

# Financial Markets and Fluctuations in Uncertainty

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# Motivation

- Recent recession has featured
  - ▶ Large contraction in output
  - ▶ Substantial increase in the dispersion of firms' growth
- Most of the recent output downturn accounted for
  - ▶ By a worsening of the labor wedge
  - ▶ Not by fall in TFP

# Goal

- Develop a model with imperfect financial markets that connects fluctuations in firm volatility to aggregate fluctuations
- Ask: Can an increase in volatility of firms' idiosyncratic shocks that generates observed increase in firms' dispersion deliver
  - ▶ Large contraction?
  - ▶ Large worsening of labor wedge?
- Today focus on current recession

# Key Elements in Model

- Firms

- ▶ Choose their scale in advance
- ▶ Issue debt uncontingent on their idiosyncratic shock and can default
- ▶ Firms pay an entry cost so ongoing firms have positive future expected profits

- Shocks

- ▶ Common shocks to the volatility of firms' idiosyncratic productivity

# Role of key elements

- Choose scale in advance
  - ▶ One scale for all states
  - ▶ In high states 'too small' and in low states 'too big'
- Uncontingent/unenforceable debt
  - ▶ If too big, might default
- Entry cost
  - ▶ In equilibrium generates costs of default

⇒ Trade-off between short-term profits and the risk of costly liquidation

# From firm volatility to aggregates

- Model: Trade-off between profits and liquidation
- Mechanism: High volatility mainly increases risk of costly liquidation
  - ▶ Firms reduce scale and output
  - ▶ Labor wedge worsens because MPL deviates more from wage

# Our answers for current recession

Can an increase in volatility of firms' idiosyncratic shocks that generates observed increase in firms' dispersion deliver

- Large contraction?
  - ▶ Model accounts for 2/3 of the output decline
- Large worsening of labor wedge?
  - ▶ The labor wedge falls by 18% in model and 15% in data

# Model

- Dynamic model of heterogeneous firms and identical households
- Households provide labor services and trade assets
- Firms use DRS technology with labor input  $\ell$  and issue debt  $b'(\sigma')$
- Firms' idiosyncratic productivity shocks  $z$  have common stochastic volatility  $\sigma$

# Firms

- Stochastic structure:

$$\log z_t = \rho_z \log z_{t-1} + \sigma_t \varepsilon_t$$

$$\log \sigma_t = (1 - \rho_\sigma) \log \mu_\sigma + \rho_\sigma \log \sigma_{t-1} + \eta_t$$

- Individual states:  $(\ell, b, z)$
- Aggregate states:  $S = (\sigma, Y)$ , where  $Y$  is measure over individual states

# Firms

- Choose  $b'(\sigma')$  and  $\ell'$  to maximize present value of dividends

$$d = z\ell^\theta - w\ell - b + \sum_{\sigma'} q'(\sigma'|.)b'(\sigma')$$

$q(\sigma'|.)$  depends on firms choices and aggregate states

- Free entry condition given fixed cost of entry  $\tilde{\zeta}$

$$\tilde{\zeta} = E_{z',\sigma'} Q(\sigma'|S) V'(\ell', b', z', S')$$

After entry the expected value is positive

- Cost of default: Firm exits so lose expected value of future profits

# Firms

- Firms must have non-negative dividends
- Debt schedule  $q(\sigma'|\cdot)$  compensates for loss in case of default
  - ▶ Schedule contains 'borrowing limits'
- For high enough debt due, firms must default:
  - ▶ Default if

$$z l^\theta - w l - b + \max \left\{ \sum_{\sigma'} q(\sigma'|\cdot) b'(\sigma') \right\} < 0$$

# Households

- Identical households with standard problem
- Choose  $c$ , and  $h$  to maximize present value of utility, where

$$u(c, h) = \log(c) - \chi \frac{h^{1+\nu}}{1+\nu}$$

# Simple Example

- Two period problem
- Firm loses exogenous future value  $V$  if liquidates

# Complete financial markets

$$\max_{\ell} \int_0^{\infty} [z\ell^{\theta} - w\ell] \phi(z) dz + V$$

Optimal scale chosen to maximize short term profits:

$$\theta \ell^{\theta-1} E(z) = w$$

Increasing volatility while preserving  $E(z)$  does not change optimal scale

## No financial markets

Without financial markets firms liquidate in low states ( $z < \hat{z}$ )

$$\max_{\ell, \hat{z}} \int_{\hat{z}}^{\infty} [z\ell^{\alpha} - w\ell]\phi(z) dz + \int_{\hat{z}}^{\infty} V\phi(z) dz$$

subject to

$$\hat{z}\ell^{\alpha} - w\ell = 0$$

- $d\hat{z}/d\ell > 0$  so higher  $\ell$  implies higher  $\hat{z}$  which generates:
  - ▶ Higher short term profits
  - ▶ Lower future value
  
- Optimal scale chosen to maximize short term profits *and* future value

# No financial markets

Optimal scale:

$$\theta \ell^{\theta-1} \frac{E(z|z \geq \hat{z})}{1 - \Phi(\hat{z})} = w + V \frac{\phi(\hat{z})}{1 - \Phi(\hat{z})} \frac{d\hat{z}}{d\ell}$$

- Marginal cost of labor equals wage plus loss in future value
- When  $V$  is high enough:
  - ▶ scale is smaller than with frictionless financial markets
  - ▶ marginal product of labor is larger than wage  $\implies$  labor wedge

## Increasing volatility

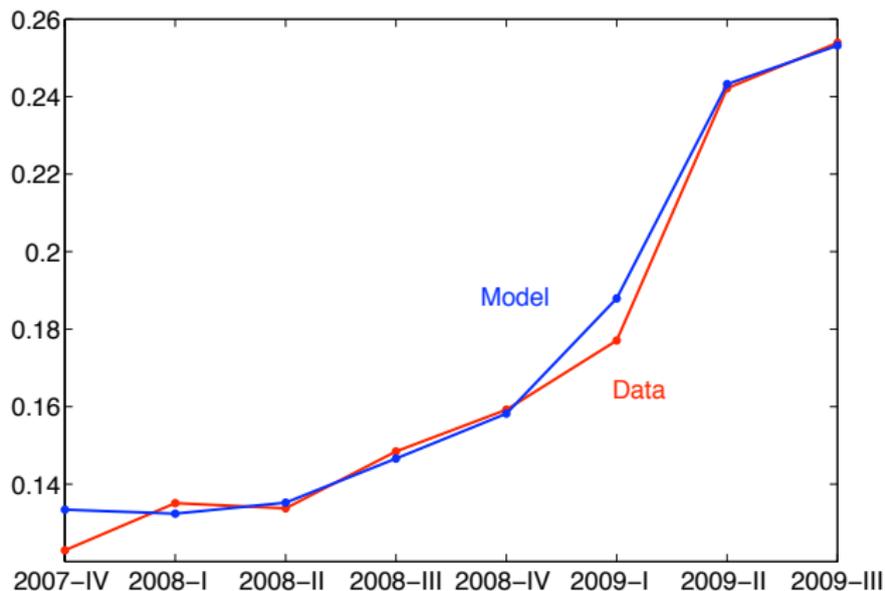
$$\theta \ell^{\theta-1} \frac{E(z|z \geq \hat{z})}{1 - \Phi(\hat{z})} = w + V \frac{\phi(\hat{z})}{1 - \Phi(\hat{z})} \frac{d\hat{z}}{d\ell}$$

- Loss in future value is larger when  $\frac{\phi(\hat{z})}{1 - \Phi(\hat{z})}$  increases with volatility:
  - ▶ scale is smaller
  - ▶ marginal product of labor is even larger than wage  
 $\implies$  even larger labor wedge

# Quantitative Exercise

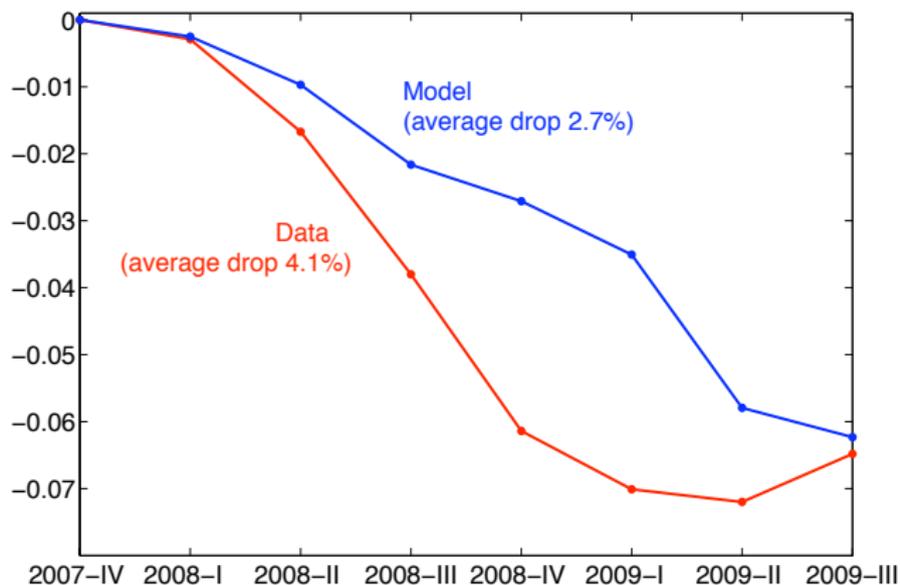
- Parameterize process for  $\sigma_t$  to the times series of IQR of sales growth in Compustat firms (1970-2009)
  - ▶ Moments: Mean, std., and autocorrelation of IQR of sales growth
  - ▶ Parameters: Mean, std. and autocorrelation of  $\sigma_t$
- Current recession: Choose the sequence of  $\sigma_t$  to match time series of IQR of sales growth

## Current recession: IQR of sales growth



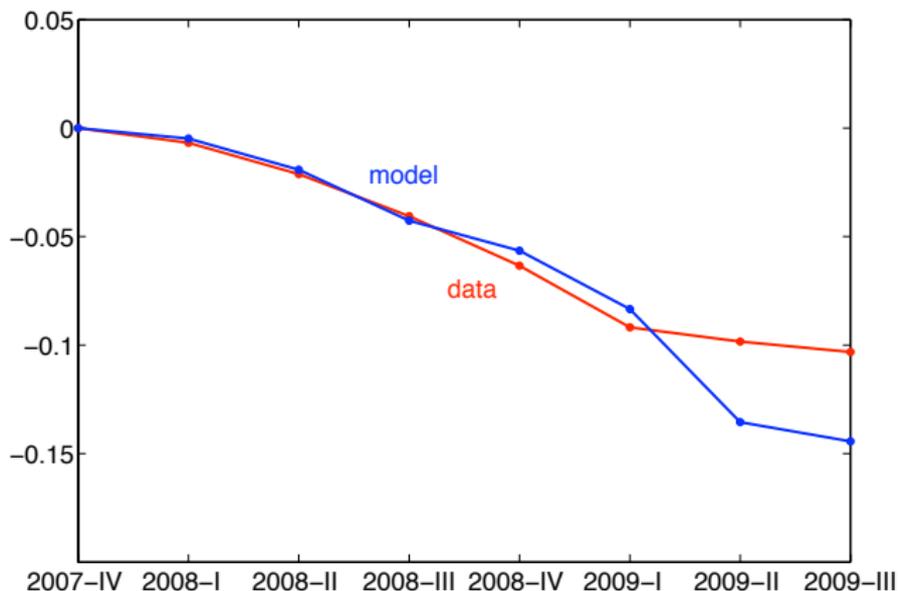
Choose sequence of  $\sigma_t$  to match time path of IQR sales growth

# Current recession: Output



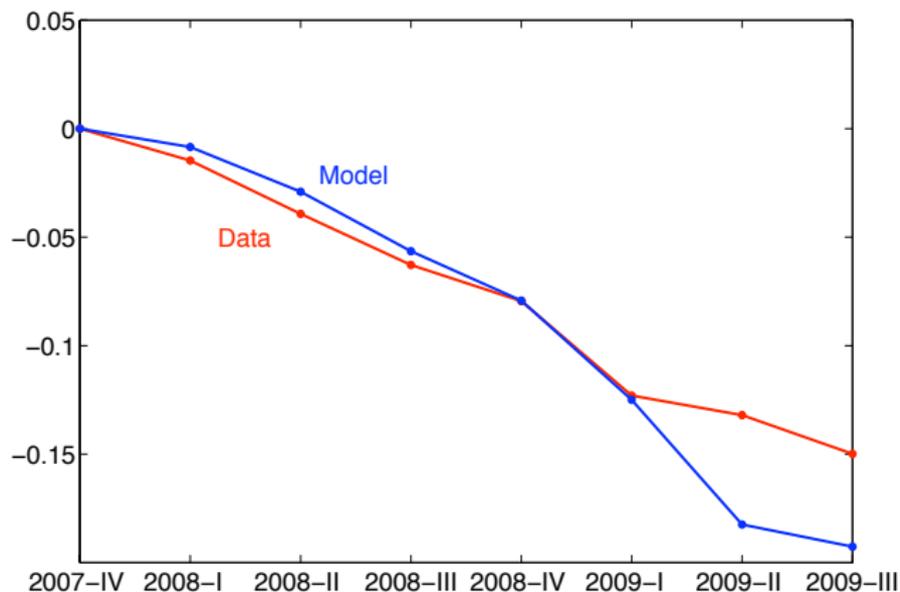
Model output matches 66% of the output decline

## Current recession: Labor



Model labor decline matches data; last couple of quarters decline is larger

## Current recession: Labor wedge



The labor wedge falls by 18% in model and 15% in data

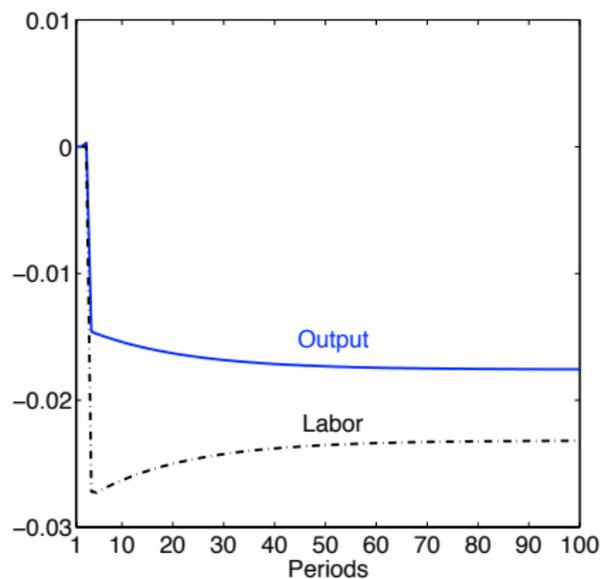
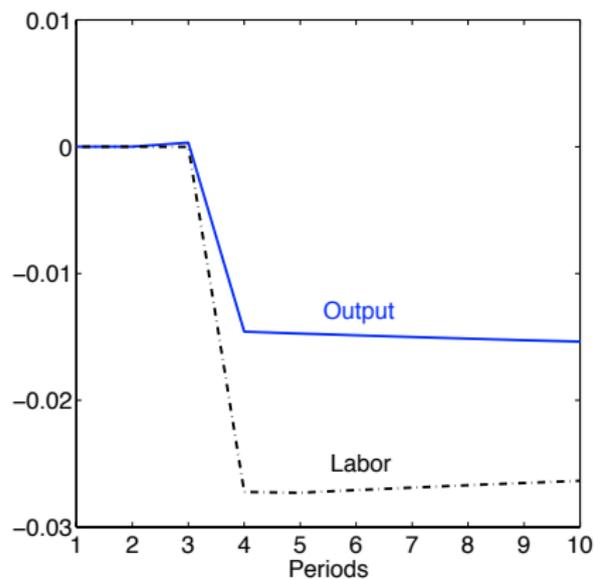
# Conclusion

- Framework that combines volatility shocks with financial markets imperfections
- Generates movements in labor wedge linked to financial frictions

# Business Cycles

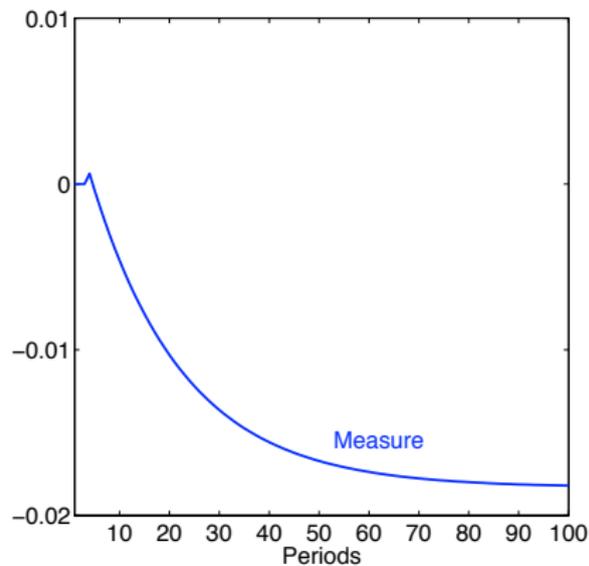
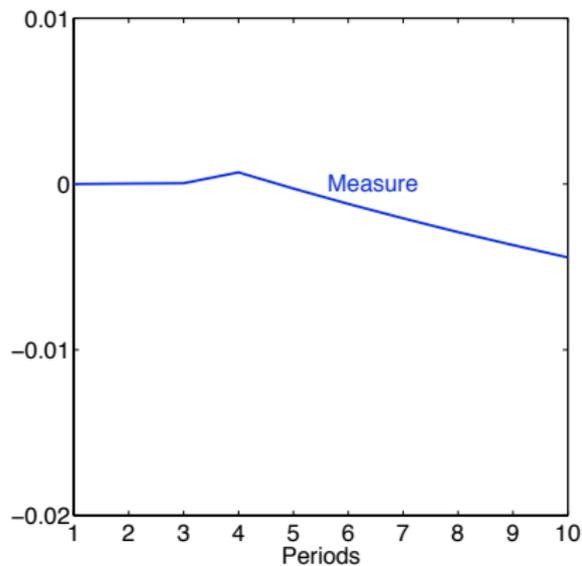
	Data			Model		
	Peak-Trough	std(x)	$\frac{\text{std}(x)}{\text{std}(\text{gdp})}$	Peak-Trough	std(x)	$\frac{\text{std}(x)}{\text{std}(\text{gdp})}$
GDP	-5.0	2.6		-3.5	1.8	
Labor	-5.1	3.4	1.3	-6.7	3.5	2.0
Consumption	-3.1	2.5	0.7	-0.6	1.1	0.6
Labor Wedge	-5.8	4.4	1.7	-7.1	4.4	2.5
TFP	-1.3	1.2	0.3	1.2	0.8	0.5

# Aggregate Impulse Response Response to High Dispersion



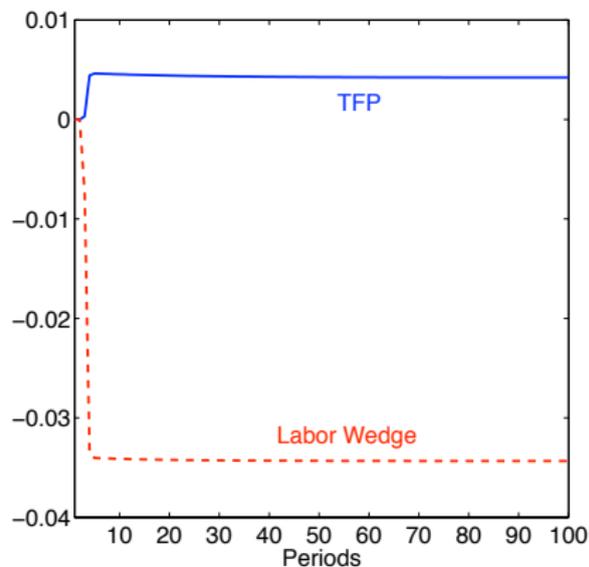
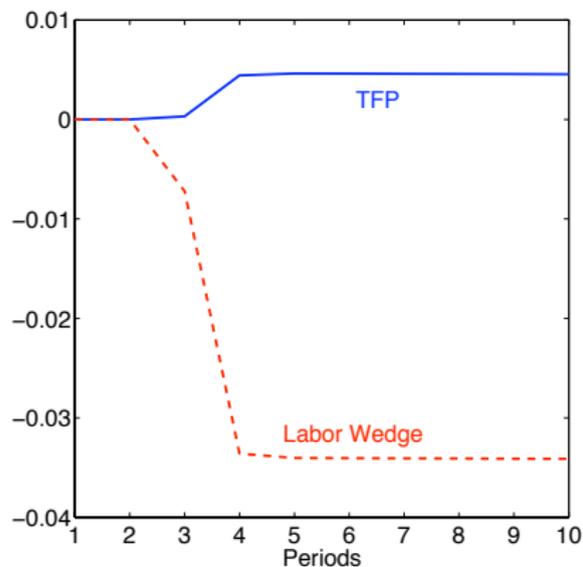
Labor falls more than output

# Aggregate Impulse Response to High Dispersion



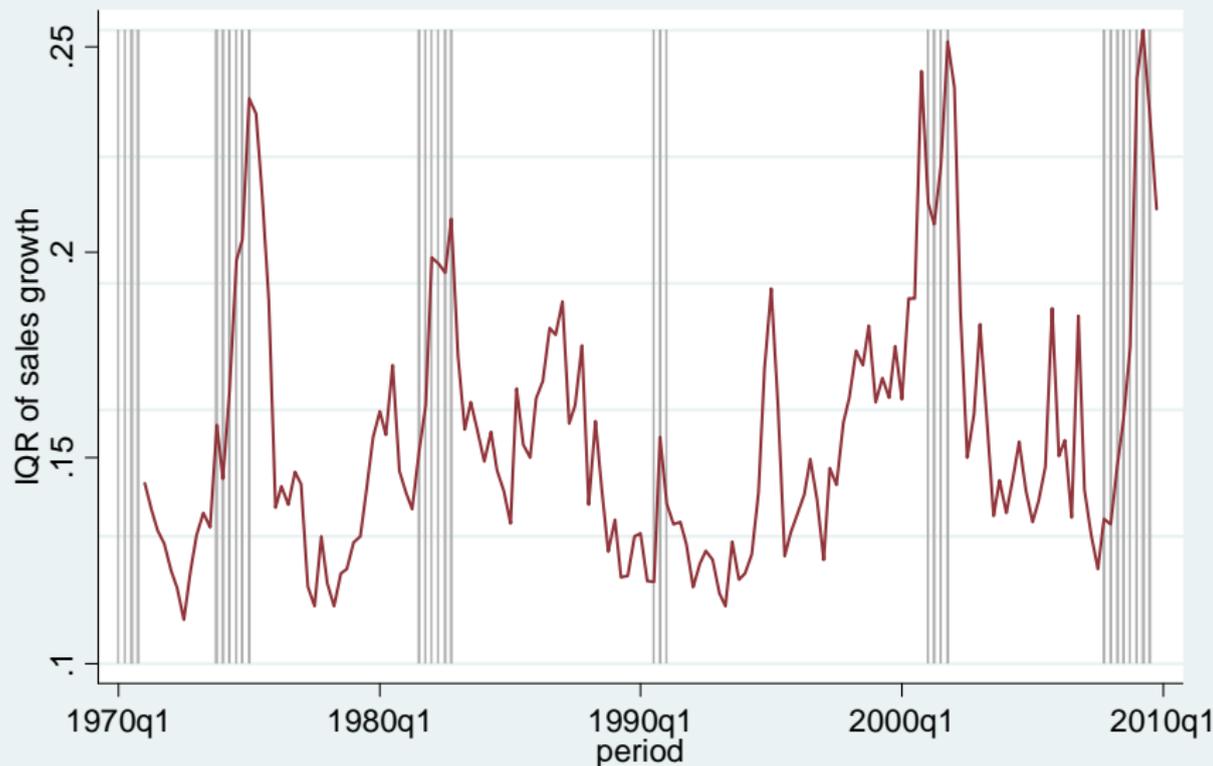
Measure of firms fall

# Aggregate Impulse Response Response to High Dispersion



Labor wedge worsens a lot, TFP rises a bit

# IQR sales growth from 1970-2010



■ NBER Recession/low    — IQR of sales growth