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Time Inconsistent Preferences and Social Security

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ABSTRACT

In this paper we examine the role of social security in an economy populated by overlapping generations of individuals with time-inconsistent preferences who face mortality risk, individual income risk, and borrowing constraints. Agents in this economy are heterogeneous with respect to age, employment status, retirement status, hours worked, and asset holdings. We consider two cases of time-inconsistent preferences. First, we model agents as quasi-hyperbolic discounters. They can be sophisticated and play a symmetric Nash game against their future selves; or they can be naive and believe that their future selves will exponentially discount. Second, we consider retrospective time inconsistency. We find that (1) there are substantial welfare costs to quasi-hyperbolic discounters of their time-inconsistent behavior, (2) social security is a poor substitute for a perfect commitment technology in maintaining old-age consumption, (3) there is little scope for social security in a world of quasi-hyperbolic discounters (with a short-term discount rate up to 15%), and, (4) the ex ante annual discount rate must be at least 10% greater than seems warranted ex post in order for a majority of individuals with retrospective time inconsistency to prefer a social security tax rate of 10% to no social security. Our findings question the effectiveness of unfunded social security in correcting for the undersaving resulting from time-inconsistent preferences.

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1 Introduction

In the United States and most other developed countries, the public pension system and associated benefit payments to retirees and their families (including disability, medical, and survivor benefits) constitute the largest item in the government budget. Partly because of their scale, these programs have during the last quarter century become the object of intense study by economists, who have become increasingly aware of the large effects such programs may have on many aspects of the economy.

The literature on unfunded public pensions has identified a variety of both costs and benefits of such systems. The major benefits arise from the fact that social security may provide avenues for risk sharing that are not otherwise available in private markets. The costs consist largely of distortions to the labor supply and saving decisions.

An important economic consequence of unfunded social security concerns its effects on the capital stock. In a model populated by overlapping generations of pure life-cycle consumers who supply labor inelastically, unfunded social security lowers the steady-state capital stock (Diamond, 1965). This effect arises because social security redistributes income away from younger agents with lower marginal propensities to consume and toward older agents with higher marginal propensities to consume.[1] Social security may also distort the labor supply decision. In an unfunded system, the mandatory contributions of current workers are immediately paid out as benefits to current retirees. These contributions in turn entitle current workers to future retirement benefits. To the extent that an additional dollar of current contributions results in less than a one-dollar increase in the present value of future benefits, these contributions constitute a tax on labor income.

On the other hand, social security may increase welfare by improving the allocation of risk bearing in the economy. It is possible that certain vehicles for allocating risk are unavailable or are very costly in private markets. Depending upon the reasons for the lack of private insurance, social security might provide a lower-cost substitute for private contracts. Annuity markets provide an example. One would expect life-cycle consumers facing uncertain death dates to utilize individual annuity contracts to smooth consumption and insure against the risk of outliving their assets. Although private annuity markets exist in the United States, the volume of contracts in these markets is surprisingly small, possibly because of adverse selection (Friedman and Warshawsky, 1990).[2] By imposing a mandatory annuity system, social security might substitute for missing private annuity markets and might at least mitigate the welfare losses due to adverse selection.[3]

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[1] The effect of social security in depressing the capital stock may be mitigated or eliminated if agents have an operative bequest motive (Barro, 1974).

[2] Individuals might choose not to annuitize all their wealth if they have operative bequest motives or wish to self-insure against large medical or nursing home expenses.

[3] Diamond (1977) discusses various rationales for a social security system qualitatively like that in the United States. Hubbard and Judd (1987), İmrohoroğlu, İmrohoroğlu, and Joines (1995,1999), and Storesletten, Telmer, and Yaron (1999) evaluate the quantitative trade-off

Unfunded social security might also improve the intergenerational allocation of risk. If there is substantial generation-specific income risk due to phenomena such as the Great Depression, fiscal policy tools like public debt or unfunded social security might be used to spread this risk across many generations. It would be impossible for private contracts to insure against these income shocks to the extent that some of the generations who are potential parties to the contracts are born only after the shock is realized.[4]

In addition to the benefits discussed above, some have argued that social security may provide welfare gains for agents who lack the foresight to save adequately for their retirement. For example, Diamond (1977, p. 281) states that a "justification for Social Security is that many individuals will not save enough for retirement if left to their own devices." Kotlikoff, Spivak, and Summers (1982) remark on the widely held belief that the "essential premise underlying the Social Security system ... is that left to their own devices, large numbers of people would fail to save adequately and find themselves destitute in their old age." And according to Feldstein (1985, p. 303), the "principal rationale for such mandatory programs is that some individuals lack the foresight to save for their retirement years."

Extensive empirical evidence is cited to support the view that many households do not save adequately, although much of this evidence is subject to alternative interpretations. Studies using a wide variety of data have documented that a substantial fraction of the U.S. population accumulates very little wealth relative to its lifetime income. For example, Avery, Elliehausen, Canner, and Gustafson (1984) examined data from the 1983 Survey of Consumer Finances and found that fewer than half of all households held more as much as $5000 in financial assets. Median financial assets of those at or near retirement age (55-64) were less than $10,000. Almost 3/4 of households in this age group had positive equity in housing, however, with a median value of $55,000 for those with positive equity. Akerlof (1991), citing evidence reported by Hurd (1990), states that the "stark absence of financial asset income [among the elderly] is consistent with the hypothesis that most households would save very little, except for the purchase of their home and the associated amortization of mortgage principal, in the absence of private pension plans." Other studies documenting low levels of wealth accumulation include Diamond (1977), Feldstein and Feenberg (1983), Diamond and Hausman (1984), Avery and Kennickell (1991), and Hubbard, Skinner, and Zeldes (1995).

The mere fact that many individuals fail to accumulate large stocks of wealth does not imply that they lack foresight, however. As Bernheim, Skinner, and

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between the insurance benefits of social security against the saving distortion and find that the cost of social security outweighs its benefits. Also see İmrohoroğlu (1999).

[4] See Gordon and Varian (1988), Gale (1990), Diamond (1996, 1997), and Bohn (1997). Recent justifications for the emergence and maintenance of the unfunded pension system are provided by Cooley and Soares (1999) and Boldrin and Rustichini (1999), who analyze social security as the outcome of majority voting. Transitional costs toward privatization also seem important. Huang, İmrohoroğlu, and Sargent (1997), De Nardi, İmrohoroğlu, and Sargent (1999), and Kotlikoff, Smetters, and Walliser (1999) document the intergenerational welfare distribution of various alternative reform schemes.

Weinberg (1997, p. 1) note, "if one takes the view that saving reflects rational, farsighted optimization, then low savers are simply expressing their preferences for current consumption over future consumption". They contrast this view with one in which "households are shortsighted, irrational, prone to regret, or heavily influenced by psychological motives". Distinguishing between these two points of view requires more detailed analysis of the data. Bernheim (1995) compares the observed saving of a group of baby boom households with that which would be required to maintain consumption after retirement at the same level as before retirement. He reports that these households' saving (in excess of Social Security and other pension assets) is only one third as much as would be required to maintain their pre-retirement levels of consumption if one assumes no reductions in future Social Security benefits. The shortfall in saving is even greater if one assumes reductions in Social Security. Because rational, farsighted households may have preferences that call for lower consumption after retirement than before, a finding that their resources are insufficient to maintain pre-retirement levels of consumption need not imply any shortsightedness on their part. Even if one accepts the pre-retirement level of consumption as a benchmark, Bernheim's findings appear to contradict those of Kotlikoff, Spivak, and Summers (1982). They perform similar calculations for a sample drawn from the Retirement History Survey in the early 1970s and find that at least 3/4 of households at retirement could finance a constant consumption stream over the remainder of their lives larger than the one they could have financed at age 30, and only a small fraction would be forced to accept a substantially smaller level of consumption. They also report (p. 1068) that if "Social Security were removed, and not replaced by private accumulation, a large fraction of the aged population would face very sharp declines in living standards."[5]

Several papers that examine the behavior of the elderly report a drop in consumption at retirement that is sometimes taken as evidence of a lack of foresight. For example, Hammermesh (1984) finds that consumption in the first year or two after retirement is larger than can be sustained by available resources, which include Social Security, private pensions, and the annuity value of physical and financial wealth. Consumption then drops by about 9% over the next two years. Mariger (1987) estimates that, after adjusting for changes in household size, consumption at retirement drops 47% below the upward trend implied by pre-retirement behavior. While such a drop in consumption might

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[5] The results of Kotlikoff, Spivak, and Summers (1982) do not contradict findings of low asset holdings by the elderly, because Social Security, private pensions, and earnings from part-time work account for the bulk of retirement resources in their sample. To the extent that this is also true of Bernheim's sample, his calculation that baby boomers are saving enough to replace only one third of their requirements unaccounted for by these three items need not imply a large decline in consumption during retirement and may in fact be broadly consistent with the Kotlikoff-Spivak-Summers results. Furthermore, if desired consumption in retirement is somewhat lower than that during working years, there may be no saving shortfall. See Gale (1997), who provides numerical examples and raises more general questions about such estimates of the adequacy of retirement saving.

Diamond (1977) also contains calculations of asset levels required to achieve various consumption targets and compares these with observed asset holdings.

be due to inadequate planning for a predictable decline in income, it might also result from unforeseen events like physical incapacity that cause individuals to retire earlier than they had planned, thus leading to an unanticipated reduction in lifetime resources. Hausman and Paquette (1987) report that workers who retire involuntarily experience a particularly sharp drop in consumption, although voluntary retirees also experience some decline. In addition, a sudden decline in consumption at retirement might be due to a substitution of leisure for market goods.

Banks, Blundell, and Tanner (1998), document a sharp drop in the average consumption of a cross section of households around the typical retirement age. They find that some, but not all, of this reduction can be attributed to changing consumption patterns associated with withdrawal from the labor force, such as reductions in work-related expenses and possibly a more general substitution of leisure for market goods. After considering several explanations, they conclude that the remainder of the consumption decline must be due to the arrival of new information, with the most likely candidate being negative innovations to the income process because workers have underestimated their retirement income.

Bernheim, Skinner, and Weinberg (1997) also document a discrete drop in consumption at retirement, with the size of the decline being negatively related to wealth and the income replacement rate. They examine expenditure by type of good and conclude that the drop in reported consumption cannot be fully accounted for by a reduction in work-related expenses, although their evidence against more general goods-leisure substitution seems tenuous. They also find that their results are largely unaffected after controlling for unplanned retirement. They conclude that "a broad range of factors operating within models of rational, farsighted, optimizing agents are collectively incapable of accounting for joint patterns of wealth and consumption" (p.3), that "the empirical patterns in this paper are more easily explained if one steps outside the framework of rational, farsighted optimization" (p. 48), and that "on average individuals who arrive at retirement with few resources experience a 'surprise' – they take stock of their finances only to discover that their resources are insufficient to maintain their accustomed standards of living" (p. 4).

Despite the apparently widespread view that many individuals may lack the foresight to save adequately for their retirement, there have been few attempts to analyze the effectiveness of social security in mitigating the welfare costs of such undersaving. Feldstein (1985) examines a two-period overlapping generations economy with inelastic labor supply and no uncertainty and analyzes the welfare consequences of social security in an environment with myopic agents. He models myopia by assuming that elderly agents attach greater weight to period-2 outcomes than do young agents. In that framework, reductions in saving constitute the only welfare cost of social security, and providing consumption for myopic agents constitutes the only benefit. His findings indicate that even if every individual is substantially myopic it may be optimal to have either no social security system or one in which the social security replacement ratio is very low.

An alternative way of modeling myopia stems from the literature on time-inconsistent behavior and more specifically from the recent literature dealing with quasi-hyperbolic discounting.[6] Strotz (1956) argues that mechanisms that constrain the future choices of agents may be desirable if their behavior exhibits time inconsistency. Social security may be viewed as such a commitment device. According to Akerlof (1998, p. 187), the "hyperbolic model explains the uniform popularity of social security, which acts as a pre-commitment device to redistribute consumption from times when people would be tempted to overspend – during their working lives – to times when they would otherwise be spending too little – in retirement ■. [S]uch a transfer is most likely to improve welfare significantly."

In this paper we examine the welfare effects of unfunded social security on individuals who exhibit two distinct forms of time-inconsistent preferences. In addition to incorporating quasi-hyperbolic discounting, our model nests the retrospective form of time inconsistency analyzed by Feldstein (1985) and extends his framework to include a wider range of benefits and costs of social security. In order to examine the role of social security in an economy with time-inconsistent preferences, we construct a model which consists of overlapping generations of 65-period-lived individuals facing mortality risk, individual income risk, and borrowing constraints. At any point in time there is a continuum of agents with total measure one. Private annuity markets and credit markets are closed by assumption. Agents in this economy choose the number of hours worked whenever they are given the opportunity to do so. If they are not given the opportunity to work, they receive unemployment insurance. Agents in this economy accumulate assets to provide for old-age consumption and, because they face liquidity constraints, to self-insure against future income shocks. Elderly agents receive social security benefits that are financed by a payroll tax on workers. At any time after reaching the normal retirement age, they may make an irreversible decision to draw social security benefits, although collection of benefits does not preclude working. Individuals in this economy are heterogenous with respect to their age, employment status, retirement status, hours worked, and asset holdings.

We consider two cases of quasi-hyperbolic discounting. In some experiments, we assume that the agents are naive in the sense that they ignore the fact that their future selves will also implement quasi-hyperbolic discounting. In most cases, however, we allow for more sophisticated behavior by assuming the agents take into account their future selves' quasi-hyperbolic discounting.

In this environment social security may provide additional utility for myopic agents who regret their saving decisions when they find themselves with low consumption after retirement. In addition, social security may substitute for missing private annuity markets in helping agents allocate consumption in the face of uncertain life spans. On the other hand, social security distorts aggregate saving and labor supply behavior and affects the wage rate and the interest rate. Consequently, whether or not social security is welfare enhancing even for

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[6] For example, see Phelps and Pollak (1968) and Laibson (1994, 1995, 1997). For time-inconsistent preferences more generally, see Strotz (1956), Pollak (1968), Kydland and Prescott (1977), Thaler and Shefrin (1981), and Goldman (1979, 1980), among others.

myopic agents is a quantitative question. We evaluate lifetime welfare from different vantage points in the life cycle.

We specify the optimization problem of the individual as a finite-state, finite-horizon, dynamic program and use numerical methods to compute stationary equilibria under alternative social security arrangements. Our calibration procedure follows Cooley and Prescott (1995) and restricts the parameters of the model using measurements from the U.S. economy.

Our findings can be summarized as follows:

- With time-consistent preferences, the actuarial reward for survival that social security provides is not quantitatively large enough to render a world with social security desirable; there is a significant welfare cost to social security, viewed at any age, although the cost declines with age. These findings are in accord with previous research.
- Quasi-hyperbolic discounting at the rate of 15% lowers the capital stock by about 20% at any social security tax rate, and there are substantial steady-state welfare costs to quasi-hyperbolic discounters of their time-inconsistent behavior.
- Social security is a poor substitute for a perfect commitment technology in maintaining old-age consumption; the capital stock would be about one-third larger in the absence of social security than with a tax rate of 10%.
- In a world of quasi-hyperbolic discounters, sophisticated or naive, unfunded social security generally does not raise welfare for short-term discount rates of at least 15%.
- Social security does raise the welfare of naive quasi-hyperbolic discounters with a short-term discount rate of 40%.
- With retrospective time inconsistency of the form considered by Feldstein (1985), the ex ante annual discount rate must be at least 10% greater than seems warranted ex post in order for a majority of the population to prefer a social security tax rate of 10% to no social security.

Overall, our quantitative findings question the efficacy of unfunded social security in correcting for the undersaving resulting from quasi-hyperbolic preferences.

The paper is organized as follows. Section 2 describes the model economy and characterizes its stationary equilibrium. Section 3 discusses calibration of the model's parameters and summarizes the solution method. Section 4 uses the model to perform a quantitative analysis of the effects of an unfunded social security program on the welfare of myopic and non-myopic agents. Section 5 concludes.

2 A Model of Social Security

2.1 The Environment

Time is discrete. The setup is a stationary overlapping generations economy. At each date, a new generation is born which is $n\%$ larger than the previous generation. Individuals face long but random lives and some live through age J, the maximum possible life span. Life-span uncertainty is described by ψ_j, the conditional survival probability from age $j-1$ to j.[7] Under our stationary population assumption, the cohort shares, $\{\mu_j\}_{j=1}^{J}$, are given by

$$\mu_j = \psi_j \mu_{j-1}/(1+n), \text{ where } \sum_{j=1}^{J} \mu_j = 1. \tag{1}$$

2.2 Preferences and Measures of Utility

Preferences are defined over sequences of consumption and labor $\{c_j, \ell_j\}_{j=1}^{J}$. The essence of myopia is that the value agents attach to these sequences depends on the agent's vantage point. In particular, the agent may value actions differently *ex post* than at the time those actions are taken, and so may later regret those actions. Social security can have potentially large effects on the average lifetime levels of consumption and labor and also on the allocation of consumption and labor over the life cycle. Reductions in consumption and leisure across the entire life cycle would presumably be disfavored by agents of all ages. The possibility that social security can improve the welfare of myopic agents, however, is primarily a question of whether the resulting intertemporal redistributions of consumption and labor would raise utility as viewed from at least some ages.[8]

If preferences are time-consistent, and assuming $\psi_j = 1.0 \ \forall j$ (i.e. ignoring life-span uncertainty), the value an agent of age j^* places on the lifetime consumption and labor sequences $\{c_1, \ell_1, c_2, \ell_2, \ldots, c_{j^*}, \ell_{j^*}, \ldots, c_J, \ell_J\}$ is independent of the agent's vantage point j^*. If preferences are time-inconsistent, this valuation depends on j^*. We are concerned with a particular type of time inconsistency in preferences that can be characterized as follows. Let U_j denote the marginal utility of consumption at age j and suppose that the values of consumption and leisure in all periods of life are fixed. Also suppose that an agent's preferences are such that the ratio of marginal utilities $U_{j'}/U_{j^*}$ for some j^* and $j' > j^*$ is larger when viewed from age j' than from age j^*. If at age j^* the agent acts so as to equate this ratio of marginal utilities (as viewed at that time) to the marginal rate of transformation, then upon reaching age j'

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[7] We assume that the survival probabilities ψ_j and the population growth rate n are time-invariant. For studies that examine the impact of time-variation in either demographic variable, see De Nardi, İmrohoroğlu and Sargent (1999).

[8] It is possible that social security results in intertemporal redistributions that are desirable, at least as viewed by individuals of some ages, but also reduces total lifetime consumption and leisure by more than enough to eliminate the welfare gains from these reallocations.

he will regret having consumed so much and saved so little at age j^*. We refer to preferences that result in this sort of regret as myopic, and we consider two features of preferences that can lead to myopia thus defined.[9]

Specifically, suppose that an individual of age j^* has preferences over lifetime consumption and labor given by

$$\begin{aligned} U_{j^*} &= \sum_{j=1}^{j^*-1} \beta_b^{j-j^*} u(c_j, \ell_j) + u(c_{j^*}, \ell_{j^*}) \\ &\quad + \delta E_{j^*} \sum_{j=j^*+1}^{J} \beta_f^{j-j^*} u(c_j, \ell_j). \end{aligned} \tag{2}$$

Here, β_f is the agent's forward-looking discount factor and β_b is the backward-looking discount factor. The expectations operator in the final term accounts for mortality risk, whereas β_f incorporates discounting only for pure time preference. Note that utility depends on consumption and leisure in the past as well as in current and future periods. The case where $\beta_f < \beta_b$ corresponds to the form of mypoia considered by Feldstein (1985).[10] The parameter $\delta \leq 1$ allows for the possibility that, viewed from today, the discount rate between this period and next may be greater than that between any two consecutive periods further into the future. The case where $\delta < 1$ corresponds to the form of time inconsistency considered by Phelps and Pollak (1968) and Laibson (1997). This case leads not only to time-inconsistent preferences but also to time-inconsistent behavior in the sense that the optimal policy functions derived at age j^* for ages $j' > j^*$ will no longer be optimal when the agent arrives age j'. In the absence of any commitment technology, the agent's future behavior will deviate from that prescribed by the earlier policy functions. Strotz (1956) showed that time-consistent behavior requires that the discount factor connecting any two periods (current or future) vary exponentially as a function of the length of the interval between the two periods. For $\beta_f < 1$, a value of δ less than unity results in discount factors that decline approximately hyperbolically from period j^* into the

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[9] It should be noted that we are not concerned with cross-sectional reallocations among agents of different ages at a point in time, with the past consumption and labor of all cohorts fixed. In particular, we are not considering instituting social security or changing an existing system, with welfare judged only by the effects on current and prospective consumption of currently alive and future agents. It seems quite reasonable to believe that individuals of different ages would disagree about the desirability of such policy changes. Instead, our experiments can be viewed as comparing the steady states of economies with different policy arrangements and asking which of these economies agents would prefer. We are concerned with whether agents of different ages would choose to live in different economies if moving from one economy to another required the admittedly unrealistic possibility of ex post changes in prior consumption and labor so as to conform to those in the newly chosen economy. Viewed in this way, agents with time-consistent preferences would never switch economies whereas agents with time-inconsistent preferences might do so.

[10] Caplin and Leahy (1999) also consider a preference structure similar to (2). They give particular attention to the case where $\beta_f < 1 < \beta_b$, implying that individuals downweight both the past and the future relative to the present. Their paper contains an extensive justification of the assumption that β_b is finite (i.e.., individuals remember and their memories matter) but less than unity (memory may be fallible).

future.[11] Phelps and Pollack (1968) analyzed the time inconsistency in behavior resulting from a quasi-hyperbolic discounting parameter δ less than unity. The form of myopia considered by Feldstein does not lead to this sort of time inconsistency in behavior.

The preference structure in equation (2) determines individual behavior and also constitutes the basis for making welfare comparisons among alternative social security arrangements. The first summation on the right-hand-side of the equation is irrelevant for determining behavior. As Deaton (1992, p. 14) notes, however, "it is important to recognize that, at best, [the remaining expression] only represents a fragment of lifetime preferences, albeit that fragment that is 'live' or 'active' for current decision-making." An analysis of the welfare effects of policies that reallocate consumption and leisure across the life cycle requires an explicit consideration of how individuals value past outcomes. If preferences are time-consistent ($\beta_f = \beta_b$ and $\delta = 1$) and there is no life-span uncertainty, then individuals of all ages agree on the welfare ranking of policies. Thus, one can make welfare comparisons solely on the basis of expected utility at birth. This procedure implicitly assumes that the elderly value the past, and in particular that they place the same value on outcomes in old age relative to those in youth as does a newborn individual. The assumption that individuals place no value on the past ($\beta_b = \infty$) constitutes time inconsistency in preferences and leads trivially to the conclusion that the elderly prefer a generous social security system.[12]

If preferences are time-inconsistent, then a single individual can be viewed as a collection of J individuals, each of a different age and each with a different set of preferences. These J individuals need not agree on their ranking of different consumption and labor sequences. Because of the well-known difficulties in making interpersonal utility comparisons, it is unclear which of these J preference orderings should be given priority in judging the welfare consequences of various social security arrangements.[13] Much of the recent literature on hyperbolic discounting is concerned primarily with characterizing behavior rather than making welfare comparisons, although a notable exception is Laibson, Repetto, and Tobacman (1998). Feldstein (1985) does make such comparisons. In his model, $J = 2$ and welfare rankings are based on the preferences of an agent in the final period of life. While arguably reasonable in the context of a 2-period model, this retrospective welfare criterion seems quite arbitrary in the 65-period model used here. Therefore, we use equation (2) to compute welfare measures at each age, denoted by W_{j^*} for $j^* = 1, 2, \ldots, J$, and we rank policy arrangements based on these measures. W_{j^*} is an average of the individual U_{j^*}, where the averaging is with respect to the stationary distribution of individuals of age j^* across employment and asset states. As might be expected, welfare measures viewed from different ages may disagree in their ranking of policy arrangements.

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[11] See Laibson (1997).

[12] There is a large literature on habit persistence in which the past is relevant not only for welfare but also for current behavior.

[13] Strotz (1956) first provided such a multi-agent interpretation of time-inconsistent preferences and pointed out the difficulty of arriving at an unambiguous welfare criterion.

In addition, we compute a weighted average of the age-specific indicators W_{j^*}, with the weight on each W_{j^*} being the unconditional probability of surviving from birth to age j^*. This aggregate welfare indicator is denoted W. With time-consistent preferences, the appropriate welfare indicator is the expected lifetime utility of a newborn individual, W_1, because all of the other indicators W_{j^*} for $j^* > 1$ are proportional to W_1. This simple proportionality relation breaks down if preferences are time-inconsistent, yet W retains a certain similarity to the expected utility of a newborn in the the time-consistent case. Throughout its lifetime, each newborn individual with time-inconsistent preferences will, depending on survival, become as many as J separate individuals, each with its own preference ordering. W is simply the expected value of the age-specific indicators W_{j^*}, where the expectation is taken with respect to the unconditional survival probabilities. This criterion obeys Ramsey's (1928) stricture against pure time discounting of the wellbeing of future generations (or, in this instance, selves), which he refers to as "a practice which is ethically indefensible [that] arises merely from the weakness of the imagination." W is also an egalitarian criterion in the following sense. If a large cohort of N newborn individuals is followed through life, it will ultimately constitute N individuals of age 1, $\pi_2 N$ individuals of age 2, $\pi_3 N$ individuals of age 3, etc., where π_j denotes the unconditional probability of surviving to age j. The welfare criterion W assigns equal weights to the preferences of these $(1 + \pi_2 + \pi_3 + ... + \pi_J)N$ individuals.[14]

Finally, we assume that the period utility function takes the form

$$u(c_j, \ell_j) = \frac{\left((c_j)^{\varphi}(1-\ell_j)^{1-\varphi}\right)^{1-\gamma}}{1-\gamma}, \tag{3}$$

where γ is the coefficient of relative risk aversion and φ is the share of consumption in utility.

2.3 Budget Constraints

Agents are subject to individual earnings uncertainty. An age-j individual faces the state vector $x_j = (a_{j-1}, s_j, \overline{e}_j, h_{j-1})$, where a_{j-1} is the stock of assets held at the end of age $j-1$, s_j denotes the individual's employment shock, $\overline{e}_j$ denotes the average past earnings at age j, and h_{j-1} indicates whether an individual has elected to collect social security benefits at age $j-1$. The individual employment state $s_j \in S = \{0, 1\}$ is assumed to follow a two-state, first-order Markov

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[14] With time-consistent preferences ($\beta_f = \beta_b = \beta$ and $\delta = 1$) and in the absence of uncertainty, equation (2) implies that the utility indicators U_{j^*} attached to a given realization of the consumption-leisure sequence are given by $U_{j^*} = \beta^{-(j^*-1)} U_1$. Thus, assuming $\beta < 1$, the U_{j^*} grow at the rate β^{-1}, so that simple aggregation of the U_{j^*} across different ages would attach greater weight to the preferences of older selves. To avoid this bias, we compute $\widehat{U}_{j^*} = \beta_b^{(j^*-1)} U_{j^*}$. With time-consistent preferences and no uncertainty, these normalized utility indicators for a given realization of the consumption-leisure sequence thus reduce to $\widehat{U}_{j^*} = U_1$ for all $j^* > 1$. The aggregate welfare measures W_{j^*} and W are computed using the normalized indicators $\widehat{U}_{j^*}$.

process. If $s_j = 1$, the agent is given the opportunity to work and if $s_j = 0$ the agent is unemployed. The transition matrix for the employment shock is given by the 2×2 matrix $\Pi(s', s) = [\pi_{kl}]$ where $\pi_{kl} = Prob\{s_{j+1} = k | s_j = l\}$. The vector of choice variables is $y_j = (a_j, c_j, \ell_j, h_j)$ where a_j indicates the stock of assets held over to the next age, c_j is consumption, ℓ_j is labor supply at age j, and h_j is the retirement decision which can only be made at or after the first of eligibility, j_R.

The budget constraint facing an age-j individual is given by

$$c_j + a_j = (1 + r)a_{j-1} + s_j w \varepsilon_j \ell_j - T_j + h_j Q_j + M_j + \xi, \tag{4}$$

where r is the real interest rate, w is the wage per efficiency unit of labor, ε_j is the efficiency index of an individual of age j, T_j is taxes paid by an age-j individual, Q_j and M_j are retirement and unemployment insurance benefits received by an age-j individual, respectively, and ξ is a per capita government transfer received by an individual. Unemployment insurance benefits are given by

$$M_j = \begin{cases} 0 & s = 1, \\ \phi w \varepsilon_j \ell_j & s = 0, \end{cases} \tag{5}$$

where ϕ is the unemployment insurance replacement ratio.

At any age $j \geq j_R - 1$ individuals may make an irreversible decision to begin collecting social security benefits next period. This choice gives rise to a state variable h_j for agents of age j_R or older. This variable takes a value of 1 for agents who have elected to receive benefits and a value of 0 for agents who have not yet elected to do so.

Note that social security benefits depend on individual earnings history. In particular, we follow Huggett and Ventura (1999) and use the old-age benefit formula employed by the U.S. Social Security Administration. This involves a two-step procedure. First, an individual's average indexed monthly earnings (AIME) are computed by keeping track of his average past earnings $\overline{e}$ and indexing it to productivity growth. Next, the primary insurance amount (PIA) is calculated using a concave formula which implements four replacement rates along four segments of AIME. The PIA replaces 90% of the AIME along the first segment, 33% of the AIME along the second segment, 15% along the third segment, and 0% beyond a maximum amount of AIME.[15] The social security policy parameter varied in our experiments is the tax rate. The replacement rates along the different segments of the benefit formula are adjusted upward or downward in equal proportion so that the system's budget balances.

This hypothetical social security system mimics the actual U.S. system in several important respects. Two members of the same cohort with the same

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[15] The first kink occurs at 16% of average total compensation, where as the second kink takes place at 99% of average total compensation. The data on total compensation are taken from *Historical Statistics of the United States, Colonial Times to 1970*, the Bureau of Labor Statistics web site (average weekly earnings), and the National Income and Product Accounts (supplements to wages and salaries).

earnings history receive identical and constant real benefits throughout their retirement years. However, an otherwise identical retiree who is one year younger receives a pension that is higher than the older retiree by a factor equal to the rate of productivity growth. The benefit formula incorporates a partial linkage between benefits and lifetime labor earnings. Finally, elderly individuals may continue to work with no reduction of benefits. Although this assumption is consistent with the most recent legislation on this issue, it appears not to have a great effect on the welfare effects of social security.[16]

Taxes paid satisfy

$$T_j = \tau_c c_j + \tau_a r a_{j-1} + (\tau_\ell + \tau_s + \tau_u) w \varepsilon_j \ell_j, \tag{6}$$

where τ_c, τ_a, τ_ℓ, τ_s, and τ_u denote the tax rates for consumption, capital income, labor income, social security and unemployment insurance, respectively.

Individuals are assumed to face borrowing constraints:

$$a_j \geq 0, \quad \forall j. \tag{7}$$

2.4 Individual's Dynamic Program

We will restrict attention to Markov Equilibria and therefore rely on recursive methods to characterize them.[17] Let $D = \{d_1, d_2, \ldots, d_m\}$ denote the discrete grid of points on which asset holdings are required to fall. For any beginning-of-period asset holding, employment status, average past earnings, and retirement status $x = (a, s, \overline{e}, h) \in D \times S \times R_+ \times \{0, 1\}$, define the constraint set of an age-j agent $\Omega_j(x) \in R_+^4$ as all quadreplets $y_j = (a_j, c_j, \ell_j, h_j)$ such that equations (4), (5), (6) and (7) are satisfied for $j = 1, 2, \ldots, J$. When preferences are time-consistent, i.e. $\delta = 1$, the individual's dynamic program is a standard backward recursion.[18]

When preferences are time-inconsistent, we have to attribute a particular belief to the individual concerning how he thinks his future selves will behave. We consider two cases. In one case, we assume that the individuals are **naive** in the sense that they think that the future selves will solve the $\delta = 1$ (time-consistent) problem despite a history of violating this belief. It turns out that the 'naive $\delta < 1$' case is not too much more difficult. Let $V_j(x)$ be the (maximized) value of the objective function of an age-j agent with state $x = (a, s, \overline{e}, h)$. $V_j(x)$ is computed as the solution to the dynamic program

$$V_j(x) = \max_{y \in \Omega_j(x)} \left\{ u(c, \ell) + \delta \beta_f \psi_{j+1} E_{s'} \widetilde{V}_{j+1}(x') \right\}, \quad j = 1, 2, \cdots, J, \tag{8}$$

---

[16] In some unreported experiments retirement is mandatory in the sense that agents are prohibited from working at age j_R or later. The welfare effects are qualitatively very similar to the endogenous retirement case.

[17] Krusell and Smith (1999) also rely on the use of Markov equilibria in their infinite-horizon, consumption-saving study, whereas Bernheim, Ray, and Yeltekin (1999) allow for historical path dependence in their study of infinite-horizon saving behavior.

[18] See Sargent (1987) and Stokey and Lucas (1989).

where the notation $E_{s'}$ means that the expectation is over the distribution of s'. In the program (8), the continuation payoff $\widetilde{V}_j(x)$ is computed for $j = 1, 2, \ldots, J$ from

$$\widetilde{V}_j(x) = \max_{y \in \Omega_j(x)} \left\{ u(c, \ell) + \beta_f \psi_{j+1} E_{s'} \widetilde{V}_{j+1}(x') \right\}.$$

Note that for $\delta = 1$, V_j and $\widetilde{V}_j$ coincide for all j and the decision rules are time-consistent. For $\delta < 1$, however, the behavior represented by the decision rules $\{A_j(x), C_j(x), L_j(x), H_j(x)\}_{j=1}^{J}$ is time-inconsistent.[19] A stationary solution to this dynamic program will consist of a set of value functions $\{V_j(x)\}_{j=1}^{J}$, decision rules $\{A_j(x), C_j(x), L_j(x), H_j(x)\}_{j=1}^{J}$ and measures of agent types $\{\lambda_j(x)\}_{j=1}^{J}$. The latter are computed using the forward recursion

$$\lambda_j(x') = \sum_{s} \sum_{a: a' \in A_j(x)} \Pi(s', s) \lambda_{j-1}(x),$$

given an initial measure of agent types $\lambda_0(x)$.

Most of our computations rely on the alternative assumption that individuals are aware that their future selves will not compute continuation payoffs $\widetilde{V}_j(x)$ according to the recursion shown above. Instead, they assume that their future selves will also engage in quasi-hyperbolic discounting. This case requires more care in computing the value functions and the policy rules. Define the value functions from the 'sophisticated $\delta < 1$' problem by $\widehat{V}_j$ and the associated policy functions by $\widehat{c}_j, \widehat{\ell}_j, \widehat{a}_j$, and $\widehat{h}_j$. We can compute these functions from the recursion

$$\widehat{V}_j(x) = \max_{y \in \Omega_j(x)} \left\{ u(c_j, \ell_j) + \delta \beta_f V^*_{j+1}(c_{j+1}, \ell_{j+1}) \right\},$$

where the V^*_j sequence is computed by

$$V^*_j(x) = u(\widehat{c}_j, \widehat{\ell}_j) + \beta_f \widehat{V}_{j+1}(\widehat{c}_{j+1}, \widehat{\ell}_{j+1}),$$

and reflects the fact that this is not the usual continuation payoff function in the dynamic program since self j has no control over the choices of self $j + 1$ and therefore must take the future self's optimal plan as given. This explains the absence of the 'max' operator in the above computation.[20]

Given these decision rules and an initial distribution of agents, we compute the measures of agent types using the forward recursion

$$\lambda_j(x') = \sum_{s} \sum_{a: a' \in \hat{a}_j(x)} \Pi(s', s) \lambda_{j-1}(x).$$

---

[19] In recent work on time-inconsistent behavior, Gül and Pesendorfer (1999) propose an alternative preference structure that is shown to satisfy the hypotheses of the Stokey and Lucas (1987) theorems on the existence and characterization of resulting dynamic programs.

[20] See the Appendix for a detailed description of the computations for the 'sophisticated $\delta < 1$' case.

2.5 Aggregate Technology

The production technology of the economy is given by a constant returns to scale Cobb-Douglas production function

$$Y = BK^{1-\alpha}L^{\alpha}, \tag{9}$$

where $\alpha \in (0,1)$ is labor's share of output, and K and L are aggregate inputs of capital and labor, respectively. The total factor productivity parameter $B > 0$ is assumed to grow at a constant, exogenously given rate, $\alpha\rho > 0$, implying that steady-state per capita output grows at rate ρ. The aggregate capital stock depreciates at the rate d. Firm maximization requires

$$r = (1-\alpha)B\left(\frac{K}{L}\right)^{-\alpha} - d, \tag{10}$$

$$w = \alpha B\left(\frac{K}{L}\right)^{1-\alpha}. \tag{11}$$

2.6 Government

There is an infinitely lived government that taxes consumption and income from labor and capital, makes purchases of goods, and maintains unfunded social security and unemployment insurance programs that are self-financing. These budget constraints are given by

$$G = \sum_{j=1}^{J}\sum_{x}\left[\tau_a r A_{j-1}(x) + \tau_\ell w L_j(x) + \tau_c C_j(x)\right]\mu_j\lambda_j(x), \tag{12}$$

$$\tau_s \sum_{j=1}^{J}\sum_{x} w\varepsilon_j L_j(x)\mu_j\lambda_j(x) = \sum_{j=j_R}^{J}\sum_{x}\mu_j\lambda_j(x)B_j, \tag{13}$$

$$\tau_u \sum_{j=1}^{j_R-1}\sum_{x:s>0} w\varepsilon_j L_j(x)\mu_j\lambda_j(x) = \sum_{j=1}^{J}\sum_{x:s=0}\mu_j\lambda_j(x)M_j. \tag{14}$$

2.7 Stationary Equilibrium

A *government policy* is a set of parameters $\{G, \tau_c, \tau_a, \tau_\ell, \tau_s, \phi\}$. An *allocation* is given by a set of decision rules $\{A_j(x), C_j(x), L_j(x), H_j(x)\}_{j=1}^{J}$, and measures of agent types $\{\lambda_j(x)\}_{j=1}^{J}$. A *price system* is a pair $\{w, r\}$. A **Stationary Recursive Equilibrium** is an *allocation*, a *price system* and a *government policy* such that

- the allocation solves the dynamic program for all individuals, given the price system and government policy,

- the allocation maximizes firms' profit by satisfying equations (10) and (11),
- the allocation and government policy satisfy the government's budget constraints (12), (13) and (14), and,
- the commodity market clears.

3 Calibration and Solution of the Model Economy

In order to obtain numerical solutions to the model, we must choose particular values for the parameters. We calibrate our model under the assumption that the model period is one year.

3.1 Demographic and Labor Market Parameters

Individuals are assumed to be born at the real-time age of 21, and they can live a maximum of $J = 65$ years. After real-time age 85, death is certain[21]. The sequence of conditional survival probabilities $\{\psi_j\}_{j=1}^{J}$ is taken from Faber (1982). The share of age groups in the population, μ_j, is calculated from the relations $\mu_j = \psi_j \mu_{j-1}/(1+n)$,where $\sum_{j=1}^{J} \mu_j = 1$ and n is the growth rate of the population, which has averaged 1.2% per year in the United States over the last fifty years. The age at which agents become eligible for social security benefits, j_R, is taken to be equal to 45, which corresponds to a real-time age of 65. The efficiency index ε_j is intended to provide a realistic cross-sectional age distribution of wages at a point in time. This index is taken from Hansen (1993), interpolated to in-between years, and normalized to average unity between ages $j = 1$ and $j = J$. Note that we extended the efficiency profile in the labor market beyond age 45 by extrapolating the Hansen (1993) data to model age 65 and then normalized the series to obtain an average of unity over $j = 1, 2, \ldots, 65$.

The unemployment insurance replacement ratio, ϕ, is taken to be 25% of the employed wage. The employment transition probabilities are chosen to make the probability of employment equal to 0.94, independent of the availability of the opportunity in the previous period. The transition probabilities matrix is then given by

$$\Pi(s, s') = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}.$$

The average duration of unemployment is therefore $1/(1 - 0.94) = 1.0638$ model periods.[22]

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[21] This assumption does not appear to be crucial; according to Faber (1982), we are leaving out less than 3% of the U.S. population.

[22] Although the unemployment rate of 0.06 is close to the postwar U.S. average, the duration clearly exceeds that in the U.S. economy. Incorporating persistence in unemployment would further increase its average duration.

3.2 Preference Parameters

In line with recent practice, we set the preference parameters β_f, δ, and γ so as to match the economy's observed wealth accumulation behavior as measured by an empirical wealth-output ratio of 2.52.[23] This single ratio is not sufficient to pin down the values of all three preference parameters. The wealth-output ratio in our model economy is positively related to the discount factors β_f and δ and negatively related to the risk aversion coefficient γ. Mehra and Prescott (1985) cite various empirical studies which suggest that the coefficient of relative risk aversion, γ, is between 1 and 2. We take $\gamma = 2$ as our base case and also consider $\gamma = 1$ and $\gamma = 3$ as alternatives. We choose various values of δ a priori, and for each combination of γ and δ we search over values of β_f to find the one which best matches the observed wealth-output ratio. In this search, we assume a social security tax rate of 10% and an unemployment insurance replacement rate of 25%. We take the share of consumption in the utility function, φ, to be 0.33. This value yields an average labor input of about 0.29 at the assumed 10% social security tax rate and 25% unemployment insurance replacement rate for all values of the other preference parameters that we considered. The parameter β_b does not affect any observable quantities, so we choose different values a priori to examine the effect of various degrees of time inconsistency on lifetime utility as viewed from different ages.

3.3 Technology Parameters

The parameters describing production technology are chosen to match long-run features of the U.S. economy along the lines suggested by Cooley and Prescott (1995). The growth rate of per capita output ρ, is set to 0.0165, which is the average growth rate of output per labor hour between 1897 and 1992. The remaining technology parameters α and d are calculated from annual data since 1954. Our calculations imply a factor share of 0.690 for labor and an aggregate depreciation rate of capital of 0.044. The technology parameter B, is normalized to obtain an output of 1.0 in the model's 'base period'. The exact value of B required for this normalization depends on the values of preference parameters but is always between 1.76 and 1.78. Per capita quantities in this economy grow at a rate of ρ per period.

3.4 Government

We calibrate the tax rate on consumption as 5.5%, the tax rate on interest earnings as 40%, and the tax rate on labor income as 20% . Government purchases of goods and services are set to 18% of output for the base case. In the experiments where we vary the social security tax rate we keep all the other tax rates and the level of government purchases constant.

Table 1 summarizes the parameter values of our benchmark model.

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[23] For a discussion of the empirical wealth output ratio see İmrohoroğlu, İmrohoroğlu, and Joines (1998).

Table 1
Calibration

Demographics		
population growth rate	n	0.012
maximum age	J	65
retirement age	j_R	45
conditional survival probabilities	ψ_j	Faber (1982)
efficiency profile	ε_j	Hansen(1993)
Technology		
labor share parameter	α	0.690
depreciation rate	d	0.044
per capita output growth rate	ρ	0.0165
Preferences		
forward-looking subjective discount factor	β_f	*
backward-looking subjective discount factor	β_b	*
quasi-hyperbolic discounting parameter	δ	*
coefficient of relative risk aversion	γ	*
consumption share parameter	φ	0.33
Government		
tax rate on consumption	τ_c	0.055
tax rate on capital income	τ_a	0.40
tax rate on labor income	τ_ℓ	0.20
unemployment insurance replacement ratio	ϕ	0.25
government purchases	G	0.18

* Parameter takes on various values as described in Sections 3.2, 4.1, and 4.2

3.5 Solution Method

In most of our simulations, the discrete set $D = \{d_1,d_2,....,d_m\}$ for asset values is chosen so that $d_1 = 0$ and $m = 4097$. The upper bound d_m is set so that it is never binding, typically a value of 20 times the annual income of an employed agent.[24] We rely on linear interpolation of the value functions so that the choice variables that enter the utility function are essentially continuous variables, yielding nearly-continuous policy functions.

To compute the measures of agent types, we do not use the forward recursion described in the definition of the recursive stationary equilibrium. Although the forward recursion leads to the same outcome and has the benefit of expositional clarity, the alternative of simulating histories of a large number of individuals has computational advantages. In particular, we follow İmrohoroğlu, İmrohoroğlu, and Joines (1998) to calculate the summary statistics of the model economies

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[24] Note that with our choice of $m = 4097$, the state space has 4097×2 points for individuals who are young and $4097 \times 2 \times 2$ points for individuals who are eligible to collect social security benefits. The discrete-state numerical method used in this paper to obtain the policy functions is quite standard. See İmrohoroğlu, İmrohoroğlu, and Joines (1995), İmrohoroğlu (1998) and the 'Practical Dynamic Programming' chapter in Ljungqvist and Sargent (1998).

from these simulations. We start with a newly born agent and randomly draw the employment state and the survival outcome. Given these realizations, we use the optimal policy functions to generate next period's endogenous state variables.[25] We recursively follow this procedure until we receive a death realization, which occurs no later than age 85. We replicate this procedure for a large number of individuals and compute the summary statistics as averages across the replications. We replicated 100,000 agent histories to match the resulting cohort shares to those calibrated for the U.S. economy.

Our computations start with a guess for the aggregate capital stock, labor input, government transfers and social security benefits, solve the individuals' dynamic program to obtain the optimal policy functions, simulate a large number of agent histories, compute the average quantities and check whether they are close to the initial guesses. If so, we have a stationary recursive equilibrium; if not, we iterate on this procedure until convergence.

4 Results

We start this section by examining some of the properties of an economy in which all individuals exhibit time-consistent preferences and behavior ($\beta_b = \beta_f$ and $\delta = 1$). This economy is calibrated to yield a capital-output ratio of 2.52 at a social security tax rate of 10%. Table 2 shows the properties of the steady state of this economy at various social security tax rates. With a 10% tax rate, the steady-state consumption-output ratio is 0.635 and the investment-output ratio is 0.183. Because this is a closed economy, the investment-output ratio is also the saving rate. As the social security tax rate is lowered toward zero, we observe a monotonic increase in the capital stock, investment, and consumption. Complete elimination of the pay-as-you-go social security system raises the saving rate to 0.216 and generates 32% more capital, 12% higher output, and 10% more consumption than an economy with a 10% social security tax rate.

Table 2
Time-Consistent Preferences

τ_s (%)	w	r	Y	C	I	K	L	CV (%)
0	2.565	0.060	1.120	0.698	0.242	3.331	0.301	0.00
2	2.522	0.064	1.092	0.684	0.227	3.127	0.298	1.06
4	2.487	0.068	1.068	0.672	0.215	2.962	0.296	2.19
6	2.455	0.071	1.046	0.661	0.205	2.817	0.294	3.39
8	2.419	0.075	1.022	0.648	0.194	2.665	0.291	4.84
10	2.384	0.079	1.000	0.635	0.183	2.522	0.289	6.91

$\gamma = 2.0$, $\beta_f = 1.00588$, $\delta = 1.0$, $B = 1.765$

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[25] We rely on linear interpolation at this stage also to ensure continuity of both the state and the control variables.

The last column of Table 2 examines the welfare at birth of an individual born into the steady state corresponding to each social security tax rate. The relevant welfare criterion is expected lifetime utility as viewed from age 21, the first period of economic life in our model. This criterion, denoted W_{21}, is described in Section 2.2 and is based on equation (2). According to this criterion, welfare is maximized at a zero tax rate. We can measure the welfare cost of being born into an economy with social security as the consumption supplement (compensating variation) needed to equate the welfare of a newborn individual in that economy to the welfare of an individual born into an economy with no social security. The compensating variation is computed as a fixed percentage increase in consumption at each age. The last column of Table 2 shows these compensating variations, denoted CV. The welfare cost increases faster than linearly in the tax rate so that at a tax rate of 10%, individuals would require an increase in annual consumption of 6.91% to compensate them for living in a world with unfunded social security.

If there were no possibility of dying before the maximum possible age J, then the compensating variation for agents with time-consistent preferences would be the same when viewed from any age. The age-specific welfare criteria W_{j^*} defined in Section 2.2, however, are contingent on survival to age j^*. These criteria sum the utility derived from realized consumption and leisure up through age j^* and the expected utility of future consumption and leisure, where the expectation discounts for mortality risk. A pay-as-you-go social security system taxes all workers but pays benefits only to those who survive to retirement, effectively raising the rate of return to survivors. Because of this actuarial reward for survival, it is possible that individuals who reach sufficiently advanced ages might prefer social security even if a newborn individual does not. This turns out not to be the case for the economy described here, as W_{j^*} is maximized at a social security tax rate of zero for all j^*. Table 3 displays the compensating variation required to make individuals of selected ages indifferent between living in an economy with a social security tax rate of 10% and an economy with no social security. Although individuals of all ages prefer a world without social security, the intensity of their aversion declines with age, reflecting the reward to survival. The fact that even the elderly do not favor unfunded social security is due primarily to the effects of such a system in lowering the aggregate capital stock and lifetime earnings and consumption.

Table 3
Welfare Costs of 10%
Social Security Tax Rate

Age	CV (%)
21	6.91
41	6.46
61	5.47
81	3.01

4.1 Time-Inconsistent Preferences and Behavior

We now consider a world populated by quasi-hyperbolic discounters ($\beta_b = \beta_f$ and $\delta < 1$) who exhibit a form of time inconsistency in preferences that leads to time inconsistency in behavior. Preferences of this sort are characterized by a current one-period discount rate that is higher than future one-period discount rates. This high short-term impatience leads quasi-hyperbolic discounters to postpone saving, and continual deferral may lead these individuals to enter retirement with substantially lower assets than exponential discounters.

4.1.1 The Effect of Quasi-Hyperbolic Discounting on Saving

If social security is to constitute a welfare-improving policy intervention in an economy with this sort of time inconsistency but not in a world of time-consistent preferences, one would expect to find an economically significant difference in saving behavior between quasi-hyperbolic discounters and exponential discounters. To see whether this is the case, we run a "counterfactual" experiment on the exponential economy summarized in Table 2. Specifically, we vary the quasi-hyperbolic discounting parameter δ while setting all other parameters to the values used in the exponential economy. The social security tax rate is set to 10%. This experiment is counterfactual in that these economies fail to replicate the empirical capital-output ratio of 2.52 when $\delta < 1$.

If $\delta < 1$, the optimal policy functions derived at age j^* for ages $j' > j^*$ will no longer be optimal when an individual arrives age j'. As a consequence the age-j' individual will in general deviate from the policy rules derived at any earlier age. We assume that individuals are aware of this feature of their own behavior and that they choose current consumption, saving, and work effort optimally, taking into account the behavior of their future selves.

Table 4 summarizes the results of varying δ. The levels of output, consumption, and capital are normalized to 100.0 in the exponential economy. The results indicate that quasi-hyperbolic discounting does indeed have a significant effect on saving. When $\delta = 0.90$, the steady-state capital stock is about 12.5% below its value in the exponential economy, and a δ of 0.85 causes the capital stock to fall almost 20% below the level in the exponential economy. The magnitude of these effects does not depend in any important way on the social security tax rate. For example, in an economy without social security, a value of $\delta = 0.90$ results in a steady-state capital stock 13% below that obtained with $\delta = 1.0$.

Table 4
Effects of Quasi-Hyperbolic Discounting

δ	Y	C	K	K/Y	w	r
1.00	100.0	100.0	100.0	2.52	2.38	0.079
0.90	94.8	95.4	87.4	2.33	2.30	0.089
0.85	91.9	93.0	81.5	2.24	2.26	0.095

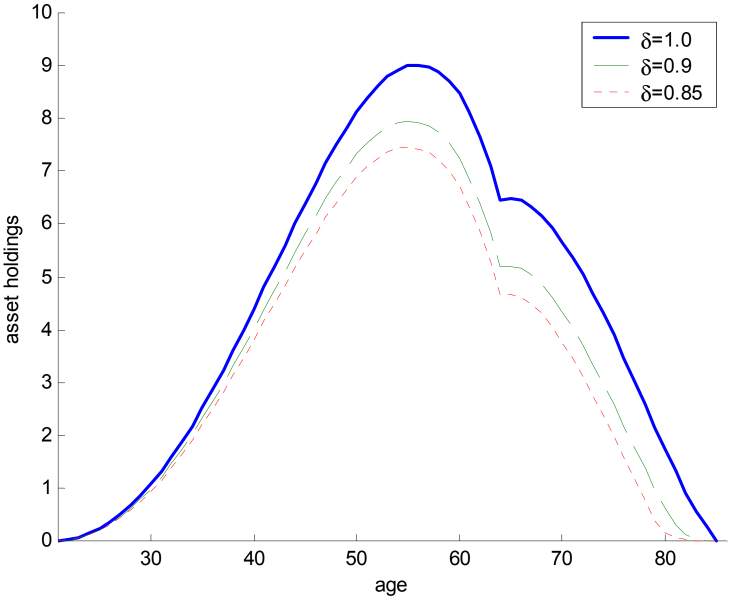

Figure 1: Age-Asset Holding Profiles for Different Values of δ

Figure 1 displays the age-asset profiles for these three economies. The highest profile corresponds to $\delta = 1.0$ and the lowest to $\delta = 0.85$.

4.1.2 Social Security in a World of Quasi-Hyperbolic Discounting

Having established the effect of quasi-hyperbolic discounting on saving, we now ask how much social security can improve welfare in such a world. As with the exponential economy described in Table 2, we require that economies with quasi-hyperbolic discounters and a 10% social security tax rate generate a capital-output ratio that matches the historical U.S. average. We do this by appropriately choosing the standard discount factor β_f for each value of δ so that each (β_f, δ) pair results in a capital-output ratio of 2.52. We then examine the effects of varying the social security tax rate.

Before reporting the results of our social security experiments, however, we first establish a benchmark against which to compare the welfare effects of different tax rates. As our benchmark we consider a world in which individuals have a technology that allows them to commit at age 20 to a state-contingent path of lifetime consumption and work effort. From age 21 until death, these individuals follow decision rules that are the same as those implied by $\delta = 1$, and they effectively overcome the short-term impatience implied by $\delta < 1$.

Table 5 summarizes the consequences of a perfect commitment technology for three configurations of preference parameters that we consider in more detail below. The table first reports the levels of capital, output, and consumption, each scaled relative to a value of 100.0 in the no-commitment case without social security. It then gives the value of the commitment technology, expressed as a fixed percentage increase in consumption at each age in the no-commitment case that makes individuals as well off as having the commitment device. These compensating variations are computed using preferences as viewed from four different ages. The commitment technology results in higher steady-state levels of capital, output, and consumption, and the increase in consumption is concentrated during retirement years.

Table 5
Perfect Commitment Technology

	$\gamma = 2.0$		$\gamma = 1.0$
	$\delta = 0.90$	$\delta = 0.85$	$\delta = 0.90$
	A. Behavior		
K	114.0	121.3	116.1
Y	105.2	107.6	105.8
C	103.4	104.9	103.7
	B. Compensating Variation		
W_{21}	3.28	4.49	3.74
W_{41}	3.20	4.62	3.75
W_{61}	3.25	4.50	3.73
W_{81}	4.68	7.07	5.09

Figure 2 shows the age-consumption profiles for the economy with $\gamma = 2.0$ and $\delta = 0.90$ in the absence of social security, both with and without the commitment device. Both behavior and welfare seem more sensitive to the quasi-hyperbolic discounting parameter than to the inverse elasticity of intertemporal substitution. These results indicate that the steady-state welfare costs to quasi-hyperbolic discounters of their time-inconsistent behavior are substantial.[26]

We now examine the effectiveness of unfunded social security as a substitute for a perfect commitment technology in maintaing old-age consumption. Table 6 summarizes the economic effects of varying the social security tax rate in economies in which the preference parameters β_f, γ, and δ take on different

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[26] Laibson (1997, p. 467) reports compensating variations that are much smaller than those in Table 5. There appear to be two reasons for the difference. First, Laibson's welfare analysis is for a partial commitment technology that takes the form of an illiquid asset. Second, his analysis is for an infinitely-lived representative agent and includes the change in consumption during the transition from one steady state to another, whereas our comparison is only of the two steady states. Barro (1999, p. 1139) examines the value of perfect commitment. Because he takes into account the transition between steady states, he reports a smaller welfare effect for a given change in steady-state capital than we do. Assuming log utility, he finds that the value of commitment is small unless the degree of short-term impatience is high.

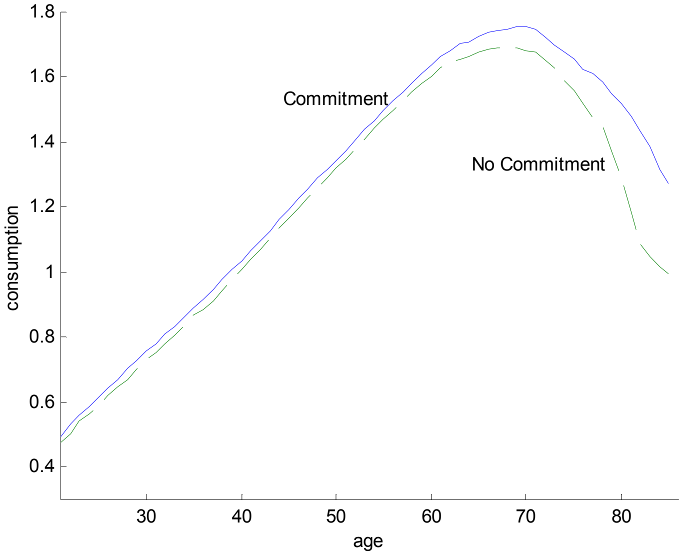

Figure 2: Age-Consumption Profiles with and without Commitment

values. The last two parameters are specified a priori, and β_f is then chosen to yield a capital-output ratio of 2.52. Our central value for γ, the inverse elasticity of intertemporal substitution, is 2.0. Given $\gamma = 2.0$, the quasi-hyperbolic discounting parameter δ takes on values of 1.00 (the exponential case from Table 1), 0.90, and 0.85. In addition, we consider values of γ of 1.0 and 3.0, each paired with a δ of 0.90. The table is normalized so that capital, output, and consumption are all 100.0 when the social security tax rate is 10%, reflecting the fact that all five economies have been calibrated to yield the same values of these three aggregates.

Social security reduces the steady-state values of capital, output, and consumption in each of the economies considered. The results for $\gamma = 2.0$ indicate that the capital stock would be about 1/3 larger in the absence of social security. The magnitude of this effect is similar across the three values of δ, although it is somewhat more pronounced with quasi-hyperbolic discounting. A lower elasticity of intertemporal substitution ($\gamma = 3.0$) implies a larger effect of social security on steady-state capital, while $\gamma = 1.0$ implies a smaller effect. The smaller the elasticity of substitution, the greater the reduction in the saving of workers when the government attempts to reallocate consumption toward retirement years through the payroll tax.

Table 6
Effects of Social Security with Quasi-Hyperbolic Discounting

	$\gamma = 2.0$			$\gamma = 3.0$	$\gamma = 1.0$
$\tau_s(\%)$	$\delta = 1.00$	$\delta = 0.90$	$\delta = 0.85$	$\delta = 0.90$	$\delta = 0.90$
			A. Capital		
0	132.0	134.0	136.4	145.7	123.1
2	123.9	125.0	126.8	133.3	119.0
4	117.4	117.7	118.7	123.2	114.6
6	111.7	111.6	111.9	114.6	109.3
8	105.6	105.6	105.5	107.1	104.3
10	100.0	100.0	100.0	100.0	100.0
			B. Output		
0	112.2	112.9	114.0	116.4	109.5
2	109.4	109.8	110.6	112.5	107.8
4	107.0	107.2	107.6	109.9	106.0
6	104.8	104.8	104.9	105.8	103.8
8	102.4	102.4	102.4	102.8	101.8
10	100.0	100.0	100.0	100.0	100.0
			C. Consumption		
0	109.9	110.5	111.3	112.6	108.4
2	107.8	108.2	108.9	110.0	106.7
4	105.9	106.2	106.5	107.3	105.3
6	104.1	104.2	104.4	104.9	103.4
8	102.1	102.1	102.1	102.4	101.5
10	100.0	100.0	100.0	100.0	100.0
			D. Other Parameter Values		
β_f	1.0058	1.0117	1.0146	1.0261	0.9980
B	1.7652	1.7740	1.7880	1.7766	1.7625

We can now use these economies to examine the welfare consequences of social security. Although the effect of social security on aggregate consumpton does not seem very sensitive to the value of the quasi-hyperbolic discounting parameter, there are two reasons why the welfare effects might still depend on δ. First, social security affects not only aggregate consumption but also its allocation over the life cycle, and these intertemporal reallocations might differ across the three values of δ considered. Second, individuals with different δ's have different preferences, so they value a given lifetime consumpton sequence differently. In addition, the welfare effects might depend on γ. A sharp decline in old-age consumption due to quasi-hyperbolic discounting might have higher welfare costs, and social security might lead to correspondingly larger welfare gains (at least as viewed from old age), the lower the elasticity of intertemporal substitution. But as we have seen, a low elasticity of substitution also raises the costs of social security, which take the form of lower steady-state capital and lifetime consumption.

As noted in Section 2.2, with time-inconsistent preferences we can view a single individual as a collection of J individuals, each of a different age and each with a different set of preferences. Since these J individuals need not agree on their ranking of different consumption and labor sequences, we use equation (2) to compute welfare measures W_{j*} for each age $j^* = 1, 2, ...J$. For social security tax rates between 2% and 10%, we determine the first age $\widehat{j}$ at which $W_{\widehat{j}}$ is greater with social security than without. (It turns out in our experiments that if social security raises welfare as viewed from age $\widehat{j}$, it also raises welfare as viewed from any age $j > \widehat{j}$.) We also calculate the fraction of the population falling into ages $j \geq \widehat{j}$.

Table 7 reports these welfare calculations for three of the five economies summarized in Table 6. We omit the exponential economy, where even a 2% social security tax rate lowers welfare as viewed from all ages, and the economy with $\gamma = 3.0$, where a social security tax rate as high as 10% lowers welfare as viewed from any age.[27] In the remaining economies, we find that social security raises the welfare of quasi-hyperbolic discounters as viewed from sufficiently advanced ages. With $\gamma = 2.0$, the fraction of the population falling into these cohorts never exceeds about 5%, however. With $\gamma = 1.0$, a 4% social security tax rate increases welfare as viewed from ages 70 and greater, corresponding to more than 10% of the population. The aggregate welfare measure W, which weights each of the age-specific indicators W_{j*} by the unconditional probability of surviving to age j^*, is always higher without social security than with any of the tax rates considered here. Overall, these results indicate that unfunded social security is not particularly effective in correcting for the undersaving resulting from quasi-hyperbolic preferences, at least for the degrees of myopia considered here.

Table 7
Who Prefers Social Security?

	$\gamma = 2.0$				$\gamma = 1.0$	
	$\delta = 0.90$		$\delta = 0.85$		$\delta = 0.90$	
τ_s(%)	Age	Share (%)	Age	Share (%)	Age	Share (%)
2	81	1.98	79	3.06	72	8.29
4	77	4.32	77	4.32	70	10.20
6	76	5.02	76	5.02	73	7.40
8	80	2.50	79	3.06	76	6.56
10	–	0.00	82	1.51	82	1.51

In light of the apparently widely held view that social security may raise the welfare of short-sighted individuals who fail to save adequately for their retirement, the question arises as to why our model implies such meager welfare

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[27] With $\gamma = 3.0$, welfare as viewed from age 85 is maximized with a tax rate of 6%, but welfare as viewed from any other age is higher with no social security than with any of the tax rates we have examined.

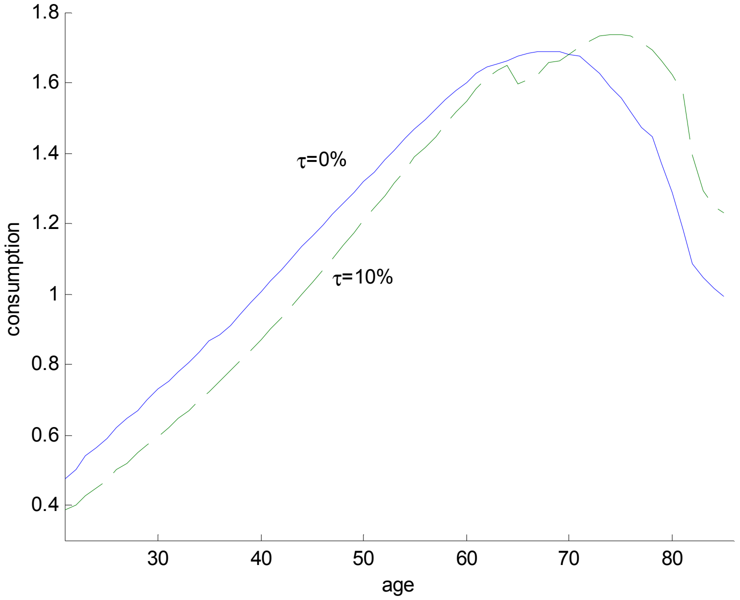

Figure 3: Age-Consumption Profiles with and without Social Security

gains. Table 6 has already documented that unfunded social security depresses steady-state capital, output, and consumption, and these effects are if anything stronger with quasi-hyperbolic discounting than in a pure exponential economy. If the consumption decline in old age were sufficiently great in the absence of social security, and if social security were sufficiently effective in reallocating consumption from working years to retirement, the welfare gains from this reallocation might outweigh the losses from lower lifetime consumption. Table 8 and Figure 3 examine this issue.

The figure shows age-consumption profiles for individuals with $\gamma = 2.0$ and $\delta = 0.90$ at social security tax rates of zero and 10%.[28] Like the commitment technology shown in Figure 2, social security raises old-age consumption, but

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[28] With social security, simulated consumption exhibits a discrete drop at retirement similar to that documented by Bernheim, Skinner, and Weinberg (1997) and Banks, Blundell, and Tanner (1998). In our model, the institutional features of social security cause an increase in the effective labor income tax rate at age 65 which is similar to that occurring when people reach their early 60s in the U.S. system. This increase in the effective tax rate causes a discrete reduction in hours worked which is not observed in the absence of social security. Because individuals smooth a composite of leisure and market goods, a sudden increase in leisure is accompanied by a drop in consumption expenditures. Bernheim, Skinner, and Weinberg have noted that any drop in consumption at retirement could be associated with a reduction in work-related expenditures that might be more properly deducted from earnings rather counted as consumption. Because of the one-good nature of our model, we are unable

unlike the commitment technology, it does so at the cost of noticeably reduced consumption during working years. The table reports average consumption levels for the last decade before retirement, the first decade after retirement, and two periods of extreme old age for each of the three economies with $\gamma = 2.0$. Consumption is normalized to 100.0 in the decade before retirement in the regime without social security, and consumption in the other cells is scaled relative to this.

Table 8
Old-Age Consumption

	$\delta = 1.00$		$\delta = 0.90$		$\delta = 0.85$	
Age	$\tau_s = 0.0$	$\tau_s = 0.1$	$\tau_s = 0.0$	$\tau_s = 0.1$	$\tau_s = 0.0$	$\tau_s = 0.1$
55-64	100.0	97.7	100.0	96.7	100.0	95.3
65-74	106.6	108.3	105.3	105.8	105.3	104.3
75-81	95.8	112.6	88.9	106.1	85.6	100.2
82-85	81.4	101.3	65.5	81.7	62.9	76.0

Without social security, consumption peaks in the first decade after retirement and then declines below pre-retirement levels. Consumption of quasi-hyperbolic discounters aged 82-85 is less than two thirds the pre-retirement level. Social security reduces pre-retirement consumption in each economy, reflecting the effects of the payroll tax and lower lifetime earnings, and the reduction is greater for quasi-hyperbolic discounters. Consumption in the first decade after retirement rises or falls slightly as a result of the 10% tax rate, depending on the value of δ. Social security has its greatest effect on consumption in extreme old age. In all three economies, a tax rate of 10% results in consumption at ages 75-81 that is noticeably above the pre-retirement level.[29] The story is different among the very oldest individuals, however. Although social security raises the consumption of quasi-hyperbolic discounters aged 82-85, the effect is not large enough to prevent a substantial shortfall relative to pre-retirement levels. Individuals with $\delta = 0.90$ drive their asset holdings to less than 20% of annual consumption by age 81, and those with $\delta = 0.85$ reach even lower asset levels even sooner. In constrast, the consumption of exponential discounters at ages

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to address this question, and our finding is due solely to a substitution between leisure and market goods in general.

[29] Consumption in our model, either with or without social security, seems to peak later than in the data. One potential explanation is age dependence in the period utility function $u(c_j, \ell_j)$, possibly due to changes in household composition. This age dependence would affect the shape of the age-consumption profile that maximizes lifetime utility as viewed from any age, and it would similarly affect the age-consumption profile that would be chosen in advance by an individual who could perfectly commit to a given path. The welfare loss from myopic behavior results from discrepancies between this optimal path and the one actually followed. We have not attempted to incorporate age-specific taste shifters because measuring them would be difficult and because it seems unlikely that they would qualitatively affect the discrepancy between the optimal and actual consumption paths.

82-85 remains above the pre-retirement level.[30] Quasi-hyperbolic discounters have higher consumption in extreme old age with the commitment technology shown in Figure 2 than with a 10% social security tax rate.

Table 8 indicates that social security does effect a redistribution of consumption from working to retirement years, but the benefits of this reallocation are apparently not sufficient to outweigh the cost of lower steady-state capital and lifetime earnings. Because these are closed economies, it is possible that the reduction in the capital stock is overstated. To examine the sensitivity of our findings to the closed-economy assumption, we analyze open-economy variants of the three economies of Table 7, in each of which social security led to welfare gains as viewed from old age. We assume that the world capital-output ratio is equal to 2.52, the value to which each of these economies is calibrated when the social security tax rate is 10%. The domestic capital-output ratio remains fixed at this level, and the wage rate and the interest rate remain fixed as we vary the tax rate. Changes in the financial wealth of domestic residents cause one-to-one changes in net foreign assets.

Table 9 shows the effects of social security on various economic aggregates in these three small, open economies. The income measure reported in the table is GNP, which includes the return on assets held abroad. Each aggregate is normalized to 100.0 with a tax rate of 10%. The effects are fairly robust across the different parameterizations reported in the table, and they indicate that the depressing effect of social security on total asset holdings is roughly three times as large in an open economy as in a closed one. As the social security tax rate is raised in a closed economy, the reduction in the capital stock raises the interest rate, which in turn mitigates the reduction in saving. The return to saving is fixed in the small, open economy, however, and does not tend to damp the change in asset accumulation. The effect of social security on GNP and aggregate consumption is about the same in these small, open economies as in their closed-economy counterparts.

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[30] It is curious that the consumption of quasi-hyperbolic discounters drops so sharply a few years before the certain death date of 85. One explanation for this phenomenon is that these individuals simply require about 15 years to exhaust their retirement assets. An alternative is that impending mortality exacerbates their high short-term impatience, causing them to run down their assets a few years before certain death. To help us distinguish between these two explanations, we simulated a version of the model with $\delta = 0.90$ and 75 periods (a maximum real-time age of 95). Individuals in this model drove their asset levels to 16% of annual consumption by age 82, tending to support the former explanation. Thus, the precipitous drop in the consumption of quasi-hyperbolic discounters in their early 80s does not seem to be merely an artifact of the certain death date.

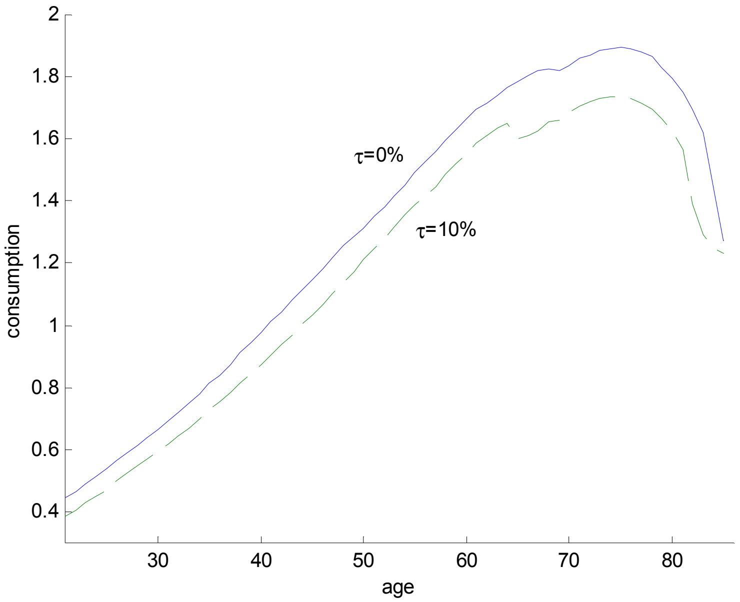

Figure 4: Age-Consumption Profiles in a Small, Open Economy

Table 9
Social Security in an Open Economy

		$\tau_s = 0.0$		
	$\tau_s = 0.1$	$\gamma = 2.0$		$\gamma = 1.0$
		$\delta = 0.90$	$\delta = 0.85$	$\delta = 0.90$
Assets	100.0	199.6	195.8	187.0
GNP	100.0	112.9	112.5	116.2
Consumption	100.0	110.5	110.3	106.5

The effects of unfunded social security in enhancing welfare and reallocating consumption from working to retirement years are weaker in an open economy than in a closed one. In each of the open economies in Table 9, lifetime utility as viewed from all ages is higher without social security than with any tax rate we have examined.

Figure 4 shows the age-consumption profiles for tax rates of zero and 10% with $\gamma = 2.0$ and $\delta = 0.90$. Because the interest rate is unchanged, these two consumption profiles have approximately the same shape, and social security results in a roughly proportional decline in consumption at all ages. Compare this with the closed-economy profiles in Figure 3. It appears that the effect of social security in reallocating consumption to retirement years in the closed economy is largely an endogenous response to the increase in the interest rate, which steepens the age-consumption profile.

4.1.3 Non-optimizing Agents

It might be argued that the individuals we have considered thus far are not very myopic. First, they are rather sophisticated in recognizing the time inconsistency resulting from their preference structure, and they optimize given those preferences. In this sense, they are not short-sighted at all. Second, the degree of myopia as represented by δ might not be large enough to generate serious welfare consequences. Concerning this second point, the results in Tables 4 and 5 above suggest that both the behavioral and the welfare consequences of a δ in the neighborhood of 0.85 to 0.90 can be significant. Nevertheless, Laibson, Repetto, and Tobacman (1998) have argued that experimental evidence supports values of δ closer to 0.60, and the scope for social security to improve welfare might be substantially greater with a lower δ. As Laibson, Repetto, and Tobacman have also pointed out, however, it is difficult if not impossible to compute equilibria of models with sophisticated agents whose values of δ are substantially lower than those we have considered.

Some of the literature on myopia has made a distinction between optimizing but time-inconsistent behavior of the sort we have considered and a more fundamental failure to plan for the future.[31] There are many ways in which individuals could fail to optimize, and we do not propose to consider a long list of possibilities. We do examine one such candidate, however. Specifically, we consider individuals with preferences as given in equation (2) who naively think that their future selves will adhere to optimal plans derived today. Because these naive individuals have the same preferences as the relatively sophisticated ones considered above, social security will have different welfare effects on the two types of agents only to the extent that they behave differently.

It is not clear a priori whether naive agents save less than sophisticated ones with the same preferences. A sophisticated individual will save more than a naive one in order to achieve any target level of wealth farther than one period into the future, because the sophisticated individual realizes that his profligate intermediate self will tend to consume any assets set aside for the more distant future. On the other hand, a dollar of saving is more valuable relative to a dollar of current consumption if the assets will be consumed optimally (as viewed by the current individual) over time rather than dissipated on consumption in the near future. Therefore, saving will appear more attractive to a quasi-hyperbolic discounter who naively believes that his future selves will optimally allocate

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[31] For example, Laibson (1996, p. 4; 1997, p. 449) and Barro (1999, p. 1127) distinguish between the model with sophisticated quasi-hyperbolic discounters and that of Akerlof (1991), in which "the standard assumption of rational, forward-looking, utility maximizing is violated." Bernheim, Skinner, and Weinberg (1997, pp. 14, 20) contrast both the standard life-cycle model and the quasi-hyperbolic model of sophisticated agents with a world in which individuals operate "outside the framework of rational, farsighted optimization" and "engage in a variety of heuristic and quasi-rational strategies to determine their saving prior to retirement (or simply procrastinate, as in Akerlof, 1991)."

additional resources over time than to a more sophisticated agent who realizes that this is not the case.

To isolate the consequences of the computational mistakes made by quasi-hyperbolic discounters who fail to take into account their future behavior, we examine three economies populated by naive agents with the same preferences as those in Table 7. It turns out to be rather straightforward to compute the policy functions for naive agents with values of δ much lower than those considered thus far. Therefore, we allow for still more severe myopia by also considering economies populated by naive individuals with $\delta = 0.60$, which is the benchmark value in Laibson (1996).

The difference in behavior between naive and sophisticated individuals is greater without than with social security, and among the three sets of preference parameters from Table 7, the difference is greatest with $\delta = 2$ and $\gamma = 0.85$. With those preferences and no social security, the capital stock is 3.4% lower and aggregate consumption is 0.9% lower in an economy with naive agents than with sophisticated ones. Furthermore, the two types of agents allocate consumption across the life cycle in an almost identical manner. As a result, the welfare consequences of social security are qualitatively the same in the two economies. A 10% social security tax rate raises the welfare of naive individuals as viewed from ages 80 and above (2.5% of the population), compared with ages 82 and above (1.5% of the population) for sophisticated individuals.

We now consider a world populated by naive quasi-hyperbolic discounters with preference parameters $\gamma = 2$ and $\delta = 0.60$. To establish the effects of such extreme myopia on saving behavior, we first conduct the counterfactual experiment of introducing $\delta = 0.60$ into the exponential economy of Table 2, holding all other parameter values fixed. The resulting capital stock is 49.5% of that in the exponential economy. To examine the welfare effects of social security in an economy with $\delta = 0.60$, however, we must recalibrate the standard time discount factor β_f to 1.03905 so that this economy generates a capital-output ratio of 2.52 when the social security tax rate is 10%.

Setting the tax rate to zero in this economy increases the steady-state values of capital, output, and consumption by 19.9%, 8.6%, and 7.9%, respectively. Thus, social security depresses saving less with $\delta = 0.60$ than with the higher values considered above. Figure 5 shows the age-consumption profiles in this economy with tax rates of zero and 10%.

As can be seen, social security raises old-age consumption substantially more than was the case with higher values of δ. This fact, combined with the smaller effect on the capital stock, means that a tax rate of 10% raises welfare as viewed from all ages. Individuals in an economy with a 10% tax rate would sacrifice a substantial fraction of annual consumption rather than give up social security entirely. The compensating variations for various ages are shown in Table 10, with negative numbers indicating a willingness to sacrifice consumption rather than forego social security. Furthermore, the optimal tax rate as viewed from all ages is substantially higher than the 10% value that approximates the current U.S. system.

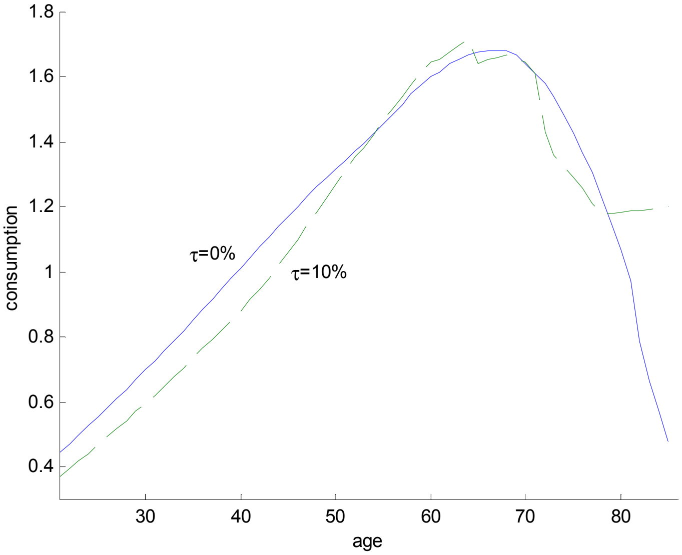

Figure 5: Age-Consumption Profiles for Naive Agents with $\delta = 0.6$

Table 10
Welfare Costs of 10%
Social Security Tax Rate

Age	*CV* (%)
21	−10.09
41	−6.07
61	−7.03
81	−14.50

In summary, our model indicates that there is little scope for unfunded social security to raise welfare in a world of sophisticated quasi-hyperbolic discounters with values of δ in the neighborhood of 0.85 to 0.90. Finding a welfare-enhancing role requires more extreme myopia. Simply replacing sophisticated agents with non-optimizing counterparts who fail to recognize the implications of their own future preferences scarcely increases the beneficial effects of social security, at least for δ in the range of 0.85 to 0.90. Social security does significantly raise the welfare of naive quasi-hyperbolic discounters with $\delta = 0.60$, however. We do not know whether this result would carry over to a world of sophisticated agents with the same δ, as we have thus far been unable to solve such a model.

4.2 Time-Inconsistent Preferences, Time-Consistent Behavior

In the previous section, individuals had time-inconsistent preferences that led to time-inconsistent behavior. In that environment, the welfare effects of social security depended much more on the time inconsistency in behavior than on any age dependency in preferences per se. According to equation (2), an age-J individual looking back from the last possible period of life places a weight on $u(c_{j^*}, \ell_{j^*})$, the period utility of consumption and leisure at age j^*, that is $\beta_b^{j^{'}-j^*}$ times the weight placed on the period utility of consumption and leisure at some later age $j^{'}$. Now consider a hypothetical individual of age 21, the first period of our planning problem, who can be assured of living to age $j^{'}$. If $\beta_b = \beta_f$, as we have assumed thus far, this individual places the same relative weights on the two period utilities as does the age-J individual unless $j^* = 21$, in which case $u(c_{j^*}, \ell_{j^*})$ gets additional weight if $\delta < 1$. In this sense, the difference in preferences between individuals in the first and last possible periods of economic life is not very great, even if $\delta < 1$. It is true that, according to equation (2), an individual looking toward the future discounts for mortality risk, giving rise to a discrepancy between his preferences and those of an older self conditional on survival. But we saw above that with time-consistent behavior, this conditional age dependence in preferences was insufficient to make even the oldest individuals prefer social security.

We now consider a form of time inconsistency in preferences that allows more scope for disagreement between the young and the old. In this specification $\delta = 1$, so that behavior is time-consistent. But now $\beta_b > \beta_f$ so that, even apart from differences due to mortality risk, an individual places more weight on $u(c_{j'}, \ell_{j'})$ relative to $u(c_{j^*}, \ell_{j^*})$ when looking back from age $j^{'}$ than when looking forward from age j^*. Thus, an old individual may regret having consumed so much when young. We define the degree of this type of myopia, $\widehat{\sigma} \geq 0$, implicitly by $\beta_b = 1/(1+\sigma-\widehat{\sigma})$, where σ is implicitly defined by $\beta_f = 1/(1+\sigma)$. This form of retrospective time inconsistency, or regret, has been considered by Feldstein (1985) and Caplin and Leahy (1999).

Behavior depends on the preference parameters β_f, γ, and δ but not on β_b. Because we take $\gamma = 2$ and $\delta = 1$, we set $\beta_f = 1.0058$ to ensure a capital-output ratio of 2.52 with a 10% social security tax rate. Thus, the behavior of this economy is identical to that displayed in Table 2. Within this environment we examine the welfare effects, as viewed from different ages, of varying the social security tax rate. We restrict our experiment to tax rates of zero, 2%, 4%, ..., 10% and ask the following questions: For different degrees of myopia $\widehat{\sigma}$, what is the earliest vantage point j^* from which lifetime welfare is higher with a positive social security tax rate than with a rate of zero and what is the earliest vantage point $j^{'}$ from which a 10% tax rate is preferred to any of the other tax rates under consideration?

Table 11 contains the answers to these questions. Individuals as young as 40 prefer some social security when the degree of myopia is as great as 8% per year. It turns out in our experiments that if an individual of age j^* prefers a given social security tax rate to zero, individuals of any greater age also prefer that tax rate. In all of the cases we have considered, the optimal tax rate as viewed from any age is either zero or at least 6%, with more modest rates never being preferred. In order for a majority of the population to view a tax rate of 10% or more as optimal, the degree of myopia must be at least 10% per year. Given our assumed value of β_f, myopia of 10% implies $\beta_b = 1.1183$, which in turn implies that the weight on past outcomes declines to 2/3 the weight on current outcomes after about 3.5 years and to 1/3 after 10 years. While we know of no empirical evidence on the magnitude of retrospective discounting, an annual rate of 10% is substantially larger than is generally assumed for ex ante discounting.

Table 11
Retrospective Time Inconsistency

	Positive tax rate preferred		10% tax rate preferred	
$\widehat{\sigma}$ (%)	Age	Share (%)	Age	Share (%)
0	–	0.0	–	0.0
2	57	26.7	64	6.6
4	46	45.0	56	28.2
6	42	52.6	49	39.7
8	40	56.6	45	46.9
10	38	60.6	43	50.7

5 Concluding Remarks

In this paper we examine the welfare effects of unfunded social security on individuals who exhibit two distinct forms of time-inconsistent preferences. In addition to incorporating quasi-hyperbolic discounting, our model nests the retrospective form of time inconsistency analyzed by Feldstein (1985) and extends his framework to include a wider range of benefits and costs of social security. In order to examine the role of social security in an economy with time-inconsistent preferences, we construct a model which consists of overlapping generations of 65-period-lived individuals facing mortality risk, individual income risk, and borrowing constraints. Private annuity markets and credit markets are closed by assumption. Individuals in this economy are heterogenous with respect to their age, employment status, retirement status, hours worked, and asset holdings.

We consider two cases of quasi-hyperbolic discounting. First, we allow for sophisticated behavior by assuming that agents take into account the fact that their future selves will also engage in quasi-hyperbolic discounting. This assumption on beliefs results in a game that the agent plays with the future selves. Second, we assume that agents are naive in the sense that they ignore quasi-hyperbolic discounting by their future selves.

In this environment social security may provide additional utility for myopic agents who regret their saving decisions when they find themselves with low consumption after retirement. In addition, social security may substitute for missing private annuity markets in helping agents allocate consumption in the face of uncertain life spans. On the other hand, social security distorts aggregate saving and labor supply behavior and affects the wage rate and the interest rate. Consequently, whether or not social security is welfare enhancing even for myopic agents is a quantitative question. Because individuals have time-inconsistent preferences, it is necessary to evaluate lifetime welfare from different vantage points in the life cycle.

Our findings can be summarized as follows:

- With time-consistent preferences, the actuarial reward for survival that social security provides is not quantitatively large enough to render a world with social security desirable; there is a significant welfare cost to social security, viewed at any age, although the cost declines with age. These findings are in accord with previous research.

- Quasi-hyperbolic discounting at the rate of 15% lowers the capital stock by about 20% at any social security tax rate, and there are substantial steady-state welfare costs to quasi-hyperbolic discounters of their time-inconsistent behavior.

- Social security is a poor substitute for a perfect commitment technology in maintaining old-age consumption; the capital stock would be about one-third larger in the absence of social security than with a tax rate of 10%.

- In a world of quasi-hyperbolic discounters, sophisticated or naive, unfunded social security generally does not raise welfare for short-term discount rates of at least 15%.

- Social security does raise the welfare of naive quasi-hyperbolic discounters with a short-term discount rate of 40%.

- With retrospective time inconsistency of the form considered by Feldstein (1985) and Caplin and Leahy (1999), the ex ante annual discount rate must be at least 10% greater than seems warranted ex post in order for a majority of the population to prefer a social security tax rate of 10% to no social security.

Overall, our quantitative findings question the efficacy of unfunded social security in correcting for the undersaving resulting from quasi-hyperbolic preferences.

Appendix

The remaining lifetime utility of an age-j agent exhibiting sophisticated quasi-hyperbolic discounting, evaluated using age-j preferences, is

$$\begin{aligned} U_j &= u(\widehat{c}_j, \widehat{\ell}_j) + \delta E_j \sum_{i=1} \beta_f^i u(\widehat{c}_{j+i}, \widehat{\ell}_{j+i}) \\ &= u(\widehat{c}_j, \widehat{\ell}_j) + \delta \beta_f E_j \left\{ u(\widehat{c}_{j+i}, \widehat{\ell}_{j+i}) + \sum_{i=1} \beta_f^i u(\widehat{c}_{j+i+1}, \widehat{\ell}_{j+i+1}) \right\}. \end{aligned}$$

The term in braces looks like the standard V_{j+1} from the $\delta = 1$ problem, in which case the solutions would be the same as in the 'naive $\delta < 1$' case. But the term in braces does not equal V_{j+1} because the consumption and leisure sequences are not the same as that resulting from the $\delta = 1$ problem, and sophisticated agents take this into account. The task is to find a convenient expression for the term in braces and hope that it can be computed recursively. Call this quantity V_{j+1}^*. Starting at age J we have $V_J^* = u(\widehat{c}_J, \widehat{\ell}_J)$ which, for this age at least, does equal V_J from the $\delta = 1$ problem. Thus

$$E_{J-1} V_J^* = E_{J-1} u(\widehat{c}_J, \widehat{\ell}_J).$$

At age $J-1$ we are looking for

$$E_{J-2}\{u(\widehat{c}_{J-1}, \widehat{\ell}_{J-1}) + \beta_f u(\widehat{c}_J, \widehat{\ell}_J)\}.$$

By the law of iterated expectations, the above expression is equal to

$$\begin{aligned} & E_{J-2} \left\{ u(\widehat{c}_{J-1}, \widehat{\ell}_{J-1}) + \beta_f E_{J-1} u(\widehat{c}_J, \widehat{\ell}_J) \right\} \\ = \; & E_{J-2} \left\{ u(\widehat{c}_{J-1}, \widehat{\ell}_{J-1}) + \beta_f E_{J-1} V_J^* \right\}. \end{aligned}$$

Thus we can define

$$V_{J-1}^* = u(\widehat{c}_{J-1}, \widehat{\ell}_{J-1}) + \beta_f E_{J-1} V_J^*. \tag{15}$$

The remaining V_j^* can be then computed recursively in a similar manner.

Unlike the 'naive $\delta < 1$' case, the 'sophisticated $\delta < 1$' cannot be solved in a simple two-pass procedure where we first get the V_j^* and then solve for the $\widehat{V}_j$ and the associated policy functions. Instead, after computing the $\widehat{V}_j$ and the corresponding policy functions at each age j, we immediately compute V_j^*, because this quantity is needed to solve for $\widehat{V}_{j-1}$. Also note that there is no 'max' operator in (15) above as the age-$J-1$ agent cannot commit his future self to a particular course of action but instead he must take the actions of his future self as given.[32]

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[32] For values of δ that are sufficiently close unity, this procedure has been successful in delivering the value functions and policy functions using the discrete-state methodology. However, for $\delta < 0.85$, we have encountered numerical problems suggesting indeterminacy or multiplicity of equilibria, similar to those reported by Laibson, Repetto, and Tobacman (1998) and Krusell and Smith (1999).

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