Information Choice Technologies

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Motivation

- Why include information choice in a model?
  - Information is not observable $\rightarrow$ theories untestable. Information choice links observables to observables.
  - Information can change over time, vary across shocks.
  - Information choice potentially affects every moment in a model.
- Many different types of information choice being used:
  1. Inattentiveness (Reis 2007)
  2. Rational inattention (Sims 2003)
  3. Information markets (Kurlat and Veldkamp, 2012)
  4. Choosing signal clarity (Myatt and Wallace 2011).
- How do they differ? Why does it matter?

Outline

- Use a common framework to describe
  - Signal content (public-private, continuous-discrete)
  - Common strategic motives in information choice
- Differences: Nature of cost function differs. Some $\rightarrow$ multiple equilibria.
- Future directions in information choice research.

A Flexible Signal Structure

- A vector of signals with public and private noise:

$$\mathbf{z}^i = \mathbf{1}_n \cdot s + A\mathbf{u} + B\mathbf{v}^i \tag{1}$$

- Cross-signal correlation is easy to undo.
- Cross-agent correlation is strategically important.
- Agent chooses diagonal $A \geq 0$ and/or $B \geq 0$, at a cost $c(A, B)$.
- Interpretation: A is sender noise, B is receiver noise.

Technology 1: Fixed Cost

- Observe no signal ($A + B = \infty$) or observe s exactly ($A = B = 0$) at a cost c.
- Captures fact checking.
- Reis (2006) calls this “inattentiveness"
  Choice variable is a time to incur this fixed cost. Makes the choice variable continuous. But, still a discrete bit of information at each date.

Technology 2: How Many Signals to Observe?

$$\mathbf{z}^i = \left[z_1^i, z_2^i, \ldots, z_N^i\right]' \quad (2)$$

- Fix A and B. Choose how many signals to observe n.
- Represents information purchased from a common source. An information market. Reading further in newspaper.
- Making signal choice quasi-continuous:
  Take limit as noise grows $A \to \infty$ and cost $c \to 0$.

Technology 3: Choosing Signal Variances

$$\mathbf{z}^i = \mathbf{1}_n \cdot s + A\mathbf{u} + B\mathbf{v}^i \qquad (3)$$

- If $A \neq 0$, signals are correlated.
  Changing either A or B changes correlation.
- Myatt and Wallace (2011): Choose B.
  Interpretation: attention to public signals.
- Hellwig, Kohls and Veldkamp (2012): Choose A.
  Fixed inattention. Can choose clear or noisy channels.
- Both cases: Arbitrary increasing, convex cost of additional precision

Technology 4: Rational Inattention

What is rational inattention?

- Adds private noise to information. ($A = 0$. Choose B.) But RI is more general than this.
- Agents choose distribution of signal noise, covariance across signals.
  Here we assume normal signals. Relaxing normality $\rightarrow$ richer results.
- A specific form of information cost, $c(\kappa)$.
  If Σ and Ω are normal prior and posterior variances,

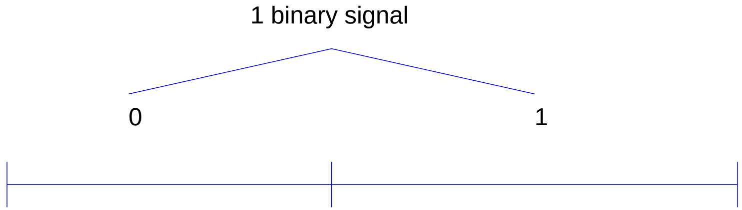
$$\frac{1}{2} log_2 \left(\frac{|\Sigma|}{|\Omega|} \right) \leq \kappa$$

For diagonal Ω, this is a *product*: $\prod_{i=1}^{N} \omega_i^{-1} \leq K$.

Technology 4: Rational Inattention

Why a product of precisions?

- Efficient binary coding bisects event space repeatedly.
- It represents an iterative search process. Knowledge is cumulative.
- Example: Uniform, each signal reduces stdev by $1/2$.



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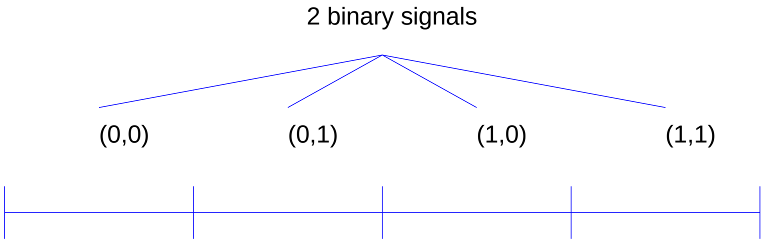


A Two-Stage Strategic Game

- Stage 1: A measure of agents have prior $s \sim N(\mu, \tau_s^{-1})$ and choose information to observe.
- Stage 2: Agents observe their information and choose a_i to max

$$u(a_i, \bar{a}, s) = -r(a_i - \bar{a})^2 - (1-r)(a_i - s)^2 - C. \quad (4)$$

  r is complementarity, C is information cost.
- Stage-2 FOC: $a_i = E[r\bar{a} + (1-r)s|\mathcal{I}_i]$.

Similarity: Strategic Motives

- The idea: If I want a_i high when $\bar{a}$ high, then I also want more information, when others choose more information. And vice-versa.
- What does “more information" mean?
  - More information about the state $[s\ u']'$
  - Information χ' is more information than χ if

$$Var\left[[s\ u']'|\chi\right] - Var\left[[s\ u']'|\chi'\right] \quad \text{is} \quad \text{p.s.d.}.$$

  - Let μ, μ' be distributions of information sets.
  - Others get more information $\mu' > \mu$ when

$$\int Var\left[[s\ u']'|\chi_i\right] di - \int Var\left[[s\ u']'|\chi_i'\right] di \quad \text{is} \quad \text{p.s.d.}$$

Strategic Motives in Information Choice

Proposition

If decisions are complementary ($r > 0$), additional information is complementary:
$EU(\chi', \mu') - EU(\chi, \mu') > EU(\chi', \mu) - EU(\chi, \mu)$.

If decisions are substitutes ($r < 0$), additional information is a substitute:
$EU(\chi', \mu) - EU(\chi, \mu) > EU(\chi', \mu') - EU(\chi, \mu')$.

Result comes from covariance risk

- When others learn, their actions covary more with the unknown state ($cov(\bar{a}, s)$ is higher).
- If actions are complements, others' learning *amplifies* risk. If you get the state wrong, your action will also be misaligned with others'.
  - Extra risk makes information more valuable.
  - *You learn more when others learn more (complementarity).*
- If actions are substitutes, this *hedges* risk. You want to align with the state, but not with others' actions.
  - Less risk makes information less valuable.
  - *You learn less when others learn more (substitutability).*

Applications of Strategic Info Choice

- Complementarity $\rightarrow$ **Inertia.**
  Allocate less attention to aggregate shocks because others do $=$ Insensitivity aggregate shocks.
  Woodford (2008), Reis (2006), Mackowiak and Wiederholt (2010)

- Substitutability $\rightarrow$ **Under-diversification.**
  Agents want to make information sets as different as possible by learning what others know least about. $\rightarrow$ home bias.
  Van Nieuwerburgh and Veldkamp (2009)

Differences between Technologies

For each technology: unique equilibrium or multiple equlibria?

- Offers practical advice about how to build a model to deliver a unique prediction.
- Opens a window in to the mechanics of each technology to tease out differences.

Technology 1: Fixed Cost Results

Proposition

There exists an equilibrium where

1. *all agents acquire information if and only if* $c \leq \tau_s^{-1}$.
2. *no agent acquires information if and only if* $c \geq (1 - r)^2 \tau_s^{-1}$.

If $r > 0$, these intervals overlap.
Complementarity + discreteness = multiple equilibria.

Technology 2: How Many Signals to See

- To eliminate discreteness:
  - Let $A = \gamma^{1/2}\delta^{-1/2}I$ and $B = (1-\gamma)^{1/2}\delta^{-1/2}I$.
  - γ is signal correlation across agents.
  - Take limit as $\delta \to 0$. Keep cost per unit precision constant.

Proposition

Observing the first n signals is an equilibrium if and only if

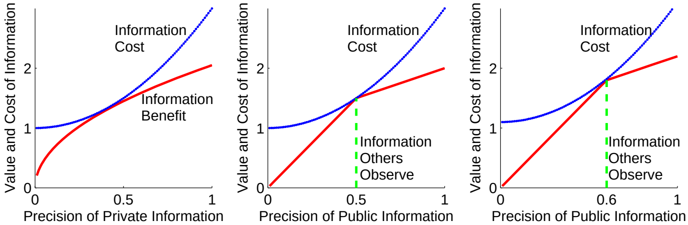
$$1 \geq C'(\delta n)\Psi \geq (1 - r\gamma^2)^2$$

where $\Psi = (1 - r\gamma + (1-r)\delta n)^2$. *(Hellwig & Veldkamp, 2009)*

- Private signals ($\gamma = 0$) $\to$ unique equilibrium n.
- Complementarity ($r > 0$) + correlated signals ($\gamma > 0$) = multiple equilibria.

Technology 2: How many Signals to See

Public Information → Multiple Equilibria



Public information, in excess of what others observe, is private. Jump in marginal value → multiplicity.

Technology 3: Choosing Signal Variances

Always a unique equilibrium:

Choose A or B in $\quad \mathbf{z}^i = \mathbf{1}_n \cdot s + A\mathbf{u} + B\mathbf{v}^i$.

Proposition

Suppose information costs are a function of the sum of private precisions: $c\left(\sum_i B_{ii}^{-2}\right)$ and c is convex. Then the equilibrium information choice is unique.

(Myatt & Wallace, 2011)

Proposition

Suppose information costs are a function of the sum of public precisions: $c\left(\sum_i A_{ii}^{-2}\right)$ and c is convex. Then the equilibrium information choice is unique.

(Hellwig, Kohls and Veldkamp, 2012)

Technology 3: Choosing Signal Variances

Why is choosing precision of public signal different from choosing how many public signals to acquire?

1. In newspaper model, agent can separate info others see from what they do not. Two types of information have discretely different marginal utility. Here, MU is continuous.
2. Lower A can increase $Var[\bar{a}|\mathcal{I}_i]$ and lower expected utility. More newspaper info is never bad.
3. Clearer signal (low A) has same s and u, with different weights. More news changes the value of u.

Technology 4: Rational Inattention

- RI can generate multiple equilibria (Myatt and Wallace '11).
- With normal signals no cross-signal correlation, mutual information $\propto |\hat{\Sigma}^{-1}| = \prod_i \hat{\sigma}_i^{-1}$.
- Ex: Suppose prior precision is 2 and 4. Add one unit.
  Learn about risk 1: cost $= 3 * 4 = 12$
  Learn about risk 2: cost $= 2 * 5 = 10$
  Learning about higher precision risks is cheaper.
  $\rightarrow$ Learn more about what you know well.
- In a model with information spillovers (ex: prices), others' learning $\uparrow$ precision, $\downarrow$ info cost.
  $\rightarrow$ multiple equilibria.

Paper Summary

- We extend HV '06 results on quantity of info to include signal precision choice.
- In all these settings, information choices inherit strategic motives in actions.
- Sources of multiple equilibria
  1. discreteness
  2. cost function properties (as in RI)
  3. choosing more (instead of clearer) public information

Where to go from here?

- Information choice and liquidity:
  Recent crisis highlights that assets that were like money, ceased to be money when info was asymmetric.
- Business cycle dynamics
  Time-varying information choices $\rightarrow$ time-varying sensitivities to shocks.
- An information-production economy
  In US, lots of GDP is acquiring and processing information.
- To examine these phenomena, we need information choice technologies. Understanding the properties of those technologies is a first step in the broader agenda.