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THE ALLOCATION OF GOODS AND TIME OVER THE BUSINESS CYCLE*

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ABSTRACT

A Beckerian model of household production is developed to study the allocation of capital and time between market and home activities over the business cycle. The adopted framework treats the business and household sectors symmetrically. In the market, labor interacts with business capital to produce market goods and services, and likewise at home the remaining time, leisure, is combined with household capital to produce home goods and services. The theoretical model presented is parameterized, calibrated, and simulated to see whether it can rationalize the observed allocation of capital and time, as well as other stylized facts, for the postwar U.S. economy.

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I. Introduction

There are two striking facts regarding the accumulation of capital in the nonmarket or household sector:

(1) The stock of household capital, defined as the combined stock of consumer durables and residential capital, is higher than the stock of business nonresidential capital. The average ratio between the two capital stocks in the 1954–1988 period is 1.13.

(2) Investment in household capital is highly procyclical. As can be seen in Figure 1, it moves together with and even leads movements in business investment. Figure 1 also shows the higher level of household investment, which is a reflection of observation (1).

These two observations indicate that household capital accumulation is both quantitatively important and closely related to business investment activity. The macroeconomic question that arises is: How is the allocation of capital between the business and household sectors over the business cycle determined? The purpose of this paper is to address this question. Specifically, a macroeconomic model that stresses household activities is developed to study the allocation of capital and time across the two sectors. The theoretical model developed is parameterized, calibrated and simulated to see whether it can rationalize the observations above, as well as other stylized facts for the postwar U.S. economy.

By and large, the business cycle literature is silent on the role of the capital stock held by households. However, some studies, such as Kydland and Prescott (1982) and Christiano (1988), do consider household capital by adding it to business capital and including its services in total consumption. The basic assumption underlying this aggregation procedure is that household and business capital are perfect substitutes. For this reason, the composition of total capital investment between business and household investment is indeterminate. Thus, this modeling strategy, which has been useful for the analysis of business fluctuations, is not well equipped to address the question at hand.

Another problem with the perfect substitution assumption arises when taxation of market activity is considered. Although both capital stocks are subject to property taxes, only business capital is subject to income taxation, which is far from being trivial [see Jorgenson and Yun (1986)]. This creates a significant distortion favoring the accumulation of household capital at the expense of business capital. This feature of the tax system, which is incorporated in the current analysis, is likely to be important for modeling the behavior of business and household investment. In a model with perfect substitution between the two capital stocks, business capital would be driven to zero.

Obviously, then, a more complete analysis of capital accumulation requires a framework that assigns to household capital a distinct role from business capital. The main methodological issue involved here is the development of a framework to model household activities. In real business cycle models, as advanced by Kydland and Prescott (1982) and Long and Plosser (1983), the household sector is encapsulated in a utility function defined over consumption and leisure that is not affected by physical capital accumulation and technological progress. A simple extension of this approach to the problem at hand would be to include the services of household capital as an additional argument in the utility function. However, given that household activities involve approximately as much capital as business activities, and three times as much (nonsleeping) time, a more detailed treatment of the household sector could prove fruitful.

To provide a natural structure to this analysis, a Beckerian (1965) view of household production is adopted. The similarities between market and home activities are stressed by following the extreme methodological strategy of symmetric treatment of both activities. There are two production functions, one for market activities and the other for nonmarket activities. In the first, labor interacts with market capital (equipment and structures) to produce market goods and services. In the second, the remaining time interacts with household capital (consumer durables and residence) to produce home goods and services. For example, watching TV, listening to music, or playing with a computer, combines time with capital to produce home goods (entertainment). Utility depends only on the consumption of market and home goods. Nonmarket time affects utility only via being an input in the production of home goods. The basic premise of this paper is that considerations of capital accumulation and technological change are important for activities carried out at home as well as in business. As in the market sector, the productivity of time spent in nonmarket activities depends on the state of knowledge and the stock of capital in the household. The only asymmetry between the two sectors is that capital goods can be produced by the business sector alone.

The paper is organized as follows: Section II presents the model and the solution technique. Section III describes the parameterization of the model and the results from the quantitative analysis. Finally, the findings are summarized in Section IV.

II. The Model and Solution Technique

A. The Economic Environment

Consider an economy in which the representative household maximizes its expected lifetime utility, as given by

$$(1) \qquad E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_t,h_t)\right] \quad 0<\beta<1,$$

where c_t is consumption of nondurable goods and services purchased in the market, and h_t is consumption of goods and services produced at home. The momentary utility function U, in addition to having the usual properties, is assumed to be homogeneous of degree q.

Market and home production technologies are described by

(2) Market: $y_t = F(k_t, z_t \ell_t)$,

(3) Home: $h_t = H(d_t, z_t(1-\ell_t))$,

where k_t is the business capital stock, d_t is the household capital stock (consumer durables and residential capital stock), 1 is the household's (normalized) endowment of time and ℓ_t is the part of it allocated to market production. The production functions F and H are both assumed to be homogeneous of degree one. The variable z_t represents labor augmenting technological progress and it evolves according to

(4) $z_t = Az_{t-1}\,\epsilon_t, \quad A > 1,$

where ϵ_t is a stationary random variable with unit mean drawn from the distribution $G(\epsilon_t | \epsilon_{t-1})$.

The two capital stocks evolve as

(5) $k_{t+1} = k_t(1-\delta_k) + i_{kt}, \quad 0 < \delta_k < 1,$

(6) $d_{t+1} = d_t(1-\delta_d) + i_{dt}, \quad 0 < \delta_d < 1,$

where δ_k and δ_d are the depreciation rates, i_{kt} is gross business investment in nonresidential market capital and i_{dt} is household investment.

The constraint applying to market output is

$$c_t + i_{kt} + i_{dt} \leq y_t.$$

Note that this condition breaks the symmetry between the two sectors. Capital goods can be produced by the business sector only.

Finally, there is a government present in the economy. It levies taxes on the market income earned by labor and capital at the rates τ_ℓ and τ_k. The revenue raised by the government in each period t is rebated back to agents in the form of lump–sum transfer payments in the amount μ_t. The government's period–t budget constraint is

(7) $$\mu_t = \tau_k r_t k_t + \tau_\ell w_t \ell_t,$$

where r_t represents the market return on capital and w_t the real wage rate.

B. Competitive Equilibrium

The "representative" household solves the dynamic programming problem shown below.

(8) $$V(k_t,d_t,z_t;\epsilon_t) = \max_{(c_t,k_{t+1},d_{t+1},\ell_t)} \{U(c_t,H(d_t,z_t(1-\ell_t)) + \beta\int V(k_{t+1},d_{t+1},z_{t+1};\epsilon_{t+1})dG(\epsilon_{t+1}|\epsilon_t)\}$$

subject to

(9) $$c_t + k_{t+1} + d_{t+1} = (1-\tau_k)r_t k_t + (1-\tau_\ell)w_t \ell_t + (1-\delta_d)d_t + \mu_t.$$

The upshot of the implied maximization routine is the following set of efficiency conditions:

(10) $$U_1(c_t,h_t) = \beta\int U_1(c_{t+1},h_{t+1})[(1-\tau_k)F_1(k_{t+1},z_{t+1}\ell_{t+1})+(1-\delta_k)]dG(\epsilon_{t+1}|\epsilon_t)$$

(11) $$U_1(c_t,h_t) = \beta\int U_1(c_{t+1},h_{t+1})[H_1(d_{t+1},z_{t+1}(1-\ell_{t+1})) \times \frac{U_2(c_{t+1},h_{t+1})}{U_1(c_{t+1},h_{t+1})} +(1-\delta_d)]dG(\epsilon_{t+1}|\epsilon_t)$$

(12) $$(1-\tau_\ell)z_t F_2(k_t,z_t\ell_t) = \frac{U_2(c_t,h_t)}{U_1(c_t,h_t)} z_t H_2(d_t,z_t(1-\ell))$$

[recall that $h_t = H(d_t,z_t(1-\ell)]$.

In competitive equilibrium the rental and wage rates, r_t and w_t, will be equal to the marginal products of capital and market time, $F_1(k_t,z_t\ell_t)$ and $z_tF_2(k_t,z_t\ell_t)$, a fact used in the presentation of (10) and (12). Also, in competitive equilibrium the market for business output must clear each period, implying

$$(13) \qquad c_t + k_{t+1} + d_{t+1} = F(k_t,z_t\ell_t) + (1-\delta_k)k_t + (1-\delta_d)d_t.$$

This condition can be obtained by substituting (7) into (9). The system of equations (10), (11), (12), and (13)—in conjunction with (3)—describe the model's general equilibrium, or provide a determination of c_t, k_{t+1}, d_{t+1}, and $z_t\ell_t$.

Let $c_t = c(k_t,d_t,z_t;\epsilon_t)$, $k_{t+1} = k(k_t,d_t,z_t;\epsilon_t)$, $d_{t+1} = d(k_t,d_t,z_t;\epsilon_t)$, and $z_t\ell_t = \ell(k_t,d_t,z_t;\epsilon_t)$ represent the solution to (10), (11), (12), and (13), taking account of (3). These *equilibrium* decision–rules are homogeneous of degree one in (k_t,d_t,z_t). This is immediate from the system of equations implicitly defining the solution since U_1 and U_2 are homogeneous of degree $q-1$, while F and H are homogeneous of degree one, and F_1, F_2, H_1, and H_2 are homogeneous of degree zero. Recall from (4) that z_t is growing across time. The model's decision–variables can be rendered stationary by deflating them by z_{t-1}. By defining $\hat{x}_t$ by $\hat{x}_t = x_t/z_{t-1}$, for $x_t = c_t$, k_t, and d_t, the above decision–rules can be transformed to get $\hat{c}_t = c(\hat{k}_t,\hat{d}_t,A\epsilon_t;\epsilon_t)$, $\hat{k}_{t+1} = k(\hat{k}_t,\hat{d}_t,A\epsilon_t;\epsilon_t)/A\epsilon_t$, $\hat{d}_{t+1} = d(\hat{k}_t,\hat{d}_t,A\epsilon_t;\epsilon_t)/A\epsilon_t$ and $\ell_t = \ell(\hat{k}_t,\hat{d}_t,A\epsilon_t;\epsilon_t)/A\epsilon_t$. Now the model's solution is expressed in a stationary form. Observe that these transformed decision–rules will also satisfy (10), (12), and (13)—together with (3)—if β and z_t are changed to $\hat{\beta}$ and $\hat{z}_t$, where $\hat{\beta} \equiv \beta(A\epsilon_t)^{q-1}$ and $\hat{z}_t \equiv A\epsilon_t$.[1]

Equation (12) governs the optimal allocation of time in the model. In order to put the implications of the present framework for the allocation of time into perspective, consider the standard paradigm used in business cycle analysis. Here a homothetic momentary utility function $U(c_t,1-\ell_t)$ defined over consumption c_t and leisure $1-\ell_t$ is used. The optimality condition governing the allocation of time is

$$\text{(14)} \qquad \frac{U_2(c_t,1-\ell_t)}{U_1(c_t,1-\ell_t)} \equiv \Gamma\left[\frac{c_t}{1-\ell_t}\right] = (1-\tau_\ell)w_t,$$

with the form of the function Γ following from the assumed homotheticity of U. Note that for this paradigm to be consistent with secular increases in real wages and consumption on the one hand, and a stationary allocation of time to market work on the other, a unit elasticity of substitution between consumption and leisure is required.[2]

A related important implication of (14), which has played a crucial role in neoclassical macroeconomic thinking, is that for real wage movements to have strong effects on market labor, they should, at least partly, be transitory. Using equation (14) and intuitive reasoning from the permanent income hypothesis, when w_t increases only temporarily it has a minor effect on c_t. Hence, (14) implies an expansion of market work. For the case when w_t moves permanently, it has a stronger effect on c_t, which reduces the extent to which market labor reacts.

By contrast from (12) the condition for optimal allocation of time in the present model reads

$$\text{(15)} \qquad z_t H_2[d_t,z_t(1-\ell_t)]\,\frac{U_2(c_t,h_t)}{U_1(c_t,h_t)} = (1-\tau_\ell)z_t F_2(k_t,z_t\ell_t) = (1-\tau_\ell)w_t.$$

Here technological progress and capital accumulation, which affect market productivity and hence the real wage, also affect home productivity. Hence, stationarity of market hours does not restrict the elasticity of substitution in utility. It requires now that technological progress and capital accumulation affect the two marginal productivities of labor in a parallel way. For this to be the case the two production functions should display constant-returns-to-scale, the utility function over c_t and h_t be homogeneous of arbitrary degree, and technological progress be representable as labor augmenting.

C. Solution Algorithm

Let the system of equations (10), (11), and (12) defining a stationary solution to the model—once $\hat{c}$ and $\hat{h}$ been solved out for using (3) and (13)—be more compactly represented by

$$(16) \qquad \Delta(\hat{k},\hat{d},\hat{k}',\hat{d}',\ell;\epsilon) = \int\Lambda(\hat{k}',\hat{d}',\ell',\hat{k}'',\hat{d}'';\epsilon,\epsilon')d\epsilon'.$$

(In the above expression time subscripts have been suppressed in the standard manner.) Here $\Delta\colon \mathbb{R}^6_+ \to \mathbb{R}^3_+$ and $\Lambda\colon \mathbb{R}^6_+ \to \mathbb{R}^3_+$. In order to simulate the model a set of policy–functions of the form $\hat{k}' = k(\hat{k},\hat{d};\epsilon)$, $\hat{d}' = d(\hat{k},\hat{d};\epsilon)$, and $\ell = \ell(\hat{k},\hat{d};\epsilon)$ must be found that solves this system of integral equations. To do this an algorithm proposed by Coleman (1989) will be employed that approximates the true equilibrium decision–rules over a grid using a multilinear interpolation scheme.[3]

To begin with assume that the technology shock, ϵ, is an element of the time invariant set $E = \{\epsilon_1,\ldots,\epsilon_p\}$. Next, restrict the permissible range of values for the stocks of capital and durables to lie in the closed intervals $[k_1,k_m]$ and $[d_1,d_n]$, respectively, and let $K = \{k_1,k_2,\ldots,k_m\}$ and $D = \{d_1,d_2,\ldots,d_n\}$ represent sets of monotonically increasing grid points that span these intervals. Now, make an initial guess for the value of the function $x = x(\hat{k},\hat{d},\epsilon)$, for $x = \hat{k}'$, $\hat{d}'$, and ℓ, at each of the $m \times n \times p$ points in the set $K \times D \times E$. Denote the value for the initial guess of the function x at the grid point (k_h,d_i,e_j) by $x^0(k_h,d_i,e_j)$. A guess for x at other points in its domain $[k_1,k_m] \times [d_1,d_n] \times E$ is then constructed through multilinear interpolation (see Press 1986). Specifically, take some point $(\hat{k},\hat{d},\epsilon_j) \in [k_1,k_m] \times [d_1,d_n] \times E$. The value of the function x^0 at the point $(\hat{k},\hat{d},\epsilon_j)$, or $x^0(\hat{k},\hat{d},\epsilon_j)$, is defined as follows:

$$(17) \qquad x^0(\hat{k},\hat{d},\epsilon_j) = (1-u)(1-v)x^0(k_h,d_i,\epsilon_j) + u(1-v)x^0(k_{h+1},d_i,\epsilon_j)$$
$$+ uvx^0(k_{h+1},d_{i+1},\epsilon_j) + (1-u)vx^0(k_h,d_{i+1},\epsilon_j),$$

where the weights u and v are given by

$$u = \frac{\hat{k} - k_h}{k_{h+1} - k_h} \text{ and } v = \frac{\hat{d} - d_i}{d_{i+1} - d_i},$$

with the grid points k_h, k_{h+1}, d_i, and d_{i+1} being chosen such that

$$k_h \leq \hat{k} \leq k_{h+1} \text{ and } d_i \leq \hat{d} \leq d_{i+1}.$$

Thus, the interpolated value of x^0 at $(\hat{k},\hat{d},\epsilon_j)$ is simply taken to be a weighted average of its values at the four nearest grid points. Note that the interpolated function x^0 is continuous on $[k_1,k_m] \times [d_1,d_n]$.

Given initial guesses for the functions, k, d, and ℓ, denoted by k^0, d^0, ℓ^0, respectively, it is straightforward to compute revised guesses, k^1, d^1, and ℓ^1. In particular for each grid point $(k_h,d_i,\epsilon_j) \in K \times D \times E$ values for $k^1(k_h,d_i,\epsilon_j)$, $d^1(k_h,d_i,\epsilon_j)$, and $\ell^1(k_h,d_i,\epsilon_j)$ can be computed by solving the following nonlinear system of equations for $\hat{k}'$, $\hat{d}'$, and ℓ'.

$$\Delta(k_h,d_i,\hat{k}',\hat{d}',\ell;\epsilon_j) = \sum_{r=1}^{p} \Lambda(\hat{k}',\hat{d}',\ell^0(\hat{k}',\hat{d}',\epsilon_r),k^0(\hat{k}',\hat{d}',\epsilon_r),d^0(\hat{k}',\hat{d}',\epsilon_r);\epsilon_j,\epsilon_r). \tag{18}$$

Given values for $\hat{k}'$, $\hat{d}'$, and $\hat{\ell}$ at each of the $m \times n \times p$ grid points in $K \times D \times E$, the functions k^1, d^1, and ℓ^1 can be extended over the entire domain $[k_1,k_m] \times [d_1,d_n] \times E$ via interpolation, as was done previously. The functions k^1, d^1, and ℓ^1 are then used as guesses on the next iteration, with the whole procedure being repeated until the decision–rules have converged.

Once the decision–rules have been obtained the model can be simulated and various sample statistics for variables of interest computed. This is discussed in further detail later on.

III. Quantitative Analysis: Calibration and Simulation

In this section of the paper, the model is calibrated, simulated, and evaluated. The dynamics of the simulated economy are compared with the behavior of annual U.S. data for the sample period 1954–1987. Specifically, the question addressed is whether the model is able to mimic the observed behavior of investment in household and market production, as well as other features of business fluctuations.

So as to impose some discipline on the simulation conducted, the calibration procedure advanced by Kydland and Prescott (1982) is adopted. In line with this approach, as many model parameters as possible are set in advance either based upon (i), a priori information about their magnitudes or (ii), so that along a deterministic balanced growth path the ratios for various endogenous variables in the model correspond to their average values for the U.S. postwar period. The shocks to the system are the z_t–process, whose moments are set to match the sample moments observed for the corresponding Solow residuals in the data.

Specifically, the procedure is the following: First the model is parameterized. Then it is transformed to obtain a stationary problem, as discussed in subsection II.B. Next, using the algorithm described in II.C, the equilibrium decision–rules for the transformed variables are computed. Using these decision–rules, 5000 artificial samples of 34 observations (the number of years in the 1954–1987 sample) are simulated. Each simulation corresponds to a randomly generated sample of 34 realizations of ϵ_t and the corresponding z_t–process. Then all variables, except ℓ_t, are transformed back by multiplying them by z_{t-1} and filtered using the Hodrick–Prescott (H–P) filter. The average moments over the 5000 samples are computed and compared to the corresponding moments of the actual H–P filtered U.S. data.

A. Parameterization of the Economy

To begin with, let tastes and technology be specified in the following way:

(19) $$U(c,h) = \frac{1}{1-\gamma}\left[\theta c^{\sigma} + (1-\theta)h^{\sigma}\right]^{(1-\gamma)/\sigma} - \frac{1}{1-\gamma}$$

(20) $$F(k,\ell) = k^{\alpha}(z\ell)^{1-\alpha}$$

and

(21) $$H(d,1-\ell) = \{\omega d^{\lambda} + (1-\omega)[z(1-\ell)]^{\lambda}\}^{1/\lambda}$$

where σ, $\lambda \leq 1$, $\gamma > 0$, and $0 < \alpha$, $\omega < 1$. Preferences and household production have been given C.E.S. functional forms, while market production has a Cobb–Douglas characterization—i.e., a C.E.S. with unit elasticity of substitution.

Next, suppose that the stochastic structure of the model is described by a two–state Markov process. Specifically, in any given period the technology shock, ϵ, is assumed to be drawn from the time–invariant two–point set

$$\Xi = \left\{e^{\xi_1}, e^{\xi_2}\right\}.$$

The distribution function governing next period's technology shock, ϵ', conditional on the current realization, ϵ, is defined by

$$\text{prob}\left[\epsilon' = e^{\xi_s} \mid \epsilon = e^{\xi_r}\right] = \pi_{rs},$$

where $0 \leq \pi_{rs} \leq 1$, for r,s = 1, 2.

B. Calibration Procedure and Benchmark Model Results

In the first stage of the quantitative analysis attention will be directed to a special version of the model, with the following properties: (i) technological change does not play any role in decisions of the household sector, and (ii) the services from household capital enter the utility function in an additive manner. Hence, this version of the model can be seen as a "straw–man" which downplays the household production structure, incorporating the minimal requirement for households to demand capital.

Features (i)–(ii) are achieved as follows: First, both preferences and household production are restricted to have unitary elasticity of substitution (i.e., $\sigma = \lambda = 0$), so that they assume the form

$$(22) \qquad U(c,h) = \frac{1}{1-\gamma}(c^{\theta}h^{1-\theta})^{1-\gamma} - \frac{1}{1-\gamma}$$

and

$$(23) \qquad H[d,z(1-\ell)] = d^{\omega}[z(1-\ell)]^{1-\omega}.$$

Substituting (23) into (22) yields

$$(24) \qquad \underline{U}\,(c,1-\ell,d,z,) = \frac{1}{1-\gamma}\left[c^{\theta}(1-\ell)^{(1-\omega)(1-\theta)}d^{\omega(1-\theta)}z^{(1-\omega)(1-\theta)}\right]^{1-\gamma} - \frac{1}{1-\gamma}.$$

The stock of household capital enters $\underline{U}$ in a similar way as in the utility function used by Macklem (1989) and others referenced in that paper.

Second, γ is chosen to be one so that $\underline{U}$ becomes

$$(25) \qquad \underline{U}(c,1-\ell,d,z) = \theta \ln c + (1-\omega)(1-\theta)\ln(1-\ell) + \omega(1-\theta)\ln d + (1-\omega)(1-\theta)\ln z.$$

Note that (25) satisfies the criteria (i)–(ii). The choice regarding ℓ will have the standard form [equation (14)] with unit elasticity of substitution between c and $(1-\ell)$, and technological change in home production will not affect any household decisions.

In order to implement the benchmark model, values for the following parameters shown below need to be chosen

Utility:	θ, β,
Market Production:	α;
Home Production:	ω,
Depreciation rates:	δ_k, δ_d,
Technology process:	A, ξ_1, ξ_2, $\pi_{11}\pi_{22}$,
Tax rates:	τ_k, τ_ℓ.

First, the number of parameters is reduced by imposing symmetry on the stochastic technology process. It is assumed that $\xi_1 = -\xi_2 = \xi$, and that $\pi_{11} = \pi_{22} = \pi$. Then, ξ is the standard deviation of the shock and $\rho = 2\pi-1$ is the coefficient of serial correlation. In market production α is chosen to be 0.3, given that the observed capital share of about 30 percent during the 1954–87 sample. For utility, since the time unit is a year, β is set equal to the standard value of 0.96.

The depreciation rate on market capital δ_k was chosen to be 7.8 percent, a value derived from the average service life of nonresidential structures and equipment for the period 1954–1985.[4] It is assumed that the average depreciation rate of household capital δ_d, which consists of components similar to the structures and equipment in business capital, is equal to δ_k.

The parameters of the z_t–process require the calculation of the Solow residuals from the U.S. data and their sample moments. This calculation was carried out using GNP and net fixed nonresidential private capital in 1982 prices and total man–hours employed. The average growth rate of z_t is 0.014, and hence A was equated to 1.014. The first difference of the log of the Solow residual, corresponding to $\ln A + \ln \epsilon_t$, has a standard deviation of 0.02 and an autocorrelation coefficient of 0.18, which is statistically insignificantly different from zero. Hence, $\xi = 0.02$ and ρ was set to zero, implying that $\pi = 0.5$.

The tax rate τ_k, applying in the model to gross capital income was set equal to 0.25. On the deterministic balanced growth path, and given the values of δ_k and β

described above, this corresponds to a tax rate on net capital income of about 50 percent. This figure is between the effective tax rates of 52 percent on corporate capital income and 40 percent on noncorporate capital income computed by Jorgenson and Yun (1986). Regarding the tax rate on labor income, prior to the Tax Reform Act of 1986 the marginal tax rates on personal income ranged from 11 percent to 50 percent. In this range $\tau_\ell =$ 0.25 was picked.[5]

Two parameters, θ in utility and ω in home production, still remain. Two first moments computed from the U.S. data are relevant for the determination of these parameters: (a) the average ratio of total hours worked to total nonsleeping hours of the working age population (16 hours per day) is 0.24, and (b) the average ratio of household capital to market capital is 1.13. The values of θ and ω were chosen so that these two first moments are satisfied along the model's balanced growth path.

Specifically, given the current parameterization for tastes and technology, the steady–state analogues to equations (10), (11), (12), and (13) are:

$$(26) \qquad 1 = \beta A^{-\gamma}(1-\tau_k)\left[\alpha(\hat{k}/\ell)^{\alpha-1} + (1-\delta_k)\right]$$

$$(27) \qquad 1 = \beta A^{-\gamma}\{\omega[(1-\theta)/\theta]\hat{c}/\hat{d} + (1-\delta_d)\}$$

$$(28) \qquad (1-\tau_\ell)(1-\alpha)A^{1-\alpha}(\hat{k}/\ell)^{\alpha} = (1-\omega)[(1-\theta)/\theta]\hat{c}/(1-\ell)$$

$$(29) \qquad \hat{c} = \hat{k}^{\alpha}(A\ell)^{1-\alpha} - Ak + (1-\delta_k)\hat{k}.$$

The above two restrictions from the long–run data imply

$$(30) \qquad \ell = 0.24$$

and

$$(31) \qquad \hat{d}/\hat{k} = 1.13.$$

Given values for β, A, γ, α, δ_k, δ_d, τ_k, and τ_d this system of six equations can be thought of as determining a solution for the six unknowns $\hat{k}$, $\hat{d}$, ℓ, $\hat{c}$, θ, and ω. The parameter values obtained for ω and θ are 0.14 and 0.26, respectively.

The steady–state values for $\hat{k}$, $\hat{d}$, ℓ and $\hat{c}$, as determined by (26), (27), (28), and (29), are sensitive to the tax parameters τ_k and τ_ℓ. As one might expect, the taxation of the market income earned by labor and capital induces substitution toward nonmarket activity. For instance, if capital taxation is eliminated in the model the ratio of household to business capital drops from 1.13 to 0.78 and the ratio of nonmarket to market time falls from 3.2 to 1.91. The marginal welfare cost of capital taxation in the benchmark steady–state is high, being 2.0 units of output in loss welfare for each extra unit of output raised in revenue.[6] The corresponding figure for labor taxation is 0.71.

Finally, the model's steady–state can be used to compute the total amount of goods and services produced in the economy. The aggregate $F(\hat{k},\ell) + [U_2(\hat{c},\hat{h})/U_1(\hat{c},\hat{h})] \times H(\hat{d},1-\ell)$, where market goods are used as the numeraire, can be denoted "Gross Economic Product" (GEP). In the model's steady–state the ratio of GEP to GNP is 2.9 The large size of this number follows from the fact that household production uses about 1.1 times the capital and 3.2 times the time of market production.

The results of the current simulation are shown in Table 1 (under Model 1) and in Figure 2. By comparing the standard deviations of the variables for Model 1 with the ones characterizing the U.S. data, one can see that, in general, the model generates too little variation. For example, the standard deviation of actual GNP around the H–P trend is 2.3 percent, while the model generates a corresponding 1.7 percent. The standard deviation of market labor is particularly low, 0.7 percent, relative to the actual figure of 1.8 percent. One cause of this result is the the nature of the technology process. Since z_t is a random walk, changes in market opportunities for labor are permanent and hence they generate a weak response of market labor effort. Also, given the focus of this paper mechanisms which enhance labor responsiveness to market opportunities were not incorporated, such as the intertemporal nonseparability of preferences in Kydland and Prescott (1982), or the indivisibility of market work in Hansen (1985).

The variability of hours and output is even smaller when the model is transformed into the standard one (with a Cobb–Douglas utility function defined over consumption and leisure) by eliminating household capital and technological change from the home production function. In this case the standard deviation of output is 1.5 percent and the standard deviation of labor is 0.4 percent. As mentioned previously, given that only market capital needs to be adjusted following a shock, one can expect market labor effort reacts by less.

The behavior of investment in business and household capital for the benchmark model is shown in Figure 2, which plots the variables i_{kt} and i_{dt} for an artificial sample of 34 years. It can be clearly seen that the two investments tend to react in opposite directions, with business investment moving much closer to output (not shown). Also, fluctuations in investment tend to be short–lived. Only when the fluctuations persist do the two investment begin to comove positively. The short nature of the fluctuations is reflected in the coefficients of serial correlation of i_{kt}, which is slightly negative, and of i_{dt}, which is only weakly positive. By contrast, in the U.S. data their coefficients of serial correlation are 0.48 and 0.44, respectively. Another lack of correspondence are the relative volatilities. Business investment is less volatile in the data with a standard deviation of 6.4 percent against 7.8 percent for household investment, while in Model 1 the corresponding figures are 7.5 percent and 4.8 percent.

The negative comovement of the two investments, which stands in contrast with the positive one displayed by actual data (Figure 1), has to do with the basic asymmetry between the two types of capital. Business capital can be used to produce household capital, but not the other way around. When an innovation to technology occurs, say a positive one, the optimal levels for both capital stocks increase. Given the asymmetry in the nature of the two capital goods, the tendency for the benchmark model is to build first business capital, and only then household capital. Capital investment requires abstention from consumption of market goods, but not (directly) of consumption of home goods. The induced short–run scarcity of market consumption goods in terms of nonmarket ones operates to reduce the benefit from immediate investment in household capital vis a vis business capital. The next section addresses this question in greater detail.

C. Departing from the Benchmark Model

In the benchmark model market and home production functions were parameterized identically. Given the asymmetric role the two types of capital play in the paradigm, and the poor outcome of the earlier simulation, it may be profitable to investigate whether relaxing the assumption of a unitary elasticity of substitution in home production, ($\lambda=0$) can improve the ability of the model to generate the pattern of investment behavior observed in the data. Note that higher values for $\lambda \in (-\infty,1]$ in (21) imply greater substitutability between d and $z(1-\ell)$ in household production.

It turns out that reducing the degree of substitution in home production, relative to the Cobb–Douglas case, has strong implications for the pattern of comovements between the two investments. The simulation results obtained when $\lambda = -1$ are reported under Model 2 in Table 1 and portrayed in Figure 3.[7] The most striking outcome is that investments now become strongly positively correlated with output. This can be seen in Table 1, where the correlations with output for market and household investment are 0.99 and 0.97, and in Figure 3 which illustrates the strong positive comovement between the two types of investment.

To interpret this result, consider the following expression which appears in (11), the Euler equation regulating the acquisition of household capital:

$$(32) \qquad H_1(d_{t+1}, z_{t+1}(1-\ell_{t+1}))\,\frac{U_2(c_{t+1}, h_{t+1})}{U_1(c_{t+1}, h_{t+1})}$$

$$= \omega \frac{d_{t+1}^{\lambda-1}}{\omega d_{t+1}^{\lambda} + (1-\omega) z_{t+1}^{\lambda}(1-\ell_{t+1})^{\lambda}}\,\frac{(1-\theta)}{\theta}\,c_{t+1}.$$

This term describes the marginal benefit from household investment (measured in terms of market goods). When a technological improvement hits the economy in period t, z_{t+1} is expected to rise since z_t follows a random walk. On the one hand, this tends to increase the marginal product of household capital, $H_1(d_{t+1}, z_{t+1}(1-\ell_{t+1}))$. On the other hand, the shadow price of home goods, $U_2(c_{t+1}, h_{t+1})/U_1(c_{t+1}, h_{t+1})$, declines because of the induced desire to build up the capital stocks (resulting in a relative scarcity of market goods). Here technological change has two opposing effects on the marginal productivity of household investment.

In the Cobb–Douglas case, where $\lambda = 0$, these two effects cancel each other out, as can be seen in equation (32). It is not surprising then that in Model 1 household investment falls following a positive shock since the latter only improves the marginal benefits of market investment. When $\lambda < 0$, the higher complementarity between d_{t+1} and $z_{t+1}(1-\ell_{t+1})$ in t + 1 household production implies a stronger impact of the shock on the marginal productivity of household capital, $H_1(d_{t+1}, z_{t+1}(1-\ell_{t+1}))$. Observe that (32) is increasing in z_{t+1} when $\lambda < 0$. Thus, in Model 2 the marginal productivity effect becomes more important relative to the relative price effect, and this is the reason behind the more procyclical behavior of household investment.

On the volatility and serial correlation properties of the two investments, Model 2 is better able to mimic the data than Model 1. Household investment is now more volatile than business investment—standard deviations of 4.5 percent and 3.8 percent—and the serial correlation coefficients are larger, with business investment exhibiting stronger serial correlation. Also, the variability of market time is increased from 0.7 percent in Model 1 to 1.0 percent, which is, however, still lower than in actual data.

The intuition behind the increase in the volatility of market time can be obtained by examining the expression shown below representing the marginal value of time *at home* [cf., equation (12)]:

$$(33) \qquad z_tH_2(d_t,z_t(1-\ell_t))\,\frac{U_2(c_t,h_t)}{U_1(c_t,h_t)} = \frac{(1-\omega)z_t^{\lambda}(1-\ell_t)^{\lambda-1}}{\omega d_t^{\lambda} + (1-\omega)z_t^{\lambda}(1-\ell_t)^{\lambda}}\,\frac{(1-\theta)}{\theta}\,c_t\,.$$

Consider first the case where the household production technology is Cobb–Douglas (λ=0). Here, when a positive technological innovation occurs, the marginal product of household time, $z_tH_2(d_t,z_t(1-\ell_t))$, increases. This effect, by itself, operates to increase the amount of time spent at home. However, this is not the end of the story, as the relative price of home goods in terms of market goods, $U_2(c_t,h_t)/U_1(c_t,h_t)$, falls due to the scarcity of market resources—since investment is expanded. This effect works in the direction of increasing market time. In the Cobb–Douglas case these two forces exactly cancel each other out, and the expression in (33) is independent of z_t. The upshot is that nonmarket time falls following a positive shock, since the marginal value of time has increases only in the market sector.

When λ is negative, the above expression is decreasing in z_t so that market time rises, a fortiori, in response to a positive innovation. As the degree of complementarity between time and capital in household production is increased, the effect of technological innovation on the marginal product of nonmarket time is dampened. (In the limiting case where $\lambda \to -\infty$, the household production function becomes Leontief and the effect that a positive shock has on nonmarket time's marginal product vanishes.)

The discussion above illustrates the relevance of technological complementarity in household production for the allocation of capital and time across sectors.

IV. Concluding Comments

Two observations about investment in durable consumption goods and housing were stressed in the introduction: (i) its high level relative to nonresidential business investment, and (ii) its procyclical behavior, leading business investment. The aim of this paper was to construct a model which treated the market and nonmarket sectors symmetrically, to see whether it could mimic both these observations, as well as other important features of business fluctuations.

The first observation about the level of home investment (arising from the first moments of the U.S. data) is satisfied by adjusting the parameters of taste and technology. This was part of the calibration procedure followed. It may be noted that in spite of the high *level* of household capital in the United States, the nonmarket sector is still strongly labor intensive. Capital in the household sector is about 1.1 times higher than in the business sector, but time spent in nonmarket activities is about 3 times larger than the time spent in market activities. (This is reflected in a higher value for α than for ω.)

In the model, the level of business capital is strongly related to taxation, which applies to business capital income but not to household capital productivity. This tax asymmetry shifts capital towards the nonmarket sector. Given the values of the parameters of the model, found under realistic tax rates, if the taxation of capital is eliminated (being replaced by lump-sum taxation), the ratio of household to business capital along the balanced growth path drops from 1.16 to 0.78.

The observation about the cyclical behavior of investment was analyzed by simulating the model. Of pivotal importance for the dynamics is the fact that capital goods can only be produced in the market sector. The ramifications of this asymmetry can be illustrated as follows: Suppose a positive technological innovation hits the economy. In response to this, the optimal levels of business and household capital increase. Given that capital goods are produced in the business sector only, the induced scarcity of market goods reduces the shadow price of home goods in terms of market goods. A shift of resources to the business sector, in terms of both time and capital, ensues. Hence, this mechanism implies a tendency for business capital to be built first, and only then household capital. This effect operates to produce negative comovement between the two investments, in contrast with the positive covariation observed in the data. The mechanism just described is not a specific feature of this framework. It would be present in any general equilibrium model with household durables. Model 1, which downplays the household production structure, is an example.

In order to overcome the tendency for household investment to move countercyclically, the degree of technical complementarity between time and capital in household production was increased (Model 2). This strengthens the positive impact that a technological improvement has on the marginal product of household capital and weakens its effect on the marginal value of household time, thereby promoting an increase in household investment and a reallocation of time toward the market. As a result, household investment moved procyclically and market time became more volatile..

A feature that the present model fails to rationalize is household investment's lead over business investment. Strengthening the effect of shocks on the marginal rate of substitution between labor and capital in home production (i.e., making λ more negative) does not affect this timing of events. The introduction of adjustment costs in business capital, which can be thought of as retarding business investment, does not produce the actual type of behavior. Such adjustment costs retard the entire build up of the two capital stocks, producing only a reduction in general volatility, including that of the two investments.

Given that investment in consumer durable goods and housing tends to lead other macroeconomic variables over the cycle, household capital plays an interesting macroeconomic role. It seems that a richer model is called for. As it is well known, fluctuations in the amount of time households devote to the market are reflected mainly in the number of jobs rather than in the number of hours worked in existing ones. Hence, reallocation of time from home to market activities may require relocation of households, purchasing another vehicle, etc., at the early stages of business cycles. Further investigation of the role of nonmarket activities, especially their implications regarding housing and consumer durable goods, could prove fruitful for understanding the origin and propagation of business cycles.

Footnotes

[1]For the transformed model, the resource constraint (13) reads:

$$\hat{c}_t + \hat{z}_t\hat{k}_{t+1} + \hat{z}_t\hat{d}_{t+1} = F(\hat{k}_t,\hat{z}_t\ell_t) + (1-\delta_k)\hat{k}_t + (1-\delta_d)\hat{d}_t.$$

This is readily verified by dividing both sides of (13) through by z_{t-1}.

[2]Kydland (1984) shows that within the C.E.S. family of utility functions, only the specification corresponding to the unitary elasticity (the Cobb–Douglas form) is consistent with balanced growth under stationary labor time. As shown in King, Plosser and Rebelo (1988), the same holds with the more general structure

$$U(c_t,1-\ell_t) = \frac{c_t^{1-\gamma}}{1-\gamma}V(1-\ell_t), \quad \text{with } 0 < \gamma < 1,\ \ \gamma > 1,\ \ V' > 0,\ \ V'' < 0.$$

[3]Coleman's (1988) technique is related to one developed by Baxter (1988) and Danthine and Donaldson (1988). In a nutshell the principle difference between the method of Baxter (1988) and Danthine and Donaldson (1988) on the one hand, and Coleman (1988) on the other, is that the former restricts the range of the functions describing the laws of motion for the state variables to lie on a grid while the latter does not. An algorithm for computing the solutions to distorted equilibrium in linear–quadratic problems is discussed in Cooley and Hansen (1989) and Kydland (1987).

[4]The source: *Fixed Reproducible Tangible Wealth in the United States, 1925–1985*, a publication of the U.S. Department of Commerce.

[5]This rate seems in line with tax rates reported in Hausman and Poterba (1987).

[6]The steady–state levels of market goals, $\hat{c}$, home goods, $\hat{h}$, welfare, $\hat{W}$, and tax revenue, $\hat{\mu}$, will be functions of the tax rates τ_k and τ_ℓ (as well as the parameters describing taste and technology). Thus, one can write $\hat{c} = \hat{c}(\tau_k,\tau_\ell)$, $\hat{h} = \hat{h}(\tau_k,\tau_\ell)$, $\hat{W} = \hat{W}(\tau_k,\tau_\ell) \equiv U(\hat{c}(\tau_k,\tau_\ell),\hat{h}(\tau_k,\tau_\ell))$, and $\hat{\mu} = \hat{\mu}(\tau_k,\tau_\ell)$. The marginal welfare cost of capital taxation is defined as $[\hat{W}_1(\tau_k,\tau_\ell)/U_1(\hat{c}(\tau_k,\tau_\ell),\hat{h}(\tau_k,\tau_\ell))]/\hat{\mu}_1(\tau_k,\tau_\ell)$.

[7]Note that changing this parameter implies recalculating ω and θ using the steady–state equations for Model 2. One now obtains $\omega = 0.11$ and $\theta = 0.26$. For the configuration of parameters values used in Model 2, the marginal welfare cost of capital taxation is 2.7 and for labor taxation is 0.65.

Table 1

	U.S. Annual Data 1954–1987			Model 1			Model 2		
Variables	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Output	2.3	0.57	1.00	1.7	0.49	1.00	1.9	0.37	1.00
Consumption	1.3	0.66	0.88	0.9	0.49	0.95	0.9	0.54	0.92
Business Investment	6.4	0.48	0.86	7.5	–0.04	0.71	3.8	0.44	0.99
Household Investment	7.8	0.44	0.63	4.8	0.17	0.37	4.5	0.24	0.97
Market Time	1.8	0.48	0.91	0.7	0.49	0.95	1.0	0.32	0.95

Note: The U.S. data was divided by the working age population (16–65) logged and Hodrick–Prescott (H–P) filtered. Output is GNP, Business Investment is Fixed Nonresidential Private Investment, and Household Investment is Private Residential Investment plus Consumer Durable Purchases, all in 1982 dollars. Market time is total hours from the Current Population Survey (which is a survey of households).

(1) = standard deviations, measured in percent

(2) = first–order autocorrelations

(3) = correlations with output

FIGURE 1

BUSINESS INVESTMENT (IK) AND HOUSEHOLD INVESTMENT (ID)

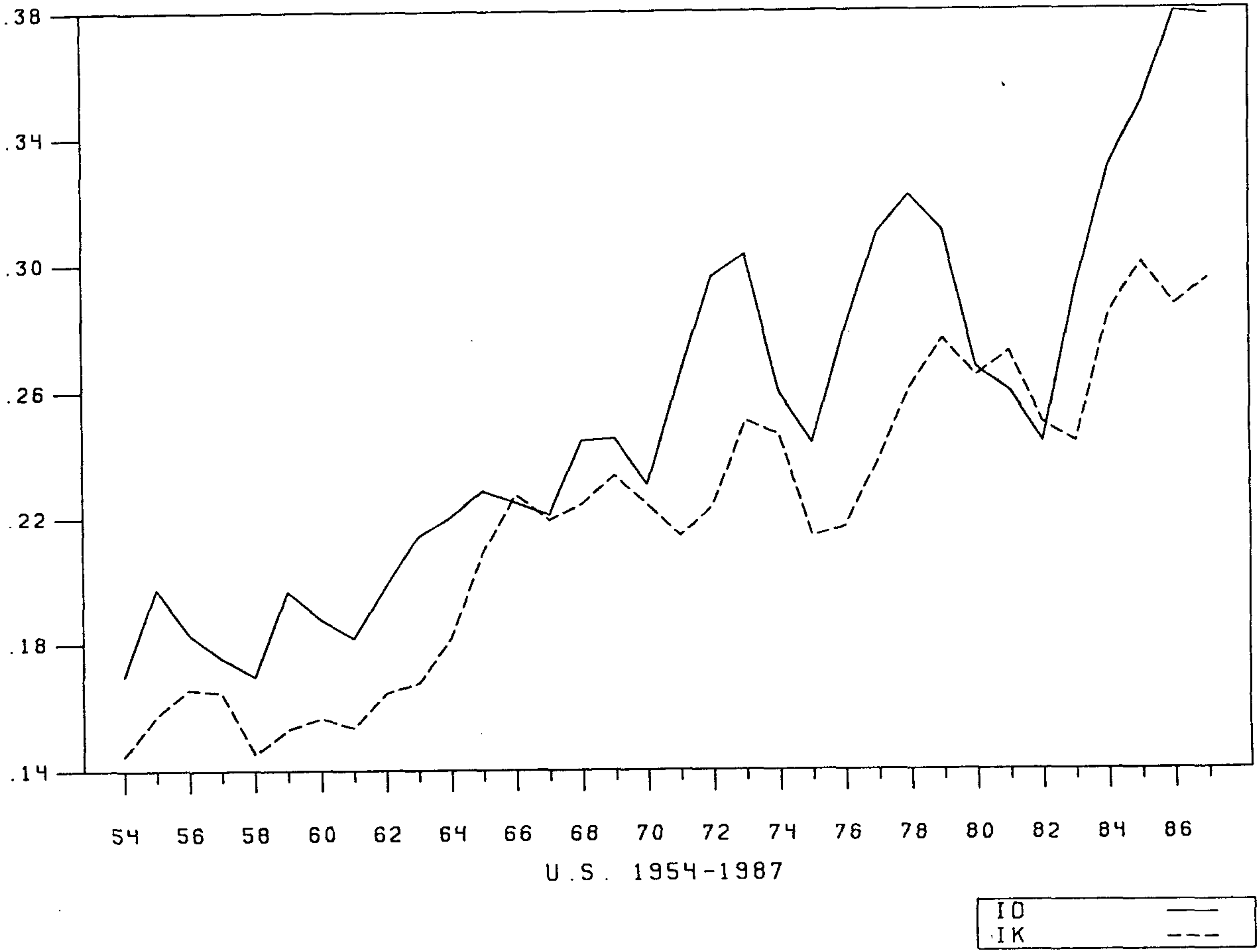


FIGURE 2

IK AND ID-MODEL 1

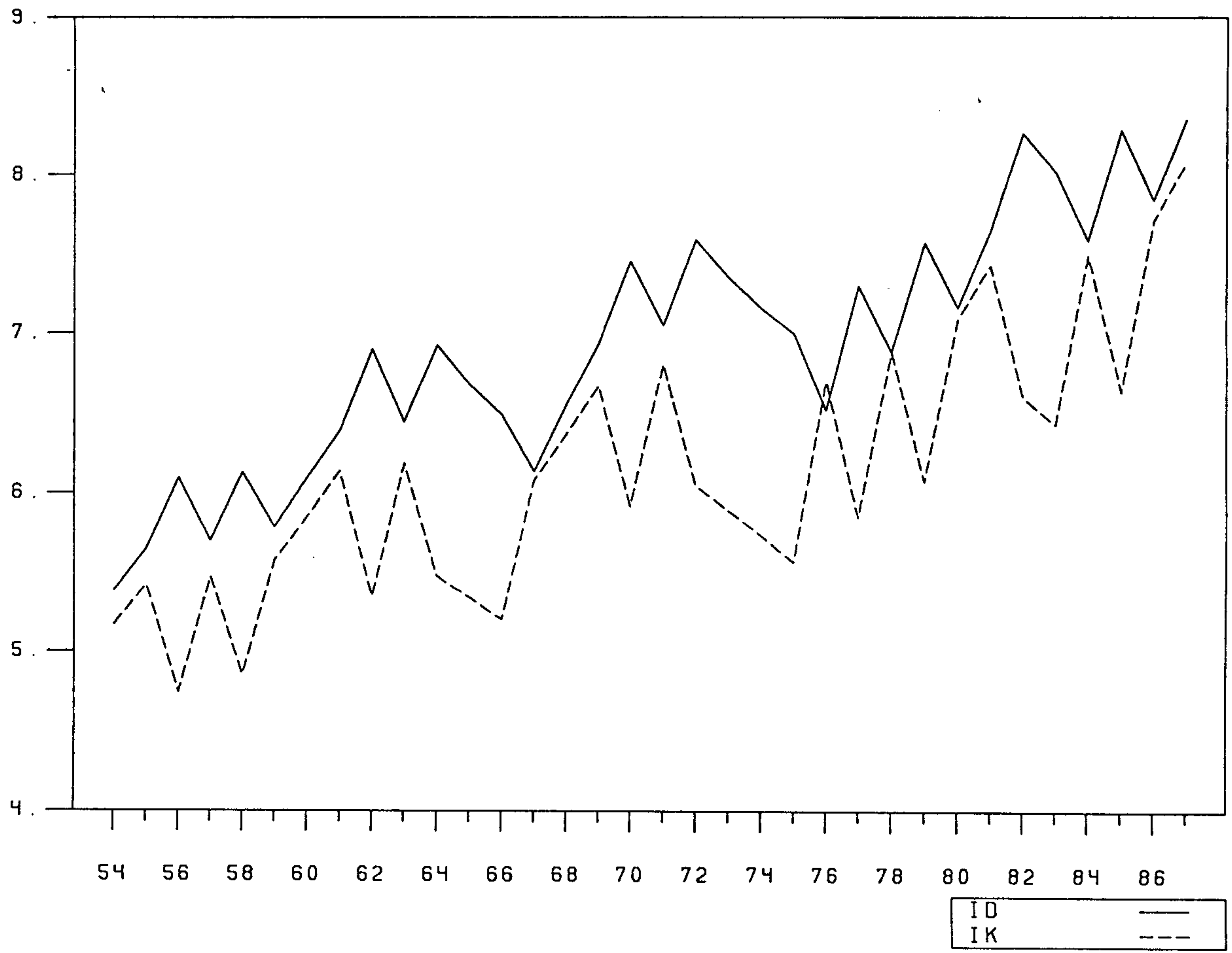
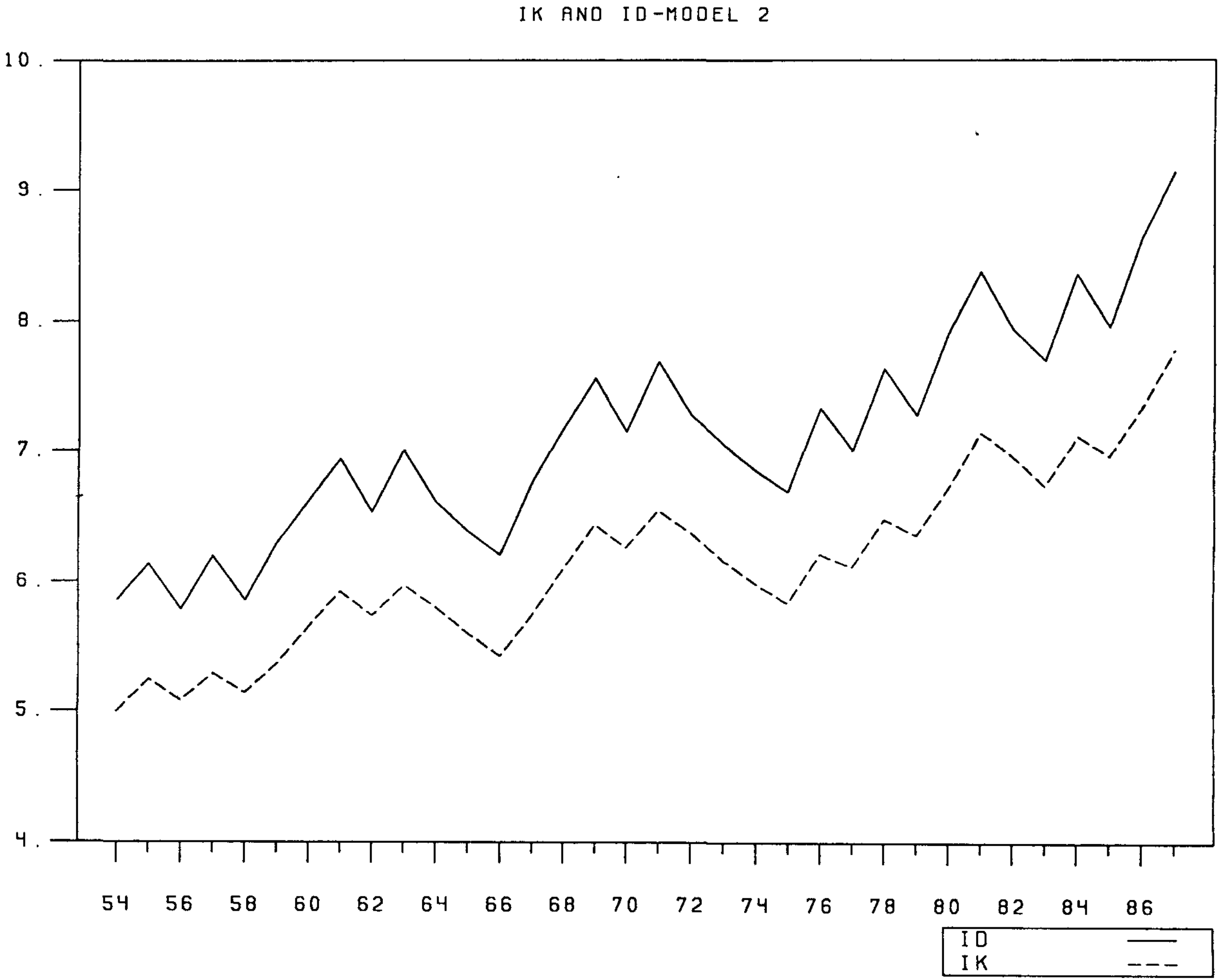


FIGURE 3


IK AND ID-MODEL 2
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