Discussion of
"Sources of entropy in representative agent models"
by Dave Backus, Mike Chernov & Stan Zin

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Conventional model diagnostics

- does your model have “reasonable” asset pricing implications?
- conventional answer: start with Euler equation

\[ E_t (m_{t+1} r_{t+1}^i) = 1 \]
\[ = \text{cov}_t (m_{t+1}, r_{t+1}^i) + E_t (m_{t+1}) E_t (r_{t+1}^i) \]
\[ = \rho_t \sigma_t (m_{t+1}) \sigma (r_{t+1}^i) + E_t (m_{t+1}) E_t (r_{t+1}^i) \]

rearrange, get Hansen-Jagannathan bounds

\[ \frac{\sigma_t (m_{t+1})}{E_t (m_{t+1})} \geq \text{Sharpe ratio for asset } i = \frac{E_t (r_{t+1}^i) - r_t^f}{\sigma_t (r_{t+1}^i)} \]

- measure RHS with data on financial returns
- compare RHS with LHS computed from your model
- nice: works even if your model does not have asset \( i \)
  e.g. Telmer 1993 heterogeneous agents with one bond
New model diagnostics proposed by Dave, Mike & Stan

1. take one-period kernel $m_{t+1}$, compute

   one-period conditional entropy $= \log E_t (m_{t+1}) - E_t (\log m_{t+1})$

   rough idea: $\sigma_t (m_{t+1})$ in HJ bounds

2. take multi-period kernel $m_{t,t+n} = m_{t+1} \cdots m_{t+n}$, compute

   multi-period conditional entropy $= \log E_t (m_{t,t+n}) - E_t (\log m_{t,t+n})$

   define

   \[
   \text{horizon dependence} = \frac{1}{n} E \left[ \log E_t (m_{t,t+n}) - E_t (\log m_{t,t+n}) \right] 
   - E \left[ \log E_t (m_{t+1}) - E_t (\log m_{t+1}) \right]
   \]

   rough idea: HJ bounds for long-horizon vs. short-horizon returns
More precisely: one-period conditional entropy

- **rough idea:** entropy is a measure of dispersion of the pricing kernel, or is like checking $\sigma_t(m_{t+1})$ in HJ bounds.

\[
\frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})} \geq \text{Sharpe ratio} = \frac{E_t(r_{t+1}^i) - r_t^f}{\sigma_t(r_{t+1}^i)}
\]

should be quantitatively large to match data on average excess returns.

- In models with lognormal $m_{t+1}$, they boil down to the same

  \[
  \text{entropy} \equiv \frac{1}{2} \sigma_t(\log m_{t+1})
  \]

- large class of representative agent models!
  - long run risk (Bansal & Yaron 2004, Hansen-Heaton-Li 2008 etc.)

- diagnostic is more interesting in models that are not lognormal
More precisely: horizon dependence

- **rough idea:** HJ bounds for long-horizon vs. short-horizon returns

  horizon dependence :=
  \[
  \frac{1}{n} E \left[ \log E_t (m_{t,t+n}) - E_t (\log m_{t,t+n}) \right] \\
  - E \left[ \log E_t (m_{t+1}) - E_t (\log m_{t+1}) \right]
  \]

  \[
  = - E \left( y_t^{(n)} - y_t^{(1)} \right)
  \]

  = \quad \text{average slope of the yield curve}

  = \quad \text{average (long-horizon return on long}

  \quad \text{short-horizon return on short bond)}

- determined by autocorrelation of the pricing kernel
- should be **quantitatively small in absolute value** in your model to
  match data on Government bond yields
Perform model diagnostics

1. power utility does badly on one-period entropy
2. long run risk models (Bansal & Yaron 2004, Hansen, Heaton, Li 2008) do well on one-period entropy, do badly on horizon dependence
3. habits ( = catching up with the Joneses) ratio habits \( u \left( \frac{c_t}{h_t} \right) \) Abel 1992 do badly on entropy difference habits \( u \left( c_t - h_t \right) \) Campbell Cochrane 1999 ✓
4. jumps/disasters: Rietz-with-time-varying-disaster-probability ✓

Are diagnostics tough enough?

- models that do best have least discipline: habit process, time variation in disaster prob calibrated to match stock prices
- get additional discipline from macro implications
- need huge disasters: 30% consumption drop
  (cf. Great Depression: "only" 10%)
- other counterfactual implications (e.g. value discount not premium)
Summary

- very nice paper
- provides organizing framework
  if you want to teach a single paper on asset pricing in rep agent models, this may be it.
- diagnostics are useful
  related to HJ bounds for short-horizon, long-horizon returns with subtle differences, mostly for models that are not lognormal
- diagnostics make two models look particularly good
  1. Campbell-Cochrane 1999
  2. Wachter (Rietz with time-varying disaster prob)
- tough enough??  discipline, numbers, other (including macro) implications
- future for rep agent models with rational expectations in asset pricing?