Financial Markets and Fluctuations in Uncertainty

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Motivation

- Recent recession has featured
  - Large contraction in output
  - Substantial increase in the dispersion of firms' growth
- Most of the recent output downturn accounted for
  - By a worsening of the labor wedge
  - Not by fall in TFP
Goal

- Develop a model with imperfect financial markets that connects fluctuations in firm volatility to aggregate fluctuations.

- Ask: Can an increase in volatility of firms’ idiosyncratic shocks that generates observed increase in firms’ dispersion deliver
  - Large contraction?
  - Large worsening of labor wedge?

- Today focus on current recession.
Key Elements in Model

- Firms
  - Choose their scale in advance
  - Issue debt uncontingent on their idiosyncratic shock and can default
  - Firms pay an entry cost so ongoing firms have positive future expected profits

- Shocks
  - Common shocks to the volatility of firms’ idiosyncratic productivity
Role of key elements

- Choose scale in advance
  - One scale for all states
  - In high states ‘too small’ and in low states ‘too big’
- Uncontingent/unenforceable debt
  - If too big, might default
- Entry cost
  - In equilibrium generates costs of default

Trade-off between short-term profits and the risk of costly liquidation
From firm volatility to aggregates

- Model: Trade-off between profits and liquidation
- Mechanism: High volatility mainly increases risk of costly liquidation
  - Firms reduce scale and output
  - Labor wedge worsens because MPL deviates more from wage
Our answers for current recession

Can an increase in volatility of firms’ idiosyncratic shocks that generates observed increase in firms’ dispersion deliver

- Large contraction?
  - Model accounts for 2/3 of the output decline

- Large worsening of labor wedge?
  - The labor wedge falls by 18% in model and 15% in data
Model

- Dynamic model of heterogeneous firms and identical households
- Households provide labor services and trade assets
- Firms use DRS technology with labor input $\ell$ and issue debt $b'(\sigma')$
- Firms’ idiosyncratic productivity shocks $z$ have common stochastic volatility $\sigma$
Firms

- Stochastic structure:

\[ \log z_t = \rho_z \log z_{t-1} + \sigma_t \epsilon_t \]

\[ \log \sigma_t = (1 - \rho_\sigma) \log \mu_\sigma + \rho_\sigma \log \sigma_{t-1} + \eta_t \]

- Individual states: \((\ell, b, z)\)

- Aggregate states: \(S = (\sigma, Y)\), where \(Y\) is measure over individual states
Firms

- Choose $b'(\sigma')$ and $\ell'$ to maximize present value of dividends

$$d = z\ell^\theta - w\ell - b + \sum_{\sigma'} q'(\sigma'|.) b'(\sigma')$$

$q(\sigma'|.)$ depends on firms choices and aggregate states

- Free entry condition given fixed cost of entry $\xi$

$$\xi = E_{z',\sigma'} Q(\sigma'|S) V'(\ell', b', z', S')$$

After entry the expected value is positive

- Cost of default: Firm exits so lose expected value of future profits
Firms

- Firms must have non-negative dividends

- Debt schedule \( q(\sigma'|.) \) compensates for loss in case of default
  - Schedule contains ‘borrowing limits’

- For high enough debt due, firms must default:
  - Default if

\[
 z\ell^\theta - w\ell - b + \max \left\{ \sum_{\sigma'} q(\sigma'|.) b'(\sigma') \right\} < 0
\]
Households

- Identical households with standard problem

- Choose $c$, and $h$ to maximize present value of utility, where

$$u(c, h) = \log(c) - \chi \frac{h^{1+\nu}}{1 + \nu}$$
Simple Example

- Two period problem
- Firm loses exogenous future value $V$ if liquidates
Complete financial markets

$$\max_{\ell} \int_{0}^{\infty} [z \ell^{\theta} - w \ell] \phi(z) dz + V$$

Optimal scale chosen to maximize short term profits:

$$\theta \ell^{\theta - 1} E(z) = w$$

Increasing volatility while preserving $E(z)$ does not change optimal scale
No financial markets

Without financial markets firms liquidate in low states \((z < \hat{z})\)

\[
\max_{\ell, \hat{z}} \int_{\hat{z}}^{\infty} [z^\alpha - w \ell] \phi(z) dz + \int_{\hat{z}}^{\infty} V \phi(z) dz
\]

subject to

\[
\hat{z}^\alpha - w \ell = 0
\]

- \(d\hat{z}/d\ell > 0\) so higher \(\ell\) implies higher \(\hat{z}\) which generates:
  - Higher short term profits
  - Lower future value

- Optimal scale chosen to maximize short term profits \textit{and} future value
No financial markets

Optimal scale:

\[ \theta \ell^{\theta-1} \frac{E(z \mid z \geq \hat{z})}{1 - \Phi(\hat{z})} = w + \frac{\phi(\hat{z})}{1 - \Phi(\hat{z})} d\hat{z} \]

- Marginal cost of labor equals wage plus loss in future value

- When \( V \) is high enough:
  - scale is smaller than with frictionless financial markets
  - marginal product of labor is larger than wage \( \longrightarrow \) labor wedge
Increasing volatility

\[ \theta \ell^{\theta-1} \frac{E(z|z \geq \hat{z})}{1 - \Phi(\hat{z})} = w + V \frac{\phi(\hat{z})}{1 - \Phi(\hat{z})} \frac{d\hat{z}}{dl} \]

- Loss in future value is larger when \( \frac{\phi(\hat{z})}{1 - \Phi(\hat{z})} \) increases with volatility:
  - scale is smaller
  - marginal product of labor is even larger than wage
  \[ \implies \text{even larger labor wedge} \]
Quantitative Exercise

- Parameterize process for $\sigma_t$ to the times series of IQR of sales growth in Compustat firms (1970-2009)
  - Moments: Mean, std., and autocorrelation of IQR of sales growth
  - Parameters: Mean, std. and autocorrelation of $\sigma_t$

- Current recession: Choose the sequence of $\sigma_t$ to match time series of IQR of sales growth
Current recession: IQR of sales growth

Choose sequence of $\sigma_t$ to match time path of IQR sales growth
Model output matches 66% of the output decline
Current recession: Labor

Model labor decline matches data; last couple of quarters decline is larger
The labor wedge falls by 18% in model and 15% in data
Conclusion

- Framework that combines volatility shocks with financial markets imperfections

- Generates movements in labor wedge linked to financial frictions
## Business Cycles

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<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
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<tbody>
<tr>
<td></td>
<td>Peak-Trough std(x)</td>
<td>std(x) std(gdp)</td>
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<tr>
<td>GDP</td>
<td>-5.0</td>
<td>2.6</td>
</tr>
<tr>
<td>Labor</td>
<td>-5.1</td>
<td>3.4</td>
</tr>
<tr>
<td>Consumption</td>
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<tr>
<td>Labor Wedge</td>
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<tr>
<td>TFP</td>
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<td>1.2</td>
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Aggregate Impulse Response to High Dispersion

Labor falls more than output
Aggregate Impulse Response to High Dispersion

Measure of firms fall
Aggregate Impulse Response to High Dispersion

Labor wedge worsens a lot, TFP rises a bit
IQR sales growth from 1970-2010

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