Discussion of "Sources of entropy in representative agent models" by Dave Backus, Mike Chernov & Stan Zin

Monika Piazzesi
Stanford & NBER

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Conventional model diagnostics

- does your model have "reasonable" asset pricing implications?
- conventional answer: start with Euler equation

$$
\begin{aligned}
E_t\left(m_{t+1} r_{t+1}^i\right) &= 1 \\
&= cov_t\left(m_{t+1}, r_{t+1}^i\right) + E_t\left(m_{t+1}\right) E_t\left(r_{t+1}^i\right) \\
&= \rho_t\, \sigma_t\left(m_{t+1}\right) \sigma\left(r_{t+1}^i\right) + E_t\left(m_{t+1}\right) E_t\left(r_{t+1}^i\right)
\end{aligned}
$$

rearrange, get Hansen-Jagannathan bounds

$$
\frac{\sigma_t\left(m_{t+1}\right)}{E_t\left(m_{t+1}\right)} \geq \text{Sharpe ratio for asset } i = \frac{E_t\left(r_{t+1}^i\right) - r_t^f}{\sigma_t\left(r_{t+1}^i\right)}
$$

- measure RHS with data on financial returns
  compare RHS with LHS computed from your model
- nice: works even if your model does not have asset i
  e.g. Telmer 1993 heterogeneous agents with one bond

New model diagnostics proposed by Dave, Mike & Stan

1. take one-period kernel m_{t+1}, compute

$$\text{one-period conditional entropy} = \log E_t\left(m_{t+1}\right) - E_t\left(\log m_{t+1}\right)$$

rough idea: $\sigma_t\left(m_{t+1}\right)$ in HJ bounds

2. take multi-period kernel $m_{t,t+n} = m_{t+1} \cdots m_{t+n}$, compute

$$\text{multi-period conditional entropy} = \log E_t\left(m_{t,t+n}\right) - E_t\left(\log m_{t,t+n}\right)$$

define

$$\begin{aligned}\text{horizon dependence} = \quad & \frac{1}{n} E\left[\log E_t\left(m_{t,t+n}\right) - E_t\left(\log m_{t,t+n}\right)\right] \\ & - E\left[\log E_t\left(m_{t+1}\right) - E_t\left(\log m_{t+1}\right)\right]\end{aligned}$$

rough idea: HJ bounds for long-horizon vs. short-horizon returns

More precisely: one-period conditional entropy

- rough idea: entropy is measure of dispersion of the pricing kernel is like checking $\sigma_t(m_{t+1})$ in HJ bounds

$$\frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})} \geq \text{Sharpe ratio} = \frac{E_t\left(r^i_{t+1}\right) - r^f_t}{\sigma_t\left(r^i_{t+1}\right)}$$

  should be quantitatively large to match data on average excess returns

- In models with lognormal m_{t+1}, they boil down to the same

$$\text{entropy} \equiv \frac{1}{2}\sigma_t(\log m_{t+1})$$

- large class of representative agent models!
  long run risk (Bansal & Yaron 2004, Hansen-Heaton-Li 2008 etc.)
  habits (Abel 1992, Constantinides 1990, Campbell & Cochrane 1999)
- diagnostic is more interesting in models that are not lognormal

More precisely: horizon dependence

- *rough idea:* HJ bounds for long-horizon vs. short-horizon returns

$$
\begin{aligned}
\text{horizon dependence} := \quad & \frac{1}{n} E\left[\log E_t\left(m_{t,t+n}\right) - E_t\left(\log m_{t,t+n}\right)\right] \\
& - E\left[\log E_t\left(m_{t+1}\right) - E_t\left(\log m_{t+1}\right)\right] \\
= \quad & -E\left(y_t^{(n)} - y_t^{(1)}\right) \\
= \quad & - \text{ average slope of the yield curve} \\
= \quad & - \text{ average (long-horizon return on long} \\
& - \text{ short-horizon return on short bond)}
\end{aligned}
$$

- determined by autocorrelation of the pricing kernel
- should be quantitatively small in absolute value in your model to match data on Government bond yields

Perform model diagnostics

1. power utility does badly on one-period entropy
2. long run risk models (Bansal & Yaron 2004, Hansen, Heaton, Li 2008) do well on one-period entropy, do badly on horizon dependence
3. habits ($=$ catching up with the Joneses)
   ratio habits $u(c_t/h_t)$ Abel 1992 do badly on entropy
   difference habits $u(c_t - h_t)$ Campbell Cochrane 1999 ✓
4. jumps/disasters: Rietz-with-time-varying-disaster-probability ✓

Are diagnostics tough enough?

- models that do best have least discipline: habit process, time variation in disaster prob calibrated to match stock prices
- get additional discipline from macro implications
- need *huge* disasters: 30% consumption drop
  (cf. Great Depression: "only" 10%)
- other counterfactual implications (e.g. value discount not premium)

Summary

- very nice paper
- provides organizing framework
  if you want to teach a single paper on asset pricing in rep agent models, this may be it.
- diagnostics are useful
  related to HJ bounds for short-horizon, long-horizon returns
  with subtle differences, mostly for models that are not lognormal
- diagnostics make two models look particularly good
  1. Campbell-Cochrane 1999
  2. Wachter (Rietz with time-varying disaster prob)
- tough enough?? discipline, numbers, other (including macro) implications
- future for rep agent models with rational expectations in asset pricing?