

Money and Banking in Search Equilibrium*

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Abstract

We develop a new theory of money and banking based on the old story about goldsmith bankers first accepting deposits for safe keeping, after which their liabilities began circulating as means of payment, and they began making loans. We first discuss the story. We then present a model where money is a medium of exchange, but subject to theft, and for safety agents may open bank accounts and pay by check. The equilibrium means of payment can consist of cash, demand deposits, or both; we show how this varies with parameters like the cost of banking and money supply. For some parameters, banks may be necessary for money to be valued. When we allow fractional reserves and a competitive loan market, we derive a money multiplier with microfoundations.

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The theory of banking relates primarily to the operation of commercial banking. More especially it is chiefly concerned with the activities of banks as holders of deposit accounts against which cheques are drawn for the payment of goods and services. In Anglo-Saxon countries, and in other countries where economic life is highly developed, these cheques constitute the major part of circulating medium. *Encyclopedia Britannica* (1954, vol.3, p.49).

1 Introduction

This paper develops a new theory of money and banking based on an old story about money and banking. This story is so well known that it is described nicely in standard reference books like *Encyclopedia Britannica*: “the direct ancestors of modern banks were, however, neither the merchants nor the scrivenors but the goldsmiths. At first the goldsmiths accepted deposits merely for safe keeping; but early in the 17th century their deposit receipts were circulating in place of money and so became the first English bank notes.” (*EB* 1954, vol. 3, p. 41). “The cheque came in at an early date, the first known to the Institute of Bankers being drawn in 1670, or so.” (*EB* 1941, vol. 3, p. 68).¹

In case one doubts the authority of general reference books on such matters,² we note that more specialized sources echo this view. As Quinn (1997,

¹To go into more detail: “To secure safety, owners of money began to deposit it with the London goldsmiths. Against these sums the depositor would receive a note, which originally was nothing more than a receipt, and entitled the depositor to withdraw his cash on presentation. Two developments quickly followed, which were the foundation of ‘issue’ and ‘deposit’ banking, respectively. Firstly, these notes became payable to bearer, and so were transformed from a receipt to a bank-note. Secondly, inasmuch as the cash in question was deposited for a fixed period, the goldsmith rapidly found that it was safe to make loans out of his cash resources, provided such loans were repaid within the fixed period.

“The first result was that in place of charging a fee for their services in guarding their client’s gold, they were able to allow him interest. Secondly, business grew to such a pitch that it soon became clear that a goldsmith could always have a certain proportion of his cash out on loan, regardless of the dates at which his notes fell due. It equally became safe for him to make his notes payable at any time, for so long as his credit remained good, he could calculate on the law of averages the exact amount of gold he needed to retain to meet the daily claims of his note-holders and depositors.” (*EB* 1941, vol. 3, p. 68).

²One probably should not, given contributors to the entries on banking include the likes of Ralph George Hawtrey (Assistant Secretary at the UK Treasury, author of “Currency and Credit”), Oliver M. W. Sprague (Harvard professor, 1937 president of the American Economics Association, author of “Theory and History of Banking”), Charles R. Whittlesey

p. 411-12) puts it, “By the restoration of Charles II in 1660, London’s goldsmiths had emerged as a network of bankers. ... Some were little more than pawn-brokers while others were full service bankers. The story of their system, however, builds on the financial services goldsmiths offered as fractional reserve, note-issuing bankers. In the 17th century, notes, orders, and bills (collectively called demandable debt) acted as media of exchange that spared the costs of moving, protecting and assaying specie.” Similarly, Joslin (1954, p.168) writes “the crucial innovations in English banking history seem to have been mainly the work of the goldsmith bankers in the middle decades of the seventeenth century. They accepted deposits both on current and time accounts from merchants and landowners; they made loans and discounted bills; above all they learnt to issue promissory notes and made their deposits transferrable by ‘drawn note’ or cheque; so credit might be created either by note issue or by the creation of deposits, against which only a proportionate cash reserve was held.”

This is not to suggest that there were no financial institutions or intermediaries of interest around other than, or prior to, goldsmith bankers.³ However, these institutions did not seem to provide anything like circulating demandable debt, and transferring funds from one account to another “generally required the presence at the bank of both payer and payee” (Kohn 1999b). That is, “In

(Penn professor, author of “Principles and Practices of Money and Banking”), and Edward Victor Morgan, (Swansea and Manchester professor, author of “The Theory and Practice of Central Banking”).

³Neal (1994) discusses some that were around along with the goldsmith bankers, including the scrivener, merchant banks, country banks, etc. Amongst others, Kohn (1999a,b,c) and Davies (2002) provide extensive discussions of various other institutions in different places and times. Particularly well known are the Italian bankers: “To avoid coin for local payments, Renaissance moneychangers had earlier developed deposit banking in Italy, so two merchants could go to a banker and transfer funds from one account to another.” (Quinn 2002). Going back further, it seems that the first group that might deserve to be called bankers were the Templars, a religious order of knights during the Crusades. Because they were fierce fighters, they specialized in moving money around safely. After this they began providing other financial services, including loans. They were quite successful for a time – until some of their leadership began losing their heads in dealings with certain kings. See Weatherford (1977) for more on their fascinating history.

order that bank credit may be used as a means of payment, it is clearly quite essential that some convenient procedure should be instituted for assigning a banker's debt from one creditor to another. In the infancy of deposit banking in mediaeval Venice, when a depositor wanted to transfer a sum to someone else, both had to attend the bank in person. In modern times the legal doctrine of negotiable instruments has been developed ... The document may take either of two forms: (1) a cheque, or the creditor's order to the bank to pay; (2) a note or the banker's promise to pay." (*EB* 1941, vol. 3, p. 44).

We take it from our sources that the story of modern banking does indeed seem to have started with London goldsmiths accepting deposits for safe keeping. However, a story – even if it is a good story or a true story – is not a theory. The goal of this paper is to build a *model* that can be used to study banks as institutions whose liabilities may substitute for money, or potentially compliment money, as a means of payment. Clearly, for this task one wants a framework where there is a role for media of exchange in the first place, and where the objects that play this role are endogenous. This framework is provided by the search-theoretic approach to monetary economics. This approach endeavors to make explicit the frictions necessary for a medium of exchange to be essential – i.e. for the set of equilibrium allocations with something like money to be bigger or better than the set without this institution – and can be used to determine endogenously which objects circulate as a medium of exchange.

Most existing search models, however, accomplish these things in environments with very severe assumptions about the way agents interact. Following Kiyotaki and Wright (1989), typically agents trade exclusively in highly decentralized markets characterized by random, anonymous, bilateral matching. This seems to leave little possibility of introducing banks in a sensible way,

although there have been a few interesting attempts.⁴ The recent model developed in Lagos and Wright (2002) deviates from previous analyses by assuming that agents interact periodically in both decentralized markets and centralized markets. This was initially used mainly to simplify outcomes in the decentralized market, as under certain assumptions all agents of a given type choose the same money balances to take out of the centralized and into the decentralized markets. But once the centralized markets are up and running it is easy to introduce labor, capital and other markets into this search model in a natural way (see e.g. Aruoba and Wright 2003).

This framework fits our needs well. The fact that agents sometimes interact in highly decentralized markets makes a medium of exchange essential, and we will determine endogenously whether this ends up being cash, bank liabilities, or both. The fact that agents sometimes interact in centralized markets allows us to think about a competitive banking industry where agents make deposits against which they can make payments, take out loans, etc. The reason agents may want to use bank deposits instead of cash as a means of payment in the model is exactly the reason they did in the historical record: *safe keeping*. That is, in the model cash will be subject to theft while assets deposited in banks' vaults will not. There were other problems with money that contributed to the development of deposit banks and bills of exchange to reduce the need for cash – among other things, coins were in short supply, were hard to transport, got clipped or worn, and were not all that easy to recognize or evaluate – but the

⁴Previous work that incorporates some notion of banks into search models includes Cavalcanti and Wallace (1999a,b), Cavalcanti et al. (1999), and Williamson (1999). We will not attempt to review the vast banking literature here, except to mention a few papers that incorporate banks into models with some microfoundation for money. Analyses in overlapping generations models include Williamson (1987), Champ et al. (1996), Schreft and Smith (1998), and Bullard and Smith (2003). Aiyagari and Williamson (2000) and Andolfatto and Nosal (2003) provide different approaches. For an extensive survey of the more conventional banking research, most of which is surprisingly far from the microfoundations of money, see Gorton and Winton (2002).

modeling the problem as one of safety is natural for our purposes, and leads to some interesting results.

Of course, formalizing banks as providing a means of payment that is relatively safe – as opposed to, say, relatively easy to transport or to recognize – means we need to take some care in the way we interpret these assets. A strict interpretation is that they are checking accounts, or maybe even better, modern travellers’ checks, which are nearly as widely accepted as cash and safer for at least two reasons: they are less valuable to thieves because they require a signature that matches the one recorded when making a deposit; and even if they are lost or stolen you can get your money back at essentially no cost. This safety aspect of our assets is consistent with the historical view of services provided by deposit banks, or more generally by bills of exchange which were not payable to the bearer.⁵ It is not so clear that our assets are much like bank notes, which presumably were about as easy to steal as money, and payable to the bearer. In any case, we think that constructing a formal model of the endogenous circulation of money and demand deposits seems worthwhile, and our basic methods should apply to constructing similar models with bank notes based on some other feature – perhaps recognizability – rather than safety.

Moreover, although checks are less important today than 50 years ago, the statement in the epigram still applies: checks remain the most common means

⁵It may be useful to define some terms. A bill of exchange is an order in writing, signed by the person giving it (the drawer), requiring the person to whom it is addressed (the drawee) to pay on demand or at some fixed time a given sum of money either to a named person (the payee) or to the bearer. A cheque is a particular form of bill, where a bank is the drawee and it must be payable on demand. Quinn (2002) also suggests that bills were “similar to a modern traveller’s check.” It is clear that safety was and is a key feature of checks. For one thing, the payee needs to endorse the check, so no one else can cash it without committing forgery; other features include the option to “stop” a check or “cross” it (make it payable only on presentation by a banker). Indeed, the word “check” or “cheque” originally signified the counterfoil or indent of an exchequer bill, on which was registered details of the principal part in order to reduce the risk of alteration or forgery. The check or counterfoil parts remained in the hands of the banker, the portion given to the customer being termed a “drawn note” or “draft.”

of payment. “There has been a 20% drop in personal and commercial check-writing since the mid 1990s, as credit cards, check cards, debit cards, and online banking services have reduced the need to pay with written checks. [But] Checks are still king. The latest annual figures from the Federal Reserve show 30 billion electronic transactions and 40 billion checks processed in the United States.” Also, “While credit cards reduce the number of checks that need to be written in retail stores, the credit-card balance still has to be paid every month. And, at least for now, that is usually done by mail – with a paper check.” (*Philadelphia Inquirer*, Feb. 14, 2003, p. A1). So the microfoundations of M_1 , cash plus demand deposits, is still relevant. More generally, our focus is on banks’ as providers of payment services, which today is as important as ever given recent moves towards hopefully safer or more convenient services like debit cards, electronic money, etc.

The rest of the paper and our main results can be summarized as follows. The analysis is organized around a sequence of formal models, each adding an additional component. Section 2 presents the basic assumptions which make a medium of exchange essential, and which allow this role to be filled by money or checks. Section 3 analyzes the simplest case, where theft is exogenous, money and goods are indivisible, and we have 100% reserve requirements. With no banking, this model is a simple extension of the textbook search-based model of monetary exchange. Once banks are added, we show there can exist equilibria where all agents, no agents, or some agents use checks, and sometimes these equilibria coexist. An interesting result is that sometimes monetary equilibria do not exist without banks but do exist with banks, so in a sense money and banking are complimentary. We also show how the equilibrium set changes as parameters like the cost of banking or the supply of outside money change.

Section 4 endogenizes theft. Without banks, this provides a slightly more interesting extension of the textbook search model, and generates some simple but intuitively reasonable results: e.g. equilibria with theft are more likely when production is more costly; too much theft is inconsistent with monetary equilibrium; and sometimes there are multiple equilibria with different amounts of theft. When we introduce banks we again expand the set of parameters for which monetary equilibria exist, so again money and banking are complementary, but now there are two reasons: as in the model with exogenous theft, the safety provided by banks makes money more valuable, but now there is an additional effect because when fewer people carry cash the number of thieves may fall. An interesting result in the model with endogenous theft is that checks can never completely drive out money. The reason is that if no one carries cash there are no thieves, but then you may as well use cash. Hence, concurrent circulation of money and bank liabilities is a natural outcome.

Section 5 generalizes the model to allow divisible goods and shows the basic results are robust.⁶ Section 6 relaxes the 100% reserve requirement by allowing banks to lend a certain fraction of their deposits. The loan rate is determined by supply and demand in the centralized market. One feature of this model is that the fee charged for checking service will be less than the resource cost of managing the account, since banks profit from making loans, and as with the goldsmiths they may end up charging a negative fee (i.e. paying interest on demand deposits). In this case, checks could possibly drive cash out of

⁶Although we allow divisible goods in this section, we retain the assumption of indivisible money and a unit upper bound on asset holdings throughout the paper. Thus, we do not exploit one of the main advantages of the Lagos-Wright structure – that under certain assumptions on preferences, it allows one to dispense with indivisible assets and the upper bound with a minimal loss in tractability. The only feature of the framework that we really use here is that agents have periodic access to centralized markets, where they can do their banking, and also periodically interact in highly decentralized markets so that some medium of exchange is essential. We wanted to explore the simplest indivisible asset models first; fully divisible asset models are being explored in a follow-up paper.

circulation. This version of the model also generates a simple money multiplier, as in undergraduate textbooks, although here the role of money and banks are explicitly modeled from microeconomic principles. Section 7 concludes.

2 Basic Assumptions

The economy is populated by a $[0, 1]$ continuum of infinitely-lived agents. Time is discrete, continues forever. Following Lagos and Wright (2002), each period is divided into two subperiods, say day and night. During the day agents will interact in a centralized (frictionless) market, while at night they will interact in a highly decentralized market characterized by random, anonymous, bilateral matching. Trade is difficult in the decentralized markets because of a standard double coincidence problem: there are many specialized goods traded in this market, and only a fraction $x \in (0, 1)$ of the population can produce a specialized good that you want. A meeting where someone can produce what you want is called a single coincidence meeting; for simplicity, there are no double coincidence meetings so that we can ignore pure barter, but it is easy to relax this. As they are nonstorable, these goods are produced for immediate trade and consumption. Goods are also indivisible for now, but this is relaxed below. Consuming a specialized good that you want conveys utility u . Producing a specialized good for someone else conveys disutility $c < u$. The rate of time preference is $r > 0$.

Due to the frictions in the decentralized market, trade would shut down if not for a medium of exchange; in particular, since agents are anonymous there can be no credit (Kocherlakota 1998; Wallace 2001). Hence, a fraction $M \in [0, 1]$ of the population are each initially endowed with one unit of money – an object that is consumed or produced by no one, but may have potential

use as a means of payment. For simplicity, if not historical accuracy, one can think of this money as fiat, although it would be easy to redo things in terms of commodity money (say, following Velde et al. 1999). Here we follow the early literature in the area and assume that money is indivisible and agents can store at most one unit at a time. A key feature of the model is that this money is unsafe – it can be stolen. We assume for simplicity that goods cannot be stolen. Also, because individuals can store at most one unit of money, only individuals without money steal.

We formalize stealing as follows.⁷ In the decentralized market, if you meet an agent holding cash you attempt to steal it with probability λ , which is endogenous in some versions of the model studied below and exogenous in others (you just can't help it). Given that you try to steal, with probability γ you are successful. Theft has a cost $z < u$. Note that theft may be more or less costly than honest trade, depending on $z - c$, and also may be more or less likely to succeed, depending on $\gamma - x$. Agents in the day subperiod can deposit their money into a bank, and can write checks on these accounts. Checks are assumed to be perfectly safe. Also, all agents believe a bank can be counted on to cash a check as long as it has been signed by the depositor.⁸ This means that everyone is as willing to accept a check in the decentralized night market as cash, since they know the former can be turned into the latter in the next day's centralized market.

In terms of the centralized day market, we make it as simple as possible to

⁷Criminal activity here is pretty simple, even when λ is endogenous, but can be thought of as following the search-theoretic models of crime in Burdett et al. (2003) and Huang et al. (2004). There is also a sense in which the present model seems closely related to the analysis of monetary exchange under private information in Williamson and Wright (1994), since stealing your money is conceptually not very different from selling you low quality merchandise. That paper did not have anything like banks, however.

⁸This belief is assumed exogenously here, say because there is a legal enforcement mechanism at work, but it should not be hard to use reputation to get banks to honor their debts endogeneously.

of money on deposit. Also, in general they are required legally to keep a fraction α of their deposits on reserve, while the rest can be loaned out. Loans may be demanded by some of the $1 - M$ agents who begin the day without purchasing power. We assume competitive banking, so the cost of a loans ρ will equate the supply and demand for loans. Thus, for example, if $\alpha = 1$ banks must keep all cash deposits in the vault, in which case competition implies $\phi = a$.

3 Exogenous Theft

In this section we study the model where stealing is exogenous: if an agent with money meets one without, the latter will try to rob the former with some fixed probability λ . With probability γ he succeeds, while with probability $1 - \gamma$ he fails and walks away empty handed. We first study the case where the only asset is money and then we introduce banking. We also assume 100% reserve requirements for now: $\alpha = 1$.

3.1 Money

Throughout the paper we use V_1 to represent the value function of an agent with 1 unit of money, called a buyer, and V_0 the value function of an agent with no money, called a seller. When there are no banks, Figure 2 shows the event trees for buyers and sellers in the night market. For example, a buyer with probability M meets another buyer and he leaves as a buyer; with probability $1 - M$ he meets a seller, and in this case with probability λ the seller tries to rob him and succeeds with probability γ , while with probability $1 - \lambda$ they try to trade and succeed with probability x . The flow Bellman equations corresponding to these

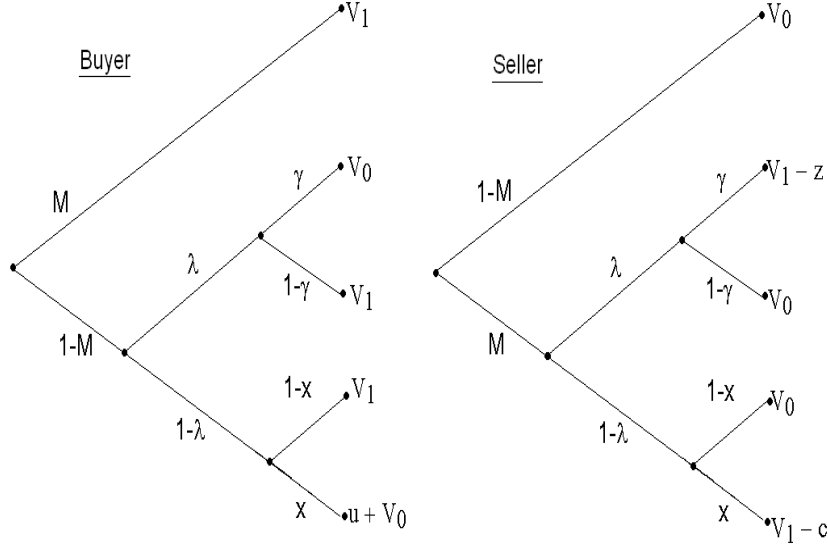


Figure 2: Event trees for buyers and sellers

trees are

$$\begin{aligned}
 rV_1 &= (1-M)(1-\lambda)x(u+V_0-V_1) + (1-M)\lambda\gamma(V_0-V_1) \\
 rV_0 &= M(1-\lambda)x(V_1-V_0-c) + M\lambda\gamma(V_1-V_0-z).
 \end{aligned}$$

We are interested in monetary equilibria (any nonmonetary equilibria are ignored from now on). The incentive condition for agents to produce in order to acquire money is $V_1 - V_0 - c \geq 0$. Note that we do not impose the symmetric condition for stealing, $V_1 - V_0 - z \geq 0$ since, as we said, theft is exogenous for now. However, we do impose participation constraints $V_0 \geq 0$ and $V_1 \geq 0$, since we allow agents to skip going to the night market. It is clear from the incentive condition $V_1 - V_0 - c \geq 0$ that the binding participation constraint is $V_0 \geq 0$. Hence, a monetary equilibrium here simply requires that that $V_0 \geq 0$ and $V_1 - V_0 - c \geq 0$ both hold.

In order to describe the regions of parameter space where these conditions

hold, and hence where a monetary equilibrium exists, define

$$C_M = \frac{(1-M)(1-\lambda)xu + M\lambda\gamma z}{r + (1-M)(1-\lambda)x + \lambda\gamma}$$

$$C_A = \frac{(1-M)[\lambda\gamma + (1-\lambda)x]u}{r + (1-M)[\lambda\gamma + (1-\lambda)x]} - \frac{\lambda\gamma z}{(1-\lambda)x}.$$

Figure 3 depicts C_M and C_A in (x, c) space using properties in the following easily verified Lemma.

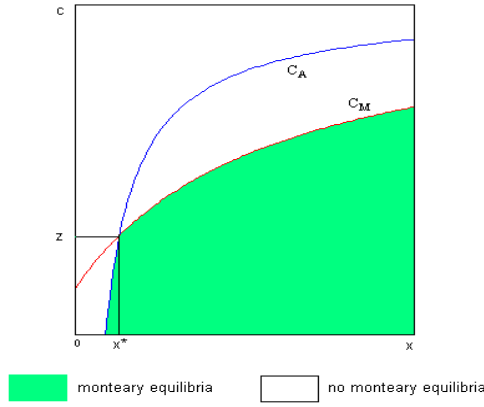


Figure 3: Existence region for monetary equilibrium

Lemma 1 (a) $x = 0 \Rightarrow C_M = \frac{M\lambda\gamma z}{r+\lambda\gamma}$, $C_A = -\infty$. (b) $C'_M > 0$, $C'_A > 0$. (c) $C_M = C_A$ iff $(x, c) = (x^*, z)$, where $x^* = \frac{[r+(1-M)\lambda\gamma]z}{(1-M)(1-\lambda)(u-z)}$.

We can now verify the following.

Proposition 1 *Monetary equilibrium exists iff $c \leq \min\{C_M, C_A\}$.*

Proof: Subtracting the Bellman equations and rearranging implies

$$V_1 - V_0 = \frac{(1-\lambda)x[(1-M)u + Mc] + \lambda\gamma Mz}{r + (1-\lambda)x + \lambda\gamma}.$$

Algebra implies $V_1 - V_0 - c \geq 0$ iff $c \leq C_M$ and $V_0 \geq 0$ iff $c \leq C_A$. ■

Naturally, monetary equilibrium is more likely to exist when c is lower or x bigger.⁹ Also notice that either of the two constraints $c \leq C_M$ and $c \leq C_A$ may bind. It is easy to see that welfare, as measured by average utility

$$W = MV_1 + (1 - M)V_0 = \frac{M(1 - M)}{r} [(1 - \lambda)x(u - c) - \lambda\gamma z],$$

is decreasing in λ and γ . The welfare cost of crime here is due to the resource cost z and the opportunity cost of thieves not producing; stealing *per se* is a transfer not an inefficiency. In any case, when $\lambda = 0$, so that $C_M = C_A = \frac{(1-M)xu}{r+(1-M)x}$, we essentially have the simple textbook search model of monetary exchange in Kiyotaki and Wright (1993). Allowing money to be subject to theft provides an simple but not unreasonable extension of the basic framework. We next show it can be used to discuss banks.

3.2 Banking

We now allow agents with money to deposit it in checking accounts. Since $\alpha = 1$ in this section, banks like the early goldsmiths simply keep the money in the vault and earn revenue by charging $\phi = a$ for this service, paid in the model in terms of general goods. Let θ be the probability an agent with money decides each day to put his money in the bank (or, if he already has an account, to not withdraw it). Let $M_0 = M(1 - \theta)$ denote the amount of cash in circulation, and let $M_1 = M_0 + M\theta = M$ denote cash plus demand deposits. Let V_m be the value function of an agent in the night market with cash, and V_d the value function of an agent at night with money deposited in his bank, exclusive of the fee a . Hence, for an agent with 1 dollar in the centralized market, $V_1 = \max\{V_m, V_d - a\}$.

⁹One might also expect $\partial C_M / \partial \lambda < 0$, but actually this is true iff $z < \tilde{z} = \frac{(1-M)(r+\gamma)xu}{M[r+(1-M)x]^\gamma}$; thus, for large z , when λ increases agents are more willing to accept money. This is because λ measures not only the probability of being robbed but also the probability of trying to rob someone else; when z is very big, if this probability goes up agents are more willing to work for money to keep themselves from crime. This effect may seem strange, but in any case it goes away when λ is endogenized.

Although checks will be perfectly safe, it facilitates the discussion to temporarily proceed more generally and let γ_m and γ_d be the probabilities that one can successfully steal from someone with money and from someone with a bank account. Bellman's equation for an agent with asset $j \in \{m, d\}$ is then¹⁰

$$rV_j = (1 - M)(1 - \lambda)x(u + V_0 - V_1) + (1 - M)\lambda\gamma_j(V_0 - V_1) + V_1 - V_j.$$

For an agent without money

$$rV_0 = (1 - \lambda)Mx(V_1 - V_0 - c) + \lambda[M_0\gamma_m + (M - M_0)\gamma_d](V_1 - V_0 - z).$$

If we set $\gamma_m = \gamma$ and $\gamma_d = 0$, then

$$\begin{aligned} rV_m &= (1 - M)(1 - \lambda)x(u + V_0 - V_1) + (1 - M)\lambda\gamma(V_0 - V_1) + V_1 - V_m \\ rV_d &= (1 - M)(1 - \lambda)x(u + V_0 - V_1) + V_1 - V_d \\ rV_0 &= (1 - \lambda)Mx(V_1 - V_0 - c) + \lambda M_0\gamma(V_1 - V_0 - z). \end{aligned}$$

We also have $V_1 = \max\{V_m, V_d - a\} = \theta(V_d - a) + (1 - \theta)V_m$, from which it is clear that

$$\theta = 1 \Rightarrow V_d - a \geq V_m; \theta = 0 \Rightarrow V_d - a \leq V_m; \text{ and } \theta \in (0, 1) \Rightarrow V_d - a = V_m.$$

Equilibrium satisfies this condition, plus the incentive condition for money to be accepted, $V_1 - V_0 - c \geq 0$, and the participation condition, $V_0 \geq 0$. To characterize the parameters for which different types of equilibria exist, define

$$\begin{aligned} C_1 &= -\frac{(1 - M)u}{M} - \frac{\lambda\gamma z}{(1 - \lambda)x} + \frac{[r + (1 - \lambda)x + \lambda\gamma]\hat{a}}{M(1 - M)\lambda(1 - \lambda)\gamma x} \\ C_2 &= \frac{(1 - M)(1 - \lambda)xu - \hat{a}}{r + (1 - M)(1 - \lambda)x} \\ C_3 &= -\frac{(1 - M)u}{M} + \frac{[r + (1 - \lambda)x + (1 - M)\lambda\gamma]\hat{a}}{M(1 - M)\lambda(1 - \lambda)\gamma x} \end{aligned}$$

¹⁰The value of entering the decentralized market with asset j is

$$V_j = \frac{1}{1 + r}[(1 - M)(1 - \lambda)x(u + V_0) + (1 - M)\lambda\gamma_j V_0 + \zeta V_1]$$

where $\zeta = 1 - (1 - M)(1 - \lambda)x - (1 - M)\lambda\gamma_j$. Multiplying by $1 + r$ and subtracting V_j from both sides yields the equation in the text.

where $\hat{a} = (1 + r)a$.

Figure 4 show the situation in (x, c) space for the two possible cases, $z < C_4$ and $z > C_4$, where

$$C_4 = \frac{\hat{a}}{(1 - M)\lambda\gamma}.$$

We assume $C_4 < u$, or $\hat{a} < (1 - M)\lambda\gamma u$, to make things interesting. The following Lemma establishes that the Figures are drawn correctly by describing the relevant properties of C_j , and relating them to C_M and C_A from the case with no banks; again the easy proof is omitted.

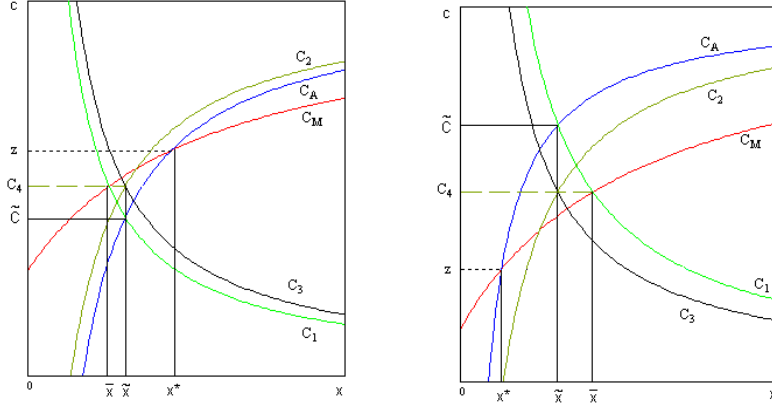


Figure 4: Conditions in Lemma 2

Lemma 2 (a) $x = 0 \Rightarrow C_1 = \infty$ or $-\infty$, $C_2 = -\hat{a}/r < 0$, $C_3 = \infty$. (b) $C_2' > 0$, $C_3' < 0$, and C_1 is monotone but can be increasing or decreasing. (c) $C_1 = C_M$ iff $(x, c) = (\bar{x}, C_4)$, $C_2 = C_3$ iff $(x, c) = (\tilde{x}, C_4)$, and $C_1 = C_A$ iff $(x, c) = (\tilde{x}, \tilde{C})$, where

$$\bar{x} = \frac{\hat{a}(r + \lambda\gamma) - M(1 - M)\lambda^2\gamma^2z}{(1 - M)(1 - \lambda)[(1 - M)\lambda\gamma u - \hat{a}]}$$

$$\tilde{x} = \frac{\hat{a}[r + (1 - M)\lambda\gamma]}{(1 - M)(1 - \lambda)[(1 - M)\lambda\gamma u - \hat{a}]}.$$

(d) $\bar{x} > \tilde{x}$ iff $z < C_4 < \tilde{C}$ and $\bar{x} < \tilde{x}$ iff $z > C_4 > \tilde{C}$.

We can now prove the following:

Proposition 2 (a) $\theta = 0$ is an equilibrium iff $c \leq \min(C_M, C_A, C_1)$. (b) $\theta = 1$ is an equilibrium iff $C_3 \leq c \leq C_2$. (c) $\theta \in (0, 1)$ is an equilibria iff either: $z < C_4$, $c \in [C_3, C_1]$ and $c \leq C_4$; or $z > C_4$, $c \in [C_1, C_3]$ and $x \geq \tilde{x}$.

Proof: Consider $\theta = 0$, which implies $M_1 = M_0 = M$ and $V_1 = V_m$. For this to be an equilibrium we require $V_m - V_0 - c \geq 0$ and $V_0 \geq 0$, which is true under exactly the same conditions as in the model with no banks, $c \leq \min(C_M, C_A)$. However, now we also need to check $V_m \geq V_d - a$ so that not going to the bank is an equilibrium strategy. This holds iff $c \leq C_1$. Now consider $\theta = 1$, which implies $M_0 = 0$ and $V_1 = V_d - a$. In this case, $V_1 - V_0 - c \geq 0$ holds iff $c \leq C_2$. We also need $V_0 \geq 0$, but this never binds. Finally, we need to check $V_d - a \geq V_m$, which is true iff $c \geq C_3$.

Finally consider $\theta \in (0, 1)$, which implies $M_0 = M(1 - \theta) \in (0, M)$ is endogenous and $V_1 = V_d - a = V_m$. The Bellman equations imply

$$(1 + r)(V_d - V_m) = (1 - M)\lambda\gamma(V_1 - V_0).$$

Inserting $V_d - V_m = a$ and

$$V_1 - V_0 = \frac{(1 - \lambda)x[(1 - M)u + Mc] + M_0\lambda\gamma z}{r + (1 - \lambda)x + (1 - M + M_0)\lambda\gamma},$$

we can solve for

$$M_0 = \frac{(1 - M)\lambda(1 - \lambda)\gamma x[(1 - M)u + Mc] - [r + (1 - \lambda)x + (1 - M)\lambda\gamma]a}{(C_4 - z)(1 - M)\lambda^2\gamma^2}.$$

We need to check $M_0 \in (0, M)$, which is equivalent to $\theta \in (0, 1)$. There are two cases, depending on the sign of the denominator: if $z < C_4$ then $M_0 \in (0, M)$ iff $c \in (C_3, C_1)$, and if $z > C_4$ then $M_0 \in (0, M)$ iff $c \in (C_1, C_3)$. We also need to check $V_1 - V_0 - c \geq 0$, which holds iff $c \leq C_4$, and $V_0 \geq 0$, which holds iff

$x \geq \tilde{x}$. When $\bar{x} > \tilde{x}$ the binding constraint is $c \leq C_4$, and when $\bar{x} < \tilde{x}$ the binding constraint is $x \geq \tilde{x}$. ■

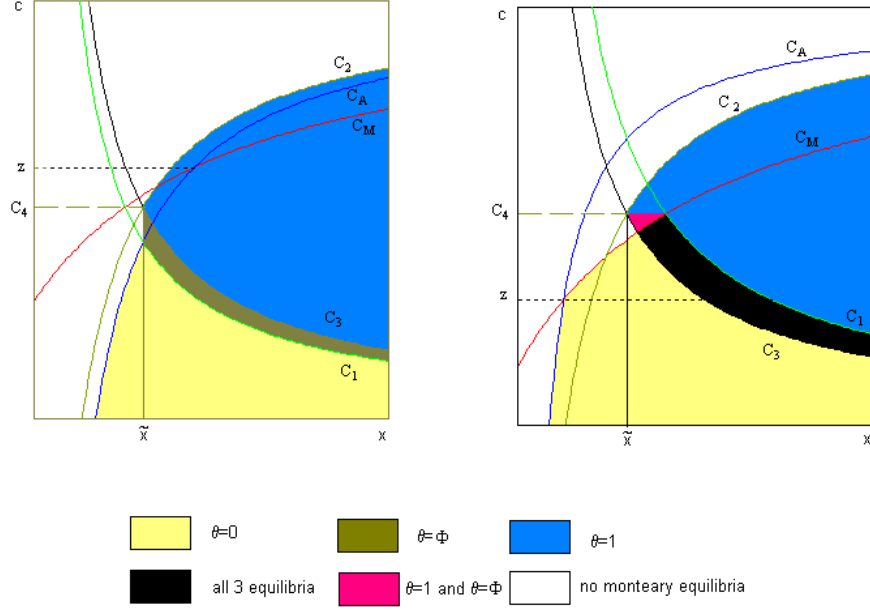


Figure 5: Existence regions with banking

Based on the above results, Figure 5 shows the situation when \bar{x} and \tilde{x} defined in Lemma 2 are in $(0, 1)$.¹¹ In the case $z > C_4$, shown in the left panel, we always have a unique equilibrium, which may entail $\theta = 0$, $\theta = 1$, or $\theta = \Phi$ where we use the notation Φ for any number in $(0, 1)$. In the case $z < C_4$, shown in the right panel, we may have a unique equilibrium but we may also have multiple equilibria (sometimes $\theta = 1$ and $\theta = \Phi$, and sometimes

¹¹The assumption $\hat{a} < (1 - M)\lambda\gamma u$ mentioned above guarantees $\tilde{x} > 0$, and it is easy to see that $\bar{x} > 0$ iff $M(1 - M)\lambda^2\gamma^2 z < (r + \lambda\gamma)\hat{a}$. We also have

$$\bar{x} < 1 \text{ iff } \hat{a} < \bar{a} = \frac{(1 - M)^2\lambda(1 - \lambda)\gamma u + M(1 - M)\lambda^2\gamma^2 z}{r + \lambda\gamma + (1 - \lambda)(1 - M)}$$

$$\tilde{x} < 1 \text{ iff } \hat{a} < \tilde{a} = \frac{(1 - M)^2\lambda(1 - \lambda)\gamma u}{r + (1 - M)(1 - \lambda + \lambda\gamma)}.$$

We do not need \bar{x} and \tilde{x} in $(0, 1)$, but if $\tilde{x} > 1$, e.g., equilibria with $\theta > 0$ disappear.

all three equilibria). Recall that without banks the monetary equilibrium exists iff $c \leq \min\{C_M, C_A\}$. Hence, if it exists without banks monetary equilibrium still exists with banks, and it may or may not entail $\theta > 0$. However, there are parameters such that there are no monetary equilibria without banks while there are with banks. In these equilibria we must have $\theta > 0$, although not necessarily $\theta = 1$. The important economic point is that for some parameters, and in particular for relatively large x and c , money cannot work without banking but it can work with it.

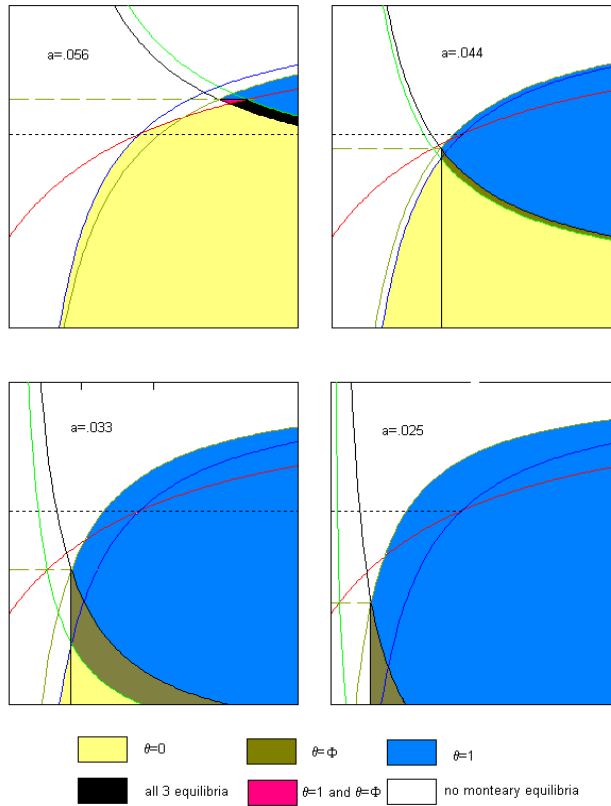


Figure 6: Equilibrium as a changes

Figure 6 shows how the set of equilibria evolves as a falls. For very large a

banking is not viable, so the only equilibrium is $\theta = 0$. As we reduce a two things happen: in the some regions where there was a monetary equilibrium without banks, agents may start using banks; and in some regions where there were no monetary equilibria without banks, a monetary equilibrium emerges. As a falls further the region where $\theta = 0$ shrinks. As a falls still further we switch from the case $z < C_4$ to the case $z > C_4$; thus, for relatively small a equilibrium must be unique, but could entail $\theta = 1$, $\theta = 0$, or $\theta = \Phi$. As a falls even further, we lose equilibrium with $\theta = 0$. Eventually checks drive currency from circulation. Note however that this is not robust: in the next section we endogenize λ and show that banking never completely drives out money. Equilibria with $\theta \in (0, 1)$ are particularly interesting because they yield the concurrent circulation of cash and checks.

A similar picture emerges if we let M fall, since reducing M also raises the demand for checking services. One arguably strange reason for this is that in the present model lower M implies a greater number of criminals $(1 - M)\lambda$. However, we can circumvent this by considering what happens as we vary M and adjust λ to keep $(1 - M)\lambda$ constant. Lower M still makes $\theta > 0$ more likely, but now the reason is simply that lower M makes money more valuable, so you are willing to pay more to keep it safe. See Figure 7, where the two rows are for $Z < C_4$ and $Z > C_4$, and in either case M falls as we move from left to right. The result that lower M makes it more likely that agents will use banking is consistent with the historical evidence that people were more likely to use demandable debt as a means of payment when cash was in short supply (Ashton 1945; Cuadras-Morato and Rosés 1998), although one can presumably tell a variety of stories consistent with this. In any case, although the model is simple, we think it captures something interesting about banking.

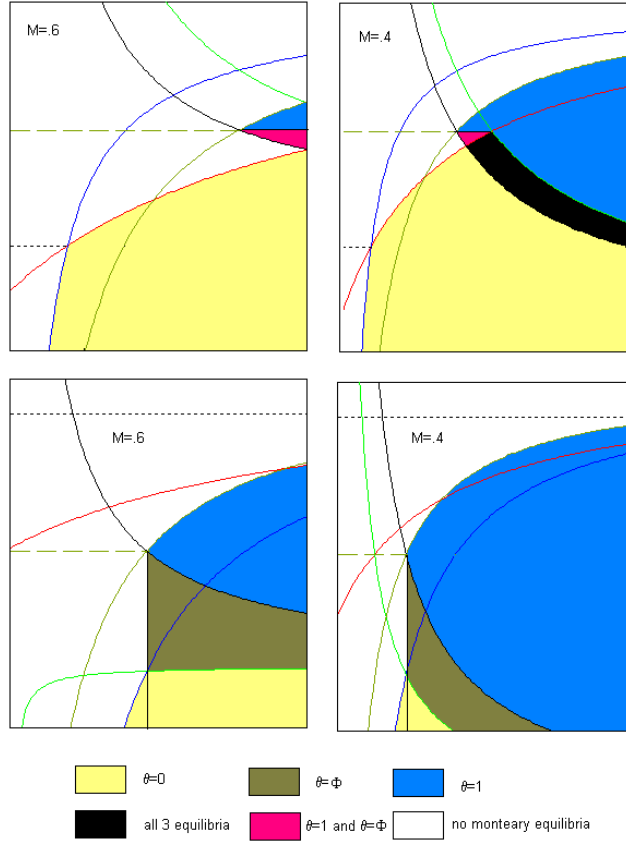


Figure 7: Equilibrium as M changes

4 Endogenous Theft

In this section we endogenize the decision to be a thief. This is useful not only for the sake of generality, but because the model with λ exogenous does have some features one may wish to avoid (e.g. when M goes down the crime rate mechanically increases). Moreover, an interesting implication of the model with λ endogenous is that checks can never completely drive currency from circulation: if no one uses cash, no one will choose crime, but then cash is safe

and no one would use checks. Hence concurrent circulation is more likely. As in the model with exogenous λ , we start with the case where money is the only asset and then introduce banks.

4.1 Money

Let λ now be the probability an individual without money chooses to be a thief before going out at night. Bellman's equation for V_1 is the same as in Section 3.1. Bellman's equations for producers and thieves are now

$$\begin{aligned} rV_p &= Mx(V_1 - V_p - c) + (1 - Mx)(V_0 - V_p) \\ rV_t &= M\gamma(V_1 - V_t - z) + (1 - M\gamma)(V_0 - V_t), \end{aligned}$$

where $V_0 = \max\{V_t, V_p\} = \lambda V_t + (1 - \lambda)V_p$. The equilibrium conditions are

$$\lambda = 1 \Rightarrow V_t \geq V_p, \lambda = 0 \Rightarrow V_t \leq V_p, \text{ and } \lambda \in (0, 1) \Rightarrow V_t = V_p,$$

plus the incentive condition $V_1 - V_0 - c \geq 0$. Notice that with λ endogenous we do not have to check the participation constraint $V_0 \geq 0$: since an agent can always set $\lambda = 0$, we have $rV_0 \geq Mx(V_1 - V_0 - c) \geq 0$ as long as the incentive condition holds.

The next observation is that we can never have equilibrium with $\lambda = 1$, since we cannot have agents accepting money when everyone is a thief.¹² To say more, define the following thresholds

$$\begin{aligned} c_0 &= \frac{(1 - M)xu}{r + (1 - M)x} \\ c_1 &= \frac{(x - \gamma)(1 - M)xu + \gamma(r + x)z}{x[r + (1 - M)x + M\gamma]} \\ c_2 &= \frac{[r + Mx + (1 - M)\gamma]\gamma z}{(r + \gamma)x}. \end{aligned}$$

¹²Formally, if $V_1 - V_0 - c \geq 0$ then $\lambda = 1 \Rightarrow rV_1 = (1 - M)\gamma(V_0 - V_1) \leq -(1 - M)\bar{\gamma}c < 0$, and therefore $V_1 - V_0 - c < 0$. Hence, $\lambda < 1$ in any equilibrium.

The next Lemma establishes properties of these thresholds illustrated in Figure 8, shown for the case $\gamma > x^* = \frac{rz}{(1-M)(u-z)}$ (in the other case the region labeled $\lambda = \Phi$ disappears). Again we omit the routine proof.

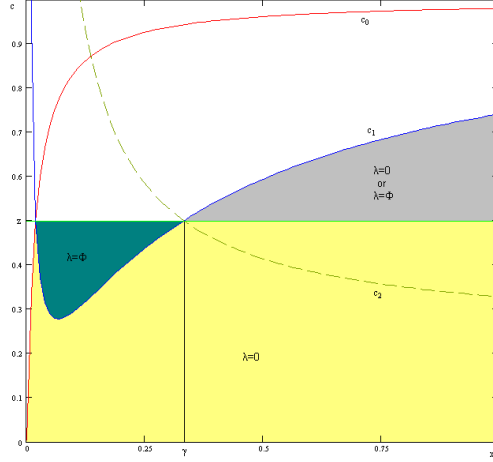


Figure 8: Equilibrium with endogenous λ , no banks

Lemma 3 (a) $x = 0 \Rightarrow c_0 = 0, c_1 = c_2 = \infty$. (b) $c'_0 > 0, c'_2 < 0$. (c) $c_0 = c_1$ iff $(x, c) = (x^*, z)$ and $c_1 = c_2$ iff $(x, c) = (\gamma, z)$. (d) $c_0, c_1 \rightarrow u$ as $x \rightarrow \infty$.

We can now establish the following:

Proposition 3 (a) $\lambda = 0$ is an equilibrium iff either: $x < x^*$ and $c \in [0, c_0]$; or $x > x^*$ and $c \in [0, c_1]$. (b) $\lambda \in (0, 1)$ is an equilibrium iff $x > \gamma$ and $c \in [z, c_1]$, or $x < \gamma$ and $c \in (c_1, z]$.

Proof: Equilibrium with $\lambda = 0$ requires $V_t \geq V_p$ and $V_1 - V_p - c \geq 0$. Inserting the value functions and simplifying, the former reduces to $c \leq c_1$ and the latter to $c \leq c_0$. Hence, $\lambda = 0$ is an equilibrium iff $c \leq \min\{c_0, c_1\}$, and the binding constraint will depend on whether x is below or above x^* . Equilibrium

with $\lambda \in (0, 1)$ requires $V_t = V_p$ and $V_1 - V_0 - c \geq 0$. We can solve $V_t = V_p$ for $\lambda = \lambda^*$, where

$$\lambda^* = \frac{(\gamma - x)x[(1 - M)u + Mc] - (r + x)(\gamma z - xc)}{(\gamma - x)(1 - M)(xu - xc + \gamma z)}.$$

It can be checked that $\lambda^* \in (0, 1)$ iff $c \in (c_1, c_2)$ when $x < \gamma$ and $\lambda^* \in (0, 1)$ iff $c \in (c_2, c_1)$ when $x > \gamma$. The condition $V_1 - V_0 - c \geq 0$ can be seen to hold iff $c \leq z$ when $x < \gamma$ and iff $c \geq z$ when $x > \gamma$. Hence, $\lambda = \lambda^* \in (0, 1)$ is an equilibrium iff $c \in (c_1, z]$ when $x < \gamma$, and iff $c \in [z, c_1)$ when $x > \gamma$. ■

As seen in the figure, a monetary equilibrium is again more likely to exist when c is low or x is high. Given a monetary equilibrium exists, it is more likely that $\lambda = 0$ when c is low or x is high, since both of these make honest production relatively attractive. Given x , it is more likely that $\lambda \in (0, 1)$ when c is bigger. When $x^* < \gamma$, there is a region of (x, c) space with $x < \gamma$ where an equilibrium with $\lambda \in (0, 1)$ exists and is unique. Regardless of x^* , as long as $c_0 > z$ at $x = 1$, there exists a region where equilibrium with $\lambda \in (0, 1)$ and $\lambda = 0$ coexist. A very interesting aspect of the results is the following. In constructing equilibrium with $\lambda \in (0, 1)$ we need to guarantee $\lambda < 1$, but this is actually never binding: as λ increases we hit the incentive constraint for money to be accepted before we reach $\lambda = 1$. For example, in the region where $\lambda \in (0, 1)$ exists uniquely, as c increases we get more thieves, but before the entire population resorts to crime people stop producing in exchange for cash and money stops circulating.

4.2 Banking

Bellman's equations are

$$\begin{aligned}
 rV_m &= (1-M)(1-\lambda)x(u+V_0-V_1) + (1-M)\lambda\gamma(V_0-V_1) + V_1 - V_m \\
 rV_d &= (1-M)(1-\lambda)x(u+V_0-V_1) + V_1 - V_d \\
 rV_p &= Mx(V_1 - V_p - c) + (1-Mx)(V_0 - V_p) \\
 rV_t &= M_0\gamma(V_1 - V_t - z) + (1 - M_0\gamma)(V_0 - V_t),
 \end{aligned}$$

where $V_1 = \max\{V_m, V_d - a\}$ and $V_0 = \max\{V_p, V_t\}$. Equilibrium requires the choice of crime to satisfy

$$\lambda = 1 \Rightarrow V_t \geq V_p; \lambda = 0 \Rightarrow V_t \leq V_p; \text{ and } \lambda \in (0, 1) \Rightarrow V_t = V_p,$$

and the choice of going to the bank satisfy

$$\theta = 1 \Rightarrow V_d - a \geq V_m; \theta = 0 \Rightarrow V_d - a \leq V_m; \text{ and } \theta \in (0, 1) \Rightarrow V_d - a = V_m.$$

In this section, since things are more complicated, we analyze possible equilibria one at a time. In principle there are nine qualitatively different types of equilibria, since each endogenous variable λ and θ can be 0, 1, or $\Phi \in (0, 1)$, but we can quickly rule out all but three possibilities.

Lemma 4 *The only possible equilibria are: $\theta = 0$ and $\lambda = 0$; $\theta = 0$ and $\lambda \in (0, 1)$; and $\theta \in (0, 1)$ and $\lambda \in (0, 1)$.*

Proof: Clearly $\lambda = 1$ cannot be an equilibrium, as then no one accepts money. If $\lambda = 0$ then there are no thieves, so money is safe and $\theta = 0$. Finally, if $\theta = 1$ then $V_t = 0$ and so we cannot have $\lambda > 0$. ■

Proposition 4 *$\theta = 0$ and $\lambda = 0$ is an equilibrium iff either: $x < x^*$ and $c \in [0, c_0]$; or $x > x^*$ and $c \in [0, c_1]$.*

Proof: Given $\lambda = 0$, it is clear that $\theta = 0$ is a best response. Hence the only conditions we need are $V_1 - V_0 - c \geq 0$ and $V_p \geq V_t$. With $\theta = 0$ these are equivalent to the conditions from the model with no banks. ■

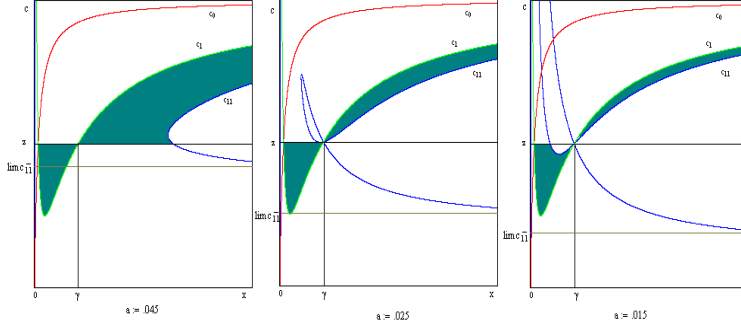


Figure 9: The function in Lemma 5

To proceed with equilibria where $\lambda > 0$, define

$$c_{11} = \frac{-B_0 \pm \sqrt{B_0^2 - 4A_0C_0}}{2A_0}$$

where

$$A_0 = \gamma x^2 [r + (1 - M)x + M\gamma]$$

$$B_0 = -[(1 - M)\gamma x u + (x - \gamma)\hat{a}](x - \gamma)x - [2(r + x) - M(x - \gamma)]\gamma^2 x z$$

$$C_0 = (x - \gamma)^2 x u \hat{a} + [(1 - M)\gamma x u + (x - \gamma)\hat{a}](x - \gamma)\gamma z + (r + x)\gamma^3 z^2.$$

As we will see, c_{11} is the solution to a quadratic equation describing the incentive condition for θ . As such, depending on the sign of $B_0^2 - 4A_0C_0$, generically c_{11} either has two real values, call them c_{11}^- and c_{11}^+ , or none. See Figure 9, which shows c_0 and c_1 as well as c_{11} . The figure is drawn assuming $\hat{a} < u\gamma(1 - M)$, which implies $\lim_{x \rightarrow \infty} c_{11}^- = \frac{\hat{a}}{\gamma(1 - M)} < u$; the other case is similar. In the left panel, drawn for large \hat{a} , c_{11} exists only for x close to 1, and in particular does not exist in the neighborhood of $x = \gamma$ (for still bigger \hat{a} , c_{11} would not even

appear in the figure). As \hat{a} shrinks, c_{11} exists for more values of x , and at some point it exists for x in the neighborhood of γ , as in the middle panel; notice c_{11}^- and c_{11}^+ happen to coalesce at $x = \gamma$. As \hat{a} shrinks further, c_{11} exists for all $x > 0$, as in the right panel. One can show the following.

Lemma 5 (a) For $x > \gamma$, if c_{11} exists then $c_{11}^+ < c_1$; for $x < \gamma$, if c_{11} exists then $c_1 < c_{11}^-$; if c_{11} exists in the neighborhood of $x = \gamma$ then $c_{11} = c_1$ at $(x, c) = (\gamma, z)$. (b) For $x > \gamma$, $c_{11}^+ \rightarrow c_1$ as $\hat{a} \rightarrow 0$; for $x < \gamma$, $c_{11}^- \rightarrow c_1$ as $\hat{a} \rightarrow 0$.

Proposition 5 $\theta = 0$ and $\lambda \in (0, 1)$ is an equilibrium iff either: $x > \gamma$, $c \in [z, c_1)$, and $c \notin (c_{11}^-, c_{11}^+)$; or $x < \gamma$, $c \in (c_1, z]$, and $c \notin (c_{11}^-, c_{11}^+)$.

Proof: In this case the conditions for $\lambda \in (0, 1)$ are exactly the same as in the model without banks, but we now have to additionally check $V_m - V_d + a \geq 0$ to guarantee $\theta = 0$. Algebra implies this condition holds iff $A_0 c^2 + B_0 c + C_0 \geq 0$, which is equivalent to $c \notin (c_{11}^-, c_{11}^+)$. ■

The region where equilibrium with $\theta = 0$ and $\lambda \in (0, 1)$ exists is shown by the shaded area in Figure 9. The economics is simple: in addition to the conditions for $\lambda \in (0, 1)$ from the model with no banks, we also have to be sure now that people are happy carrying cash instead of checks, which reduces to $c \notin (c_{11}^-, c_{11}^+)$. For large \hat{a} this is not much of a constraint. As \hat{a} gets smaller we eliminate more of the region where $\lambda \in (0, 1)$ is an equilibrium. When $\hat{a} \rightarrow 0$ the relevant branch of c_{11} converges to c_1 , and this equilibrium vanishes.

Now consider equilibria with $\theta \in (0, 1)$ and $\lambda \in (0, 1)$. To begin, we have the following result.

Lemma 6 If there exists an equilibrium with $\lambda \in (0, 1)$ and $\theta \in (0, 1)$, then

$$\lambda = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A(1 - M)} \text{ and } \theta = 1 - \frac{x[\hat{a} - (1 - M)\lambda\gamma c]}{\gamma[\hat{a} - (1 - M)\lambda\gamma z]}$$

where $A = \gamma x u$, $B = -[(1 - M)\gamma x u + M\gamma x c + (x - \gamma)\hat{a}]$, and $C = (r + x)\hat{a}$.

Proof: In this equilibrium we have $V_1 = V_m = V_d - a$ and $V_0 = V_p = V_t$. Solving Bellman's equations and inserting the value functions into these conditions gives us two equations in λ and θ . One is a quadratic that can be solved for $\lambda = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A(1-M)}$. The other gives us θ as a function of the solution for λ . ■

In order to reduce the number of possibilities, we concentrate on the smaller root for λ in Lemma 6.¹³ To see when such an equilibrium exists, define

$$\begin{aligned}
c_3 &= \frac{-k + \sqrt{4(r+x)\gamma x u \hat{a}}}{M\gamma x} \\
c_4 &= \frac{-(1-M)^2\gamma x u + [2(r+x) - (x-\gamma)(1-M)]\hat{a}}{(1-M)M\gamma x} \\
c_5 &= \frac{[r + Mx + (1-M)\gamma]\hat{a}}{(1-M)M\gamma x} \\
c_6 &= \frac{k}{[2(r+x) - Mx]\gamma} \\
c_7 &= \frac{k_{\pm} \sqrt{k^2 - 4[r + (1-M)x]\gamma x u \hat{a}}}{2[r + (1-M)x]\gamma} \\
c_8 &= \frac{-k + 2(r+x)\gamma z}{Mx\gamma} \\
c_9 &= \frac{xu\hat{a} - kz + (r+x)\gamma z^2}{M\gamma xz} \\
c_{10} &= \frac{(x-\gamma)k + 2(r+x)\gamma^2 z}{[2(r+x) - M(x-\gamma)]x\gamma}
\end{aligned}$$

where we let $k = (1-M)\gamma x u + (x-\gamma)\hat{a}$ to reduce notation. We now prove some properties of the c_j 's and relate them to c_0 , c_1 and c_{11} , continuing to assume $\hat{a} < \gamma(1-M)u$.

Lemma 7 (a) $x = 0 \Rightarrow c_3 = c_4 = c_5 = c_8 = c_9 = c_{10} = \infty$ and $c_6 = -\frac{\hat{a}}{2r} < 0$.
(b) $c'_3 < 0$, $c'_4 < 0$, $c'_5 < 0$, $c'_6 > 0$, $c'_8 < 0$, $c'_9 < 0$. (c) $c_7 \geq 0$ exists iff $x \geq x_7 \in (0, 1)$; if c_7 exists then $c_7^{-'} < 0$, $c_7^{+'} > 0$ and $(c_7^+, c_7^-) \rightarrow \left(\frac{\hat{a}}{\gamma(1-M)}, u\right)$ as $x \rightarrow \infty$; $c_7 \in (c_3, c_0)$ and $(c_7^+, c_7^-) \rightarrow (c_0, 0)$ as $\hat{a} \rightarrow 0$. (d) $c_{10} = c_1$ at

¹³In examples we found it was not impossible to have an equilibrium with λ given by the larger root, but only for a very small set of parameters.

$(x, c) = (\gamma, z)$; c_6, c_8 and c_{10} all cross at $c = z$, and c_7, c_9 and c_{11} all cross at $c = z$, although the values of x at which these crossing occur need not be in $(0, 1)$. (e) $c_3 = c_6 = c_7$ at a point where c_7 is tangent to c_3 ; $c_3 = c_{10} = c_{11}$ at a point where c_{11} is tangent to c_3 .

The c_j are shown in (x, c) space in Figure 10 for various values of a progressively decreasing to 0. The shaded area is the region where equilibrium with $\theta \in (0, 1)$ and $\lambda \in (0, 1)$ exist, as proved in the next Proposition.

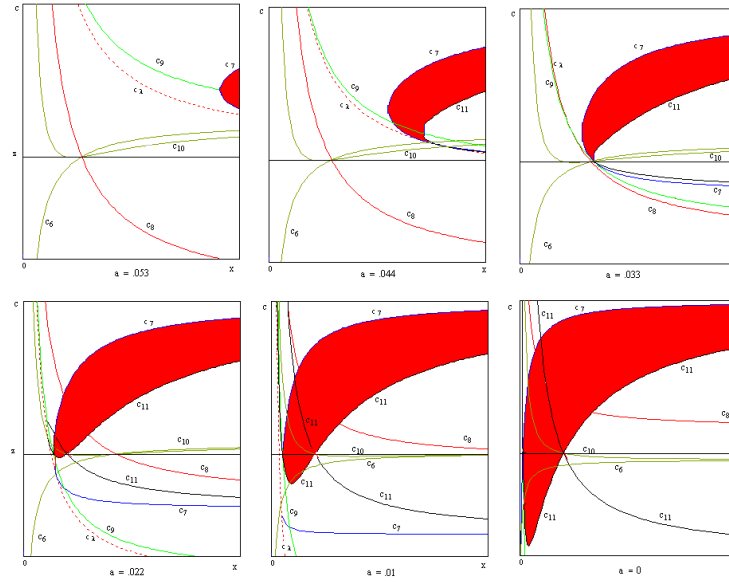


Figure 10: Proposition 6 equilibria

Proposition 6 $\theta \in (0, 1)$ and $\lambda \in (0, 1)$ is an equilibrium iff all of the following conditions are satisfied: (i) $c > c_\lambda = \max\{c_3, \min\{c_4, c_5\}\}$; (ii) $c > \max\{c_8, c_9\}$ and either $c < c_6$ or $c \in (c_7^-, c_7^+)$; and either (iii-a) $x > \gamma$, $c > \max\{z, c_{10}\}$, and $c \notin (c_{11}^-, c_{11}^+)$, or (iii-b) $x < \gamma$ and either $c > c_{10}$ or $c \in (c_{11}^-, c_{11}^+)$.

Proof: The previous lemma gives us λ as a solution to a quadratic equation

and θ as a function of λ , assuming that an equilibrium with $\theta \in (0, 1)$ and $\lambda \in (0, 1)$ exists. We now check the following conditions: when does a solution to this quadratic in λ exist; when is that solution in $(0, 1)$; when is the implied θ in $(0, 1)$; and when is $V_1 - V_0 \geq c$.

First, a real solution for λ exists iff $B^2 - 4AC \geq 0$, which holds iff either of the following hold:

$$\begin{aligned} c &\geq \frac{-[(1-M)\gamma xu + (x-\gamma)(1+r)a] + \sqrt{4(r+x)(1+r)\gamma xua}}{M\gamma x} = c_3 \\ c &\leq \frac{-[(1-M)\gamma xu + (x-\gamma)(1+r)a] - \sqrt{4(r+x)(1+r)\gamma xua}}{M\gamma x} = c'_3 \end{aligned}$$

Second, given that it exists, $\lambda > 0$ iff $B < 0$ iff

$$c > \frac{-(1-M)\gamma xu - (x-\gamma)(1+r)a}{M\gamma x} = c''_3.$$

It is easy to check that $c_3 > c''_3 > c'_3$, and so $\lambda > 0$ exists iff $c \geq c_3$.

It will be convenient to let $W = V_1 - V_0$. Subtracting the first two Bellman equations, we have $V_m - V_d = \frac{(1-M)\lambda\gamma}{1+r}W$, and hence by the equilibrium condition for $\theta \in (0, 1)$, $V_m - V_d = -a$, we have $\omega = \frac{\hat{a}}{\lambda(1-M)\gamma} = \frac{-B + \sqrt{B^2 - 4AC}}{2(r+x)\gamma}$ after inserting λ . We can now see that $\lambda < 1$ is equivalent to $\omega > \frac{\hat{a}}{(1-M)\gamma}$. Analysis shows this holds iff $c > \min\{c_4, c_5\}$. Hence, there exists a $\lambda \in (0, 1)$ satisfying the equilibrium conditions iff

$$c > c_\lambda = \max\{c_3, \min\{c_4, c_5\}\}.$$

We now proceed to check $\theta \in (0, 1)$ and $\omega \geq c$. Rearranging Bellman's equations gives us

$$1 - \theta = \frac{M_0}{M} = \frac{x(\omega - c)}{\gamma(\omega - z)}.$$

Hence, we conclude the incentive condition $\omega \geq c$ and $\theta < 1$ both hold iff

$\omega > \max(c, z)$. Algebra shows that $\omega > c$ holds iff $c < c_6$ or $c_7^- < c < c_7^+$, and that $\omega > z$ holds iff $c > \min(c_8, c_9)$.

For the last part, $\theta > 0$ holds iff $(x - \gamma)\omega < xc - \gamma z$. When $x > \gamma$, $\theta > 0$ is therefore equivalent to $\omega < \frac{xc - \gamma z}{x - \gamma}$, which holds iff $c > c_{10}$ and $c \notin (c_{11}^-, c_{11}^+)$. Moreover, notice that when $x > \gamma$, $c < z$ implies $\omega < \frac{xc - \gamma z}{x - \gamma} < c$, and therefore, we need $c > z$ as an extra constraint. When $x < \gamma$, $\theta > 0$ is equivalent to $\omega > \frac{xc - \gamma z}{x - \gamma}$, which is equivalent to $c > c_{10}$ or $c_{11}^- < c < c_{11}^+$. This completes the proof. ■

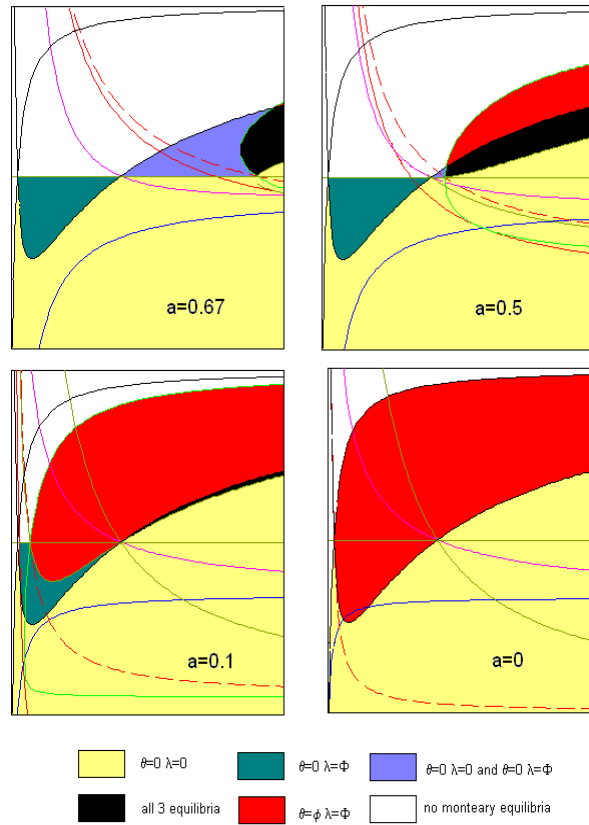


Figure 11: Equilibrium set for different a

Figure 11 puts together everything we have learned in this section and shows

the equilibrium set for decreasing values of a . For big a the equilibrium set is like the model with no banks. As a decreases, equilibria with $\theta > 0$ emerge, and we expand the set of parameters for which there exists a monetary equilibrium. Thus, for relatively high values of c there cannot be a monetary equilibrium without banks, because too many people would be thieves, but once banks are introduced and a is not too big agents will deposit their money into checking accounts and monetary equilibria can exist. It is important to emphasize that the fall in a actually has *two* effects in this regard: the direct effect is that it makes it cheaper for agents to keep their money safe; the indirect effect is that as more agents put their money in the bank the number of thieves changes.

Figure 12 shows what happens as M decreases. As in Section 3.2, lower M raises the demand for banking and makes it more likely to have equilibria with $\theta > 0$; however, in this model this result cannot be due to the number of thieves $(1 - M)\lambda$ mechanically increasing with a fall in M , since λ is endogenous. Perhaps the most interesting thing about the model with endogenous λ is that as long as $a > 0$, no matter how small, we can never have $\theta = 1$. The reason is simple: $\theta = 1$ implies $\lambda = 0$, but then no one would pay for checking. Hence cash will always circulate, and whenever $\theta > 0$ cash and checks coexist. The more general point is that as the cost of money substitutes falls there can be general equilibrium effects that make the demand for these substitutes fall, and the net effect may be that cash will never be driven entirely out of circulation.

5 Prices

So far we have been dealing with indivisible goods and money in the decentralized market, which means that every trade is a one-for-one swap. Here we follow the approach in Shi (1995) and Trejos and Wright (1995) and endogenize prices

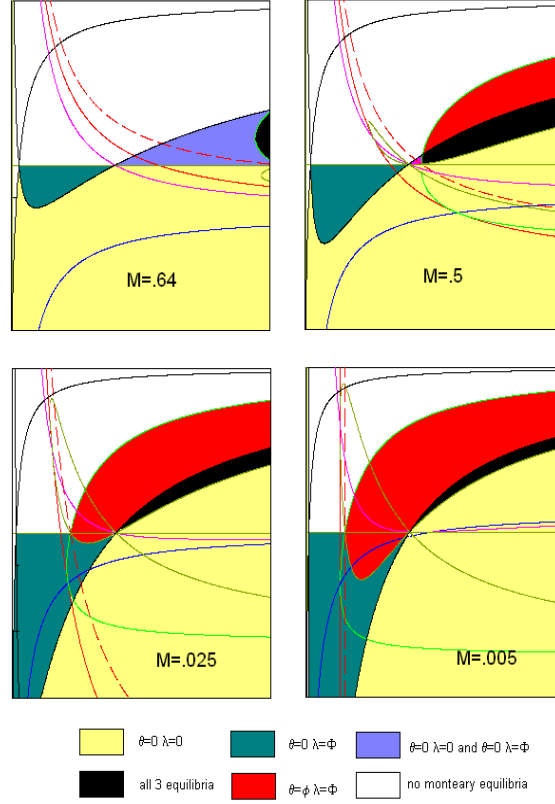


Figure 12: Equilibrium set for different M

by making all goods divisible. Since we mainly want to illustrate the method, and show that the basic results carry through, we do this only for exogenous λ . As above, we start with money and then add banks. Without banks,

$$\begin{aligned}
 rV_1 &= (1-M)(1-\lambda)x[u(q) + V_0 - V_1] + (1-M)\lambda\gamma(V_0 - V_1) \\
 rV_0 &= M(1-\lambda)x[V_1 - V_0 - c(q)] + M\lambda\gamma(V_1 - V_0 - z),
 \end{aligned}$$

where $u(q)$ is the utility from consuming and $c(q)$ the disutility from producing q units. We assume $u(0) = 0$, $u(\bar{q}) = \bar{q}$ for some $\bar{q} > 0$, $u' > 0$, $u'' < 0$, and normalize $c(q) = q$.

For simplicity we assume the agent with money gets to make a take-it-or-leave-it offer.¹⁴ Given $c(q) = q$, this implies $q = V_1 - V_0$, and so

$$rV_0 = M\lambda\gamma(V_1 - V_0 - z).$$

Rearranging Bellman's equations, we have $q = C_M(q)$ where

$$C_M(q) = \frac{(1-M)(1-\lambda)xu(q) + M\lambda\gamma z}{r + (1-M)(1-\lambda)x + \lambda\gamma}$$

is the same as the threshold C_M defined in the model with indivisible goods, except that $u(q)$ replaces u . We also need to check the participation condition $V_0 \geq 0$, which holds iff $q \leq C_A(q)$ where

$$C_A(q) = \frac{(1-M)[\lambda\gamma + (1-\lambda)x]u(q)}{r + (1-M)[\lambda\gamma + (1-\lambda)x]} - \frac{\lambda\gamma z}{(1-\lambda)x}$$

is the same as the threshold C_A , except $u(q)$ replaces u . One can show $q \leq C_A(q)$ reduces to $q \geq z$.

Hence, a monetary equilibrium exists iff the solution to $q = C_M(q)$ satisfies $q \leq C_A(q)$, or equivalently $q \geq z$. A particularly simple special case is the one with $z = 0$, since the participation condition holds automatically and there always exists a unique monetary equilibrium $q \in (0, \bar{q})$. The equilibrium price level is $p = 1/q$, and one can check that, as long as z is not too big, $\partial q/\partial \lambda < 0$ and $\partial q/\partial \gamma < 0$. Hence, more crime means money is less valuable and prices are higher.

With banks, we have

$$rV_m = (1-M)(1-\lambda)x[u(q) + V_0 - V_1] + (1-M)\lambda\gamma(V_0 - V_1) + V_1 - V_m$$

$$rV_d = (1-M)(1-\lambda)x[u(q) + V_0 - V_1] + V_1 - V_d$$

$$rV_0 = M(1-\theta)\lambda\gamma(V_1 - V_0 - z),$$

¹⁴Shi and Trejos and Wright actually assume symmetric Nash bargaining; the model with generalized Nash bargaining, of which take-it-or-leave-it offers constitutes a special case, it analyzed in detail by Rupert et al. (2001).

where $V_1 = \max\{V_m, V_d - a\}$. Equilibrium again requires $q = V_1 - V_0$ and $V_0 \geq 0$, and now also

$$\theta = 1 \Rightarrow V_d \geq V_m; \theta = 0 \Rightarrow V_d \leq V_m; \text{ and } \theta \in (0, 1) \Rightarrow V_d = V_m.$$

Consider first $\theta = 0$. We need the same conditions as in the model with no banks, $q = C_M(q)$ and $q \geq z$, but now we additionally need $V_m \geq V_d - a$. This latter condition holds iff $q \leq C_1(q)$, where C_1 is the same as in the model with indivisible goods, except $u(q)$ replaces u . One can show $q \leq C_1(q)$ reduces to $q \leq \hat{a}/(1 - M)\lambda\gamma = C_4$. In what follows we write q_0 for the value of q in equilibrium with $\theta = 0$. Then the previous condition has a natural interpretation as saying that for $\theta = 0$ we need the cost of banking \hat{a} to exceed the benefit, which is avoiding the expected loss $(1 - M)\lambda\gamma q_0$.

Now consider $\theta = 1$. Then $q = V_1 - V_0$ implies $q = C_2(q)$, where

$$C_2(q) = \frac{(1 - M)(1 - \lambda)xu(q) - \hat{a}}{r + (1 - M)(1 - \lambda)x}$$

is the same as above, except $u(q)$ replaces u . For small values of \hat{a} there are two solutions to $q = C_2(q)$ and for large \hat{a} there are none. Hence, we require \hat{a} below some threshold, say \hat{a}_2 , in order for there to exist a $q = C_2(q)$ consistent with this equilibrium. Since $\theta = 1$ the participation condition $V_0 \geq 0$ holds automatically, but we still need to check $V_m \leq V_d - a$. This holds iff $q \geq C_3(q)$, where C_3 is the same as in the model with indivisible goods, except $u(q)$ replaces u . The condition $q \geq C_3(q)$ can be reduced to $q \geq C_4 = \hat{a}/(1 - M)\lambda\gamma$. Writing q_1 for the equilibrium value of q when $\theta = 1$, this says that we need the cost of banking \hat{a} to be less than the benefit, which is again avoiding the expected loss $(1 - M)\lambda\gamma q_1$. Again this requires \hat{a} to be below some threshold, say \hat{a}_1 . Hence, whenever $\hat{a} \leq \min\{\hat{a}_1, \hat{a}_2\}$, $q = C_2(q)$ exists and satisfies all the equilibrium conditions.¹⁵

¹⁵ As we said above, there can be two different values of q in equilibrium with $\theta = 1$, although

Note that the conditions for the two equilibria considered so far are not mutually exclusive: $\theta = 0$ requires $C_4 \geq q_0$ and $\theta = 1$ requires $C_4 \leq q_1$, but the equilibrium values q_0 and q_1 are not the same. Hence these equilibria may overlap, or there could be a region of parameter space where neither exists. In either case it is interesting to consider $\theta \in (0, 1)$. This requires $V_m = V_d - a$, which reduces to $q = C_4$. As in the model with q fixed, we can now solve for M_0 and check $M_0 \in (0, M)$. Recall that with q fixed there were two possibilities for $M_0 \in (0, M)$: either $z < C_4$, $c \in [C_3, C_1]$ and $c \leq C_4$; or $z > C_4$, $c \in [C_1, C_3]$ and $x \geq \tilde{x}$. It is easy to check that now the latter possibility violates the condition $V_0 \geq 0$, leaving the former possibility. Hence, $\theta \in (0, 1)$ is an equilibrium when $q = C_4 > z$ and

$$C_3(C_4) \leq C_4 \leq C_1(C_4).$$

Much more can be said about this model. For instance, it is not hard to describe the regions of parameter space where the various equilibria exist, and how these regions change with a or M , as in the previous sections. We leave this as an exercise. The main point here is to show the key economic insights in the simple money and banking model are robust to having divisible goods.

6 Fractional Reserves

We now relax the 100% required reserve ratio and allow banks to make loans. For simplicity, for this extension we only consider the case where λ is exogenous and specialized goods are indivisible. Now, any agent without money can go to the bank and ask for a loan, which consists of a unit of money in cash – although he can directly deposit it in the bank. For ease of presentation we assume the loan is never repaid; rather, the agent pays ρ units of general goods it is also possible that one solution exceeds C_4 , in which case there is only one equilibrium q .

up front when the loan is extended. This is for ease of exposition, and it would be equivalent to have e.g. one-period loans. Let V_n denote the value function of an agent with no money deciding whether to take a loan: $V_n = \max\{V_0, V_1 - \rho\}$. We also let χ denote the fraction of such agents who decide to take a loan. Clearly, in any monetary equilibrium we must have $\chi < 1$, or $V_n = V_0 \geq V_1 - \rho$. In equilibrium, ρ will be determined to equate demand and supply for loans.

There is an exogenous required reserve ratio $\alpha \in (0, 1)$. While there is still a cost a for managing each dollar on deposit, we assume there is no cost for managing loans, although this is really without loss in generality. Hence, banks charge ϕ for deposit services, but since they can lend money it is not necessarily the case that $\phi = a$. In fact, zero bank profit implies $r\rho L + \phi D = aD$, where L is the measure of agents with loans and D is the measure with deposits.¹⁶ It is obvious that banks will lend out as much as possible, since there is no uncertainty regarding withdraws, so the required reserve ratio is binding: $L = (1 - \alpha)D$. As long as $D > 0$, zero profit requires

$$(1 - \alpha)r\rho + \phi = a.$$

This implies $\phi < a$, and ϕ can even be negative (interest on checking accounts).

We next present some accounting identities. As above, M_0 denotes the measure of agents with cash and M_1 the measure with cash or demand deposits. Loans plus the original stock of money sum to $L + M = M_1$, as do deposits plus cash held by individuals, $D + M_0 = M_1$. Combining these equations with $L = (1 - \alpha)D$ leads to the identity

$$\alpha M_1 + (1 - \alpha)M_0 = M,$$

which we will use below. If θ is again the proportion of agents with money who

¹⁶Since the fee for deposit services ϕ is received each period while the revenue from a loan ρ is received only once, we need to multiply the latter by r to get the units right.

deposit it, given α and M banks can “initially” make $\theta(1-\alpha)M$ loans, but then a fraction θ of these get deposited, and so on. We therefore have the textbook money multiplier equation,

$$M_1 = M + \theta(1-\alpha)M + \theta^2(1-\alpha)^2M + \dots = \frac{M}{1-\theta(1-\alpha)}.$$

Also, $M_0 = (1-\theta)M_1 = (1-\theta)M/[1-\theta(1-\alpha)]$.

Bellman’s equations can be written

$$rV_m = (1-\lambda)(1-M_1)x(u+V_0-V_1) + \lambda(1-M_1)\gamma(V_0-V_1) + V_1 - V_m$$

$$rV_d = (1-\lambda)(1-M_1)x(u+V_0-V_1) + V_1 - V_d,$$

$$rV_0 = (1-\lambda)M_1x(V_1-V_0-c) + \lambda M_0\gamma(V_1-V_0-z),$$

where $V_n = \max\{V_0, V_1 - \rho\} = V_0$, $V_1 = \max\{V_m, V_d - \phi\}$, $V_1 - V_0 \geq c$ and $V_0 \geq 0$. Notice that these equations are the same as Section 3.2 except M_1 replaces M (the previous model was the special case where $\alpha = 1$). In principle, there are nine qualitatively different types of equilibria, since χ and θ can each be 0, 1, or $\Phi \in (0, 1)$, but we can quickly rule out several combinations.

Lemma 8 *The only possible equilibria are: $\theta = 0$ and $\chi = 0$; $\theta \in (0, 1)$ and $\chi \in (0, 1)$; and $\theta = 1$ and $\chi \in (0, 1)$.*

Proof: Clearly $\chi = 1$ cannot be an equilibrium. For $\chi = 0$ to be an equilibrium we require $\theta = 0$ (for the loan market to clear). For $\chi \in (0, 1)$ we require $\theta > 0$. ■

We study the three possible cases $\theta = 0$, $\theta = 1$ and $\theta \in (0, 1)$, where in each case we know χ from the above Lemma. In the first case, there are no one deposits, so $M_0 = M_1 = M$, and $V_1 = V_m$. Recall that in Section 3, where loans were not considered, the condition for such an equilibrium is $c \leq \min\{C_M, C_A, C_1\}$, which corresponds to conditions $V_1 - V_0 \geq c$, $V_0 \geq 0$ and

$V_m \geq V_d - \phi$. With the opening of the loan market, the third condition needs to be modified. The maximum amount a borrower is willing to pay is $\bar{\rho} = V_1 - V_0$, and the maximum a depositor is willing to pay is $\bar{\phi} = V_d - V_m$. If the cost a exceeds potential revenue the loan market clears at $D = L = 0$. This happens iff

$$(1 - \alpha)r\bar{\rho} + \bar{\phi} \leq a,$$

which simplifies to

$$c \leq C_1 \equiv \frac{[r + (1 - \lambda)x + \lambda\gamma]\hat{a}}{(1 - \lambda)Mx[\lambda\gamma(1 - M) + (1 - \alpha)r(1 + r)]} - \frac{(1 - M)u}{M} - \frac{\lambda\gamma z}{(1 - \lambda)x}.$$

If $\alpha = 1$, then C_1 reduces to the expression for C_1 in Section 3.2. Since C_1 is increasing in α , it is more difficult to have equilibrium with $\theta = 0$ when the required reserve ratio is low. Intuitively this is because when banks can make loans, the equilibrium service fee ϕ goes down, making agents more inclined to use banking. We can rearrange $c \leq C_1$ as

$$\hat{a} \geq \hat{A}_1 \equiv \frac{\lambda\gamma(1 - M) + (1 - \alpha)r(1 + r)}{r + (1 - \lambda)x + \lambda\gamma} \{(1 - \lambda)x[(1 - M)u + Mc] + \lambda\gamma Mz\}$$

and present results for this version of the model in terms of thresholds for \hat{a} rather than c . Intuitively, when \hat{a} is too high banking is not viable, but the possibility of lending lessens this problem (\hat{A}_1 is decreasing in α) because borrowers share in the cost.

We summarize the results for the case $\theta = 0$ as follows.

Proposition 7 *A unique equilibrium with $\theta = \chi = 0$ exists iff $c \leq \min\{C_M, C_A\}$ and $\hat{a} \geq \hat{A}_1$.*

Now consider the case $\theta = 1$, where every individual with money, including those who just borrowed money, deposits it in the bank.¹⁷ This implies $M_0 = 0$,

¹⁷Recall that, when $\alpha = 1$, $\theta = 1$ could never be an equilibrium in the model with λ

and $M_1 = M/\alpha$, at least given $M < \alpha$ (if $M \geq \alpha$ an equilibrium with $\theta = 1$ cannot exist). Since $\theta = 1$, we have $V_1 = V_d - \phi \geq V_m$. Moreover, the previous lemma implies that $\chi \in (0, 1)$ when $\theta = 1$, so we must have $\rho = V_1 - V_0$. Solving for $V_1 - V_0$ and using the zero profit condition, we get

$$\rho = \frac{(1 - \lambda)x[(1 - M_1)u + M_1c] - \hat{a}}{(1 - \lambda)x - r[(1 + r)(1 - \alpha) - 1]}.$$

This is the value of ρ that clears the loan market.

For such an equilibrium to exist we need to check two things. First, the incentive condition $c \leq V_1 - V_0 = \rho$, which reduces to

$$\hat{a} \leq \hat{A}_2 \equiv (1 - \lambda)\left(1 - \frac{M}{\alpha}\right)x(u - c) + r[r - \alpha(1 + r)]c.$$

Second, $V_d - V_m \geq \phi$, which using the zero profit condition and the equilibrium value of ρ reduces to

$$\hat{a} \leq \hat{A}_3 \equiv \frac{(1 - \lambda)x[r(1 + r)(1 - \alpha) + \lambda\left(1 - \frac{M}{\alpha}\right)\gamma][(1 - \frac{M}{\alpha})u + \frac{M}{\alpha}c]}{r + (1 - \lambda)x + \lambda\gamma\left(1 - \frac{M}{\alpha}\right)}.$$

When \hat{a} is small enough, checks completely replace money. A low reserve ratio facilitates the circulation of checks, as reflected in the fact that \hat{A}_2 and \hat{A}_3 are decreasing in α .

Proposition 8 *A unique equilibrium with $\theta = 1$ and $\chi \in (0, 1)$ exists iff $\hat{a} \leq \min\{\hat{A}_2, \hat{A}_3\}$.*

Finally, consider the case $\theta \in (0, 1)$. An individual with money is indifferent between holding cash and a bank account if $\phi = V_d - V_m$. Using the value functions, $\rho = V_1 - V_0$, and the zero profit condition, we get

$$\rho = s(M_1) \equiv \frac{\hat{a}}{r(1 + r)(1 - \alpha) + \lambda\gamma(1 - M_1)}.$$

endogenous. Here we are looking at the case with λ exogenous, but even if it were endogenous it may now be possible to have $\theta = 1$, because ϕ can be negative once we allow $\alpha < 1$. It is still the case, however, that endogenizing λ could make it more likely for $\theta < 1$, since when θ is big λ will be small.

We interpret this as the (inverse) loan supply function, since it is the result of comparing V_m and V_d . Notice $s'(M_1) > 0$. Intuitively, a higher loan rate ρ leads to a lower ϕ through the zero profit condition, which induces more deposits and larger M_1 .

We can reduce the indifference condition for taking out a loan $\rho = V_1 - V_0$ to

$$\rho = d(M_1) \equiv \frac{(1-\lambda)(1-M_1)xu + (1-\lambda)M_1xc + \lambda\gamma zM_0}{r + (1-\lambda)x + \lambda\gamma(1-M_1) + \lambda\gamma M_0}.$$

We interpret this as the (inverse) loan demand function, since it tells us the what the loan rate ρ has to be to get the number of borrowers consistent with M_1 . One should interpret M_0 as a function of M_1 in this relation, using the identity $\alpha M_1 + (1-\alpha)M_0 = M$ derived above. It can be shown that $d'(M_1) < 0$ iff c is below some threshold c^* .¹⁸ In the analysis below we assume this is satisfied.

Figure 13 shows the supply and demand curves in (M_1, ρ) space.

In order to have $\theta \in (0, 1)$ we must have $M_1 \in (M, M/\alpha)$, which as can be seen in Figure 13 means

$$s(M) < d(M) \text{ and } s(M/\alpha) > d(M/\alpha).$$

These conditions reduce to $\hat{a} < \hat{A}_1$ and $\hat{a} > \hat{A}_3$, where \hat{A}_1 and \hat{A}_3 were defined above. We also need the incentive condition $c \leq V_1 - V_0$, and the participation condition $V_0 \geq 0$; for simplicity we assume here that $c > z$, so that the former implies the latter automatically. In terms of demand and supply, we need

$$d^{-1}(c) \geq s^{-1}(c)$$

since then $\rho \geq c$ at the intersection of supply and demand, and since $\rho = V_1 - V_0$

¹⁸For the record, c^* is given by

$$\frac{(1-\lambda)xu\{(1-\alpha)[r + (1-\lambda)x] + \lambda\gamma(M-\alpha)\} + \lambda\gamma z\{\alpha[r + (1-\lambda)x] + \lambda\gamma(\alpha-M)\}}{(1-\lambda)x(1-\alpha)[r + (1-\lambda)x + \lambda\gamma] + \lambda\gamma M}.$$

the incentive condition holds. It is easy to check $d^{-1}(c) \geq s^{-1}(c)$ iff

$$\hat{a} \geq \hat{A}_4 \equiv r(1+r)(1-\alpha)c + \lambda\gamma c \frac{r(1-\alpha)c + \lambda\gamma(M-\alpha)(z-c)}{(1-\alpha)(1-\lambda)x(u-c) - \lambda\gamma c + \lambda\gamma\alpha z}.$$

Proposition 9 *Assume $c > \max\{z, c^*\}$. A unique equilibrium with $\theta \in (0, 1)$ and $\chi \in (0, 1)$ exists iff $\hat{A}_3 < \hat{a} < \hat{A}_1$ and $\hat{a} \geq \hat{A}_4$.*

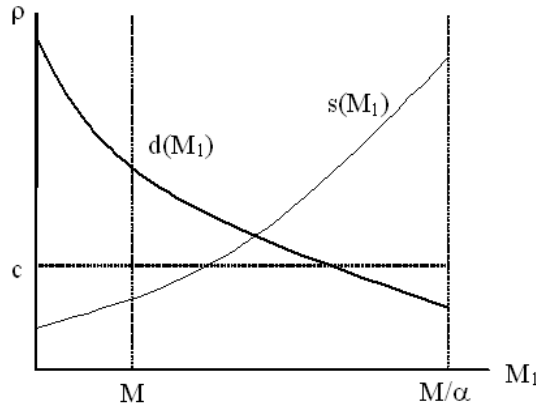


Figure 13: Loan supply and demand

We can use Figure 13 to describe how the equilibrium changes with parameters like a or α . First, $d(M_1)$ is independent of the banking cost a , while $s(M_1)$ shifts up as a increases. Hence M_1 goes down and ρ goes up with an increase in a ; intuitively, the loan volume decreases as some of the increase in cost is passed on to borrowers. With an increase in the reserve ratio α , $s(M_1)$ shifts up, while $d(M_1)$ shifts up iff $(1-\lambda)[(1-M_1)u + M_1c] > [r + (1-\lambda)x + \lambda\gamma(1-M_1)]z$.¹⁹ This condition is equivalent to $V_1 - V_0 > z$. For example, if $z < c$ as assumed in the previous Proposition, this condition must be satisfied, and so both supply and demand shift up with an increase in α . This means that ρ increases but the

¹⁹Demand shifts with α according to $\partial d/\partial\alpha = (\partial d/\partial M_0)(\partial M_0/\partial\alpha)$. Since $\partial M_0/\partial\alpha < 0$, $\partial d/\partial\alpha > 0$ iff $\partial d/\partial M_0 < 0$, which is equivalent to the condition in the text.

effect on M_1 is actually ambiguous. Other results can be derived, but we leave this as an exercise.

7 Conclusion

We have analyzed some models of money and banking based on explicit frictions in the exchange process. Although simple, we think these models capture something interesting and historically accurate about banking. Various extensions may contribute to our understanding of financial institutions more generally. Possible extensions include using versions of the model to study many of the phenomena addressed in the existing literature (e.g. bank runs, delegated monitoring, etc.). The goal here was to provide a first pass at fairly simpler class of models where money and banking arise endogenously and in accord with the economic history.

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