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Plant Level Irreversible Investment and Equilibrium Business Cycles

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ABSTRACT

This paper studies a version of the neoclassical growth model where heterogenous establishments are subject to partial irreversibilities in investment. Under such investment technology, the optimal decision rules of establishments are of the (S,s) variety. A novel contribution of the paper is the analysis of the general equilibrium dynamics arising from aggregate productivity shocks. This is a difficult task given the high dimensionality of the state vector, which includes the distribution of establishments across capital levels and idiosyncratic shocks. The paper overcomes this difficulty by developing a suitable computational approach. The model is used to study the importance of investment irreversibilities for macroeconomic dynamics. It is found that investment irreversibilities have no major implications for aggregate fluctuations, even though they are crucial for establishment level dynamics. This result contradicts previous conclusions in the literature which rely on partial equilibrium analysis.

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Abstract: This paper studies a version of the neoclassical growth model where heterogeneous establishments are subject to partial irreversibilities in investment. Under such investment technology, the optimal decision rules of establishments are of the (S,s) variety. A novel contribution of the paper is the analysis of the general equilibrium dynamics arising from aggregate productivity shocks. This is a difficult task given the high dimensionality of the state vector, which includes the distribution of establishments across capital levels and idiosyncratic shocks. The paper overcomes this difficulty by developing a suitable computational approach. The model is used to study the importance of investment irreversibilities for macroeconomic dynamics. It is found that investment irreversibilities have no major implications for aggregate fluctuations, even though they are crucial for establishment level dynamics. This result contradicts previous conclusions in the literature which rely on partial equilibrium analysis.

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1. Introduction

Since the early and influential paper by Arrow [2] there has been a considerable amount of theoretical microeconomic work on irreversible investment (see Dixit and Pindyck [9] for a survey). Arrow [2] recognized that investment is in general costly reversible, the purchase price of capital being larger than its resale price (due, for example, to the costs of detaching and moving machinery). For simplicity though, he suggested concentrating in the case where the sale price of capital is zero, i.e. where investment is completely irreversible. Most of the literature followed this suggestion. In an interesting paper, Abel and Eberly [1] have recently analyzed the more realistic and general case of partial irreversibilities in investment. In particular, they studied the problem of a firm facing a positive resale price of capital, which is lower than its purchase price. They showed that the optimal investment decision of the firm is a two-triggers (S,s) policy, and characterized the range of inactivity as a function of the wedge between the purchase and resale price of capital. A surprising result stemming from their quantitative analysis, is that even relatively small wedges between the purchase and sale price of capital can make the range of inactivity extremely similar to the one under complete irreversibility. This finding leads to the conclusion that small irreversibilities can matter a lot, and that modelling investment as being completely irreversible probably provides a better description of actual investment behavior than assuming it to be perfectly reversible.

There are not only theoretical reasons to emphasize investment irreversibilities: the empirical literature has also found evidence that investment is lumpy and infrequent at the establishment level. Doms and Dunne [10] using LRD data on manufacturing plants over 17 years found that the distribution of investment rates across plants is highly skewed, with 80% of the plants displaying annual capital growth rates below 10% (accounting for 45% of aggregate investment), while only 6% of the plants displaying capital growth rates over 30% (accounting for 25% of aggregate investment). They also found that over 50% of the establishments display capital growth rates of at least 37% in a single year, and that about 25% of a plant's cumulative investment over the 17 years is concentrated in a single year (suggesting sporadic investment spikes at the plant level). Moreover, they report that the number of plants going through large investment episodes is closely related to aggregate investment. Cooper, Haltiwanger and Power [7] found analogous results using a similar data set.

More direct evidence of the empirical importance of investment irreversibilities has been provided by Ramey and Shapiro [19]. Using data from an equipment auction performed by an aerospace firm, they estimated the wedge between the purchase price and resale price for different types of capital. They found that machine tools sell at about 31% of their purchase value, while structural equipment sell at only 5%. These estimates suggest substantial levels of investment irreversibilities.

An important issue which received much attention in the literature is the relevance of microeconomic irreversibilities for macroeconomics dynamics (e.g., Caballero and Engel [5], Caballero, Engel and Haltiwanger [6], and Bertola and Caballero [3], [4]). Aggregating the behavior of heterogeneous establishments subject to aggregate shocks, this literature found considerable support for the view that microeconomic irreversibilities are important for aggregate dynamics. For example, Caballero, Engel and Haltiwanger [6] using plant level LRD data found that non-linear adjustment rules at the plant level substantially improves the ability of their aggregate investment equation to keep track of actual aggregate investment behavior. In particular, the non-linearities appear to be crucial at periods of large deviations from trend in actual investment.

All the studies mentioned above are partial equilibrium models of sectoral investment. To fully analyze the macroeconomic implications of microeconomic irreversibilities a general equilibrium analysis is required. The literature has long avoided studying stochastic general equilibrium economies with heterogeneous agents following (S,s) decision rules since this class of problems seem extremely difficult to solve. This paper develops a methodology suitable to conduct such an analysis and explores the quantitative importance of plant level irreversibilities for equilibrium business cycle dynamics.[1]

Previous general equilibrium studies have focused on investment irreversibilities at the aggregate level, i.e. the case where capital goods have no use in consumption (e.g. Sargent [21], Olson [17], and Dow and Olson [11]). Sargent [21] considered a standard one sector growth model subject to i.i.d. productivity shocks, where agents supply labor inelastically, and aggregate investment is subject to a non-negativity constraint. However, his emphasis was in the problems associated with using a q-theory investment function for econometric policy evaluations, and not in the implications of irreversibilities for aggregate dynamics. Olson [17] generalized the class of productivity shocks considered by Sargent [21] and established the existence of a unique invariant distribution for the stock of

[1] A notable exception is Fisher and Hornstein [12], who study the general equilibrium dynamics of an economy with retailers which follow one sided (S,s) inventory policies.

capital. Interestingly, he showed that irreversibilities can dampen the response of investment to large aggregate productivity shocks.

The closest precedent to the current paper is Dow and Olson [11], who introduced aggregate investment irreversibilities into the real business cycle studied by Hansen [14]. For a similar aggregate productivity process as Hansen's, they found that aggregate investment irreversibilities had no effects. Given the small variability of aggregate productivity shocks, the nonnegative constraint on investment never became binding. Arguing that aggregate uncertainty underestimates the uncertainty faced at more disaggregate levels, they proceded to analyze a two sectors business cycle model. They found similar results: irreversibilities mattered only when shocks displayed an implausible large variance. This paper goes a few steps further with the level of disagregation. Davis and Haltiwanger [8] showed that there are large employment flows across establishments in the manufacturing sector, suggesting a considerable amount of idiosyncratic uncertainty at the establishement level. It seems natural to ask wether irreversibilities at this level of disagregation can have major implications for the aggregate dynamics implied by the theory. This paper pursues this question.

The model considered here is similar to the one in Veracierto [22] except that it allows for partial investment irreversibilities at the establishment level. The basic framework is analogous to the neoclassical stochastic growth model with indivisible labor analyzed by Hansen [14] and for a particular parametrization the model reduces to his. Output, which can be consumed or invested, is produced by a large number of establishments that use capital and labor as inputs under a decreasing returns to scale production technology. Establishments receive idiosyncratic productivity shocks that determine their expansion, contraction or death. They are also subject to an aggregate productivity shock (common to all establishments) that generates aggregate fluctuations in the economy. For simplicity, both entry and exit are treated as exogenous.

Labor is perfectly mobile across establishments but capital is not. Once capital is in place at an establishment there are costs associated with detaching and moving it. These costs imply that a fraction of the productive services of capital are lost in the process of uninstalling it. This is analogous to the general case noted by Arrow [2] and analyzed by Abel and Eberly [1], where the sale price of capital is smaller than its purchase price. In the face of these partial irreversibilities, the optimal investment behavior of establishments is to follow two sided (S,s) decision rules. The dynamics of the model are complicated since one must keep track as an endogenous state variable the full distribution of establishments across idiosyncratic productivity levels and capital stocks. I show how to overcome this difficulty below.

The model economy is used to study the quantitative importance of investment irreversibilities for macroeconomic dynamics. For this purpose, economies with different degrees of irreversibilities are calibrated to U.S. data and their aggregate fluctuations simulated and compared. Parameter values are selected so that their deterministic steady states reproduce key observations from U.S. data. These observations come from the National Income Accounts and from establishment level dynamics, as reported by Davis and Haltiwanger [8]. The process for aggregate productivity shocks is chosen so that measured Solow residuals display similar properties in the model economy as in the U.S. data.

The equilibrium fluctuations of the economy with perfectly reversible investment are found to be broadly consistent with U.S. business cycles, displaying similar features as those found in previous real business cycle models. The main result in the paper is that economies with different degrees of investment irreversibilities display somewhat different aggregate fluctuations, but that these differences are quantitatively unimportant. We conclude that for studying aggregate fluctuations we can safely abstract from investment irreversibilities at the establishment level. This result seems striking since Abel and Eberly [1] found that relatively small degrees of irreversibilities lead establishments to ranges of inactivity that are similar to those corresponding to complete irreversibility. On the other hand, Caballero, Engel and Haltiwanger [6] concluded that non-linearities are extremely important for aggregate dynamics. In light of these findings, it seemed natural to speculate that even small degrees of investment irreversibilities would have mattered for equilibrium aggregate dynamics.

The paper is organized as follows: Section 2 describes the economy, Section 3 discusses the competitive equilibrium and the solution strategy used to solve for it, Section 4 describes the observations used to calibrate the model, and Section 5 presents the results of the experiments.

2. The model economy

The economy is populated by a continuum of ex-ante identical agents with names in the unit interval. Their preferences are described by the following utility function:

$$E \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t\right) + v\left(l_t\right)\right] \tag{2.1}$$

where c_t and l_t are consumption and leisure respectively, and $0 < \beta < 1$ is the subjective time discount factor. Every period agents receive a time endowment equal to ω. Following Rogerson [20] and Hansen [14], it is assumed that there is an institutionally determined workweek of fixed length which is normalized to one, so leisure can only take values ω or ω-1.

Output, which can be consumed or invested, is produced by a large number of establishments. Each establishment uses capital (k) and labor (n) as inputs into a production technology given by:

$$y_t = e^{z_t} s_t k_t^{\theta} n_t^{\gamma} \tag{2.2}$$

where $\theta + \gamma < 1$, s_t is an idiosyncratic productivity shock and z_t is an aggregate productivity shock common to all establishments. Realizations of the idiosyncratic productivity shock s_t take values in the set $\{0, 1, \lambda\}$ and are independent across establishments. Over time, s_t follows a first order Markov process with transition matrix Π, where $\pi(s, s')$ is the probability that $s_{t+1} = s'$ conditional on $s_t = s$. This process is assumed to be such that: 1) starting from any initial value, with probability one s_t reaches zero in finite time, and 2) once s_t reaches zero, there is zero probability that s_t will receive a positive value in the future. Given these assumptions, it is natural to identify a zero value for the productivity shock with the death of an establishment.[2] The aggregate productivity shock z_t follows a law of motion given by:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \tag{2.3}$$

where $0 < \rho < 1$, and ε_t is i.i.d. with variance σ_ε^2 and zero mean.

Labor is perfectly mobile in this economy, but capital is not. On one hand, the amount of capital k_{t+1} in place at an establishment at date $t+1$ must be decided at period t before the realization of s_{t+1} becomes known. On the other hand, investment is partially irreversible at the establishment level. In particular, whenever capital is detached from the floor of an establishment it loses a fraction $(1-q)$ of its remaining productive services. To be more precise, let $0 < \delta < 1$ be the depreciation rate of capital. In order to increase an establishment's next period capital k_{t+1} above its current level net of depreciation $(1-\delta)k_t$, an investment of $k_{t+1} - (1-\delta)k_t$ units is needed. On the contrary, when an establishment decreases its next period capital k_{t+1} below its current level net of depreciation $(1-\delta)k_t$, the amount of investment goods obtained from the establishment is only a fraction q

[2]Given that there are no fixed costs to operate an establishment already created, exit will take place only when the idiosyncratic productivity shock takes a value of zero.

of $(1-\delta)k_t - k_{t+1}$. The parameter q is a measure of the degree of the investment irreversibilities in the economy and will play a crucial role in our analysis.

Every period, agents receive an endowment of new establishments which arrive with zero initial capital in place. Initial values for s across new establishments are distributed according to ψ. This exogenous birth of new establishments compensates the ongoing death of existing establishments (as they get absorbed into zero productivity) and results in a constant long run number of plants.[3]

The presence of idiosyncratic productivity shocks and irreversible investment at the establishment level suggests indexing establishments according to their current productivity shocks s and current stock of capital k. In what follows, a measure x_t over current productivity shocks and capital levels will describe the number of establishments of each type at period t. Also, a measurable function n_t will describe the number of workers across establishment types, a measurable function g_{t+1} will describe the next period stock of capital across establishment types, and η_t will denote the fraction of the population that works.

Feasibility constraints consumption as follows:

$$c_t \leq \int \left\{ e^{z_t} s \; k^{\theta} n_t(k,s)^{\gamma} - \left[g_{t+1}(k,s) - (1-\delta)k\right] \; Q\left[g_{t+1}(k,s) - (1-\delta)k\right] \right\} dx_t$$

$$+ \int (1-\delta) \; g_t(k,s) \; q \; \pi(s,0) \; dx_{t-1} \tag{2.4}$$

where $Q()$ is an indicator function that takes value 1 if its argument is positive, and value q (the irreversibility parameter) otherwise. The first term, is the sum of output minus investment across all types of establishments, taking into account the capital losses due to the investment irreversibilities. The second term on the right hand side corresponds to all those establishments that where in operation the previous period and die during the current period (transit to an idiosyncratic shock equal to 0), getting to sell a fraction q of their stock of capital $g_t(k,s)$ net of depreciation.

Similarly, the total number of workers at plants is constrained not to exceed the fraction of the population that works η_t:

$$\int n_t(k,s) \; dx_t \leq \eta_t \tag{2.5}$$

[3]Even though the entry and exit decisions of establishments are not endogenously determined in this economy, it seems important to incorporate them at least exogenously. A significant probability of death will probably affect how establishments respond to aggregate productivity shocks in the presence of investment irreversibilities.

Finally, the law of motion for the measure x_t must be consistent with the capital decisions at the plant level. That is, for every Borel set B:

$$x_{t+1}(B, s') = \int_{(k,s): g_{t+1}(k,s) \in B} \pi(s, s') \; dx_t \; + \; v \; \psi(s') \; \chi(0 \in B) \tag{2.6}$$

where $\chi()$ is an indicator function that takes value 1 if its argument is true, and a value of zero otherwise. In words, the number of establishments that next period have a stock of capital in the set B and a productivity shock s', is given by the sum of two terms: 1) all those establishments that transit from their current shocks to the shock s' and choose a next period stock of capital in the set B, and 2) in the case that $0 \in B$, all new establishments that arrive with an initial productivity shock s' (note that new establishments are borned with a zero initial stock of capital).

3. Competitive equilibrium and solution method

Following Hansen [14] and Rogerson [20], agents are assumed to trade employment lotteries. These are contracts that specify probabilities of working, and allow agents to perfectly diversify the idiosyncratic risk they face. Since agents are ex-ante identical, they all chose the same lottery. The economy therefore has a representative agent with utility function:

$$\ln c_t - \alpha \; \eta_t \tag{3.1}$$

i.e. utility becomes linear with respect to the probability of working η_t (for details see Hansen [14]; and Rogerson [20]). Since this is a convex economy with no externalities nor other distortions, its competitive equilibrium allocation can be solved by analyzing the Social Planner's problem with equal weights.

The state of the economy is given by the current aggregate productivity shock z, the current measure x across establishment types, the previous period measure y across establishment types, and the previous period investment decisions d across establishment types (z_t, x_t, x_{t-1} and g_t respectively in our previous notation). The Social Planner's Problem is described by the following Bellman equation:

$$V(d, x, y, z) = MAX \left\{ \ln c - \alpha \; \eta + \beta \; E \; V(d', x', y', z') \right\} \tag{3.2}$$

subject to

$$c \leq \int \left\{ e^{z} s\ k^{\theta} n\,(k,s)^{\gamma} - [g\,(k,s) - (1-\delta)k]\ \ Q\,[g\,(k,s) - (1-\delta)k]\ \right\} dx$$

$$+ \int (1-\delta)\ d\,(k,s)\ \ q\ \pi\,(s,0)\ \ dy \tag{3.3}$$

$$\int n\,(k,s)\ \ dx \leq \eta \tag{3.4}$$

$$x'\,(B,s') = \int_{(k,s):g(k,s)\in B} \pi\,(s,s')\ \ dx\ +\ v\ \psi\,(s')\ \ \chi\,(0 \in B) \tag{3.5}$$

$$d' = g \tag{3.6}$$

$$y' = x \tag{3.7}$$

$$z' = \rho z + \varepsilon' \tag{3.8}$$

where the maximization is over $n\,()$ and $g\,()$. Note the high dimensionality of the state space which seems to preclude any possibilities of computing a solution. Below, I will show that this difficulty is only apparent: the problem becomes fully tractable once it is redefined in terms of a convenient set of variables.

To understand the rationale for the transformed problem, it will be convenient to analyze the structure of the problem that establishments face at the competitive equilibrium. The individual state of an establishment is given by its current productivity shock s and its current stock of capital k. The problem of an establishment with individual state (k,s) when the aggregate state is (d,x,y,z) is given by:

$$J(k,s,d,x,y,z) = MAX\ \{\ e^{z} s\, k^{\theta} n^{\gamma} - w\,(d,x,y,z)\, n - [k' - (1-\delta)k]\, Q\,[k' - (1-\delta)\,k]$$

$$+ E\,[i\,(d,x,y,z;d',x',y',z')\, J\,(k',s',d',x',y',z')]\ \} \tag{3.9}$$

subject to:

$$s' \sim \Pi \tag{3.10}$$

$$z' = \rho z + \varepsilon' \tag{3.11}$$

$$(d', x', y') = H(d, x, y, z) \tag{3.12}$$

where $w()$ is the equilibrium wage rate, $i()$ are the equilibrium prices of Arrow securities, $H()$ is the equilibrium law of motion for the aggregate state of the economy, and where the maximization is over the scalars n and k' . Note that the decision rule for capital that corresponds to the solution to this Bellman equation is of the (s,S) type. It is characterized by a pair of lower and upper capital thresholds $a(s)$, $A(s)$ such that:

$$\begin{aligned} k' &= a(s), && \text{if } (1-\delta)k < a(s) \\ &= A(s), && \text{if } (1-\delta)k > A(s) \\ &= (1-\delta)k, && \text{otherwise} \end{aligned} \tag{3.13}$$

where the dependence of $a(s)$ and $A(s)$ on the aggregate state of the economy has been suppressed to simplify notation (Figure 1 shows a picture of these decision rules). Note that there is a pair of lower and upper threshold $(a(s), A(s))$ for every possible idiosyncratic productivity shock s. Hereon we will denote (a, A) as being the vector $(a(s), A(s))_{s=1,\lambda}$ across idiosyncratic shocks.

Our strategy will be to keep track of long histories of (a, A) as state variables instead of the actual distributions x and y and use them to construct approximate distributions for x and y using the law of motion in equation (2.6).[4] In principle, as we make the length of the history of (a, A) arbitrarily large we would obtain an arbitrarily good approximation for x and y. An important question will be how large to make this length in practice (I will return to this issue below). Our solution method will require solving independently for the deterministic steady state of the economy. Appendix A describes how this is performed.

Let $(\underline{a}, \underline{A})$ denote the history of thresholds $\{a_t, A_t\}_{t=1,\ldots,T}$, for some large horizon T, where (a_t, A_t) were the thresholds chosen t periods before the current date. Also, let (a^c, A^c) be the thresholds for the current period. Since we know that the optimal decision rules of establishments are of the (S,s) variety, there is no loss of generality in defining the Social Planner's problem directly in terms of the current thresholds (a^c, A^c) and the fraction of people that work η as follows:[5]

$$V(\underline{a}, \underline{A}, z) = MAX \{\ln [c(\underline{a}, \underline{A}, z, a^c, A^c, \eta) - \alpha\ \eta + \beta EV(\underline{a}', \underline{A}', z')]\} \tag{3.14}$$

[4]Note that yesterday's (a, A) defines yesterday's decision rule d.

[5]Note that problem (3.14) reduces to the original problem (3.2) as T goes to infinity.

subject to:

$$\begin{aligned} a'_{t+1}(s) &= a_t(s), && \text{for } t=1,2,...,T-1 \text{ and } s=1,\lambda && (3.15) \\ a'_1(s) &= a^c(s), && \text{for } s=1,\lambda \\ A'_{t+1}(s) &= A_t(s), && \text{for } t=1,2,...,T-1 \text{ and } s=1,\lambda \\ A'_1(s) &= A^c(s), && \text{for } s=1,\lambda && (3.16) \end{aligned}$$

$$z' = \rho z + \varepsilon' \tag{3.17}$$

where equations (3.15) update tomorrow's histories given the current choices. The function $c(\underline{a}, \underline{A}, z, a^c, A^c, \eta)$ gives the maximum consumption that can be obtained given the history of thresholds $(\underline{a}, \underline{A})$, the current aggregate productivity shock z, the current choices of thresholds (a^c, A^c), and the decision of how many agents to currently put to work η. Formally, $c(\underline{a}, \underline{A}, z, a^c, A^c, \eta)$ is given as the solution to the following problem:

$$c(\underline{a}, \underline{A}, z, a^c, A^c, \eta) = MAX \int \{ e^z s\ k^\theta n(k,s)^\gamma$$

$$- [g(k,s) - (1-\delta)k]\ Q[g(k,s) - (1-\delta)k]\ \}\ dx + \int (1-\delta)\ d(k,s)\ \ q\ \pi(s,0)\ dy \tag{3.18}$$

subject to:

$$\int n(k,s)\ dx \leq \eta \tag{3.19}$$

where the maximization is with respect to the function $n(k,s)$, and where g, x, d, and y are obtained from $(\underline{a}, \underline{A}, z, a^c, A^c)$ in the following way:

(i) The current investment decision rule are the ones implied by the current thresholds (a^c, A^c):

$$\begin{aligned} g(k,s) &= a^c(s), && \text{if } (1-\delta)k < a^c(s) \\ &= A^c(s), && \text{if } (1-\delta)k > A^c(s) \\ &= (1-\delta)k, && \text{otherwise} \end{aligned} \tag{3.20}$$

(ii) The (approximate) current measure across establishment types x is obtained by initializing this measure T periods before the current period (x_T) to be the deterministic steady state measure x^*, and updating it recursively by iterating on the law of motion given by:

$$x_{t-1}(B, s') = \int_{(k,s):g_t(k,s)\in B} \pi(s, s')\ dx_t\ +\ \upsilon\,\psi(s')\ \chi(0 \in B) \tag{3.21}$$

for $t = T, T-1, ..., 1$. The (approximate) current x is then given by x_0.

The investment decision rules (g_t), t periods away (for $t = T, T-1, ..., 1$), that are used in this law of motion are the ones implied by the corresponding thresholds (a_t, A_t) in the history $(\underline{a}, \underline{A})$:

$$\begin{aligned} g_t(k, s) &= a_t(s), && \text{if } (1-\delta)k < a_t(s) \\ &= A_t(s), && \text{if } (1-\delta)k > A_t(s) \\ &= (1-\delta)k, && \text{otherwise} \end{aligned} \tag{3.22}$$

(iii) The previous period measure across establishment types y, and the previous period decisions over current capital levels across establishment types are those returned as x_1 and g_1 in (ii).

Note that the Social Planner's problem in equation (3.14) has linear constraints, and that the deterministic steady state values for the (endogenous) state variables are all strictly positive. We can then perform a quadratic approximation to the return function about the deterministic steady state, leaving us with a standard linear quadratic (L-Q) problem which can be solved by ordinary value function iteration.[6]

Let now return to the question of how long the history of thresholds $(\underline{a}, \underline{A})$ should be to get a good approximate solution to the original problem (3.2). Appendix B shows that there exists a length J for thresholds histories such that solving by L-Q methods the planner's problem (3.14) corresponding to length J,

[6] The quadratic approximation is obtained by imposing zero errors of approximation of the return function at the grid points that lie just above and below the steady state grid points computed in the Appendix. The procedure to obtain numerical derivatives follows closely the one described in Kydland and Presott [13]. We are left with a standard linear quadratic problem if the return function is concave with respect to the thresholds (a, A). It hapens to be that the problem is actually not concave with respect to these variables. But if instead of directly working with the thresholds $(a(s), A(s))$ we work with a transformation $(a(s)^{\tau}, A(s)^{\tau})$ of these original variables, the problem does become concave. In all the experiments analyzed a value of $\tau = 0.999$ was sufficient to make the problem concave. I must thank Larry Jones for making me this suggestion.

gives exactly the same solution as solving by L-Q methods the planner's problem 3.14 corresponding to any other length $T > J$.[7] It follows that the only approximation error introduced by the solution method stems from applying L-Q methods and not from keeping track of a finite history of thresholds.

4. Calibration

This section describes the steady state observations used to calibrate the parameters of the model. In this section, the irreversibility parameter q will be assumed fixed at some particular value. Given a fixed q, the rest of the parameters we need to calibrate are $\beta, \theta, \gamma, \delta, \alpha, \upsilon, \psi(1), \lambda$, the transition matrix Π, and the parameters determining the driving process for the aggregate productivity shock: ρ and σ_ε^2.

The first issue we must address is what actual measure of capital will our model capital correspond to. Since we are interested in investment irreversibilities at the establishment level it seems natural to abstract from capital components such as land, residential structures and consumer durables. The empirical counterpart for capital was consequently identified with plant and equipment. As a result, investment was associated in the National Income and Product Accounts with non-residential investment. On the other hand, the empirical counterpart for consumption was identified with personal consumption expenditures in non-durable goods and services. Output was then defined to be the sum of these investment and consumption measures. The annual capital-output ratio and the investment-output ratio corresponding to these measures were found to be 1.7 and 0.15 respectively. The depreciation rate δ was selected to be consistent with these two magnitudes.

The annual interest rate was selected to be 4 per cent. This is a compromise between the average real return on equity and the average real return on short-term debt for the period 1889 to 1978 as reported by Mehra and Prescott [16]. The discount factor β was chosen to generate this interest rate at steady state. Given the interest rate i and the depreciation rate δ, the parameter θ was selected to match the capital-output ratio in the U.S. economy. The labor share parameter was in turn selected to replicate a labor share in National Income of 0.64 (this is the standard value used in the business cycle literature). On the other hand, the preference parameter α was picked such that 80% of the population works

[7]Moreover, J is easily determined and in all experiments performed below, it happens to be a relatively small number (never exceeded 45).

at steady state (roughly the fraction of the U.S. working age population that is employed).

The transition matrix Π was chosen to be of the following form:

$$\begin{pmatrix} 1 & 0 & 0 \\ \zeta & \phi(1-\zeta) & (1-\phi)(1-\zeta) \\ \zeta & (1-\phi)(1-\zeta) & \phi(1-\zeta) \end{pmatrix} \tag{4.1}$$

i.e. a process that treats the low and the high productivity shocks symmetrically. The rest of the parameters to calibrate are then $\phi, \zeta, \upsilon, \psi(1)$, and λ. The parameters ζ, ϕ, and λ were selected to reproduce important observations on job creation and job destruction reported in Davis and Haltiwanger [8]. These are: (i) that the average annual job creation rate due to births and the average annual job destruction rate due to deaths are both about 2.35%, (ii) that the average annual job creation rate due to continuing establishments and the average annual job destruction rate due to continuing establishments are both about 7.9%, and (iii) that about 82.3% of the jobs destroyed during a year are still destroyed the following year. The parameter determining the number of establishments being created every period was chosen so that the average establishment size in the model economy is about 65 employees, same magnitude as in the data.

Next, we must determine the distribution ψ over initial productivity shocks. If we would allow for a large number of possible idiosyncratic productivity shocks, it would be natural to chose a ψ to reproduce the same size distribution of establishments as in the data. With only two values for the idiosyncratic shocks this approach does not seem restrictive enough since we can pick any two arbitrary employment ranges in the actual size distribution to calibrate to. For this reason I chose to follow the same principle as in the choice of Π and pick $\psi = (0.5, 0.5)$, i.e. a distribution that treats the low and the high productivity shock symmetrically (note that these choices of Π and ψ imply that at steady state there will be as many establishments with the low shock as with the high shock).

Finally, we must determine values for ρ and σ_ε^2. The strategy for selecting values for these parameters was to chose them so that measured Solow residuals in the model economy replicate the behavior of measured Solow residuals in the data. Proportionate changes in measured Solow residual are defined as the proportionate changes in aggregate output minus the sum of the proportionate change in labor times the labor share γ, minus the sum of the proportionate change in capital times $(1-\gamma)$. Note that these changes in measured Solow residuals do not coincide with changes in the aggregate productivity variable z in the model (the aggregate production function in the model economy is not a constant returns Cobb-Douglas function in labor and aggregate capital). Using the measure of output described above and a share of labor of 0.64, measured Solow residuals (in the data) were found to be as highly persistent as in Prescott [18] but the standard deviation of technology changes came up somewhat smaller: 0.0063 instead of the usual 0.0076 value used in the literature. Given a fixed irreversibility parameter q and the rest of the parameters calibrated as above, values for ρ and σ_ε^2 were selected so that measured Solow residuals in the model economy displayed similar persistence and variability as in the data. It happened to be the case that values of $\rho = 0.95$ and $\sigma_\varepsilon^2 = 0.0063^2$ were consistent with these observations in all the experiments reported below.

Parameters values corresponding to economies with several different possible values for q are reported in Table 1.

5. Results

To have an idea of the quantitative behavior of our model, let first consider the economy with perfectly reversible investment ($q = 1$) as a benchmark case and analyze the business cycles that it generates. Table 2 reports summary statistics (standard deviations and correlations with output) for the aggregate fluctuations of this benchmark economy and compares them to those of the actual U.S. economy. Before any statistics were computed, all time series were logged and detrended using the Hodrick-Prescott filter. The statistics reported for the U.S. economy correspond to the output, investment and consumption measures described in the previous section, and refer to the period between 1960:3 and 1993:4. For the artificial economy, time series of a length of 136 periods (same as in the data) were computed for 100 simulations, the reported statistics being averages across these simulations. We see that the benchmark economy displays salient features of the U.S. business cycle. Output fluctuates about as much in the model economy as in actual data. Investment is about 5 times more variable than output in the model while it is about 4 times as variable in the U.S. economy. Consumption is less variable than output in both economies (though consumption is less variable in the model than in the U.S.). The variability of the aggregate stock of capital is about the same in both economies. On the other hand, hours variability is only 70% the variability of output in the model while they vary as much as output in the actual economy. Productivity fluctuates less in the model than in the U.S. economy. In terms of correlations with output, we see that almost all variables are highly procyclical both in the model and in U.S. data. The only exceptions are capital (which is acyclical both in the model and the actual economy) and productivity (which is highly procyclical in the model while it is acyclical in the data). In a broad sense, these features are similar to those commonly found in previous real business cycle models.

We turn next to the main question addressed in this paper, that is: what are the implications of investment irreversibilities for aggregate fluctuations? In terms of our model economy the question can be restated as follows: are the equilibrium fluctuations in our benchmark economy (perfect reversible investment) substantially different from those corresponding to an economy calibrated to an empirically plausible value of q?

Instead of arguing in favor of the empirical plausibility of any particular value for q, the strategy here will be to report results for different economies with q's in a wide range of values. Table 3 summarizes the equilibrium fluctuations of economies with q's ranging between 1 and 0. The results are striking. Irreversibilities tend to decrease the variability of output, investment and hours, and increase the variability of consumption.[8] However, these differences are suprisingly small. For example, the standard deviation of output decreases monotonically as q goes from 1 to 0 (as one would expect given the adjustment costs introduced), but it goes from 1.41 when $q = 1$ to only 1.39 when $q = 0$. This is a small difference considering that we are moving from the perfectly reversible case to the complete irreversibilities scenario. Overall, the properties of the business cycles generated by all these economies are extremely similar. We conclude that, at least in terms of the standard statistics which the real business cycle literature focuses on, investment irreversibilities at the plant level play no crucial role for aggregate dynamics.

Let now consider the importance of irreversibilities for plant level investment dynamics. Figure 2 shows the distribution of plant level gross investment rates (for continuing establishments) across all realizations, under different values for the irreversibility parameter q.[9] We observe that when there are no irreversibilities ($q = 1.0$), there is a large number of establishments which do not adjust their stock of capital (their net investment is zero), and there is a small number of firms

[8]Interestingly, Dow and Olson [11] found that when aggregate productivity shocks are variable enough, aggregate investment irreversibilites bind and output, consumption, investment and hours are affected in a qualitatively similar way as here.

[9]These histograms correspond to the same simulations as those underlying Table 3.

with both sharp increases and sharp decreases in their stock of capital.[10] On the contrary, when the irreversibility parameter q becomes zero: 1) the number of establishments displaying close-to-zero adjustment is larger, 2) there are no establishments with sizable negative investment rates, and 3) capital increases are not as sharp as they were under $q = 1.0$. It is interesting to note that when $q = 0$, although only 2% of the establishments have investment rates larger than 20%, they contribute to 38% of aggregate gross investment. Investment irreversibilities then bring the distribution of investment rates closer to the features emphasized by Doms and Dunne [10]. Also, note that the histogram of investment rates that arises when $q = 0.95$ is very similar to the one under $q = 0$. This is related to the finding by Abel and Eberly [1], that relatively small degrees of irreversibility give rise to similar plant level adjustments as under complete irreversibilities. From this figure, we see that even though irreversibilities are not important in terms of the standard RBC statistics, they appear to be crucial for establishment level investment dynamics.

A feature of the data which has been believed to have important implications for aggregate investment dynamics, is that the fraction of plants going through large investment episodes is positively correlated with aggregate investment (see Doms and Dunne [10], and Cooper, Haltiwanger and Power [7]). Table 4 reports (for the different economies) the correlations with aggregate investment of: 1) the fraction of plants with investment rates greater than 20%, and 2) the average investment rate among plants with investment rates greater than 20%. We see that for economies with low irreversibilities, the fraction of plants making large capital adjustment is uncorrelated with aggregate investment, while the average investment rate of those making large adjustment is positively correlated. The opposite is true in economies with large irreversibilities, bringing them closer to actual data in this dimension. Since all these economies look similar in terms of their RBC statistics we conclude this feature does not necesarily have major implications for aggregate dynamics.

As a matter of fact, the importance of plant level non-linearities for aggregate dynamics has not been emphasized in terms of the standard RBC statistics. Caballero and Engel [5] and Caballero, Engel and Haltiwanger [6] have stressed that for a given sequence of shocks, their statistical models keep track much better of actual aggregate investment when non-linear plant level adjustments are allowed for. They report that non-linearities are particularly important in periods of large

[10]This is due to the fact that only two (positive) idiosyncratic shocks are considered, and that they are very persistent.

departures of aggregate investment from its mean, being able to generate brisk expansions and (to a lesser extent) sharp contractions. Figure 3 explores this argument. It reports the histograms for the deviations of aggregate investment from trend across all realizations, for the economies with $q = 1$ and $q = 0$. We see from the pictures that investment irreversibility at the plant level does not generate noticeable asymmetries nor lumpiness in aggregate investment behavior. Figure 4 goes one step further. It reports the realizations of aggregate investment for these two economies, which arise from feeding into them the empirical realization of Solow residuals for the period 1960:1 to 1993:4.[11] We observe that the time series for aggregate investment originated by the economy with large irreversibilities is virtually identical to the one generated by the economy with perfectly reversible investment.

Finally, Figure 5 shows the impulse response functions for output (Y), consumption (c), investment (I), and labor (η) to a one-time aggregate productivity shock of one standard deviation, which correspond to the economies with $q = 1$ and $q = 0$. It shouldn't be surprising by now that they look almost exactly the same: the business cycles generated by economies with different degrees of irreversibilities are extremely similar.

We'll now give a closer look to the lack of aggregate asymmetries associated with investment irreversibilities. For the economy with $q = 0.99$, Figure 6 shows the impulse response of the capital support of the distribution x_t to a one-time aggregate productivity shock of one standard deviation, starting from the steady state support (the steady state capital distribution x^* is displayed in Figure 7).[12] We see that in response to a positive shock, the thresholds $a(1)$, $A(1)$ and $a(2)$ increase on impact, continue to increase for a number of periods and eventually decrease, returning gradually to their steady state levels (not shown). Instead, the capital levels pertaining to the range of inactivity between $a(1)$ and $A(1)$ are not affected on impact. They follow the same dynamics as the upper threshold $A(1)$ but with a lag, which depends on the number of periods it takes $A(1)$ to depreciate to the corresponding capital level. Note that the support of the distri-

[11] There are 136 periods of observations for the Solow residuals. In this experiment, the distribution across establishment types is inititalized to be the deterministic steady state distribution. Then, 136 periods are generated from the model economy but only the last 85 periods are reported in the figure. As a consequence, the effects of initializing with the deterministic steady state distribution vanish out.

[12] We chose to show the behavior of the economy with $q = 0.99$ over those with a smaller q, since it has a relatively small capital support (simplifying the figures considerably). However, similar patterns can be found in the other economies.

bution responds symmetrically to positive and negative shocks. Since the state of the economy behaves symmetrically, it is not surprising that the business cycles generated by these shocks will inherit similar features.[13]

It seems safe to conjecture that this symmetry would be lost if aggregate shocks had an (empirically implausible) large variance. To be concrete, let consider how the largest point in the capital support would respond to a large negative shock, starting from its steady state value $a^*(2)$. Suppose that the shock is so low that the threshold $a(2)$ decreases on impact below $(1-\delta)a^*(2)$. The highest point in the support would then become $(1-\delta)a^*(2)$, since it would fall in the range of inactivity defined by the new value of $a(2)$. What is important to note is that negative shocks of larger magnitude would generate no further effects on impact, since $(1-\delta)a^*(2)$ would still fall in a range of inactivity. On the contrary, there would be no counterpart to this lack of further responsiveness when shocks are positive. If a positive shock drives $a(2)$ above $(1-\delta)a^*(2)$, the highest point in the support would always jump on impact to the new value of $a(2)$. This would be true no matter how large the positive shock is.

Figure 8 illustrates these ideas by showing the impulse responses of the highest point in the capital support to one-time aggregate shocks, ranging from one to twenty standard deviations in magnitude.[14] Let consider the responses in period one to each of these shocks. We see that when shocks are negative, the largest capital level in the support moves to smaller values as the shock becomes larger. However, once the shock reaches fifteen standard deviations, it stops responding to further shocks. On the contrary, when shocks are positive, this capital level always moves to higher values as the shock gets larger. This pattern of response opens interesting possibilities for the creation of asymmetries in aggregate fluctuations. In particular, it suggests that aggregate investment would tend to decrease slowly in response to large negative shocks, and increase sharply in response to large positive shocks.[15]

[13]Stictly speaking, describing the response of the capital support is not enough. The number of establishments at each of these capital levels and idiosyncratic productivity shocks should also be considered. However, at any point in time, the number of establishments at each of these capital levels can be read directly from the corresponding point in the steady state distribution in Figure 7. The reason is that the process for the idyosincratic shocks is exogenous and the paths illustrated in Figure 6 do not cross.

[14]This figure is drawn only for heuristic purposes. If shocks were as large as those shown, the linear quadratic aproximation performed in the paper would probably be of poor quality.

[15]It should be clear that $a(1)$ would generate similar asymmetries, since it would mimic the behavior of $a(2)$. The analysis would be somewhat more complicated though, since capital levels

In view of these arguments, we must view the lack of asymmetries the theory predicts as arising purely from measurement. Measured solow residuals are not variable enough for investment irreversibilities to create asymmetries in aggregate business cycles: the associated fluctuations in capital thresholds are too small compared with the drift introduced by depreciation.[16]

A. Appendix

This appendix describes the algorithm used to compute the steady state of the deterministic version of the economy. We will show that the problem is reduced to solving one equation in one unknown (after the relevant substitutions have been made). First, it must be noticed that (similarly to the neoclassical growth model) the steady state interest rate is given by:

$$1 + i = \frac{1}{\beta} \tag{A.1}$$

Fixing the wage rate at an arbitrary value w, the value of the different types of establishments (as a function of w) can be obtained by solving the following functional equation:

$$J(k, s; w) = MAX \ \{ \ s \ k^{\theta} n^{\gamma} - wn - [k' - (1-\delta)k] \, Q \, [k' - (1-\delta) \, k]$$

in the lower portion of the range of inactivity would be affected by large fluctuations in $a(1)$. In particular, these points would collapse into $a(1)$ under sufficiently large increases in $a(1)$, but will not be affected when $a(1)$ decreases. This effect would tend to reinforce the asymmetries described above.

The behavior of $A(1)$ seems to work against these arguments. The capital levels in the upper portion of the range of inactivity would collapse into $A(1)$ under a sufficiently large decrease in $A(1)$, but will not be affected when $A(1)$ increases. However, when q becomes less than 0.9 the ranges of inactivity overlap and no establishment has a capital level close to $A(1)$. For these higher degree of irreversibilities, the behavior of $A(1)$ becomes irrelevant for aggregate fluctuations.

[16]Actually in none of the simulations reported, the rate of change of thresholds ever exceeded the rate of depreciation. This result is closely related to Dow and Olson [11]. They found that in the real business cycles model of Hansen [14], aggregate irreversibilities play no role since productivity shocks are not variable enough to make the non-negativity constraint in aggregate investment binding. An important difference in this paper is that plant level irreversibilities do bind. However, they bind due to the amount of idiosyncratic risk that establishments face, not because of the level of aggregate uncertainty in the economy. Aggregate productivity shocks play only a minor role.

$$+\frac{1}{1+i}\sum_{s'} J\left(k', s'; w\right)\pi\left(s, s'\right)\} \tag{A.2}$$

The solution to this problem is computed using standard recursive methods. Note that the solution to this problem also gives the decision rules $n(k,s;w)$ and $g(k,s;w)$ as a function of w.

Given a w and the corresponding $g(k,s;w)$, a measure $x(w)$ across productivity shocks and capital levels can be obtained from the law of motion for x:

$$x\left(B, s'; w\right) = \int_{(k,s):g(k,s;w)\in B} \pi\left(s, s'\right)\ dx(w) + v\ \psi\left(s'\right)\ \chi\left(0 \in B\right) \tag{A.3}$$

In practice, this is solved by iterating on this law of motion starting from an arbitrary initial guess for $x(w)$.

Once a $x(w)$ is obtained and given the previous $n(k,s;w)$ and $g(k,s;w)$ found, we can solve for the corresponding consumption $c(w)$ implied by the feasibility condition:

$$c\left(w\right) = \int\ s\ k^{\theta} n\left(k, s; w\right)^{\gamma} - \left[g\left(k, s; w\right) - (1-\delta)k\right]\ Q\left[g\left(k, s; w\right) - (1-\delta)k\right]\ dx(w)$$

$$+\int (1-\delta)\ g\left(k, s; w\right)\ q\ \pi\left(s, 0\right)\ dx(w) \tag{A.4}$$

A wage rate w corresponds to the steady state value if the marginal rate of substitution between consumption and leisure is satisfied, i.e.:

$$c(w) = \frac{w}{\alpha} \tag{A.5}$$

This is one equation in one unknown and is solved using standard root finding methods.

The actual computer implementation of this algorithm requires working with a finite grid of capital levels. In all experiments reported in the paper, the number of grid points were between 1,000 and 1,800.

B. Appendix

This appendix shows that, when the planner's problem (3.14) is solved by L-Q methods, carrying a finite history of thresholds $(a_t, A_t)_{t=1}^{T}$ in the state vector leads to exactly the same solution as carrying an infinite history.

Define h to be the vector $\left[z, \eta, (a^c, A^c), (a_t, A_t)_{t=1}^{T}\right]$, and h^* the corresponding deterministic steady state values. Let $r(h) = \ln[c(h)] - \alpha\,\eta$ be the return function to the Social Planner's Problem (3.14). Following Kydland and Prescott [13], the quadratic approximation to $r(h)$ is given by:

$$R(h) = r(h^*) + b'(h - h^*) + (h - h^*)'Q\;(h - h^*) \tag{B.1}$$

where the elements of b and Q are given by:

$$b_i = \frac{r(h^* + h^i) - r(h^* - h^i)}{2h_i} \tag{B.2}$$

$$q_{i,i} = \frac{r(h^* + h^i) - r(h^*) + r(h^* - h^i) - r(h^*)}{2h_i^2} \tag{B.3}$$

$$q_{i,j} = \frac{r(h^* + h^i + h^j) - r(h^* + h^i - h^j) - r(h^* - h^i + h^j) + r(h^* - h^i - h^j)}{8h_i h_j} \tag{B.4}$$

and where h^i is a vector with all components equal to zero, except for its *ith* component which is equal to $h_i > 0$, a small number.

To simplify notation, we'll assume that the idiosyncratic shock s takes values 1 and 2. In addition, we'll suppose that $(1-\delta)\,a^*(2) < A^*(1)$, where (a^*, A^*) are steady state values.[17] Noting that $a^*(1) < a^*(2)$, define J to be the smallest natural number such that:

$$(1-\delta)^J a^*(2) < a^*(1) \tag{B.5}$$

Suppose that $T > J + 1$. We'll show that the coefficients of b and Q corresponding to the thresholds $(a_t, A_t)_{t=J+2}^{T}$ are all equal to zero. Before we proceed, it will be useful to show two Lemmas.

Lemma B.1. *The capital support for the steady state measure x^* is:*

$$K^* = \{0,\; a^*(1)\} \bigcup_{j=0}^{J-1} \left\{(1-\delta)^j\, a^*(2)\right\} \tag{B.6}$$

[17] The other case can be handled along similar lines.

Proof. Since establishments are borned with $k = 0$, we know that $\{0\} \subset K^*$.

Depending on their current productivity shock s, establishments with $k = 0$ chose $k' = a^*(s)$. It follows that $\{0,\ a^*(1),\ a^*(2)\} \subset K^*$.

We'll proceed by induction. Suppose that K^* includes the set:

$$K_N = \{0,\ a^*(1)\} \bigcup_{j=0}^{N} \left\{(1-\delta)^j\, a^*(2)\right\} \tag{B.7}$$

for some N (zero or natural) such that $(1-\delta)^N a^*(2) > a^*(1)$. We'll show that:

(1) if $(1-\delta)^{N+1} a^*(2) > a^*(1)$, then K^* includes the set:

$$K_{N+1} = K_N \cup \left\{(1-\delta)^{N+1} a^*(2)\right\} \tag{B.8}$$

(2) if $(1-\delta)^{N+1} a^*(2) \leq a^*(1)$, then $K^* = K_N$.

Case 1: Suppose that $(1-\delta)^{N+1} a^*(2) > a^*(1)$.

An establishment of type (k, s), with $k \in K_N$, will chose k' as follows:

(i) Suppose that $s = 1$:

If $k \leq a^*(1)$, then $k' = a^*(1)$.

If $k = (1-\delta)^j\, a^*(2)$ for some $0 \leq j \leq N$, then $a^*(1) < (1-\delta)k < A^*(1)$. It follows that $k' = (1-\delta)^{j+1} a^*(2)$.

Note that for $k = (1-\delta)^N\, a^*(2) : k' = (1-\delta)^{N+1} a^*(2) \notin K_N$.

(ii) Suppose that $s = 2$:

For every $k \in K_N : (1-\delta)\ k < a^*(2)$. Hence, $k' = a^*(2) \in K_N$.

Since $K_N \subset K^*$, (i) and (ii) imply: $K_{N+1} = K_N \cup \left\{(1-\delta)^{N+1} a^*(2)\right\} \subset K^*$.

Case 2: Suppose that $(1-\delta)^{N+1} a^*(2) \leq a^*(1)$.

The analysis in case 1) applies, except for $k = (1-\delta)^N a^*(2)$ and $s = 1$. We now have that $(1-\delta)\ k \leq a^*(1)$ and consequently, that $k' = a^*(1) \in K_N$. It follows that $K_N = K^*$ ■

Let $\left\{x_t^h\right\}$ for $t = T, T-1, ..., 0$, be the sequence of measures which satisfies equation (3.21) for the history of thresholds $(a_t, A_t)_{t=1}^T$ given in h. More precisely:

(i) $x_T^h = x^*$

(ii) for $t = T, T-1, ..., 1$, x_{t-1}^h is given by:

$$x_{t-1}^h(k', s') = \sum_{(k,s) \in K_t \times S : g_t^h(k,s) = k'} \pi(s, s')\ x_t^h(k, s) + v\ \psi(s')\ \chi(k' = 0) \tag{B.9}$$

where:

$$
\begin{aligned}
g_t^h(k,s) &= a_t(s), && \text{if } (1-\delta)k < a_t(s) \\
&= A_t(s), && \text{if } (1-\delta)k > A_t(s) \\
&= (1-\delta)k, && \text{otherwise}
\end{aligned}
\tag{B.10}
$$

and K_t is the (finite) capital support of x_t^h.

Noting that we'll be interested in histories of the form $h = h^* + h^i$, we can now proceed to our next Lemma.

Lemma B.2. *Let $h = h^* + h^i$. Suppose that i corresponds to a lower threshold $a_j(2)$, where $J+1 < j \le T$. Suppose that for some $1 \le t \le J$, x_{j-t}^h is given by:*

$$
\begin{aligned}
x_{j-t}^h\left((a^*(2)+h_i)(1-\delta)^{t-1}, s\right) &= x^*\left(a^*(2)(1-\delta)^{t-1}, s\right) \\
x_{j-t}^h\left(a^*(2)(1-\delta)^{t-1}, s\right) &= 0 \\
x_{j-t}^h(k,s) = x^*(k,s), &\quad \textit{for every other } k
\end{aligned}
\tag{B.11}
$$

Then,
1) if $a^*(2)(1-\delta)^t < a^*(1)$: $x_{j-t-1}^h = x^*$.
2) if $a^*(2)(1-\delta)^t \ge a^*(1)$, x_{j-t-1}^h satisfies:

$$
\begin{aligned}
x_{j-t-1}^h\left((a^*(2)+h_i)(1-\delta)^{t}, s\right) &= x^*(a^*(2)(1-\delta)^{t}, s) \\
x_{j-t-1}^h\left(a^*(2)(1-\delta)^{t}, s\right) &= 0 \\
x_{j-t-1}^h(k,s) = x^*(k,s), &\quad \textit{for every other } k
\end{aligned}
\tag{B.12}
$$

Proof. Note that $g_{j-t}^h = g^*$. Then, x_{j-t-1}^h is given by:

$$
\begin{aligned}
x_{j-t-1}^h(k',s') = {} & \upsilon\psi(s')\,\chi(k'{=}0) + \sum_{(k,s)\in\{K^*\setminus\{a^*(2)(1-\delta)^{t-1}\}\}\times S : g^*(k,s)=k'} \pi(s,s')\,x^*(k,s) \\
& + \pi(1,s')\,x_{j-t}^h\left((a^*(2)+h_i)(1-\delta)^{t-1},1\right)\chi\left(k' = g^*\left((a^*(2)+h_i)(1-\delta)^{t-1},1\right)\right) \\
& + \pi(2,s')\,x_{j-t}^h\left((a^*(2)+h_i)(1-\delta)^{t-1},2\right)\chi\left(k' = g^*\left((a^*(2)+h_i)(1-\delta)^{t-1},2\right)\right)
\end{aligned}
\tag{B.13}
$$

Also note that x^* satisfies:

$$x^*\left(k',s'\right)=\upsilon\psi\left(s'\right)\chi\left(k'{=}0\right)+\sum_{(k,s)\in\{K^*\setminus\{a^*(2)(1-\delta)^{t-1}\}\}\times S:g^*(k,s)=k'}\pi\left(s,s'\right)x^*(k,s)$$

$$+\ \pi\left(1,s'\right)x^*\left(a^*(2)(1-\delta)^{t-1},1\right)\chi\left(k'=g^*\left(a^*(2)(1-\delta)^{t-1},1\right)\right)$$

$$+\ \pi\left(2,s'\right)x^*\left(a^*(2)(1-\delta)^{t-1},2\right)\chi\left(k'=g^*\left(a^*(2)(1-\delta)^{t-1},2\right)\right)\tag{B.14}$$

Since h_i is small: $a^*(2)(1-\delta)^t<(a^*(2)+h_i)(1-\delta)^t<a^*(2)$. Therefore,

$$g^*\left((a^*(2)+h_i)(1-\delta)^{t-1},2\right)=g^*\left(a^*(2)(1-\delta)^{t-1},2\right)=a^*(2)\tag{B.15}$$

From (B.11) and (B.15), it follows that the last term in (B.13) is identical to the last term in (B.14).

Case 1): $a^*(2)(1-\delta)^t<a^*(1)$.

Since h_i is small: $a^*(2)(1-\delta)^t<(a^*(2)+h_i)(1-\delta)^t<a^*(1)$. Therefore,

$$g^*\left((a^*(2)+h_i)(1-\delta)^{t-1},1\right)=g^*\left(a^*(2)(1-\delta)^{t-1},1\right)=a^*(1)\tag{B.16}$$

From (B.11) and (B.16), it follows that the third term in (B.13) is identical to the third term in (B.14). Hence, $x^h_{j-t-1}=x^*$.

Case 2): $a^*(2)(1-\delta)^t\geq a^*(1)$.

Note that: $a^*(1)\leq a^*(2)(1-\delta)^t<(a^*(2)+h_i)(1-\delta)^t$. Therefore:

$$g^*\left((a^*(2)+h_i)(1-\delta)^{t-1},1\right)=(a^*(2)+h_i)(1-\delta)^t\tag{B.17}$$

$$g^*\left(a^*(2)(1-\delta)^{t-1},1\right)=a^*(2)(1-\delta)^t\tag{B.18}$$

a) Suppose that $k'\notin\{(a^*(2)+h_i)(1-\delta)^t,a^*(2)(1-\delta)^t\}$.

Then, the third terms of both (B.13) and (B.14) are equal to zero. It follows that: $x^h_{j-t-1}(k,s)=x^*(k,s)$.

b) Suppose that $k'=a^*(2)(1-\delta)^t$.

Then, all terms in (B.13) are zero. It follows that: $x^h_{j-t-1}\left(a^*(2)(1-\delta)^t,s\right)=0$.

c) Suppose that $k' = (a^*(2) + h_i)(1-\delta)^t$.
From (B.13) it follows that $x^h_{j-t-1}(k', s')$ is given by:

$$\pi(1, s')\, x^h_{j-t}\left((a^*(2) + h_i)(1-\delta)^{t-1}, 1\right) \chi\left(k' = g^*\left((a^*(2) + h_i)(1-\delta)^{t-1}, 1\right)\right) \tag{B.19}$$

Note from (B.14) that $x^*(a^*(2)(1-\delta)^t, s')$ is given by:

$$\pi(1, s')\, x^*\left(a^*(2)(1-\delta)^{t-1}, 1\right) \chi\left(a^*(2)(1-\delta)^t = g^*\left(a^*(2)(1-\delta)^{t-1}, 1\right)\right) \tag{B.20}$$

From (B.11), (B.17), (B.18), (B.19), and (B.20), we have that:
$x^h_{j-t-1}((a^*(2) + h_i)(1-\delta)^t, s) = x^*(a^*(2)(1-\delta)^t, s)$. ■
We are now ready to proceed to the main result of this appendix.

Proposition B.3. Let $i > \dim\left[z, \eta, (a^c, A^c), (a_t, A_t)_{t=1}^{J+1}\right]$. Then,[18]

$$b_i = q_{ii} = 0 \tag{B.21}$$

Proof. From equations (B.2) and (B.3), it suffices to show that:

$$r\left(h^* + h^i\right) = r\left(h^* - h^i\right) = r\left(h^*\right) \tag{B.22}$$

Let $h = h^* + h^i$. We'll prove that $r(h) = r(h^*)$.[19] It will suffice to show that $x^h_1 = x^*$. To see why, suppose that $x^h_1 = x^*$. Since i corresponds to a capital threshold more than one period in the past: $(a_1, A_1) = (a^*, A^*)$. Equation (B.9) then implies that $x^h_0 = x^*$. Since $(a^c, A^c) = (a^*, A^*)$ and $\eta = \eta^*$, equation (3.18) implies that both h and h^* lead to identical consumption levels.

Assume w.l.o.g. that i corresponds to some lower threshold $a_j(\overline{s})$, for some period j (where $J + 1 < j \leq T$) and some shock $\overline{s}$.[20] Since $(a_t, A_t) = (a^*, A^*)$, for

[18] For reasons of space, we will omit the proof that $q_{i,j} = 0$ for every j. Such a proof would follow similar arguments as here.

[19] To show that $r\left(h - h^i\right) = r\left(h^*\right)$ is exactly analogous.

[20] If i corresponds to some upper threshold, it would be easy to show that $x^h_1 = x^*$. The reason is that the deterministic capital support K^* is finite, contains no upper thresholds and h^i is small. Making small perturbations to an upper threshold will not affect the actual investment of any steady state establishment.

every t such that $j < t \leq T$, it follows that $x^h_j = x^*$. We'll proceed to find x^h_{j-1}. Note that: (i) $g^h_j(k,s) = g^*(k,s)$, for every k and every $s \neq \overline{s}$, and (ii) :

$$\begin{aligned} g^h_j(k,\overline{s}) &= a^*(\overline{s}) + h_i, && \text{if } (1-\delta)k < a^*(\overline{s}) + h_i \\ &= A^*(\overline{s}), && \text{if } (1-\delta)k > A^*(\overline{s}) \\ &= (1-\delta)k, && \text{otherwise} \end{aligned} \tag{B.23}$$

Then, x^h_{j-1} is given as follows:

$$\begin{aligned} x^h_{j-1}(k',s') &= \upsilon\,\psi(s')\;\chi(k'=0) + \sum_{(k,s)\in K^*\times\{S\backslash\{\overline{s}\}\}:g^*(k,s)=k'} \pi(s,s')\;x^*(k,s) \\ &\quad + \sum_{k\in K^*:g^h_j(k,\overline{s})=k'} \pi(\overline{s},s')\;x^*(k,\overline{s}) \end{aligned} \tag{B.24}$$

Also, note that x^* satisfies:

$$\begin{aligned} x^*(k',s') &= \upsilon\,\psi(s')\;\chi(k'=0) + \sum_{(k,s)\in K^*\times\{S\backslash\{\overline{s}\}\}:g^*(k,s)=k'} \pi(s,s')\;x^*(k,s) \\ &\quad + \sum_{k\in K^*:g^*(k,\overline{s})=k'} \pi(\overline{s},s')\;x^*(k,\overline{s}) \end{aligned} \tag{B.25}$$

Noticing that (B.24) and (B.25) differ only in their third terms, we'll proceed to characterize x^h_{j-1} in terms of x^*.

Case 1): $k' < a^*(\overline{s})$ or $k' > a^*(\overline{s}) + h_i$.

Since h_i is small: $\left\{k \in K^* : g^h_j(k,\overline{s}) = k'\right\} = \{k \in K^* : g^*(k,\overline{s}) = k'\}$. Hence, the third terms in (B.24) and (B.25) are the same. Consequently, $x^h_{j-1}(k',s') = x^*(k',s')$.

Case 2): $a^*(\overline{s}) < k' < a^*(\overline{s}) + h_i$:

First, note that: $\left\{k \in K^* : g^h_j(k,\overline{s}) < a^*(\overline{s}) + h_i\right\} = \emptyset$. Since $a^*(\overline{s}) \in K^*$ and K^* is finite, h_i can always be chosen small enough so that no k' in the interval $(a^*(\overline{s}), a^*(\overline{s}) + h_i)$ belongs to K^*. It follows that: $\{k \in K^* : g^*(k,\overline{s}) = k'\} = \emptyset$. Consequently: $x^h_{j-1}(k',s') = x^*(k',s')$.

Case 3): $k' = a^*(\overline{s})$

Note that: $\left\{k \in K^* : g^h_j(k,\overline{s}) < a^*(\overline{s}) + h_i\right\} = \emptyset$. Therefore, the third term in (B.24) is zero. Since $a^*(\overline{s}) > 0$, it follows that the first term in (B.24) is zero. In what follows, we'll argue that the second term in (B.24) is also zero.

Suppose first that $\overline{s} = 1$ and $s = 2$. Then, for every $k \in K^* : g^*(k,s) > a^*(\overline{s}) = k'$

Suppose now that $\overline{s} = 2$ and that $s = 1$. Suppose that there exists a $k \in K^*$ such that $g^*(k,1) = a^*(2)$. Then, it must be true that $k = \frac{a^*(2)}{(1-\delta)} \in K^*$. A contradiction.

It follows that: $\{(k,s) \in K^* \times \{S \backslash \{\overline{s}\}\} : g^*(k,s) = k'\} = \emptyset$.

Therefore, $x^h_{j-1}(a^*(\overline{s}), s') = 0$.

Case 4): $k' = a^*(\overline{s}) + h_i$:

Case 3) showed that:

$$\{(k,s) \in K^* \times \{S \backslash \{\overline{s}\}\} : g^*(k,s) = a^*(\overline{s})\} = \emptyset \tag{B.26}$$

Using identical arguments as in case 3), we can also conclude that:

$$\{(k,s) \in K^* \times \{S \backslash \{\overline{s}\}\} : g^*(k,s) = a^*(\overline{s}) + h_i\} = \emptyset \tag{B.27}$$

Note that, since h_i is small and K^* is finite:

$$\left\{k \in K^* : g^h_j(k,\overline{s}) = a^*(\overline{s}) + h_i\right\} = \{k \in K^* : g^*(k,\overline{s}) = a^*(\overline{s})\} \tag{B.28}$$

Also, since $a^*(\overline{s}) + h_i > a^*(\overline{s}) > 0$, we have that:

$$\chi\left(a^*(\overline{s}) + h_i = 0\right) = \chi\left(a^*(\overline{s}) = 0\right) = 0 \tag{B.29}$$

From equations (B.24), (B.25),(B.26), (B.27), (B.28), and (B.29), it follows that: $x^h_{j-1}(a^*(\overline{s}) + h_i, s') = x^*(a^*(\overline{s}), s')$.

From cases 1) through 4), we conclude that for every s':

$$\begin{aligned} x^h_{j-1}(a^*(\overline{s}) + h_i, s') &= x^*(a^*(\overline{s}), s') \\ x^h_{j-1}(a^*(\overline{s}), s') &= 0 \\ x^h_{j-1}(k, s') &= x^*(k, s'), \text{ for every other k.} \end{aligned} \tag{B.30}$$

If $\overline{s} = 2$, Lemma B.2 applies (for $t = 1$). It follows that $x^h_{j-J-1} = x^*$. Since $(a_t, A_t) = (a^*, A^*)$ for $t = 1, ..., j - J - 1$, we have that $x^h_1 = x^*$.

Suppose that $\overline{s} = 1$. We'll show that $x^h_{j-2} = x^*$. Since $g^h_{j-1} = g^*$, then:

$$x^h_{j-2}\left(k',s'\right) \;=\; \upsilon\,\psi\left(s'\right)\;\chi\left(k'=0\right)+\sum_{(k,s)\in\{K^*\setminus\{a^*(\overline{s})\}\}\times S:g^*(k,s)=k'}\pi\left(s,s'\right)\;x^*(k,s)$$

$$\sum_s \pi\left(s,s'\right)\;x^h_{j-1}(a^*(\overline{s})+h_i,s)\;\chi\left[g^*\left(\left(a^*(\overline{s})+h_i,s\right)\right)=k'\right] \quad \text{(B.31)}$$

Note that $x^h_{j-1}(a^*(\overline{s})+h_i,s)=x^*(a^*(\overline{s}),s)$ for every s. Then suffices to show that for every s:

$$g^*\left(a^*(\overline{s})+h_i,s\right)=g^*\left(a^*(\overline{s}),s\right) \quad \text{(B.32)}$$

Consider first that $s=\overline{s}=1$. Since h_i is small, we have that:

$$(1-\delta)a^*(\overline{s})<(1-\delta)(a^*(\overline{s})+h_i)<a^*(\overline{s}) \quad \text{(B.33)}$$

Then, $g^*\left(a^*(\overline{s}),s\right)=g^*\left(a^*(\overline{s})+h_i,s\right)=a^*(\overline{s})$.

Suppose now that $s=2$ (and $\overline{s}=1$). Since h_i is small:

$$(1-\delta)a^*(\overline{s})<(1-\delta)(a^*(\overline{s})+h_i)<a^*(s) \quad \text{(B.34)}$$

Then, $g^*\left(a^*(\overline{s}),s\right)=g^*\left(a^*(\overline{s})+h_i,s\right)=a^*(s)$.

Therefore, $x^h_{j-2}=x^*$. Since $(a_t,A_t)=(a^*,A^*)$ for $t=1,...,j-2$, it follows that $x^h_1=x^*$ ∎

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TABLE 1

Parameter Values

	q = 1.0	q = 0.99	q = 0.95	q = 0.90	q = 0.75	q = 0.50	q = 0.00
β	0.99	0.99	0.99	0.99	0.99	0.99	0.99
θ	0.2186	0.2185	0.2179	0.2174	0.2173	0.2172	0.2168
γ	0.64	0.64	0.64	0.64	0.64	0.64	0.64
δ	0.02206	0.02184	0.02163	0.02146	0.02066	0.01914	0.01622
α	0.94	0.94	0.94	0.94	0.94	0.94	0.94
ν	7.257E-5	7.260E-5	7.257E-5	7.273E-5	7.278E-5	7.280E-5	7.287E-5
$\psi(1)$	0.5	0.5	0.5	0.5	0.5	0.5	0.5
λ	1.112	1.127	1.195	1.208	1.210	1.213	1.220
ϕ	0.92	0.92	0.92	0.92	0.92	0.92	0.92
ζ	0.00593	0.00593	0.00593	0.00593	0.00593	0.00593	0.00593
ρ	0.95	0.95	0.95	0.95	0.95	0.95	0.95
σ_ε^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2	0.0063^2

TABLE 2

U.S. and benchmark fluctuations

	U.S. Economy (60:1-93:4)		**Reversible Investment Economy**	
	Std. Deviation	Correlation	Std. Deviation	Correlation
Output	1.33	1.00	1.41	1.00
Consumption	0.87	0.91	0.49	0.91
Investment	4.99	0.91	7.09	0.98
Capital	0.63	0.04	0.51	0.08
Hours	1.42	0.85	0.98	0.98
Productivity	0.76	-0.16	0.49	0.91

TABLE 3

Business cycles across economies

Standard Deviations:

	q = 1.0	q = 0.99	q = 0.95	q = 0.90	q = 0.75	q = 0.50	q = 0.00
Output	1.41	1.40	1.39	1.39	1.39	1.39	1.39
Cons.	0.49	0.50	0.51	0.51	0.51	0.51	0.52
Investm.	7.09	7.01	6.89	6.85	6.83	6.83	6.82
Capital	0.51	0.50	0.49	0.49	0.49	0.49	0.48
Hours	0.98	0.97	0.95	0.94	0.94	0.94	0.94
Product.	0.49	0.50	0.51	0.51	0.51	0.51	0.52

Correlations with Output:

	q = 1.0	q = 0.99	q = 0.95	q = 0.90	q = 0.75	q = 0.50	q = 0.00
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Cons.	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Investm.	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Capital	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Hours	0.98	0.98	0.98	0.98	0.98	0.98	0.98
Product.	0.91	0.91	0.91	0.92	0.92	0.92	0.92

TABLE 4

Correlations with I_t:

Model	Fraction of plants with i/k > 0.20	Average i/k across plants with i/k > 0.20
q = 1.00	0.00	0.75
q = 0.99	0.01	0.61
q = 0.95	0.49	-0.07
q = 0.90	0.39	-0.12
q = 0.75	0.38	-0.13
q = 0.50	0.36	-0.15
q = 0.00	0.31	-0.17

FIGURE 1
Decision Rules

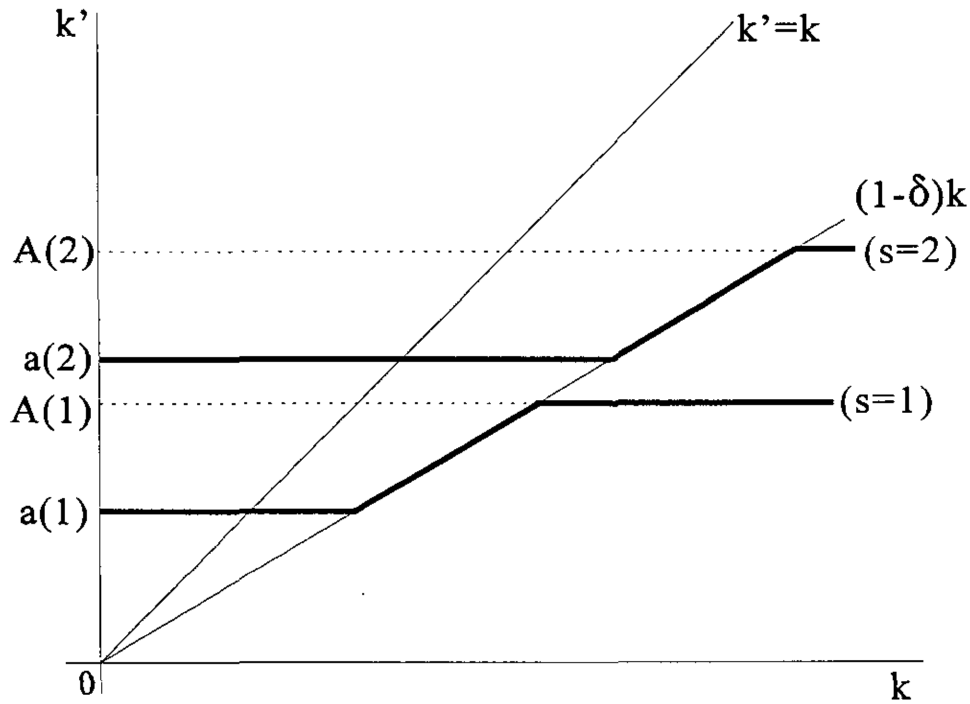
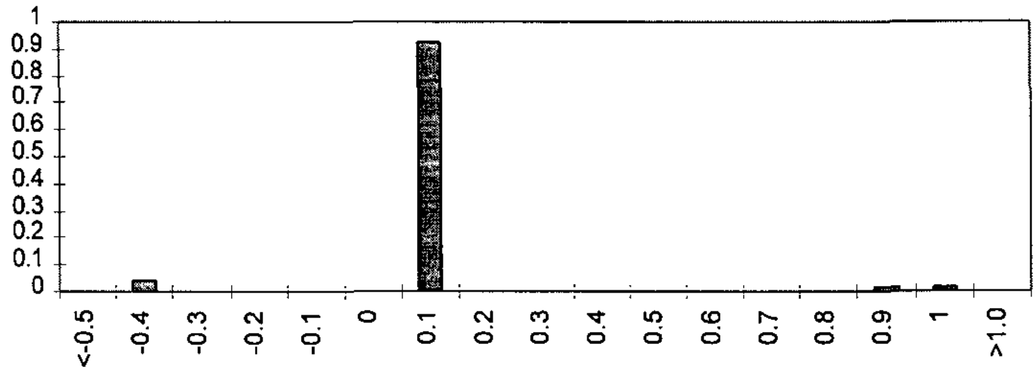
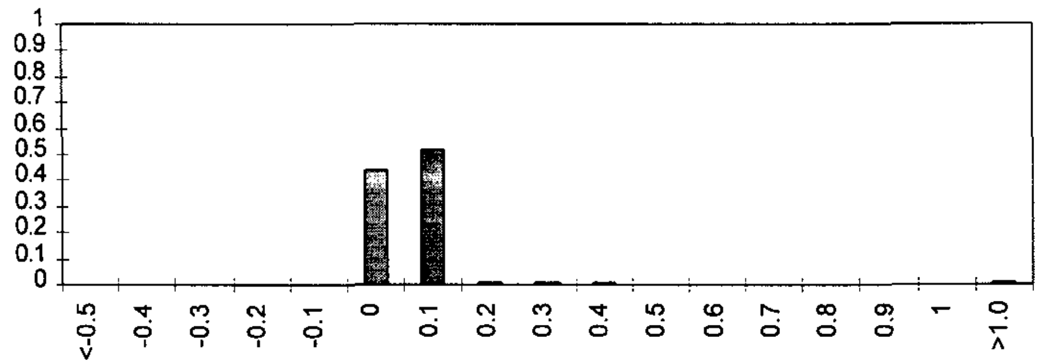


FIGURE 2
Investment Rate Distributions

(q=1.0)



(q=.95)



(q=.00)

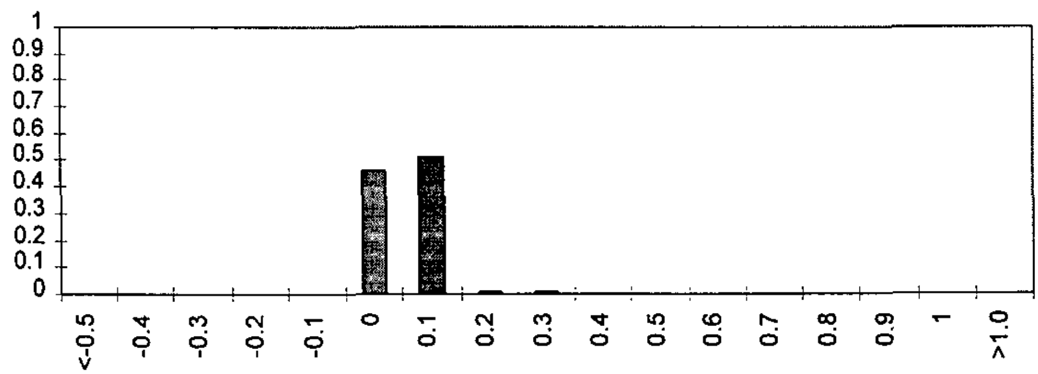
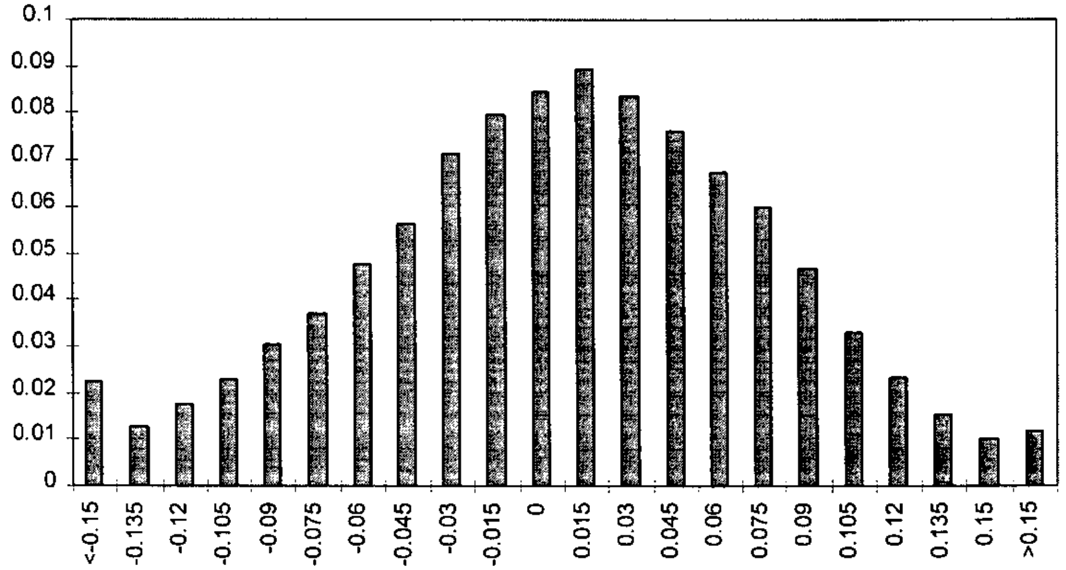


FIGURE 3
Distributions of Investment Deviations from Trend

(q=1.0)



(q=.00)

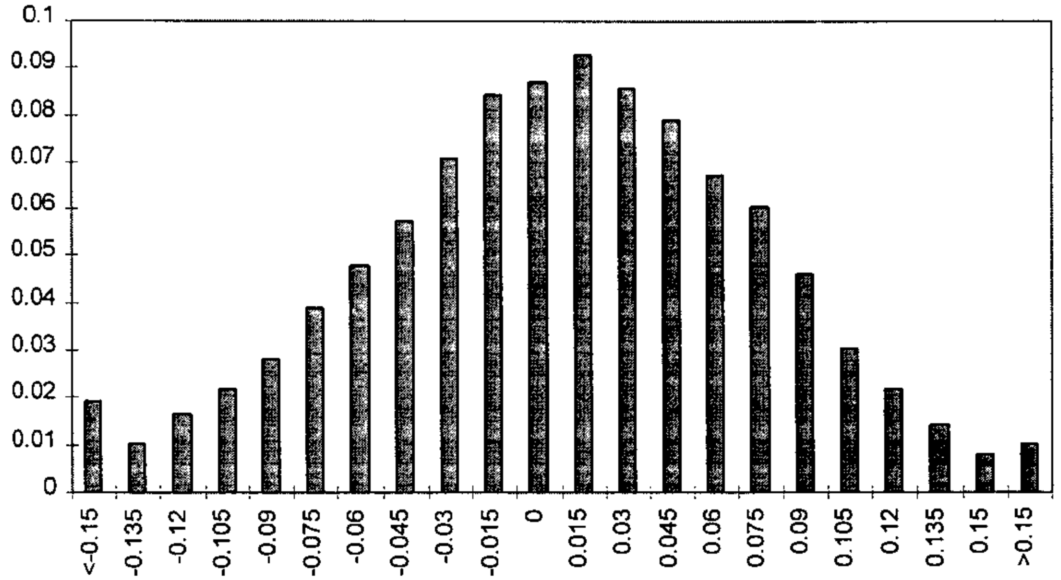


FIGURE 4
Realizations of Aggregate Investment

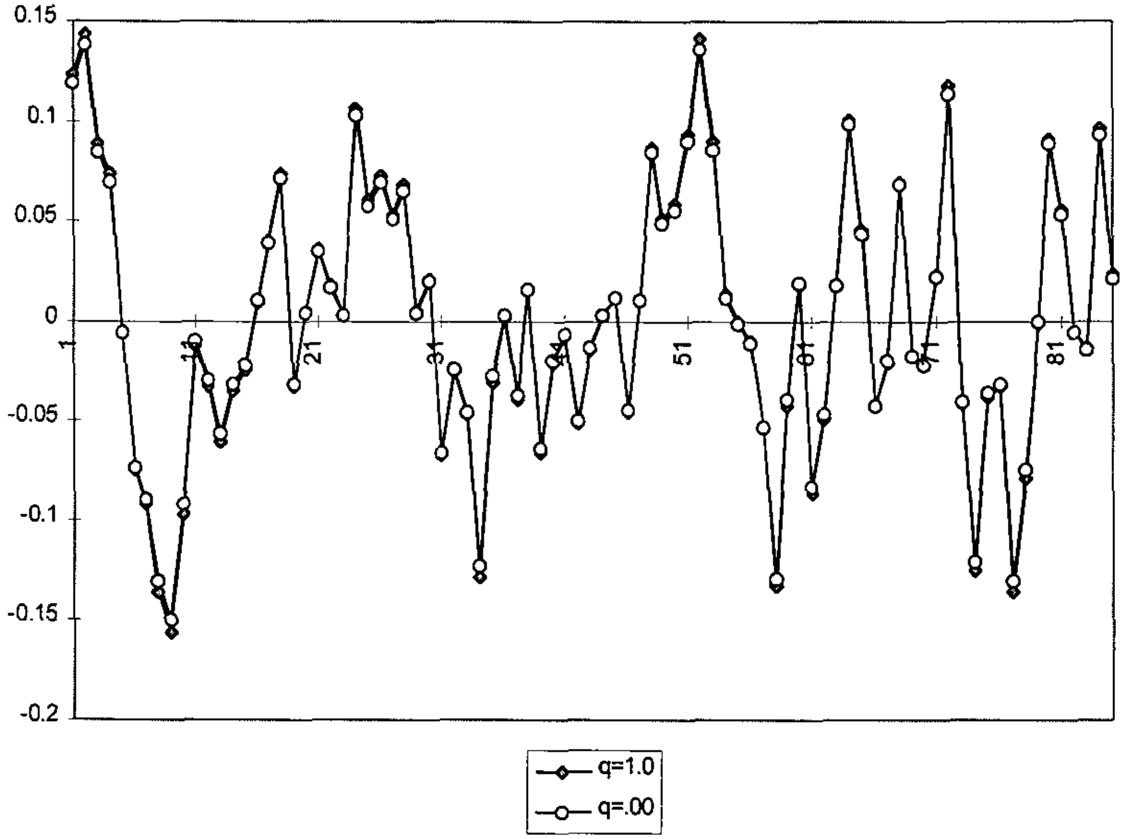
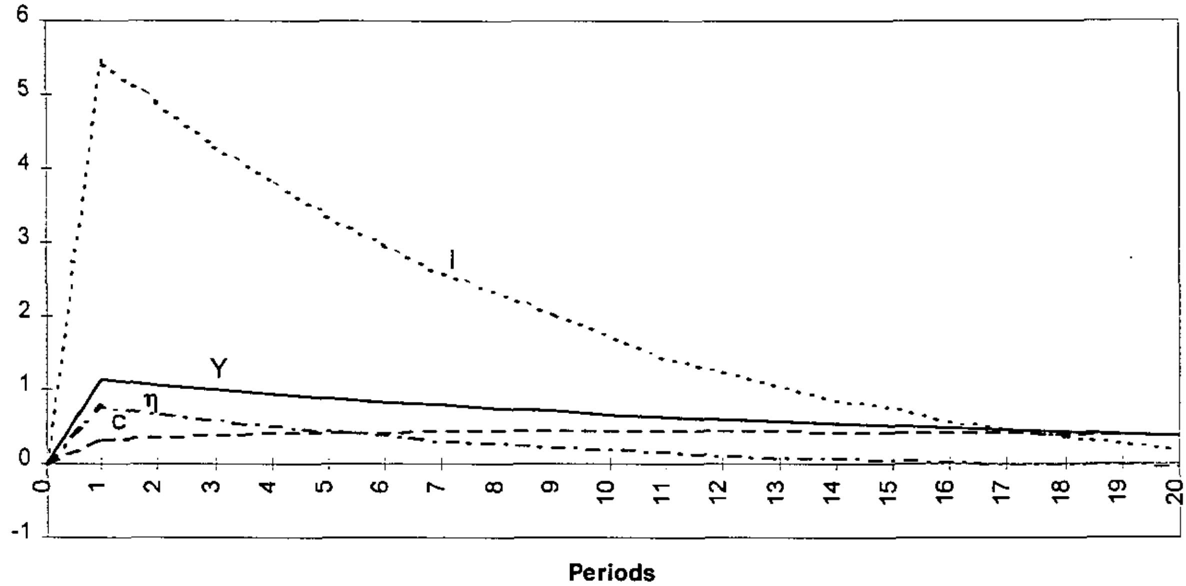


FIGURE 5
Impulse Response Functions

(q=1.0)



(q=.00)

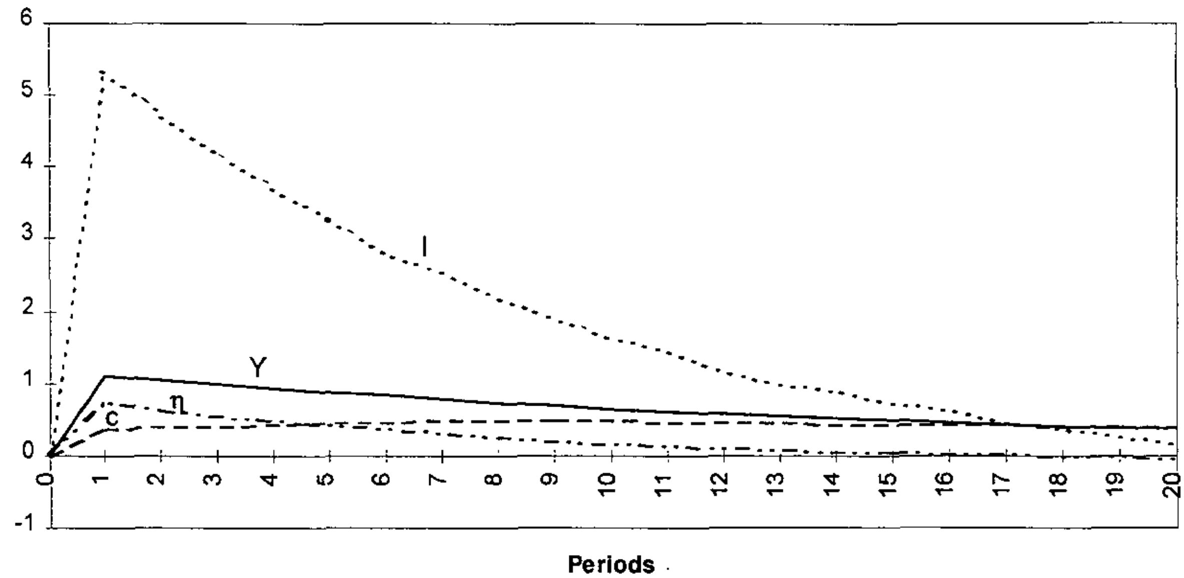


FIGURE 6
Impulse Response-Capital Support

Negative Shock: **Positive Shock:**

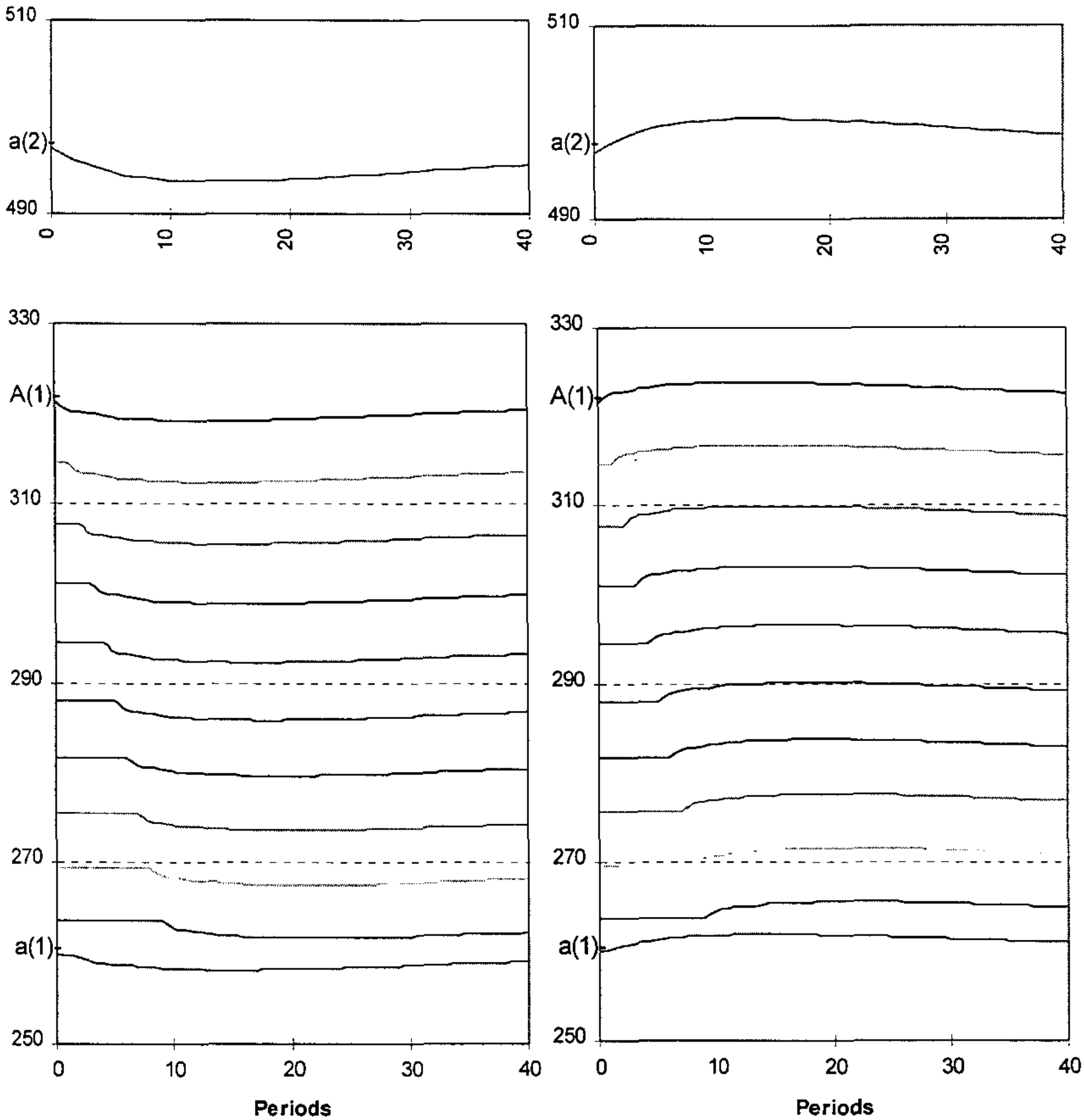


FIGURE 7
Steady State Capital Distribution

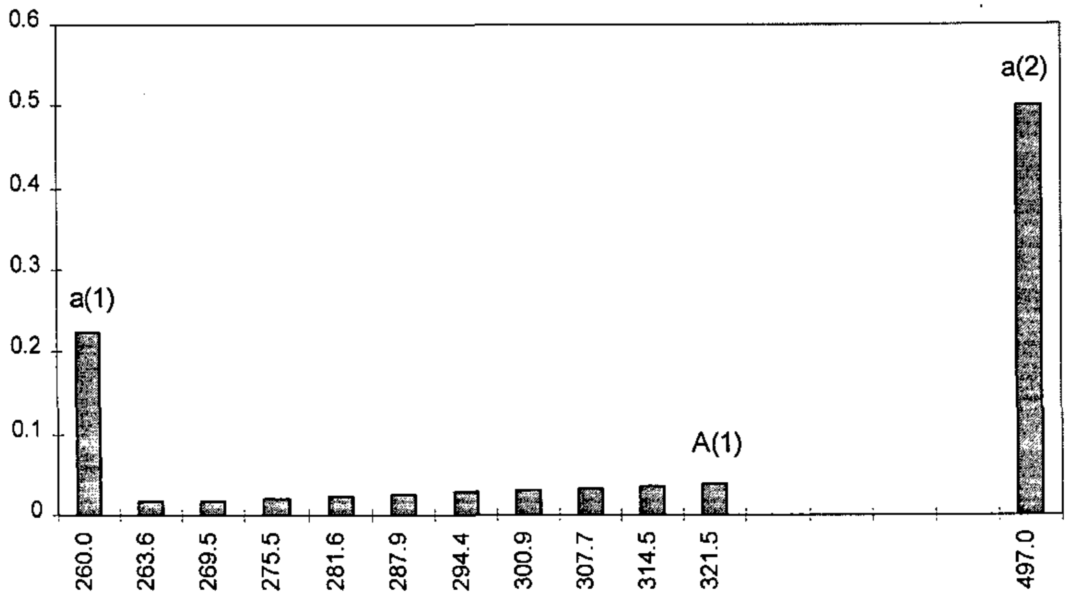
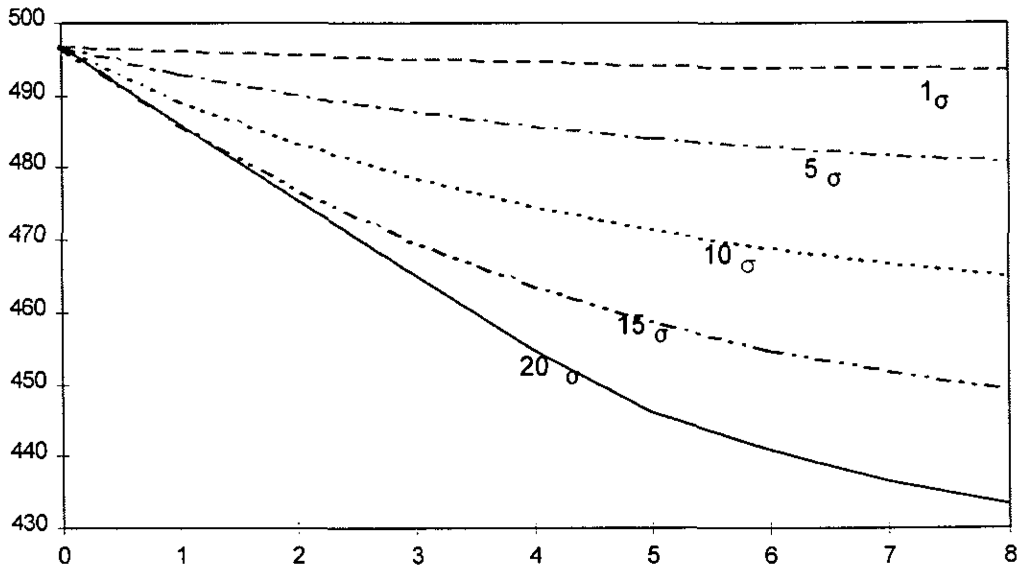


FIGURE 8

Impulse Responses - Highest Point in Support

Negative Shock



Positive Shock