ft. The oldest building was built in 1868 and the newest 1987. (Each building was in every year of the 9-year sample).

Table 1

Variable	Mean	Std. Dev.	Minimum	Maximum
size (square feet)	312,403	330,065	15,683	2,529,269
repair and main./s.f. (1996)	$1.42	$0.776	$0.353	$5.40
rent/s.f. (1996)	$15.23	$6.93	$1.52	$56.6
age	25.9	22.2	2	128
floors	16.0	13.4	2	80

Figure 3.1 plots the kernel estimate of (the ln of) rent per square foot as a function of age. As can be seen in the figure, the decline in rent is monotonic until the building is approximately 46 years old. Then there is a sharp increase, returning to a monotonic decline a few years later. This may reflect extensive remodelling or refurbishing of a building. To the extent that this is the case, it could in essence be considered a new or different building. Therefore, the estimating equation is based on a restricted sample of those buildings 46 years old or younger. Figure 3.2 plots the kernel estimate of (the ln of) repair and maintenance as a function of age. Repair and maintenance costs rise over time.

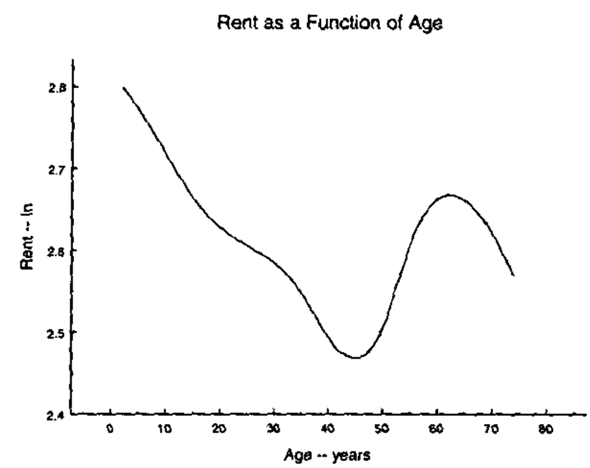

Figure 3.1:

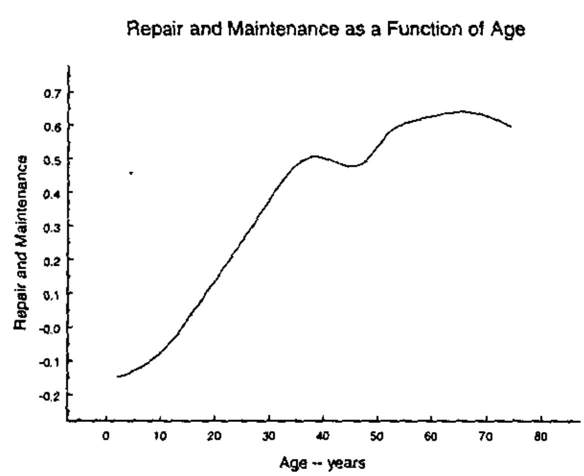

Figure 3.2:

Given that the data set contains observations on the same buildings over time, an obvious choice would be to use an estimator that controls for the building-specific effects. That is, a fixed effects estimator with buildings as the unit of observation. Now, the rent gradient is a measure of how the rent on a building changes as the building ages by one additional year. However, to determine the value for the rent gradient, δ_s, a building fixed effects estimator would essentially turn the age variable into a time trend, and would remove all of the cross section variation in age. This is because as one year passes the age of the building also increases by one year.

Therefore, the specification used consists of a cross section time series with the age of the building, the ln of repair and maintenance expenditures, total square footage, and dummy variables for time, region, and whether the building is located downtown or in the suburbs. If repair and maintenance expenditures counteract the effects of physical wear and tear, then the coefficient on age in the regression captures the effect of depreciation due to obsolescence alone. The results for δ_s are given in Table 2.

Table 2	
Dep. Var.: ln (Rent/ft)	
Variable*	Estimate
	(Std. Error)
constant	1.55
	(0.133)
age	-0.015
	(0.001)
size (sq. ft.)	0.092
	(0.011)
downtown	0.059
	(0.024)
r^2	.38
* plus time and regional dummies	

Table 3	
Dep. Var.: ln (Rep. and Maint./ft)	
Variable*	Estimate
	(Std. Error)
constant	-2.27
	(0.152)
age	0.020
	(0.001)
size (sq. ft.)	0.155
	(0.012)
downtown	0.111
	(0.028)
r^2	0.45
* plus time and regional dummies	

The value of ϕ was chosen to represent the ratio of repair and maintenance to rents in newer buildings. From the data the number for buildings 5 years old or younger was calculated. The value of ϕ is 0.055. To obtain a parameter estimate for γ_μ a similar approach was taken as for δ_s. The results are reported in Table 3.[10]

[10]The fact that expenditures on repair and maintenance begin to decline with age suggests that such expenditures may be endogenous, i.e., at some point less and less is spent, and the

3.2. Calibration

The model's parameters are assigned either (i) on the basis a priori information about their values or (ii) so that the model's balanced growth is consistent with certain features displayed in the U.S. data over the postwar period.[11]

3.2.1. A Priori Information

1. $\gamma_q = 0.032$. This number represents the average annual decline in the relative price of equipment price for the postwar period based on data taken from Gordon [2].[12]

2. $\delta_e = 0.12$. This is an estimate of average depreciation rate for equipment used in constructing NIPA's equipment stock figures.[13]

3. $\beta = 0.68$. Labor's share of income, as estimated from the NIPA for the period 1959-1996. Here labor income is defined as total compensation of employees in nominal terms. Income is taken to be nominal GDP minus nominal gross housing product.

4. $\phi = 0.055$. This is the estimate from Section 3.1

5. $\gamma_\mu = 0.020$. Again, as estimated from Section 3.1.

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building is allowed to deteriorate. Therefore, the estimation is based on the sample where expenditures are increasing, since the maintained assumption is that this is the amount it would take to keep the building in its original condition. Note that since expenditures are increasing over time it costs more each year to keep up the building.

[11] Cooley and Prescott [1] provide a guide to calibration within the context of the standard neoclassical growth model.

[12] As computed by Greenwood, Hercowitz and Krusell [3].

[13] Again, as calculated by Greenwood, Hercowitz and Krusell [3].

3.2.2. Restrictions on Balanced Growth

Parameter values still need to be determined for α_e, α_s, $\mu(0)$, γ_v, γ_z and ρ. These six parameters will be pinned down using six long-run restrictions from the data.

1. Over the postwar period 1959-1996 output-per-manhour worked in the U.S. economy has grown at an average rate of 1.22%. Again, nominal output is measured as nominal GDP minus nominal gross housing product. Since the numeraire in the model was consumptions goods, this series was divided through by the implicit price deflator for personal consumption expenditures on nondurables and nonhousing services. Total private sector manhours was calculated as an annual average of average weekly hours of total private production or nonsupervisory workers multiplied by the number of civilians employed. If the model is to be consistent with this fact, then

$$\gamma_y = 0.0122. \tag{3.1}$$

2. In the U.S. the equipment-investment-to-GDP ratio averaged 7.3% for the period 1959-1996. Using the law of motion for equipment it is easy to see that in balanced growth date-0 investment in equipment is given by $i_{e,0} = (\gamma_y + \gamma_q + \delta_e)k_{e,0}/q_0$. Now, from (2.8) it apparent that $k_{e,0} = \int n(j)k_{e,0}(j)dj = (\alpha_e w)/(\beta r_{e,0}) \int l(j)dj = (\alpha_e w)/(\beta r_{e,0})$ (since the supply of labor is one). Finally, date-0 GDP is given by w_0/β. Hence, the following restriction on the model's balanced growth path obtains:

$$\frac{i_e}{y} = \frac{\alpha_e}{r_{e,0}} = 0.073. \tag{3.2}$$

3. The ratio of structure investment to GDP in the U.S. economy is 4.1% (for the 1959-1996 sample period). If this restriction is imposed on the model then[14]
$$\frac{i_s}{y} = \frac{[k_s(0)/v_0]/T}{w_0/\beta} = 0.041. \tag{3.3}$$
In a world with investment-specific technological progress conventional measures of capital stocks are flawed since adjusting for quality improvements is difficult. Therefore, measures of $(k_e/q)/y$ and $(k_s/v)/y$ taken from NIPA are likely to be unreliable. Nominal investments, however, do not suffer from this problem so that i_e/y and i_s/y can be measured with reasonable accuracy.

4. The average age of buildings in the sample is 26 years. Now, recall that the lure of profits was a central factor in the firm's replacement decision. Thus, the returns to scale, as given by α_e and α_s, should be critical in determining T. The following restriction on the average age of buildings is added to the model's balanced growth path:
$$\frac{1}{T}\int_0^T j dj = \frac{T}{2} = 26. \tag{3.4}$$

5. In Section 3.1 it was found that the average ratio of repair and maintenance to rents in newer buildings is 0.055. This dictates the following condition on

[14]Repair and maintenance is netted out of GNP in the National Income and Product Accounts. Ideally this should be added back to GNP, because it is a type of investment spending. Since a series on repair and maintenance for structures wasn't available this couldn't be done here. Some sensitivity analysis showed that for reasonable estimates of repair and maintenance the results barely changed.

$\mu(0)$:

$$\mu(0) = 0.055 \times \frac{\pi_0(0)v_0}{k_{s,0}(0)}. \tag{3.5}$$

6. The estimation results from Section 3.1 show that a one year increase in a building's age reduces its rent by 1.5%. Recall that the rent gradient is a measure of obsolescence of structures. Hence, it should provide useful information for calculating γ_v. Using the rent gradient formula (2.26), together with (2.21), leads to the last restriction.

$$\gamma_v = -\frac{(1-\alpha_e-\beta)}{\alpha_s}\delta_s - \gamma_y = \frac{(1-\alpha_e-\beta)}{\alpha_s} \times 0.017 - \gamma_y. \tag{3.6}$$

Counting establishes that equations (2.10), (2.15), (2.19), (2.22), (2.24), (2.27) to (2.32), and (3.1) to (3.6) represent a system of 17 equations in the 17 unknowns $\pi_0(0)$, ι, $r_{e,0}$, γ_y, γ_s, T, $V(k_{s,0}(0))$, $V_{k_s}(k_{s,0}(0))$, $\pi_{ks,0}(0)$, $k_{s,0}(0)$, w_0, α_e, α_s, $\mu(0)$, γ_v, γ_z and ρ. The results will now be reported.

3.3. Findings

Values of 0.10 and 0.15 are found for α_e and α_s, respectively. This implies that $\alpha_e + \alpha_s + \beta = 0.93$, so pure rents (before maintenance costs) are about 7% of income. The rate of time preference, ρ, has a value of 0.072. This yields an interest rate of 8.4%, a number somewhat larger than that of 6.9% calculated by Cooley and Prescott [1] for the 1954-1992 period.[15] Cooley and Prescott's [1] number is probably too low for the purposes here, though, since they included the value of land in the definition of the physical capital stock which works to reduce their estimated return on capital.

[15] Note that 10% is the interest rate that Taubman and Rasche [5] used in their study of office buildings.

The rate of technological progress in structures is found to be 1% a year; that, is $\gamma_v = 0.01$. Consequently, a forgone unit of consumption can purchase 1% more efficiency units of structures each year. This is smaller than the 3.2% estimated for equipment, but casual empiricism suggests that technological progress has been much less in the building sector. The rate of neutral technological progress is 0.43%, or $\gamma_z = 0.0043$.

The contribution of each source of technological progress to economic growth can be calculated using (2.22) as follows:

$$f_q = \frac{\alpha_e/[1-\alpha_e-\alpha_s]\gamma_q}{\gamma_y} = 0.37,$$

$$f_v = \frac{\alpha_s/[1-\alpha_e-\alpha_s]\gamma_v}{\gamma_y} = 0.15,$$

and

$$f_z = \frac{1/[1-\alpha_e-\alpha_s]\gamma_z}{\gamma_y} = 0.48,$$

where f_q, f_v, and f_z denote the fractions of output growth that is accounted for by equipment-specific, structure-specific and neutral technological progress. As can be seen, structure-specific technological progress accounts for 15%. Overall investment-specific technological progress, or technological progress in the capital goods sectors, generates sixty percent of overall growth.

3.3.1. Capital Stock and Depreciation Measures

The numbers in the NIPA imply that the real stock of structures per manhour worked grew at an annual rate of 0.75% over the 1959-1996 period. The current analysis suggests, on the basis of equation (2.21), that it grew at 2.2% over this period. Likewise, the NIPA figures indicate that the annual growth rate in the stock of equipment per manhour worked was 2.5%. The estimate obtained from (2.20) is 4.42%. The failure to incorporate technological progress in the production of new capital goods, or neglecting the terms q, γ_q, v and γ_v in (2.4), (2.20), (2.5), and (2.21), has significant consequences for the measurement of the effective capital stock.

The numbers in the NIPA do not measure physical depreciation, as is conventionally assumed in macroeconomics. The NIPA measures are based on straight-line depreciation over the *economic* service life of an asset (and not its physical service life). Hotelling [4] introduced the concept of economic depreciation, defining it to be the rate of decline in the value of the asset over time. Let $\Pi_0(j)$ be the date-0 present value of rents (net of maintenance costs) for an age$-j$ building until the next replacement date $T - j$.[16] Now, imagine constructing an annual measure of depreciation. The annual rate of economic depreciation that transpires between year 0 and year -1 is simply given by $[\Pi_0(j) - \Pi_{-1}(j-1)]/\Pi_{-1}(j-1)$. The rate of straight-line depreciation would be $(1/T)/[1-(j-1)/T]$; note the importance of the replacement date, T, in this formula. Table 4 gives these depreciation rates for selected ages of a building. Observe how the rate of depreciation grows slowly at first and then accelerates rapidly toward the end of the building's life.[17]

[16]That is, $\Pi_0(j) = \int_0^{T-j}[\pi_t(j+t) - \mu(j+t)k_{s,-j}(0)/v_{-j}]e^{-\iota t}dt$. Assume that at the time of construction the owner purchases the structure and obtains a lease to the land for T years. The date-0 cost and benefit of doing this would be $\Pi_0(0)$.

[17]The rate of economic depreciation is very low early on. Taubman and Rasche [5] constructed a similar table to argue that tax laws allowed depreciation allowances that were too generous, because of this fact.

Table 4			
Age (years)	Econ. Dep., %	St. Line Dep., %	Physical Dep. %
1	1.89	1.92	0.83
5	1.97	2.08	0.94
15	2.29	2.63	1.30
30	3.55	4.35	2.10
40	6.52	7.69	2.90
50	32.1	33.3	4.00
51	49.1	50.00	4.13
52	100.00	100.00	4.26
Mean Dep.	6.63%	7.24%	1.93%

Note that the average rates of depreciation, both economic and straight line, are somewhat higher than the 5.6% used for structures in the NIPA.[18,19]

With an *additional* assumption, the rate of physical depreciation occurring over time can be calculated. Suppose that the stock of structures for an age-j building follows the law of motion $dk_s(j)/dj = -\delta(j)k_s(j) + v_{-j}i_m(j)$, where v_{-j} denotes value that v had j periods ago.[20] Here a dollar of repair and maintenance investment can offset one dollar of depreciation, where the latter is measured in terms of the original cost of the building. (Recall that the original cost for a

[18]Once again, as calculated by Greenwood, Hercowitz and Krusell [3].

[19]A weighted average is used to calculate the mean rates of depreciation in Table 4. The weights are based on the purchase prices for the buildings of various ages.

[20]This law of motion only holds when $k_s(j) < k_{s,-j}(0)$; that is, investment in repair and maintenance cannot be used to augment the scale of the original structure. Also, note that repair and maintenance is effected with the same level of efficiency, v_{-j}, as the original investment in structures was.

unit of structures was $1/v_{-j}$). Under the maintained hypothesis that repair and maintenance expenditures exactly offset physical wear and tear in each and every period it will transpire that $\delta(j) = v_{-j}i_m(j)/k_s(j) = v_{-j}i_m(j)/k_{s,-j}(0) = \mu(j)$. The physical depreciation rate is shown in the last column of Table 4. Note that different models of the depreciation process will lead to different estimates of the rate of physical depreciation. For a new (one-year old) building this is 0.8% while it steadily rises to 4.3% for an old (fifty-two year) structure. Observe that physical depreciation, measured this way, is considerably less than economic depreciation.

3.3.2. Statistical Robustness

The analysis hinges on the estimated value for the rent gradient. How sensitive are the results to this parameter? To answer this question note that the model defines two mappings Γ_v and F_v such that $\gamma_v = \Gamma_v(\delta_s)$ and $f_v = F_v(\delta_s)$; that is, the model returns values for the rate of structure-specific technological progress and its contribution to economic growth, given an estimate for the rent gradient. It turns out that (numerically) these mappings are monotonically decreasing in δ_s. In other words, the steeper the rent gradient is (or the smaller is δ_s), the faster is the pace of structure-specific technological progress and the larger is its contribution to growth. Now, in Bayesian fashion suppose that one has some beliefs about the value of δ_s, as summarized by a probability distribution. This will imply some associated beliefs about the rate of structure-specific technological progress and its contribution to growth. Specifically, $\Pr[\gamma_v \geq x] = \Pr[\delta_s \leq \Gamma_v^{-1}(x)]$ and $\Pr[f_v \geq x] = \Pr[\delta_s \leq F_v^{-1}(x)]$. The estimate of the rent gradient is a normally distributed random variable with mean 0.015 and standard deviation 0.001. Take this for the belief over δ_s. Given this belief, what do the distributions

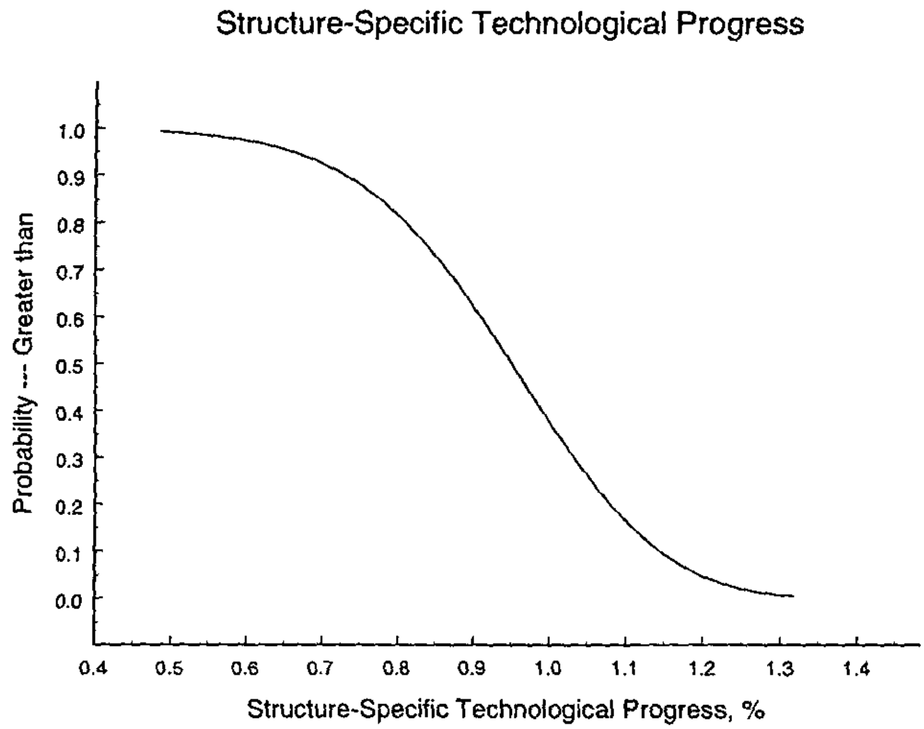

Figure 3.3:

for $\Pr[\gamma_v \geq x]$ and $\Pr[f_v \geq x]$ look like?

Figures 3.3 and 3.4 plot the distributions for $\Pr[\gamma_v \geq x]$ and $\Pr[f_v \geq x]$. As can be seen from Figure 3.3, the probability that structure-specific technological progress is greater than 0.50% is very high. But it is almost certainly true too that it is less than equipment-specific technological progress. Likewise, Figure 3.4 shows the odds that structure-specific progress accounts for at least 10% of growth are excellent; yet, that it contributes more than 20% to growth looks remote.

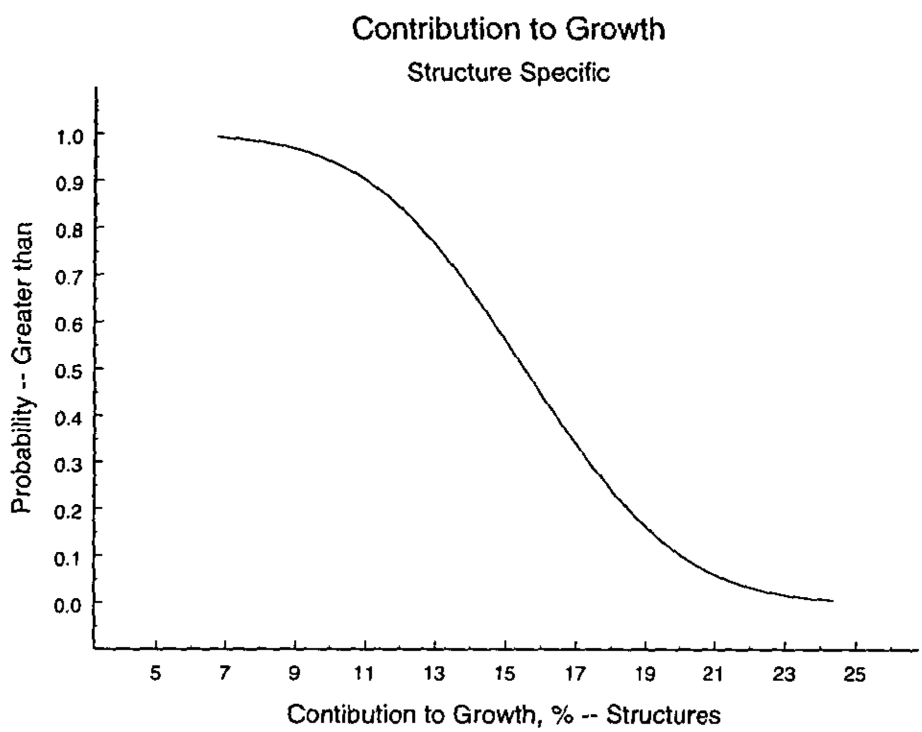
Contribution to Growth
Structure Specific
Probability -- Greater than
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
5
7
9
11
13
15
17
19
21
23
25
Contibution to Growth, % -- Structures

Figure 3.4:

4. Conclusion

The analysis here takes a different route to measuring technological progress than the one typically travelled by growth accountants. Price data is used to shed information on the sources of economic growth. Over the postwar period, the relative price of equipment has fallen dramatically. This suggests that there is technological advance in the equipment-producing sector of the economy. Similarly, rents decline with the age of a building, holding fixed factors such as repair and maintenance. Perhaps this is because new buildings embody new and improved technology in their structures. By casting the analysis in a general equilibrium setting, a link can be established between the observed rent gradient and the rate of technological progress in buildings. Likewise, the tie between the decline in the relative price of equipment and equipment-specific technological progress is made explicit. Similarly, the connection can be derived between, on the one hand, the observed data on the average age of structures, the structures investment-to-GDP ratio and the equipment investment-to-GDP ratio, and on the other one hand, the implied shares of structures and equipment in GDP and the interest rate.

The upshot of the analysis is that the rate of structure-specific technological progress is about 1% a year. This implies that 15% of economic growth can be attributed to structure-specific technological progress. Given that it is also found that equipment-specific technological progress accounts for 37% of growth, the conclusion is that about 52% of economic growth is due to technological progress embodied in the form of new capital goods.

The National Income and Product Accounts compute the rate of economic depreciation for capital goods, and not the rate of physical depreciation as is typically assumed. The current analysis assumes that structures are kept in good condition through repair and maintenance. This assumption may not be that unrealistic. Building codes, for instance, regulate the condition of business structures. Due to technological progress buildings eventually become obsolete, however, and are replaced. The model generated a rate of economic depreciation of about 6.6%, not far from the 5.6% used in the NIPA. Future work may be better able to decompose the rate of economic depreciation into the rate of obsolescence and the rate of physical depreciation.

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