Beyond Search: Fiat Money in Organized Exchange

Peter Howitt
Brown University

May 17, 2004

Abstract

A model of fiat money is constructed in which spatial separation and the logistics of communication are made explicit as in search theory, but in which exchange is organized by profit-seeking business enterprises as in all market economies. Firms mitigate search costs by opening shops that are easily located. The model endogenizes the number of shops and which objects are traded in each shop. It permits the double coincidence of wants and deals easily with divisible commodities. Equilibria may exist in which fiat money is used as a universal medium of exchange. When such an equilibrium exists, fiat money is essential, in the sense that it is the universal medium of exchange in any efficient network of shops. The model provides a foundation to cash-in-advance theory, without specifying in advance that one object will be used as the universal medium of exchange.

For presentation to the conference on "Models of Monetary Economies II: The Next Generation" at the Federal Reserve Bank of Minneapolis, May 21, 2004. Helpful comments have been received from Daron Acemoglu, Nezih Guner, Christian Hellwig, Gustavo Ventura, Neil Wallace and Randy Wright. Many of the basic ideas in the paper were worked out with Bob Clower. I thank all of the above without holding them responsible for the paper's shortcomings.

One of the objectives of monetary theory is to produce models of endogenous fiat money; that is, models that do not assign to fiat money any *a priori* role in the transactions process, but in which fiat money is nonetheless used as a universal medium of exchange. (Wallace, 1998). In recent years, such models have been produced using search theory (Kiyotaki and Wright, 1989; and Aiyagari and Wallace 1991), according to which exchange takes place only when traders with complementary tastes and endowments are randomly matched with one another. These models have the virtue that they specify explicit environments in which spatial separation limits the ability to communicate and to trade. As Kocherlakota (1998) argues, fiat money is a record-keeping device that helps overcome these limitations.

In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters with non-specialists. The purpose of this paper is to provide a model of endogenous fiat money that specifies explicitly the spatial limitations to communication and trade as in search theory, but in which trade is organized by specialist traders, through facilities called "shops."

The model is based on recent work on *commodity* money, in which exchange is organized by an endogenous network of shops. Starr and Stinchcombe (1999) showed that under a variety of circumstances an optimal network of shops for trading n commodities has a monetary structure; i.e., that it has $n-1$ shops and there is one commodity c such that for every other commodity j there is one shop trading j and c. Starr and Stinchcombe (1998) show that such an arrangement characterizes some equilibria in an environment with free entry of shops. Howitt and Clower (2000) show by means of computer simulation that this is the only equilibrium that can be reached by a particular decentralized mechanism in which people act according to simple adaptive rules.

Two features of the technology of operating a shop are invoked to generate a shop network with a monetary structure in these theories. One is that at least part of the cost is *fixed* independently of the volume of trade. The other is that each shop can trade only a limited variety of commodities. In the model below these features are critical to the existence of an equilibrium with valued fiat money. A shop that did not trade fiat money would be unable to operate on a large enough scale to cover its fixed cost when competing with shops that do trade money, because its clientele would be limited to people that satisfy the double condition of being endowed with one of the objects traded in the shop and having a taste for consuming another of them. If fiat money is expected to be traded in all shops then this restriction does not apply to a shop that trades it, and the expectation will be self-fulfilling.

The present paper go beyond existing work on organized commodity exchange, by paying attention to spatial separation and communication costs at the same level as modern search theory, and by offering people the alternative of engaging in either random search or organized trade. The paper also specifies that people cannot transfer objects from one shop to another within a week, thus requiring material balance constraints to apply at all stages of a multi-step exchange. As I argue in section 4.4 below, this no-cross-hauling restriction is crucial for supporting fiat money, as opposed to the commodity money studied in the earlier papers. Finally, in order to deal with fiat money I study an intertemporal economy with an infinite horizon rather than the atemporal one-period model of earlier studies.

The analysis is also related to the "trading-post" models of Shapley and Shubik (1977), dynamic versions of which have been studied by Hayashi and Matsui (1996) and Alonzo (1999), both of whom discuss conditions under which a fiat money will have value and will be used.[1] Those models assume that trade can take place in specific locations, in each of

[1] The shops of the present paper are also like the "trading zones" of Iwai (1996) except that in Iwai's analysis someone who visits the ij-trading zone must continue to search for a trading partner in that zone, whereas in the present analysis the shops obviate the need for search. Iwai also assumes there is no fixed cost to operating a trade facility.

which a specific pair of objects can be traded for each other. The exchange rate at each trading post is determined as the ratio of quantities delivered on either side of the market. These models are characterized by a coordination problem giving rise to multiple equilibria. Specifically, if no one delivers commodities to a particular trading post then no one will want to do so. Thus in Alonzo's model, for any given choice of trading posts there will be an equilibrium in which those trading posts are inactive. Hayashi and Matsui eliminate this multiplicity by assuming that prices are posted at all trading posts, and that all participants assume they can trade all they want at those prices, even if some trading posts are inactive, but this begs the questions of who posts the prices at the inactive trading posts and why people think they can trade all they want when there is no one else to trade with.

The present paper can be seen as providing a micro-foundation to the trading-post story, by allowing profit-seeking business firms to create and operate them, and by making explicit the constraints that limit the objects that can be traded in them and determine the way people can visit them. I also suppose, as indicated above, that there is a fixed cost to operating a trading post; this allows fiat money to be used and to have value under circumstances where it would not in the other dynamic trading-post models. The paper analyzes subgame perfect equilibria and suggests a refinement under which a unique monetary equilibrium may exist.[2]

The model shares several features with search-theoretic models. It starts from the same premises of spatial separation and costly communication. Also, the above-mentioned "double condition" that handicaps a shop that does not trade a generally accepted medium of exchange is nothing but the double coincidence of wants, the unlikelihood of which underlies the use of money in search theory. However, the model deals easily with perfectly divisi-

[2] Several other papers are related. Banerjee and Maskin (1996) is discussed below in footnote 6. Camera (2000) portrays organized exchange as an alternative to monetary exchange. Corbae, Temzelides and Wright (2000) replace the stochastic pairwise matching of search theory with deterministic pairwise matching, but without intermediaries or specialist trading facilities. Hellwig (2000) analyzes equilibria with intermediation under stochastic matching. Laing, Li and Wang (1997) assume that each worker is matched randomly with an infinity of different shops.

ble money, which has led to considerable difficulties in search theory.[3] Also, unlike most search models (and the dynamic trading-post models discussed above) the model analyzed below does not depend on special assumptions regarding the joint distribution of tastes and endowments (such as the well-known "Wicksellian triangle") that rule out the possibility of a double coincidence of wants. Indeed I assume a symmetric distribution under which everyone could potentially find a double-coincidence trading partner at the same time.

Moreover, although the model starts from the same premises as search theory, it bears a stronger resemblance to existing "cash-in-advance" models of money than to search-theoretic models. It has long been recognized (Howitt, 1974; and Kohn, 1981) that cash-in-advance models can be rationalized by assuming that people trade exclusively with shops that will accept only money in exchange for commodities and will only pay only money for commodities. These rationalizations do not however provide models of endogenous money, because they specify *a priori* the unwillingness of shops to exchange commodities directly for other commodities. The present paper provides a cash-in-advance theory that does away with this *a priori* specification. Rather the specification emerges endogenously from the underlying logistical assumptions of the model.

I show below that a monetary equilibrium in this model is Pareto efficient whenever it exists. Moreover, fiat money is "essential," (Hahn, 1973), in the sense that whenever a monetary equilibrium exists fiat money is the universal medium of exchange in any efficient network of shops. The reason for this is the saving in transaction costs; that is, in the fixed costs of operating shops. As others have noted, a monetary pattern of shops allows everyone to trade using the least possible number of shops, and hence paying the least possible fixed costs.

Money is even essential in cases where a robust monetary equilibrium does not exist.

[3] Lagos and Wright (2002) provide a possible resolution of these difficulties by assuming that trade takes place partly through unorganized search and partly through organized markets. In their model, unlike in the present model, no medium of exchange is needed in the organized markets.

The reason is related to the wedge between the private and social costs of holding money, which underlies Friedman's (1969) argument regarding the optimum quantity of money. Specifically, from the private point of view there is a time cost to trading using money, because it takes two transactions instead of one. Thus when the cost of setting up a shop is low enough, barter shops can break a monetary equilibrium, despite their suboptimal scale of operation, by offering immediate gratification. But from a social point of view the time cost of using money is non-existent; in equilibrium current consumption equals current endowment minus the fixed costs of operating shops, regardless of the method by which commodities are distributed among people.[4]

1 Basics

1.1 Preferences and endowments

Time is discrete, with an infinite sequence of "weeks" indexed by t. There are $n \geq 4$ distinct commodities, all perfectly divisible, and none storable from one week to another. In addition there is money, a perfectly divisible and perfectly durable object. There are two groups of households; N "workers" and a large number of "merchants".

Each worker-household has a constant endowment flow y per week of one commodity, called its "manna", and can consume only one other commodity, its "food". Assume that tastes and endowments are distributed symmetrically, with the same number b of workers of each "type"; that is, of each of the $n(n-1)$ possible manna-food pairs. Thus instead of ruling out a double coincidence of wants we are going to the opposite extreme of making such a coincidence possible for everyone simultaneously. Accordingly:

$$N = bn(n-1) \tag{1}$$

Each merchant-household has a unique food it can consume, but receives no manna.

[4] Engineer and Bernhardt (1991) make a similar argument and note that this implies a rationale for legal restrictions against barter transactions.

Instead it is endowed with the capability of creating a trading facility, or "shop," whose operation will be described below. Thus the total endowment of the economy, summed across all commodities, is:

$$Y = Ny \tag{2}$$

For each commodity i we refer to those households endowed with i as "i-makers" and to those for whom i is food as "i-eaters." Every household derives utility from consumption, according to the linear lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t x_t, \ 0 < \beta < 1$$

where x_t is the amount of food it consumes at t. The economy begins with all money in the hands of workers, each one holding m units. There is a fixed amount M of money, where:

$$M = mN \tag{3}$$

1.2 Locations and shops

The logistical constraints imposed on people are similar to those of search theory. Each household begins each week in a private home not accessible to anyone else. This is where endowments are received. There is a continuum of locations at which traders can meet to exchange objects. Each household consists of two traders, each of whom can visit at most one location per week.[5] A trader can trade only with other traders who visit the same location that week. No record can be kept of who visited what locations, or who traded what amounts, in previous weeks.

A merchant-trader who visits a location can create a shop there. Only one shop can operate on any given location. Any number of traders can visit a location with a shop, but only to trade with the merchant who created the shop (the "shopkeeper").

As discussed in the introduction above, it is critical to recognize that each shop can trade only a limited variety of objects. As Alchian (1977) argued, if there were no such limitation

[5] The device of assuming a "divided household" is borrowed from Lucas (1990).

then there would be no need for anyone to trade indirectly or to hold a temporary abode of purchasing power. Such a limitation is empirically plausible, given the casual observation that no retail outlet (even Walmart) in any economy of record trades more than a small fraction of all tradeable objects. Moreover, the limitation can be rationalized by the costs of acquiring the specialized knowledge needed to assess commodities, as Alchian also argued.[6] Accordingly, I suppose that only two objects can be traded at a given shop. I refer to a shop as "monetary" if money is one of those two objects; otherwise it is a "barter shop."

For simplicity I assume that for each merchant k there is only one pair of objects it can trade in its shop. Thus each merchant belongs to one of $n + (n-1)\,n/2$ possible "classes" - monetary-i for some commodity i or barter-(i,j) for some unordered pair (i,j) of commodities. A shop is said to belong to the same class as its shopkeeper. Assume that:

$$\text{There exist at least 2 merchants in each class,} \tag{4}$$

and:

$$\left\{ \begin{array}{l} \text{Each monetary-}i\text{ merchant is an }i\text{-eater and} \\ \text{each barter-}(i,j)\text{ merchant is either an }i\text{-eater or a }j\text{-eater} \end{array} \right\}. \tag{5}$$

If no shop exists on a location, then no one can distinguish it from any other location that lacks a shop, without first visiting it. In particular, no one can tell if anyone else has chosen to visit it. Thus anyone who visits a location without a shop must choose one at random. A shop, however, can be seen from anyone's home. Thus a trader can choose to visit any given location on which a shop has been created that week. When choosing what location to visit, the trader can see what objects are traded in each shop, and can also see the terms at which each shopkeeper offers to trade.

[6] Banerjee and Maskin (1996) present a formal argument along the same lines, in which one good emerges as the universal medium of exchange in a competitive equilibrium with asymmetric information concerning the quality of specific commodities. They assume no fixed cost of operating a shop. As a result, a monetary equilibrium would not exist in their analysis if they were to assume a symmetric distribution of types as in our analysis. Instead they assume the Wicksellian triangle (A-makers eat B, B-makers eat C and C-makers eat A) that many authors have shown leads to monetary exchange by eliminating all double coincidence of wants.

Shops compete for customers, à la Bertrand, by posting these terms. Specifically, each monetary shop posts a pair of offer prices: $(w_0, w_1) > 0$. By doing so, it offers to pay w_0 units of money for each unit of the commodity delivered to it by a trader visiting the shop, and w_1 units of the commodity for each unit of money delivered. Thus w_0 is the shop's wholesale price and w_1 is the inverse of its retail price. Each barter shop posts a uniform offer price w_b, thus promising to pay w_b units of one commodity for each unit of the other commodity delivered to it.[7]

As mentioned in the introduction, it is crucial to assume a fixed cost of operating a shop. Suppose accordingly that an active shop must give up the amount σ of the commodities traded at the shop. More specifically, an active monetary shop must give up σ of the only commodity it trades, whereas an active barter shop must give up the amount $\sigma/2$ of each commodity it trades. Assume that:

$$\sigma > 2y \tag{6}$$

That is, the endowment of a single household is never enough to defray the setup cost of a shop.

To cover this cost each firm will of course charge different bid and ask prices. That is, for a monetary shop the retail price will exceed the wholesale price and for a barter shop the offer price will be less than unity. In principle a worker could avoid shops by engaging in random search for another worker with at least a single coincidence of wants. However, the assumption that there is a continuum of locations and just a finite number workers reduces the probability of such random encounters to zero. Thus, as is almost always the case in reality, people who trade will choose to pay the intermediaries' costs rather than undergo the (pointless) inconvenience of random search, and any trade that occurs will take place within the network of shops organized by merchants rather than through disorganized search.[8]

[7] Nothing substantive is lost by resticting barter shops to a uniform offer price.

[8] This is not to deny that search is important in real-world monetary economies, just that it is important in understanding why money is used. A fuller analysis would reintroduce search as it was initially conceived

1.3 The sequencing of actions

Events transpire in a fixed sequence each week, with three stages. In stage 1, endowments are received, and merchants visit locations to establish their shops and post their prices. In stage 2, each trader can visit a single location with some amount of money or of newly received commodity. In stage 3, shops pay for these deliveries, households consume the food acquired through trade, and money holdings are stored until the beginning of the next period.

2 Trading within each week

Formally I model the choices made each week as a two-stage game with observable actions. In stage 1, each merchant k chooses what prices to post. At stage 2 each household chooses what quantity of commodity endowment to deliver to a shop, which shop to deliver it to, what quantity of money to deliver to a shop and which shop to deliver it to.

2.1 Designated shops

The second stage game will generally have multiple equilibria, for reasons that are related to the "zero-activity" problem of trading-post models (see Alonzo, 1999). That is, there is a strong strategic complementarity in the choice of which shops to patronize. More specifically, a household h will have no incentive to deliver anything to shop k if no other household chooses to deliver anything to that shop, even if that shop is offering the most advantageous prices, because the shop, which begins the week with no inventories, will be unable to pay anything to h. Thus in general it is possible to have equilibria of the second-stage delivery subgame in which for example each i-maker delivers all its endowment to the monetary-i shop offering the lowest wholesale price and each i-eater delivers money only to the same monetary-i shop, or equilibria in which all i-makers and i-eaters patronize just the merchant

by Stigler (1961), with people searching not for random encounters with other searchers but for shops. The assumption above to the effect that each worker can costlessly observe each shop's location, objects and prices each week rules out this kind of search. The dynamic analysis of Howitt and Clower (2000), which analyzed the emergence of commodity money, depends very much on the cost of searching for shops.

charging the second highest retail price, and so forth.

To circumvent this particular version of the zero-activity problem, the only equilibria I will consider to the second-stage game are those in which there is at most one active shop (that is, one shop receiving positive deliveries) within each class, and that only the shops offering the most favorable prices within their class will be active. Thus in stage 1 merchants will compete to become the "designated" shop within their class in the second stage.

Specifically, the designated shops will be determined as follows. Within each class there is a preset ranking that is used to settle ties. The top-ranked shop can be thought of as the incumbent firm. Within any monetary class we say that shop k's prices "dominate" those of shop k' if $\left(w_0^k, w_1^k\right) \geqq \left(w_0^{k'}, w_1^{k'}\right)$. The rule is that the designated shop within each monetary class will be the highest ranked shop whose prices are not dominated by any other shop in the class, and the designated shop within each barter class will be the highest ranked shop whose offer price is the maximum within the class.

In stage 2 of each week, each household of type (i, j) takes as given the triple $w = (w_0, w_1, w_b)$, where w_0 is the wholesale price of the designated monetary-i shop, w_1 is the inverse retail price of the designated monetary-j shop and w_b is the offer price of the designated barter-(i, j) shop. The household then chooses (1) how much of its money holding q to deliver to the designated monetary-j shop and (2) whether to deliver its endowment to the designated monetary-i shop or to the designated barter-(i, j) shop. The second choice is represented by the variable z, which equals 0 if the endowment is delivered to the monetary shop or 1 if delivered to the barter shop.

2.2 Equilibrium trading-mode strategies

Even with the restriction to designated shops, there is still some multiplicity in the second-stage game because of the strategic complementarity involved in the choice of z by different households with the same manna. That is, condition (6) above guarantees that no i-maker will have an incentive to deliver its endowment to the designated i-shop when no other i-maker has chosen to do so. Likewise no household will have an incentive to deliver its endowment to the designated barter shop of its type when no other household of the same type has chosen to do so. Because of this strategic complementarity we lose no generality by imposing a common choice of z on each type of household. In fact we will go further by imposing a common choice of z on each household with the same manna commodity. In our study of robust monetary equilibria in section 4.3 below, we allow this common choice to depend on the shops' offer prices. Since no trader will ever choose to deviate from such a delivery strategy we suppose that in stage 2 the household's only decision is what quantity q of money to deliver.

2.3 Merchant choices

In stage 1 of each week, each merchant-household k chooses an offer price that its shop will pay for each unit delivered in case it becomes the designated shop in its class. For this offer to be admissible the shop must be able to honor these promises in the event that it receives the maximal possible deliveries. When less than the maximal amount is delivered to a shop, the shopkeeper has the right to refuse some or all deliveries, in which case we assume that on each side of the market the shop accepts the same proportion of each trader's delivery.

2.3.1 Barter merchants

Thus for a merchant in any barter-(i, j) class, the action set is the set of all offer prices $w_b \geq 0$ such that:

$$w_b by \leqq by - \sigma/2 \tag{7}$$

of such a price exists, or else $w_b = 0$. The left-hand side of (7) is what the merchant's shop must pay, in the form of commodity j, to receive delivery of the maximal amount y of commodity i from each of the b households of type (i, j), whereas the right-hand side is how much the shop will have available in the form of commodity j if it receives the maximal amount y from each household of type (j,i), after the amount $\sigma/2$ is used up in defraying the setup cost of activating the shop. Thus (7) is the shop's material balance constraint for commodity j in the event that it becomes the designated shop and receives the maximal deliveries. By symmetry it is also the shop's material balance constraint for commodity i in the same event.

Define:

$$\widehat{w}_b \equiv 1 - \sigma n (n-1)/2Y. \tag{8}$$

It follows from (1) and (2) that $\widehat{w}_b$ is the "breakeven price" of a barter shop - the offer price that just satisfies its material balance constraint (7) with equality. Accordingly, the action set of each barter merchant can be expressed as:

$$A_{1b} = \left\{ \begin{array}{ll} [0, \widehat{w}_b] & \text{if } \widehat{w}_b \geq 0 \\ \{0\} & \text{otherwise} \end{array} \right\}$$

2.3.2 Monetary merchants

For a merchant in any monetary-i class, the action set is the set of all offer prices (w_0, w_1) such that:

$$\left\{ \begin{array}{lll} w_0 (n-1) by & \leqq & (n-1) bm, \text{ and} \\ w_1 (n-1) bm & \leqq & (n-1) by - \sigma \end{array} \right\} \tag{9}$$

The first of these inequalities is the material balance constraint for money, in which the left-hand side is what the merchant's shop must pay for commodity deliveries in the event that all $(n-1)\, b$ i-makers deliver y to it, and the right-hand side is the money it will have available for making those payments in the event that all $(n-1)\, b$ money-holding i-eaters deliver all their money to it. Similarly the second inequality is the material balance constraint for commodity i under the same circumstances.

Define:

$$\widehat{w}_0 \equiv \frac{M}{Y} \tag{10}$$

and:

$$\widehat{w}_1 \equiv \frac{Y - n\sigma}{M}. \tag{11}$$

It follows from (1), (2) and (3) that $\widehat{w}_0$ is the breakeven wholesale price of a monetary shop - the wholesale price that just satisfies the first of the material balance constraints (9) with equality, and $\widehat{w}_1$ is the breakeven inverse-retail price of a monetary shop - the inverse-retail price that just satisfies the second material balance constraint with equality. Accordingly, the action set of each monetary merchant can be expressed as:

$$A_{1m} = \left\{ \begin{array}{ll} [0, \widehat{w}_0] \times [0, \widehat{w}_1] & \text{if } \widehat{w}_1 \geq 0 \\ \{(0,0)\} & \text{otherwise} \end{array} \right\}.$$

3 Expectations and Payoffs

3.1 Expectation of active monetary shops.

Suppose there is a triple of offer prices: $w' = (w_0', w_1', w_b')$ such that each household expects w' to be the prices that it will take as given at the beginning of each future week, regardless of what actions the household takes this week. Suppose also that each household expects that everyone in the future will choose $z = 0$ - that is, no one will make deliveries to a barter shop, and that the designated monetary shops will accept all deliveries. Accordingly each worker-household h of type (i, j) will rationally plan to deliver, in every future week, all its beginning-of-week money holding to the designated monetary-j shop and all of its y units of endowment to the designated monetary-i shop. Thus if the household enters next week holding m' units of money, its expected lifetime utility from then on will be given by the value function:

$$V(m', w_0', w_1') = w_1' m' + \Sigma_{t=1}^{\infty} \beta^t w_0' w_1' y$$

and its expected lifetime utility from this week on can be expressed, up to an additive constant, as:

$$U = x + \beta w_1' m' \tag{12}$$

where x is this week's consumption. The same expression represents the expected the expected lifetime utility from this week on of a merchant that consumes x this period and carries m' units of money into next week, again up to an additive constant.

Under these expectations, the amount of money a worker-household of type (i, j) will carry into next period will be:

$$m' = m + \alpha_0 w_0 (1 - z) y - \alpha_1 q \tag{13}$$

where q is the amount of money the household delivers to the designated monetary-j shop this period, z is the trading mode of all i-makers this period, w_0 is the wholesale price of the designated monetary-i shop, α_0 is the fraction of commodity deliveries accepted by this shop and α_1 is the fraction of money deliveries accepted this period by the designated monetary-j shop.

The amount consumed this week by the household is:

$$x = \alpha_1 w_1 q + \alpha_b w_b z y \tag{14}$$

where a_b is the fraction of deliveries of commodity i accepted by the designated barter-(i, j) shop. It follows from (12) $\sim$ (14) that the household will find it optimal to choose:

$$q = \left\{ \begin{array}{ll} m & \text{if } w_1 \geq \beta w_1' > 0 \\ 0 & \text{otherwise} \end{array} \right\}. \tag{15}$$

3.2 Expectation of inactive monetary shops.

Suppose alternatively that each household expects that in each future week all households will choose $z = 1$ and all designated barter shops will accept all deliveries. Then its expected lifetime utility from next week on will be independent of what happens this week, since the only carryover will be of money, which will have no value, since no one will be delivering commodities to any monetary shop.

3.3 Merchant payoffs

Under either set of expectations, the payoff of a designated barter-(i, j) merchant-household will depend on its own offer price w_b, the trading-mode decision z of the i-makers and the trading-mode decision z' j-makers, according to the function:

$$\Pi_b(w_b, z, z') = zz'(by(1 - w_b) - \sigma/2)^+$$

where the notation x^+ denotes $\max\{x, 0\}$. That is, if both types of household decide for barter the shopkeeper will be able to consume the slack in the material balance constraint (7) for its food, provided that this slack is non-negative. Otherwise it will be unable to accept deliveries and will have to consume zero.

Next, consider the designated monetary-i shop, and suppose that it is offering at least as much for money deliveries as the discounted expected future value: $w_1 \geq \beta w_1' > 0$. (Otherwise it will receive no money and will therefore be unable to be active.) The amount of money that the shopkeeper will carry over to the next period will be the slack in the shop's material balance constraint for money, which will depend on its own offer prices (w_0, w_1) and the respective trading mode decisions (z, z') of the i-eaters and i-makers, according to:

$$m' = (1 - z)(1 - z')(n - 1)b(m - w_0 y)$$

The same merchant's current consumption will equal the slack in the shop's material balance constraint for commodity:

$$x = (1 - z)(1 - z')((n - 1)b(y - w_1 m) - \sigma).$$

So, under the assumption of active monetary shops its payoff will be given by $x + \beta w_1' m'$:

$$\Pi_m = (1 - z)(1 - z')((n - 1)b(y - w_1 m) - \sigma + \beta w_1'(n - 1)b(m - w_0 y)).$$

The payoff of any inactive merchant is zero.

4 Equilibrium

4.1 Barter Equilibrium

We first establish conditions under which an equilibrium exists in which people visit just barter shops, trading without the use of a common medium of exchange, and in which money has no exchange value. Suppose that people have the expectation of inactive monetary shops as described in section 3.2 above. That is, each household expects that in each future week all households will choose $z = 1$ and all designated barter shops will accept all deliveries. Formally, a "Barter Equilibrium" is an offer price $w_b > 0$ such that when people have these expectations there is an equilibrium to this week's two-stage game in which all barter shops post w_b as their offer prices in the first stage and all households choose barter exchange in the second stage ($z = 1$) regardless of the prices posted in the first stage. It is easy to show that there exists a unique Barter Equilibrium, in which the offer price is the breakeven prices defined by (8) above, provided only that this breakeven price is positive. The following proposition is proved in the Appendix:

Proposition 1 *If*

$$Y/\sigma n > (n-1)/2 \tag{16}$$

then

$$\widehat{w}_b \equiv 1 - \sigma n (n-1)/2Y$$

constitutes the unique Barter Equilibrium. Otherwise there does not exist a Barter Equilibrium.

The existence of this equilibrium is quite intuitive. That the designated shops should just break even is a standard consequence of Bertrand price competition. The existence condition (16) requires just that a barter shop that received the maximal possible delivery $Y/n(n-1)$ of each commodity traded would have enough to defray the operating cost $\sigma/2$ in that commodity. If it were violated then clearly the economy would be unable to operate barter shops.

The existence of this equilibrium even when money exists is like the existence of an equilibrium with valueless fiat money in other standard monetary models such as cash-in-advance or the overlapping generations model. The difference is that in this case people are still able to trade even without using money, provided condition (16) is satisfied.

4.2 Monetary Equilibrium

Next we show conditions under which there is an equilibrium in which all traders visit monetary shops and trade using money. Formally, a "Monetary Equilibrium" is a pair of offer prices $(w_0, w_1) > 0$ such that if people expect active monetary shops with future prices $(w_0', w_1') = (w_0, w_1)$, as described in section 3.1 above, then there is an equilibrium to this week's two-stage game in which all monetary shops post (w_0, w_1) as their offer prices in the first stage and all households choose the second-stage strategy:

$$\left\{ \begin{array}{l} z = 0 \text{ for all posted prices} \\ q = \left\{ \begin{array}{ll} m & \text{if } w_1 \geq \beta \widehat{w}_1 > 0 \\ 0 & \text{otherwise} \end{array} \right\} \end{array} \right\} \tag{17}$$

It is straightforward to show that there exists a unique Monetary Equilibrium, in which the offer prices are the two breakeven prices defined by (10) and (11) above, provided only that these breakeven prices are both positive. In the appendix we prove:

Proposition 2 *If*

$$Y/\sigma n > 1 \tag{18}$$

then

$$(\widehat{w}_0, \widehat{w}_1) = \left(\frac{M}{Y}, \frac{Y - n\sigma}{M} \right)$$

constitutes the unique Monetary Equilibrium. Otherwise there does not exist a Monetary Equilibrium.

That the designated shops should just break even is again a standard consequence of Bertrand price competition. The existence condition (18) requires just that a monetary shop that received the maximal possible delivery Y/n of the commodity it trades would have enough to defray the operating cost σ. If (18) were violated then clearly the economy would be unable to operate monetary shops.

The equilibrium prices are those implied by the quantity theory of money. That is, the equilibrium retail price level is $P_r = 1/\widehat{w}_1$, while the equilibrium volume of retail transactions per period is $T_r = Y - n\sigma$. Likewise the equilibrium wholesale price level and volume of wholesale transactions are $P_w = \widehat{w}_0$ and $T_w = Y$. Thus (10) and (11) can be written as:

$$MV = P_w T_w = P_r T_r$$

with $V = 1$. The velocity of circulation is indeed equal to unity in this equilibrium, since each unit of money changes hands once per week in a retail transaction (when a worker-household delivers it to a shop) and once per week in a wholesale transaction (when a worker-household receives it in exchange for its delivery of commodity).

Moreover, the expectation that monetary shops will be active in each future week with all future offer prices equal to $(\widehat{w}_0, \widehat{w}_1)$ is a rational one, because the situation at the beginning of the next week will be the same as at the beginning of this week. All the money will be in the hands of workers, since merchants earn no surpluses, and each household will be holding its carryover m' of money which, by (13) above is just the same amount m as it started with this week.[9] Thus the equilibrium can be repeated indefinitely.

4.3 Robust Monetary Equilibrium

The zero-activity problem makes the use of monetary exchange almost automatic given the setup I have assumed. For it makes the expectation that only monetary shops will be active self-fulfilling, and makes the strategy of always ignoring barter shops, no matter what their posted prices, an equilibrium strategy. But to explain the ubiquitous use of money on the basis of such reasoning is less than compelling. That traders would not take seriously the possibility of a barter shop being able to accept deliveries at competitive prices may seem a reasonable approximation to reality in any economy of record, but it remains to be seen

[9] That is, from (15) and the fact that all deliveries are accepted in the third stage: $m' = m + \widehat{w}_0 y - q$. From this and (17): $m' = \widehat{w}_0 y$. From this, (2), (3) and (10), $m' = m$.

under what conditions an Monetary Equilibrium would exist even if people did take barter alternatives seriously, as they presumably would learn to do, at least in some places and times, if advantageous barter alternatives were potentially available.

Thus it is useful to refine our notion of equilibrium in an effort to eliminate any Monetary Equilibria that would not exist without the self-enforcing custom of ostracizing barter shops offering better deals, and to ensure that fiat money will be used even when people take barter alternatives seriously. To this end, I propose restricting attention to equilibria that are robust, in that they exist even when people are trying to take advantage of the best deals available, in the following sense.

Suppose that a worker-household of type (i, j) believed that its delivery of endowment would be accepted by whichever shop it chose - either the designated monetary-i shop or the designated barter-(i, j) shop. Then its payoff would be given by (12) $\sim$ (14) above with $\alpha_0 = \alpha_b = 1$:

$$U = \alpha_1 w_1 q + w_b z y + \beta w_1' \left(m + w_0 \left(1 - z\right) y - \alpha_1 q\right)$$

Under these beliefs, clearly an optimal second-stage strategy would be:

$$\left\{ \begin{array}{l} z = \left\{ \begin{array}{ll} 0 & \text{if } \beta w_1' w_0 \geq w_b \\ 1 & \text{otherwise} \end{array} \right\} \\ q = \left\{ \begin{array}{ll} m & \text{if } w_1 \geq \beta \widehat{w}_1 > 0 \\ 0 & \text{otherwise} \end{array} \right\} \end{array} \right\} \tag{19}$$

That is, a unit of i sold to a monetary shop yields a monetary revenue w_0, which can be converted into $w_1' w_0$ units of consumption next week when spent at the expected retail price, so the expected benefit of choosing monetary exchange, discounted back to this week, is $\beta w_1' w_0$. The same unit sold to a barter shop yields an immediate consumption of w_b, which is the expected discounted benefit of choosing barter. Thus the first part of (19) indicates that the household will choose the trading mode that yields the higher expected discounted benefit.

Of course it would typically not make sense for a household to assume that both alternatives were available with no possibility of rationing. But it would make sense for households to follow a convention that took into account the relative benefits of monetary exchange versus barter rather than always choosing monetary exchange whatever the relative benefits. Consider, for example, the alternative second-stage strategy.

For each i-maker household, define $w^i = (w_0^i, w_1^i, w_a^i)$ where w_0^i and w_1^i are the posted prices of the designated monetary-i shop and w_a^i is the maximum offer price of a designated barter-(i,j) shops for all j. The alternative second-stage strategy for each i-maker is thus:

$$\left\{ \begin{array}{l} z = \left\{ \begin{array}{ll} 0 & \text{if } \beta w_1' w_0^i \geq w_a^i \\ 1 & \text{otherwise} \end{array} \right\} \\ q = \left\{ \begin{array}{ll} m & \text{if } w_1^j \geq \beta \widehat{w}_1 > 0 \\ 0 & \text{otherwise} \end{array} \right\} \end{array} \right\} \tag{20}$$

This alternative strategy differs from the strategy (19) of the naive price-taker only in being even more eager to adopt barter. For it requires all households of type (i,j) to choose barter if some barter shop trading i offers a better deal than the monetary-i shop, even if the designated barter-(i,j) shop does not.

Formally, a "Robust Monetary Equilibrium" is a pair of offer prices $(w_0, w_1) > 0$ such that if people expect active monetary shops with future prices $(w_0', w_1') = (w_0, w_1)$, as described in section 3.1 above, then there is an equilibrium to this week's two-stage game in which all monetary shops post (w_0, w_1) as their offer prices in the first stage, all households follow the second-stage strategy (20), and all households choose monetary trade ($z = 0$) on the equilibrium path.

A necessary condition for the Monetary Equilibrium of Proposition 2 to be robust is that $\beta \widehat{w}_0 \widehat{w}_1 \geq \widehat{w}_b$. That is, when all shops charge their breakeven prices and people expect the same breakeven prices to prevail in the future then monetary trade will be more advantageous than barter. This necessary condition can be expressed as:

$$Y/\sigma n \leq \frac{n-3}{2(1-\beta)} + 1. \tag{21}$$

In the Appendix we prove:

Proposition 3 *If*

$$Y/\sigma n > 1$$

and

$$Y/\sigma n \leq \frac{n-3}{2\left(1-\beta\right)} + 1$$

then

$$\left(\widehat{w}_0, \widehat{w}_1\right) = \left(\frac{M}{Y}, \frac{Y - n\sigma}{M}\right)$$

constitutes the unique Robust Monetary Equilibrium. Otherwise there does not exist a Robust Monetary Equilibrium.

4.4 Commentary

Proposition 2 outlines necessary and sufficient conditions for fiat money to potentially play the role of universal medium of exchange. This requires only that the fixed cost of operating a shop be less than the economy's total endowment of each commodity. However, by Proposition 3 a monetary equilibrium will be fragile if this fixed cost is too small, for if households are confident that others will visit barter shops that offer favorable prices, then a monetary shop will not be able to compete effectively with barter shops.

The fragility of monetary shops when operating costs are small can be explained as follows. Someone who sells to a monetary shop must wait until another transaction is completed before consuming the proceeds of the sale, whereas no such wait is required if it sells instead to a barter shop. So to compete against barter shops, a monetary shop must offer an advantage in the form of a relative wholesale price $w_0 w_1'$ that is sufficiently larger than any barter shop's offer price. The reason why it can offer this advantage is the economy of scale that arises from the fixed cost of operating a shop. That is, because a monetary-j shop can service all makers of j, not just those who also consume some particular commodity i, it can have a smaller average cost than any (i, j) shop. Without the fixed cost, however, there would be no economy of scale, and monetary exchange would offer no advantage over barter.[10]

Proposition 3 also shows that fiat money is more robust, the larger is the variety n of commodities in the economy, given the aggregate supply Y/n of each commodity. This is because as variety increases, the market that can be served by a barter shop becomes increasingly fragmented, making it less likely that such a shop can operate on a large enough scale to break a monetary equilibrium by stealing the customers of monetary shops. Thus as economies become increasingly complex the likelihood increases that fiat money will be used. Since economic development tends to be characterized by increasing product variety, this result accords with the observation[11] that the degree of monetization rises as economic development proceeds.

Proposition 3 implies further that fiat money is less likely to be used as output per commodity increases, ceteris paribus. This is because in the limit as the market for any given commodity becomes infinitely large, the economy of scale implied by a fixed operating cost vanishes, and hence the advantage of monetary shops vanishes. Thus with economic growth either an economy eventually reaches the point where fiat money becomes unstable or the effects of increasing output per commodity are continually offset by the effects of increasing product variety. Asymptotically, if product variety grows faster than the square root of aggregate output then beyond some point a robust monetary equilibrium will always exist.

The spatial/logistical constraints imposed in section 1.2 above are also important for the existence of fiat money in a sense that cannot be seen immediately in the statement of Propositions 2 and 3. For suppose that each trader could visit two locations during any given

[10] This is why in the dynamic trading-post model of Hayashi and Matsui (1996), which has no costs of operating a trading post, there does not exist a monetary equilibrium under the configuration of tastes and endowments considered here. Likewise, in the related model of Alonzo (1999), which also does not have any costs of operating trading posts, a monetary equilibrium exists under such a configuration, but only what she calls a "cash-in-advance" equilibrium, in which no one ever considers visiting a barter trading post.

[11] See, for example, Bordo and Jonung (1987).

week,[12] trading in one shop the object acquired in another, instead of being restricted to one location per week. Then a stationary monetary equilibrium could not exist. For in this case each worker would choose to sell the proceeds of its current sale this week rather than incur a waiting cost; but then there would always exist an excess supply of money, with no one willing to hold it at the end of the week. This excess supply of money would correspond to an excess demand for commodities, which would prevent a stationary equilibrium from existing at any finite price level.

Note that a Monetary Equilibrium and a Barter Equilibrium must coexist over some range of parameter space. Indeed even a Robust Monetary Equilibrium must coexist with a Barter Equilibrium over some range. This is because the upper limit on $Y/\sigma n$ specified by the robustness condition (21) exceeds the lower limit specified by the existence condition (16) for a Barter Equilibrium. That is, condition (16) simply requires that the breakeven price of a barter shop be positive, whereas in order to break a Robust Monetary Equilibrium a barter shop must be capable of offering a price that is not just positive but large enough to steal customers from the designated monetary shops.

Of course it would be surprising not to find this kind of multiplicity in a model of fiat money. To analyze whether the economy might achieve a barter equilibrium or a monetary equilibrium when both exist would take us beyond the scope of the present paper, for it would require an adaptive analysis like that of Howitt and Clower (2000) intended to portray an economy's coordination mechanisms. In particular, Howitt and Clower take into account that entrepreneurship and the building of a customer base are not instantaneous, costless activities as in the present analysis. The analysis of that paper suggests that in cases of multiple equilibria only the monetary equilibrium would be reachable. For the equilibria of Propositions 2 and 3 above corresponds to the unique absorbing state of the adaptive mechanism studied by Howitt and Clower, except that the universal medium of exchange

[12] As is assumed by Starr and Stinchcombe (1998, 1999) and by Howitt and Clower (2000) in their models of commodity money.

that emerges there is a commodity money rather than fiat money.

5 Efficiency

To address the question of social efficiency we can assume that no one will deliver endowments to a shop unless given a positive reward.[13] By assumption no record-keeping device exists other than money. Thus there is no way that someone can consume the endowment of a type (i, j) worker unless there is an active monetary-i shop and an active monetary-j shop, through which the worker can be compensated via monetary exchange, or an active barter-(i, j) shop through which the worker can be compensated directly in the form of commodity j.

Suppose that the condition (16) for the existence of a Barter Equilibrium is not satisfied. Then there are not enough households of type (i, j) or (j, i) for their combined endowments to cover a shop's operating cost. So an (i, j) shop could operate only if workers of types other than (i, j) and (j, i) were to make deliveries to it. But nothing could induce them to do this. Thus there is no way for anyone to experience positive utility except by the use of monetary shops.

In the Monetary Equilibrium, by contrast, every worker experiences a positive lifetime utility and every merchant experiences a zero utility. Thus, whenever the Monetary Equilibrium exists and a Barter Equilibrium does not exist, the Monetary Equilibrium Pareto-dominates any socially feasible allocation that doesn't have money being used as a medium of exchange. In other words, when (18) is satisfied but not (16), then money is essential, in the sense that any feasible Pareto-efficient allocation must use money as a medium of exchange.

In the case where both (18) and (16) are satisfied, money is still essential in a utilitarian

[13] Strictly speaking an endowed worker would be indifferent between staying home and delivering endowment for a zero price, but that indifference would be broken by an epsilon cost of visiting a shop, which would not materially affect any other result of the paper.

sense. That is, any feasible allocation that maximizes the sum of lifetime utilities of all households must use money as a medium of exchange. We can demonstrate this as follows.

The sum of lifetime utilities is:

$$W = \Sigma_{t=0}^{\infty} \beta^t W_t = \Sigma_{t=0}^{\infty} \beta^t \Sigma_{i=1}^{n} C_{it}$$

where C_{it} is total consumption of commodity i by i-eaters in week t. The level of $W_t = \Sigma_{i=1}^{n} C_{it}$ attainable in week t depends on what shops are active that week. Suppose that money is never used. Then only barter shops are ever active. Workers of type (i, j) can be induced to deliver some commodity i only if an (i, j) shop is active that week, in which case the total amount that can be delivered by such workers is $Y/n\,(n-1)$, of which the amount $\sigma/2$ will be used up by the shop's operating cost. Therefore each week:

$$W_t \leq \left[\frac{Y}{n\,(n-1)} - \frac{\sigma}{2} \right] N^b$$

where N^b is the number of distinct worker types (i, j) for which an (i, j) shop is active in week t. Since N^b cannot exceed the total number of distinct worker types $n\,(n-1)$:

$$W_t \leq Y - \left(\frac{n-1}{2} \right) n\sigma.$$

Since $n > 3$:

$$W_t < Y - n\sigma \tag{22}$$

Since the right-hand side of (22) is the level of W_t attained in the Monetary Equilibrium, this shows that in any allocation in which only barter shops are active, the sum of lifetime utilities is strictly less than in the Monetary Equilibrium. Thus money is essential in the utilitarian sense, as claimed. Intuitively, monetary exchange is required for social optimality because it minimizes the number of shops, and hence the total operating cost, of allowing all potential GDP to be consumed each period.

Indeed we can go further and show that any allocation that uses any barter shops at all is suboptimal by the utilitarian criterion W. For consider any week t in which N^m monetary shops and $N^b/2$ barter shops are active. Assume with no loss of generality that there exist no two active shops of the same type. Let $N^{b/m}$ be the number of distinct worker types (i, j) such that a barter-(i, j) shop is active but not a monetary-i shop. Each one of these types can be induced to deliver endowments to a barter shop. In addition, for each of the N^m commodities for which a monetary shop exists there are $(n-1)$ types that can be induced to deliver to a monetary shop. Since each shop incurs a total set up cost of σ, therefore:

$$W_t \leq \left[\frac{Y}{n(n-1)}\right]\left[(n-1)N^m + N^{b/m}\right] - \sigma\left[N^m + N^b/2\right]$$

Note that $N^{b/m}$ is no greater than twice the number of barter shops:

$$N^{b/m} \leq N^b, \tag{23}$$

and no greater than the number of types (i, j) such that a monetary-i shop is not active:

$$N^{b/m} \leq (n-1)(n-N^m). \tag{24}$$

From (23) and the assumption that $n-1>2$:

$$W_t \leq \frac{Y-n\sigma}{n(n-1)}\left[(n-1)N^m + N^{b/m}\right], \text{ with strict inequality if } N^b > 0.$$

From this and (24):

$$W_t \leq Y - n\sigma, \text{ with strict inequality if } N^b > 0. \tag{25}$$

Hence the sum of lifetime utilities attainable when barter shops are active (the discounted sum of W_t's) is less than the sum attainable in the Monetary Equilibrium (the discounted sum of $Y-n\sigma$ each week), provided that the Monetary Equilibrium exists. On the other hand if the Monetary Equilibrium does not exist, then it is not even feasible to have active barter shops. Therefore it is never socially optimal by the utilitarian criterion to have an active barter shop. It also follows from (25) that whenever a Monetary Equilibrium exists it produces an optimal allocation by the utilitarian criterion. *A fortiori* an Monetary Equilibrium produces a Pareto-efficient allocation. In summary:

Proposition 4 *If a Monetary Equilibrium exists then (a) it produces a Pareto-efficient allocation, (b) money must be used in every transaction in order to maximize the sum of lifetime utilities across all households, and (c) if a Barter Equilibrium does not exist then money must be used in every transaction in order to achieve Pareto efficiency.*

It does not follow that a Robust Monetary Equilibrium always exists when it is needed for utilitarian welfare maximization. On the contrary, as we have seen, if the transaction cost σ is too low then a Barter Equilibrium exists but no Robust Monetary Equilibrium exists, even though Proposition 4 implies that the Barter Equilibrium is not efficient in the utilitarian sense. The reason is related to Friedman's (1969) argument regarding the optimal quantity of money. Specifically, from the private point of view there is a time cost to trading using money, because it takes two transactions instead of one. This is why, as we have seen, when the cost of setting up a shop is low enough barter shops can break a monetary equilibrium. But from a social point of view this time cost is non-existent; in equilibrium current consumption equals current endowment minus the fixed costs of operating shops, regardless of the method by which commodities are redistributed. Thus the attempt to avoid a private cost for which there is no corresponding social cost can destroy the only mechanism by which an efficient allocation of resources can be achieved. This problem could presumably be solved either by following Friedman's proposal to engineer a deflation through lump sum taxes that contracted the money supply over time, or to make barter transactions illegal, as suggested by Engineer and Bernhardt (1991).

6 Conclusion

One of the oldest problems of monetary economics is that the models of money that are most useful in applied work are far removed from those in which the institution of money can be shown to arise spontaneously. This gap undermines the credibility of useful models, by raising the question of the extent to which their conclusions rest on *ad hoc* assumptions needed to fit money into a scheme where it doesn't really belong. My aim in this paper was to take a step toward reducing this gap between useful and well-founded models, by showing that competition between firms that create facilities for trading limited subsets of objects can spontaneously generate a network of markets in which, to use Clower's (1967) aphorism, money buys goods and goods buy money but goods do not buy goods. The resulting equilibrium bears a strong resemblance to that of the cash-in-advance model that has been one of the profession's useful workhorses. The existence of this equilibrium does not depend on any special configuration of tastes and endowments.

According to the argument developed in the paper, fiat money rests on the same spatial/logistical foundations as in search theory, but those foundations result in organized trade, through trade facilities that are easily located, as is the case in most economies of record, rather than the disorganized random exchange depicted by search theory. In reality, most people trade using money as a universal medium of exchange because the enterprises that organize trade give them no other option. The paper proposed an account of how such a restrictive pattern of market organization might exist as an equilibrium phenomenon.

The model is simple but could be generalized in a number of useful directions. Allowing a reservation demand for endowments would allow a more meaningful efficiency analysis; to do this it would make sense to allow shops to set two-part tariffs, with a lump-sum entry fee and a positive marginal cost of trading, since otherwise the equilibrium marginal cost of producing sold consumption goods would be strictly less than the retail price (as it is in the Monetary Equilibrium above) and therefore the shops would generally not be allowing their customers fully to exploit all potential gains from trade. This would result in a general-equilibrium version of the Baumol-Tobin model of money-demand[14], in which households face a lumpy setup cost paid with each transaction regardless of the size of the transaction.

[14]More precisely, a general-equilibrium Clower-Howitt (1978) model, since the lumpy transactions costs faced by households are costs of trading commodities, not financial assets.

Another useful extension would be to generalize the model so that a shop can trade more than two objects. Preliminary work on the above model that makes only this change indicates that analogues to Propositions $1 \sim 3$ can be derived. But there will be barter trades even in an Monetary Equilibrium, because there will be worker types (i, j) for which both i and j can be traded in the same shop. There is clearly no need for such a worker to hold or use money in an Monetary Equilibrium. However, as long as the maximal number of objects traded in a shop is small relative to the total number of commodities n, most trades will be monetary in an Monetary Equilibrium.

A more interesting way to allow for a shop to trade more than one object would be to suppose that people have only one endowment good, which they must sell to a producer of a corresponding consumption good, and that they have a taste for consuming all possible consumption goods, as in the analysis of Laing, Li and Wang (1997). Then if workers were faced with the same logistical constraints as assumed above they would like to visit "supermarket" shops that traded a large variety of consumption goods. We could then allow such supermarkets to form by assuming that merchants have the possibility to visit many producers and to sell a large variety of consumption goods (but not endowment goods).

Another extension would be to allow for durable goods. Preliminary work indicates that analogues of propositions $1 \sim 3$ go through under perfect durability. Stationary equilibria in which fiat money is used and has value continue to exist, because in a stationary equilibrium the storage option is not used. Indeed this would even be true if storage were a productive activity, because a demand for money would continue to be supported by the cash-in-advance constraint implied by the basic logistical assumptions of the model. However there will also exist various commodity-money equilibria, in which fiat money is not used but one of the goods emerges as the universal medium of exchange.

Yet another extension would be to include credit markets. Merchants could be allowed, for example, to set up banks, which are facilities in which money can be exchanged for promises to deliver money in the future. Workers who decide to enter the credit market would have to forego the opportunity to buy or sell goods that week, because they have decided instead to visit the shop on which a bank is situated. The result would be another variant of Baumol-Tobin, in which the setup cost of visiting a bank is the foregone opportunity to visit another shop that week in which goods could be bought or sold.

Likewise, by adding stochastic endowment shocks, we could introduce out-of-steady-state dynamics, along with precautionary savings and precautionary money demand.

By putting business firms at the heart of a theory of money, this approach is also well suited for studying an economy's adjustment dynamics. The coordination mechanisms that allow a decentralized economy to regulate itself cannot be studied by conventional equilibrium methods, because to assume that the economy is in an equilibrium presupposes an unspecified coordination mechanism, involving unspecified agents. In reality, the agents of coordination are the enterprises that organize trade, for they are typically the agents that set prices, arrange for goods to be available at predictable times in known locations, and more generally undertake to coordinate supply and demand. By putting these agents at the center of monetary theory, the approach of this paper aims at laying the foundation not only for equilibrium monetary theories but also for disequilibrium theories. The eventual goal is to study the behavior of a stochastic decentralized market system in which competing shops must decide on prices, inventory holdings, advertising, entry and exit, without the full system-wide information that would be needed to carry out equilibrium actions.[15]

[15] Howitt and Clower (2000) represents another step in this same direction.

References

Aiyagari, S. Rao, and Neil Wallace. "Existence of Steady States with Positive Consumption in the Kiyotaki-Wright Model." *Review of Economic Studies* 58 (October 1991): 901-16.

Alchian, Armen. "Why Money?" *Journal of Money, Credit and Banking* 9 (February 1977, Part 2): 133-40.

Alonzo, Irasema. "Patterns of Exchange, Fiat Money, And the Welfare Costs of Inflation." unpublished, University of Rochester, June 1999.

Banerjee, Abhijit V., and Eric S. Maskin."A Walrasian Theory of Money and Barter." *Quarterly Journal of Economics* 111 (November 1996): 955-1005.

Bordo, Michael D., and Lars Jonung. *The Long-Run Behaviour of the Velocity of Circulation.* Cambridge UK: Cambridge University Press, 1987.

Camera, Gabriele. "Money, Search and Costly Matchmaking." *Macroeconomic Dynamics* (2000).

Clower, Robert W. "A Reconsideration of the Microfoundations of Monetary Theory." *Western Economic Journal* 6 (December 1967): 1-9. (now *Economic Inquiry*).

Clower, Robert, and Peter Howitt. "The Transactions Theory of the Demand for Money: A Reconsideration." *Journal of Political Economy* 86 (June 1978): 449-66.

Corbae, Dean, Ted Temzelides, and Randall Wright. "Matching and Money." unpublished 2000.

Engineer, Merwan, and Dan Bernhardt. "Money, Barter, and the Optimality of Legal Restrictions." *Journal of Political Economy* 99 (1991):743-73.

Friedman, Milton. "The Optimum Quantity of Money." In *The Optimum Quantity of Money and Other Essays*, edited by Milton Friedman. Chicago: Aldine Press, 1969.

Hahn, Frank. "On Transaction Costs, Inessential Sequence Economies and Money." *Review of Economic Studies* 40 (October 1973): 449-61.

Hayashi, Fumio, and Akihiko Matsui. "A Model of Fiat Money and Barter." *Journal of Economic Theory* 68 (January 1996): 111-32.

Hellwig, Christian. "Money, Intermediaries and Cash-in-Advance Constraints." unpublished, London School of Economics, March 2000.

Howitt, Peter. "Stability and the Quantity Theory." *Journal of Political Economy* 82 (JanuaryFebruary 1974): 133-51.

Howitt, Peter, and Robert Clower. "The Emergence of Economic Organization." *Journal of Economic Behavior and Organization* 41 (January 2000): 55- 84.

Iwai, Katsuhito. "The Bootstrap Theory of Money: A Search-Theoretic Foundation of Monetary Economics." *Structural Change and Economic Dynamics* 7 (December 1996): 451-77.

Kiyotaki, Nobuhiro, and Randall Wright. "On Money as a Medium of Exchange." *Journal of Political Economy* 97 (August 1989): 927-54.

Kocherlakota, Narayana. "Money is Memory." *Journal of Economic Theory* 81 (August 1998): 232-51.

Kohn, Meir. "In Defense of the Finance Constraint." *Economic Inquiry* 19 (April 1981): 177-95.

Lagos, Ricardo, and Randall Wright. "A Unified Framework for Monetary Theory and Policy Analysis." unpublished, February 2002.

Laing, Derek, Victor Li, and Ping Wang. "Money and Prices in a Multiple-Matching Search Model." unpublished, Penn State University, September 1997.

Lucas, Robert E. Jr. "Liquidity and Interest Rates." *Journal of Economic Theory* 50 (April 1990): 237-64.

Shapley, Lloyd, and Martin Shubik. "Trade Using One Commodity as a Means of Payment." *Journal of Political Economy* 85 (October 1977): 937-68.

Starr, Ross M., and Maxwell B. Stinchcombe. "Monetary Equilibrium with Pairwise Trade and Transaction Costs." Unpublished, University of California San Diego, September 3, 1998.

Starr, Ross M., and Maxwell B. Stinchcombe. "Exchange in a Network of Trading Posts." In *Markets, Information and Uncertainty: Essays in Economic Theory in Honor of Kenneth J. Arrow*, edited by Graciela Chichilnisky, 217-34. Cambridge UK: Cambridge University Press, 1999.

Stigler, George. "The Economics of Information." *Journal of Political Economy* 69 (June 1961): 213-25.

Wallace, Neil. "A Dictum for Monetary Theory." *Federal Reserve Bank of Minneapolis Quarterly Review* 22 (Winter 1998): 20-26.

Appendix

Proof of Proposition 1 (Existence of Barter Equilibrium)

(a) Suppose (16) is satisfied. Then $\widehat{w}_b > 0$. It follows from the discussion of section 2.2 above that under the expectation of inactive monetary shops the strategy:

$$z = 1 \text{ and } q = 0 \text{ for all posted prices} \tag{26}$$

constitutes an equilibrium to the second stage game for all posted prices. By construction the price $\widehat{w}_b$ lies in each barter merchant's action set A_{1b}. Given the household strategies (26), every barter merchant will earn a zero payoff in stage 1 if they all post $\widehat{w}_b$; the non-designated shops earn zero because they are inactive and the designated shops earn zero because by construction $\widehat{w}_b$ constitute a breakeven price. So $\widehat{w}_b$ will constitute a Barter Equilibrium if no barter merchant can earn a positive payoff by deviating from this uniform offer price. This follows because, by the construction of A_{1b}, the only possible deviation would be to a price that is strictly less than $\widehat{w}_b$, which would prevent the deviator from being designated and hence would yield a zero payoff. To show that $\widehat{w}_b$ is the unique barter equilibrium, consider any alternative price $\widetilde{w}_b \in A_{1b}$, with $\widetilde{w}_b \neq \widehat{w}_b$. If all barter merchants posted $\widetilde{w}_b$ then there would be at least one non-designated merchant (by assumption (4)) earning a zero payoff who could earn a positive payoff by posting $\widehat{w}_b - \varepsilon$,where ε is positive but small enough that the deviant becomes designated. The deviant would then receive the maximal deliveries as described above and would earn a strictly positive payoff because its trading surplus (the slack in its material balance constraint) would be positive for both commodities that it trades, including the one it eats.

(b) Suppose (16) is not satisfied. Then there is an no offer price in the action sets A_{1b} with $w_b > 0$, and hence there can be no Barter Equilibrium. ||

Proof of Proposition 2 (Existence of Monetary Equilibrium)

(a) Suppose (18) is satisfied. Then $(\widehat{w}_0, \widehat{w}_1) > 0$. It follows from (15) that and the discussion of section 2.2 above that having all households following the strategy (17) constitutes an equilibrium to the second stage game for all posted prices. By construction the pair $(\widehat{w}_0, \widehat{w}_1)$ lies in each monetary merchant's action set A_{1m}. Given the household strategies (17), every monetary merchant will earn a zero payoff in stage 1 if they all post $(\widehat{w}_0, \widehat{w}_1)$; the non-designated shops earn zero because they are inactive and the designated shops earn zero because by construction $(\widehat{w}_0, \widehat{w}_1)$ constitute breakeven prices. So $(\widehat{w}_0, \widehat{w}_1)$will constitute a

Monetary Equilibrium if no monetary merchant can earn a positive payoff by deviating from this uniform pair of offer prices. This follows because, by the construction of A_{1m}, the only possible deviation would be to a pair that is strictly dominated by $(\widehat{w}_0, \widehat{w}_1)$, which would prevent the deviator from being designated and hence would yield a zero payoff. To show that $(\widehat{w}_0, \widehat{w}_1)$ is the unique monetary equilibrium, consider any alternative pair $(\widetilde{w}_0, \widetilde{w}_1) \in A_{1m}$, with $(\widetilde{w}_0, \widetilde{w}_1) \neq (\widehat{w}_0, \widehat{w}_1)$. If all monetary merchants posted $(\widetilde{w}_0, \widetilde{w}_1)$ then there would be at least one non-designated merchant (by assumption (4)) earning a zero payoff who could earn a positive payoff by posting either $(\widehat{w}_0 - \varepsilon, \widehat{w}_1)$ or $(\widehat{w}_0, \widehat{w}_1 - \varepsilon)$ where ε is positive but small enough that the deviant becomes designated and $\widehat{w}_1 - \varepsilon \geq \beta \widehat{w}_1$. The deviant would then receive the maximal deliveries as described above and would earn a strictly positive payoff because its trading surplus (the slack in its material balance constraint) would be positive either for the commodity that it trades and eats or for money.

(b) Suppose (18) is not satisfied. Then there is no pair of offer prices in the action sets A_{1m} with $w_1 > 0$, and hence there can be no Monetary Equilibrium. ||

Proof of Proposition 3 (Existence of Robust Monetary Equilibrium)

(a) Suppose that (18) and (21) are both satisfied. Then it follows from the same line of reasoning that established Propositions 1 and 2 that under the expectations of active monetary shops posting breakeven prices, having all monetary shops posting $(\widehat{w}_0, \widehat{w}_1)$ and all barter shops posting:

$$\overline{w}_b \equiv \max \{\widehat{w}_b, 0\}$$

constitutes a first-stage equilibrium. If in addition all households follow the strategy (20) then they will choose $z = 0$ on the equilibrium path because (21) assures that under the given expectations $\beta w_0 w_1' \geq w_b$. Thus $(\widehat{w}_0, \widehat{w}_1)$ is a Robust Monetary Equilibrium. Uniqueness follows because with merchants charging any other uniform offer price a monetary merchant could earn a positive surplus by deviating and charging a little less than the breakeven prices as in the proof of Proposition 2.

(b) Suppose that (18) and (21) are not both satisfied. Then either (18) fails, in which case there is no pair of offer prices in the monetary merchants' action sets A_{1m} with $w_1 > 0$, or else (18) is satisfied but (21) fails, in which case whatever pair of offer prices were being charged by all monetary merchants a barter merchant could deviate from an equilibrium in which no one was visiting barter shops by charging a little below $\widehat{w}_b$ thus earning a positive surplus under the household strategy (20). ||