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ENERGY PRICE SHOCKS, CAPACITY UTILIZATION AND BUSINESS CYCLE FLUCTUATIONS

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ABSTRACT

This study focuses on the analysis of energy price shocks in the generation of business cycle phenomena. These shocks are transmitted through endogenous fluctuations in capital utilization. The production structure of the model gives rise to an empirical measure of 'true' technology growth that is exempt from recent criticisms levelled at the standard measure, i.e., Solow residual growth. The model is calibrated and evaluated for the U.S. economy using annual data over the 1960–1988 period. At business cycle frequencies, the model accounts for 74–91 percent of the volatility of U.S. output; closely matches the strong negative correlation between output and energy prices manifested in the U.S. data; and is generally consistent with other facts characterizing U.S. business cycles. Energy price shocks make a significant quantitative contribution to the model's ability to explain the data.

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I. Introduction

In popular discussion it is common to hear reference to energy price movements as shocks, shocks that are (in some sense) equivalent to production technology shocks and important sources of fluctuations in economic activity. Hamilton (1983) provides statistical evidence for the postwar U.S. that is supportive of the exogeneity of nominal oil price changes and of the significance of the relationship wherein oil price increases preceded most recessions. Hall (1989) shows for the U.S. (1953-84) that there are significantly negative correlations between sectoral technology growth, as measured by Solow residual growth, and the rate of change of the nominal price of oil.[1] Other facts, documented here for the U.S. (1960-88), include:

(i) the growth rates of an economy-wide measure of the Solow residual and the real price of energy show a correlation of -0.70;

(ii) at business cycle frequencies, aggregate output and the real price of energy exhibit a correlation of -0.77.

Figures 1-3 show the time series plots of the Solow residual, aggregate output and the real price of energy. These considerations prompt the questions: How are energy price shocks transmitted and propagated? How is the correlation between Solow residual growth and energy price changes to be explained? Relatedly, how is the correlation between growth rates of the Solow residual and government purchases to be rationalized?[2] Does the explicit accounting of energy price shocks reduce the variance of 'true' technology growth to an negligible amount? What is the quantitative importance of energy price shocks in the generation of business cycle phenomena (as defined by Lucas (1977))? These questions are the focus of this study.

In order to address these questions the methodology advanced by Kydland and

Prescott (1982) is followed. A model is specified, calibrated, numerically solved and simulated. It is then evaluated in terms of its ability to match the correlations in (i) and (ii) above as well as other facts characterizing U.S. business cycle fluctuations. The empirical regularities concerning the Solow residual constitute a new dimension for the evaluation of business cycle models.[3]

Previous analyses, based on traditional macroeconomic models, have established the quantitative importance of energy price shocks in their explanations of economic activity. Rasche and Tatom (1981) review and contribute to this literature. The first study to analyze the role of energy price shocks in generating business cycle phenomena, based on a dynamic, stochastic, computable general-equilibrium model is Kim and Loungani (1991). They estimate the percentage of output variability accounted for by energy price shocks as 10% or 4% depending on whether the value of the elasticity of substitution between the stock of capital and energy use is set at 1 or 0.6 respectively. Here also a dynamic, stochastic, computable general-equilibrium framework is adopted. One of the crucial differences between this model and that of Kim and Loungani (1991), as well as the traditional models, is that energy is <u>not</u> viewed as an input into production with a positive marginal product (ceteris paribus).[4] Rather, the present model maintains that energy is <u>essential</u> to the utilization of capital and thus to the flow of capital services that enters into the production function.[5] Accordingly, energy price shocks are transmitted <u>through</u> endogenous variations in capital utilization and the flow of capital services.[6]

There are three additional reasons for considering endogeneity of the capital utilization decision. First, its quantitative significance is advanced in Kydland and Prescott (1988, 1991) and Greenwood, Hercowitz and Huffman (1988).

In the Kydland-Prescott view, there is a fixed proportionate relationship (i.e., zero elasticity of substitution) between the hours worked by capital and labor and the only cost of capital utilization is the associated labor-hours cost. In the Greenwood-Hercowitz-Huffman view, there is a flexible proportionate relationship (i.e., unitary elasticity of substitution) between the hours of capital and labor and the only cost of capital utilization is a depreciation cost. Here the Greenwood-Hercowitz-Huffman view is extended to admit an energy cost to the capital utilization decision. The reason for choosing the latter view is also the second additional reason for considering endogenous utilization. Specifically, it results in a production structure which, when imposed on the data, gives a measure of true technology growth that is uncorrelated with the rate of change of real energy prices and government spending--as true technology growth should be. This was not the case for the production specification consistent with the Kydland-Prescott view of utilization. Third, the potential to explain the empirical regularities concerning the Solow residual exists. Solow residual growth can significantly mismeasure true technology growth by incorrectly including the rate of change of capital utilization. Given the endogeneity of the latter, all shocks will impact in equilibrium on the Solow residual. In particular, shocks to energy prices and government spending will be correlated with Solow residual growth.

The remainder of the paper is organized as follows. Section II outlines the model and solution technique. Section III describes the empirical data and measures of technology growth. Section IV describes the calibration and evaluation procedure. Section V presents and discusses the findings. Section VI concludes the paper.

II The Model and Solution Technique

A. The Economic Environment

Consider an environment inhabited by a large number of identical, infinitely-lived agents. The representative agent has preferences defined over consumption and leisure given by:

$$(1) \qquad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \quad , \; u(c_t, h_t) = \log c_t + \gamma \log(1-h_t)$$

$$0 < \beta < 1 \quad , \; \gamma > 0$$

where c_t is per-capita consumption at time t, h_t is per-capita hours worked at time t, β is the discount factor, γ is a preference parameter and the time endowment is normalized at unity. The momentary utility function, u, satisfies standard properties and a unitary elasticity of substitution between consumption and leisure. The latter restriction ensures that the model is consistent with balanced growth and a stationary allocation of time to market work.[7]

Output is produced in accordance with the production function:

$$(2) \qquad y_t = F(z_t h_t \; , \; k_t h_{kt}) = (z_t h_t)^{\theta} \; (k_t h_{kt})^{(1-\theta)}$$

$$0 < \theta < 1$$

where y_t is per-capita output at time t, z_t is exogenous technology at time t, k_t is the per-capita stock of capital in place at the beginning of time t, h_{kt} is an index of the utilization rate of k_t at time t and θ is the labor share of output. The production function, F, satisfies standard properties, constant returns to scale and a unitary elasticity of substitution between h_t and k_t.[8]

Given constant returns to scale, permanent technological change must be expressed in labor-augmenting form to ensure that the model is consistent with balanced growth.[9] This rationalizes why z_t enters (2) as specified. This production function differs from the standard neo-classical function solely by the inclusion of h_{kt}, representing the intensity of capital utilization (i.e., the number of hours per period and/or the speed per hour at which the capital stock is operated). For a given k_t, h_{kt} determines the flow of capital services, $k_t h_{kt}$. The manner in which h_{kt} enters (2) follows Taubman and Wilkinson (1970) and Greenwood, Hercowitz and Huffman (1988)--admitting flexible proportions (or a unitary elasticity of substitution) between h_t and h_{kt} and a direct relationship between h_{kt} and labor productivity.

Capital evolves according to:

$$(3) \qquad k_{t+1} = [1 - \delta(h_{kt})]k_t + i_t \quad , \quad \begin{array}{l} \delta(h_{kt}) = \omega_1 h_{kt}^{\omega_2} \\ 0 < \delta(\cdot) < 1 \\ \omega_1 > 0 \quad , \quad \omega_2 \geq 1 \end{array}$$

where i_t is per-capita gross investment at time t and ω_1, ω_2 are parameters. Equation (3) differs from the standard capital accumulation equation by allowing variable depreciation--specifying the depreciation function, δ, as an increasing convex function of h_{kt}. This specification also follows that in Taubman and Wilkinson (1970), and Greenwood, Hercowitz and Huffman (1988).[10] It captures Keynes's notion of the user cost to capital i.e., a higher utilization rate causes faster depreciation of the capital stock (at an increasing rate) because of wear and tear.

In the present environment, utilization also involves an energy cost.

Specifically:

$$(4) \qquad e_t/k_t = a(h_{kt}) \quad , \quad a(h_{kt}) = \nu_1 h_{kt}^{\nu_2}$$
$$\nu_1 > 0 \quad , \nu_2 \geq 1$$

where e_t is per-capita energy usage and ν_1, ν_2 are parameters. Equation (4) is a technical relationship, capturing the idea that energy is essential to the utilization of capital, with an increase in the utilization rate increasing energy usage (per unit of capital) at an increasing rate. The convexity of the function, a, is motivated by considerations of diminishing marginal energy efficiency. The resource constraint for the economy is:

$$(5) \qquad y_t = c_t + i_t + g_t + p_t e_t$$

where g_t is per-capita exogenous government spending at time t and p_t is the exogenous relative price of energy at time t. Hence, as in Christiano and Eichenbaum (1990), government enters the economy only as a pure resource drain in order to isolate its effects through spending per se.[11] Including $p_t e_t$ in the resource constraint may be interpreted as the output that is exchanged with the rest of the world for energy, e_t, at the exogenous terms of trade, p_t.[12] It is the only international trade that occurs.

The description of the environment is completed by describing the stochastic exogenous shock structure:

$$(6) \qquad \log(z_{t+1}) = \log(z_t) + \log(\bar{z}) + u_{zt+1}$$

$$(7) \qquad \log(\bar{g}_{t+1}) = \rho_g \log(\bar{g}_t) + (1 - \rho_g)\log(\bar{g}) + u_{gt+1} \quad , \quad \bar{g}_t \equiv g_t/z_t$$
$$0 < \rho_g < 1$$

$$(8) \qquad \log(p_{t+1}) = \rho_p \log(p_t) + (1 - \rho_p)\log(\bar{p}) + u_{pt+1} \quad , \quad 0 < \rho_p < 1$$

where $\log(\bar{z})$ is the mean growth of z_t, $\log(\bar{g})$ is the mean of $\log(\bar{g}_t)$, $\log(\bar{p})$ is the mean of $\log(p_t)$ and ρ_g, ρ_p are parameters. The innovations u_{zt+1}, u_{gt+1} and u_{pt+1} are assumed to be realized at the beginning of time (t+1), have zero means and are generated from the stationary Markov distribution function $\Phi(u_{t+1} \mid u_t)$, where u_{t+1} is a vector comprising of the three innovations. The specification of the g_t process is the same as that in Christiano and Eichenbaum (1990)--movements in z_t give rise to permanent movements in g_t, while changes in $\bar{g}_t$ cause temporary fluctuations in g_t.

B. The Competitive Equilibrium

In order to determine the equilibrium process of this economy, the coincidence between competitive equilibria and Pareto optima in the absence of externalities is invoked. The relevant Pareto optimum is the one that maximizes the utility of the representative agent in (1) subject to the technology/resource constraints in (2) - (5) and the stochastic exogenous shock structure in (6) - (8). This is a standard discounted programming problem:

$$(9) \qquad v(k_t \,;\, z_t, g_t, p_t, u_t) =$$
$$\max \{u(c_t, h_t) + \beta \int v(k_{t+1} \,;\, z_{t+1}, g_{t+1}, p_{t+1}, u_{t+1}) d\Phi(u_{t+1} \mid u_t)\}$$

subject to:

$$(10) \qquad c_t = F(z_t h_t,\ k_t h_{kt}) - k_{t+1} + [1 - \delta(h_{kt})]k_t - g_t - p_t k_t a(h_{kt})$$

and (6) - (8), where the transition equation (10) is obtained by substituting (2) - (4) into (5). The choice variables are c_t, h_t, k_{t+1}, h_{kt}.

The first-order conditions of the problem consist of equation (10) and the following three conditions:

$$(11) \qquad -u_2(c_t,\ h_t) = u_1(c_t,\ h_t)\ z_t F_1(z_t h_t,\ k_t h_{kt})$$

$$(12) \qquad \delta'(h_{kt})k_t + a'(h_{kt})p_t k_t = F_2(z_t h_t,\ k_t h_{kt})k_t$$

$$(13) \qquad u_1(c_t,\ h_t) = \beta \int u_1(c_{t+1},\ h_{t+1})[F_2(z_{t+1}h_{t+1},\ k_{t+1}h_{kt+1})h_{kt+1} + 1 - \delta(h_{kt+1}) - a(h_{kt+1})p_{t+1}]d\Phi(u_{t+1} \mid u_t)$$

Equation (11) is the efficiency condition governing h_t--extending the standard one by incorporating h_{kt}. Equation (12) determines the efficient value of h_{kt}, setting the sum of the marginal depreciation and energy costs equal to the marginal productivity of an increase in h_{kt}. Equation (13) is the efficiency condition governing capital accumulation. It differs from the standard one not only by allowing variable utilization but also by subtracting the marginal energy cost from the (gross) marginal productivity of an increase in k_{t+1}.

In this economy a positive shock to p_t will directly cause a negative income effect (see (10)) that works to reduce c_t and increase h_t. From (12), the optimal value of h_{kt} falls, which in turn reduces the marginal productivity of labor and promotes an intratemporal substitution effect out of work and consumption and into leisure (see (11)). In addition, the fall in h_{kt} directly impacts on the production function, working to reduce output and to enhance the negative income effect of the shock to p_t. This is the sense, then, in which a positive energy price shock is tantamount to a negative production technology shock in the present environment. If the increase in p_t is temporary but persistent, intertemporal substitution margins are also affected as follows (see (13))--capital accumulation is discouraged as agents smooth consumption and anticipate reduced returns to investment.

A positive shock to g_t will also directly cause a negative income effect (see (10)) that tends to reduce c_t and increase h_t. The increase in h_t increases the marginal productivity of utilization and thus the optimal value of h_{kt} (see (12)). The increases in h_t and h_{kt} cause output to increase. Given the direct link between h_{kt} and the marginal productivity of labor, it follows that labor faces less sharply diminishing returns than it otherwise would. If the marginal productivity of labor falls, an intratemporal substitution effect enhances the decrease in consumption. To the extent that the shock is temporary, it is likely that investment falls as agents attempt to smooth consumption. Due to the response of h_{kt}, the propagation mechanism of shocks to g_t is quite different to that in Christiano and Eichenbaum (1990).

Equations (10) - (13) implicitly define the competitive allocation/decision rules for the economy. Exact solutions are not possible. Instead, the decision rules for an approximate economy are numerically computed.

C. The Solution Technique

The decision rules are obtained using the solution technique advanced by

Kydland and Prescott (1982) and modified by Christiano (1988). It is implemented here following the approach outlined in Hansen and Sargent (1988) and Pace (1989). The key steps are indicated in Appendix 1.

III The Empirical Data and Measures of Technology Growth

(i) Data

The empirical data are annual, real, per capita data for the U.S. over the period 1960-1988. They are used at the calibration and evaluation stages of the study. The choice of an annual periodicity was predicated on the desire to use the most relevant and longest data series on energy usage available. A brief description of the data ensues, full details are presented in Appendix 2.

Energy usage is the sum of electricity, coal, natural gas and petroleum usage by the private non-energy production sector of the economy. The four components of this energy good serve as weights in the construction of the energy price deflator. The real price of energy is the ratio of the energy price deflator to the gross domestic product price deflator. Output is gross domestic product less the sum of gross housing and government products as well as value added by the energy-producing sector. Consumption is personal consumption expenditure on nondurables and services minus housing services and spending on energy goods. Investment is gross private domestic fixed investment in nonresidential capital. Government spending is government purchases of goods and services. Labor hours are the product of employment and average hours per worker per year, where employment is private non-energy sector employment. One measure of the capital stock is the net stock of fixed nonresidential private capital, denoted henceforth by k_t^*.

The reason for subtracting gross housing product and consumer spending on housing services above derives from the consideration in Greenwood and Hercowitz (1991)--these are activities associated with household production.[13] The rationale for subtracting gross government product and for choosing private sector measures of investment, capital and employment is that the present model is more relevant to explanations of private sector behavior than it is to that of government. Finally, energy-sector production activity and household energy consumption is excluded from the foregoing measures since they are not explained by the present model.[14]

The capital stock measure, k_t^*, is constructed using the Perpetual Inventory Method which assumes a constant depreciation rate. Accordingly, k_t^* is <u>not</u> the empirical counterpart to the model's capital stock. Existing data also do not provide satisfactory counterparts for the capital utilization rate. As pointed out in Greenwood, Hercowitz and Huffman (1988), existing utilization measures reflect the cyclical behavior of manufacturing output. Therefore, empirical counterparts for k_t and h_{kt} are constructed here by imposing equations (3) and (12) on the data i.e., by using:

$$(3) \qquad k_{t+1} = [1 - \delta(h_{kt})]k_t + i_t$$

$$(12') \qquad \delta'(h_{kt}) + a'(h_{kt})p_t = (1 - \theta)y_t/(k_t h_{kt})$$

where (12') is obtained from (12) having noted equation (2) and simplifying. The initial starting value of k_t is set equal to the 1960 value of k_t^*. Values for the parameters in (3) and (12') are chosen as part of the calibration exercise. The actual data counterparts for i_t, p_t and y_t are used in (3) and (12').

(ii) Measures of Technology Growth

The imposition of equation (2) on the data gives rise to the measure of 'true' technology growth (as defined by the present model):

$$(14) \qquad \Delta\log z_t = [\Delta\log y_t - \theta\Delta\log h_t - (1-\theta)\Delta\log k_t - (1-\theta)\Delta\log h_{kt}]/\theta$$

In contrast, the standard measure of technology growth, Solow residual growth (or just the Solow residual) is:

$$(15) \qquad \Delta\log sr_t = [\Delta\log y_t - \theta\Delta\log h_t - (1-\theta)\Delta\log k^*_t]/\theta$$

The two measures can be seen to differ in their measurement of capital and treatment of utilization.

The distinction is an important one. Consider the time series properties:

Table 1

	%SD	Corr with $\Delta\log z_t$	Corr with $\Delta\log sr_t$
$\Delta\log z_t$	2.1782	1	0.6389
$\Delta\log sr_t$	3.4790	0.6389	1
$\Delta\log p_t$	9.4770	-0.0346	-0.7003
$\Delta\log g_t$	3.0870	0.0688	0.2456

Key: %SD denotes the percentage standard deviation. Corr with x_t denotes correlation with x_t, $x_t = \Delta\log z_t$, $\Delta\log sr_t$.

Correlations between Solow residual growth and the rates of change of energy prices and government spending, of similar sign and magnitude to those in Table 1, are pointed out by Hall (1989).[15] Such correlations make nonsense of the interpretation of Solow residual growth as true technology growth. Hall

(1989) interprets the correlations as evidence of market imperfection and increasing returns to scale. Given the negligible correlations between $\Delta\log z_t$ and both $\Delta\log p_t$ and $\Delta\log g_t$ in Table 1, an alternative interpretation, offered here, is that the Solow residual correlations can be explained by the endogenous variations of utilization in an environment of perfect competition and constant returns to scale. In addition, they can be explained while the variance of true technology growth is a significant number.

The difference in the time series properties of $\Delta\log z_t$ and $\Delta\log sr_t$ stems, in essence, from the behavior of h_{kt}. The correlations between $(\Delta\log k_t - \Delta\log k_t^*)$ and each of $\Delta\log p_t$ and $\Delta\log g_t$ are very small-- -0.0074 and 0.0604, respectively. The correlations between $\Delta\log h_{kt}$ and each of $\Delta\log p_t$ and $\Delta\log g_t$ are -0.7726 and 0.1841, respectively. The latter correlations are consistent with the qualitative discussion in Section II concerning the effect of energy price and governing spending shocks on h_{kt}.

Maintaining the more restrictive assumption of a fixed proportionate relationship between the hours worked by capital and labor, as in Kydland and Prescott (1988, 1991), does not result in a generated $\Delta\log z_t$ series that is satisfactory. Specifically, the generated $\Delta\log z_t$ series exhibits very similar dynamics to those of $\Delta\log sr_t$. The correlation between the two series is 0.9971 and $\Delta\log z_t$ shows a correlation with $\Delta\log p_t$ ($\Delta\log g_t$) equal to -0.7075 (0.2356). In essence, this result obtains not only because of the small capital share but also because the correlation between the growth rate of hours per worker and $\Delta\log p_t$ is not sufficiently negative at -0.4129.[16]

Hall (1989) rules out fluctuations in the utilization rate of capital as being quantitatively capable of explaining the Solow residual correlations. The reason for the apparent inconsistency between that argument and the one advanced here concerns the modelling of the utilization rate. Hall maintains a fixed proportionate relationship between the rates of change of the utilization rate of capital and total labor hours per unit of the measured capital stock. No such restriction is imposed by the present model.

IV Calibration and Evaluation Procedures

(i) Calibration

The calibration and evaluation procedures advanced by Kydland and Prescott (1982) are followed in this study. Therefore, model parameters are set in advance based upon: prior information; equality between parameters or steady state values of model variables and the average values of their data counterparts that characterize the U.S. growth experience; or upon estimation of the stochastic processes governing the evolution of exogenous variables. Specific details ensue.

Preliminary notes include the following. The time period of the model is defined as one year. Steady state values of model variables will be denoted using earlier notation except that time subscripts are omitted and, when relevant, a bar denotes that the stationarity-inducing transformation defined in Appendix 1 has been undertaken. In order to promote parsimony and to treat the (multiplicative) coefficients of h_k in each of the two marginal cost terms in equation (12) symmetrically (when evaluated at the steady state), $\omega_1 = \omega_2^{-1}$ and $\nu_1 = (\nu_2 p)^{-1}$.

β is set equal to its standard value of 0.96. The balanced growth hypothesis and equation (2) are imposed on the data to give the value of $\bar{z}$ = 1.0164, the average gross growth rate of U.S. output. h is set equal 0.3529, the average value in the data for the ratio of hours worked to total nonsleeping hours (per worker). p is set equal to 0.6373, which is the average value of the relative price of energy for the U.S. The average ratio of government spending to output in the U.S., 0.2890, is chosen for $(\bar{g}/\bar{y})$. θ is set equal to 0.64, which is an average data measure of labor's share of output.[17]

No guide is available in the literature as to the values of ω_2 and ν_2. Accordingly, these values are determined here by using the values for $\bar{z}$, β, p and θ, together with the steady state condition that determines h_k, to ensure simultaneously that steady state depreciation, $\delta(h_k)$, equals 0.078 and that the steady state energy-capital ratio is 0.071. The number, 0.078, is derived from the average service life of nonresidential structures and equipment for the 1954-1985 period in the U.S.[18] The number, 0.071, is regarded as a reasonable estimate of the true average energy-capital ratio in the U.S. data--given that the actual measured average ratio equals 0.051 and the reasons for why the energy measure is an understatement of true energy usage in Appendix 2.[19]

The foregoing values and the remaining steady state conditions of the model are next used to solve for $\gamma = 2.7011$ (as well as the steady state values of the remaining endogenous variables). The parameters of the stochastic processes for the exogenous variables, in (6) - (8), are consistently estimated using the Box-Jenkins three-step--identification, estimation and diagnostics--univariate analysis.[20] This suggested that the most parsimonious and adequate specifications are:

$$(6') \qquad \log(z_{t+1}) = \log(z_t) + \log(\bar{z}) + u_{zt+1}, \; u_{zt+1} = \epsilon_{zt+1} + \eta_z \epsilon_{zt}$$

$$(7') \qquad \log(\bar{g}_{t+1}) = \rho_g \log(\bar{g}_t) + (1-\rho_g)\log(\bar{g}) + u_{gt+1}, \; u_{gt+1} = \epsilon_{gt+1} + \eta_g \epsilon_{gt}$$

(8') $\log(p_{t+1}) = \rho_p \log(p_t) + (1-\rho_p)\log(p) + u_{pt+1}, \quad u_{pt+1} = \epsilon_{pt+1} + \eta_p \epsilon_{pt}$

where ϵ_{it} is a stationary, zero-mean, serially-uncorrelated innovation (i = z, g, p) and η_i is a parameter (i = z, g, p).[21] The key findings are presented in Table 2 (with new notation specified in the key).

Table 2

Coefficient Estimates

$\hat{\eta}_z = 0.4981$	(0.1924)		
$\hat{\rho}_g = 0.9278$	(0.0336)	$\hat{\eta}_g = 0.6898$	(0.2015)
$\hat{\rho}_p = 0.9143$	(0.0597)	$\hat{\eta}_p = 0.3809$	(0.2087)

Residual Properties

$\hat{\sigma}_z = 0.0195$	$\hat{\sigma}_g = 0.0279$	$\hat{\sigma}_p = 0.0871$
$\hat{\sigma}_{zg} = -0.0003$	$\hat{\sigma}_{zp} = -0.0002$	$\hat{\sigma}_{gp} = -0.0001$
$\hat{c}_{zg} = -0.5120$	$\hat{c}_{zp} = -0.1012$	$\hat{c}_{gp} = -0.0236$

Autocorrelations

S.E. = 0.189

	$\hat{\epsilon}_{zt}$	$\hat{\epsilon}_{gt}$	$\hat{\epsilon}_{pt}$
Lag 1	-0.005	0.070	0.034
Lag 2	0.145	0.130	0.162
Lag 3	0.098	-0.053	0.157
Lag 4	0.223	0.029	-0.017
Lag 5	0.072	0.002	0.022
	Q(5) = 2.399	Q(5) = 0.711	Q(5) = 1.479
	$\chi^2_4 = 9.49$	$\chi^2_3 = 7.81$	$\chi^2_3 = 7.81$

Key: (i) A ^ denotes an estimated quantity.
(ii) Standard errors are in parentheses.
(iii) σ_i is the standard deviation of ϵ_{it} $(i = z,g,p)$.

σ_{ij} is the covariance between ϵ_{it} and ϵ_{jt} $(i,j = z,g,p)$.

c_{ij} is the correlation coefficient between ϵ_{it} and ϵ_{jt} $(i,j = z,g,p)$.

(iv) S.E. denotes standard error. Q is the Box-Pierce statistic and χ^2_i is the critical value of the chi-square statistic at the 5% significance level and i degrees of freedom.

(v) Sample period: 1961-1988.

The coefficient estimates are significantly greater than zero.[22] Thus, $\rho_g = \hat{\rho}_g$, $\rho_p = \hat{\rho}_p$, $\eta_z = \hat{\eta}_z$, $\eta_g = \hat{\eta}_g$ and $\eta_p = \hat{\eta}_p$. The values: $\sigma_z = \hat{\sigma}_z$, $\sigma_g = \hat{\sigma}_g$ and $\sigma_p = \hat{\sigma}_p$ are also chosen. Of the covariances only $\hat{\sigma}_{zg}$ is significantly different from zero.[23],[24] Therefore, $\sigma_{zg} = \hat{\sigma}_{zg}$, $\sigma_{zp} = 0$ and $\sigma_{gp} = 0$. The analysis of the autocorrelations of the residuals and Box-Pierce test suggests that the residuals are serially-uncorrelated.

Table 3 summarizes the parameter and steady state values of the model.

Table 3

<u>Preferences</u>	<u>Steady State</u>
$\beta = 0.9600$	$\bar{y} = 0.2098$
$\gamma = 2.7011$	$\bar{c} = 0.0912$
<u>Production</u>	$\bar{i} = 0.0390$
$\theta = 0.6400$	$h = 0.3529$
$\omega_2 = 1.3880$	$\bar{k} = 0.4204$
$\omega_1 = \omega_2^{-1}$	$h_k = 0.2013$
$\nu_2 = 1.6200$	$\bar{e} = 0.0299$
$\nu_1 = (\nu_2 p)^{-1}$	$\delta(h_k) = 0.0779$
	$(\bar{e}/\bar{k}) = 0.0710$

Stochastic Structure

$\bar{z} = 1.0164$	$\eta_z = 0.4981$	
$\sigma_z = 0.0195$	$\sigma_{zg} = -0.0003$	$\sigma_{zp} = 0$
$\bar{g} = 0.0606$	$\rho_g = 0.9278$	$\eta_g = 0.6898$
$\sigma_g = 0.0279$	$\sigma_{gz} = \sigma_{zg}$	$\sigma_{gp} = 0$
$p = 0.6373$	$\rho_p = 0.9143$	$\eta_p = 0.3809$
$\sigma_p = 0.0871$	$\sigma_{pz} = 0$	$\sigma_{pg} = 0$

(ii) Evaluation

The steps involved in the evaluation procedure are now described.

(a) The Markovian decision rules for the nonstationary economy, the laws of motion of the exogenous variables and the definitions of relevance are used to simulate time paths for (logarithmic levels of) the variables of interest.[25] The time paths have 29 observations (the size of the U.S. data sample). Any one simulation, then, corresponds to one sample of 29 realizations of $\epsilon_t \equiv [\epsilon_{zt} \quad \epsilon_{gt} \quad \epsilon_{pt}]'$. Two alternative approaches are taken in obtaining the latter sample:

[1] a normal random number generator is used to give a sequence for ϵ_t that satisfies the assumed properties for the innovations.

[2] the actual sequence of residuals from the estimation exercise is used for ϵ_t.

The approach in [1] is generally the one taken in the existing literature. Its advantages include the possibility of reducing dependency on the initial conditions of the simulation as well as on sampling uncertainty. Its disadvantage is that it imposes the assumption of normally distributed innovations. The approach in [2] reverses this scenario. Specifically, its disadvantages lie in the dependency on initial conditions and exposure to the idiosyncracies of a sample realization. Its advantage is that it does not impose a distributional assumption of ϵ_t. This may be an important advantage in the present context, where the innovations to real energy prices are unlikely to be normally distributed.[26] Due to these considerations, both approaches are pursued here. In order to reap the advantages mentioned for approach [1], 200 independent samples, each initially consisting of 200 observations, are simulated; then, the first 171 observations are discarded from each sample. For each simulation (in each approach), the steady state values of state variables and $z_0 = 1$ are used as the initial conditions.

(b) For each sample, the data is filtered using the Hodrick-Prescott (H-P) or first-difference filter.[27]

(c) Summary statistics are computed for the filtered model data. When approach [1] is followed, the statistics are averages of the statistic in question across the 200 samples.

(d) The statistics for the model data are compared to the corresponding statistics for the U.S. filtered (logarithmic level) data.

The foregoing evaluation procedure is undertaken for the model described earlier (henceforth referred to as the basic model). It is also undertaken for two special variants--one that abstracts from energy price shocks (so $p_t \equiv p \ \forall t$ and $\sigma_p = 0$) and one that abstracts from shocks to the stationary component of government spending (so $\tilde{g}_t = \bar{g} \ \forall t$ and $\sigma_g = 0$). The latter two experiments permit the isolation of the contribution of the energy-price and the temporary government spending shocks to the basic model.[28]

As mentioned above, two alternative filtering methods are employed--since, generally, it is interesting to examine behavior at the associated different frequencies. Specifically, the two methods are used in obtaining all model and U.S. data statistics except those pertaining to the dynamic properties of the Solow residual and true technology. In the latter case, only the first-difference filter is used since the interest in these properties is motivated exclusively by the documented regularities (in Section 3 and in Hall (1989)) at first-difference frequencies. When both filtering methods are employed, the focus of the discussion is on the 'H-P filtered' statistics because of their advantages over 'first-differenced' statistics, as emphasized by Kydland and Prescott (1991).

V Quantitative Findings

The findings to be discussed are presented in Tables 4-9.[29]

(i) The Basic Model

Consider the basic model, starting with Table 4.

In the U.S. data, the salient features of the standard deviations are: the well-known facts that investment is more volatile and consumption and hours are less volatile than output; the capital stock (this is the 'true' one) and utilization rate are quite volatile. The model accounts for 74%/91% of the volatility of U.S. output (depending on whether normal or actual innovations are used). It also captures the rankings of the relative volatilities of investment, consumption and hours as well as that of the average factor productivities. The model significantly overstates (understates) the variability of investment (capital).[30] The former is especially true when the actual innovations are used. It is not surprising that the model predicts energy-use volatility in excess of that in the data, given the under-measurement of the latter.

Each series in the U.S. data exhibits high persistency. The model mimics this quite well--only the autocorrelation of consumption is somewhat understated.

Regarding correlations between the endogenous variables and output, the U.S. data show all series to be strongly procyclical except for capital and the average productivity of capital services, which are acyclical and countercyclical, respectively. These features are generally captured closely by the model--exceptions being that consumption is not procyclical enough while investment and the average productivity of capital are too procyclical.

The negative correlations in the U.S. data between each of capital services and capital and its respective productivity are well predicted by the model. The positive correlation in the U.S. data between hours and its productivity is significantly understated by the model, especially when normal innovations are used.[31]

The remaining correlations involving the three exogenous variables, show that the model matches the U.S. data closely along these dimensions. In particular, for the correlations between output and energy prices, the model predictions of -0.52/-0.77 are close or equal to the data's -0.77. The apparent overstatement by the model of the magnitude of the correlation between energy use and prices does not seem to be a real one--again due to the underestimate of energy use in the U.S. data.

In Table 5, the model is seen to closely capture the empirical regularities relating to Solow residual growth. Notice especially, for the correlation between Solow residual and energy price growth, the model predictions of -0.55/ -0.63 are close to the data's -0.70.

In short, Tables 4 and 5 suggest that the model explains a high fraction of U.S. output variability; closely matches the U.S. regularities that involve capital utilization, energy use, energy prices and Solow residual growth; and is generally consistent with the other facts of U.S. business cycles.[32] Discrepancies between the model and data that seem significant include: (a) the overstatement (understatement) of investment (capital) volatility; (b) consumption persistency that is too low; (c) the overestimate (underestimate) of the procyclicality of investment and the average productivity of capital (consumption) and (d) the correlation between hours and its productivity that is too low.

It is possible that some of these discrepancies stem from a lack of support for the assumption of a unitary elasticity of intertemporal substitution in consumption. Lower values of this elasticity imply agents are less willing to substitute consumption intertemporally, making consumption (investment) more (less) procyclical, consumption more persistent and investment less volatile. It is difficult to assess apriori the implications for capital volatility and the procyclicality of the average product of capital, since these depend (inter alia) crucially on how the covariance between utilization and investment is affected. In view of these considerations and the fact that independent evidence gives imprecise estimates of the elasticity of intertemporal substitution, the foregoing possibility will be investigated in further work.[33],[34] The discrepancy in (iv) will be returned to below.

(ii) The Contribution of Energy-Price Shocks

Next compare the findings of the model with $\sigma_p = 0$ to those for the basic model, starting with Tables 6 and 4.

First consider the standard deviations. Energy-price shocks contribute

13%/31% to the fraction of U.S. output volatility that is accounted for by the basic model (depending on whether normal or actual innovations are used). In addition, they strongly impact on the volatilities of investment, capital, utilization, energy use and the average productivity of capital services.

Second, energy-price shocks contribute little to the basic model predictions with respect to persistency. This suggests rapid adjustment to these shocks.

Third, examine the correlations between the endogenous variables and output. The important and quite dramatic effect of energy-price shocks is on the correlation between the average productivity of capital services and output.[35] These shocks are a strong source of negative covariation between capital services and output and are primarily responsible for the basic model's ability to explain this dimension of the data. Intuitively, the productive input most strongly and negatively affected by a positive energy-price shock is utilization; promoting a decline in output and an increase in the average productivity of capital services.

Fourth, consider the correlations between factors and their average productivities. The important effect of energy-price shocks is on the correlations involving hours and capital services. In each case, these shocks promote the basic model's ability to explain the data. The latter finding is consistent with that discussed in the last paragraph. The energy-price shocks are a strong source of positive covariation between hours and its productivity (but only when actual innovations are used). Intuitively, a positive energy-price shock reduces utilization and thus the marginal product of hours, creating an intratemporal substitution force to reduce hours.

Fifth, energy-price shocks have little effect on the basic model's predictions for the correlations between output and each of technology and government spending.

Finally, compare Tables 7 and 5. The key effects here are: energy-price shocks enhance the volatility of Solow residual growth and reduce the correlation between the Solow residual and technology growth. In so doing, the basic model better captures the data. The dramatic increase in the volatility of the utilization growth rate, induced by energy-price shocks (shown in Appendix 3), would seem to be primarily responsible for these effects.

In short, energy price shocks contribute a significant percentage to the fraction of U.S. output variability accounted for by the basic model. They strongly impact on the volatilities of investment, capital, utilization, energy use and the average productivity of capital services; and are an important source of negative (positive) comovement between capital services (hours) and its productivity. The effects along each of the dimensions constitute improvements in the basic model's ability to match the regularities in the U.S. data, with the one exception of investment volatility. In addition, it is only by including energy-price shocks that the basic model can predict the strong negative correlations between output and energy prices, energy use and energy prices, Solow residual and energy price growth that are manifest in the U.S. data.

(iii) The Contribution of Temporary Government Spending Shocks

The comparison of Tables 8 and 4 and of Tables 9 and 5 permits isolation of the contribution of (temporary) government spending shocks.[36] The quantitatively important effects for the predictions of the basic model arising from the inclusion of these shocks are:

(a) The increased volatility of hours.

(b) The reduced procyclicality of consumption and hours, which hampers the basic model's ability to explain the data. The explanation is as follows. From the discussion in Section II, government spending shocks induce negative (positive) comovement between consumption (hours) and output. To explain the reduced procyclicality of hours one must further recall that innovations to technology and government spending are negatively correlated and each type of innovation will have differential quantitative impacts on hours and output.

(c) The reduction in the correlation between hours and its productivity. In fact, absent government spending shocks, the model would no longer perform poorly along this dimension of the data. The explanation of the reduction, is direct from the law of diminishing returns and the discussion in Section II, where it was seen that these shocks directly impact on hours.

(d) The decrease in the correlation between output and government spending (but only when normal innovations are used), which mitigates the basic model's ability to explain the data. The reason for this lies in the negative correlation between innovations to technology and government spending.

(e) The reduced correlation between Solow residual and government spending growth, which enhances the basic model's explanation of the data. The reduction derives from the facts that temporary government spending shocks break the perfect link between technology and government spending growth that would otherwise obtain (by construction) and technology and Solow residual growth are highly positively correlated.

In view of these considerations it seems that a richer model of government spending is called for, but this is beyond the scope of the present study.

(iv) Other Findings

The most striking differences in the basic model's predictions across the innovation strategies taken concern the fraction of U.S. output volatility accounted for by the model and the volatility of investment. The former (latter) difference is mostly eliminated when σ_p or σ_g (σ_p) are set to zero. This suggests that the innovations to energy prices strongly deviate from a normal distribution.

The first-difference counterparts to Tables 4, 6 and 8 are Tables 4A, 6A and 8A and are shown in Appendix 3. As a general rule, the model does not perform quite as well at this frequency and the findings are qualitatively similar to those discussed above. The former is not entirely surprising, however, since the model assumes exogenous growth given its focus on explaining cyclical behavior. An extension of the model to admit endogenous growth seems an interesting avenue for future research.

VI Conclusion

This study focuses on the analysis of energy price shocks in the generation of business cycle phenomena. The model maintains that energy is essential to the utilization of capital. Accordingly, energy price shocks are transmitted through endogenous fluctuations in capital utilization and the flow of capital services that enter production.

When the production structure envisaged by this model is imposed on the U.S. data, it gives rise to a measure of 'true' technology growth that differs in important respects from the standard measure, i.e., Solow residual growth.

Specifically, the former measure is uncorrelated with changes in energy prices and government spending--as true technology should be--while the latter measure exhibits strong correlations with these variables.

The model is calibrated and evaluated for the U.S. economy using annual data over the 1960-1988 period. At business cycle frequencies, the model accounts for 74% - 91% of the volatility of U.S. output; closely matches the strong negative correlation between output and energy prices manifested in the U.S. data; and is generally consistent with the other facts characterizing U.S. business cycles. In addition, the model can explain the aforementioned empirical regularities concerning Solow residual growth. Energy price shocks make a significant quantitative contribution to the model's ability to explain the data.

Extensions of this model to address questions of economic growth (see e.g. Hercowitz and Sampson (1991) and Gomme (1991)); questions concerning the dynamics of small open economies (see e.g. Finn (1990) and Mendoza (1991)), particularly real exchange rate dynamics and, more generally, international business cycle behavior (see e.g. Stockman and Tesar (1990)) seem exciting avenues for future research.

Fig. 1

Logarithm of Solow Residual

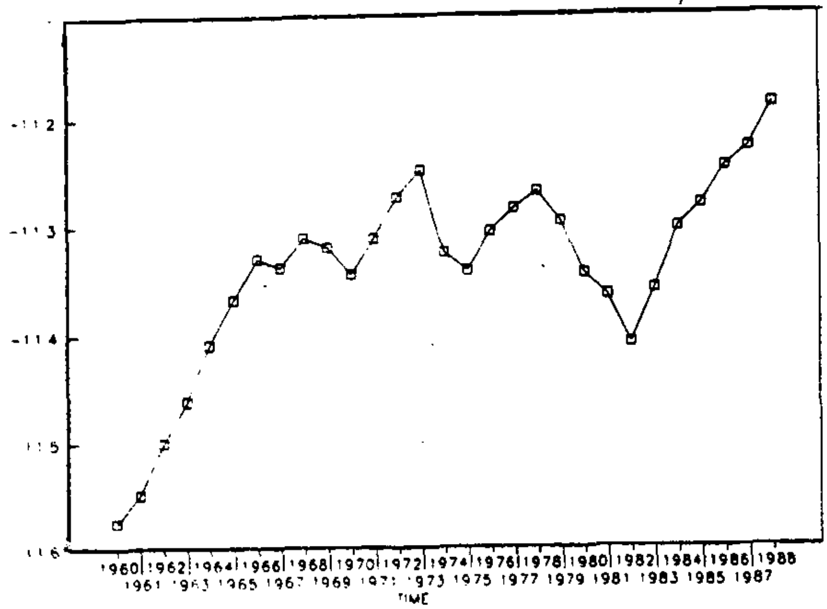

Fig. 2

Logarithm of Output

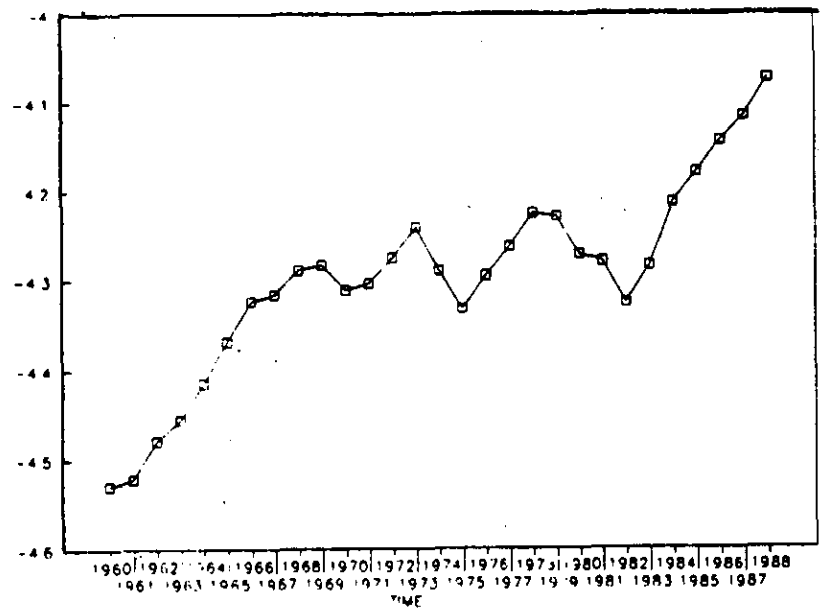

Fig. 3

Logaritm of Real Energy Price

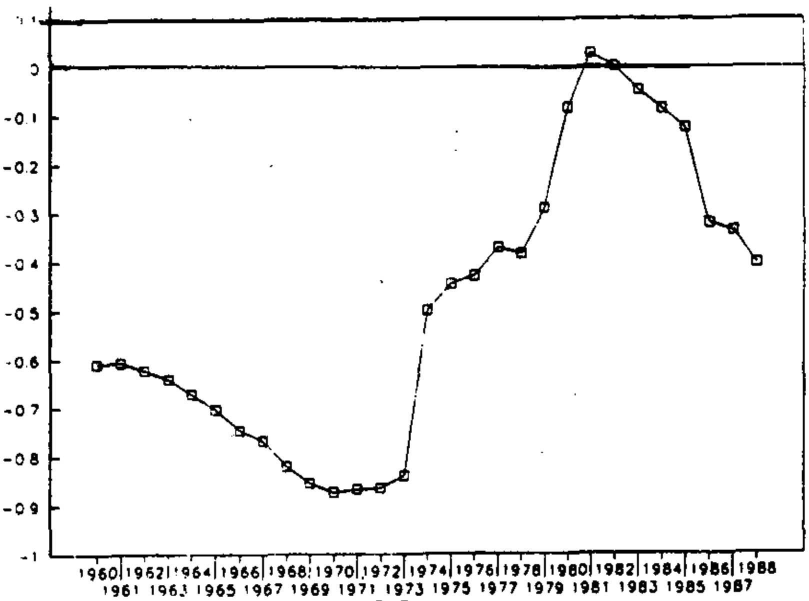

Table 4: Basic Model, U.S. data (H-P filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Autol	Corry	% SD	Autol	Corry	% SD	Autol	Corry
yf	3.0286	0.6688	1.0000	3.7501	0.8259	1.0000	4.1180	0.7189	1.0000
cf	2.6906	0.5484	0.6557	2.7695	0.5014	0.4244	2.0522	0.8161	0.8786
if	11.2706	0.5256	0.8707	14.4279	0.6164	0.9139	6.7180	0.5031	0.7351
hf	1.4461	0.3125	0.5279	2.2814	0.6700	0.7416	2.2894	0.6152	0.8585
ksf	4.4894	0.6661	0.8009	6.0084	0.8512	0.9022	7.0532	0.8305	0.8858
kf	1.2727	0.8248	-0.0932	1.7465	0.8693	-0.2686	4.8166	0.9305	0.0356
$h_k f$	4.6196	0.6438	0.7968	6.5572	0.8520	0.8983	7.3325	0.8059	0.8286
ef	7.2845	0.6542	0.8067	9.9889	0.8512	0.9083	4.4025	0.6152	0.8227
APhf	2.5579	0.6508	0.8661	2.5648	0.7385	0.8025	2.4518	0.7738	0.8779
APksf	2.6547	0.6243	-0.2288	3.0831	0.7520	-0.5420	3.9054	0.6133	-0.5452
APkf	3.3678	0.6438	0.9228	4.5423	0.8193	0.9289	6.2245	0.7843	0.6340
	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)
	0.0620	-0.7379	-0.4535	0.1948	-0.8514	-0.6063	0.5082	-0.8720	-0.7502
	c(yf,zf)	c(yf,pf)	c(yf,gf)	c(yf,zf)	c(yf,pf)	c(yf,gf)	c(yf,zf)	c(yf,pf)	c(yf,gf)
	0.7221	-0.5242	0.3974	0.5194	-0.7676	0.7627	0.4426	-0.7722	0.6628
		c(ef,pf)			c(ef,pf)			c(ef,pf)	
		-0.9165			-0.9651			-0.6641	

Key: (1) The variables yf, cf, if, hf, ksf, kf, $h_k f$, ef, APhf, APksf and APkf are the HP-filtered logarithmic levels of y_t, c_t, i_t, h_t, $(k_t h_{kt})$, k_t, h_{kt}, e_t and the average products of h_t, $(k_t h_{kt})$ and k_t, respectively.

(2) % SD denotes the percentage standard deviation.
Autol denotes autocorrelation at lag 1.
Corry denotes correlation with output.
$c(\cdot,\cdot)$ denotes the correlation coefficient between the indicated variables.

Table 5: Basic Model, U.S. data (first-difference filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Corrs	Corrz	% SD	Corrs	Corrz	% SD	Corrs	Corrz
srΔ	3.5278	1.0000	0.7799	3.3612	1.0000	0.7278	3.4790	1.0000	0.6389
zΔ	2.1140	0.7799	1.0000	2.1740	0.7278	1.0000	2.1782	0.6389	1.0000
pΔ	9.4202	-0.5503	0.0074	9.5211	-0.6307	-0.0582	9.4770	-0.7003	-0.0346
gΔ	2.9641	0.1874	0.1649	3.1777	0.3906	0.1444	3.0870	0.2456	0.0688

Key: (1) srΔ, zΔ, pΔ, gΔ denote the first-differences of the logarithmic levels of the Solow residual, z_t, p_t and g_t, respectively.

(2) % SD denotes percentage standard deviation.
Corrs denotes correlation with the Solow residual.
Corrz denotes correlation with z_t.

Table 6: Model with $\sigma_p = 0$, U.S. data (H-P filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Autol	Corry	% SD	Autol	Corry	% SD	Autol	Corry
yf	2.5220	0.6586	1.0000	2.4806	0.7094	1.0000	4.1180	0.7189	1.0000
cf	2.3029	0.5300	0.5704	2.3981	0.4395	0.1071	2.0522	0.8161	0.8786
if	7.3262	0.3528	0.8467	7.7698	0.1884	0.8117	6.7180	0.5031	0.7351
hf	1.3824	0.3023	0.5315	2.0243	0.6120	0.6852	2.2894	0.6152	0.8585
ksf	1.7147	0.6956	0.9821	1.6054	0.7182	0.9597	7.0532	0.8305	0.8858
kf	0.9877	0.7989	-0.0814	1.2441	0.7695	-0.2767	4.8166	0.9305	0.0356
$h_k f$	1.8618	0.6206	0.9307	2.0321	0.6988	0.9275	7.3325	0.8059	0.8286
ef	2.7621	0.6530	0.9994	2.7135	0.7020	0.9984	4.4025	0.6152	0.8227
APhf	2.1214	0.6471	0.8167	1.8356	0.6828	0.5957	2.4518	0.7738	0.8779
APksf	0.8984	0.6207	0.9307	1.0427	0.6892	0.9015	3.9054	0.8133	-0.5452
APkf	2.7602	0.6206	0.9307	3.0674	0.6982	0.9209	6.2245	0.7843	0.6340
	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)
	-0.0182	0.8465	-0.4219	-0.1768	0.7434	-0.6294	0.5082	-0.8720	-0.7502
	c(yf,zf)	c(yf,gf)		c(yf,zf)	c(yf,gf)		c(yf,zf)	c(yf,gf)	
	0.8686	0.4810		0.7534	0.6340		0.4426	0.6628	

Key: (1) The variables yf, cf, if, hf, ksf, kf, $h_k f$, ef, APhf, APksf and APkf are the HP-filtered logarithmic levels of y_t, c_t, i_t, h_t, $(k_t h_{kt})$, k_t, h_{kt}, e_t and the average products of h_t, $(k_t h_{kt})$ and k_t, respectively.

(2) % SD denotes the percentage standard deviation.
Autol denotes autocorrelation at lag 1.
Corry denotes correlation with output.
c(·,·) denotes the correlation coefficient between the indicated variables.

Table 7: Model with $\sigma_p = 0$, U.S. data (first-difference filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Corrs	Corrz	% SD	Corrs	Corrz	% SD	Corrs	Corrz
srΔ	2.8609	1.0000	0.9712	2.7964	1.0000	0.9558	3.4790	1.0000	0.6389
zΔ	2.1140	0.9712	1.0000	2.1740	0.9558	1.0000	2.1782	0.6389	1.0000
gΔ	2.9641	0.2312	0.1649	3.1777	0.2647	0.1444	3.0870	0.2456	0.0688

Key: (1) srΔ, zΔ, pΔ, gΔ denote the first-differences of the logarithmic levels of the Solow residual, z_t, p_t and g_t, respectively.

(2) % SD denotes percentage standard deviation.
Corrs denotes correlation with the Solow residual.
Corrz denotes correlation with z_t.

Table 8: Model with $\sigma_g = 0$, U.S. data (H-P filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Autol	Corry	% SD	Autol	Corry	% SD	Autol	Corry
yf	3.2637	0.6928	1.0000	3.2396	0.7815	1.0000	4.1180	0.7189	1.0000
cf	2.1734	0.5653	0.8877	2.6160	0.6094	0.8208	2.0522	0.8161	0.8786
if	10.7437	0.6325	0.8883	13.4323	0.7744	0.8860	6.7180	0.5031	0.7351
hf	1.0166	0.4519	0.8245	0.9587	0.4687	0.7759	2.2894	0.6152	0.8585
ksf	4.5719	0.6736	0.7991	5.6088	0.8261	0.8696	7.0532	0.8305	0.8858
kf	1.2183	0.8521	-0.1065	1.8519	0.8958	-0.0660	4.8166	0.9305	0.0356
$h_k f$	4.6952	0.6547	0.7980	6.1035	0.8300	0.8192	7.3325	0.8059	0.8286
ef	7.4206	0.6633	0.8053	9.2873	0.8261	0.8590	4.4025	0.6152	0.8227
APhf	2.4829	0.6560	0.9713	2.5680	0.7620	0.9719	2.4518	0.7738	0.8779
APksf	2.6764	0.6258	-0.1658	3.2172	0.7865	-0.5092	3.9054	0.8[illegible]53	-0.5452
APkf	3.5727	0.6780	0.9382	3.8362	0.7794	0.8763	6.2245	0.7843	0.6340
	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)	c(hf,APhf)	c(ksf,APksf)	c(kf,APkf)
	0.6694	-0.6916	-0.4304	0.6056	-0.8677	-0.5385	0.5082	-0.8720	-0.7502
	c(yf,zf)	c(yf,pf)	c(yf,gf)	c(yf,zf)	c(yf,pf)	c(yf,gf)	c(yf,zf)	c(yf,pf)	c(yf,gf)
	0.8361	-0.4948	0.8361	0.7312	-0.6880	0.7312	0.4426	-0.7722	0.6628
		c(ef,pf)			c(ef,pf)			c(ef,pf)	
		-0.9018			-0.9623			-0.6641	

Key: (1) The variables yf, cf, if, hf, ksf, kf, $h_k f$, ef, APhf, APksf and APkf are the HP-filtered logarithmic levels of y_t, c_t, i_t, h_t, $(k_t h_{kt})$, k_t, h_{kt}, e_t and the average products of h_t, $(k_t h_{kt})$ and k_t, respectively.

(2) % SD denotes the percentage standard deviation.
Autol denotes autocorrelation at lag 1.
Corry denotes correlation with output.
c(·,·) denotes the correlation coefficient between the indicated variables.

Table 9: Model with $\sigma_g = 0$, U.S. data (first-difference filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Corrs	Corrz	% SD	Corrs	Corrz	% SD	Corrs	Corrz
srΔ	3.6277	1.0000	0.8041	3.4325	1.0000	0.7583	3.4790	1.0000	0.6389
zΔ	2.1140	0.8041	1.0000	2.1740	0.7583	1.0000	2.1782	0.6389	1.0000
pΔ	9.4202	-0.5338	0.0074	9.5211	-0.6304	-0.0582	9.4770	-0.7003	-0.0346
gΔ	2.1140	0.8041	1.0000	2.1740	0.7583	1.0000	3.0870	0.2456	0.0688

Key: (1) srΔ, zΔ, pΔ, gΔ denote the first-differences of the logarithmic levels of the Solow residual, z_t, p_t and g_t, respectively.

(2) % SD denotes percentage standard deviation.
Corrs denotes correlation with the Solow residual.
Corrz denotes correlation with z_t.

Appendix 1: The Solution Technique

[1] A stationarity-inducing transformation of model variables must be undertaken since exogenous growth occurs, stemming from the growth of z_t. Let the new, stationary variables be defined as:

$$\bar{c}_t = c_t/z_t \quad , \quad \bar{y}_t = y_t/z_t \quad , \quad \bar{k}_t = k_t/z_{t-1}$$

$$\bar{i}_t = i_t/z_t \quad , \quad \bar{e}_t = e_t/z_t$$

The variables h_t, h_{kt}, $\bar{g}_t$ and p_t and innovation vector u_t (defined previously) are also stationary.

The discounted programming problem for the stationary economy is:

$$\text{(A1)} \quad v^s(\bar{k}_t; \bar{g}_t, p_t, u_t) =$$

$$\max_{(\bar{c}_t, h_t, \bar{k}_{t+1}, h_{kt})} \{\log\bar{c}_t + \log\dot{z}_t + \gamma\log(1-h_t) + \beta\int v^s(\bar{k}_{t+1}; \bar{g}_{t+1}, p_{t+1}, u_{t+1})d\Phi(u_{t+1}|u_t)\}$$

subject to:

$$\text{(A2)} \quad \bar{c}_t = h_t^{\theta}(\bar{k}_t h_{kt})^{(1-\theta)}\, \dot{\bar{z}}^{(\theta-1)}\exp[(\theta-1)u_{zt}] - \bar{k}_{t+1} + [1-\delta(h_{kt})]\bar{k}_t\dot{\bar{z}}^{-1}\exp[-u_{zt}]$$
$$- \bar{g}_t - p_t a(h_{kt})\bar{k}_t\dot{\bar{z}}^{-1}\exp[-u_{zt}]$$

and equations (7) - (8). It is equivalent to the problem for the nonstationary economy, specified earlier in equations (9), (10), (6) - (8), having noted equation (6) and the functional forms for u and F. The term, $\log z_t$, can be discarded without loss of generality in solving this problem since it is exogenous.

[2] Determine the deterministic steady state of the stationary economy.

[3] Equation (A2) is used to substitute for $\bar{c}_t$ in the one-period return function in (A1). The logarithmic transformation suggested by Christiano (1988) is undertaken. The MA(1) structures of the innovation processes are noted i.e.,

$$u_{zt} = \epsilon_{zt} + \eta_z \epsilon_{zt-1}$$
$$u_{gt} = \epsilon_{gt} + \eta_g \epsilon_{gt-1}$$
$$u_{pt} = \epsilon_{pt} + \eta_p \epsilon_{pt-1}$$

where ϵ_{it} is a zero mean, white noise innovation process and η_i is a parameter $(i = z,g,p)$. These specifications are discussed in Section 4. Finally, various algebraic rearrangements are undertaken so that the one-period return function can be represented (in general form) by $r(x_t)$, where $x_t \in R^{10}$ is:

$$x_t' = (\epsilon_{zt}, \epsilon_{zt-1}, \log(\bar{g}_t/\bar{g}), \epsilon_{gt}, \log(p_t/p), \epsilon_{pt}, \log\bar{k}_t, \log\bar{k}_{t+1}, \log h_t, \log h_{kt})$$

For the convenience of this exposition, ϵ_{gt} and ϵ_{pt} are entered as arguments even though they do so with zero coefficients. The approximate quadratic return function is:

$$R(x_t) = r(\bar{x}) + B'(x_t-\bar{x}) + (x_t-\bar{x})'Q(x_t-\bar{x}) \tag{A3}$$

where $\bar{x}$, $B \epsilon R^{10}$, Q is a (10 x 10) symmetric matrix, $\bar{x}$ is the steady state counterpart of x_t and the elements of B and Q are parameters, determined by requiring that the approximation in (A3) be good not only at $\bar{x}$ but also for other values of x_t close to $\bar{x}$. Kydland and Prescott (1982) specify the formulas for the calculation of the latter parameters.

[4] The outline of this step closely follows that of Hansen and Sargent (1988). The optimization problem for the approximate, stationary economy can be shown to reduce to that of maximizing:

(A4) $$E_0 \sum_{t=0}^{\infty} \beta^t \, [s_t' W_1 s_t + l_t' W_2 l_t + 2 \, l_t' W_3 s_t]$$

subject to:

(A5) $$s_{t+1} = A_1 s_t + A_2 l_t + A_3 \epsilon_{t+1}$$

$s_t \epsilon R^8$ is the state vector at time t: $s_t' = (1, \epsilon_{zt}, \epsilon_{zt-1}, \log(\bar{g}_t/\bar{g}), \epsilon_{gt}, \log(p_t/p), \epsilon_{pt}, \log\bar{k}_t)$.

$l_t \epsilon R^3$ is the control vector at time t: $l_t' = (\log\bar{k}_{t+1}, \log h_t, \log h_{kt})$.

$\epsilon_{t+1} \epsilon R^3$ is the innovation vector at time (t+1): $\epsilon_{t+1}' = (\epsilon_{zt+1}, \epsilon_{gt+1}, \epsilon_{pt+1})$.

W_1, W_2 and W_3 are matrices (of appropriate size) whose elements are scalar functions of the parameters in (A3).

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_g & \eta_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_p & \eta_p & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

This is a discounted optimal linear regulator problem whose solution gives the time-invariant, linear decision rules:

$$\text{(A6)} \quad l_t = -f\, s_t, \quad f = (A_2'PA_2 + W_2)^{-1}(A_1'PA_2 + W_3)$$

where P is the unique, limit matrix emerging from iterations on the matrix Ricatti equation:

$$\text{(A7)} \quad P_{t+1} = W_1 + A_1'P_tA_1 - (A_1'P_tA_2 + W_3')(W_2 + A_2'P_tA_2)^{-1}(A_2'P_tA_1 + W_3)$$

starting from P_0 equal to the zero matrix. In practice, convergence of the decision rules, (A6), obtains more quickly than that of P--whose solution is of no intrinsic interest for present purposes. Accordingly, the tolerance level is set only for the change in f (here, specifically at 1.0E-05).

Substituting (A6) into (A5) gives the 'optimal closed loop system':

(A8) $s_{t+1} = (A_1 - A_2 f)s_t + A_3 \epsilon_{t+1}$

Equation (A6) can be used together with (A2), the stationary versions of (2) - (4) to solve for $\log\bar{c}_t$, $\log\bar{y}_t$, $\log\bar{i}_t$ and $\log\bar{e}_t$ as time-invariant, nonlinear functions of s_t. These functions, (A6) and (A8) thus specify the equilibrium stochastic process of the approximate stationary economy. Finally, the equilibrium stochastic process of the approximate nonstationary economy can be obtained by taking account of equation (6) and adding $\log z_t$ to $\log\bar{k}_{t+1}$ ($\log\bar{v}_t$) to give the nonstationary variables $\log k_{t+1}$ ($\log v_t$), where $\bar{v}_t = \bar{c}_t, \bar{y}_t, \bar{i}_t, \bar{e}_t, \bar{g}_t$.

Appendix 2: The Data

The data are annual real per-capita data for the U.S. over the period 1960-1988. The individual series used are described here.

(i) Energy Usage, Price and Value-Added Data

The sources for this data are:

(1) State Energy Data Report: Consumption Estimates 1960-1988, Energy Information Administration. Here it will be denoted by SEDR.

(2) Annual Energy Review 1989, Energy Information Administration. Here it will be denoted by AER.

The data derivations use the conversion factors published in the Appendices of SEDR and AER.

Due to data limitations, the energy usage series derived here is an understatement of the true energy usage. The key omissions are indicated as follows.

(a) Commercial sector (defined here as wholesale, retail and other service businesses) usage of petroleum, coal and natural gas is omitted since it could not be separated from government usage.

(b) Transportation sector usage of motor gasoline is omitted since it is not possible to isolate that component which is consumed by private (household sector) motor vehicles.

Energy Usage: the sum of electricity (elec), coal (coal), petroleum (petr) and natural gas (natg) usage by the private, non-energy production sector of the economy; measured in trillion BTUs. Specific details follow.

elec: CSE + ISE + TSE

CSE: commercial sector electricity usage (Table 92 AER).

ISE: industrial sector electricity (including hydroelectricity) usage (Table 12 SEDR).

TSE: transportation sector electricity usage (Table 13 SEDR).

coal:.ISC + TSC

ISC: industrial sector coal usage, including net imports of coal coke (Table 12 SEDR), minus the component consumed by coke plants (Table 81 AER).

TSC: transportation sector coal usage (Table 13 SEDR).

petr: ISP + TSP

ISP: industrial sector petroleum usage less that component listed in the asphalt and road oil, lubricants and 'other' categories (Table 12 SEDR).

TSP: transportation sector petroleum usage less that component listed in the lubricants and motor gasoline categories (Table 13 SEDR).

natg: ISG

ISG: industrial sector natural gas usage (Table 12 SEDR) minus that component used as lease and plant fuel by the natural gas industry (Table 75 AER).

Value of energy usage: the nominal value of the above energy usage, billions of current dollars. It is the sum:

(prelec x elec) + (prcoal x coal) + (prnatg x natg) + (prpetr x petr)

prelec: dollar price of electricity per trillion BTUs (Table 95 AER).

prcoal: dollar price of coal per trillion BTUs (Table 86 AER).

prnatg: dollar price of natural gas per trillion BTUs (Table 77 AER).

prpetr: dollar price of petroleum per trillion BTUs, derived as follows: $prpetr = (x_1 + x_2)/x_3$ where:

x_1: dollar value of total production plus net imports of oil and petroleum products (Tables 32, 33, 34 AER).

x_2: dollar value of natural gas plant liquids production (Table 50 AER), evaluated at domestic crude oil prices (Table 29 AER).

x_3: total economy-wide consumption of petroleum, measured in trillion BTUs (Table 9 SEDR).

Energy price deflator: 1982=100, derived as follows:

It is the value of energy usage (defined above) in current dollars divided by its constant 1982 dollar counterpart.

Energy sector value added: billions of dollars, derived as follows:

It is the sum of the total value of fossil fuel (oil, coal and natural gas) production (Table 32 AER) and value added by the electricity producing sector (Table 90 AER). The latter is defined as sales minus the values of oil, coal and natural gas inputs, where use is made of the price series established above.

(ii) All Other Data

The sources for the remaining data are Citibase and:

John C. Musgrave, "Fixed Reproducible Tangible Wealth in the United States", Survey of Current Business, U.S. Department of Commerce, BEA; Jan. 1986

(Vol. 66, No. 1, pp. 51-75) and Aug. 1989 (Vol. 69, No. 8, pp. 89-92). Here it will be denoted by SCB.

Unless otherwise stated, the source is Citibase.

Population: civilian non-institutional population aged 16 and over; thousands of persons.

Aggregate price level: gross domestic product price deflator; 1982 = 100

Output: gross domestic product minus gross housing product minus gross government product minus value-added by the energy-producing sector (source indicated in section (i) above); billions of 1982 dollars.

Consumption: personal consumer expenditure on: nondurable goods plus services minus housing services minus the sum of gasoline and oil, fuel oil and coal, electricity and gas; billions of 1982 dollars.

Investment: gross private domestic fixed investment in nonresidential capital (i.e., structures plus producers durable equipment); billions of 1982 dollars.

Government spending: government purchases of goods and services; billions of 1982 dollars.

Labor hours: the product of employment and hours per worker per year. Employment is total employment (civilian plus resident armed forces) minus employment by government plus armed forces overseas minus the sum of employment in coal mining, oil and gas extraction, petroleum and coal product manufacturing and electricity, gas plus sanitary services; thousands of persons. Hours per worker per year is an average across all workers in all industries.

Capital stock: net stock of fixed nonresidential private capital (equipment plus structures); billions of 1982 dollars. [Source: SCB].

Proprietor's income: proprietor's income with inventory valuation adjustment and capital consumption adjustment, billions of 1982 dollars.

Employee compensation: compensation of employees in domestic industries minus that of employees in coal mining, oil and gas extraction, petroleum and coal product manufacturing, electricity, gas and sanitary services and government; billions of 1982 dollars.

Real energy price: ratio of the energy price deflator (described in section (i) above) to the aggregate price level (described in this section, above), 1982 = 1.

Appendix 3: Additional Tables

Table 4A: Basic Model, U.S. data (first-difference filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Autol	Corry	% SD	Autol	Corry	% SD	Autol	Corry
yΔ	2.4967	0.3500	1.0000	2.7499	0.5913	1.0000	3.2337	0.3489	1.0000
cΔ	2.5518	0.0383	0.4395	2.9622	0.0083	0.3111	1.3655	0.4516	0.7956
iΔ	10.8475	0.0359	0.8328	13.2691	-0.0668	0.7903	6.9006	0.1678	0.7414
hΔ	1.6746	-0.2400	0.5162	1.9531	-0.1157	0.5451	2.1373	0.2664	0.8226
ksΔ	3.7034	0.3256	0.7858	3.8976	0.5585	0.8201	4.6381	0.4771	0.8044
kΔ	0.8839	0.6366	-0.2587	1.2810	0.7908	-0.0088	3.8323	0.9172	-0.1726
h_kΔ	3.8889	0.2934	0.8012	4.0772	0.5416	0.7867	5.3907	0.5715	0.8148
eΔ	6.0756	0.3081	0.7969	6.3473	0.5460	0.8169	4.1193	0.1890	0.7951
APhΔ	2.1705	0.2752	0.7434	2.3499	0.4248	0.7172	1.9115	0.3925	0.7719
APksΔ	2.2998	0.2134	-0.1865	2.2744	0.1723	-0.1963	2.7999	0.3528	-0.1776
APkΔ	2.8399	0.2930	0.9503	3.0438	0.4929	0.9071	5.4242	0.6280	0.7181
	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)
	-0.1742	-0.7421	-0.5365	-0.1933	-0.7221	-0.4288	0.2735	-0.7275	-0.8094
	c(yΔ,zΔ)	c(yΔ,pΔ)	c(yΔ,gΔ)	c(yΔ,zΔ)	c(yΔ,pΔ)	c(yΔ,gΔ)	c(yΔ,zΔ)	c(yΔ,pΔ)	c(yΔ,gΔ)
	0.6428	-0.4991	0.4013	0.6597	-0.5194	0.5887	0.5490	-0.6186	0.3291
		c(eΔ,pΔ)			c(eΔ,pΔ)			c(eΔ,pΔ)	
		-0.9172			-0.9164			-0.3739	

Key: (1) The variables yΔ, cΔ, iΔ, hΔ, ksΔ, kΔ, h_kΔ, eΔ, APhΔ, APksΔ, and APkΔ are the first-differenced logarithmic levels of y_t, c_t, i_t, h_t, $(k_t h_{kt})$, k_t, h_{kt}, e_t and the average products of h_t, $(k_t h_{kt})$ and k_t, respectively.

(2) % SD denotes the percentage standard deviation.
Autol denotes autocorrelation at lag 1.
Corry denotes correlation with output.
c(·,·)denotes the correlation coefficient between the indicated variables.

Table 6A: Model with $\sigma_p = 0$, U.S. data (first-difference filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Autol	Corry	% SD	Autol	Corry	% SD	Autol	Corry
yΔ	2.1096	0.3160	1.0000	2.2875	0.4421	1.0000	3.2337	0.3489	1.0000
cΔ	2.2317	0.0078	0.3156	2.6576	-0.0560	0.1395	1.3655	0.4516	0.7956
iΔ	8.2692	-0.2009	0.7688	9.9809	-0.2369	0.7712	6.9006	0.1678	0.7414
hΔ	1.6136	-0.2545	0.5520	1.8572	-0.1493	0.5997	2.1373	0.2664	0.8226
ksΔ	1.3826	0.3895	0.9865	1.5356	0.5116	0.9691	4.6381	0.4771	0.8044
kΔ	0.7174	0.5380	-0.2793	0.9392	0.6048	-0.1227	3.8323	0.9172	-0.1726
h_kΔ	1.6170	0.2367	0.9546	1.7068	0.2993	0.9395	5.3907	0.5715	0.8148
eΔ	2.3231	0.3050	0.9996	2.4898	0.4073	0.9970	4.1193	0.1890	0.7951
APhΔ	1.8120	0.2605	0.6601	1.8938	0.3810	0.6198	1.9115	0.3925	0.7719
APksΔ	0.7802	0.2370	0.9546	0.8845	0.3413	0.9037	2.7999	0.3528	-0.1776
APkΔ	2.3971	0.2368	0.9546	2.5772	0.3154	0.9323	5.4242	0.6280	0.7181
	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)
	-0.2451	0.8935	-0.5420	-0.2563	0.7702	-0.4733	0.2735	-0.7275	-0.8094
	c(yΔ,zΔ)	c(yΔ,gΔ)		c(yΔ,zΔ)	c(yΔ,gΔ)		c(yΔ,zΔ)	c(yΔ,gΔ)	
	0.7643	0.4784		0.7467	0.5417		0.5490	0.3291	

Key: (1) The variables yΔ, cΔ, iΔ, hΔ, ksΔ, kΔ, h_kΔ, eΔ, APhΔ, APksΔ, and APkΔ are the first-differenced logarithmic levels of y_t, c_t, i_t, h_t, $(k_t h_{kt})$, k_t, h_{kt}, e_t and the average products of h_t, $(k_t h_{kt})$ and k_t, respectively.

(2) % SD denotes the percentage standard deviation.
Autol denotes autocorrelation at lag 1.
Corry denotes correlation with output.
c(·,·)denotes the correlation coefficient between the indicated variables.

Table 8A: Model with $\sigma_g = 0$, U.S. data (first-difference filtered results)

Variable	I Model, normal innovations			II Model, actual innovations			III U.S. data, 1960-1988		
	% SD	Autol	Corry	% SD	Autol	Corry	% SD	Autol	Corry
yΔ	2.6025	0.4368	1.0000	2.7044	0.5948	1.0000	3.2337	0.3489	1.0000
cΔ	2.0308	0.0715	0.7602	2.5198	0.1681	0.6985	1.3655	0.4516	0.7956
iΔ	9.1721	0.2683	0.8384	10.0342	0.3998	0.7727	6.9006	0.1678	0.7414
hΔ	1.0577	-0.0845	0.6399	1.0199	-0.1770	0.5760	2.1373	0.2664	0.8226
ksΔ	3.7379	0.3470	0.7861	3.8928	0.5159	0.8291	4.6381	0.4771	0.8044
kΔ	0.8131	0.7549	-0.2363	1.3012	0.8887	0.1833	3.8323	0.9172	-0.1726
h_kΔ	3.8982	0.3279	0.7957	3.9894	0.5000	0.7492	5.3907	0.5715	0.8148
eΔ	6.1217	0.3347	0.7941	6.2836	0.5001	0.8085	4.1193	0.1890	0.7951
APhΔ	2.0925	0.2914	0.9192	2.2752	0.4739	0.9304	1.9115	0.3925	0.7719
APksΔ	2.3099	0.2222	-0.1528	2.2385	0.2409	-0.2337	2.7999	0.3528	-0.1776
APkΔ	2.8828	0.4083	0.9589	2.7778	0.5148	0.8877	5.4242	0.6280	0.7181
	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)	c(hΔ,APhΔ)	c(ksΔ,APksΔ)	c(kΔ,APkΔ)
	0.2909	-0.7175	-0.4918	0.2362	-0.7374	-0.2899	0.2735	-0.7275	-0.8094
	c(yΔ,zΔ)	c(yΔ,pΔ)	c(yΔ,gΔ)	c(yΔ,zΔ)	c(yΔ,pΔ)	c(yΔ,gΔ)	c(yΔ,zΔ)	c(yΔ,pΔ)	c(yΔ,gΔ)
	0.8090	-0.4774	0.8090	0.8384	-0.5090	0.8384	0.5490	-0.6186	0.3291
		c(eΔ,pΔ)			c(eΔ,pΔ)			c(eΔ,pΔ)	
		-0.9085			-0.9173			-0.3739	

Key: (1) The variables yΔ, cΔ, iΔ, hΔ, ksΔ, kΔ, h_kΔ, eΔ, APhΔ, APksΔ, and APkΔ are the first-differenced logarithmic levels of y_t, c_t, i_t, h_t, $(k_t h_{kt})$, k_t, h_{kt}, e_t and the average products of h_t, $(k_t h_{kt})$ and k_t, respectively.

(2) % SD denotes the percentage standard deviation.
Autol denotes autocorrelation at lag 1.
Corry denotes correlation with output.
c(·,·)denotes the correlation coefficient between the indicated variables.

Endnotes

1. Solow residual growth is output growth less the share weighted growth rates of total labor hours and the capital stock. The shares are factor shares in a Cobb-Douglas production function. This approach to measuring technology growth is due to Solow (1957).

2. Hall (1989) also shows that for the U.S. (1953-84) there is some evidence of significantly positive correlations between sectoral Solow residual growth and real military purchases growth. This study documents, for the U.S. (1960-88), a correlation of 0.25 between an economy-wide measure of Solow residual growth and the growth of aggregate real government purchases.

3. Hornstein (1990) extends the neoclassical growth model to incorporate monopolistic competition and increasing returns to scale and shows that the variance of Solow residual growth will overstate the variance of true technology growth. Burnside, Eichenbaum and Rebelo (1990) extend the Hansen (1985) indivisible labor model to admit labor hoarding behavior. They show that the model can account for the observed correlation between the growth rates of the Solow residual and government spending.

4. Other key differences between the present model and that of Kim and Loungani (1991) (abbreviated as KL) include:

   (i) here it is assumed that labor is divisible while KL assume indivisible labor;

   (ii) here technology is modelled as an integrated process while KL maintain a stationary process;

   (iii) here shocks to government spending are also included.

   Due to the difference in the production functions across the two studies, the empirical measurement of technology growth differs very sharply and importantly, as discussed later in the text. Finally, the empirical measurement of energy use and nominal energy price differs significantly. KL use total U.S. consumption of the fossil fuels (petroleum, coal and natural gas) and dollar (U.S. production-weighted) price of this composite. Here an attempt is made to measure that component of U.S. consumption of the fossil fuels plus electricity that is consumed by the private non-energy production sector of the economy as well as the associated price deflator. So, e.g., energy consumption by households (and their motor vehicles!), government and the energy-producing sector is excluded, as is non-fuel usage of the fossil fuels. The data appendix presents details.

5. This idea is due to Jeremy Greenwood and Zvi Hercowitz. Jorgenson and Griliches (1967) espoused a similar idea in arguing that electricity and utilized capital are very complementary in production.

6. In addition, transmission occurs through a negative wealth effect.

7. Kydland (1984) shows that within the class of C.E.S. utility functions, only those maintaining a unitary elasticity of substitution between consumption and leisure are consistent with balanced growth and stationary hours worked. King, Plosser and Rebelo (1988) show that a unitary elasticity is also required within a more general class of utility functions.

8. Kydland and Prescott (1991) cite empirical studies supporting the assumption of a unitary elasticity of substitution between capital and labor.

9. This is shown by Swan (1963), Phelps (1966) and King, Plosser and Rebelo (1988).

10. Greenwood, Hercowitz and Huffman (1988) cite other studies employing this specification.

11. Aiyagari, Christiano and Eichenbaum (1990) analyze the effects of government spending in a more general environment than is considered here. The present model implicitly assumes lump-sum taxation. Greenwood and Huffman (1991) analyze the effects of distortionary taxation.

12. An alternative interpretation is that $p_t e_t$ is the output absorbed by energy extraction at the exogenous extraction cost, p_t.

13. An implicit assumption here is that utility is additively separable across the consumption of market and household goods.

14. It would be interesting to extend the model in order to explain household energy consumption. The approach to household production activity advanced by Greenwood and Hercowitz (1991) could be followed. Specifically, a household production structure that is symmetrical to the market production structure in the text could be specified for this purpose.

15. As mentioned in the introduction and in endnote 2, Hall (1989) uses different measures of Solow residual growth and rates of change of energy prices and government spending to those used here.

16. The correlation between the growth rate of hours per worker and $\Delta \log g_t$ is 0.2615, which is close to the correlation between $\Delta \log h_{kt}$ and $\Delta \log g_t$ reported in the text. Also, given the tiny correlation between ($\Delta \log k_t$ - $\Delta \log k_t^*$) and $\Delta \log p_t$ mentioned earlier, it is unlikely that an extension of the Kydland-Prescott view of utilization to admit endogenous depreciation

would result in a generated $\Delta \log z_t$ that is satisfactory.

17. Specifically, $\theta = 0.64$ is an average the two averages in the U.S. data: (i) the share of employee compensation in output net of proprietor's income and (ii) the share of employee compensation in output.

18. The source is the John C. Musgrave article referenced in Appendix 2.

19. In computing the measured average ratio, energy usage is measured by the dollar value of energy usage multiplied by the reciprocal of the real price of energy rather than by the BTU measure of energy usage. This ensures comparable units in the numerator and denominator of the ratio.

20. In view of the significant correlations between innovations to the $\Delta \log z_t$ and $\log(\bar{g}_t)$ processes, documented below, it may be more efficient to estimate these processes jointly. This is difficult to assess in view of the small sample size.

21. The calibrated values of $\bar{z}$ and p were imposed on (6') and (8') during the estimation. The sample mean of $\bar{g}_t$ was imposed on (7') during its estimation. This sample mean is different from the calibrated $\bar{g}$ that is used in the model specification--since what is relevant for the model is the value of $\bar{g}$ implied by the average share, i.e., the mean of $(\bar{g}_t/\bar{y}_t)$.

22. Significance is judged at the 5% level throughout this discussion.

23. The test of significance is a t-test on the coefficient of a least-squares regression of $\hat{\epsilon}_{it}$ on $\hat{\epsilon}_{jt}$ ($i = z, g, p$).

24. There is no (necessary) inconsistency between the findings of a significant correlation between innovations to the $\Delta \log z_t$ and $\log \bar{g}_t$ processes and an insignificant correlation between the $\Delta \log z_t$ and $\Delta \log g_t$ processes (documented in Section 3). This point may be highlighted by reference to the Granger Representation Theorem.

Suppose, as maintained in the model, that $\log g_t$ and $\log z_t$ are I(1) processes whose innovations are independent of one another. Further suppose, as maintained in the model, that $\log \bar{g}_t$ is an I(0) process, i.e., $\log g_t$ and $\log z_t$ are cointegrated with cointegrating vector, [1 -1].

Consider an example of the error-correction-form for the vector stochastic process, $[\Delta \log g_t \quad \Delta \log z_t]'$:

$$\text{(i)} \quad \begin{bmatrix} \Delta \log g_t \\ \Delta \log z_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} - \begin{bmatrix} \gamma_1 \\ 0 \end{bmatrix} [1 \quad -1] \begin{bmatrix} \log g_{t-1} \\ \log z_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

where u_{1t} and u_{2t} are stationary, mean zero, independent innovations; α_1, α_2 and γ_1 are positive scalars.

From (i) it follows:

$$\text{(ii)} \quad \log \bar{g}_t = (1-\gamma_1) \log \bar{g}_{t-1} + (\alpha_1-\alpha_2) + (u_{1t}-u_{2t})$$

Under balanced growth, $E[\Delta \log z_t] = E[\Delta \log g_t]$. Using this hypothesis and taking expectations in (i) gives:

$$\text{(iii)} \quad \alpha_1-\alpha_2 = \gamma_1 \log \bar{g}$$

Substituting (iii) into (ii) implies:

$$\text{(iv)} \quad \log \bar{g}_t = (1-\gamma_1) \log \bar{g}_{t-1} + \gamma_1 \log \bar{g} + (u_{1t}-u_{2t})$$

The second equation in (i) and equation (iv) have exactly the same structures as equations (6') and (7'). From (i) and (iv) it is clear that their innovations will exhibit negative covariation even though u_{1t} and u_{2t} are independent processes. Since equations (6') and (7') maintain the same assumptions as (i) and (iv), it is also clear that their innovations will exhibit negative covariation even when innovations to $\Delta \log g_t$ and $\Delta \log z_t$ are independent. I thank Bob Rasche for discussing this issue with me.

25. The definitions of relevance are those for average factor productivities and the Solow residual. In order to simulate a time path for the Solow residual it is necessary to simulate and use a time path for k^*_t. This is the model's counterpart to the capital stock series (of same notation)

measured directly in the data. Therefore, k_t^* is generated for the model from the assumed law of motion:

$$k_{t+1}^* = (1 - 0.078)k_t^* + i_t$$

where i_t is the simulated investment series, depreciation is assumed constant at its steady state value and the initial value of k_t^* is set equal to the steady state value of k_t.

26. Existing tests of normality are asymptotic tests, whose properties are unknown for small samples. As Hamilton (1983) points out, nonnormality of the innovation distribution may invalidate tests of parameter significance pertaining to the stochastic process in question.

27. The smoothing parameter for the H-P filter is set at 400, the value commonly used for annual data.

28. In order to keep this isolation pure, the same sets of innovations are used across experiments.

29. The coefficients of the three linear Markovian decisions rules for the stationary, basic, economy are:

$$\begin{bmatrix} \log(\bar{k}_{t+1}/\bar{k}) \\ \log(h_t/h) \\ \log(h_{kt}/h_k) \end{bmatrix} =$$

$$\begin{bmatrix} -1.0026 & -0.4274 & -0.0044 & 0.0794 & -0.0312 & 0.0207 & 0.8579 \\ 0.0117 & 0.1586 & 0.2964 & 0.1684 & -0.0304 & 0.0439 & -0.3184 \\ 0.5767 & 0.3744 & 0.1691 & 0.0960 & -0.3809 & 0.0251 & -0.7517 \end{bmatrix} \begin{bmatrix} \epsilon_{zt} \\ \epsilon_{zt-1} \\ \log(\bar{g}_t/\bar{g}) \\ \epsilon_{gt} \\ \log(p_t/\bar{p}) \\ \epsilon_{pt} \\ \log(\bar{k}_t/\bar{k}) \end{bmatrix}$$

The eigenvalues of the optimal closed loop matrix, (A_1-A_2f), are: [1 0 0 0.9278 0 0.9143 0 0.8579]. These values are consistent with a stationary system. The stationary value of $\log\bar{k}_t$ implied by the nonstochastic optimal closed loop system is the same as that implied by the original nonlinear deterministic model.

Notice the following. The innovations ϵ_{zt} and ϵ_{zt-1} enter the stationary economy as negative, transitory technology shocks. The innovations ϵ_{gt} and ϵ_{pt} only influence the economy indirectly--by influencing expectations of future government spending and energy price shocks. The adjustment coefficient, 0.8579, is quite smaller than that reported in other studies which assume a fixed utilization rate (e.g., 0.95 in Christiano and Eichenbaum (1990)). This suggests that endogenous utilization results in faster adjustments to disturbances, since it provides an additional margin along which agents can respond. The signs of the above coefficients can be rationalized by considering the interaction between wealth, intertemporal and intratemporal substitution effects.

30. Statements concerning significance used throughout this section do not have any formal statistical connotation.

31. The U.S. data employed here give c(hf,APhf) = 0.5082. This number is quite higher than the 0.282 reported by Kim and Loungani (1991) (using annual data) and the approximately zero number reported by Christiano and Eichenbaum (1990) (using quarterly data) for the same correlation. The data measures, sample period and (as indicated) data periodicity differ across the studies.

32. The findings were robust to
    (i) the replacement of the MA(1) representation of innovations to technology by an AR(1) representation.
    (ii) the replacement of $\omega_2 = 1.39$ and $\nu_2 = 1.62$ by the pair: $\omega_2 = 1.40$ and $\nu_2 = 1.70$ which implied the *same* steady state depreciation as before but a lower steady state energy to capital ratio (equal to 6.1%).

33. For the independent evidence see, e.g., Finn, Hoffman and Schlagenhauf (1990).

34. This will involve the adoption of a solution technique that undertakes an approximation of the first-order conditions of the model rather than of the utility function, as is the case here. The reason is that, under the assumption of nonstationary in the z_t process, a stationarity-inducing transformation must be employed prior to approximation. For utility-function approximation methods, this restricts one to a logarithmic function of consumption in order to preserve a quadratic objective function. For first-order-condition approximation methods there is no such restriction.

35. There is also some evidence that the omission of energy price shocks results in a fairly sharp decline in the correlation between output and consumption. However, this decline is not huge and is not consistent across the innovation strategies taken. It is therefore unlikely that much is read from it.

36. The inclusion of government spending shocks reduces the volatility of output when normal innovations are used. The reason for this apparently bizarre finding is that the basic model assumes a negative covariance between innovations to technology and government spending and each of these innovations cause output movements in the same direction.

References

Aiyagari, S. Rao; Christiano, Lawrence J. and Eichenbaum, Martin. "The Output and Employment Effects of Government Spending", manuscript, Federal Reserve Bank of Minneapolis, January 1989.

Burnside, Craig; Eichenbaum, Martin and Rebelo, Sergio. "Labor Hoarding and the Business Cycle", manuscript, Queen's University and Northwestern University, October 1990.

Christiano, Lawrence J. "Why Does Inventory Investment Fluctuate So Much?", Journal of Monetary Economics, 21, March/May 1988, pp. 247-280.

Christiano, Lawrence J. and Eichenbaum, Martin. "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations", manuscript, Federal Reserve Bank of Minneapolis and Northwestern University, January 1990.

Finn, Mary G. "On Savings and Investment Dynamics in a Small Open Economy", Journal of International Economics, 29, 1990, pp. 1-21.

Finn, Mary G; Hoffman, Dennis L. and Schlagenhauf, Don E. "Intertemporal Asset-Pricing Relationships in Barter and Monetary Economies: An Empirical Analysis", Journal of Monetary Economics, 25, May 1990, pp. 431-451.

Gomme, Paul. "Money and Growth", manuscript, University of Montreal, 1991.

Greenwood, Jeremy; Hercowitz, Zvi and Huffman, Gregory. "Investment, Capacity Utilization and the Real Business Cycle", American Economic Review, 78(3), 1988, pp. 402-417.

Greenwood, Jeremy and Huffman, Gregory W. "Tax Analysis in a Real-Business-Cycle Model: On Measuring Harberger Triangles and Okun Gaps", Journal of Monetary Economics, 27, April 1991, pp. 167-190.

Greenwood, Jeremy and Hercowitz, Zvi. "The Allocation of Capital and Time Over the Business Cycle", Journal of Political Economy, forthcoming 1991.

Hall, Robert E. "Invariance Properties of Solow's Productivity Residual", National Bureau of Economic Research, Working Paper No. 3034, July 1989.

Hamilton, James D. "Oil and the Macroeconomy Since World War II", Journal of Political Economy, 91, 1983, pp. 228-48.

Hansen, Gary D. "Indivisible Labor and the Business Cycle", Journal of Monetary Economics, 16, November 1985, pp. 309-28.

Hansen, Gary D. and Sargent, Thomas J. "Straight Time and Overtime in Equilibrium", Journal of Monetary Economics, 21, March/May 1988, pp. 281-308.

Hercowitz, Zvi and Sampson, Michael. "Output Growth, The Real Wage, and Employment Fluctuations, American Economic Review, forthcoming 1991.

Hornstein, Andreas. "Monopolistic Competition, Increasing Returns to Scale, and the Importance of Productivity Changes", manuscript, University of Western Ontario, November 1990.

Jorgenson, Dale W. and Griliches, Z. "The Explanation of Productivity Change", Review of Economic Studies, 34, April 1967, pp. 249-83.

Kim, In-Moo and Loungani, Prakash. "The Role of Energy in Real Business Cycle Models", Federal Reserve Bank of Chicago, Working Paper 91-6, March 1991.

King, Robert E.; Plosser, Charles and Rebelo, Sergio. "Production, Growth and Business Cycles I", Journal of Monetary Economics, 21, March/May 1988, pp. 195-232.

Kydland, Finn E. "Labor-Force Heterogeneity and the Business Cycle", Carnegie-Rochester Conference Series on Public Policy, 21, Autumn 1984, pp. 173-208.

Kydland, Finn E. and Prescott, Edward C. "The Workweek of Capital and Its Cyclical Implications", Journal of Monetary Economics, March/May 1988, pp. 343-360.

Kydland, Finn E. and Prescott, Edward C. "Time to Build and Aggregate Fluctuations", Econometrica, 50, November 1982, pp. 1345-70.

Kydland, Finn E. and Prescott, Edward C. "Hours and Employment Variation in Business Cycle Theory", forthcoming, 1991.

Mendoza, Enrique G. "Real Business Cycles in a Small Open Economy", American Economic Review, 81, September 1991.

Lucas, Robert E. "Understanding Business Cycles", Carnegie-Rochester Conference Series on Public Policy, 5, 1977, pp. 7-29.

Pace, Richard R. "Temporary vs. Permanent: Evaluating Innovation Permanence in a Nonlinear Stochastic Growth Model", manuscript, University of Rochester, June 1989.

Phelps, E. "Golden Rules of Economic Growth, (Norton, New York), 1966.

Rasche, Robert H. and Tatom, John A. "Energy Price Shocks, Aggregate Supply and Monetary Policy: The Theory and the International Evidence", Carnegie-Rochester Conference Series on Public Policy, 14, 1981, pp. 9-94.

Solow, Robert M. "Technical Change and the Aggregate Production Function", Review of Economics and Statistics, 39, August 1957, pp. 312-320.

Stockman, Alan C. and Tesar, Linda L. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements", manuscript, University of Rochester and University of Santa Barbara, October 1990.

Swan, T. "On *Golden Ages and Production Functions*" *in* Kenneth Berril, ed., Economic Development with Special Reference to Southeast Asia, (Macmillan, London), 1963.

Taubman, Paul and Wilkinson, Maurice. "User Cost, Capital Utilization and Investment Theory", International Economic Review, 11, June 1970, pp. 209-15.

Fig. 1

Logarithm of Solow Residual

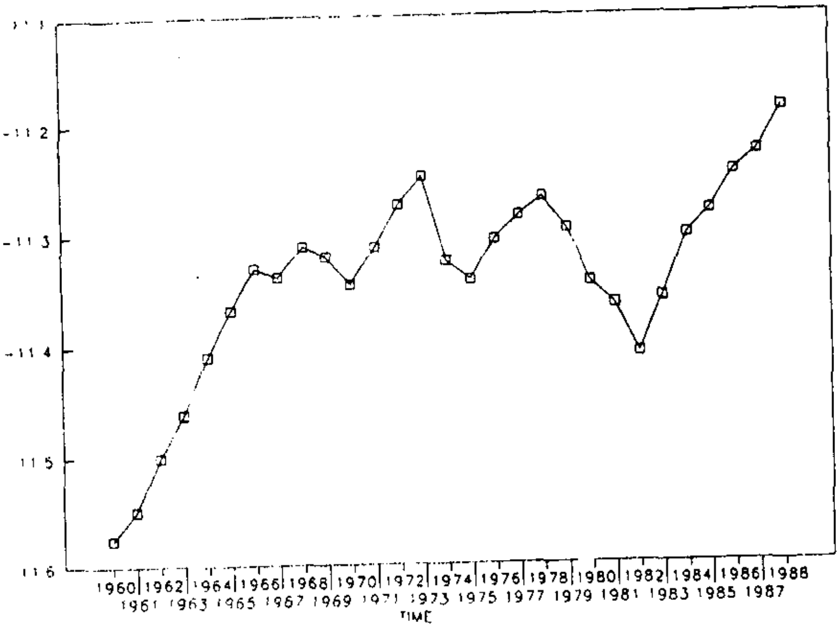

Fig. 2

Logarithm of Output

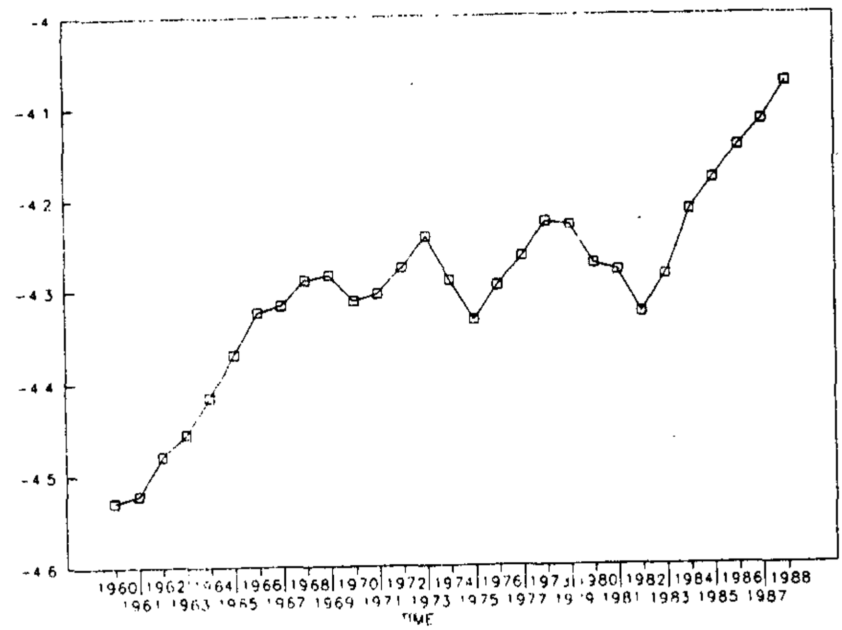

Fig. 3

Logaritm of Real Energy Price