Monetary Policy in a Channel System

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EONIA - Euro OverNight Index Average
Source: ECB
Features of “pure” channel systems

- Standing facilities
- All CB loans are secured with collateral (typically REPOS)
- Few or no open market operations
- Money market allocates reserves; reserves management
This Paper

- ... is on the optimal design of the MP implementation framework, given the CB uses a "pure" channel system.

- ...is not on the optimal monetary policy response to shocks.
Fed Fund Rate - 8 September - 28 October 2008
(Source: BOG)

Eonia - 8 September - 28 October 2008
(source EBA and ECB)
Objectives

- Optimal interest-rate corridor.
- Shift of corridor vs. changing the size.
- Implications of collateral requirements for the optimal policy.
- Steering money market rates without open market operations.
Environment

- Based on current CB practice as much as possible.

- Version of Lagos and Wright (2005).

- Time discrete and infinite.

- $[0, 1]$ continuum of $\infty$-lived agents (banks/households). Anonymity.

- Walrasian markets open/close sequentially, in each $t$. 
Environment

Settlement market: Settle claims by trading a general good. Adjust money and collateral holdings.

Money market: Signals on liquidity needs. Borrow/lend money.

Goods market (liquidity shock):

- Produce with probability $n$ at costs $c(q_s) = q_s$
- Consume with probability $1 - n$ and get $u(q)$

Standing facilities:
Open before and after the goods market.

- Borrow from lending facility against collateral at rate $i_l$,
- Deposit money at rate $i_d$
Collateral

- General goods can be ‘stored’ at the CB.
- Return in $t + 1$ is $R \geq 1$ with $\beta R < 1$.
- $1 + r = 1/\beta$ implies $R < 1 + r$. 
Benchmark: First Best Allocation

Expected lifetime utility of a representative agent

\[(1 - \beta)W = (1 - n)[u(q) - q] + (\beta R - 1)b\]

First best allocation \((q^*, b^*)\), where:

\[u'(q^*) = 1, \text{ and } b^* = 0.\]

Decentralization:
Anonymity implies money is necessary.
Money

- Central bank prints/burns paper money at no cost. Fiat.
- CB has no fiscal authority. No lump-sum transfers.

- Endogenous growth rate

\[ M_t = M_{t-1} - (1 - n)i_L L_{t-1} + ni_d D_{t-1}. \]

- Stationary equilibrium

\[ \phi M = \phi_{+1} M_{+1} \text{ and denote } \gamma = \frac{M_{+1}}{M} \]
No Money Market
(signal totally uninformative)
Symmetric Stationary Equilibrium

Settlement stage:

\[ W(m_{-1}, b_{-1}, \ell, d) = \max_{h,m,b} -h + V(m, b) \]

subject to \( \phi m + b = h + \phi m_{-1} + Rb_{-1} + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell. \)

First-order conditions \((m, b = \bar{m}, \bar{b} \text{ for all agents})\)

\[ V_m \leq \phi \ (= \text{if } m > 0) \tag{1} \]
\[ V_b \leq 1 \ (= \text{if } b > 0) \tag{2} \]

Envelope conditions

\[ W_m = \phi; W_b = R; W_\ell = -\phi(1 + i_\ell); W_d = \phi(1 + i_d). \]
Equilibrium

Goods market:

\[ V(m, b) = (1 - n)V^b(m, b) + nV^s(m, b) \]

Sellers' problem

\[ V^s(m, b) = \max_{q_s} \left\{ -q_s + \beta Rb + \beta \phi_+ (m + pq_s) (1 + i_d) \right\} \]
\[ + \beta \left[ V(\bar{m}, \bar{b}) - \phi_+ \bar{m} - \bar{b} \right] \]

\[ \text{FOC: } p \beta \phi_+(1 + i_d) = 1 \] (3)
Equilibrium

Buyers’ problem

\[ V^b (m, b) = \max_{q, \ell} \left\{ u(q) + \beta R b + \beta \phi_+ (m + \ell - pq) (1 + i_d) \right \} \]

\[ \quad - \beta \phi_+ \ell (1 + i_\ell) \]

\[ + \beta \left[ V (\bar{m}, \bar{b}) - \phi_+ \bar{m} - \bar{b} \right] \]

s.t. \hspace{1cm} pq \leq m + \ell \hspace{0.5cm} \text{and} \hspace{0.5cm} \phi_+ \ell (1 + i_\ell) \leq R b

FOC: \hspace{1cm} u'(q) = \beta p \phi_+ (1 + i_\ell) + \lambda_\ell \tag{4}
Equilibrium

**Marginal value of money** in the good market:

\[ \phi \geq V_m = (1 - n)u'(q) / p + n(1 + i_d) \beta \phi + 1 \]  

(5)

\[ \frac{\gamma / \beta - (1 + i_d)}{(1 + i_d)} \geq (1 - n) [u'(q) - 1] \]  

(6)

**Marginal value of collateral** in the good market:

\[ 1 \geq V_b = (1 - n) \lambda \beta R / (1 + i_\ell) + \beta R \]  

(7)

\[ \frac{1 / \beta - R}{R} \geq (1 - n) [u'(q) / \Delta - 1] \]  

(8)

\[ \Delta = (1 + i_\ell) / (1 + i_d). \]
Equilibrium

Definition
A symmetric stationary monetary equilibrium is a list $(\gamma, q, z_\ell, z_m, b)$ satisfying (9)-(13) with $z_\ell \geq 0$ and $z_m \geq 0$.

\begin{align*}
\frac{1/\beta - R}{R} &\geq (1 - n) \left[ u'(q) / \Delta - 1 \right] \\
\frac{\gamma / \beta - (1 + i_d)}{(1 + i_d)} &\geq (1 - n) \left[ u'(q) - 1 \right] \\
\gamma &= 1 + i_d - (1 - n)(i_\ell - i_d) \frac{z_\ell}{z_m} , \\
q &= z_m + z_\ell \\
z_\ell &= \beta R b / \Delta
\end{align*}
Equilibrium

Proposition

For any $\Delta \geq 1$ there exists a unique symmetric stationary equilibrium such that

\[
\begin{align*}
z_\ell &> 0 \text{ and } z_m = 0 \quad \text{if and only if} \quad \Delta = 1 \\
z_\ell &> 0 \text{ and } z_m > 0 \quad \text{if and only if} \quad 1 < \Delta < \tilde{\Delta} \\
z_\ell &= 0 \text{ and } z_m > 0 \quad \text{if and only if} \quad \Delta \geq \tilde{\Delta}.
\end{align*}
\]

where

\[
\tilde{\Delta} = \frac{1 - n\beta}{1/R - n\beta} \quad \text{and} \quad \Delta = \frac{1 + i_\ell}{1 + i_d}.
\]
Optimal Policy

Equilibrium with a positive amount of collateral $1 \leq \Delta < \tilde{\Delta}$.
This defines constraints on $q$:
- $\hat{q}$ is the level of consumption when $\Delta = 1$.
- $\tilde{q}$ is the level of consumption when $\Delta > \tilde{\Delta}$

The central bank’s problem is

$$\max_{q,b} \quad (1 - n) \left[ u(q) - q \right] + (\beta R - 1) b$$

subject to

$$q = \beta b R F \left( \frac{R \beta (1 - n) u'(q)}{1 - n R \beta} \right)$$

$$\hat{q} \geq q \geq \tilde{q}$$
Optimal Policy

Proposition

There exists a critical value $\bar{R}$ such that if $R < \bar{R}$, then the optimal policy is $\Delta \geq \tilde{\Delta}$. Otherwise the optimal policy is $\Delta \in (1, \tilde{\Delta})$.

- Since $\beta R < 1$ it is never optimal to set a zero band.

\[
\gamma = 1 + i_d - (1 - n)(i_\ell - i_d)\ell / m
\]

- Set $i_d = i_\ell$ (0 corridor) so that $\gamma = 1 + i_d$. Return on cash is $\beta / \gamma$.
  Return on collateral is $\beta R > \beta / \gamma$, use only collateral, no money.
  Collateral is socially inefficient.

- Rather, set $i_\ell > i_d$: Makes borrowing less attractive, reduce inflation, holding money more attractive.
Money Market
(signal contains some information)
Record Keeping

- Operated by the CB. Identifies participants and verifies collateral.
- Cannot keep record of goods market transactions.
Trade on the Money Market

Two Types: $H$ (likely to be seller) and $L$ (likely to be buyer)

- $H$-types lend money / $L$-types borrow money
- $\sigma^k$: probability of $k$-type.
- $n^k$: probability that a $k$-type turns seller.
Agents’ Problems

essentially the same as before, except

- Short selling constraints on the money market:

\[ \phi_+ y^k (1 + i_m) \leq Rb \] and \[ m + y^k \geq 0. \]

- Borrowing constraint in the goods market:

\[ \ell^k \leq \bar{\ell}^k \equiv \frac{Rb}{\phi_+ (1 + i_\ell)} - y^k \frac{(1 + i_m)}{(1 + i_\ell)} \]
Stationary equilibrium is determined by

\[ \hat{\Delta} = \frac{1 + i_\ell}{1 + i_m} \quad \text{and} \quad \Delta = \frac{1 + i_\ell}{1 + i_d} \]
Equilibrium

Short-selling constraints are nonbinding, then

$$\hat{\Delta} = \frac{\Delta}{n\beta R (1 - \Delta) + \Delta}$$  \hfill (14)

$$u'(q^k) = \frac{n^k}{1 - n^k} \frac{1 - n\beta R}{n\beta R}, \quad k = H, L.$$  \hfill (15)

Definition

A symmetric stationary equilibrium where no short-selling constraint is binding in the money market is a time-invariant list \((\hat{\Delta}, q^L, q^H)\) satisfying (14) - (15) with \(b \geq 0, z^L < \beta R b \hat{\Delta}/\Delta\) and \(z^H > -z_m\).
Equilibrium

Let $n^H - n^L = \epsilon$. So that $n^H = n + \sigma^L \epsilon$ and $n^L = n - \sigma^H \epsilon$.

Proposition

For any $1 < \Delta < \tilde{\Delta}$ there exists a critical value $\epsilon_1 > 0$ such that if $\epsilon < \epsilon_1$ a symmetric monetary equilibrium exists where no short-selling constraint in the money market binds.
Results

1) Money market rate and the corridor

\[ \hat{\Delta} = \frac{\Delta}{n \beta R (1 - \Delta) + \Delta} \]

\[ i_m = i_\ell - n \beta R (i_\ell - i_d) \]

- If \( n = 1/2 \) and \( \beta R \to 1 \), then \( i_m \to (i_\ell + i_d)/2 \).
- If \( n = 1/2 \) and \( \beta R < 1 \), then \( i_m > (i_\ell + i_d)/2 \).

2) Collateral requirement

\[ u'(q^k) = \frac{n^k}{1 - n^k \Delta} \frac{1 - n \beta R}{n \beta R} \]

- Collateral modifies the real allocation.
3) **Need to specify a corridor rule**

- Symmetric increase of the corridor width leaves \( i_m \) constant but *has* real effects:

\[
u'(q^k) = \frac{n^k}{1 - n^k} \Delta \frac{1 - n\beta R}{n\beta R}\]

- Need to specify a corridor rule as well as an interest rate rule.
Summary and final remarks

- Details of implementation framework matter.

- The more costly the collateral, the larger the band optimally. The least costly the collateral, $i_m \rightarrow (i_\ell + i_d)/2$.

- Shifting the corridor $\delta = i_\ell - i_d$ up increases the money market rate $i_m$.

- It does not matter whether the deposit rate is set to zero (i.e. deposits are not allowed).
EONIA - Euro OverNight Index Average
and Eurepo - reference rate for the Euro GC repo market

Source: European Banking Federation and ECB