DSGE Model-Based Forecasting

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Abstract

Dynamic stochastic general equilibrium (DSGE) models use modern macroeconomic theory to explain and predict comovements of aggregate time series over the business cycle and to perform policy analysis. We explain how to use DSGE models for all three purposes – forecasting, story-telling, and policy experiments – and review their forecasting record. We also provide our own real-time assessment of the forecasting performance of the Smets and Wouters (2007) model data up to 2011, compare it with Blue Chip and Greenbook forecasts, and show how it changes as we augment the standard set of observables external information from surveys (nowcasts, interest rates, and long-run inflation and output growth expectations). We explore methods of generating forecasts in the presence of a zero-lower-bound constraint on nominal interest rates and conditional on counterfactual interest rate paths. Finally, we perform a post-mortem of DSGE model forecasts of the Great Recession, and show that forecasts from a version of the Smets-Wouters model augmented by financial frictions and with spreads as an observable compare well with Blue Chip forecasts.

Contents

1 Introduction

[sec:intro] Dynamic stochastic general equilibrium (DSGE) models use modern macroeconomic theory to explain and predict comovements of aggregate time series over the business cycle. The term *DSGE model* encompasses a broad class of macroeconomic models that spans the standard neoclassical growth model discussed in King, Plosser, and Rebelo (1988) as well as New Keynesian monetary models with numerous real and nominal frictions that are based on the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). A common feature of these models is that decision rules of economic agents are derived from assumptions about preferences, technologies, and the prevailing fiscal and monetary policy regime by solving intertemporal optimization problems. As a consequence, the DSGE model paradigm delivers empirical models with a strong degree of theoretical coherence that are attractive as a laboratory for policy experiments.

DSGE models are increasingly being used by central banks around the world as tools for macroeconomic forecasting and policy analysis. Examples of such models include the small open economy model developed by the Sveriges Riksbank (Adolfson, Lindé, and Villani (2007) and Adolfson, Andersson, Lindé, Villani, and Vredin (2007)), the New Area-Wide Model developed at the European Central Bank (Coenen, McAdam, and Straub (2008) and Christoffel, Coenen, and Warne (2010)), and the Federal Reserve Board's new Estimated, Dynamic, Optimization-based model (Edge, Kiley, and Laforte (2009)). DSGE models are frequently estimated with Bayesian methods (see, for instance, An and Schorfheide (2007a) or Del Negro and Schorfheide (2010) for a review), in particular if the goal is to track and forecast macroeconomic time series. Bayesian inference delivers posterior predictive distributions that reflect uncertainty about latent state variables, parameters, and future realizations of shocks conditional on the available information.

The contribution of this paper has a methodological and a substantive dimension. On the methodological side, we provide a collection of algorithms that can be used to generate forecasts with DSGE models that have been estimated with Bayesian methods. In particular, we focus on novel methods that allow the user to incorporate external information into the DSGE-model-based forecasts. This external information could take the form of forecasts for the current quarter (nowcasts) from surveys of professional forecasters, short-term and medium-term interest rate forecasts, or long-run inflation and output-growth expectations.

We also study the use of unanticipated and anticipated monetary policy shocks to generate forecasts conditional on desired interest rate paths.

On the substantive side, we are providing detailed empirical applications of the forecasting methods. The empirical analysis features small and medium-scale DSGE models estimated on U.S. data. The novel aspects of the empirical analysis are to document how the forecast performance of the Smets and Wouters (2007) model can be improved by incorporating data on long-run inflation expectations as well as nowcasts from the Blue Chip survey. We also show that data on short- and medium-horizon interest rate expectations improves the interest rate forecasts of the Smets-Wouters model with anticipated monetary policy shocks, but has some adverse effects on output growth and inflation forecasts. Finally, we provide new insights in the real-time forecasting performance of the Smets-Wouters model and a DSGE model with financial frictions during the 2008-09 recession.

The remainder of this paper is organized as follows. Section 2 provides a description of the DSGE models used in the empirical analysis of this paper. The mechanics of generating DSGE model forecasts within a Bayesian framework are described in Section 3. We review well-known procedures to generate draws from posterior parameter distributions and posterior predictive distributions for future realizations of macroeconomic variables. From these draws one can then compute point, interval, and density forecasts. The first set of empirical results is presented in Section 4. We describe the real-time data set that is used throughout this paper and examine the accuracy of our benchmark point forecasts. We also provide a review of the sizeable literature on the accuracy of DSGE model forecasts.

The accuracy of DSGE model forecasts is affected by how well the model captures low frequency trends in the data and the extent to which important information about the current quarter (nowcast) is incorporated into the forecast. In Section 5 we introduce shocks to the target-inflation rate, long-run productivity growth, as well as anticipated monetary policy shocks into the Smets and Wouters (2007) model. With these additional shocks, we can use data on inflation, output growth, and interest rate expectations from the Blue Chip survey as observations on agents' expectations in the DSGE model and thereby incorporate the survey information into the DSGE model forecasts. We also consider methods of adjusting DSGE model forecasts in light of Blue Chip nowcasts. In Section 6 we use unanticipated and anticipated monetary policy shocks to generate forecasts conditional on a desired interest rate path.

Up to this point we have mainly focused on point forecasts generated from DSGE models. In Section 7 we move beyond point forecasts. We start by using the DSGE model to decompose the forecasts into the contribution of the various structural shocks. We then generate density forecasts throughout the 2008-09 financial crisis and recession, comparing predictions from a DSGE model without and with financial frictions. We also present some evidence on the quality of density forecasts by computing probability integral transformations. Finally, Section 8 concludes and provides an outlook. As part of this outlook we point the reader to several strands of related literature in which forecasts are not directly generated from DSGE models but the DSGE model restrictions nonetheless influence the forecasts.

Throughout this paper we use the following notation. $Y_{t_0:t_1}$ denotes the sequence of observations or random variables $\{y_{t_0}, \dots, y_{t_1}\}$. If no ambiguity arises, we sometimes drop the time subscripts and abbreviate $Y_{1:T}$ by Y. θ often serves as generic parameter vector, $p(\theta)$ is the density associated with the prior distribution, $p(Y|\theta)$ is the likelihood function, and $p(\theta|Y)$ the posterior density. We use *iid* to abbreviate independently and identically distributed. If $X|\Sigma \sim MN_{p\times q}(M, \Sigma \otimes P)$ is matricvariate Normal and $\Sigma \sim IW_q(S, \nu)$ has an Inverted Wishart distribution, we say that $(X, \Sigma) \sim MNIW(M, P, S, \nu)$. Here $\otimes$ is the Kronecker product. We use I to denote the identity matrix and use a subscript indicating the dimension if necessary. $tr[A]$ is the trace of the square matrix A, $|A|$ is its determinant, and $vec(A)$ stacks the columns of A. Moreover, we let $\|A\| = \sqrt{tr[A'A]}$. If A is a vector, then $\|A\| = \sqrt{A'A}$ is its length. We use $A_{(.j)}$ ($A_{(j.)}$) to denote the j'th column (row) of a matrix A. Finally, $\mathcal{I}\{x \geq a\}$ is the indicator function equal to one if $x \geq a$ and equal to zero otherwise.

2 The DSGE Models

[sec:models] We consider three DSGE models in this paper. The first model is the Smets and Wouters (2007), which is based on earlier work by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) (Section 2.1). It is a medium-scale DSGE model, which augments the standard neoclassical stochastic growth model by nominal price and wage rigidities as well as habit formation in consumption and investment adjustment costs. The second model is obtained by augmenting the Smets-Wouters model with credit frictions as in the financial accelerator model developed by Bernanke, Gertler, and Gilchrist (1999)

(Section 2.2). The actual implementation of the credit frictions closely follows Christiano, Motto, and Rostagno (2009). Finally, we consider a small-scale DSGE model, which is obtained as a special case of the Smets and Wouters (2007) model by removing some of its features such as capital accumulation, wage stickiness, and habit formation (Section 2.3).

2.1 The Smets-Wouters Model

[subsec:swmodel] We begin by briefly describing the log-linearized equilibrium conditions of the Smets and Wouters (2007) model. We deviate from Smets and Wouters (2007) in that we detrend the non-stationary model variables by a stochastic rather than a deterministic trend. This approach makes it possible to express almost all equilibrium conditions in a way that encompasses both the trend-stationary total factor productivity process in Smets and Wouters (2007), as well as the case where technology follows a unit root process. We refer to the model presented in this section as SW model. Let $\tilde{z}_t$ be the linearly detrended log productivity process which follows the autoregressive law of motion

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{1}$$

We detrend all non stationary variables by $Z_t = e^{\gamma t + \frac{1}{1-\alpha}\tilde{z}_t}$, where γ is the steady state growth rate of the economy. The growth rate of Z_t in deviations from γ, denoted by z_t, follows the process:

$$z_t = \ln(Z_t/Z_{t-1}) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t}. \tag{2}$$

All variables in the subsequent equations are expressed in log deviations from their non-stochastic steady state. Steady state values are denoted by $*$-subscripts and steady state formulas are provided in a Technical Appendix (available upon request). The consumption Euler equation takes the form:

$$\begin{aligned} c_t = &-\frac{(1-he^{-\gamma})}{\sigma_c(1+he^{-\gamma})}\left(R_t - I\!E_t[\pi_{t+1}] + b_t\right) + \frac{he^{-\gamma}}{(1+he^{-\gamma})}\left(c_{t-1} - z_t\right) \\ &+ \frac{1}{(1+he^{-\gamma})} I\!E_t\left[c_{t+1} + z_{t+1}\right] + \frac{(\sigma_c - 1)}{\sigma_c(1+he^{-\gamma})}\frac{w_* L_*}{c_*}\left(L_t - I\!E_t[L_{t+1}]\right), \end{aligned} \tag{3}$$

where c_t is consumption, L_t is labor supply, R_t is the nominal interest rate, and π_t is inflation. The exogenous process b_t drives a wedge between the intertemporal ratio of the marginal utility of consumption and the riskless real return $R_t - I\!E_t[\pi_{t+1}]$, and follows an AR(1) process with parameters ρ_b and σ_b. The parameters σ_c and h capture the relative degree of risk aversion and the degree of habit persistence in the utility function, respectively. The next condition follows from the optimality condition for the capital producers, and expresses the relationship between the value of capital in terms of consumption q_t^k and the level of investment i_t measured in terms of consumption goods:

$$q_t^k = S'' e^{2\gamma}(1+\beta e^{(1-\sigma_c)\gamma})\Big(i_t - \frac{1}{1+\beta e^{(1-\sigma_c)\gamma}}(i_{t-1}-z_t) - \frac{\beta e^{(1-\sigma_c)\gamma}}{1+\beta e^{(1-\sigma_c)\gamma}} I\!E_t\left[i_{t+1}+z_{t+1}\right] - \mu_t\Big), \quad (4)$$

which is affected by both investment adjustment cost (S'' is the second derivative of the adjustment cost function) and by μ_t, an exogenous process called "marginal efficiency of investment" that affects the rate of transformation between consumption and installed capital (see Greenwood, Hercovitz, and Krusell (1998)). The latter, called $\bar{k}_t$, indeed evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right)\left(\bar{k}_{t-1} - z_t\right) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} S'' e^{2\gamma}(1+\beta e^{(1-\sigma_c)\gamma})\mu_t, \quad (5)$$

where $i_*/\bar{k}_*$ is the steady state ratio of investment to capital. μ_t follows an AR(1) process with parameters ρ_μ and σ_μ. The parameter β captures the intertemporal discount rate in the utility function of the households. The arbitrage condition between the return to capital and the riskless rate is:

$$\frac{r_*^k}{r_*^k + (1-\delta)} I\!E_t[r_{t+1}^k] + \frac{1-\delta}{r_*^k + (1-\delta)} I\!E_t[q_{t+1}^k] - q_t^k = R_t + b_t - I\!E_t[\pi_{t+1}], \quad (6)$$

where r_t^k is the rental rate of capital, r_*^k its steady state value, and δ the depreciation rate. Capital is subject to variable capacity utilization u_t. The relationship between $\bar{k}_t$ and the amount of capital effectively rented out to firms k_t is

$$k_t \;=\; u_t - z_t + \bar{k}_{t-1}. \quad (7)$$

The optimality condition determining the rate of utilization is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \quad (8)$$

where ψ captures the utilization costs in terms of foregone consumption. From the optimality conditions of goods producers it follows that all firms have the same capital-labor ratio:

$$k_t \;=\; w_t - r_t^k + L_t. \quad (9)$$

Real marginal costs for firms are given by

$$mc_t \;=\; w_t + \alpha L_t - \alpha k_t, \tag{10}$$

where α is the income share of capital (after paying markups and fixed costs) in the production function.

All of the equations so far maintain the same form whether technology has a unit root or is trend stationary. A few small differences arise for the following two equilibrium conditions. The production function is:

$$y_t \;=\; \Phi_p\left(\alpha k_t + (1-\alpha)L_t\right) + \mathcal{I}\{\rho_z < 1\}(\Phi_p - 1)\frac{1}{1-\alpha}\tilde{z}_t, \tag{11}$$

under trend stationarity. The last term $(\Phi_p - 1)\frac{1}{1-\alpha}\tilde{z}_t$ drops out if technology has a stochastic trend, because in this case one has to assume that the fixed costs are proportional to the trend. Similarly, the resource constraint is:

$$y_t = g_t + \frac{c_*}{y_*}c_t + \frac{i_*}{y_*}i_t + \frac{r^k_* k_*}{y_*}u_t - \mathcal{I}\{\rho_z < 1\}\frac{1}{1-\alpha}\tilde{z}_t, . \tag{12}$$

The term $-\frac{1}{1-\alpha}\tilde{z}_t$ disappears if technology follows a unit root process. Government spending g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz}\sigma_z\varepsilon_{z,t}.$$

Finally, the price and wage Phillips curves are, respectively:

$$\begin{aligned}\pi_t = {} & \frac{(1-\zeta_p\beta e^{(1-\sigma_c)\gamma})(1-\zeta_p)}{(1+\iota_p\beta e^{(1-\sigma_c)\gamma})\zeta_p((\Phi_p - 1)\epsilon_p + 1)}mc_t \\ & + \frac{\iota_p}{1+\iota_p\beta e^{(1-\sigma_c)\gamma}}\pi_{t-1} + \frac{\beta e^{(1-\sigma_c)\gamma}}{1+\iota_p\beta e^{(1-\sigma_c)\gamma}}I\!\!E_t[\pi_{t+1}] + \lambda_{f,t},\end{aligned} \tag{13}$$

and

$$\begin{aligned}w_t = {} & \frac{(1-\zeta_w\beta e^{(1-\sigma_c)\gamma})(1-\zeta_w)}{(1+\beta e^{(1-\sigma_c)\gamma})\zeta_w((\lambda_w - 1)\epsilon_w + 1)}\left(w^h_t - w_t\right) \\ & - \frac{1+\iota_w\beta e^{(1-\sigma_c)\gamma}}{1+\beta e^{(1-\sigma_c)\gamma}}\pi_t + \frac{1}{1+\beta e^{(1-\sigma_c)\gamma}}\left(w_{t-1} - z_t - \iota_w\pi_{t-1}\right) \\ & + \frac{\beta e^{(1-\sigma_c)\gamma}}{1+\beta e^{(1-\sigma_c)\gamma}}I\!\!E_t\left[w_{t+1} + z_{t+1} + \pi_{t+1}\right] + \lambda_{w,t},\end{aligned} \tag{14}$$

where ζ_p, ι_p, and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameters in the Kimball aggregator for prices, and ζ_w, ι_w, and ϵ_w are the corresponding parameters for wages. The variable w_t^h corresponds to the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{1}{1 - he^{-\gamma}} \left(c_t - he^{-\gamma} c_{t-1} + he^{-\gamma} z_t \right) + \nu_l L_t, \tag{15}$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in absence of wage rigidities). The mark-ups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow exogenous ARMA(1,1) processes

$$\lambda_{f,t} = \rho_{\lambda_f} \lambda_{f,t-1} + \sigma_{\lambda_f} \varepsilon_{\lambda_f,t} + \eta_{\lambda_f} \sigma_{\lambda_f} \varepsilon_{\lambda_f,t-1}, \text{ and}$$

$$\lambda_{w,t} = \rho_{\lambda_w} \lambda_{w,t-1} + \sigma_{\lambda_w} \varepsilon_{\lambda_w,t} + \eta_{\lambda_w} \sigma_{\lambda_w} \varepsilon_{\lambda_w,t-1},$$

respectively. Last, the monetary authority follows a generalized feedback rule:

$$\begin{aligned} R_t \;=\; & \rho_R R_{t-1} + (1 - \rho_R) \left(\psi_1 \pi_t + \psi_2 (y_t - y_t^f) \right) \\ & + \psi_3 \left((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f) \right) + r_t^m, \end{aligned} \tag{16}$$

where the flexible price/wage output y_t^f obtains from solving the version of the model without nominal rigidities (that is, Equations (3) through (12) and (15)), and the residual r_t^m follows an AR(1) process with parameters ρ_{r^m} and σ_{r^m}.

The SW model is estimated based on seven quarterly macroeconomic time series. The measurement equations for real output, consumption, investment, and real wage growth, hours, inflation, and interest rates are given by:

$$\begin{array}{lcl} \textit{Output growth} & = & \gamma + 100 \left(y_t - y_{t-1} + z_t \right) \\ \textit{Consumption growth} & = & \gamma + 100 \left(c_t - c_{t-1} + z_t \right) \\ \textit{Investment growth} & = & \gamma + 100 \left(i_t - i_{t-1} + z_t \right) \\ \textit{Real Wage growth} & = & \gamma + 100 \left(w_t - w_{t-1} + z_t \right) \;, \\ \textit{Hours} & = & \bar{l} + 100 l_t \\ \textit{Inflation} & = & \pi_* + 100 \pi_t \\ \textit{FFR} & = & R_* + 100 R_t \end{array} \tag{17}$$

where all variables are measured in percent, π_* and R_* measure the steady state level of net inflation and short term nominal interest rates, respectively, and $\bar{l}$ captures the mean of hours (this variable is measured as an index). The priors for the DSGE model parameters is the same as in Smets and Wouters (2007), and is summarized in Panel I of Table 1.

2.2 A Medium-Scale Model with Financial Frictions

[subsec:ffmodel] We now add financial frictions to the SW model following the work of Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2009). This amounts to replacing (6) with the following conditions:

$$E_t\left[\tilde{R}^k_{t+1} - R_t\right] = -b_t + \zeta_{sp,b}\left(q^k_t + \bar{k}_t - n_t\right) + \tilde{\sigma}_{\omega,t} \tag{18}$$

and

$$\tilde{R}^k_t - \pi_t = \frac{r^k_*}{r^k_* + (1-\delta)} r^k_t + \frac{(1-\delta)}{r^k_* + (1-\delta)} q^k_t - q^k_{t-1}, \tag{19}$$

where $\tilde{R}^k_t$ is the gross nominal return on capital for entrepreneurs, n_t is entrepreneurial equity, and $\tilde{\sigma}_{\omega,t}$ captures mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano, Motto, and Rostagno (2009)) and follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω}. The second condition defines the return on capital, while the first one determines the spread between the expected return on capital and the riskless rate.[1] The following condition describes the evolution of entrepreneurial net worth:

$$\begin{array}{rcl} \hat{n}_t & = & \zeta_{n,\tilde{R}^k}\left(\tilde{R}^k_t - \pi_t\right) - \zeta_{n,R}\left(R_{t-1} - \pi_t\right) + \zeta_{n,qK}\left(q^k_{t-1} + \bar{k}_{t-1}\right) + \zeta_{n,n} n_{t-1} \\ & & -\frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1} \end{array}. \tag{20}$$

In addition, the set of measurement equations (17) is augmented as follows

$$\text{Spread} = SP_* + 100 I\!\!E_t\left[\tilde{R}^k_{t+1} - R_t\right], \tag{21}$$

where the parameter SP_* measures the steady state spread. We specify priors for the parameters SP_*, $\zeta_{sp,b}$, in addition to ρ_{σ_ω} and σ_{σ_ω}, and fix the parameters $\bar{F}_*$ and γ_* (steady state default probability and survival rate of entrepreneurs, respectively). A summary is provided in Panel V of Table 1. In turn, these parameters imply values for the parameters of (20), as shown in the Technical Appendix. We refer to the DSGE model with financial frictions as SW-FF.

2.3 A Small-Scale DSGE Model

[subsec:smallmodel] The small-scale DSGE model is obtained as a special case of the SW model, by removing some of its features such as capital accumulation, wage stickiness, and

[1] Note that if $\zeta_{sp,b} = 0$ and the financial friction shocks are zero, (6) coincides with (18) plus (19).

Table 1: Priors for the Medium-Scale Model

	Density	Mean	St. Dev.		Density	Mean	St. Dev.
Panel I: SW Model							
Policy Parameters							
ψ_1	Normal	1.50	0.25	ρ_R	Beta	0.75	0.10
ψ_2	Normal	0.12	0.05	ρ_{r^m}	Beta	0.50	0.20
ψ_3	Normal	0.12	0.05	σ_{r^m}	InvG	0.10	2.00
Nominal Rigidities Parameters							
ζ_p	Beta	0.50	0.10	ζ_w	Beta	0.50	0.10
Other "Endogenous Propagation and Steady State" Parameters							
α	Normal	0.30	0.05	π^*	Gamma	0.62	0.10
Φ	Normal	1.25	0.12	γ	Normal	0.40	0.10
h	Beta	0.70	0.10	S''	Normal	4.00	1.50
ν_l	Normal	2.00	0.75	σ_c	Normal	1.50	0.37
ι_p	Beta	0.50	0.15	ι_w	Beta	0.50	0.15
r_*	Gamma	0.25	0.10	ψ	Beta	0.50	0.15
(Note $\beta = (1/(1 + r_*/100))$)							
ρs, σs, and ηs							
ρ_z	Beta	0.50	0.20	σ_z	InvG	0.10	2.00
ρ_b	Beta	0.50	0.20	σ_b	InvG	0.10	2.00
ρ_{λ_f}	Beta	0.50	0.20	σ_{λ_f}	InvG	0.10	2.00
ρ_{λ_w}	Beta	0.50	0.20	σ_{λ_w}	InvG	0.10	2.00
ρ_μ	Beta	0.50	0.20	σ_μ	InvG	0.10	2.00
ρ_g	Beta	0.50	0.20	σ_g	InvG	0.10	2.00
η_{λ_f}	Beta	0.50	0.20	η_{λ_w}	Beta	0.50	0.20
η_{gz}	Beta	0.50	0.20				
Panel II: SW with Loose π_* Prior ($SW - Loose$)							
π^*	Gamma	0.75	0.40				
Panel III: Model with Long Run Inflation Excpetations ($SW\pi$)							
ρ_{π^*}	Beta	0.50	0.20	σ_{π^*}	InvG	0.03	6.00
Panel IV: Model with Long Run Output Excpetations ($SW\pi Y$)							
ρ_{z^p}	Beta	0.98	0.01	σ_{z^p}	InvG	0.01	4.00
Panel V: Financial Frictions ($SW - FF$)							
SP_*	Gamma	2.00	0.10	$\zeta_{sp,b}$	Beta	0.05	0.005
ρ_{σ_w}	Beta	0.75	0.15	σ_{σ_w}	InvG	0.05	4.00

Notes: The following parameters are fixed in Smets and Wouters (2007): $\delta = 0.025$, $g_* = 0.18$, $\lambda_w = 1.50$, $\varepsilon_w = 10.0$, and $\varepsilon_p = 10$. In addition, for the model with financial frictions we fix $\bar{F}_* = .03$ and $\gamma_* = .99$. The columns "Mean" and "St. Dev." list the means and the standard deviations for Beta, Gamma, and Normal distributions, and the values s and ν for the Inverse Gamma (InvG) distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The effective prior is truncated at the boundary of the determinacy region. The prior for $\bar{l}$ is $\mathcal{N}(-45, 5^2)$.

habit formation. After setting $h = 0$ and eliminating the shock b_t the consumption Euler equation simplifies to:

$$c_t = I\!\!E_t\left[c_{t+1} + z_{t+1}\right] - \frac{1}{\sigma_c}\left(R_t - I\!\!E_t[\pi_{t+1}]\right). \tag{22}$$

After setting the capital share α in the production function to zero, the marginal costs are given by the wage: $mc_t = w_t$. In the absence of wage stickiness the wage equals the households' marginal rate of substitution between consumption and leisure, which in equilibrium leads to $w_t = c_t + \nu_l L_t$. In the absence of fixed costs ($\Phi_p = 1$) detrended output equals the labor input $y_t = L_t$. Overall, we obtain

$$mc_t = c_t + \nu_l y_t. \tag{23}$$

The Phillips curve simplifies to

$$\pi_t = \frac{(1-\zeta_p\beta)(1-\zeta_p)}{(1+\iota_p\beta)\zeta_p} mc_t + \frac{\beta}{1+\iota_p\beta} I\!\!E_t[\pi_{t+1}] + \frac{\iota_p}{1+\iota_p\beta}\pi_{t-1}. \tag{24}$$

We assume that the central bank only reacts to inflation and output growth and that the monetary policy shock is iid. This leads to a policy rule of the form

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\big[\psi_1\pi_t + \psi_2(y_t - y_{t-1} + z_t)\big] + \sigma_R\epsilon_{R,t}. \tag{25}$$

Finally, the aggregate resource constraint simplifies to

$$y_t = c_t + g_t. \tag{26}$$

Here we have adopted a slightly different definition of the government spending shock than in the SW model.

The model is completed with the specification of the exogenous shock processes. The government spending shock evolves according to

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{g,t}. \tag{27}$$

We slightly generalize the technology process from an AR(1) process to an AR(2) process

$$\tilde{z}_t = \rho_z(1-\varphi)\tilde{z}_{t-1} + \varphi\tilde{z}_{t-2} + \sigma_z\epsilon_{z,t}, \tag{28}$$

which implies that the growth rate of the trend process evolves according to

$$z_t = \ln(Z_t/Z_{t-1}) - \gamma = (\rho_z - 1)(1-\varphi)\tilde{z}_{t-1} - \varphi(\tilde{z}_{t-1} - \tilde{z}_{t-2}) + \sigma_z\epsilon_{z,t}.$$

The innovations $\epsilon_{z,t}$, $\epsilon_{g,t}$, and $\epsilon_{R,t}$ are assumed to be iid standard normal.

The small-scale model is estimated based on three quarterly macroeconomic time series. The measurement equations for real output growth, inflation, and interest rates are given by:

$$\begin{array}{lcl} \textit{Output growth} & = & \gamma + 100\,(y_t - y_{t-1} + z_t) \\ \textit{Inflation} & = & \pi_* + 100\pi_t \\ \textit{FFR} & = & R_* + 100R_t \end{array} \tag{29}$$

where all variables are measured in percent and π_* and R_* measure the steady state level of inflation and short term nominal interest rates, respectively. For the parameters that are common between the SW model and the small-scale model we use the same marginal prior distributions as listed in Table 1. The additional parameter φ_z has a prior distribution that is uniform on the interval $(-1, 1)$ because it is a partial autocorrelation. The joint prior distribution is given by the products of the marginals, truncated to ensure that the DSGE model has a determinate equilibrium.

3 Generating Forecasts with DSGE Models

[sec:dsgeforecasts] Before examining the forecast performance of DSGE models we provide a brief overview of the mechanics of generating such forecasts in a Bayesian framework. A more comprehensive review of Bayesian forecasting is provided by Geweke and Whiteman (2006). Let θ denote the vector that stacks the DSGE model parameters. Bayesian inference starts from a prior distribution represented by a density $p(\theta)$. The prior is combined with the conditional density of the data $Y_{1:T}$ given the parameters θ, denoted by $p(Y_{1:T}|\theta)$. This density can be derived from the DSGE model. According to Bayes Theorem, the posterior distribution, that is the conditional distribution of parameters given data, is given by

$$p(\theta|Y_{1:T}) = \frac{p(Y_{1:T}|\theta)p(\theta)}{p(Y_{1:T})}, \quad p(Y_{1:T}) = \int p(Y_{1:T}|\theta)p(\theta)d\theta, \tag{30}$$

where $p(Y_{1:T})$ is called the marginal likelihood or data density. In DSGE model applications it is typically not possible to derive moments and quantiles of the posterior distribution analytically. Instead, inference is implemented via numerical methods such as MCMC simulation. MCMC algorithms deliver serially correlated sequences $\{\theta^{(j)}\}_{j=1}^{n_{sim}}$ of n_{sim} draws from the density $p(\theta|Y_{1:T})$.

In forecasting applications the posterior distribution $p(\theta|Y_{1:T})$ is not the primary object of interest. Instead, the focus is on predictive distributions, which can be decomposed as follows:

$$p(Y_{T+1:T+H}|Y_{1:T}) = \int p(Y_{T+1:T+H}|\theta, Y_{1:T})p(\theta|Y_{1:T})d\theta. \tag{31}$$

This decomposition highlights that draws from the predictive density can be obtained by simulating the DSGE model conditional on posterior parameter draws $\theta^{(j)}$ and the observations $Y_{1;T}$. In turn, this leads to sequences $Y^{(j)}_{T+1:T+H}$, $j = 1, \ldots, n_{sim}$ that represent draws from the predictive distribution (31). These draws can then be used to obtain numerical approximations of moments, quantiles, and the probability density function of $Y_{T+1:T+H}$. In the remainder of this section, we discuss how to obtain draws from the posterior distribution of DSGE model parameters (Section 3.1) and how to generate draws from the predictive distribution of future observations (Section 3.2).

3.1 Posterior Inference for θ

[subsec:posteriortheta] Before the DSGE model can be estimated, it has to be solved using a numerical method. In most DSGE models, the intertemporal optimization problems of economic agents can be written recursively, using Bellman equations. In general, the value and policy functions associated with the optimization problems are nonlinear in terms of both the state and the control variables, and the solution of the optimization problems requires numerical techniques. The implied equilibrium law of motion can be written as

$$s_t = \Phi(s_{t-1}, \epsilon_t; \theta), \tag{32}$$

where s_t is a vector of suitably defined state variables and ϵ_t is a vector that stacks the innovations for the structural shocks. In this paper, we proceed under the assumption that the DSGE model's solution is approximated by log-linearization techniques and ignore the discrepancy between the nonlinear model solution and the first-order approximation:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t. \tag{33}$$

The system matrices Φ_1 and Φ_ϵ are functions of the DSGE model parameters θ, and s_t spans the state variables of the model economy, but also might contain some redundant elements that facilitate a simple representation of the measurement equation:

$$y_t = \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_2(\theta)s_t. \tag{34}$$

Equations (33) and (34) provide a state-space representation for the linearized DSGE model. This representation is the basis for the econometric analysis. If the innovations ϵ_t are Gaussian, then the likelihood function $p(Y_{1:T}|\theta)$ can be evaluated with a standard Kalman filter.

We now turn to the prior distribution represented by the density $p(\theta)$. An example of such a prior distribution is provided in Table 1. The table characterizes the marginal distribution of the DSGE model parameters. The joint distribution is then obtained as the product of the marginals. It is typically truncated to ensure that the DSGE model has a unique solution. DSGE model parameters can be grouped into three categories: (i) parameters that affect steady states; (ii) parameters that control the endogenous propagation mechanism of the model without affecting steady states; and (iii) parameters that determine the law of motion of the exogenous shock processes.

Priors for steady-state related parameters are often elicited indirectly by ensuring that model-implied steady states are commensurable with pre-sample averages of the corresponding economic variables. Micro-level information, e.g. about labor supply elasticities or the frequency of price and wage changes, is often used to formulate priors for parameters that control the endogenous propagation mechanism of the model. Finally, beliefs about volatilities and autocovariance patterns of endogenous variables can be used to elicit priors for the remaining parameters. A more detailed discussions and some tools to mechanize the prior elicitation are provided in Del Negro and Schorfheide (2008).

A detailed discussion of numerical techniques to obtain draws from the posterior distribution $p(\theta|Y_{1:T})$ can be found, for instance, in An and Schorfheide (2007a) and Del Negro and Schorfheide (2010). We only provide a brief overview. Because of the nonlinear relationship between the DSGE model parameters θ and the system matrices Ψ_0, Ψ_1, Ψ_2, Φ_1 and Φ_ϵ of the state-space representation in (33) and (34), the marginal and conditional distributions of the elements of θ do not fall into the well-known families of probability distributions. Up to now, the most commonly used procedures for generating draws from the posterior distribution of θ are the Random-Walk Metropolis (RWM) Algorithm described in Schorfheide (2000) and Otrok (2001) or the Importance Sampler proposed in DeJong, Ingram, and Whiteman (2000). The basic RWM Algorithm takes the following form

Algorithm 1. Random-Walk Metropolis (RWM) Algorithm for DSGE Model. [algo:rwm]

1. Use a numerical optimization routine to maximize the log posterior, which up to a constant is given by $\ln p(Y_{1:T}|\theta) + \ln p(\theta)$. Denote the posterior mode by $\tilde{\theta}$.

2. Let $\tilde{\Sigma}$ be the inverse of the (negative) Hessian computed at the posterior mode $\tilde{\theta}$, which can be computed numerically.

3. Draw $\theta^{(0)}$ from $N(\tilde{\theta}, c_0^2\tilde{\Sigma})$ or directly specify a starting value.

4. For $j = 1, \ldots, n_{sim}$: draw ϑ from the proposal distribution $N(\theta^{(j-1)}, c^2\tilde{\Sigma})$. The jump from $\theta^{(j-1)}$ is accepted ($\theta^{(j)} = \vartheta$) with probability $\min\{1, r(\theta^{(j-1)}, \vartheta|Y_{1:T})\}$ and rejected ($\theta^{(j)} = \theta^{(j-1)}$) otherwise. Here,

$$r(\theta^{(j-1)}, \vartheta|Y_{1:T}) = \frac{p(Y_{1:T}|\vartheta)p(\vartheta)}{p(Y_{1:T}|\theta^{(j-1)})p(\theta^{(j-1)})}. \quad \square$$

If the likelihood can be evaluated with a high degree of precision, then the maximization in Step 1 can be implemented with a gradient-based numerical optimization routine. The optimization is often not straightforward because the posterior density is typically not globally concave. Thus, it is advisable to start the optimization routine from multiple starting values, which could be drawn from the prior distribution, and then set $\tilde{\theta}$ to the value that attains the highest posterior density across optimization runs. In some applications we found it useful to skip Steps 1 to 3 by choosing a reasonable starting value, such as the mean of the prior distribution, and replacing $\tilde{\Sigma}$ in Step 4 with a matrix whose diagonal elements are equal to the prior variances of the DSGE model parameters and whose off-diagonal elements are zero.

While the RWM algorithm in principle delivers consistent approximations of posterior moments and quantiles even if the posterior contours are highly non-elliptical, the practical performance can be poor as documented in An and Schorfheide (2007a). Recent research on posterior simulators tailored toward DSGE models tries to address the shortcomings of the "default" approaches that are being used in empirical work. An and Schorfheide (2007b) use transition mixtures to deal with a multi-modal posterior distribution. This approach works well if the researcher has knowledge about the location of the modes, obtained, for instance, by finding local maxima of the posterior density with a numerical optimization algorithm. Chib and Ramamurthy (2010) propose to replace the commonly used single block RWM algorithm with a Metropolis-within-Gibbs algorithm that cycles over multiple, randomly selected blocks of parameters. Kohn, Giordani, and Strid (2010) propose an adaptive hybrid Metropolis-Hastings samplers and Herbst (2010) develops a Metropolis-within-Gibbs algorithm that uses information from the Hessian matrix to construct parameter blocks that maximize within-block correlations at each iteration and Newton steps to tailor proposal distributions for the various conditional posteriors.

3.2 Evaluating the Predictive Distribution

[subsec:preddistribution] Bayesian DSGE model forecasts can computed based on draws from the posterior predictive distribution of $Y_{T+1:T+H}$. We use the parameter draws $\{\theta^{(j)}\}_{j=1}^{n_{sim}}$ generated with Algorithm 1 in the previous section as a starting point. Since the DSGE model is represented as a state-space model with latent state vector s_t, we modify the decomposition of the predictive density in (31) accordingly:

$$\begin{aligned} & p(Y_{T+1:T+H}|Y_{1:T}) \\ & = \int_{(s_T,\theta)} \left[\int_{S_{T+1:T+H}} p(Y_{T+1|T+H}|S_{T+1:T+H}) p(S_{T+1:T+H}|s_T,\theta,Y_{1:T}) dS_{T+1:T+H} \right] \\ & \quad \times p(s_T|\theta,Y_{1:T}) p(\theta|Y_{1:T}) d(s_T,\theta) \end{aligned} \tag{35}$$

Draws from the predictive density can be generated with the following algorithm:

Algorithm 2. Draws from the Predictive Distribution. [algo:preddraws] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$. Generate a draw $s_T^{(j)}$ from this distribution.

2. A draw from $S_{T+1:T+H}|(s_T,\theta,Y_{1:T})$ is obtained by generating a sequence of innovations $\epsilon^{(j)}_{T+1:T+H}$. Then, starting from $s_T^{(j)}$, iterate the state transition equation (33) with θ replaced by the draw $\theta^{(j)}$ forward to obtain a sequence $S^{(j)}_{T+1:T+H}$:

$$s_t^{(j)} = \Phi_1(\theta^{(j)}) s_{t-1}^{(j)} + \Phi_\epsilon(\theta^{(j)}) \epsilon_t^{(j)}, \quad t = T+1,\dots,T+H.$$

3. Use the measurement equation (34) to obtain $Y^{(j)}_{T+1:T+H}$:

$$y_t^{(j)} = \Psi_0(\theta^{(j)}) + \Psi_1(\theta^{(j)}) t + \Psi_2(\theta^{(j)}) s_t^{(j)}, \quad t = T+1,\dots,T+H. \quad \square$$

Algorithm 2 generates n_{sim} trajectories $Y^{(j)}_{T+1:T+H}$ from the predictive distribution of $Y_{T+1:T+H}$ given $Y_{1:T}$. The algorithm could be modified by executing Steps 2 and 3 m times for each j, which would lead to a total of $m \cdot n_{sim}$ draws from the predictive distribution. A point forecast $\hat{y}_{T+h}$ of y_{T+h} can be obtained by specifying a loss function $L(y_{T+h}, \hat{y}_{T+h})$ and determining the prediction that minimizes the posterior expected loss:

$$\hat{y}_{T+h|T} = \text{argmin}_{\delta \in \mathbb{R}^n} \int_{y_{T+h}} L(y_{T+h}, \delta) p(y_{T+h}|Y_{1:T}) dy_{T+h}. \tag{36}$$

For instance, under the quadratic forecast error loss function

$$L(y, \delta) = tr[W(y-\delta)'(y-\delta)],$$

where W is a symmetric positive-definite weight matrix and $tr[\cdot]$ is the trace operator, the optimal predictor is the posterior mean

$$\hat{y}_{T+h|T} = \int_{y_{T+h}} y_{T+h} p(y_{T+h}|Y_{1:T}) dy_{T+h} \approx \frac{1}{n_{sim}} \sum_{j=1}^{n_{sim}} y^{(j)}_{T+h}, \tag{37}$$

which can be approximated by a Monte Carlo average.

Pointwise (meaning for fixed h rather than jointly over multiple horizons) $1-\alpha$ credible interval forecasts for a particular element $y_{i,T+h}$ of y_{T+h} can be obtained by either computing the $\alpha/2$ and $1-\alpha/2$ percentiles of the empirical distribution of $\{y^{(j)}_{i,T+h}\}_{j=1}^{n_{sim}}$ or by numerically searching for the shortest connected interval that contains a $1-\alpha$ fraction of the draws $\{y^{(j)}_{i,T+h}\}_{j=1}^{n_{sim}}$. By construction, the latter approach leads to sharper interval forecasts.[2] Finally, density forecasts can be obtained by applying a density estimator (see Silverman (1986) for an introduction) to the set of draws $\{y^{(j)}_{i,T+h}\}_{j=1}^{n_{sim}}$.

As a short-cut, practitioners sometimes replace the numerical integration with respect to the parameter vector θ in Algorithm 2 by a plug-in step. Draws from the plug-in predictive distribution $p(y_{T+1:T+H}|\hat{\theta}, Y_{1:T})$ are obtained by setting $\theta^{(j)} = \hat{\theta}$ in Steps 2 and 3 of the algorithm. Here $\hat{\theta}$ is a point estimator such as the posterior mode or the posterior mean. While the plug-in approach tends to reduce the computational burden, it does not deliver the correct Bayes predictions and, importantly, interval and density forecasts will understate the uncertainty about future realizations of y_t.

[2] In general, the smallest (in terms of volume) set forecast is given by the highest-density set. If the predictive density is uni-modal the second above-mentioned approach generates the highest-density set. If the predictive density is multi-modal, then there might exist a collection of disconnected intervals that provides a sharper forecast.

4 Accuracy of Point Forecasts

[sec:pointforecasts] We begin the empirical analysis with the computation of RMSEs for our DSGE models. The RMSEs are based on a pseudo-out-of-sample forecasting exercise in which we are using real-time data sets to recursively estimate the DSGE models. The construction of the real-time data set is discussed in Section 4.1. Empirical results for the small-scale DSGE model of Section 2.3 are presented in Section 4.2. We compare DSGE model-based RMSEs to RMSEs computed for forecasts of the Blue Chip survey. A similar analysis is conducted for the SW model in Section 4.3. Finally, Section 4.4 summarizes results on the forecast performance of medium-scale DSGE models published in the literature.

4.1 A Real Time Data Set for Forecast Evaluation

[subsec:realtimedata] Since the small-scale DSGE model is estimated based on a subset of variables that are used for the estimation of the SW model, we focus on the description of the data set for the latter. Real GDP (GDPC), the GDP price deflator (GDPDEF), nominal personal consumption expenditures (PCEC), and nominal fixed private investment (FPI) are constructed at a quarterly frequency by the Bureau of Economic Analysis (BEA), and are included in the National Income and Product Accounts (NIPA).

Average weekly hours of production and nonsupervisory employees for total private industries (PRS85006023), civilian employment (CE16OV), and civilian noninstitutional population (LNSINDEX) are produced by the Bureau of Labor Statistics (BLS) at the monthly frequency. The first of these series is obtained from the Establishment Survey, and the remaining from the Household Survey. Both surveys are released in the BLS Employment Situation Summary (ESS). Since our models are estimated on quarterly data, we take averages of the monthly data. Compensation per hour for the nonfarm business sector (PRS85006103) is obtained from the Labor Productivity and Costs (LPC) release, and produced by the BLS at the quarterly frequency.

Last, the federal funds rate is obtained from the Federal Reserve Board's H.15 release at the business day frequency, and is not revised. We take quarterly averages of the annualized daily data. All data are transformed following Smets and Wouters (2007). Specifically:

$$
\begin{array}{lcl}
\textit{Output growth} & = & LN((GDPC)/LNSINDEX) * 100 \\
\textit{Consumption growth} & = & LN((PCEC/GDPDEF)/LNSINDEX) * 100 \\
\textit{Investment growth} & = & LN((FPI/GDPDEF)/LNSINDEX) * 100 \\
\textit{Real Wage growth} & = & LN(PRS85006103/GDPDEF) * 100 \\
\textit{Hours} & = & LN((PRS85006023 * CE16OV/100)/LNSINDEX) * 100 \\
\textit{Inflation} & = & LN(GDPDEF/GDPDEF(-1)) * 100 \\
\textit{FFR} & = & FEDERAL\ FUNDS\ RATE/4
\end{array}
$$

In the estimation of the DSGE model with financial frictions we measure *Spread* as the annualized Moody's Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity. Both series are available from the Federal Reserve Board's H.15 release, and averaged over each quarter. Spread data is also not revised.

Many macroeconomic time series get revised multiple times by the statistical agencies that publish the series. In many cases the revisions reflect additional information that has been collected by the agencies, in other instances revisions are caused by changes in definitions. For instance, the BEA publishes three releases of quarterly GDP in the first three month following the quarter. Thus, in order to be able to compare DSGE model forecasts to real-time forecasts made by private-sector professional forecasters or the Federal Reserve Board, it is important to construct vintages of real time historical data. We follow the work by Edge and Gürkaynak (2010) and construct data vintages that are aligned with the publication dates of the Blue Chip survey and the Federal Reserve Board's Greenbook/Tealbook.

Blue Chip's survey of professional forecasters is published on the 10th of each month, based on responses that have been submitted at the end of the previous month. For instance, forecasts published on April 10 are based on information that was available at the end of March. Whenever we evaluate the accuracy of Blue Chip forecasts in this paper, we focus on the so-called Consensus Blue Chip forecast, which is defined as the average of all the forecasts gathered in the Blue Chip Economic Indicators (BCEI) survey. While there are three Blue Chip forecasts published every quarter, we restrict our attention to the month in which the last forecast is made in each quarter. Given the approximate two week delay between the survey and the publication of the results on the 10th of each month, this means that we are constructing data sets that are aligned with the information available for the January, April, July, and October Blue Chip publications. For concreteness, consider the

April 1992 Blue Chip release date. In late March the NIPA series for 1992:Q1 are not yet available, which means that the DSGE model can only be estimated based on a sample that ends in 1991:Q4. Our selection of Blue Chip dates maximizes the informational advantage for the Blue Chip forecasters, who can in principle utilize high-frequency information about economic activity in 1992:Q1 that is available by late March. The first forecast origin considered in the subsequent forecast evaluation is January 1992 and the last one is April 2011. We refer to the collection of data vintages aligned with the Blue Chip publication dates as Blue Chip sample.

The Greenbook/Tealbook contains macroeconomic forecasts from the staff of the Board of Governors in preparation for a FOMC meeting. There are typically eight FOMC meetings per year. For the comparison of Greenbook versus DSGE model forecasts we also only consider a subset of four Greenbook publication dates, one associated with each quarter: typically from the months of March, June, September, and December.[3] We refer to the collection of vintages aligned with the Greenbook dates as Greenbook sample. The first forecast origin in the Greenbook sample is March 1992 and the last one is September 2004, since the Greenbook forecasts are only available with a 5 year lag. Table 2 summarizes the Blue Chip and Greenbook forecast origins in 1992 for which we are constructing DSGE model forecasts. Since we always use real time information, the vintage used to estimate the DSGE model for the comparison to the March 1992 Greenbook may be different from the vintage that is used for the comparison with the April 1992 Blue Chip forecast, even though in both cases the end of the estimation sample for the DSGE model is T=1991:Q4.

The Blue Chip Economic Indicators survey only contain quarterly forecasts for one calendar year after the current one. This implies that on January 10 the survey will have forecasts for eight quarters, and only for six quarters on October 10. When comparing forecast accuracy between Blue Chip and DSGE models, we use seven- and eight-quarter ahead forecasts only when available from the Blue Chip survey (which means we only use the January and April forecast dates when computing eight-quarter ahead RMSEs). For consistency, when comparing forecast accuracy across DSGE models we use the same approach (we refer to this set of dates/forecast horizons as the "Blue Chip dates"). Similarly, the horizon of Greenbook

[3] As forecast origins we choose the last Greenbook forecast date before an advanced NIPA estimate for the most recent quarter is released. For instance, the advanced estimate for Q1 GDP is typically released in the second half of April, prior to the April FOMC meeting.

Table 2: Blue Chip and Greenbook Forecast Dates for 1992

Forecast Origin		End of Est.	Forecast			
Blue Chip	Greenbook	Sample T	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Apr 92	Mar 92	91:Q4	92:Q1	92:Q2	92:Q3	92:Q4
Jul 92	Jun 92	92:Q1	92:Q2	92:Q3	92:Q4	93:Q1
Oct 92	Sep 92	92:Q2	92:Q3	92:Q4	93:Q1	93:Q2
Jan 93	Dec 92	92:Q3	92:Q4	93:Q1	93:Q2	93:Q3

forecasts also varies over time. In comparing DSGE model and Greenbook forecast accuracy we only use seven- and eight-quarter ahead whenever available from both.

For each forecast origin our estimation sample begins in 1964:Q1 and ends with the most recent quarter for which a NIPA release is available. Historical data were taken from the FRB St. Louis' ALFRED database. For vintages prior to 1997, compensation and population series were unavailable in ALFRED. In these cases, the series were taken from Edge and Gürkaynak (2010).[4] In constructing the real time data set, the release of one series for a given quarter may outpace that of another. For example, in several instances, Greenbook forecast dates occur after a quarter's ESS release but before the NIPA release. In other words, for a number of data vintages there is, relative to NIPA, an extra quarter of employment data. Conversely, in a few cases NIPA releases outpace LPC, resulting in an extra quarter of NIPA data. We follow the convention in Edge and Gürkaynak (2010) and use NIPA availability to determine whether a given quarter's data should be included in a vintage's estimation sample. When employment data outpace NIPA releases, this means ignoring the extra observations for hours, population, and employment from the Employment Situation Summary. In cases where NIPA releases outpace LPC releases, we include the next available LPC data in that vintage's estimation sample to "catch up" to the NIPA data.

There is an ongoing debate in the forecasting literature as to whether the "actuals" used in computing forecast errors should be the values of the variables according to the last available vintage, or the so-called "first finals", which for output corresponds with the

[4]We are very grateful to Rochelle Edge and Refet Gürkaynak for giving us this data, and explaining us how they constructed their dataset.

"Final" NIPA estimate (available roughly three months after the quarter is over). We show results according to the first approach.

Finally, the various DSGE models only produce forecasts for per-capita output, while Blue Chip and Greenbook forecasts are in terms of total GDP. When comparing RMSEs between the DSGE models and Blue Chip/Greenbook we therefore transform per-capita into aggregate output forecasts using (the final estimate of) realized population growth.[5]

4.2 Forecasts from the Small-Scale Model

[subsec:rmsesmallmodel] We begin by comparing the point forecast performance of the small-scale DSGE model described in Section 2.3 to that of the Blue Chip and Greenbook forecasts. RMSEs for output growth, inflation, and interest rates (Federal Funds) are displayed in Figure 1. Throughout this paper, GDP growth rates, inflation rates, and interest rates are reported in Quarter-on-Quarter (QoQ) percentages. The RMSEs in the first row of the figure are for forecasts that are based on the information available prior to the January, April, July, and October Blue Chip publication dates over the period 1992 to 2011. The RMSEs in the bottom row correspond to forecasts generated at the March, June, September, and December Greenbook dates over the period from 1992 to 2004.

The small-scale model attains a RMSE for output growth of approximately 0.65%. The RMSE is fairly flat with respect to the forecast horizon, which is consistent with the low serial correlation of U.S. GDP growth. At the nowcast horizon ($h = 1$), the Blue Chip forecasts are much more precise, their RMSE is 0.42, because they incorporate information from the current quarter. As the forecast horizon increases to $h = 4$ the RMSEs of the DSGE model and the Blue Chip forecasts are approximately the same. The accuracy of inflation and, in particular, interest rate forecasts of the small scale DSGE model is decreasing in the forecast horizon h due to the persistence of these series. The inflation RMSE is about 0.25% at the nowcast horizon and 0.35% for a two-year horizon. For the Federal Funds rate the RMSE increases from about 0.15 to 0.5. The inflation and interest rate Blue Chip forecasts tend to be substantially more precise than the DSGE model forecasts both at the nowcast as well as the one-year horizon.

[5] Edge and Gürkaynak (2010) follow a similar approach, except that their population "actuals" are the "first finals", consistently with the fact that they use "first finals" to measure forecast errors.

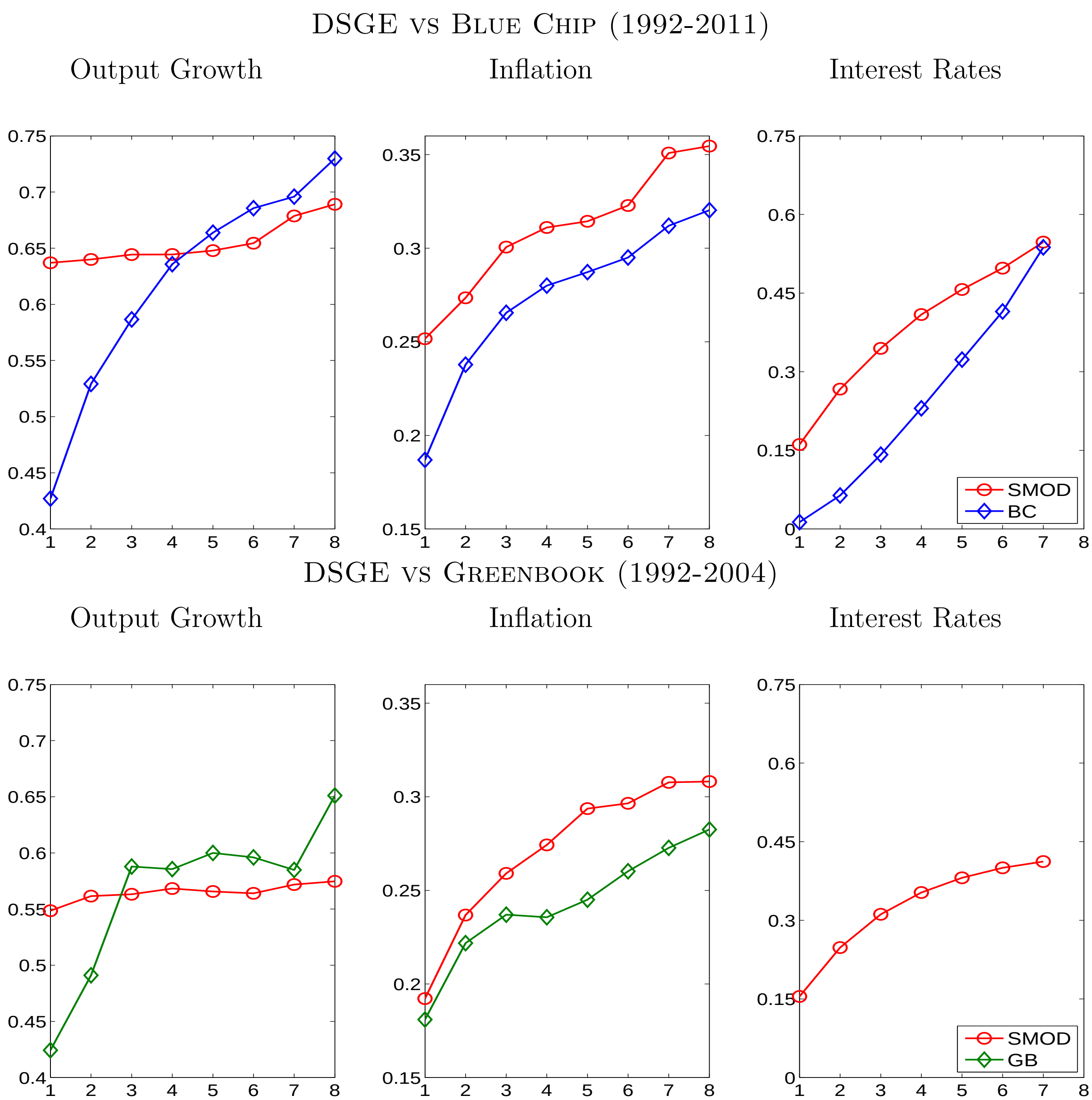
In comparison to the Greenbook forecasts the output growth forecasts of the small-scale DSGE model are more precise for horizons $h \geq 3$. Moreover, the inflation forecast of the DSGE model at the nowcast horizon is about as precise as the Greenbook inflation nowcast, but for horizons $h \geq 1$ the Greenbook forecasts dominate. We do not report RMSEs for Greenbook interest rate projections because the FOMC sets the nominal interest rate in part based on the information provided in the Greenbook.

4.3 Forecasts from the Smets-Wouters Model

[subsec:rmsemediummodel] We proceed by computing forecast error statistics for the SW model reviewed in Section 2.1. The results are reported in Figure 2. The top panels provide a comparison to Blue Chip forecasts from 1992 to 2011 and the bottom panels a comparison to Greenbook forecasts from 1992 to 2004. The accuracy of the output growth and inflation forecasts from the SW model forecasts for the Blue Chip dates is commensurable with the accuracy of the forecasts generated by the small-scale DSGE model. The inflation forecast of the SW model, however, are more precise than the inflation forecasts of the small-scale model, which can be attributed to a more sophisticated Phillips curve relationship and the presence of wage stickiness. The SW interest rate forecasts are slightly more accurate in the short run but slightly less precise in the long run. In the short-run the Blue Chip forecasts of output growth and inflation are more precise than the forecasts from the SW model, but for horizons $h = 5$ to $h = 8$, the DSGE model dominates. In general the DSGE model forecast errors are smaller for the Greenbook sample than for the Blue Chip sample. While the Blue Chip sample spans the period from 1992 to 2011, the forecasts for the Greenbook sample end in 2004 and thereby exclude the most recent recession. Except at the nowcast horizon, the SW model produces slightly more precise point forecasts than the Greenbook, though the differences in forecast accuracy tend to be small.

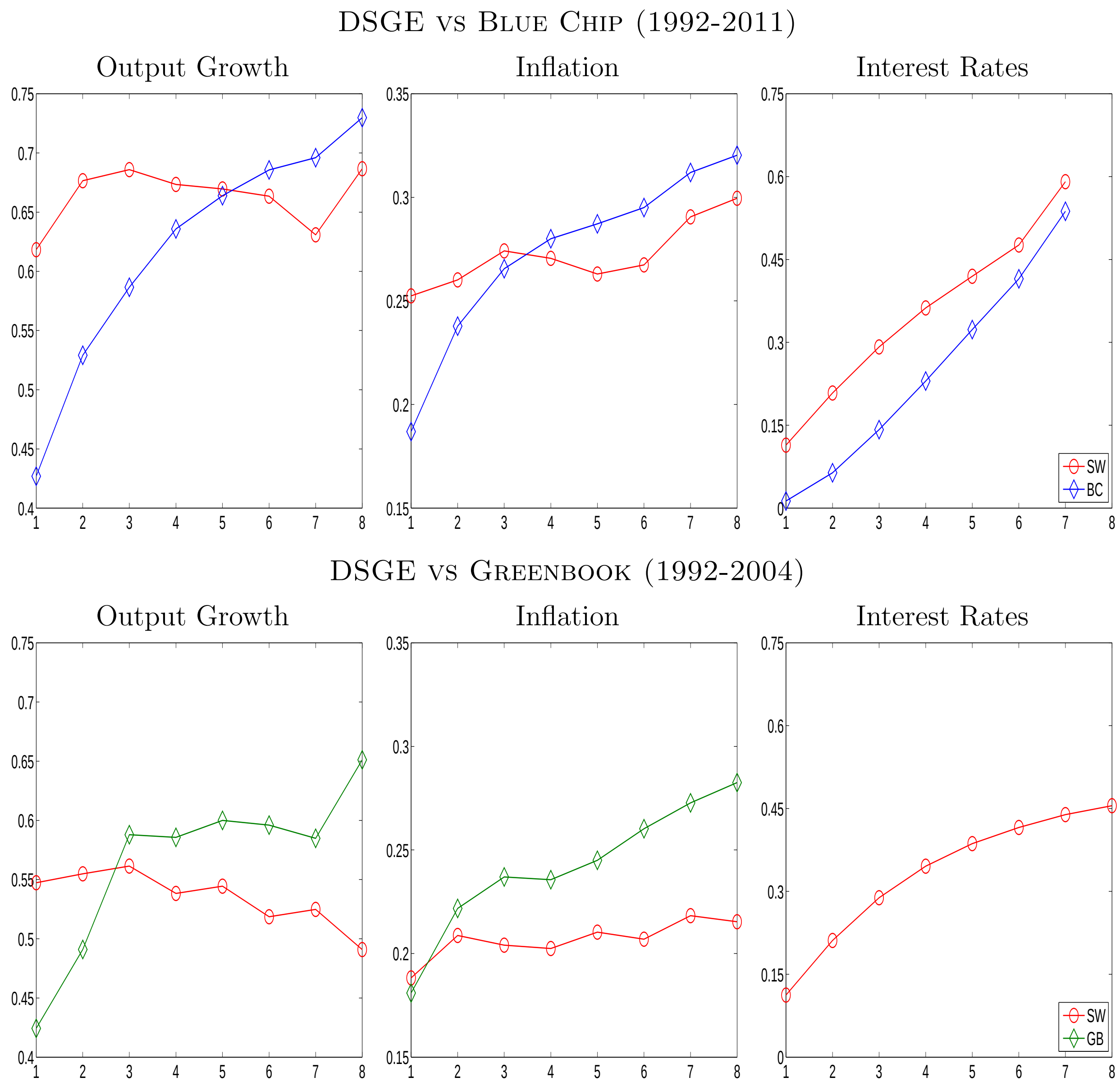
Up to this point we considered multi-step-ahead forecasts of growth rates of output and prices, as well as multi-step-ahead forecast of interest rates. Alternatively, the model can be used to forecast average growth rates and average interest rates over the next h-periods. In many instance, forecasts of averages might be more appealing than forecasts of a growth rate between period $T+h-1$ and $T+h$. RMSEs associated with forecasts of averages tend to have a different profile as a function of h. To fix ideas, suppose that y_t, say inflation,

Figure 1: RMSEs for Small-Scale Model



Notes: The top and bottom panels compare the RMSEs for the Small-Scale DSGE model (circles) with the Blue Chip (blue diamonds, top panel) and Greenbook (green diamonds, bottom panel) for one through eight quarters ahead for output growth, inflation, and interest rates. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

Figure 2: RMSEs for SW Model



Notes: The top and bottom panels compare the RMSEs for the SW DSGE model (circles) with the Blue Chip (blue diamonds, top panel) and Greenbook (green diamonds, bottom panel) for one through eight quarters ahead for output growth, inflation, and interest rates. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

evolves according to an AR(1) process

$$y_t = \theta y_{t-1} + u_t, \quad u_t \sim iidN(0,1), \quad 0 < \theta < 1. \tag{38}$$

To simplify the exposition, we will abstract from parameter uncertainty and assume that θ is known. The time T h-step forecast of y_{T+h} is given by $\hat{y}_{T+h|T} = \theta^h y_T$. The h-step ahead forecast error is given by

$$e_{T+h|T} = \sum_{j=0}^{h-1} \theta^j u_{T+h-j}. \tag{39}$$

In turn, the population RMSE is given by

$$\sqrt{\mathbb{E}[e^2_{T+h|T}]} = \sqrt{\frac{1-\theta^{2h}}{1-\theta^2}} \longrightarrow \frac{1}{\sqrt{1-\theta^2}} \text{ as } h \to \infty. \tag{40}$$

If θ is close to zero, the RMSE as a function of h is fairly flat, whereas it is strongly increasing for values of θ close to one. The RMSEs associated with the DSGE model forecasts aligned with the Blue Chip publication dates in the top panels of Figure 2 are broadly consistent with this pattern. The serial correlation of output growth and inflation is fairly small, which leads to a fairly flat, albeit slightly increasing RMSE function. Interest rates, on the other hand, follow a highly persistent process ($\theta \approx 1$), which generates RMSEs that are essentially linearly increasing in the forecast horizon.

The error associated with a forecast of an h-period average is given by

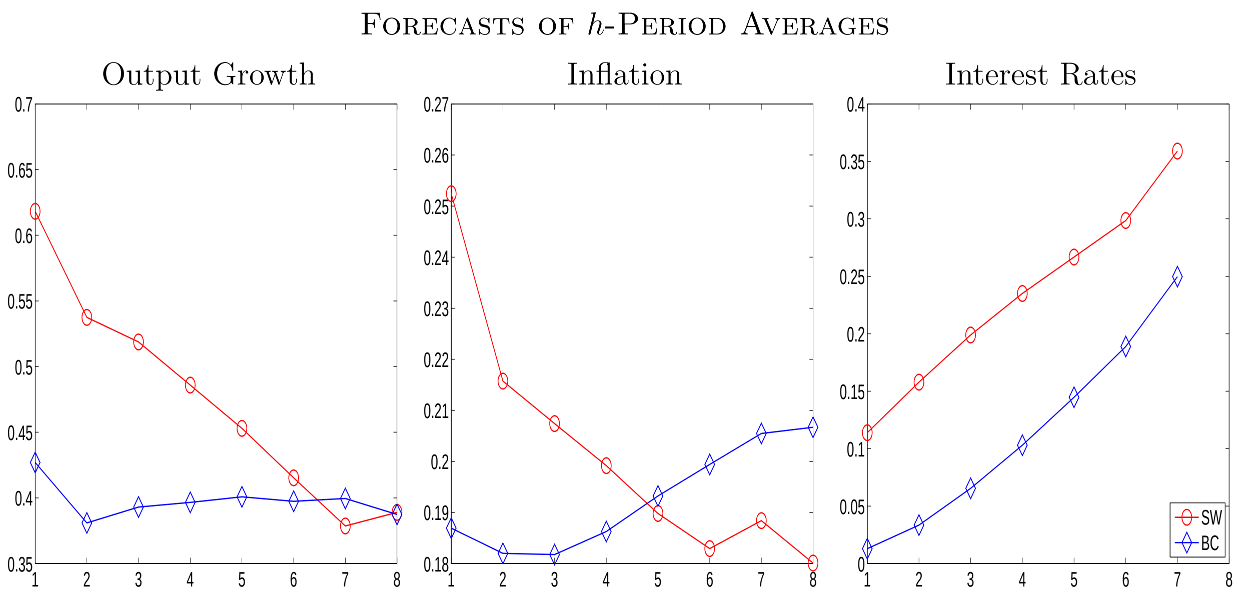
$$\bar{e}_{T+h|T} = \frac{1}{h}\sum_{s=1}^{h}\left(\sum_{j=0}^{s-1}\theta^j u_{T+s-j}\right) = \frac{1}{h}\sum_{j=0}^{h-1}\frac{1-\theta^{j+1}}{1-\theta} u_{t+h-j}. \tag{41}$$

The second equality is obtained by re-arranging terms and using the formula $\sum_{s=0}^{j-1}\theta^s = (1-\theta^j)/(1-\theta)$. The resulting population RMSE is given by

$$\sqrt{\mathbb{E}[\bar{e}^2_{T+h|T}]} = \frac{1}{\sqrt{h(1-\theta)^2}}\sqrt{1 - 2\theta\frac{1-\theta^h}{h(1-\theta)} + \theta^2\frac{1-\theta^{2h}}{h(1-\theta^2)}}. \tag{42}$$

Thus, the RMSE of the forecast of the h-period average decays at rate $1/\sqrt{h}$. Based on results from the Blue Chip sample, we plot RMSEs for the forecasts of average output growth, average inflation, and average interest rates in Figure 3. In assessing the empirical results, it is important to keep in mind that the population RMSE calculated above abstracts from parameter uncertainty and potential misspecification of the forecasting model. The GDP growth and inflation RMSEs for the DSGE model are indeed decreasing in the forecast horizon. The interest rate RMSEs remain increasing in h, but compared to Figure 2 the slope is not as steep. Since the Blue Chip forecasts are more precise at short horizons, the averaging favors the Blue Chip forecasts in the RMSE comparison.

Figure 3: RMSEs for SW Model vs Blue Chip: Forecasting Averages



Notes: The figure compares the RMSEs for the SW DSGE model (circles) with the Blue Chip forecasts (blue diamonds) for one through eight quarters-ahead averages for output growth, inflation, and interest rates. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

4.4 Literature Review of Forecasting Performance

[subsec:rmseliterature] By now there exists a substantial body of research evaluating the accuracy of point forecasts from DSGE models. Some of the papers are listed in Table 3. Many of the studies consider variants of the Smets and Wouters (2003, 2007) models. Since the studies differ with respect to the forecast periods, that is, the collection of forecast origins, as well as the choice of data vintages, direct comparisons of results are difficult. Smets and Wouters (2007) report output growth, inflation, and interest rate RMSEs of 0.57%, 0.24%, and 0.11% QoQ. The forecast period considered by Smets and Wouters (2007) ranges from 1990:Q1 to 2004:Q2 and is comparable to our Greenbook sample. The corresponding RMSEs obtained in our analysis in Section 4.3 using real-time data are 0.55%, 0.19%, and 0.11%.

In order to make the RMSE results comparable across studies we generate forecasts from a simple AR(2), using the variable definitions, forecast origins, and estimation samples that underly the studies listed in Table 3. In particular, we use real-time data whenever the original study was based on real-time data and we use the corresponding vintage for studies that were based on the analysis of a single vintage. The AR(2) model is estimated using

Table 3: A Sample of Studies Reporting RMSEs for Medium-Scale DSGE Models

Study	Forecast Origins	Real Time
Rubaszek and Skrzypczynski (2008)	1994:Q1 - 2005:Q3	Yes
Kolasa, Rubaszek, and Skrzypczyński (2010)	1994:Q1 - 2007:Q4	Yes
Graeve, Emiris, and Wouters (2009)	1990:Q1 - 2007:Q1 (h=1)	No
Wolters (2010), Del Negro-Schorfheide Model	1984:Q1 - 2000:Q4	Yes
Wolters (2010), Fuhrer-Moore Model	1984:Q1 - 2000:Q4	Yes
Wolters (2010), SW Model	1984:Q1 - 2000:Q4	Yes
Wolters (2010), EDO Model	1984:Q1 - 2000:Q4	Yes
Edge and Gürkaynak (2010)	1992:Jan - 2004:Q4	Yes
Edge, Kiley, and Laforte (2009)	1996:Sep - 2002:Q4	Yes
Smets and Wouters (2007)	1990:Q1 - 2004:Q4 (h=1)	No
Del Negro, Schorfheide, Smets, and Wouters (2007)	1985:Q4 - 2000:Q2 (h=1)	No
Schorfheide, Sill, and Kryshko (2010)	2001:Q1 - 2007:Q4 (h=1)	No

Bayesian techniques with the improper prior $p(\sigma^2) \propto (\sigma^2)^{-1}$, where σ^2 is the innovation variance.

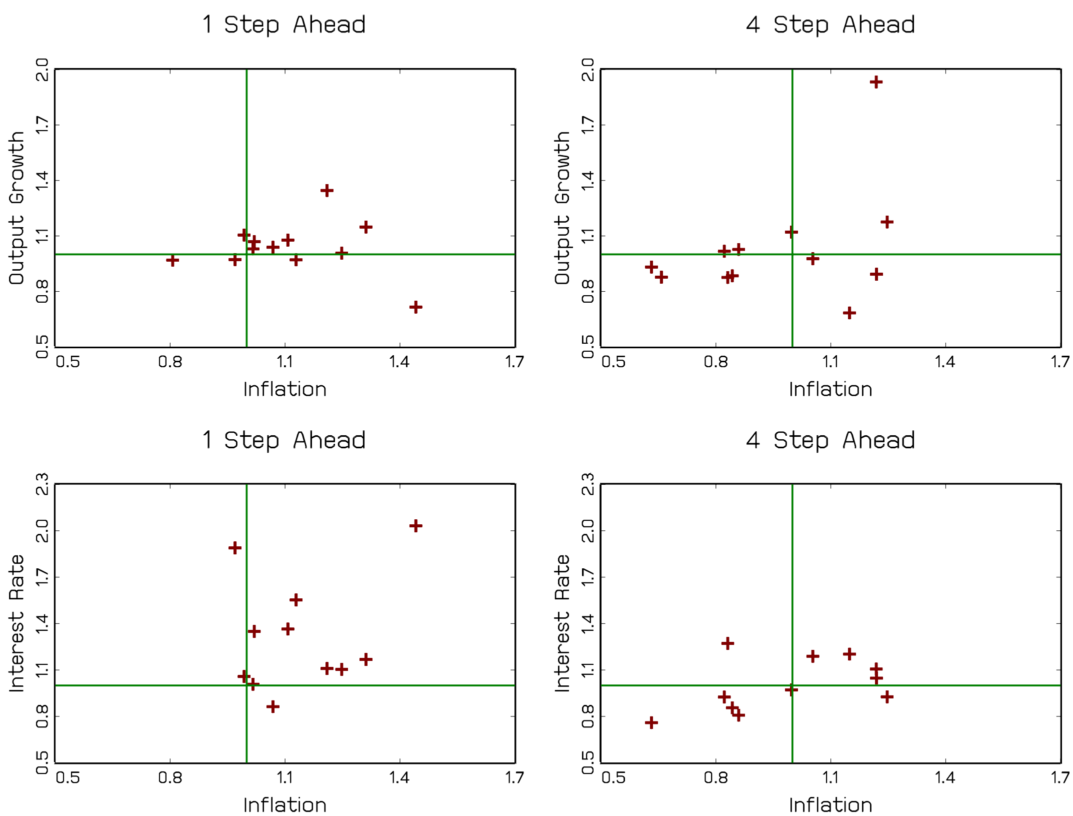
Figure 4 depicts RMSE ratios for DSGE model forecasts versus AR(2) forecasts. Each cross corresponds to one of the studies listed in Table 3. A value less than one indicates that the RMSE of the DSGE model forecast is lower than the RMSE of the benchmark AR(2) forecast. The solid lines indicate RMSE ratios of one. The top panels summarize the accuracy of output growth and inflation forecasts, whereas the bottom panel summarize the accuracy of interest rate and inflation forecasts. In general, the DSGE models perform better at the $h = 4$ horizon than at the one-quarter-ahead horizon as there are fewer observations in the upper-right quadrant.

While the one-step-ahead output growth forecasts from the DSGE models are by and large at par with the AR(2) forecasts, the bottom left panel indicates that the DSGE model inflation and interest rate forecasts in general tend to be worse than the AR(2) forecasts. At the one-year horizon, more than half of the DSGE model output growth forecasts are more accurate than the corresponding AR(2) forecasts. One outlier (the RMSE ratio is close to 2.0) is the output growth RMSE reported in Del Negro, Schorfheide, Smets, and Wouters (2007), which is computed from an infinite-order VAR approximation of the state-space representation of the DSGE model. Growth rate differentials between output, investment, consumption, and real wages might contribute to the poor forecast performance of the DSGE model. Finally, about half of the estimated DSGE models considered here are able to produce inflation and interest rate forecasts that attain a lower RMSE than the AR(2) forecasts.

Our interpretation of Figure 4 is that DSGE model forecasts can be competitive in terms of accuracy with simple benchmark models, in particular for medium-run forecasts. This statement, however, has two qualifications. First, the DSGE model needs to carefully specified to optimize forecast performance. Second, if the AR(2) model is replaced by a statistical model that is specifically designed to forecast a particular macroeconomic time series well, DSGE model forecasts can be dominated in terms of RMSEs by other time series models.

Many of the papers in the DSGE model forecasting literature offer direct comparisons of DSGE model forecasts to other forecasts. Edge and Gürkaynak (2010) compare univariate forecasts from the SW model estimated with real-time data against forecasts obtained from

Figure 4: RMSEs Reported in the Literature



Notes: Figure depicts RMSE ratios: DSGE (reported in various papers) / AR(2) (authors calculation).

the staff of the Federal Reserve, the Blue Chip survey, and a Bayesian vector autoregression (VAR). Based on RMSEs, they conclude that the DSGE model delivers forecasts that are competitive in terms of accuracy with those obtained from the alternative prediction methods. Comparisons between DSGE model and professional forecasts are also reported in Wieland and Wolters (2011) and Wieland and Wolters (2012). The evidence from Euro Area data is similar. Adolfson, Lindé, and Villani (2007) assess the forecasting performance of an Open Economy DSGE model during the period of 1994 to 2004 based on RMSEs, log determinant of the forecast-error covariance matrix[6], predictive scores, and the coverage frequency of interval forecasts. Overall, the authors conclude that the DSGE model compares well with more flexible time series models such as VARs.

Christoffel, Coenen, and Warne (2010) examine the forecasting performance of the New Area Wide Model (NAWM), the DSGE model used by the European Central Bank. The authors evaluate the model's univariate forecast performance through RMSEs and its multivariate performance using the ln-det statistic. They find that the DSGE model is competitive with other forecasting models such as VARs of various sizes. The authors also find that the assessment of multivariate forecasts based on the ln-det statistic can sometimes be severely affected by the inability to forecast just one series, nominal wage growth.

The Bayesian VARs that serve as a benchmark in the aforementioned papers use a Minnesota prior but are typically not optimized with respect to their empirical performance. For instance, some of the dummy observations described in Sims and Zha (1998) and more recently discussed in Del Negro and Schorfheide (2010) that generate *a priori* correlations among VAR coefficients and have been found useful for prediction have been excluded from the construction of the prior distribution. Del Negro and Schorfheide (2004) and DSSW compare the forecasting performance of a three-equation New Keynesian DSGE model and a variant of the SW model to Bayesian VARs that use a prior distribution centered at the DSGE model restrictions. Both papers find that the resulting DSGE-VAR forecasts significantly better than the underlying DSGE model.

In addition to comparing point forecasts across different models, Edge and Gürkaynak

[6]The so-called "ln-det" statistic had been proposed by Doan, Litterman, and Sims (1984). The eigenvectors of the forecast error covariance matrix generate linear combinations of the model variables with uncorrelated forecast errors. The determinant equals the product of the eigenvalues and thereby measures the product of the forecast error variances associated with these linear combinations. The more linear combinations exist that can be predicted with small forecast error variance, the smaller the ln-det statistic.

(2010) also examine the overall quality of DSGE model forecasts. To do so, they estimate regressions of the form

$$y_{i,t} = \alpha^{(h)} + \beta^{(h)} \hat{y}_{i,t|t-h} + e_t^{(h)}. \tag{43}$$

If the predictor $\hat{y}_{i,t|t-h}$ is the conditional mean of $y_{i,t}$ then the estimate of $\alpha^{(h)}$ should be close to zero and the estimate of $\beta^{(h)}$ close to one. In the simple AR(1) example in Equation (38) of Section 4.3 the residual $e_t^{(h)}$ would be equal to the h-step-ahead forecast error $e_{t|t-h}$ in (39) and the population R^2 of the regression (43) would be θ^{2h}. For inflation forecasts of varying horizons h Edge and Gürkaynak (2010) find that $\alpha^{(h)}$ is significantly positive, $\beta^{(h)}$ is significantly less than one, and R^2 is near zero. The output growth forecasts are better behaved in that the authors are unable to reject the hypotheses that $\alpha^{(h)} = 0$ and $\beta^{(h)} = 1$. Moreover, R^2 is between 0.07 and 0.2. While the fairly low R^2 is qualitatively consistent with the low persistent in inflation and output growth during the forecasting period, the estimates of $\alpha^{(h)}$ and $\beta^{(h)}$ indicate that the DSGE model forecasts are deficient.

Herbst and Schorfheide (2011) examine whether the realized pseudo-out-of-sample RMSE of DSGE model forecasts is commensurable with the RMSE that would be expected given the posterior distribution of DSGE model parameters. By simulating the estimated DSGE model and then generating recursive forecasts on the simulated trajectories, one can obtain a DSGE model-implied predictive distribution for RMSEs. The authors find that for a small-scale DSGE model, similar to the model of Section 2.3, the actual RMSEs of output and inflation forecasts are within the bands of the predictive distribution. The actual interest rate RMSEs, on the other hand, exceed the predictive bands, indicating a deficiency in the law of motion of the interest rate. For the Smets and Wouters (2007) model, the inflation and interest rate RMSEs fall within the bands of the predictive distribution, but the realized output growth RMSE is smaller than the RMSE predicted by the model. A possible explanation is that some of the estimated shock processes are overly persistent because they need to absorb violations of the balanced growth path restrictions of the DSGE model. This would lead to excess volatility in the simulated output paths.

To summarize, the empirical evidence supports our claim that DSGE model forecasts are comparable to standard autoregressive or vector autoregressive models but can be dominated by more sophisticated univariate or multivariate time series. Nonetheless DSGE models present advantages relative to reduced form models as tools for predictions because they provide an intelligible econonomic story for their projections, as we discuss in Section 7.

Moreover, these models also provide a framework for policy analysis. In the forecasting context this is important as they can be used to make projections based on alternative paths for the policy instrument (see Section 6).

5 DSGE Model Forecasts using External Information

[sec:externalinfo] In the previous section we generated baseline forecasts from two DSGE models. For the small-scale model these forecasts were based on output growth, inflation, and interest rate data. For the SW model we also used data on consumption, investment, hours worked, and real wages. However, these series reflect only a subset of the information that is available to a forecaster in real time. While quarterly NIPA data are released with a lag of more than four weeks, other series, e.g. interest rates, are observed at a much higher frequency and without publication lag. Thus, in this section we present methods of improving DSGE model forecasts by incorporating what we call external information. This external information can take various forms. We consider long-run inflation expectations (Section 5.1), long-run output growth expectations (Section 5.2), nowcasts of output and inflation from professional forecasters (Section 5.3), as well as expectations of the short-term interest rate over various horizons (Section 5.4).

Two distinctly different approaches of incorporating the external information are considered. First, in Sections 5.1, 5.2, and 5.4 we treat some of the (rational) expectations held by agents within the DSGE model as observable and equate them with multi-step forecasts published by the Blue Chip Survey. Discrepancies between DSGE-model implied expectations and professional forecasts are bridged by introducing additional structural shocks into the DSGE models presented in Section 2: shocks to the target inflation rate, the growth rate of technology, and anticipated monetary policy shocks. Second, in Section 5.3 we consider methods that amount to interpreting nowcasts from professional forecasters as a noisy measure of (or as news about) current quarter macroeconomic variables. In turn, the external nowcasts provide information about the exogenous shocks that hit the economy in the current quarter and thereby alter the DSGE model forecasts. These methods do not require the DSGE model to be modified and augmented by additional structural shocks.

5.1 Incorporating Long-Run Inflation Expectations

[subsec:inflationexpectations] The level of inflation and interest rates has shifted substantially in the post-war period. In our DSGE models the estimated target inflation rate roughly corresponds to the sample average of the inflation rate. If the sample includes observations from the 70s and 80s, then this sample average tends to be higher than what can be thought of as a long-run inflation target of the past decade, which is around 2%. In turn, this leads to a poor forecast performance.

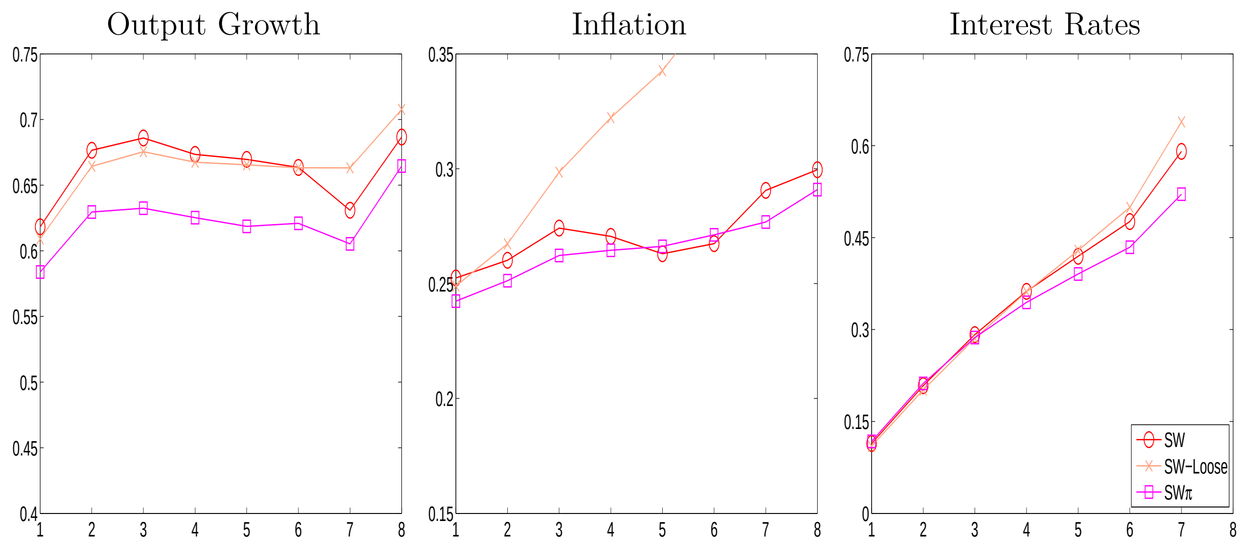
In Figure 5 we are plotting the RMSE of the output growth, inflation, and interest rate forecasts from the SW model under the prior distribution used in Smets and Wouters (2007) as well as an alternative prior. The original prior for the quarterly steady state inflation rate used by Smets and Wouters (2007) is tightly centered around 0.62% (which is about 2.5% annualized) with a standard deviation of 0.1%. Our alternative prior is centered at 0.75% and is less dogmatic with a standard deviation of 0.4% (see Panel II of Table 1). We refer to the model with "loose" prior as SW-Loose. Under the Smets and Wouters (2007) prior the estimated target inflation rate is around 2.7% to 3.0%, whereas the "loose" prior yields posterior estimates in the range of 4% to 5% As a consequence, the medium-run forecast accuracy is worse for the SW-Loose model than for the SW model, in particular for inflation but also for interest rates and output growth.

The forecast inaccuracy caused by the gradual decline of inflation and interest rates post 1980 had been recognized by Wright (2011), who proposed to center the prior distribution for the vector of long-run means in a Bayesian VAR at the five-to-ten year expectations of professional forecasters. This approach turned out to be particularly helpful for inflation forecasts, because of the ability of survey forecasts to capture shifting end points. Faust and Wright (2011) use a similar approach to improve inflation forecasts from a DSGE model. Instead of simply centering a tight prior for π_* with hindsight, they center the prior at the most recent long-run inflation forecast.

Our approach is similar in spirit to Faust and Wright (2011), but differs in regard to the implementation. In order to capture the rise and fall of inflation and interest rates in the estimation sample we replace the constant target inflation rate by a time-varying target inflation. While time-varying target rates have been frequently used for the specification of monetary policy rules in DSGE model (e.g., Erceg and Levin (2003), Smets and Wouters

Figure 5: Using Inflation Expectations

SW vs SW with "loose" prior (SW-Loose)
vs SW with "loose" prior and long run inflation expectations (SWπ)



Notes: The figure compares the one through eight quarters-ahead RMSEs for the SW DSGE model (SW, circles) with the SW model with a "loose" prior on the parameter π_* (SW-Loose, crosses) and the SW model with observed long run inflation expectations (SWπ, squares) for output growth, inflation, and interest rates. The comparison is done for the same vintages/forecast dates as the Blue Chip/DSGE comparison discussed in Section 4.3. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

(2003) and Justiniano, Primiceri, and Tambalotti (2009), among others), we follow the approach of Aruoba and Schorfheide (2010) and Del Negro and Eusepi (2011) and include data on long-run inflation expectations as an observable into the estimation of the DSGE model. At each point in time, the long-run inflation expectations essentially determine the level of the target inflation rate.

More specifically, for the SW model the interest-rate feedback rule of the central bank (16) is modified as follows:[7]

$$\begin{aligned} R_t &= \rho_R R_{t-1} + (1-\rho_R)\Big(\psi_1(\pi_t - \pi_t^*) + \psi_2(y_t - y_t^f)\Big) \\ &\quad + \psi_3\Big((y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)\Big) + r_t^m. \end{aligned} \tag{44}$$

[7]We follow the specification in Del Negro and Eusepi (2011), while Aruoba and Schorfheide (2010) assume that the inflation target also affects the intercept in the feedback rule.

The time-varying inflation target evolves according to:

$$\pi^*_t = \rho_{\pi^*}\pi^*_{t-1} + \sigma_{\pi^*}\epsilon_{\pi^*,t}, \tag{45}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{\pi^*,t}$ is an iid shock. We follow Erceg and Levin (2003) and model π^*_t as following a stationary process, although our prior for ρ_{π^*} will force this process to be highly persistent (see Panel III of Table 1). The set of measurement equations (17) is augmented by

$$\begin{aligned} \pi^{O,40}_t &= \pi_* + 100 I\!E_t\left[\frac{1}{40}\sum_{k=1}^{40}\pi_{t+k}\right] \qquad (46)\\ &= \pi_* + \frac{100}{40}\Psi_2(\theta)_{(\pi,.)}(I-\Phi_1(\theta))^{-1}\left(\Phi_1(\theta)-\Phi_1(\theta)^{41}\right)s_t, \end{aligned}$$

where $\pi^{O,40}_t$ represents observed long run inflation expectations obtained from surveys (in percent per quarter), and the right-hand-side of (46) corresponds to expectations obtained from the DSGE model (in deviation from the mean π_*). The second line shows how to compute these expectations using the transition equation (33), where $\Psi_2(\theta)_{(\pi,.)}$ is the row of the matrix $\Psi_2(\theta)$ entering the measurement equation (34) corresponding to inflation.

The long-run inflation forecasts are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters (SPF) available from the FRB Philadelphia. Long-run inflation expectations (average CPI inflation over the next 10 years) are available from 1991:Q4 onwards. Prior to 1991:Q4, we use the 10-year expectations data from the Blue Chip survey to construct a long time series that begins in 1979:Q4. Since the Blue Chip survey reports long-run inflation expectations only twice a year, we treat these expectations in the remaining quarters as missing observations and adjust the measurement equation of the Kalman filter accordingly. Long-run inflation expectations $\pi^{O,40}_t$ are therefore measured as

$$\pi^{O,40}_t = (\text{10-YEAR AVERAGE CPI INFLATION FORECAST} - 0.50)/4.$$

where .50 is the average difference between CPI and GDP annualized inflation from the beginning of the sample to the 1992, the starting point for our forecasting exercise, and where we divide by 4 since the data are expressed in quarterly terms.

Importantly from a real-time forecasting perspective, the inflation expectation data used in the DSGE model estimation is available to both Blue Chip and Greenbook forecasters by the time they make their forecasts. The timing of the SPF Survey is geared to the release of the BEA's Advance NIPA report, which is released at the end of the first month of each quarter. This implies that, for instance, when producing DSGE forecasts with T =1991:Q4 we use long-run inflation expectation data that is public by the end of January 1992, that is, well before the associated Greenbook and Blue Chip forecasts are made (March and April 1992, respectively, see Table 2).

RMSEs from the modified SW model with time-varying inflation target and inflation expectation data, henceforth SWπ, are also plotted in Figure 5. While the RMSEs associated with forecasts from the SWπ model are only slightly lower than those of the SW model, the former is much more appealing because it is not based on a prior distribution that from an *a priori* perspective is rather tight. Moreover, the SWπ is much more flexible. If the average level of inflation as well as inflation expectations will rise again in the future, then the estimated inflation target will increase and the forecasts will adapt to a higher level of inflation.

5.2 Incorporating Output Expectations

[subsec:outputexpectations] Over the past six decades the U.S. economy has experienced several shifts in the long-run growth rates of productivity and output, e.g. the productivity slowdown of the 1970s. While the changes in long-run growth rates are not as pronounced as the changes in the inflation rate during the late 1970s and early 1980s, capturing low frequency movements in productivity growth is potentially important for DSGE model forecasts of output. Thus, we now introduce long-run output growth expectations as an observable variable in the DSGE model following the same approach we used for the long-run inflation expectations in Section 5.1

The measurement equations are augmented with an expression equating the model-implied long-run output expectation with the long-run growth expectations data obtained from a combination of Blue Chip Financial Forecasts (BCFF), Blue Chip Economic Indicators (BCEI), Livingstone Survey, and the SPF:[8]

$$Growth_t^{O,40} = \gamma + 100 I\!E_t \left[\frac{1}{40} \sum_{k=1}^{40} (y_{t+k} - y_{t+k-1} + z_{t+k}) \right], \tag{47}$$

where $Growth_t^{O,40}$ represents the observed long-run-growth expectation (in percent per quarter) obtained from the two surveys and the right-hand-side of (47) is the corresponding expectation computed from the DSGE model. 10-year GDP forecasts are given in aggregate annualized growth rates. They are transformed into quarterly per capita rates using the 5-year (backward looking) moving average of the population series from the ESS Household Survey:

$$\begin{aligned} Growth_t^{O,40} &= \text{10-YEAR AVERAGE GDP GROWTH FORECAST}/4 \\ &\qquad - 100 * (LN(LNSINDEX/LNSINDEX(-20))/20). \end{aligned}$$

In order to generate time-variation in the DSGE model's implied long-run output growth expectations we introduce very persistent changes to the growth rate of productivity in the SW model described in (2). Specifically, we assume that z_t, the growth rate of the stochastic trend Z_t in deviations from γ, follows the process:

$$z_t = \log(Z_t/Z_{t-1}) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t} + z_t^p, \tag{48}$$

where

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p}\epsilon_{z^p,t}. \tag{49}$$

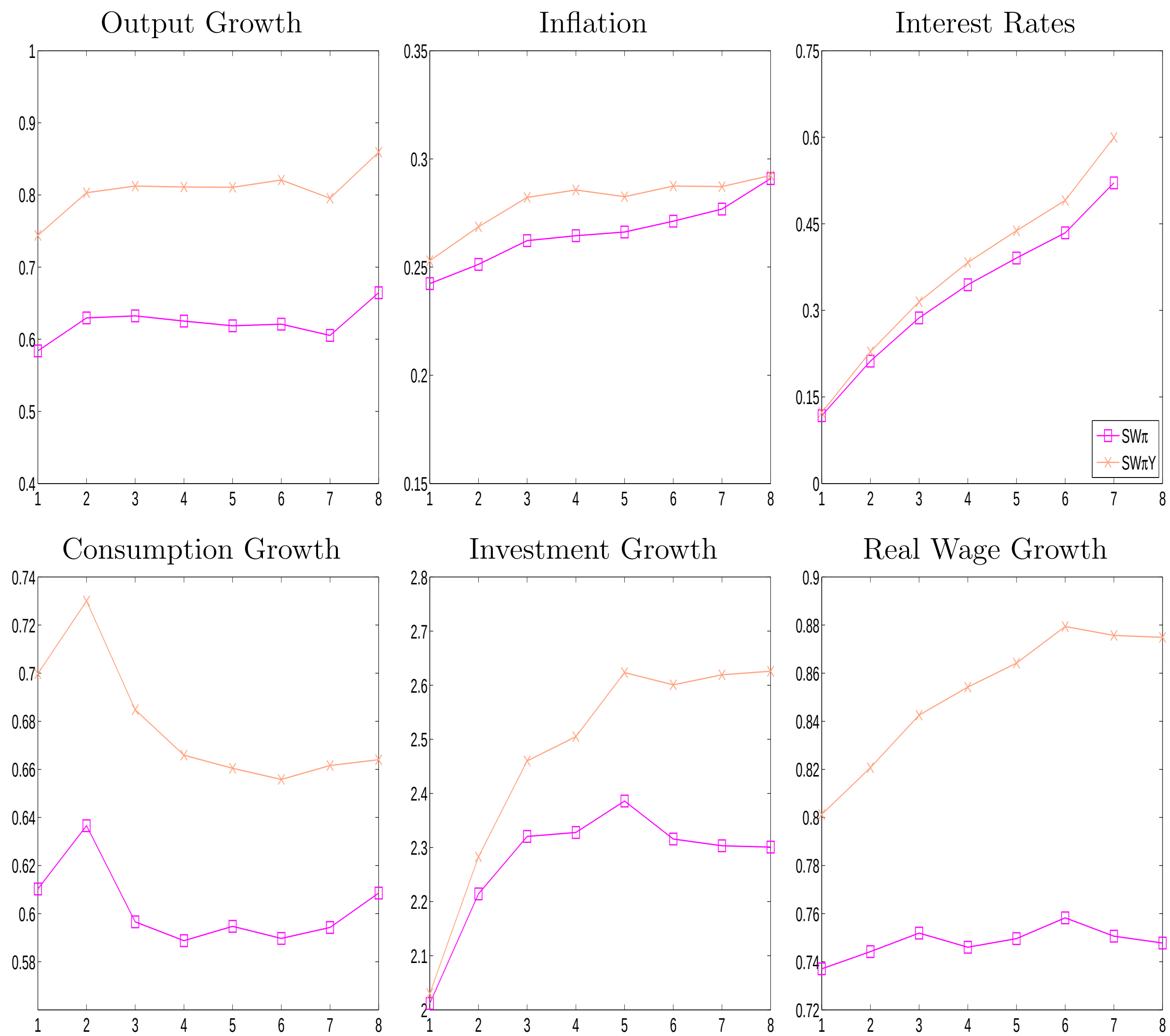
The prior for ρ_{z^p} is chosen to ensure that the local level process z_t^p is highly persistent (see Panel IV of Table 1).

RMSEs for the SWπ model versus the DSGE model with inflation and output expectations, denoted by SWπY, are depicted in Figure 6. Unlike the incorporation of long-run inflation expectations, the use of long-run output growth expectations does not lead to an

[8] We are very grateful to Stefano Eusepi and Emanuel Mönch for providing us with this data, which are described in Eusepi and Mönch (2011). Specifically, Eusepi and Mönch obtain a monthly time series of 10-years ahead output growth forecasts using the data from SPF for February, BCFF for May and November, BCEI for March and October, the Livingston survey for June and December. We take quarterly averages of the monthly data, whenever available. We adjust the observation equation in the Kalman filter to deal with missing observations.

Figure 6: Using Inflation and Output Expectations

SW with long run inflation expectations (SWπ) vs
SW with long run inflation and output growth expectations (SWπY)



Notes: The figure compares the one through eight quarters-ahead RMSEs for the SW model with observed long run inflation expectations (SWπ, squares) and the SW model with observed long run inflation and output growth expectations (SWπY, crosses) for output growth, inflation, interest rates, consumption, investment, and real wage growth. The comparison is done for the same vintages/forecast dates as the Blue Chip/DSGE comparison discussed in Section 4.3. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

improvement in the forecast performance of the DSGE model. In fact, the forecasts for output, consumption, investment, and real wage growth deteriorated substantially. While the SWπ model attains an RMSE for output growth of 0.58% at $h = 1$ and 0.67% at $h = 8$, the corresponding RMSEs for the SWπY model are 0.74% and 0.86%. Including long-run output growth expectations does not improve DSGE model forecasts partly because these forecasts appear to be overly optimistic. Of course, we do not have a large-enough sample to test the accuracy of 10-years ahead forecasts. However, we observe that the eight-quarter ahead forecast bias (defined as actual value minus forecast) for output growth is -0.5% for the SWπY model, whereas it is only -0.1% for the SWπ model.

5.3 Conditioning on External Nowcasts

[subsec:externalnowcasts] As explained in Section 4.1, the NIPA data that enter the estimation of the DSGE model only become available with a lag of more than four weeks. During this time period, a lot of other important information about the state of the economy is released, e.g. interest rates, unemployment rates, inflation data. Some of this information is implicitly incorporated in the current quarter forecasts surveyed by Blue Chip because the professional forecasters included in the survey are not required to use quarterly-frequency time series models and potentially make subjective adjustments to model-based forecasts in view of high frequency economic data. In this section we use the nowcasts obtained from the Blue Chip survey to improve the forecasts from the SWπ DSGE model. We proceed in four steps. First, the timing of the nowcast release is described. Second, we consider two approaches of incorporating external information: (i) nowcasts are interpreted as noisy measures of variables dated $T+1$ (recall that T corresponds to the end of estimation sample and beginning of forecasting origin); (ii) nowcasts are interpreted as news about $T+1$ data. We provide algorithms to generate draws from the predictive density of the DSGE models under these two interpretations of external information. Third, using the noise interpretation, nowcasts are incorporated into forecasts from the SWπ model. Finally, we discuss alternative methods that have been proposed in the literature.

To fix ideas, the timing of the DSGE model forecasts and the Blue Chip nowcasts for the year 1992 is illustrated in Table 4. Columns 1 and 2 of Table 4 are identical to Columns 1 and 3 of Table 2. Consider for instance the forecast origin that corresponds to the July 1992 Blue Chip release. Due to the timing of the NIPA GDP release the estimation sample ends in 1992:Q1. In our notation, the first quarter of 1992 corresponds to period T. We modify the DSGE model forecast by incorporating the nowcast for 1992:Q2 ($T+1$) published in July 1992. To fix notation, assume that the variables used for the DSGE model estimation

Table 4: Blue Chip Forecast Dates and Nowcast Information for 1992

Forecast Origin	End of Est. Sample T	External Nowcast $T+1$	Forecast $h=1$	$h=2$
Apr 92	91:Q4	92:Q1 based on Apr 92 BC	92:Q1	92:Q2
Jul 92	92:Q1	92:Q2 based on Jul 92 BC	92:Q2	92:Q3
Oct 92	92:Q2	92:Q3 based on Oct 92 BC	92:Q3	92:Q4
Jan 93	92:Q3	92:Q4 based on Jan 93 BC	92:Q4	93:Q1

are partitioned into $y'_{T+1} = [y'_{1,T+1}, y'_{2,T+1}]$, where $y_{1,T+1}$ is the subvector for which external information z_{T+1} is available. The $T+1$ subscript highlights that the information in z pertains to $t = T+1$ variables.

To understand how external information alters the DSGE model forecasts, consider the following factorization of the one-step-ahead predictive density:

$$p(y_{T+1}|Y_{1:T}) = \int_\theta \left[\int_{s_T, s_{T+1}} p(y_{T+1}|s_{T+1}, \theta) p(s_T, s_{T+1}|\theta, Y_{1:T}) d(s_T, s_{T+1}) \right] p(\theta|Y_{1:T}) d\theta. \quad (50)$$

We adopted the timing convention that the Blue Chip nowcasts z_{T+1} become available after period T, but prior to the release of y_{T+1}. In view of (50), z_{T+1} provides information about the latent states (s_{T+1}, s_T) and the DSGE model parameters θ. Thus, $p(s_{T+1}, s_T|\theta, Y_{1:T})$ and $p(\theta|Y_{1:T})$ should be replaced by $p(s_{T+1}, s_T|\theta, Y_{1:T}, z_{T+1})$ and $p(\theta|Y_{1:T}, z_{T+1})$, respectively. In the remainder of this section we focus on $p(s_{T+1}, s_T|\theta, Y_{1:T}, z_{T+1})$, assuming that

$$p(\theta|Y_{1:T}, z_{T+1}) \approx p(\theta|Y_{1:T}). \quad (51)$$

Thus, unlike in the work on conditional forecasting with Bayesian VARs by Waggoner and Zha (1999), we disregard the information contents of the external nowcasts with respect to the model parameters θ. This assumption is compelling in applications in which the information in the sample $Y_{1:T}$ and prior distribution strongly dominates the information contained in z_{T+1}. For the SWπ model considered below, the shortest estimation sample contains about 110 observations for 8 macroeconomic time series, whereas z_{T+1} is comprised of only 3 observations.

We now turn our attention to the construction of $p(s_{T+1}, s_T|\theta, Y_{1:T}, z_{T+1})$. Since we adopted the convention that z_{T+1} provides information about $y_{1,T+1}$ we can write without loss of generality

$$y_{1,T+1} = z_{T+1} + (y_{1,T+1} - z_{T+1}) = z_{T+1} + \eta_{T+1}. \tag{52}$$

An assumption about the joint distribution of z_{T+1} and η_T determine the joint distribution of $y_{1,T+1}$ and z_{T+1}. For now, we consider two specific assumptions that we classify as *Noise* and *News*. Under the *Noise* assumption the external information z_{T+1} is interpreted as a noisy measure of $y_{1,T+1}$, that is

$$Noise: \quad z_{T+1} = y_{1,T+1} - \eta_{T+1}, \quad y_{1,T+1} \perp \eta_{T+1}. \tag{53}$$

Here η_{T+1} is a measurement error that is independent ($\perp$) of the actual value $y_{1,T+1}$. Under the *News* assumption it is the nowcast z_{T+1} that is independent of the error term η_{T+1}

$$News: \quad y_{1,T+1} = z_{T+1} + \eta_{T+1}, \quad z_{T+1} \perp \eta_{T+1}. \tag{54}$$

Such a correlation structure arises if, for instance, z_{T+1} is a conditional expectation of $y_{1,T+1}$ given $Y_{1:T}$ and other information.

The *Noise* assumption can be easily incorporated into the Kalman-filter-based analysis of the DSGE model. After the time T Kalman filter iterations have been completed and $p(s_T|Y_{1:T}, \theta)$ has been computed, (53) is used as period $T+1$ measurement equation. This leads to the following algorithm:

Algorithm 3. Draws from the Predictive Distribution Conditional on External Nowcast (Noise Assumption). [algo:preddrawsnowcastnoise] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$.

2. In period $T+1$ use Equation (53) as measurement equation for the nowcast z_{T+1} assuming $\eta_{T+1} \sim N(0, \sigma_\eta^2)$. Use the Kalman filter updating to compute $p(s_{T+1}|\theta^{(j)}, Y_{1:T}, z_{T+1})$ and generate a draw $s_{T+1}^{(j)}$ from this distribution.

3. Draw a sequence of innovations $\epsilon_{T+2:T+H}^{(j)}$ and, starting from $s_{T+1}^{(j)}$, iterate the state transition equation (33) forward to obtain the sequence $S_{T+2:T+H}^{(j)}$.

4. Use the measurement equation (34) to compute $Y_{T+1:T+H}^{(j)}$ based on $S_{T+1:T+H}^{(j)}$. □

So far, we have assumed that the external information only pertains to observations dated $T+1$. Algorithm 3 has a straightforward generalization to the case in which the external information spans multiple horizons, e.g. $T+1,\dots,T+\bar{H}$. Denoting this information by $Z_{T+1:T+\bar{H}} = \{z_{T+1},\dots,z_{T+\bar{H}}\}$, Step 2 can be replaced by using the simulation smoother described in Carter and Kohn (1994)) to generate a draw $S^{(j)}_{T+1:T+\bar{H}}$ from $p(S_{T+1:T+\bar{H}}|\theta^{(j)}, Y_{1:T}, Z_{T+1:T+\bar{H}})$. Associated with each simulated sequence of latent states $S^{(j)}_{T+1:T+\bar{H}}$ is a sequence of structural shocks $\epsilon^{(j)}_{T+1:T+\bar{H}}$. The distribution of the structural shocks conditional on the external information has no longer mean zero. Thus, an external nowcast of output growth that is larger than the DSGE model forecast might be rationalized by a particular combination of technology, government spending, and monetary policy shocks.[9]

According to the *News* assumption in (54), the nowcast is interpreted as a predictive distribution for $y_{1,T+1}$ that incorporates both the information $Y_{1:T}$ used in the DSGE model estimation as well as some additional, not explicitly specified information that has been processed by the professional forecasters included in the Blue Chip survey. We will describe an algorithm that is based on the following representation of the predictive density

$$\begin{aligned} p(y_{T+1}|Y_{1:T}, z_{T+1}) \;&=\; \int_\theta \Bigg[\int_{\tilde{y}_{1,T+1}} p(y_{T+1}|s_{T+1},\theta) p(s_{T+1}|\tilde{y}_{1,T+1}, Y_{1:T},\theta) \\ &\quad \times p(\tilde{y}_{1,T+1}|Y_{1:T}, z_{T+1}) d\tilde{y}_{1,T+1} \Bigg] p(\theta|Y_{1:T}). \end{aligned} \tag{55}$$

We assume that conditional on the Blue Chip nowcast $Y_{1:T}$ contains no additional information that is useful for predicting $y_{1,T+1}$, that is,

$$p(\tilde{y}_{1,T+1}|Y_{1:T}, z_{T+1}) = p(\tilde{y}_{1,T+1}|z_{T+1}) \tag{56}$$

and the density on the right-hand-side is given by (54). The density $p(s_{T+1}|\tilde{y}_{1,T+1}, Y_{1:T},\theta)$ in (55) captures the information about the latent state s_{T+1}, accounting through $\tilde{y}_{1,T+1}$ for the information contained in z_{T+1}. Since the DSGE model is represented as a linear Gaussian state-space model, the one-step-ahead forecast generated by (55) of $y_{1,T+1}$ equals z_{T+1}. The following algorithm implements the conditional forecast.

[9] Beneš, Binning, and Lees (2008) interpret the likelihood of the structural shocks that are needed to attain the path of observables implied by the external information as a measure of how plausible this external information is in view of the model.

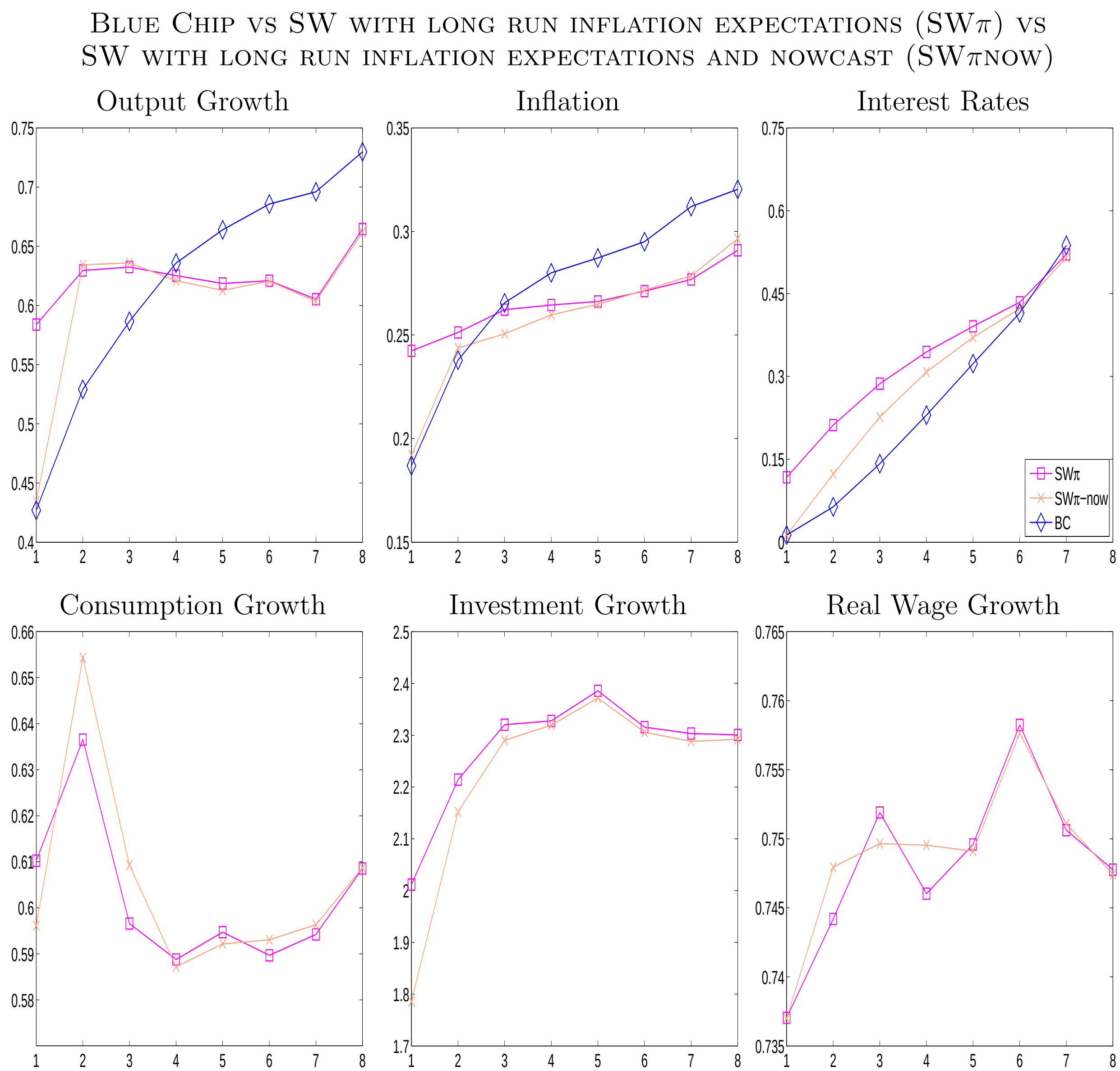
Algorithm 4. Draws from the Predictive Distribution Conditional on External Nowcast (News Assumption). [algo:preddrawsnowcastnews] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$.
2. Generate a draw $\tilde{y}^{(j)}_{1,T+1}$ from the distribution $p(\tilde{y}_{1,T+1}|Y_{1:T}, z_{T+1})$ using (54), assuming $\eta_{T+1} \sim N(0, \sigma^2_\eta)$.
3. Treating $\tilde{y}^{(j)}_{1,T+1}$ as observation for $y_{1,T+1}$ use the Kalman filter updating step to compute $p(s_{T+1}|\theta^{(j)}, Y_{1:T}, \tilde{y}^{(j)}_{1,T+1})$ and generate a draw $s^{(j)}_{T+1}$ from this distribution.
4. Draw a sequence of innovations $\epsilon^{(j)}_{T+2:T+H}$ and, starting from $s^{(j)}_{T+1}$, iterate the state transition equation (33) forward to obtain the sequence $S^{(j)}_{T+2:T+H}$.
5. Use the measurement equation (34) to obtain $Y^{(j)}_{T+1:T+H}$ based on $S^{(j)}_{T+2:T+H}$.

Using (56) in Step 2 of Algorithm 4, we impose that $\tilde{y}_{1,T+1} \sim N(z_{T+1}, \sigma^2_\eta)$. This step can be modified to allow for a more general conditional distribution of $\tilde{y}_{1,T+1}$. For instance, instead of imposing that the conditional mean of $\tilde{y}_{1,T+1}$ equals the Blue Chip nowcast z_{T+1}, one could use a weighted average of the Blue Chip nowcast and the one-step-ahead DSGE model forecast from $p(y_{1,T+1}|Y_{1:T}, \theta)$. Finally, hard conditioning on external nowcasts, i.e. imposing the equality $y_{1,T+1} = z_{T+1}$, can be implemented by setting $\sigma^2_\eta = 0$. In this case Algorithms 3 and 4 are identical.

We now use Algorithm 3 to incorporate information from Blue Chip nowcasts of output growth, inflation, and interest rates into the DSGE model forecasts. We refer to the resulting forecasts as SWπ-now. The vector of measurement errors σ^2_η in (53) associated with the Blue Chip nowcasts are calibrated to match the size of the nowcast error. Figure 7 depicts RMSEs for SWπ and SWπ-now forecasts as well as the Blue Chip forecasts. The top panels of the figure depict RMSEs for output growth, inflation, and interest rates, which are the three series for which we add external information. At the nowcast horizon $h = 1$ the RMSEs associated with SWπ-now and Blue Chip forecasts are essentially identical and dominate the SWπ forecasts by a considerable margin. The nowcasts reduce the RMSEs of output growth forecasts horizon $h = 1$ from 0.58% to 0.43%, but essentially have no effect on RMSEs

Figure 7: Using Inflation Expectations and External Nowcasts



Notes: The figure compares the one through eight quarters-ahead RMSEs for Blue Chip (diamonds), the SW model with observed long run inflation expectations (SWπ, squares) and the SW model with observed long run inflation expectations and output growth, inflation, and interest rate nowcasts (SWπ-now, crosses) for output growth, inflation, interest rates, consumption, investment, and real wage growth. The comparison is done for the same vintages/forecast dates as the Blue Chip/DSGE comparison discussed in section 4.3. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

for $h > 1$. At horizons $h = 2$ and $h = 3$ the Blue Chip forecasts dominate the SWπ-now forecasts. This ranking is reserved for horizons $h > 3$.

The positive effect of the external information on inflation and interest rate forecasts is more persistent. For instance, for $h = 1$ the interest rate RMSE is reduced from 0.12 (SWπ) to 0.01 (SWπ-now). For $h = 4$ the RMSE is lowered from 0.35 (SWπ) to 0.31 (SWπ-now). For horizons $h = 2$ to $h = 5$ the Blue Chip interest rate forecasts remain more accurate than the SWπ-now forecasts. For inflation, on the other hand, SWπ predictions weakly dominate Blue Chip forecasts at all horizon. Although the Blue Chip nowcasts include no information about consumption and investment growth, we observe a RMSE reduction for $h = 1$. To the extent that the joint predictive distribution correctly captures non-zero correlations between output, inflation, and interest rates on the one hand and consumption and investment growth on the other hand, information about the first set of variables can sharpen the predictions for the second set of variables.

A number of alternative approaches of incorporating external information in DSGE model forecasts have been considered in the literature. Herbst and Schorfheide (2011) take the output of a simulator that generates draws from the unconditional predictive density, e.g. Algorithm 2, and use Kernel weights to convert draws from the unconditional predictive density into draws from a predictive density. This nonparametric approach can in principle be applied to draws from any kind of joint predictive distribution to hard-condition on $y_{1,T+1} = z_{T+1}$. However, if the dimension of z_{T+1} is moderately large or if the external nowcast lies far in the tails of the model-implied predictive distribution $p(Y_{1,T+1}|Y_{1:T})$ a precise Kernel-based approximation of the conditional distribution could require a large number of draws from the predictive distribution $p(y_{T+1:T+H}|Y_{1:T})$. One benefit of the Kernel-based method is that the posterior distribution of θ implicitly also is updated in light of the external information z_{T+1}.

Robertson, Tallman, and Whiteman (2005) propose a nonparametric method that allows users to soft-condition on external information. Rather than imposing that $y_{1,T+1} = z_{T+1}$, the authors' goal is to impose the restriction $\mathbb{E}[y_{1,T+1}] = z_{T+1}$. Building on insights from the empirical likelihood literature, see Owen (2001), the authors apply an exponential tilting procedure to the draws from the unconditional predictive distribution $p(Y_{T+1:T+H}|Y_{1:T})$. Each draw $Y^{(j)}_{T+1:T+H}$ receives a weight w_j such that the empirical distribution associated with the weighted draws minimizes the Kullback-Leibler distance to the unweighted empirical distribution subject to the moment constraint $\sum_{j=1}^{n_{sim}} w_j y^{(j)}_{1,T+1} = z_{T+1}$. The procedure allows the user to remain agnostic about all aspects of the distribution of η_{T+1} in (54), except the constraint $\mathbb{E}[\eta_{T+1}] = 0$.

Monti (2010) develops an approach of incorporating external professional forecasts into a DSGE model, which combines aspects of what we previously referred to as news and noise assumption. She assumes that the professional forecasters have additional information (news) about the structural shocks that are hitting the economy in the current period. However, the professional forecasters also add some personal judgement to their forecasts which works like a noise term. Monti (2010) derives a set of measurement equations for current period observations and multi-step professional forecasts and estimates the DSGE model on this joint information. Rather than conditioning on external forecasts, Giannone, Monti, and Reichlin (2009) directly incorporate monthly information in the estimation of a DSGE model. In a nutshell, the authors first estimate the DSGE model parameters based on the usual quarterly observations and then transform the state-transition equations to monthly frequency. In addition to the usual quarterly variables, the authors then also use monthly variables to make inference about the current state of the economy and to improve the accuracy of short-horizon forecasts.

5.4 Incorporating Interest Rate Expectations

[subsec:interestrateexpectations] Discrepancies between DSGE model-based interest rate forecasts on the one hand and external forecasts or financial-market based expectations of future interest rates pose a challenge for the DSGE model analysis, in particular, if it is evident that the latter are more accurate than the former. The state-space representation of the DSGE model given by (33) and (34) implies that

$$\mathbb{E}[y_{t+h}|s_t] = \psi_0(\theta) + \Psi_1(\theta)(t+h) + \Psi_2(\theta)[\Phi_1(\theta)]^h s_t.$$

Thus, adding observations that are equated with the DSGE model-implied expectations of endogenous variables generates a singularity problem because there are fewer shocks in the model than observables in the measurement equation. In Section 5.1 we overcame the singularity problem by adding an additional structural shock to the model: we replaced the constant target inflation rate by a stochastically varying target inflation rate which was driven by a new innovation $\epsilon_{\pi^*,t}$. We followed a similar approach in Section 5.2 by adding a shock to the growth rate of technology. In this section we will introduce so-called anticipated monetary policy shocks to absorb discrepancies between observed and DSGE-model-implied interest rate expectations.

Equation (16) characterizes the monetary policy rule for the SW model with constant target inflation rate and (44) is the modified version with the time-varying inflation target. The disturbance r_t^m captures deviations from the systematic part of the policy rule. While in many DSGE models these deviations are assumed to be iid, the SW model allows for a serially correlated process:

$$r_t^m = \rho_{r^m} r_{t-1}^m + \sigma_{r^m} \epsilon_t^m. \tag{57}$$

We now augment the process r_t^m by anticipated shocks that capture future expected deviations from the systematic part of the monetary policy rule:

$$r_t^m = \rho_{r^m} r_{t-1}^m + \sigma_{r^m} \epsilon_t^m + \sum_{k=1}^{K} \sigma_{r^m,k} \epsilon_{k,t-k}^m, \tag{58}$$

where the policy shocks $\epsilon_{k,t-k}^m$, $k = 1, \ldots, K$, are known to agents at time $t - k$, but affect the policy rule with a k period delay in period t. Thus, agents are assumed to expect certain deviations from the systematic part of the interest-rate feedback rule several quarters in advance.

To the extent that the SWπ-now model with a policy rule given by (44) and (57) is unable to match the observed interest rate expectations in the data (see Figure 7), the anticipated monetary policy shocks can absorb the discrepancies between actual and DSGE model-implied expectations. As central banks around the world have been experimenting with so-called forward guidance, that is, sending signals about the future path of interest rates, we would expect the external interest rates forecasts to become more accurate and the use of anticipated shocks to rationalize the interest rate expectations in DSGE models to become attractive and plausible.

It is convenient to express the anticipated shocks in recursive form. For this purpose, we augment the state vector s_t with $\bar{H}$ additional states ν_t^m,...,$\nu_{t-\bar{H}}^m$ whose law of motion is as follows:

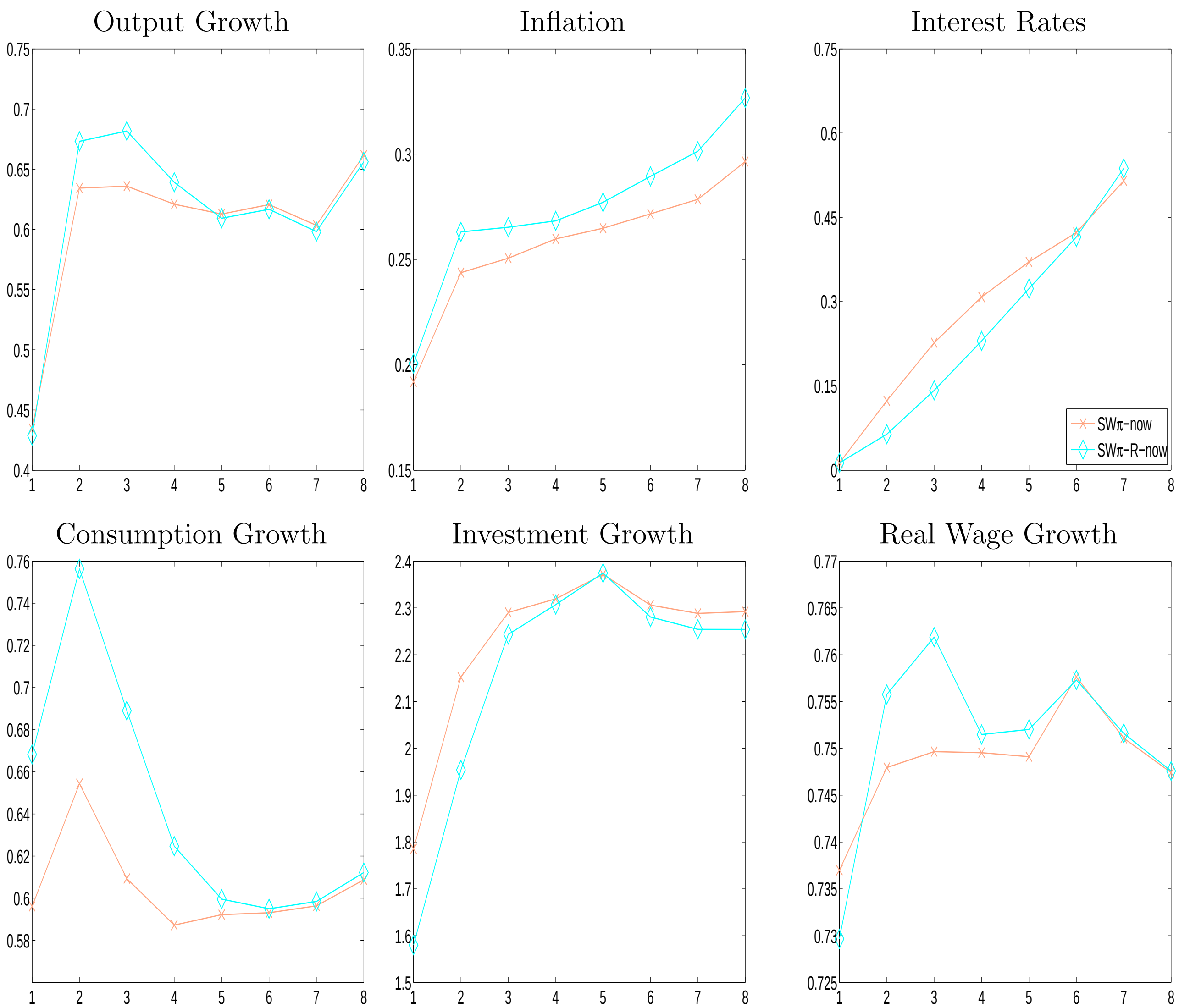
$$\nu_{1,t}^m = \nu_{2,t-1}^m + \sigma_{r^m,1}\epsilon_{1,t}^m, \quad \nu_{2,t}^m = \nu_{3,t-1}^m + \sigma_{r^m,2}\epsilon_{2,t}^m, \quad \ldots \quad \nu_{K,t}^m = \sigma_{r^m,K}\epsilon_{K,t}^m \tag{59}$$

and rewrite the process r_t^m in (58) as

$$r_t^m = \rho_{r^m} r_{t-1}^m + \sigma_{r^m} \epsilon_t^m + \nu_{1,t-1}^m. \tag{60}$$

Figure 8: Using Inflation Expectations, External Nowcasts, and Interest Rate Expectations

Blue Chip vs SW with long run inflation expectations and nowcast (SWπ-now) vs SW with long run inflation and interest rate expectations, and nowcast (SWπ-R-now)



Notes: The figure compares the one through eight quarters-ahead RMSEs for the SW model with observed long run inflation expectations and output growth, inflation, and interest rate nowcasts (SWπ-now, crosses) and the SW model with observed long run inflation and interest rate expectations and nowcasts (SWπR-now, diamonds) for output growth, inflation, interest rates, consumption, investment, and real wage growth. The comparison is done for the same vintages/forecast dates as the Blue Chip/DSGE comparison discussed in Section 4.3. All variables are expressed in terms of QoQ rates in percentage. Section 4.1 provides the details of the forecast comparison exercise.

Table 5: Blue Chip Forecast Dates and Nowcast Information and Interest Rate Expectations for 1992

Forecast Origin	End of Est. Sample T	External Nowcast $T+1$	Interest Rate Exp $R^e_{T+2\|T+1}, \ldots, R^e_{T+5\|T+1}$	Forecast $h=1$	$h=2$
Apr 92	91:Q4	92:Q1 based on Apr 92 BC	92:Q2 - 93:Q1	92:Q1	92:Q2
Jul 92	92:Q1	92:Q2 based on Jul 92 BC	92:Q3 - 93:Q2	92:Q2	92:Q3
Oct 92	92:Q2	92:Q3 based on Oct 92 BC	92:Q4 - 93:Q3	92:Q3	92:Q4
Jan 93	92:Q3	92:Q4 based on Jan 93 BC	93:Q1 - 93:Q4	92:Q4	93:Q1

It is easy to verify that $\nu^m_{1,t-1} = \sum_{k=1}^K \sigma_{r^m,k}\epsilon^m_{k,t-k}$, that is, $\nu^m_{1,t-1}$ is a "bin" that collects all anticipated shocks that affect the policy rule in period t. The model's solution can then again be expressed in terms of the transition equation (33).

While one could in principle estimate the anticipated shock model based on an augmented data set that includes interest rate expectations, we start from estimates of the SWπ model based on $Y_{1:T}$ and then switch to the anticipated shocks model, denoted by SWπR-now, to generate forecasts. This shortcut facilitates the comparison between forecasts from the SWπ-now and the SWπR-now because the forecasts are generated based on the same posterior distribution of DSGE model parameters θ.[10] The timing of the forecasts and the external information is explained in Table 5. The first three columns are identical to Columns 1 to 3 of Table 4. Consider the forecast origin that corresponds to the July 1992 Blue Chip release. The July 10 Blue Chip Economic Indicator survey is based on forecasts that were generated at the end of June. At this point, the forecasters essentially know the average interest rate for 1992:Q2, which is period $T+1$. We interpret Blue Chip interest forecasts for 1992:Q3 through 1993:Q2 as observations of interest rate expectations $R^e_{T+2|T+1}$ to $R^e_{T+5|T+1}$:

$$R^e_{T+1+k|T+1} = R_* + I\!\!E_{T+1}\left[R_{T+1+k}\right], \quad k = 1, \ldots. \tag{61}$$

R_* is the steady state interest rate and $I\!\!E_{T+1}\left[R_{T+1+k}\right]$ is the DSGE model-implied k-period-ahead interest rate expectation.

[10]We do not have estimates for the standard deviations $\sigma_{r^m,k}$ of the anticipated shocks. In the implementation, we assume that these shocks have the same standard deviation as the contemporaneous shock: $\sigma_{r^m,k} = \sigma_{r^m}$.

Federal funds rate expectations are taken from Blue Chip Financial Forecasts survey, which is published on the first of each month.[11] They are given in annual rates and are transformed in the same manner as the interest rate series in the estimation sample:

$$R^e_{T+1+k|T+1} = \text{BLUE CHIP } k\text{-QUARTERS AHEAD FFR FORECAST}/4.$$

Since Blue Chip Financial Forecasts extend to at most two calendar year including the current one, the horizon k for the interest forecasts varies from seven to five quarters, depending on the vintage. We use all available data. In addition to the interest rate expectations, we also incorporate the Blue Chip nowcasts into the forecasting procedure, using the *Noise* approach described in Section 5.3. This leads to the following algorithm to generate draws from the predictive distribution:

Algorithm 5. Draws from the Predictive Distribution Conditional on External Nowcast (Noise Assumption) and Interest Rate Expectations. [algo:preddrawsinterestexpect] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Based on the DSGE model without anticipated shocks, use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$.

2. Forecast the latent state s_{T+1} based on T information using the DSGE model without anticipated shocks.

3. Switch to DSGE model with anticipated shocks. Augment the state vector by the additional state variables $\nu^m_{1,t}, \dots, \nu^m_{K,t}$. Set mean and variances/covariances of these additional states to zero. Denote the augmented state vector by $\tilde{s}_t$.

4. Adjust the measurement equation such that it lines up with the available Blue Chip nowcasts, z_{T+1}, as well as the interest rate expectations $R^e_{T+2|T+1}, \dots, R^e_{T+5|T+1}$. Use the Kalman filter updating to compute $p(\tilde{s}_{T+1}|\theta^{(j)}, Y_{1:T}, z_{T+1}, R^e_{T+2|T+1}, \dots, R^e_{T+5|T+1})$ Generate a draw $\tilde{s}^{(j)}_{T+1}$ from this distribution.

5. Draw a sequence of innovations $\epsilon^{(j)}_{T+2:T+H}$ and, starting from $\tilde{s}^{(j)}_{T+1}$, iterate the state transition equations of the DSGE model forward to obtain a sequence $\tilde{S}^{(j)}_{T+2:T+H}$.

[11] There is a ten day gap between the BCFF and the BCEI survey, so the two are not quite based on the same information set. Also, the survey participants are not the same, although there is a substantial overlap. We ignore these differences. We thank Stefano Eusepi and Emanuel Mönch for providing us with this data, and their RA, Jenny Chan, for helping us find out how they were constructed.

6. Use the measurement equation to obtain $Y^{(j)}_{T+1:T+H}$ based on $\tilde{S}^{(j)}_{T+1:T+H}$.

In Figure 8 we compare forecasts from the SWπ-now model, which only utilizes current quarter interest rates, and the model that utilizes interest rate expectations up to four quarters ahead, SWπR-now. The interest-rate expectations modify the DSGE model forecasts as follows. The use of interest-rate expectations in the measurement equation affects the inference about the latent state $\tilde{s}_{T+1}$ in Step 4 of Algorithm 5. This latent state vector has two components, namely s_{T+1} and the additional state variables $\nu^m_{k,t}$, $k = 1, \dots, K$, specified in (59). Since the anticipated monetary policy shocks only affect the exogenous component of the monetary policy rule, the output growth, inflation, and interest rate dynamics generated by the reversion of s_{T+1} to its steady state of zero are the same as in the SWπ-now model. However, the inferred period $T+1$ level of the state vector differs across models. In addition, the forecasts of the SWπR-now model are influenced by the impulse-responses to the anticipated monetary policy shocks that align the model-based interest rate forecasts in period $T+1$ with the observed interest rate expectations.

The use of interest-rate expectations reduces the RMSE for the Federal Funds rate for horizon $h = 2$ to $h = 5$. For instance, while the RMSE associated with the SWπ-now model is 0.23% for $h = 2$, it drops to 0.14% if interest rate expectations are included. Unfortunately, the interest rate expectations have an adverse effect on output growth and inflation forecasts. For instance, at $h = 3$ the output growth RMSE rises from 0.63% to 0.68% and the inflation RMSE increases from 0.25% to 0.27%. While consumption and real wage growth forecasts also deteriorate over the two- to five-quarter horizon, only the investment growth forecast improves. For $h = 2$ the investment growth RMSE for the SWπ-now model is 2.15% whereas it is only 1.95% for the SWπR-now model. While it is difficult to disentangle which feature of SWπR-now is responsible for the observed deterioration in the forecast performance, we provide a detailed discussion of responses to anticipated monetary policy shocks in the next section.

6 Forecasts Conditional on Interest Rate Paths

[sec:forecastgivenR] In this section we are generating forecasts conditional on a particular interest path. In particular, we assume that in periods $t = T+1, \dots, T+\bar{H}$ the interest rate takes (in expectation or actually) the values $\bar{R}_{T+1}$, $\ldots$, $\bar{R}_{T+\bar{H}}$, where $\bar{H} \leq H$. We consider two methods: using unanticipated monetary policy shocks (Section 6.2) and using anticipated monetary policy shocks (Section 6.3). Before engaging in conditional forecasting, we briefly review the effects of monetary policy shocks in a simple DSGE model (Section 6.1) and provide impulse response functions for the SWπ model. At last, we provide an empirical illustration of conditional forecasting (Section 6.4).

6.1 The Effects of Monetary Policy Shocks

[subsec:mpolshocks] In order to understand the effects of unanticipated and anticipated monetary policy shocks, we begin by solving an analytical example.[12] The subsequent example is based on a further simplification of the small-scale model presented in Section 2.3. The simplified version of the model consists of the linearized Euler equation:

$$y_t = \mathbb{E}[y_{t+1}] - (R_t - \mathbb{E}[\pi_{t+1}]) \,, \tag{62}$$

a Phillips curve,

$$\pi_t = \beta \mathbb{E}[\pi_{t+1}] + \kappa y_t, \tag{63}$$

and a monetary policy rule with unanticipated and anticipated monetary policy shocks:

$$R_t = \frac{1}{\beta}\pi_t + \epsilon_t^R + \sum_{k=1}^{K} \epsilon_{k,t-k}^R. \tag{64}$$

As in Section 5.4, we use $\epsilon_{k,t-k}^R$ as a shock that is realized in period $t-k$ and affects the interest rate k periods later. The inflation coefficient in the policy rule is restricted to be equal to $1/\beta$, which facilitates the analytical solution of the model.

We first determine the law of motion of output. The Euler equation (62) implies that output is the sum of expected future real rates. Of course future real rates are endogenous and further manipulations are needed to express output as the sum of expected future monetary policy shocks. Using (64) to eliminating the nominal interest rate from the Euler equation yields

$$y_t = \mathbb{E}_t[y_{t+1}] - \left(\frac{1}{\beta}\pi_t - \mathbb{E}_t[\pi_{t+1}] \right) - \epsilon_t^R - \sum_{k=1}^{K} \epsilon_{k,t-k}^R. \tag{65}$$

[12] See also Milani and Treadwell (2011) for a discussion and some empirical results.

The restriction imposed on the inflation coefficient in the monetary policy rule implies that we can express next period's real return on a nominal bond as a function of current output. More specifically, we can re-write the Phillips curve (63) as

$$\frac{1}{\beta}\pi_t - \mathbb{E}[\pi_{t+1}] = \frac{\kappa}{\beta}y_t \tag{66}$$

and combine (66) with (65) to obtain

$$y_t = \mathbb{E}[y_{t+1}] - \frac{\kappa}{\beta}y_t - \epsilon_t^R - \sum_{k=1}^{K}\epsilon_{k,t-k}^R. \tag{67}$$

Defining $\psi = (1+\kappa/\beta)^{-1}$ and solving (67) forward yields

$$y_t = -\psi\mathbb{E}_t\left[\sum_{j=0}^{\infty}\psi^j\left(\epsilon_{t+j}^R + \sum_{k=1}^{K}\epsilon_{k,t+j-k}^R\right)\right].$$

Since the expected value of $\epsilon_{k,t+j}^R$ is zero for $j > 0$ we deduce

$$y_t = -\psi\left(\epsilon_t^R + \sum_{k=1}^{K}\epsilon_{k,t-k}^R + \sum_{j=1}^{K}\sum_{k=j}^{K}\psi^j\epsilon_{k,t+j-k}^R\right). \tag{68}$$

This equation implies that the impulse response function for a K-period anticipated shock takes the form

$$\frac{\partial y_{t+h}}{\partial \epsilon_{K,t}^R} = \frac{\partial y_t}{\partial \epsilon_{K,t-h}^R} = -\psi^{1+K-h}, \quad h = 0,\dots,K \tag{69}$$

and is zero thereafter. The anticipated monetary policy shock raises the expected real return on government bonds and through the consumption Euler equation leads to a decrease in output. Output drops upon impact. Since $0 < \psi < 1$ the output effect increases over time and peaks at $-\psi$ K periods after impact, before it drops to zero.

The law of motion of inflation can be obtained by solving (63) forward. After calculating $\mathbb{E}_t[y_{t+i}]$ based on (68), it can be shown that inflation has the representation

$$\begin{aligned}\pi_t \quad = \quad & -\kappa\psi\left(\epsilon_t^R + \sum_{k=1}^{K}\epsilon_{k,t-k}^R + \sum_{j=1}^{K}\sum_{k=j}^{K}\psi^j\epsilon_{k,t+j-k}^R\right) \\ & -\kappa\psi\sum_{i=1}^{K}\beta^i\left(\sum_{k=i}^{K}\epsilon_{k,t+i-k}^R + \sum_{j=1}^{K}\sum_{k=j+i}^{K}\psi^j\epsilon_{k,t+i+j-k}^R\right).\end{aligned} \tag{70}$$

It can be verified that inflation responds to a K-period anticipated shock according to

$$\frac{\partial \pi_{t+h}}{\partial \epsilon^R_{K,t}} = \frac{\partial \pi_t}{\partial \epsilon^R_{K,t-h}} = -\kappa\psi \left(\psi^{K-h} + \beta^{K-h} + \psi^{K-h} \sum_{i=1}^{K-1-h} \left(\frac{\beta}{\psi}\right)^i \right), \quad h = 0, \ldots, K, \tag{71}$$

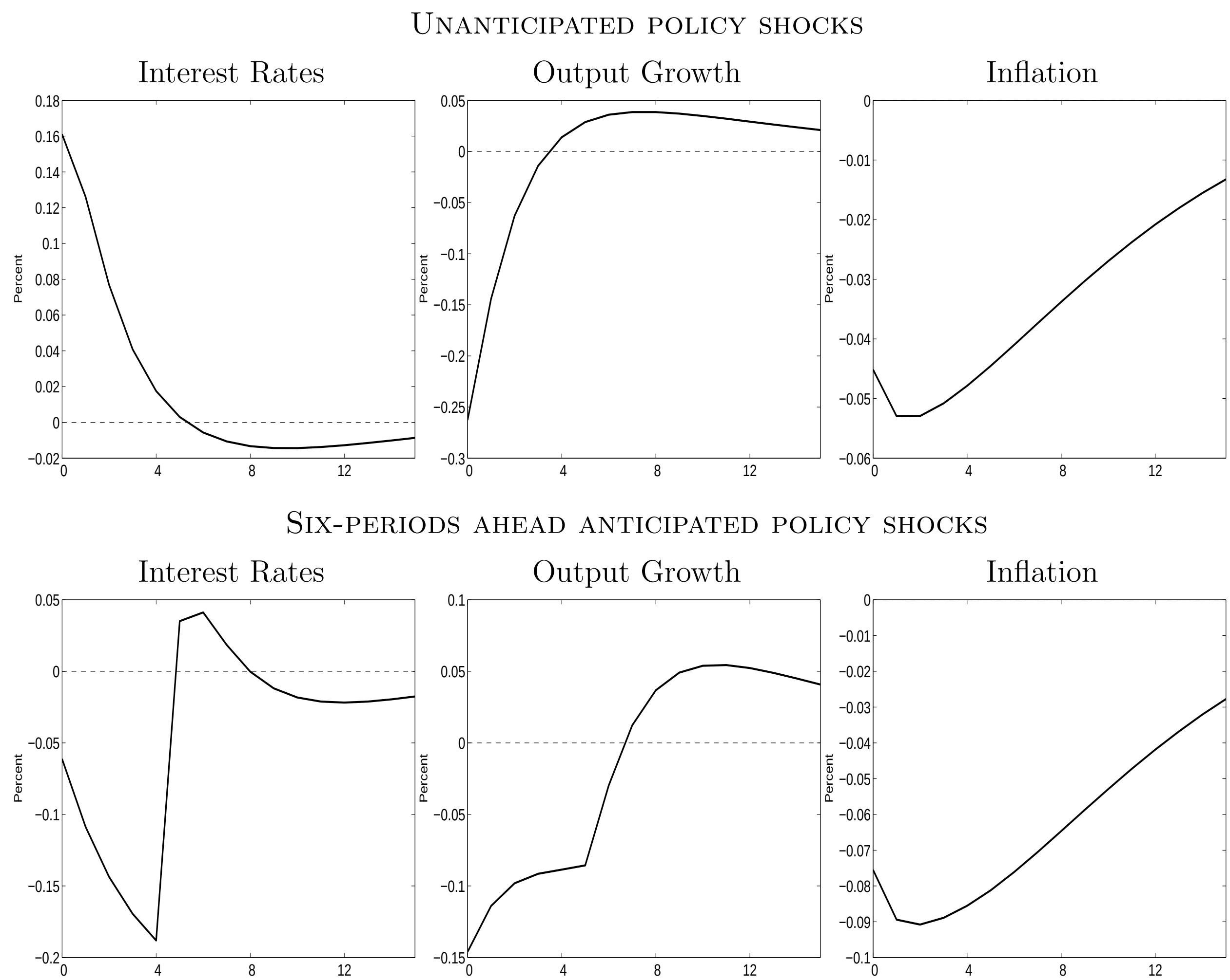
where $\beta/\psi = \beta + \kappa$. Inflation also drops on impact of the anticipated monetary policy shock and remains below steady state until period $t + K$, after which it reverts to zero. The shape of the inflation response depends on whether $\beta + \kappa$ is less than or greater than unity. Finally, the law of motion of the interest rates is obtained by plugging (70) into the monetary policy rule (64). The anticipated future increase in interest rates leads to a drop in interest rates prior to $h = K$ because the central bank lowers interest rates in response to the below-target inflation rate.

We now compute impulse response functions of interest rates, output growth, and inflation to an unanticipated and an anticipated contractionary policy shock based on the estimated SWπ model. This model exhibits more elaborate dynamics than the simple analytical model. The impulse response functions depicted in Figure 9 are computed using the posterior mode estimates from the May-2011 vintage. The anticipated shock is known to the agents in the model $k = 6$ periods in advance. The size of both shocks is the same, and equal to the estimated standard deviation of the unanticipated shocks.

The response to the unanticipated monetary policy shock (top panels) follows the familiar pattern. Interest rates rise by about 16bp, whereas output falls by 25bp upon impact. Over time, output growth reverts back to zero and eventually becomes negative, as the long-run effect of an unanticipated shock on the level of output is zero. Inflation falls by 5bp and slowly reverts back to zero. The strong response of output relative to inflation is a reflection of the magnitude of estimated nominal rigidities in the model.

As foreshadowed by the analytical calculations, the effect of the anticipated policy shock is quite different from the response to the unanticipated shock. To understand the pattern it is helpful to reconsider (69) and (71). Upon impact, the anticipated monetary policy shock lowers output and inflation, and via the systematic part of the monetary policy rule also interest rates. This pattern is also evident in the bottom panels of Figure 9. Output and inflation drop by 15bp and 8bp, respectively, interest rates fall by 5bp due to the endogenous policy response. In the simple analytical model output keeps on falling after the impact of the anticipated policy shock because $0 < \psi < 1$. This implies that output growth remains

Figure 9: Impulse Responses to Anticipated and Unanticipated Policy Shocks



Notes: The figure shows the impulse response functions of interest rates, output growth, and inflation to a one-standard deviation unanticipated (top panel) and anticipated (bottom panel) policy shock. The anticipated shock is known to agents six periods in advance. The impulse responses are computed using the modal estimates for the last available vintage (May 2011) for model SWπ.

negative, which we also see in Figure 9. According to (71) the shape of the inflation response is ambiguous. The SWπ produces a hump-shaped response that reaches its trough at $h = 2$. The interest rate jumps after six periods, when the anticipated deviation from the rule is actually realized. Unlike the simple analytical model, the SWπ model has endogenous state variables, which generate fairly persistent dynamics even after the policy shock is realized.

Generally, the effect of the anticipated shock on output is much more persistent than that of the unanticipated shock. Since inflation depends on the present discounted value of future marginal costs, this persistence implies that the impact on inflation is almost twice as strong, even though the size of the shock is the same. After having examined the responses to unanticipated and anticipated monetary policy shocks, we now turn to the problem of generating forecasts conditional on a desired interest-rate path by the central bank.

6.2 Using Unanticipated Shocks to Condition on Interest Rates

[subsec:unanticipatedshocks] Many central banks generate model-based forecasts conditional on hypothetical interest rate paths. One popular scenario in policy discussions is the constant-interest-rate scenario which assumes that the nominal interest rate stays fixed at its current level over the forecast horizon. Since in DSGE models as well as vector autoregressive models interest rates are endogenous, it is by no means guaranteed that the model predicts the interest rate to be constant. For concreteness, suppose that the current level of the nominal interest rate is 2% and the posterior mean predictions for periods $T+1$ and $T+2$ are 2.25% and 2.50%, respectively. In this case, a constant-interest rate path would, in the logic of the DSGE model, require an intervention that lowers the interest rate by 25bp in period $T+1$ and by 50bp in period $T+2$.

One approach of generating forecasts conditional on hypothetical interest rate with DSGE models is to utilize a sequence of unanticipated monetary policy shocks as in Leeper and Zha (2003) and Smets and Wouters (2005). Mechanically, it is straightforward to compute such forecasts. Without loss of generality assume that the interest rate R_t is ordered first in the vector y_t, that is, $y_{1,t} = R_t$. Moreover, use $\epsilon_t^{<-p>}$ to denote the sub-vector of ϵ_t that contains all structural innovations, except for the monetary policy innovation that is used to attain the desired interest rate path. Lastly, assume that the monetary policy shock ϵ_t^p is ordered first, such that $\epsilon_t = [\epsilon_t^p, \epsilon_t^{<-p>'}]'$. Typically, the policy shock ϵ_t^p would correspond to a short-lived deviation from the systematic part of the monetary policy rule, but in the SWπ model ϵ_t^p could also correspond to the innovation of the target-inflation process.

Let $\bar{R}_{T+1}, \dots, \bar{R}_{T+\bar{H}}$ denote the desired interest rate path, where $\bar{H} \leq H$. Using the unanticipated monetary policy shocks we can modify the predictive density in two ways: (i) the expected value of the interest rates in periods $T+1, \dots, T+\bar{H}$ is equal to $\bar{R}_{t+1}, \dots, \bar{R}_{T+\bar{H}}$; (ii) the simulated values of the interest rates along each trajectory are exactly equal to

$\bar{R}_{t+1}, \ldots, \bar{R}_{T+\bar{H}}$. The following algorithm can be used to generate draws from the predictive distribution conditional on the desired interest rate path.

Algorithm 6. Draws from the Counterfactual Predictive Distribution via Unanticipated Shocks. [algo:pred:unanticipatedshocks] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$. Generate a draw $s_T^{(j)}$ from this distribution.

2. Draw a sequence of innovations $\epsilon_{T+1:T+H}^{<-p>(j)}$ for the non-policy shocks.

3. **Case (i):** Compute the sequence $\bar{\epsilon}_t^p$, $t = T+1, \ldots, T+\bar{H}$ as follows. For $t = T+1$ to $t = T+\bar{H}$:

   (a) Determine $\bar{\epsilon}_t^p$ as the solution to

   $$\bar{R}_t = \Psi_{1.,1}(\theta^{(j)}) + \Psi_{1.,1}(\theta^{(j)})t + \Psi_{1.,2}(\theta^{(j)})\big(\Phi_1(\theta^{(j)})s_{t-1} + \Phi_\epsilon(\theta^{(j)})[\bar{\epsilon}_t^p, 0]'\big).$$

   (b) Let $s_t = \Phi_1 s_{t-1} + \Phi_\epsilon[\bar{\epsilon}_t^p, 0]'$.

   Case (ii): Compute the sequence $\bar{\epsilon}_t^p$, $t = T+1, \ldots, T+\bar{H}$ as follows. For $t = T+1$ to $t = T+\bar{H}$:

   (a) Determine $\bar{\epsilon}_t^p$ as the solution to

   $$\bar{R}_t = \Psi_{1.,1}(\theta^{(j)}) + \Psi_{1.,1}(\theta^{(j)})t + \Psi_{1.,2}(\theta^{(j)})\big(\Phi_1(\theta^{(j)})s_{t-1} + \Phi_\epsilon(\theta^{(j)})[\bar{\epsilon}_t^p, \epsilon_t^{<-p>(j)}]'\big).$$

   for $\bar{\epsilon}_t^p$.

   (b) Let $s_t = \Phi_1 s_{t-1} + \Phi_\epsilon[\bar{\epsilon}_t^p, \epsilon_t^{<-p>(j)}]'$.

4. Starting from $s_T^{(j)}$, iterate the state transition equation (33) forward to obtain a sequence $s_{T+1:T+H}^{(j)}$:

   $$s_t^{(j)} = \Phi_1(\theta^{(j)})s_{t-1}^{(j)} + \Phi_\epsilon(\theta^{(j)})[\epsilon_t^p, \epsilon_t^{<-p>(j)'}]', \quad t = T+1, \ldots, T+H.$$

   For $t = T+1, \ldots, T+\bar{H}$ use $\epsilon_t^p = \bar{\epsilon}_t^p$. For $t > T+\bar{H}$, generate a draw $\epsilon_t^p \sim N(0,1)$.

5. Use the measurement equation (34) to compute $y_{T+1:T+H}^{(j)}$ based on $s_{T+1:T+H}^{(j)}$. $\square$

There are two conceptual drawbacks associated with the use of unanticipated monetary policy shocks. First, if the interest rate path $\bar{R}_{T+1:T+\bar{H}}$ is credibly announced by the central bank, then the deviations from the systematic part of the monetary policy rule are not *unanticipated*. Consequently, the use of unanticipated monetary policy shocks might lead to inaccurate predictions. Second, suppose that the interest rate path is not announced to the public but its implementation requires a sequence of strongly positively correlated unanticipated monetary policy shocks. Over time, the agents in the DSGE model might be able to detect the persistence in the deviation from the systematic part of the monetary policy rule and suspect that the policy rule itself might have changed permanently, which, in turn, creates an incentive to update decision rules. Of course, none of this is captured in the DSGE model itself. Leeper and Zha (2003) recommend to analyze the effect of monetary policy interventions with unanticipated shocks only if the interventions are modest. Here modest essentially means that in a larger model in which agents assign positive probability to occasional shifts in policy regimes, the intervention would not trigger the learning mechanism and lead the agent to belief that the policy regime has shifted.

6.3 Using Anticipated Shocks to Condition on Interest Rates

[subsec:anticipatedshocks] More recently, the literature has considered the use of anticipated monetary policy shocks to generate forecasts conditional on an interest rate path that deviates from the model-implied path, e.g. Laseen and Svensson (2011), Blake (2011), and Milani and Treadwell (2011). This approach is appealing because several central banks have changed their communication strategy and started to announce interest rate paths. Consider the modified policy rule (58) that includes anticipated shocks $\epsilon^R_{k,t-k}$ as discussed in Section 5.4.

Suppose that after time T shocks are realized, the central bank announces the interest rate path. For the agents the announcement in a one-time surprise in period $T+1$, which corresponds to the realization of a single unanticipated monetary policy shock ϵ^R_{T+1} and a sequence of anticipated shocks

$$\epsilon^R_{1:K,T+1} = \left[\epsilon^R_{1,T+1},\ \epsilon^R_{2,T+1},\ \ldots,\ \epsilon^R_{K,T+1}\right]',$$

where $K = \bar{H} - 1$. Notice that unlike in Section 6.2 all policy shocks that are used to implement the interest rate path are dated $T+1$. We will subsequently use ϵ_t to denote the vector that collects the innovation of the unanticipated shocks and $\epsilon^R_{1:K,t}$, the vector of anticipated shocks. In slight abuse of notation, we denote the expanded state vector that includes cumulative effects of anticipated shocks, see (59), also by s_t and use the same notation for the state transition equation, which is now driven by the combined innovation vector $[\epsilon_t', \epsilon^{R'}_{1:K,t}]'$. The following algorithm determines the time $T+1$ monetary policy shocks as a function of the desired interest rate sequence $\bar{R}_{T+1}, \ldots, \bar{R}_{T+\bar{H}}$ to generate predictions conditional on an announced interest rate path. The announced interest rate path will be attained in expectation.

Algorithm 7. Draws from the Counterfactual Predictive Distribution via Anticipated Shocks. [algo:predanticipatedshocks] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the Kalman filter to compute mean and variance of the distribution $p(s_T|\theta^{(j)}, Y_{1:T})$. Generate a draw $s_T^{(j)}$ from this distribution.

2. Draw a sequence of innovations $\epsilon^{(j)}_{T+1:T+H}$.

3. Consider the following system of equations, omitting the $\theta^{(j)}$ argument of the system matrices:

$$
\begin{aligned}
\bar{R}_{T+1} &= \Psi_{1.,0} + \Psi_{1.,1}(T+1) + \Psi_{1.,2}\Phi_1 s_T + \Psi_{1.,2}\Phi_\epsilon[\underbrace{\bar{\epsilon}^R_{T+1}, 0, \ldots, 0}_{\epsilon'_{T+1}}, \bar{\epsilon}^{R'}_{1:K,T+1}]' \\
\bar{R}_{T+2} &= \Psi_{1.,0} + \Psi_{1.,1}(T+2) + \Psi_{1.,2}(\Phi_1)^2 s_T + \Psi_{1.,2}\Phi_1\Phi_\epsilon[\underbrace{\bar{\epsilon}^R_{T+1}, 0, \ldots, 0}_{\epsilon'_{T+1}}, \bar{\epsilon}^{R'}_{1:K,T+1}]' \\
&\vdots \\
\bar{R}_{T+\bar{H}} &= \Psi_{1.,0} + \Psi_{1.,1}(T+\bar{H}) + \Psi_{1.,2}(\Phi_1)^{\bar{H}} s_T + \Psi_{1.,2}(\Phi_1)^{\bar{H}-1}\Phi_\epsilon[\underbrace{\bar{\epsilon}^R_{T+1}, 0, \ldots, 0}_{\epsilon'_{T+1}}, \bar{\epsilon}^{R'}_{1:K,T+1}]'
\end{aligned}
$$

   This linear system of $\bar{H}$ equations with $\bar{H}$ unknowns can be solved for for $\bar{\epsilon}^R_{T+1}$ and $\bar{\epsilon}^R_{1:K,T+1}$.

4. Starting from $s_T^{(j)}$, iterate the state transition equation (33) forward to obtain a se-

quence $s^{(j)}_{T+1:T+H}$:

$$s_t^{(j)} = \Phi_1(\theta^{(j)})s_{t-1}^{(j)} + \Phi_\epsilon(\theta^{(j)})[\underbrace{\epsilon_t^R, \epsilon_t^{<-R>'}}_{\epsilon_t'}, \epsilon^{R'}_{1:K,t}]', \quad t = T+1,\ldots,T+H,$$

where (i) $\epsilon_t^{<-R>} = \epsilon_t^{<-R>(j)}$ for $t = T+1,\ldots,T+\bar{H}$ (we are using simulated values throughout); (ii) $\epsilon^R_{T+1} = \bar{\epsilon}^R_{T+1}$ and $\epsilon^R_t = \epsilon_t^{R(j)}$ for $t = T+2,\ldots,T+\bar{H}$ (use solved-for value in period $T+1$ and simulated values thereafter); (iii) $\epsilon^R_{1:K,T+1} = \bar{\epsilon}^R_{1:K,T+1}$ and $\epsilon^R_{1:K,t} = 0$ for $t = T+2,\ldots,T+\bar{H}$ (use solved-for values in period $T+1$ and zeros thereafter).

5. Use the measurement equation (34) to compute $y^{(j)}_{T+1:T+H}$ based on $s^{(j)}_{T+1:T+H}$. $\square$

To shed some light on the algorithm it is instructive to revisit the analytical example of Section 6.1. For $K = 1$ output, inflation, and interest rates are given by

$$\begin{aligned} y_t &= -\psi\big(\epsilon_t^R + \epsilon^R_{1,t-1} + \psi\epsilon^R_{1,t}\big) \\ \pi_t &= -\kappa\psi\big(\epsilon_t^R + \epsilon^R_{1,t-1} + (\psi+\beta)\epsilon^R_{1,t}\big) \\ R_t &= \psi\epsilon_t^R + \psi\epsilon^R_{1,t-1} - \frac{1}{\beta}\kappa\psi(\psi+\beta)\epsilon^R_{1,t}. \end{aligned} \tag{72}$$

Suppose that the central bank wants to raise interest rates by 25 basis points (bp) for periods $T+1$ and $T+2$. The unanticipated policy shock ϵ^R_{T+1} and the anticipated policy shock $\epsilon^R_{1,T+1}$ are determined by solving the system

$$\begin{aligned} \bar{R}_{T+1} = 0.25 &= \psi\epsilon^R_{T+1} - \frac{1}{\beta}\kappa\psi(\psi+\beta)\epsilon^R_{1,T+1} \\ \bar{R}_{T+2} = 0.25 &= \psi\epsilon^R_{1,T+1}. \end{aligned}$$

For $\kappa = 0.1$ and $\beta = 0.99$, which leads to $\psi = 0.91$, the second equation implies that the anticipated policy shock needs to be equal to $\epsilon^R_{1,T+1} = 0.275$. The anticipated shock lowers the interest rate in the first period by 2.5bp. To compensate for this effect, the unanticipated monetary policy shock has to be equal to 30bp. Once the policy shocks have been determined, Algorithm 7 amounts to simulating the system (72) holding the time $T+1$ monetary policy shocks fixed.

One can solve for the effect of a policy that raises interest rates 25bp above the steady state level in periods $T+1$ and $T+2$ in an alternative manner. Since there is no persistence in the model, the economy returns to the rational expectations equilibrium in period $t = T+3$. Thus, in the absence of further shocks $y_{T+3} = \pi_{T+3} = R_{T+3} = 0$. In turn, $\mathbb{E}_{T+2}[y_{T+3}] = \mathbb{E}_{T+2}[\pi_{T+3}] = 0$. The Euler equation (62) for period $T+2$ implies that output is determined by $y_{T+2} = -R_{T+2}$. Using $\mathbb{E}_{T+2}[\pi_{T+3}] = 0$ once more, the Phillips curve (63) implies that $\pi_{T+2} = \kappa y_{T+2}$. Now that period $T+2$ output and inflation are determined, (62) and (63) can be solved to find y_{T+1} and π_{T+1} conditional on R_{T+1}. The solution is identical to the one obtained with the anticipated monetary policy shocks.

The effect of keeping the interest rate constant at, say $\bar{R} = 25bp$, for an extended period of time can be determined by proceeding with the backward solution of the difference equations:

$$
\begin{aligned}
y_{t-j} &= y_{t-j+1} - \bar{R} + \pi_{t-j+1}, \quad j = 0, 1, \ldots, K \\
\pi_{t-j} &= (1+\beta)\pi_{t-j+1} + y_{t-j+1} - \bar{R}
\end{aligned}
$$

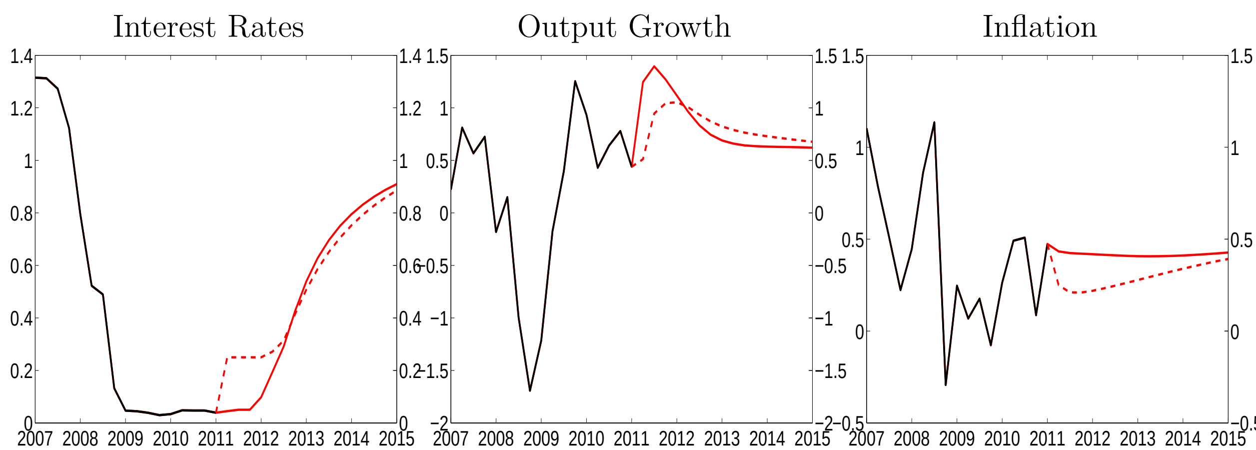
As explained in detail in Carlstrom, Fuerst, and Paustian (2012) the backward iterations generate explosive paths for output and inflation which leads to potentially implausibly large initial effects of extended periods of fixed interest rates. In larger systems the explosive roots could also be complex such that fixed interest rates cause oscillating dynamics. Carlstrom, Fuerst, and Paustian (2012) interpret the explosive dynamics as a failure of New Keynesian monetary DSGE models. This sentiment is shared by Blake (2011) who proposes an alternative method of simulating DSGE models conditional on an interest rate path that is pre-determined for $\bar{H}$ periods. His solution evolves a modification that introduces indeterminacy into the model and then selecting an equilibrium path that delivers *a priori* reasonable responses.

In sum, it remains an open research question how to best generate DSGE model forecasts conditional on a fixed interest rate paths. While, the use of anticipated shocks is appealing at first glance and easy to implement, it might produce unreasonable dynamics. We view this as a reason to exercise caution when generating predictions of interest rate announcements and recommend to carefully examine the responses to anticipated monetary policy shocks before engaging in this type of policy analysis.

6.4 Forecasting Conditional on an Interest Rate Path: An Empirical Illustration

[subsec:forecastgivenRillustration] Figure 10 provides an example of forecasting conditional on an interest rate path, where the new path is implemented via anticipated policy shocks using Algorithm 7. The figure shows the May-2011 vintage data for interest rates, output growth, and inflation (black lines), the DSGE model mean forecasts for these variables conditional on the Blue Chip expectations for the FFR (red solid lines), and the forecasts conditional on the announcement that the quarterly FFR will instead be 0.25% for the next four quarters (red dashed lines). The exercise is conducted with model SWπR.

Figure 10: Forecasting Conditional on an Interest Rate Path



Notes: The figure shows the May 2011 vintage data for interest rates, output growth, and inflation (black lines), the DSGE model mean forecasts for these variables conditional on the Blue Chip expectations for the FFR (red solid lines), and the forecasts conditional on the announcement that the quarterly FFR will instead be .25% over the next four quarters (red dashed lines). The exercise is conducted with model SWπ.

The left panel shows the expected interest rate path pre- and post-intervention. The pre-intervention interest rate forecast (solid) incorporates the market Federal Funds rate expectations for the subsequent six quarters, as measured by the Blue Chip forecasts available on May 10, 2011. Markets expect the interest rate to remain at (or near) the effective zero lower bound through the end of 2011, and liftoff to occur only in 2012:Q1. The post-intervention path (dashed line) captures the effect of an hypothetical announcement by the monetary authorities that they intend to raise rates immediately. Specifically, the intervention consists of an announcement at the beginning of period $T + 1$ (2011:Q2 in our case) that the quarterly FFR will be .25% (1% annualized) for the next four quarters (through 2012:Q1). In terms of the model mechanics, such an announcement amounts to a vector of time $T+1$ unanticipated and anticipated shocks computed in such a way to obtain $R_{T+1} = \mathbb{E}_{T+1}[R_{T+2}] = .. = \mathbb{E}_{T+1}[R_{T+4}] = 0.25$, as described in Algorithm 7.

Consistent with the impulse responses shown in Figure 9, the announcement that policy will be more contractionary than expected leads to lower inflation and lower output growth. The effect of the announcement on output growth is front-loaded, as we discussed in Section 6.1. On impact (2011:Q2) the difference between the solid and dashed lines is about 75 basis points, that is, roughly 3% annualized. The difference narrows over the following two quarters and is about zero in 2012:Q1, even though the difference in interest rates in that quarter is almost as large as it was in 2011:Q2. After 2012:Q1 output growth following the contractionary announcement is actually higher than otherwise. This is not surprising in light of the fact that monetary policy is still neutral in this model. Slower growth in the short-run must be compensated by higher growth later, since eventually the effect of the announcement on the level of output must be zero. Nonetheless the post-intervention *level* of output remains below the pre-intervention level at least through 2015, leading to lower real marginal costs and lower inflation, as shown in the last panel of Figure 10.

7 Moving Beyond Point Forecasts

[sec:densforecast] Thus far, this paper has focused on point forecasts generated from DSGE models and on how to improve their accuracy by using external information. For the remainder of this paper we will explore other aspects of DSGE model forecasts. First, an important feature that distinguishes DSGE models from many other time series models, is that DSGE models attribute macroeconomic fluctuations to orthogonal structural shocks. Thus, the models can provide decompositions of historical time series as well as the predicted path of the economy. We illustrate the use of shock decompositions in Section 7.1. Second, the algorithms described in the preceding sections generate draws from the predictive distribution which, as discussed in Section 3.2, can also be used to obtain interval or density forecasts. In Section 7.2 we generate real-time density forecasts from the SWπ model as well as the DSGE model with financial frictions introduced in Section 2.2 and examine the extent to which the forecasts capture the evolution of output growth and inflation during the 2008-09 recession. Third, in Section 7.3 we examine more systematically whether DSGE model density forecasts are well calibrated in the sense that stated probabilities are commensurable with actual frequencies.

7.1 Shock Decompositions

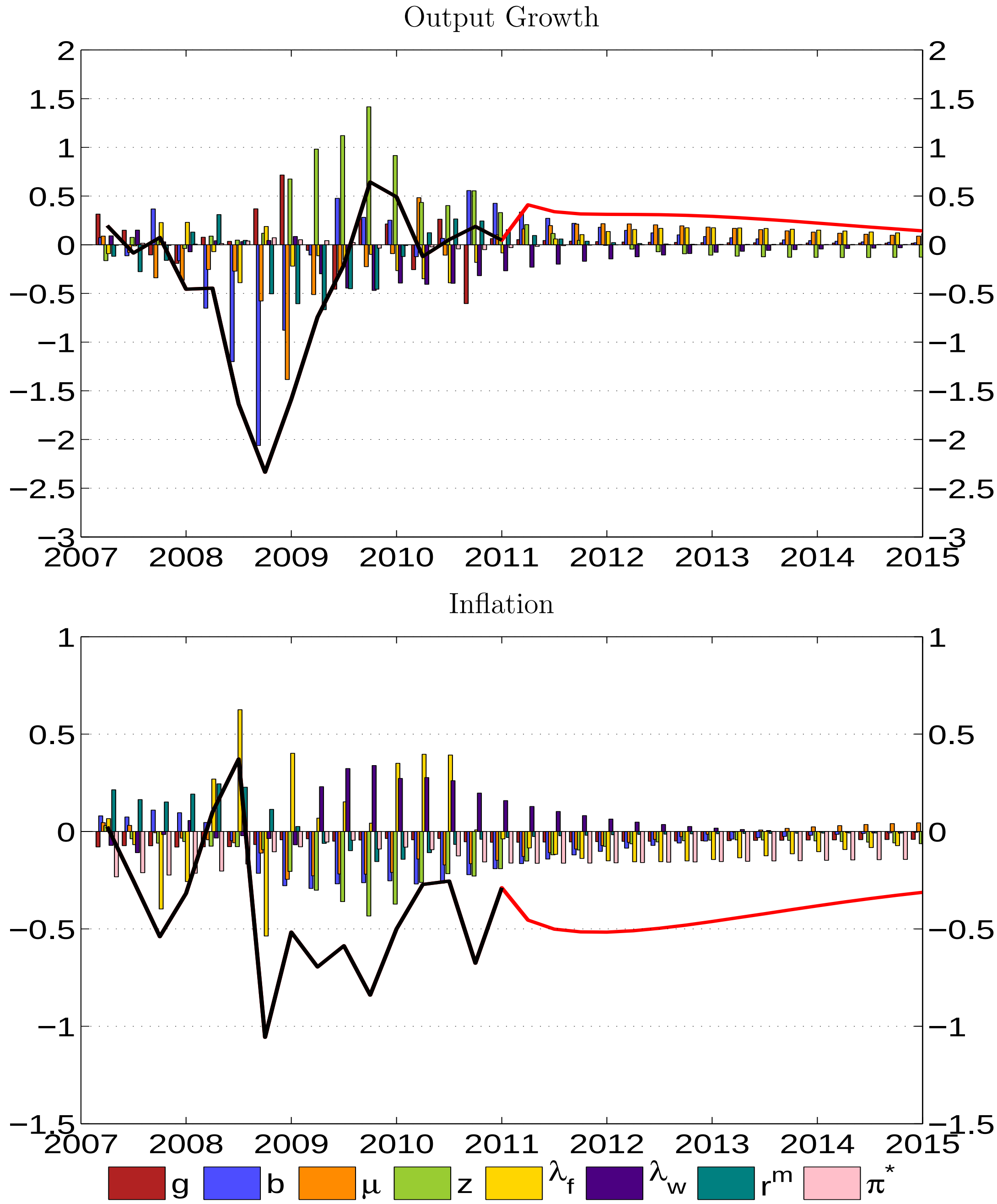
[subsec:shockdecompositions] DSGE models deliver a structural interpretation for both the history and the forecasts of macroeconomic time series. Figure 11 displays so-called shock decompositions for output growth and inflation, that illustrate the contribution of the various structural shocks to the historical and projected evolution of the two series. Before discussing the results in detail, we present the algorithm that is used to construct the decomposition.

Algorithm 8. Draws from the Posterior Distribution of a Shock Decomposition. [algo:shockdecomposition] For $j = 1$ to n_{sim}, select the j'th draw from the posterior distribution $p(\theta|Y_{1:T})$ and:

1. Use the simulation smoother (see, for instance, the textbook by Durbin and Koopman (2001) for a description) to generate a draw $S_{0:T}^{(j)}$ from the distribution $p(S_{0:T}^{(j)}|Y_{1:T}, \theta^{(j)})$.
2. For each structural shock $i = 1, \ldots, m$ (which is an element of the vector s_t):
   (a) Compute the sequence of shock innovations $\epsilon_{i,1:T}^{(j)}$, for instance, by solving $s_{i,t}^{(j)} = \rho_i^{(j)} s_{i,t-1}^{(j)} + \sigma_i^{(j)} \epsilon_{i,t}^{(j)}$ for $\epsilon_{i,t}^{(j)}$.
   (b) Define a new sequence of innovations $e_{1:T}$ (e_t is of the same dimension as ϵ_t) by setting the i'th element $e_{i,t} = \epsilon_{i,t}^{(j)}$ for $t = 1, \ldots, T$ and $e_{i,t} \sim N(0, \sigma_i^2)$ for $t = T+1, \ldots, T+H$. All other elements of e_t, $t = 1, \ldots, T+H$, are set equal to zero.
   (c) Starting from $\tilde{s}_0 = s_0^{(j)}$, iterate the state transition equation (33) forward using the innovations $e_{1:T+H}$ to obtain the sequence $\tilde{S}_{1:T+H}^{(j)}$.
   (d) Use the measurement equation (34) to compute $\tilde{Y}_{1:T+H}^{(j)}$ based on $\tilde{S}_{1:T+H}^{(j)}$. $\square$

In practice, researchers sometimes take some or all of the following short-cuts: the parameter draws $\theta^{(j)}$ are replaced by the posterior mean or mode, say $\hat{\theta}$; draws of the

Figure 11: Shock Decompositions



Notes: The shock decompositions for output growth (top) and inflation (bottom) are computed using model SWπ estimated on the last available vintage (May 2011). The black and red lines represent the data and the forecasts, both in deviation from the steady state. The colored bars represent the contribution of each shock to the evolution of the variables.

sequence $S^{(j)}_{0:T}$ are replaced by the mean of the distribution $p(S_{0:T}|Y_{1:T}, \hat{\theta})$; and future values of $e_{i,t}$ (Step 2(b)) are set to zero.

The shock decomposition for output growth and inflation in Figure 11 is obtained from the SWπ model based on the May-2011 data vintage and provides an interpretation of the 2008-09 recession through the lens of a DSGE model. The black and red lines represent the data and the forecasts, both in deviations from the steady state. The colored bars represent the contribution of each shock in the model to the evolution of the historical and the projected path of the two series. The bars in Figure 11 show the posterior mean of the output generated with Algorithm 8. The two shocks chiefly responsible for the drop in output (top panel of Figure 11) are shocks that captures imperfections in financial markets, namely the discount rate (b) and the marginal efficiency of investment (μ) shocks.

As discussed in Smets and Wouters (2007), the discount rate shock has similar effects as a shock to the external finance premium in a model with explicit financial frictions as in Section 2.2. This is evident from the no-arbitrage condition (18). All else equal, a negative b shock coincides with an increase in the expected return of capital. Likewise, an increase in the riskiness of entrepreneurial projects (positive $\tilde{\sigma}_\omega$ shock) raises the spread between the expected return on capital and the riskless rate. The μ shock captures, in a broad sense, the headwinds from the crisis. More precisely, the shock shifts the efficiency with which savings are turned into future capital, and therefore serves as a proxy for the efficiency of financial intermediation (see Justiniano, Primiceri, and Tambalotti (2009)).

Wage mark-up (λ_w) and monetary policy (r^m) shocks also play a significant role in the 2008-09 recession. Wage mark-up shocks capture imperfections in the labor market, whereas the monetary policy shocks capture unanticipated deviations from the systematic part of the interest rate feedback rule. During the recession output and inflation were very low compared to their target value. According to the systematic part of the interest-rate feedback rule nominal interest rates should have been below zero during this period. Since the linearized version of the DSGE model ignores the zero-lower-bound constraint on the nominal interest rate, contractionary monetary policy shocks are necessary to rationalize the observed 25bp interest rates. The contractionary monetary policy shocks contributed to the depth of the recession. Finally, positive productivity shocks are the main drivers of the recovery in GDP after the trough, which is consistent with the behavior of measured productivity.

The same shocks that drive fluctuations in economic activity – μ and b shocks – also explain much of the business-cycle frequency movements in inflation. This is not surprising in light of the New Keynesian Phillips curve. These shocks depress the level of output, and therefore real marginal costs, for a long period of time, and consequently lead to inflation below trend. Productivity shocks also play an important role, as positive shocks lead to lower real marginal costs, *ceteris paribus*. High frequency movements in inflation, e.g., due to oil price shocks, are captured by price mark-up shocks (λ_f). Conversely, movements in the inflation target (π_*), which are disciplined by the use of long-run inflation expectations in the SWπ model, capture low frequency inflation movements.

Figure 11 is also helpful in explaining the forecasts. For instance, the SWπ model forecasts above trend output growth throughout the forecast horizon largely because of b and μ shocks: as the economy recovers from the Great Recession, the negative impact of these shocks on the *level* of output diminishes, and this results in a boost in terms of *growth rates*. Because of their protracted effect on economic activity, these shocks also keep inflation lower than steady state.

7.2 Real-Time DSGE Density Forecasts During the Great Recession: A Post-Mortem

[subsec:densforecastcrisis] After having provided an ex-post rationalization of the 2008-09 recession in the previous section, we now examine ex-ante forecasts of the SWπ model as well as two variants of the DSGE model with financial frictions discussed in Section 2.2, henceforth SWπ-FF. Figure 12 shows the DSGE models' and Blue Chip's forecasts for output growth (in Q-o-Q percent) obtained at three different junctures of the financial crisis that lead to the recession. The dates coincide with Blue Chip forecasts releases: (i) October 10, 2007, right after turmoil in financial markets had begun in August of that year; (ii) July 10, 2008, right before the default of Lehman Brothers; and (iii) January 10, 2009, at or near the apex of the crisis. Specifically, each panel shows the current real GDP growth vintage (black line), the DSGE model's mean forecasts (red line) and percentile bands of the predictive distribution (shaded blue areas indicate 50% (dark blue), 60%, 70%, 80%, and 90% (light blue) bands), the Blue Chip forecasts (blue diamonds), and finally the actual realizations according to the May-2011 vintage (black dashed line).

In interpreting the results, the reader should bear in mind that the information used to generate the forecasts depicted in Figure 12 consists only of data that was available at the beginning of October 2007, July 2008, and January 2009, respectively. For instance, by January 10, 2009 the forecaster would have access to NIPA samples that end in 2008:Q3 data. The information set of the Blue Chip forecasters, on the other hand, contains additional information such as economic indicators for 2008:Q4, information from financial markets, and a plethora of qualitative information provided by the media, speeches of government officials, *et cetera*.

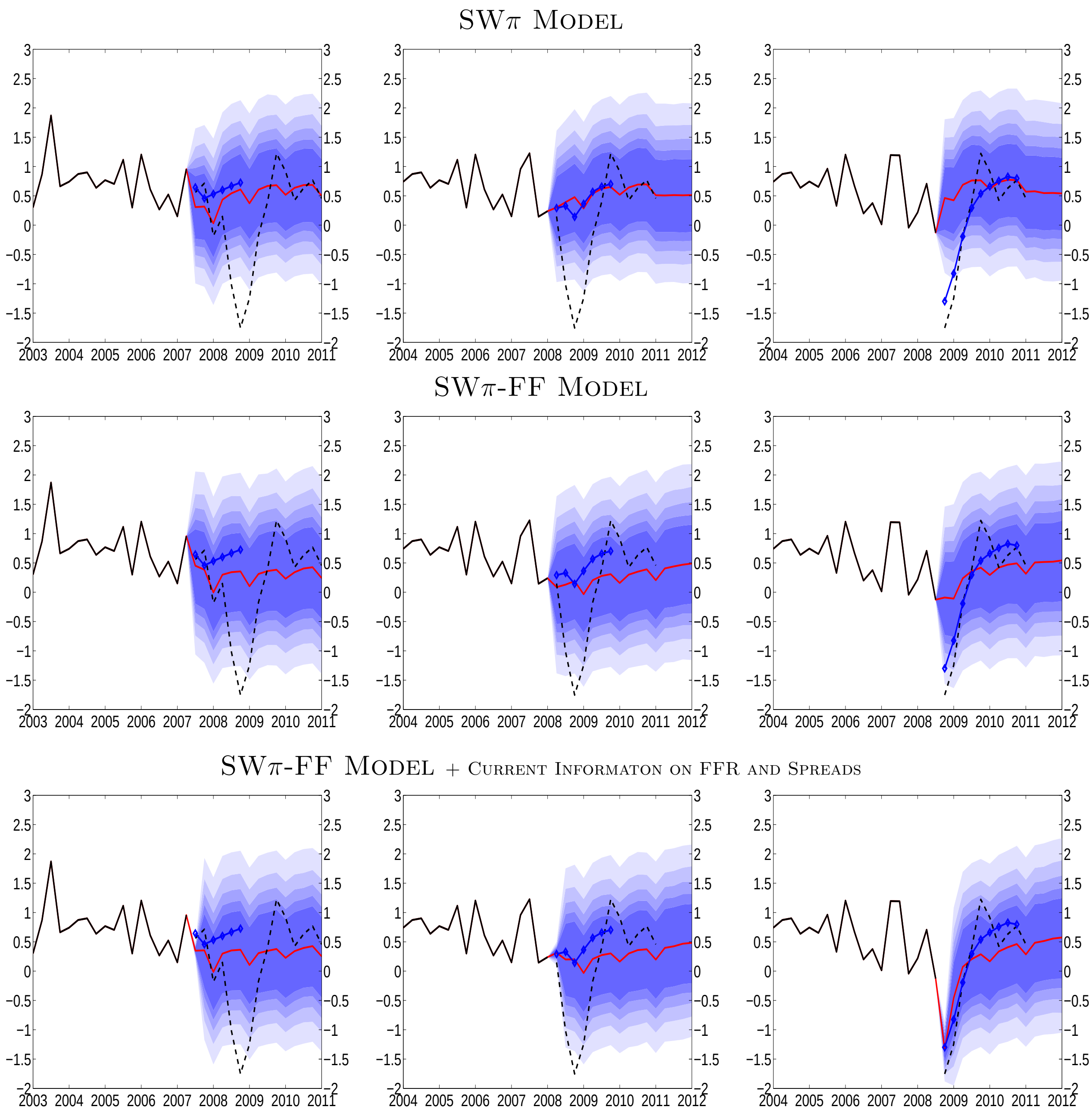
Each row of Figure 12 contains forecasts from a different model. We consider the SWπ model, the SWπ-FF model, and a specification that we label as SWπ-FF-Current. The two key differences between the SWπ and the SWπ-FF model are the introduction of financial frictions as in Bernanke, Gertler, and Gilchrist (1999) and the use of the Baa-10 year Treasury rate spread as an observable, which arguably captures distress in financial markets.[13] The difference between the SWπ-FF and the SWπ-FF-Current specification is that forecasts from the latter also utilize the Federal Funds rate and spreads of the most recent quarter, that is, of the quarter for which NIPA data is not yet available. For instance, the January 2009 SWπ-FF-Current forecast incorporates the average Federal Funds rate and the average spread for 2008:Q4. At any point in time, the information used to generate predictions from the SWπ-FF-Current model remains a subset of the information that has been available to the Blue Chip forecasters.

The October 10, 2007, Blue Chip Consensus forecasts for output are relatively upbeat, at or above 0.5% Q-o-Q, that is, 2% annualized, as shown in the panels in the first column of Figure 12. The SWπ forecasts are less optimistic, especially in the short run. The model's mean forecasts for output growth are barely above zero in 2008:Q1, with a non-negligible probability of sustained negative growth (recession) throughout the year. The forecasts for the two SWπ-FF specifications are in line with those of the SWπ model, although a bit more subdued. Quarter by quarter, the SWπ-FF specifications assign a probability of 50% or more to the occurrence of negative growth. While the DSGE models capture the slowdown that occurred in late 2007 and early 2008, they do not anticipate the subsequent post-Lehman

[13] Gilchrist and Zakrajsek (forthcoming) use secondary market prices of corporate bonds to construct a credit spread index that, they argue, is a a considerably more powerful predictor of economic activity than the measure of spreads we use. Their finding suggests that using an improved measure of spreads may further improve the SWπ-FF model's predictive ability.

Figure 12: Predicting the Crisis: Model and Blue Chip Forecasts for Output Growth

October 10, 2007 (2007Q2 data) July 10, 2008 (2008Q1 data) January 10, 2009 (2008Q3 data)



Notes: The panels show for each model/vintage the available real GDP growth data (black line), the DSGE model's mean forecasts (red line) and bands of its forecast distribution (shaded blue areas; these are the 50, 60, 70, 80, and 90 percent bands, in decreasing shade), the Blue Chip forecasts (blue diamonds), and finally the actual realizations according to the last available vintage (May 2011, black dashed line) . All the data are in percent, Q-o-Q.

collapse of economic activity. The decline in real GDP that occurred in 2008:Q4 lies far in the tails of the predictive distribution generated by the SWπ model. While the DSGE models with financial frictions place more probability on growth rates below -1% than the SWπ model, the 2008:Q4 growth rate still falls outside of the 90% credible prediction intervals.

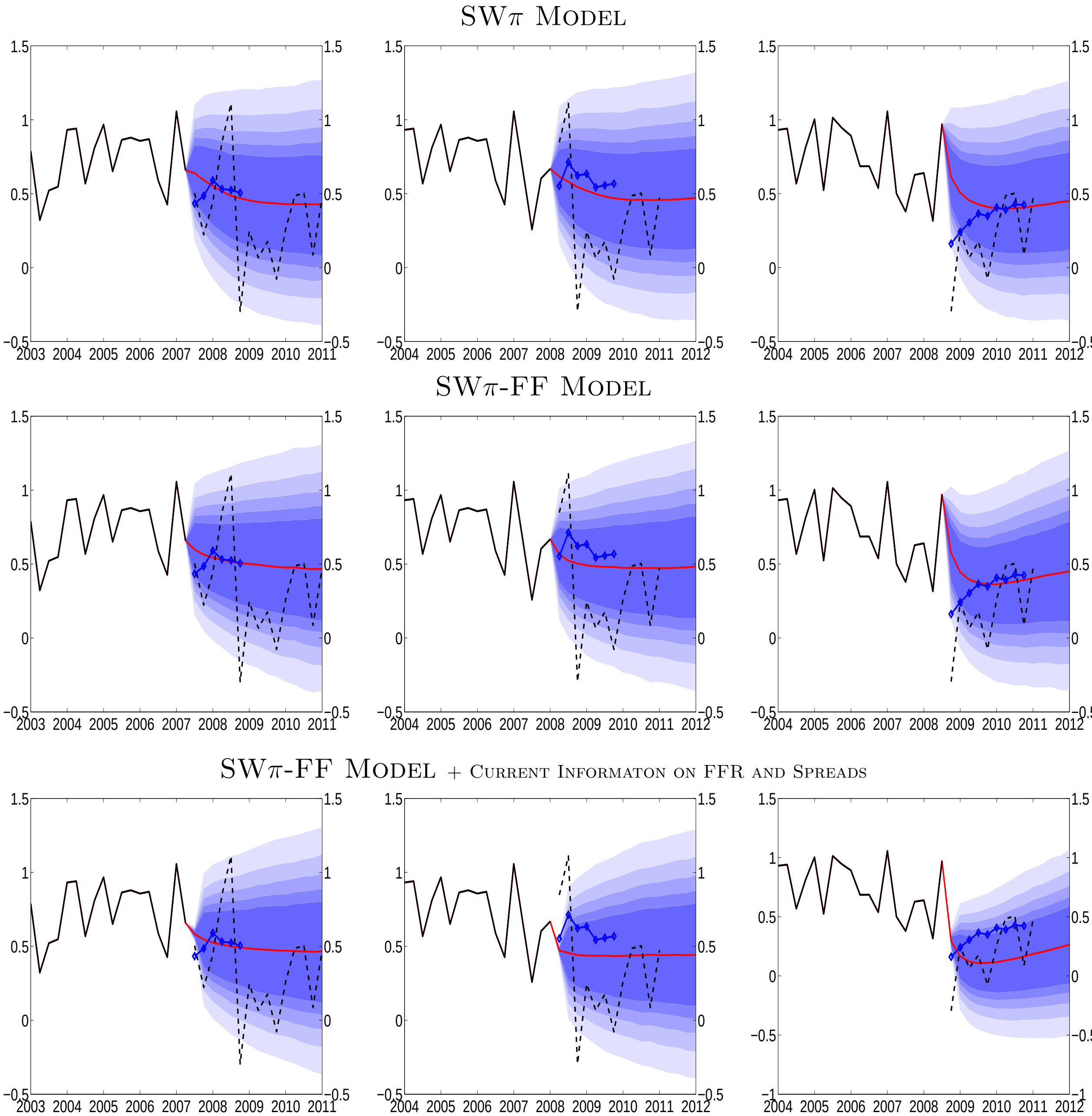
In July 2008 the Blue Chip forecast and the mean forecast for the SWπ model are roughly aligned. Both foresaw a weak economy – but not negative growth – in 2008, and a rebound in 2009. The two SWπ-FF specifications are less sanguine. Their forecasts for 2008 are only slightly more pessimistic than the Blue Chip forecast for 2008, but, unlike Blue Chip, the financial frictions models do not predict a strong rebound of the economy in 2009. While the two SWπ-FF deliver point forecasts of essentially zero growth in 2008:Q4, the models assign a lot of probability to strongly negative growth rates. As a consequence, the realized -1.7% growth rate in the last quarter of 2008 falls almost within the 90% credible interval associated with the predictive distribution.

By January 2009 the scenario has changed dramatically: Lehman Brothers has filed for bankruptcy a few months earlier (September 15, 2008), stock prices have fallen, financial markets are in disarray, and various current indicators have provided evidence that real activity was tumbling. None of this information was available to the SWπ model, which for the January 10, 2009 forecast round uses data up to 2008:Q3. Not surprisingly, the model is out of touch with reality with regard to the path of economic activity in 2008:Q4 and thereafter. It predicts a positive growth rate of 0.5% for the fourth quarter, while the actual growth rate is approximately -1.7%. The SWπ-FF model is less optimistic, it forecasts zero growth for 2008:Q4, but also misses the steep decline. The SWπ-FF uses spreads as an observable, but since the Lehman bankruptcy occurred toward the end of the third quarter, it had minor effects on the average Baa-10 year Treasury rate spread for 2008:Q3. As a consequence, the SWπ-FF model has little direct information on the turmoil in financial markets.

Finally, we turn to the forecasts from the SWπ-FF-Current specification, which uses 2008:Q4 observations on spreads and the Federal Funds rate. This model produces about the same forecast as Blue Chip for 2008:Q4. Unlike Blue Chip forecasters, the agents in the laboratory DSGE economy have not seen the Fed Chairman and the Treasury Secretary on television painting a dramatically bleak picture of the U.S. economy. Thus, we regard it as a significant achievement that the DSGE model forecasts and the Blue Chip forecasts are both

Figure 13: Predicting the Crisis: Model and Blue Chip Forecasts for Inflation

October 10, 2007 (2007Q2 data) July 10, 2008 (2008Q1 data) January 10, 2009 (2008Q3 data)



Notes: The panels show for each model/vintage the available GDP deflator data (black line), the DSGE model's mean forecasts (red line) and bands of its forecast distribution (shaded blue areas; these are the 50, 60, 70, 80, and 90 percent bands, in decreasing shade), the Blue Chip forecasts (blue diamonds), and finally the actual realizations according to the last available vintage (May 2011, black dashed line) . All the data are in percent, Q-o-Q.

around -1.3%. More importantly, we find this to be convincing evidence on the importance of using appropriate information in forecasting with structural models.

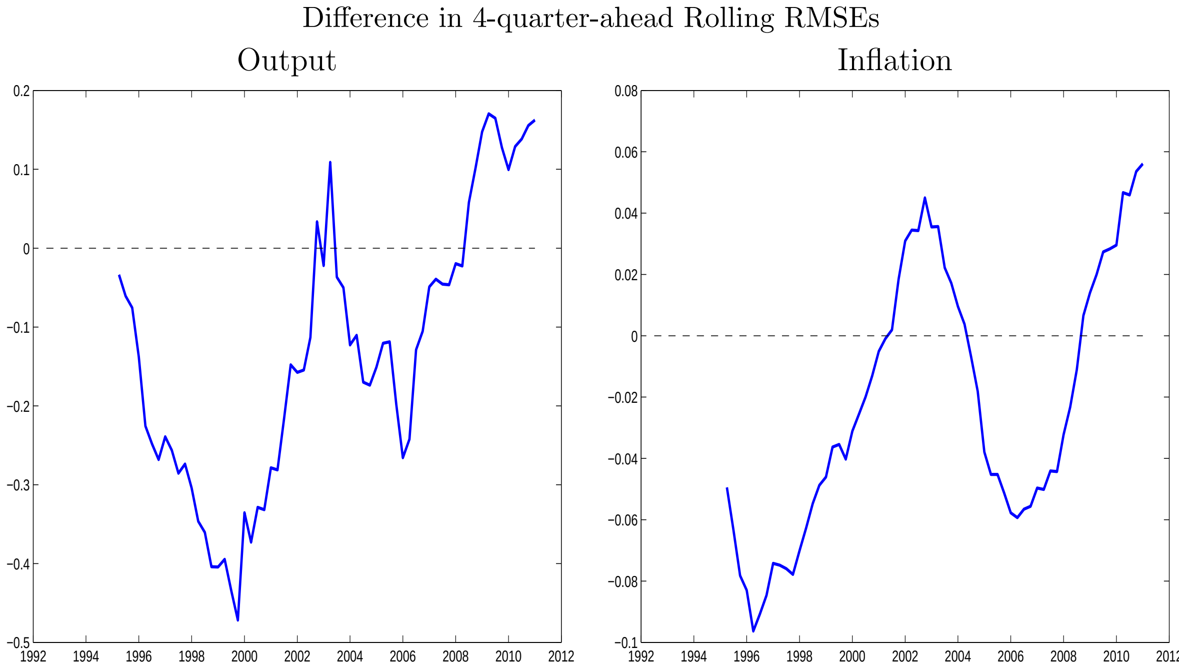
Figure 13 conducts the same post-mortem for inflation. On October 10, 2010, all three specifications generate similar forecasts of inflation. The mean quarterly forecasts are above 0.5% (2% annualized) in 2007, and slightly below 0.5% throughout the rest of the forecast horizon. Blue Chip forecasts are more subdued for 2007, and correctly so, but essentially coincide with the DSGE models' forecasts thereafter. The DSGE model point forecasts overstate inflation in 2009 and 2010. As the structural models miss the severity of the Great Recession, they also miss its impact on the dynamics of prices. In terms of density forecasts however, the forecasts of inflation are not as bad as those for output: In 2009 and 2010 the ex-post realizations of inflation are mostly within the 70 and 50% bands, respectively.

The July 10, 2008, Blue Chip forecasts for inflation increase quite a bit compared to 2007. The U.S. economy has just been hit by a commodity shock and, moreover, the Federal Reserve has lowered rates in response to the financial crisis, leading to Blue Chip quarterly inflation forecasts above 0.5% (2% annualized) throughout the forecast horizon. The mean DSGE model forecasts are instead slightly more subdued than in 2007, especially for the models with financial frictions. This occurs for two reasons. First, the DSGE models have no information on the recent rise in commodity prices. Second, the models perceive a weakness in aggregate activity and translate that into a moderate inflation outlook.

By 2008:Q3 inflation has risen following the commodity shock. Nonetheless, the January 10, 2009, inflation forecasts of the DSGE models are low. While all three models correctly assume the high 2008Q3 inflation to be temporary, there exist significant differences in the inflation forecasts across models, which reflect their different assessment of the state of the economy. The SWπ and SWπ-FF models' mean forcasts for inflation are generally above the actuals for 2009 and 2010. Blue Chip consensus forecasts are – correctly – more subdued than these forecasts for 2008:Q4 and the first half of 2009, but are no different afterwards: quarterly inflation quickly reverts to 0.5% in their outlook. Conversely, the SWπ-FF-current model predicts inflation to remain low throughout the forecast horizon, which is consistent with the actual evolution of this variable in the aftermath of the Great Recession.

The examination of DSGE model forecasts during the 2008-09 recession suggests that the DSGE models with financial frictions are preferable to the SWπ model. It turns out that this ranking is not stable over time. Figure 14 depicts RMSE differentials for the SWπ

Figure 14: Difference in Forecasting Accuracy Over Time: SWπ and SWπFF-Current



Notes: The figure shows the difference over time in 4-quarter-ahead rolling RMSEs between the SWπ and SWπFF-Current models for output growth and inflation. At each point in time, the RMSEs are computed using the previous 12 quarters, that is, the figure shows $RMSE_t(\text{SW}\pi) - RMSE_t(\text{SW}\pi\text{FF-Current})$, where $RMSE_t(\mathcal{M}_i) = \sqrt{\frac{1}{12}\sum_{j=0}^{11}\left(y_{t-j} - \hat{y}^{\mathcal{M}_i}_{t-j|t-j-4}\right)^2}$ and $\hat{y}^{\mathcal{M}_i}_{t-j|t-j-4}$ is the 4-quarter ahead forecast of y_{t-j} obtained using model $\mathcal{M}_i$, $\mathcal{M}_i = \{\text{SW}\pi, \text{SW}\pi\text{FF-Current}\}$.

model and the SWπ-FF-Current model for $h = 4$-step-ahead forecasts (the results for model SWπ-FF are very similar). At each point in term the RMSEs are computed using the 12 previous quarters. A value greater than zero indicates that the financial-frictions model attains a lower RMSE. The Figure indicates that on average over the forecast period the model without financial frictions generates more accurate forecasts. However, during the recent financial crisis, the ordering is reversed. The SWπ-FF-Current model contains an additional mechanism that associates high spreads with low economic activity and helps the model to track and forecast aggregate output and inflation throughout the crisis.

7.3 Calibration of Density Forecasts

[subsec:densforecastcalibration] We previously presented a variety of interval and density forecasts which begs the question of how accurate these forecasts are. While, strictly speaking, predictive distributions in a Bayesian framework are subjective, the statistics reported below provide a measure of the extent to which the predicted probabilities of events are consistent with their observed frequencies. Dawid (1984) views this consistency as a minimal desirable property for probability forecasts. Bayarri and Berger (2004) refer to the notion that in repeated practical use of a sequential forecasting procedure the long-run average level of accuracy should be consistent with the long-run average reported accuracy as *frequentist principle*. The literature, see for instance Dawid (1982), refers to sequences of (subjective) density forecasts that adhere to the frequentist principle as *well calibrated*. To assess whether DSGE model density forecasts are well calibrated we generate histograms for probability integral transformations (PITs).

Starting with Dawid (1984) and Kling and Bessler (1989) the use of probability integral transformations (PITs) has a fairly long tradition in the literature on density forecast evaluation. The PIT of $y_{i,T+h}$ based on its time T predictive distribution as

$$z_{i,h,T} = \int_{-\infty}^{y_{i,T+h}} p(\tilde{y}_{i,T+h}|Y_{1:T})d\tilde{y}_{i,T+h}. \tag{73}$$

Thus, the PIT is defined as the cumulative density of the random variable $y_{i,T+h}$ evaluated at $y_{i,T+h}$. Based on the output of Algorithm 2 the PITs can be easily approximated by (recall that $\mathcal{I}\{x \geq a\}$ denotes the indicator function)

$$z_{i,h,T} \approx \sum_{j=1}^{n_{sim}} \mathcal{I}\{y^{(j)}_{i,T+h} \leq y_{i,T+h}\},$$

where $y_{i,T+h}$ is now the value of y_i observed in period $T+h$. It is straightforward to show that the marginal distribution of PITs is uniform. Consider a random variable X with density $F(x)$. Then

$$\mathbb{P}\{F(X) \leq z\} = \mathbb{P}\{X \leq F^{-1}(z)\} = F\big(F^{-1}(z)\big) = z$$

Building on results by Rosenblatt (1952), Diebold, Gunther, and Tay (1998) show that for $h = 1$ the $z_{i,t,h}$'s are not just uniformly distributed, but they are also independent across time: $z_{i,t,h} \sim iid\, \mathcal{U}[0,1]$. For this reason, PITs are often called generalized residuals.

Below, we plot PIT histograms and informally assess the distance of the unconditional empirical distribution of the PITs from the uniform distribution. A more formal assessment, via posterior predictive checks, is provided in Herbst and Schorfheide (2011).[14] It is important to stress that the uniformity of PITs does not imply that a forecast is sharp. Abstracting from parameter uncertainty, suppose that y_t evolves according to

$$y_t = \theta y_{t-1} + u_t, \quad u_t \sim iidN(0,1), \quad 0 \leq \theta < 1. \tag{74}$$

Moreover, suppose that Forecaster F_1 reports the predictive density $N(\theta y_{t-1}, 1)$, whereas forecaster F_2 reports the density forecast $N\big(0, 1/(1-\theta^2)\big)$. Both forecasts lead to PITs that are *unconditionally* uniformly distributed. The uniformity of the PITs associated with F_2 follows immediately. Let $\Phi(\cdot)$ denote the cdf of a $N(0,1)$ random variable. For F_1 it can be verified as follows:

$$\mathbb{P}^{(y_t, y_{t-1})}\big\{\Phi(y_t - \theta y_{t-1}) \leq z\big\} = \mathbb{E}^{y_{t-1}}\Big[\mathbb{P}^{y_t}_{y_{t-1}}\big\{\Phi(y_t - \theta y_{t-1}) \leq z\big\}\Big] = \mathbb{E}^{y_{t-1}}[z] = z.$$

The unconditional probability needs to be computed under the joint distribution of (y_t, y_{t-1}). It can be obtained by first conditioning on y_{t-1} and subsequently integrating out y_{t-1}, which leads to the first equality. The second equality follows from (74). However, as long as $\theta > 0$, F_1's forecast will be more precise than F_2's forecast because it exploits conditioning information that reduces the variance of the predictive distribution from $1/(1-\theta^2)$ to 1. In fact, *conditional* on y_{t-1} the cdf of the PIT computed from F_2 is given by

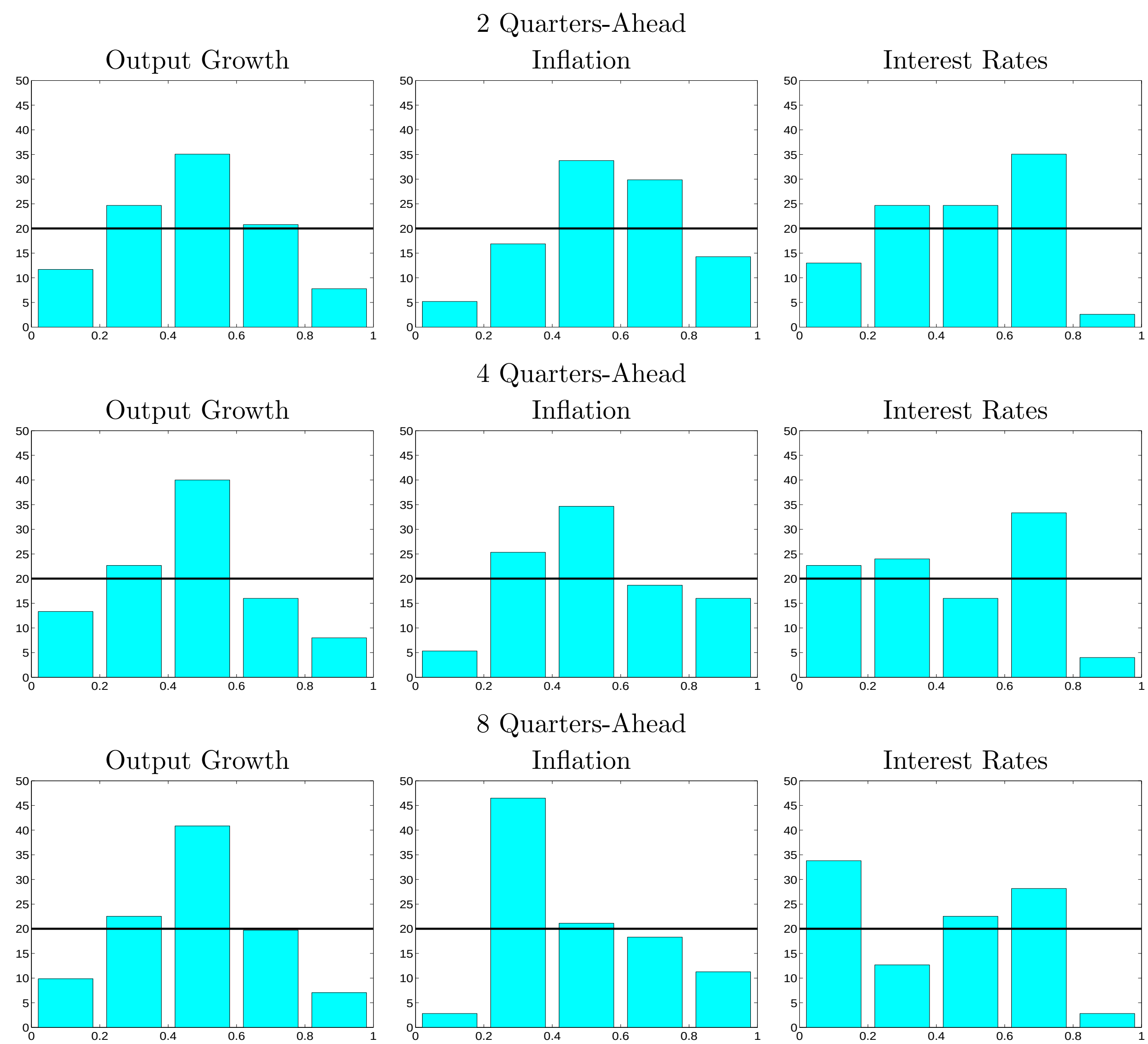
$$\mathbb{P}^{y_t}_{y_{t-1}}\left\{\Phi\big(\sqrt{1-\theta^2}\, y_t\big) \leq z\right\} = \Phi\left(\frac{\Phi^{-1}(z)}{\sqrt{1-\theta}} - \theta y_{t-1}\right),$$

which implies that the sequence of PITs from F_2 is not independently distributed.

As we have seen in Section 5, DSGE model forecasts often do not exploit all the available information and therefore might not be as sharp as other forecasts. Nonetheless, it remains interesting to assess whether the predictive distributions are well-calibrated in the sense that PITs have an unconditional distribution that is approximately uniform. Figure 15 depicts

[14] As emphasized by Geweke and Whiteman (2006), Bayesian approaches to forecast evaluation are fundamentally different from frequentist approaches. In a Bayesian framework there is no uncertainty about the predictive density given the specified collection of models, because predictive densities are simply constructed by the relevant conditioning. Non-Bayesian approaches, see Corradi and Swanson (2006), tend to adopt the notion of a "true" data-generating process (DGP) and try to approximate the predictive density inherent in the DGP with a member of a collection of probability distributions indexed by a parameter θ. To the extent that the forecaster faces uncertainty with respect to θ, there is uncertainty about the density forecast itself, and non-Bayesian assessments try to account for this uncertainty.

Figure 15: PITs: SWπ model



Notes:

histograms for PITs based on forecasts generated with the SWπ model. We group the PITs into five equally sized bin. Under a uniform distribution, each bin should contain 20% of the PITs, indicated by the solid horizontal lines in the figure. The empirical distribution looks quite different from a uniform distribution and the discrepancy increases with forecast horizon h. For output growth, an overly large fraction of PITs fall into the 0.4-0.6 bin. This indicates that the predictive distribution is too diffuse.

One potential explanation is that all of the forecasts are generated post 1984 and most of them fall into the period of the so-called Great Moderation. The estimation sample, on the other hand, contains a significant fraction of observations from the pre-1984 period. Thus, the shock standard deviations, roughly speaking, are estimated to capture an average of the pre- and post-moderation volatility, which means that they tend to overpredict the volatility during the forecast period. While the empirical distribution of the output growth PITs is essentially symmetric, the PITs associated with inflation and interest rate forecasts have a slightly skewed distribution. The DSGE model assigns substantial probability to low inflation rates that never materialize. Vice versa, the model also assigns positive probability to relatively high interest rates that are not observed during the forecast period. Further results on the evaluation of density forecasts of medium-scale DSGE models can be found in Herbst and Schorfheide (2011) and Wolters (2010).

8 Conclusion and Outlook

[sec:conclusion] This paper reviewed the recent literature on forecasting with DSGE models, discussed numerous useful algorithms, and provided empirical illustrations of the various methods considered. We presented some novel methods that allow modelers to incorporate external information and that may increase the accuracy of DSGE model forecasts. Moreover, we compared methods of generating forecasts conditional on desired interest rate paths, and studied the forecasting performance of DSGE models with and without financial frictions during the 2008-09 recession. In closing, we provide some discussion of why we think that DSGE-model-based forecasts are useful, we review empirical approaches that relax some of the DSGE model-implied restrictions to improve forecast accuracy, and lastly we engage in a wild speculation about the future of DSGE model forecasting.

8.1 Why DSGE Model Forecasting?

A macroeconomic forecaster can in principle choose from a large pool of econometric models. Some models are univariate, others are multivariate; some are linear, others are nonlinear; some are based on economic theory whereas others simply exploit correlations in the data. Empirical DSGE models are multivariate, in most instances they are linearized, and they build upon modern dynamic macroeconomic theory which emphasizes intertemporal decision making and the role of expectations. The benefit of building empirical models on sound theoretical foundations is that the model delivers an internally consistent interpretation of the current state and future trajectories of the economy and enables a sound analysis of policy scenarios. The potential cost is that theory-implied cross-coefficient restrictions might lead to a deterioration in forecast performance.

While a decade ago the costs outweighed the benefits, the scale has tipped in favor of DSGE models in recent years. First, DSGE models have been enriched with endogenous propagation mechanisms, e.g. Christiano, Eichenbaum, and Evans (2005), and exogenous propagation mechanisms, e.g. Smets and Wouters (2003, 2007), which allow the models to better capture the autocovariance patterns in the data. Second, as demonstrated in Section 5, DSGE models can be easily modified to incorporate external information into the forecasts, both real-time information about the current state of the economy as well information about its long run trends. Real-time information is interpreted by the models as information about the realization of the structural shocks, and is useful to improve the accuracy of short-horizon forecasts. Moreover, long-run inflation expectations can be used to anchor long-horizon forecasts of nominal variables.

The case for DSGE model forecasting ultimately rests on the fact that these models provide a good package. Granted, there exist time series models that generate more accurate univariate forecasts of output growth and inflation, but these models might miss comovements between these two variables. Bayesian VARs tend to be good multivariate forecasting models but it is difficult to identify more than one or two structural shocks and to provide a narrative for the current and future state of the economy. Moreover, VARs typically do not have enough structure to generate predictions about anticipated changes in interest rates. Dynamic factor models are able to extract information from a large cross section of macroeconomic variables and to explain the comovements among these series as being generated by a low-dimensional vector of latent variables. While the forecasting record of these models is strong, the policy experiments that could be carried out with these models are very limited. Finally, none of the aforementioned models would allow the user to measure the degree of distortion in the economy that ought to be corrected through monetary policy.

Estimated DSGE models can perform a lot of tasks simultaneously. They generate multivariate density forecasts that reflect parameter and shock uncertainty. They provide a device of interpreting the current state and the future path of the economy through the lens of modern dynamic macroeconomics and provide decompositions in terms of structural shocks. Moreover, the models enable the user to generate predictions of the effect of alternative policy scenarios. While a successful decathlete may not be the fastest runner or the best hammer thrower, she certainly is a well-rounded athlete.

8.2 Beyond DSGE Models

Throughout this paper have focused on forecasts generated from specific DSGE models. In closing we briefly mention some strands of the literature that either relax some of the DSGE model restrictions to improve their forecast performance or combine different classes of econometric models. Ingram and Whiteman (1994) were the first to use DSGE models to construct a prior distribution for vector autoregressions that is centered at the DSGE model-implied parameter restrictions. This approach has the advantage that the DSGE model restrictions are imposed in a non-dogmatic manner, allowing for modest violations of the DSGE model restrictions. Del Negro and Schorfheide (2004) developed this approach further and constructed a hierarchical Bayes model, called DSGE-VAR, that takes the form of a structural VAR and allows the researcher to simultaneously estimate the parameters of the DSGE model and the VAR. A hyperparameter determines the scale of the prior covariance matrix. If the prior covariance matrix is zero, then the DSGE model restrictions are dogmatically imposed on the VAR.

In the context of a small-scale DSGE model Del Negro and Schorfheide (2004) document that the best forecasting performance is obtained for an intermediate value of the hyperparameter that implies that prior distribution and likelihood of the VAR are about equally informative about the parameters. The DSGE-VAR produces substantially more accurate pseudo-out-of-sample forecasts than the underlying DSGE model. A similar empirical result is reported in Del Negro, Schorfheide, Smets, and Wouters (2007) for a variant of the Smets-Wouters model.[15]

[15]Kolasa, Rubaszek, and Skrzypczyński (2010) provide a less favorable assessment of the DSGE-VAR approach, however. Using the Smets and Wouters (2007) model to generate a prior distribution, the authors found that the DSGE model actually outperforms the DSGE-VAR. This result might be due to the fact that the 2007-version of the Smets-Wouters model contains a number of features that are designed to boost its forecast performance. Also, the DSGE-VAR specification that they use is in first differences.

An alternative way of combining VARs and DSGE models for macroeconomic forecasting applications is explored by Amisano and Geweke (2011). The authors consider a pool of macroeconomic models that incorporates, among others, DSGE models and VARs. A combined forecast is generated from a convex combination of the predictive densities associated with the models included in the pool. The weights are estimated such that asymptotically the Kullback-Leibler discrepancy between the convex combination of models and some underlying "data generating process" is minimized. The authors find that while the DSGE model receives a non-trivial weight in the model mixture, the forecasting performance of the pool is substantially better than the forecasting performance of any of the individual models in the pool. Waggoner and Zha (2010) extend the Amisano-Geweke approach by allowing for time-varying model weights that follow a regime-switching process. Moreover, model parameters and mixture weights are estimated simultaneously rather than sequentially. The authors identify episodes in which the DSGE model is useful for macroeconomic forecasting and episodes in which the combined forecasts are dominated by the VAR. The same approach could be used to combine different DSGE models. As documented in Section 7.2, the relative ranking of DSGE models without and with financial frictions seems to shift over time.

Finally, there is a strand of literature that combines DSGE models and dynamic factor models (DFM). The goal of this literature is to link the DSGE model with a large cross section of macroeconomic indicators rather than a small set of seven or eight observables as was done in this paper. One the one hand, the large set of macroeconomic variables might provide sharper inference about the current state of the economy. On the other hand, this framework allows the modeler to assess the effect of structural shocks, e.g. monetary policy shocks, on variables that are not explicitly modeled in the DSGE model. The resulting empirical specification is called DSGE-DFM. It is essentially a DFM in which the latent factors are equated with the state variables of a DSGE model and follow the DSGE model-implied law of motion. The DSGE-DFM was first developed by Boivin and Giannoni (2006) and studied further by Kryshko (2010) who documents that the space spanned by the factors of a DSGE-DFM is very similar to the space spanned by factors extracted from an unrestricted DFM. Schorfheide, Sill, and Kryshko (2010) used a DSGE-DFM to generate DSGE-model-based forecasts for variables that do not explicitly appear in the DSGE model.

8.3 The Future

While the literature on forecasting with DSGE models was practically non-existent a decade ago, it has become a vibrant area of research. A lot of progress has been made in the specification of DSGE models, as well as in the development of methods that enable the incorporation of real-time information, the relaxation of overly tight cross-equation restrictions, and the combination of DSGE models with other macroeconometric models. The progress is in part driven by the desire of central banks to incorporate modern macroeconomic equilibrium into their decision making process. In this regard, the recent crisis with the emergence of nonconventional monetary policies and interest rates near the zero-lower bound has supplied new challenges for DSGE model-based forecasting that need to be tackled in future research.

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A Details for Figure 4

Table A-1 lists the RMSEs that are plotted in Figure 4 by study. The specific references for each study can be found in Table 3.

Table A-2 contains details on the computation of AR(2) forecasts that are used to construct RMSE ratios.

Table A-1: RMSEs for DSGE and AR(2) Models in Figure 4

		$h = 1$			$h = 4$		
Study	Model	GDP	INFL	INT	GDP	INFL	INT
RS	DSGE	0.496	0.224	0.130	0.448	0.287	0.335
	AR (2)	0.512	0.198	0.083	0.502	0.235	0.320
KRS	DSGE	0.485	0.240	0.108	0.477	0.255	0.335
	AR (2)	0.471	0.236	0.107	0.465	0.297	0.416
GEW	DSGE	0.610	0.290	0.138	0.385	0.350	0.405
	AR (2)	0.532	0.221	0.118	0.328	0.281	0.437
W-DS	DSGE	0.525	0.262	0.163	0.532	0.272	0.372
	AR (2)	0.491	0.257	0.121	0.523	0.331	0.402
W-FM	DSGE	0.494	0.321	0.133	0.586	0.330	0.391
	AR (2)	0.491	0.257	0.121	0.523	0.331	0.402
W-SW	DSGE	0.542	0.255	0.127	0.462	0.279	0.344
	AR (2)	0.491	0.257	0.121	0.523	0.331	0.402
W-Edo	DSGE	0.529	0.285	0.164	0.511	0.349	0.478
	AR (2)	0.491	0.257	0.121	0.523	0.331	0.402
EG	DSGE	0.550	0.180	0.110	0.510	0.200	0.100
	AR (2)	0.568	0.223	0.122	0.582	0.304	0.425
EKL	DSGE	0.448	0.294	0.208	0.502	0.292	0.465
	AR (2)	0.626	0.204	0.102	0.734	0.254	0.387
SW	DSGE	0.566	0.245	0.108	0.327	0.183	0.354
	AR (2)	0.546	0.229	0.126	0.352	0.289	0.467
DSSW	DSGE	0.664	0.249	0.123	0.657	0.243	0.394
	AR (2)	0.493	0.206	0.111	0.340	0.199	0.357
SSK	DSGE	0.510	0.220	0.177	0.410	0.190	0.532
	AR (2)	0.525	0.227	0.094	0.469	0.229	0.419

Table A-2: A Sample of Studies Reporting RMSEs for Medium-Scale DSGE Models: Part 1

Study	**Real-time Data Set?**	**Actuals for Forec. Eval Used for DSGE Model**	**Actuals for Forec. Eval Used for AR(2) Model**	**GDP Data Per Capita?**	**Forecast Multi-Averages?**
RS	Yes	One-year after the forecasting origin	First final release	No	No
KRS	Yes	Fixed vintage (2009:Q1)	First final release	No	No
GEW	No	N/A	Fixed vintage (Jan 2008)	Yes	Yes (Output)
W-DS	Yes	Vintage released two quarters after	Same as original study	No	No
W-FM	Yes	Vintage released two quarters after	Same as original study	No	No
W-SW	Yes	Vintage released two quarters after	Same as original study	No	No
W-Edo	Yes	Vintage released two quarters after	Same as original study	No	No
EG	Yes	First final release	First final release	Yes	No
EKL	Yes	First final release	First final release	Yes	No
SW	No	N/A	Fixed vintage (Jan 2006)	Yes	Yes (Output)
DSSW	No	N/A	Fixed vintage (Apr 2005)	Yes	Yes (Output, Inflation)
SSK	No	N/A	Fixed vintage (Apr 2009)	Yes	No

Table A-2: A Sample of Studies Reporting RMSEs for Medium-Scale DSGE Models: Part 2

Study	**Forecast Origins**	**Forecasting Dates**	**Data for AR (2) Forecasting**	**Estimation Sample**
RS	1994:Q1-2005:Q3	Middle of each quarter	Our realtime dataset (GB dates)	Most recent 60 quarters
KRS	1994:Q1 - 2007:Q4	Middle of each quarter	Our realtime dataset (GB dates)	Fixed starting point (1964:Q3)
GEW	1990:Q1-2007:Q1 (h=1)	N/A	Vintage 4Q after last date of forecasting origin	Fixed starting point (1966:Q1)
W-DS	1984:Q1 - 2000:Q4	Greenbook	Use Faust and Wright dataset	Most recent 80 quarters
W-FM	1984:Q1 - 2000:Q4	Greenbook	Use Faust and Wright dataset	Most recent 80 quarters
W-SW	1984:Q1 - 2000:Q4	Greenbook	Use Faust and Wright dataset	Most recent 80 quarters
W-Edo	1984:Q1 - 2000:Q4	Greenbook	Use Faust and Wright dataset	Most recent 80 quarters
EG	1992:Q1(Jan)-2004:Q4(Dec)	Greenbook	Our realtime dataset (GB dates)	Fixed starting point (1965:Q1)
EKL	1996:Q3(Sep)-2002:Q4(Dec)	Greenbook	Our realtime dataset (GB dates)	Fixed starting point (1983:Q1)
SW	1990:Q1-2004:Q4 (h=1)	N/A	Vintage 4Q after last date of forecasting origin	Fixed starting point (1966:Q1)
DSSW	1986:Q1-2000:Q2 (h=1)	N/A	Vintage 4Q after last date of forecasting origin	Most recent 120 quarters
SSK	2001:Q1-2007:Q4 (h=1)	N/A	Vintage 4Q after last date of forecasting origin	Fixed starting point (1984:Q1)