An Equilibrium Theory of Learning, Search and Wages

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1. Tasks and Motivation

- Formulate the problem of learning by unemployed workers about themselves
- characterize equilibrium with such learning
- examine how reemployment wages and rates depend on search history

What's the story?

- workers do not know their ability/productivity
  - some are lucky to find jobs $\Longrightarrow$ revise beliefs upward
  - some are not so lucky $\Longrightarrow$ discouragement

- divergence in histories $\Longrightarrow$ **endogenous heterogeneity** in:
  - workers' beliefs about their job-finding process
  - search decisions
  - job-finding rates and wages

Specific facts:

- average job-finding prob decreases with duration

- wage losses increase with unemployment duration:
  - US Displaced Worker Survey (Addison and Portugal 89): increasing duration by 100% reduces wages by 10%

  - UK Labour Force Survey (Gregg and Wadsworth 00): duration of 7-12 months $\Longrightarrow$ wage loss of 27 log points

Other complementary theories:

- unobserved worker heterogeneity,
  and long duration is a signal of low productivity

- skill depreciation during unemployment

- declining wealth/benefit during unemployment

Why use an equilibrium?

- need to explain the above facts as market outcomes

- firms can adjust offers and vacancies to respond to learning:
  - with exogenous wages, low-wage jobs would be filled more quickly as reservation wages fall

2. Model Environment

Workers and jobs:

- firms or jobs: free entry

- workers (risk neutral):
  - unemployed workers search
  - employed workers produce $y > 0$, shock of separation into unemployment: δ
  - shock of exit from market: σ

Worker's unknown ability:

- new worker draws ability $i \in \{H, L\}$:
  - unknown, permanent, $prob(H) = p$

- worker's productivity is a random variable:

$$\begin{cases} y > 0, & \text{prob } a_i \\ 0, & \text{prob } 1 - a_i \end{cases}$$

  - H is more "productive": $0 < a_L < a_H < 1$
  - realized immediately after contact

Directed search:

- continuum of submarkets $x \in X = [0, 1/a_H]$

  x: prob of getting productive match (per search unit);

  $W(x)$: wage level; $\lambda(x)$: tightness

- search choice:
  a submarket x to enter (tradeoff between x and W)

Matching in submarket x:

- total number of (productive) matches: $F(u_e(x), v(x))$

- total productive units of search in submarket x:

$$u_e(x) = a_H \times u_H(x) + a_L \times u_L(x)$$

$u_i(x)$: # of type-i workers in submarket x

Matching in submarket x (continued):

- matching probability per productive (search) unit:

$$x = \frac{F(u_e(x), v(x))}{u_e(x)} = F(1, \underbrace{\frac{v(x)}{u_e(x)}}_{\text{tightness } \lambda(x)})$$

- matching probabilities for participants:

type-H	type-L	vacancy
$a_H\ x$	$a_L\ x$	$\frac{F}{v} = \frac{x}{\lambda(x)}$

Wage function $W(x)$:

- free-entry of vacancies:

$$J_v(x) \leq 0 \quad \text{and} \quad v(x) \geq 0 \quad \text{for all } x \in X$$

with complementary slackness

- firm's expected profit of vacancy in market x:

$$J_v(x) = -c \ + \ \frac{x}{\lambda(x)} \times (1 - \sigma) \times J_f(W(x))$$

- firm's value of employing worker at w, discounted to the end of previous period:

$$J_f(w) = \frac{1}{1+r} \left[y - w \ + \ (1 - \sigma) \times (1 - \delta) \times J_f(w) \right]$$

Wage function $W(x)$:

- free entry implies wage function:

$$W(x) = y - cA \, \frac{\lambda(x)}{x}, \qquad A \equiv \delta + \frac{r+\sigma}{1-\sigma}$$

- $W'(x) < 0$ (tradeoff between W and matching prob x)

3. Learning in directed search equilibrium

Information and learning:

- match success and failure contain info about a_i
  - info content depends on x: $\quad a_L x < a_H x$
- firms do not face signal extraction:
  matching prob $x/\lambda(x)$ and wage $W(x)$ are known
- all participants know all statistics in all submarkets

Worker's beliefs: expected value of his a

- common initial belief: $\mu_0 = p\, a_H + (1-p)\, a_L$

- belief before search in a period: $\mu = P_H\, a_H + P_L\, a_L$

- posterior prob after search outcome:

$$P(a_i \mid x,\ \text{success}) = \frac{a_i\ x}{\mu\ x} P_i = \frac{a_i}{\mu} P_i$$

$$P(a_i \mid x,\ \text{failure}) = \frac{1 - x a_i}{1 - x\mu} P_i$$

Updating beliefs:

- beliefs before and after search:

$$\mu \longrightarrow \begin{cases} \mathbb{E}\left(a \mid x, \text{success}\right) = a_H + a_L - a_H a_L/\mu \ \equiv \phi(\mu) \\ \\ \mathbb{E}(a \mid x, \text{failure}) = a_H - \frac{1-xa_L}{1-x\mu}(a_H - \mu) \ \equiv H(x,\mu) \end{cases}$$

- properties of updating:
  - beliefs obey a Markov process
  - μ is sufficient statistic for search history
  - search in market with higher x is more informative

Rule out experimentation (sufficient condition):

$$\frac{y-b}{c} > [A + a_H x^*]\,\lambda'(x^*) - a_H \lambda(x^*)$$

$$x^* \text{ is defined by: } \lambda'(x^*) = a_H \lambda(\frac{1}{a_H})$$

Search decision of a worker with belief μ:

- value of being employed at wage w, discounted to the end of previous period:

$$J_e(\mu, w) = \frac{1}{1+r} \{w \; + \; (1-\sigma) \, [(1-\delta) \; J_e(\mu, w) \; + \; \delta \, V(\mu)]\}$$

- return to search in market x:

$$R(x, \mu) \equiv x\mu \; J_e(\phi(\mu), W(x)) \; + \; (1 - x\mu) \; V(H(x, \mu))$$

Search decision of a worker with belief μ (continued):

- search decision:

$$(1 + r)\ V(\mu) = b\ +\ (1 - \sigma) \times \max_{x \in X} R(x, \mu)$$

- policy functions:
  - search choice (of submarket): $x = g(\mu) \in G(\mu)$
  - desired wage: $w(\mu) = W(g(\mu))$

Stationary symmetric equilibrium:

- Block 1: individual decisions and market tightness

  (i) given $W(.)$, workers with belief μ choose $x = g(\mu) \in G(\mu)$

  (ii) workers update beliefs according to $\phi(\mu)$ and $H(g(\mu), \mu)$

  (iii) $W(.)$ satisfies free-entry condition

  (iv) consistency: $\lambda(x) = \frac{v(x)}{u_e(x)}$ for all x with $v(x) > 0$

- Block 2:

  (v) distribution of workers consistent with law of motion

Equilibrium is block recursive (as in Shi 09)

4. Monotonicity of desired wages

Want to show:

- policy function, $w(\mu) = W(g(\mu))$, is strictly increasing
  - i.e., wages fall as beliefs deteriorate
  - i.e., search decision $x = g(\mu)$ strictly decreases in μ

Problems:

- value $V(\mu)$ is convex;
- $V'(\mu)$ may not exist; FOC may not be applicable

A map of our approach:

- use lattice-theoretic methods to prove: policy function is monotone

- monotone policy function + convex value function $\Longrightarrow$ validate first-order condition

- the above results + first principles of calculus $\Longrightarrow$ envelope condition + differentiability of V

Topkis' Theorems (98):

$$\max_{z \in -X} \; f(z,\mu), \qquad z = -x; \; \mu \in M$$

- If f is supermodular in (z,μ),
  (and if $(-X) \times M$ is a lattice),
  then $\max Z(\mu)$ and $\min Z(\mu)$ are increasing in μ

- If f is strictly supermodular in (z,μ),
  then every selection $z(\mu) \in Z(\mu)$ is increasing in μ.

Use lattice-theoretic techniques:

- transform payoff function:

$$\hat{R}(z, \mu) \equiv \frac{R(x, \mu)}{\mu}, \qquad z \equiv -x$$

- optimal search decision $z(\mu) \in Z(\mu)$:

$$(1 + r)\, V(\mu) = b + (1 - \sigma) \times \mu \times \max_{z \in -X}\ \hat{R}\,(z, \mu)$$

Theorem 4.1: monotonicity of desired wages

Assume separation rate satisfies $0 \leq \delta \leq \bar{\delta}$. Then

- $\hat{R}(z, \mu)$ is strictly supermodular in (z, μ)

- every selection $z(\mu) \in Z(\mu)$ is an increasing function; every selection $x = g(\mu)$ is a decreasing function

- $w(\mu) = W\left(g\left(\mu\right)\right)$ is an increasing function

Why is $\hat{R}(z, \mu)$ strictly supermodular?

$$\mu \begin{cases} & \qquad\qquad\qquad\qquad \text{expected value:} \\ \xrightarrow{\text{prob. } x\mu} \quad \phi(\mu); & \delta \times x\mu\phi(\mu) \ + \mu x W(x) \\ \\ \xrightarrow{\text{prob. } (1-x\mu)} \quad H(x,\mu); & m \equiv (1-x\mu)H(x,\mu) \end{cases}$$

High x submarkets have low wages:

- failure in higher $x \implies$ deeper discouragement: $\frac{\partial m}{\partial x} < 0$
- marginal "damage" of x increases in μ: $\frac{\partial}{\partial \mu}\left[\frac{\partial m}{\partial x}\right] < 0$

Why is $\hat{R}(z,\mu)$ strictly supermodular? (cont'd)

- convexity of V is important:
  properties above carry over to payoff only for convex V:

$$\hat{R} = \underbrace{\frac{-z\,W(-z)}{A(1-\sigma)} - \frac{\delta}{A}\,z\,V(\phi(\mu))}_{\text{expected payoff to success}} + \underbrace{(\frac{1}{\mu} + z)\,V(H(z,\mu))}_{\text{to failure}}$$

- assumption $\delta \leq \bar{\delta}$ is needed

Theorem 4.1 (continued): strict monotonicity

Assume $0 < \delta \leq \bar{\delta}$. Statements below are equivalent:

- (i) $V(\mu)$ is **strictly** convex for all μ
- (ii) every selection $z(\mu) \in Z(\mu)$ is **strictly** increasing
- (iii) corner $z = -1/a_H$ is not optimal for any $\mu > a_L$
- (iv) corner $z = -1/a_H$ is not optimal for $\mu = a_H$
- (v) $\frac{y-b}{c} < (A+1)\lambda'(\frac{1}{a_H}) - a_H\lambda(\frac{1}{a_H})$

Why linear V over some beliefs $\Longrightarrow$
even most optimistic workers search for lowest wage?

- $V(\mu)$ being linear in $[\mu_a, \mu_b]$
$\Longrightarrow$ decision problem is strictly concave for such μ
$\Longrightarrow$ optimal choice of z is unique for such μ

- strict supermodularity of $\hat{R}$
$\Longrightarrow$ monotonicity of optimal decisions
$\Longrightarrow$ unique maximizer is corner, $\{-1/a_H\}$, for such μ

Why linear V over some beliefs $\Longrightarrow$
even most optimistic workers search for lowest wage?

- $V(\mu)$ linear in $[\mu_a, \mu_b] \Longrightarrow$ unique maximizer is $\{-1/a_H\}$

- Same argument applies to $\mu \in \left[\phi^i(\mu_a), \phi^i(\mu_b)\right]$, $i \geq 1$: unique maximizer is $\{-1/a_H\}$ for all such μ

- $\lim_{i \to \infty} \phi^i(\mu) \to a_H$,
and $Z(\mu)$ is upper hemicontinuous
$\Longrightarrow \{-1/a_H\} \in Z(a_H)$.

5. Uniqueness and differentiability

Theorem 5.1:

- optimal choices obey first-order condition
- generalized envelope theorem holds
- from the point where a worker has a match failure,
  - value function is differentiable
  - optimal choice is unique

Why is V differentiable at a match failure?

- Suppose V not differentiable at $\mu_{\mathcal{T}+1} = H(z(\mu_{\mathcal{T}}), \mu_{\mathcal{T}})$

  $\implies$ multiple choices will be optimal in $(\tau + 1)$,

  $\implies V'(\mu^{-}_{\mathcal{T}+1}) < V'\left(\mu^{+}_{\mathcal{T}+1}\right)$

- worker can gain by raising z slightly above $z\left(\mu_{\mathcal{T}}\right)$ (i.e., searching in submarket with slightly higher w)
  - next period beliefs slightly above $\mu_{\mathcal{T}+1}$
  - marginal benefit increases by a discrete amount
  - matching prob decreases continuously

5. Implications

- unemployment duration $\Longrightarrow$ wage losses, discouragement

- wage dispersion among identical workers

- what about average job-finding prob in a cohort?
  - searching for easier jobs increases job-finding
  - but average ability in a cohort deteriorates with duration

Implications (continued):

- reemployment and wages depend on **entire** history: past occurrences of unemployment, past spells, etc.

- history can be summarized by beliefs entering unemployment and, hence, by worker's pre-unemployment wage

- even without skill differences, higher pre-unemp wage
  - increases reemployment wages;
  - may induce longer duration

6. Conclusion

- tractable **equilibrium theory of learning:**
  block recursivity;   lattice-theoretic in dynamic prog

- discouragement during search:
  longer search $\Longrightarrow$ more pessimistic $\Longrightarrow$ wage losses

- endogenous heterogeneity useful for:
  understanding wage formation, duration dependence, etc.

- learning + aggregate fluctuations?