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Federal Reserve Bank of Minneapolis
Research Department
250 Marquette Avenue
Minneapolis, Minnesota 55480

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PRODUCTIVE EXTERNALITIES AND BUSINESS CYCLES

Marianne Baxter* Robert G. King*

Institute for Empirical Macroeconomics
Federal Reserve Bank of Minneapolis

and University of Rochester

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Abstract

This paper begins with the observation that the volatility of factor input growth is insufficient to explain the volatility in the growth rate of output, and explores the empirical plausibility of the hypothesis that this fact is due to the presence of productive externalities and increasing returns to scale. We construct a quantitative equilibrium macroeconomic model which incorporates these features, and allows for demand shocks operating at the level of the consumer. We employ the method of Hall (1986) and Parkin (1988) to measure these demand shocks, and use these measured disturbances to conduct stochastic simulations of the model. We find that the model with increasing returns, when driven by measured demand shocks, generates time series which replicate the basic stylized facts of U.S. business cycles, although with lower amplitude. However, in the absence of increasing returns the measured demand shocks do not produce a characteristic business cycle response. When preference shocks are combined with productivity shocks, we find that both the increasing returns and the constant returns models correctly predict a weak correlation between hours and wages, while the predictions of the increasing returns model provide the better overall match with the data.

1. Introduction

Since the work of Solow [1957], economists have recognized that measured growth in factor inputs is insufficient to explain output growth. Figure 1 plots annual output growth over the postwar period against annual growth in total factor input (defined as growth in labor and capital weighted by factor income shares). Factor input growth is positively correlated with output growth, but fails to explain it in two important ways. First, the growth rate of total input averaged only 2.45 percent per year over the postwar period, while output grew at an average rate of 3.22 percent. Second, total factor input growth is less volatile than output growth, with a standard deviation of 1.75 percent for inputs, compared with 2.96 percent for output.

As a related matter, labor productivity is well-known to be procyclical. Real business cycle theory explains this fact via procyclical movements in total factor productivity; other explanations that have been advanced include market power, increasing returns to scale and the existence of labor hoarding (Hall [1987,1988] and Bernanke and Parkinson [1990]). Further, a great deal of recent theoretical research has stressed the potential role of external economies in generating long-term economic growth (Romer [1986] and Lucas [1988]). In addition, recent empirical research has suggested the existence of external economies operating at the national level; see, for example, the work of Caballero and Lyons [1989], [1990]. While the exact form of the external economies is still a matter of active debate in the profession, the idea that such externalities may be potentially important for growth and business cycles is well-entrenched. However, there has been no previous investigation of the quantitative implications of externalities for the character of business cycles.

The goal of the present paper is to begin to fill this gap.[1,2] As a starting point, we employ a specification of technology in which the production technology is constant returns to scale at the individual level, but that the existence of productive externalities operating at the aggregate level means that increasing returns operate at the level of the economy as a whole. This specification has been previously used in theoretical work by Romer [1986] on economic growth and by Murphy, Schleifer and Vishny [1989] on business cycle. The empirical analyses of Caballero and Lyons [1989], [1990] support the hypothesis that these "Marshallian externalities" exist.[3]

By incorporating productive externalities, this research represents a major departure from standard "real business cycle" theory, which has so far focused exclusively on models in which the production function exhibits (individual and social) constant returns to scale. Until very recently, this research program focused on studying the properties of business cycles driven by exogenous shocks to productivity.[4] In the Keynesian tradition, however, business cycles were thought to be be generated primarily as a response to demand shocks, interpreted in that tradition as shocks to consumers' saving propensities at given levels of wealth and relative prices. More recently, Mankiw, Rotemberg and Summers [1985] and Eichenbaum, Hansen, and Singleton [1988] have shown that the Euler equations of the representative agent model provide poor descriptions of the U.S. macroeconomy. One interpretation of this finding is that there are large and persistent shocks to preferences that are important at the aggregate level. Nevertheless, modern equilibrium business cycle research has generally not concerned itself with the question of whether demand shocks are capable of generating business cycles.[5]

The paper is organized as follows. Section 2 lays out our business cycle model, in which a single parameter governs returns to scale. We also discuss the method used to solve and simulate this economy when increasing returns and productive externalities are present. Section 3 is devoted to issues of estimation and calibration. First, we discuss the estimation of the parameter governing the extent of increasing returns. Second, we follow Hall's [1986] and Parkin's [1988] suggestion that demand shocks can be measured as residuals from the Euler equation governing efficient consumption/leisure choice. We use this procedure to generate a time series of demand shocks from the U.S. data, and estimate a stochastic process for the demand shock. Finally, we briefly discuss our calibration of the remaining components of the model, using parameters that are for the most part standard. In Section 4, we study the dynamic response of our model to demand shocks. Using the sequence of demand shocks generated from the data, we conduct a stochastic simulation of the increasing returns model. We find that the simulated responses of output, consumption, investment, and labor supply in the increasing returns model resemble actual U.S. aggregate data to a surprising extent. However, when the same sequence of measured demand shocks is fed through the constant returns real business cycle model, the responses do not mimic actual business cycle phenomena. In particular, the constant returns model does not produce the highly volatile behavior of investment response that is typical of actual business cycles.

In section 5 we study the implications of combining preference shocks with productivity shocks. Using standard production function measures of productivity shocks, we estimate the parameters of stochastic processes for productivity shocks under the alternative model structures (increasing returns and constant returns). When driven by shocks to preferences and technology, both the increasing returns and constant returns models generate realistic business cycle behavior. However, the increasing model performs better than the constant returns model along several dimensions. First, the increasing returns model is better at explaining the level of output volatility and the relative volatility of consumption and labor supply. Second, the increasing returns model is more effective in reducing the correlation between output and labor productivity (output per manhour) toward the level observed in the data. Our most striking finding, however, is that both models—when driven by the two shocks together—are capable of generating the observed weak cyclical comovement between labor input and measures of the return to labor (real wages and output per manhour). Existing business cycle theories based on a single source of shocks have not been able to generate empirically reasonable predictions for these labor market variables. Further, Christiano and Eichenbaum's [1992] multi-shock real business cycle model cannot capture these "Dunlop-Tarshis" correlations in the absence of significant measurement error in labor input. Section 7 concludes with a discussion of the paper's main results, and directions for future research.

2. A Business Cycle Model with Increasing Returns

Our model departs in two important ways from the basic neoclassical macroeconomic model.[6] First, we introduce productive externalities leading to increasing returns to scale. Second, we alter preferences to allow for shifts to the marginal utility of consumption. Although our quantitative analysis incorporates growth in population and technology, for simplicity we have transformed the model to eliminate growth in our presentation below. This transformed model possesses a stable stationary state in the absence of shocks to preferences or technology, so long as the returns to scale parameter (defined below) is not too large. The value that we assign to this parameter, based on our own and other researchers' empirical analyses, is small enough so that there is a stable steady state in the model. That is, the externality is not strong enough to generate endogenous growth, in contrast to Romer [1986].

2.1 The Model

The building blocks of our model are specifications of preferences, technology, resource constraints, and accumulation equations for capital. These are spelled out below; we focus mainly on those aspects of this model which differ from more standard equilibrium business cycle models.

Preferences. Each agent has preferences over consumption and leisure as summarized by (1a) and (1b):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \, u(C_t, L_t) \tag{1a}$$

$$u(C_t, L_t) = \log(C_t - \Delta_t) + \theta_L \log(\nu(L_t)), \tag{1b}$$

where consumption is C_t; leisure is L_t, with $\nu(L)$ a positive and increasing function; and Δ_t is a stochastic component of preferences that permits us to analyze demand shifts. A positive innovation to Δ_t represents a positive demand shock, i.e., an increased urgency to consume. In our analysis below, we set $\nu(L) = L$ so that the labor supply elasticity is determined by the stationary level of hours as in Prescott [1986] and Plosser [1989].

The stochastic preference term Δ_t is interpretable a demand shift in the following specific sense. Consider the "Frisch" demand function for consumption which describes date t consumption demand as a function of its price p_t and a measure of lifetime wealth, the multiplier (Λ) on the intertemporal budget constraint.[7] Under the preference specification (1), that demand function is $C_t = (\Lambda p_t)^{-1} + \Delta_t$. Thus Δ_t can be interpreted as an additive demand shift, holding fixed prices and the wealth measure.

Technology. We assume that an individual agent (a representative worker-producer) combines capital (K_t) and labor input (N_t) to produce output according to:

$$Y_t = A_t F(K_t, N_t) \underline{Y}_t^{\epsilon}. \tag{2}$$

In equation (2), $F(K_t, N_t)$ is a constant returns to scale production function of the Cobb-Douglas form, $F(K,N) = K^{\theta_K} N^{\theta_N}$; A_t is a total factor productivity shock; and $\underline{Y}_t^{\epsilon}$ is per capita output ($\underline{Y}_t$) raised to the power ϵ. Thus ϵ controls the magnitude of the external effect. Throughout, we use underbars to denote endogenous variables which private individuals view as being beyond their control. Thus, as is standard in competitive models, the representative worker-producer is assumed to treat $\underline{Y}_t$ as exogenous. Yet the actions of all the (identical) agents taken together determine the per capita capital stock $\underline{K}_t$, labor input $\underline{N}_t$, and output $\underline{Y}_t$. Thus, equilibrium output is:

$$\underline{Y}_t = [A_t F(\underline{K}_t, \underline{N}_t)]^{\eta}, \tag{3}$$

where $\eta = 1/(1-\epsilon)$ indexes the extent of increasing returns.

Private and Social Marginal Products: In the presence of productive externalities, it is necessary to distinguish between private and social marginal product schedules. We continue to let underbars denote aggregate quantities beyond the control of the individual. Given the production function facing the individual, $Y_t = A_t K_t^{\theta_K} N_t^{\theta_N} \underline{Y}_t^{\epsilon}$, the private marginal product schedules for labor and capital are:

$$MPN_t = \theta_N(Y_t/N_t)\underline{Y}_t^{\epsilon} = \theta_N\underline{Y}_t/\underline{N}_t \tag{4a}$$

$$MPK_t = \theta_K(Y_t/K_t)\underline{Y}_t^{\epsilon} = \theta_K\underline{Y}_t/\underline{K}_t, \tag{4b}$$

where the latter equality reflects the fact that all agents will be producing the same quantities and selecting the same input choices in equilibrium. The social marginal product schedules for labor and capital are

$$SMPN_t = \eta\theta_N\underline{Y}_t/\underline{N}_t \tag{5a}$$

$$SMPK_t = \eta\theta_K\underline{Y}_t/\underline{K}_t, \tag{5b}$$

which are higher at given values of $\underline{K}_t$ and $\underline{N}_t$ so long as $\eta>1$. While the levels of these schedules are different, the (constant) elasticities with respect to capital, labor and technology shocks are equal for private and social marginal products. For example, the labor elasticity of the marginal product of labor is $\eta\theta_N-1$ in both cases.

Accumulation Technology. Capital evolves according to:

$$K_{t+1} = [(1-\delta_K)K_t + I_t], \tag{6}$$

where I_t is gross investment (i.e. the amount of current output to be used in next period's production) and δ_K is the rate of depreciation of capital.

Government. The government imposes a tax on net output at the rate τ; it uses the proceeds for expenditure on goods which do not yield utility directly to individuals and which do not affect private marginal products on the technology side. Government expenditure is assumed constant at the level $\underline{G}$. Variations in revenues associated with fluctuations in output are returned to private agents in the form of lump-sum transfers, T_t. Thus the government's flow budget constraint is:

$$\underline{G} + T_t \leq \tau Y_t \ . \tag{7}$$

Resource Constraints. In each period, there are resource constraints on goods and time:

$$L_t + N_t \leq 1 \tag{8}$$

$$C_t + I_t + \underline{G} \leq Y_t \tag{9}$$

Equation (9) need not hold for an individual agent, who may borrow and lend. However, the aggregate resource constraint $\underline{C}_t + \underline{I}_t + \underline{G} \leq \underline{Y}_t$ must hold in equilibrium, and will also hold for each individual in our representative agent economy. We therefore impose equation (9) as an equilibrium condition in our analysis.

2.2 Analysis of Dynamic Equilibrium

The standard method of solving real business cycle models with constant returns to scale technology and no government-imposed distortions is to solve an associated planner's problem, and to reinterpret as competitive market outcomes the planner's optimal decisions and the associated shadow prices. In our setting, the presence of productive externalities makes that methodology inapplicable. We therefore use an alternative, Euler-equation-based approach. Within this "Euler equation" approach to finding suboptimal dynamic equilibria, there are a variety of methods for approximating the equilibrium laws of motion for macroeconomic prices and quantities.[8] In this paper we employ the log-linear approximation methods of King, Plosser and Rebelo [1987], which produce certainty-equivalent decision rules describing deviations from steady state values.[9] The basic logic behind the Euler-equation approach is as follows. In any competitive equilibrium problem, individuals make privately-efficient decisions which are summarized by first-order necessary conditions. In making these decisions, individuals take as given the paths of per capita quantities. Next, aggregate consistency conditions (resource constraints and rational expectations) are imposed on the first-order conditions. This two-stage procedure generates conditions that restrict the dynamic evolution of the economy, and describes competitive equilibrium even in distorted economies.

Our representative consumer makes consumption, leisure and investment decisions in a manner that is privately efficient: he equates the marginal utility of date t consumption to its opportunity cost; the marginal utility of leisure to the value of foregone earnings; and the opportunity cost of investment to its expected future return. Under certainty equivalence, these conditions are:

$$D_1u(C_t-\Delta_t,L_t) = \lambda_t \tag{10}$$

$$D_2u(C_t-\Delta_t,L_t) = \lambda_t(1-\tau)A_tD_2F(K_t,N_t)\underline{Y}_t^{\epsilon} \tag{11}$$

$$\beta E_t\left\{ \lambda_{t+1}[A_{t+1}(1-\tau)D_1F(K_{t+1},N_{t+1})\underline{Y}_{t+1}^{\epsilon} + 1-\delta_K] \right\} = \lambda_t . \tag{12}$$

where λ_t is the Lagrange multiplier attached to the flow budget constraint (9), and is interpretable as the shadow value of private consumption at date t. We use the notation $D_1u(C_t,\ldots)$ to represent the marginal utility of consumption (the partial derivative of utility with respect to its first argument), and we use corresponding notation for other marginal utilities and marginal products throughout the paper. By combining these efficiency conditions with the macroeconomic equilibrium conditions, (6)–(9) and the production function (2), we obtain a dynamic system that can be solved to trace out the response of the economy to shifts in A_t or Δ_t.

The log-linear system that we obtain describes the evolution of a vector of state variables, $s_t = [\hat{K}_t, \hat{A}_t, \hat{\Delta}_t]'$, where the circumflex denotes the proportionate deviation from the stationary level for capital and productivity, $\hat{K}_t = \log(K_t/K)$ and $\hat{A}_t = \log(A_t/A)$. For the demand shock, deviations are computed relative to stationary consumption. The state vector evolves according to $s_t = M\, s_{t-1} + \xi_t$, where $\xi_t = [0,\ a_t,\ d_t]'$ is a vector containing the innovations to technology and demand, and where the matrix M is given by

$$M = \begin{bmatrix} \mu_1 & \pi_{KA} & \pi_{K\Delta} \\ 0 & \rho_A & 0 \\ 0 & 0 & \rho_\Delta \end{bmatrix} .$$

The coefficients in this matrix determine the evolution of the economy's state variables. Specifically, ρ_A and ρ_Δ determine the persistence of exogenous shocks; and the implied reduced form for capital is $\hat{K}_{t+1} = \mu_1\, \hat{K}_t + \pi_{KA}\, \hat{A}_t + \pi_{K\Delta}\, \hat{\Delta}_t$. Hence μ_1 determines the speed of transition-path dynamics. The impulse responses of capital and other variables to an innovation in ξ are jointly determined by the exogenous propagation mechanisms of the model (parameterized by ρ_A and ρ_K) and the endogenous propagation mechanism (governed by μ_1).

The remainder of the model's variables are simply functions of the state variables. Letting $Z_t = [\hat{C}_t\ \hat{N}_t\ \hat{Y}_t\ \hat{W}_t\ r_t\ \ldots.]$ be the vector of these variables, the model implies that $z_t = \Pi s_t$ with particular numerical values for the elements of the matrix Π. For example, consumption is governed by the relation $\hat{C}_t = \pi_{CK}\, \hat{K}_t + \pi_{CA}\, \hat{A}_t + \pi_{C\Delta}\, \hat{\Delta}_t$. With this "state space" system in hand, it is direct to compute the stochastic simulations, population moments, and impulse responses discussed in the paper.

3. Estimation and Calibration

In order to obtain quantitative predictions from our model, we must assign numerical values to the parameters of the model. This investigation has introduced two new elements into the quantitative business cycle literature—increasing returns and preference shocks—and we consequently must address the question of how to parameterize the model along these dimensions. We consider each in turn, and conclude this section with a brief discussion of the calibration of the remaining parameters of the model.

3.1 Measuring the returns-to-scale parameter

An estimate of the returns-to-scale parameter η can readily be obtained from a regression of output growth, $\gamma_{Y,t} = \log(Y_t/Y_{t-1})$, on input growth, $\gamma_{Z,t} = [\theta_K \log(K_t/K_{t-1}) + \theta_N \log(N_t/N_{t-1})]$. If there are no random variations in technology, $\hat{A}_t=0$ for all t, then we can estimate η consistently with least squares. Table 1 presents statistics on this regression and others to be discussed below. In this regression equation, the OLS point estimate of η is 1.45, corresponding to a value of ϵ of about 1/3.

If there are technology shocks, however, these will induce movements in factor inputs, so that the required orthogonality condition for consistency of the least-squares estimator is not satisfied. Thus an instrumental variables estimator of η must be constructed. We experimented with some measures of public expenditure as instruments. First, we employed three military spending measures as suggested by the work of Hall [1987], [1988]. With these instruments, we obtain an estimate of η equal to 1.81. However, the poor performance of the first stage regression made us concerned about the precision of this estimate. We therefore explored two other sets of instruments: (i) two measures of defense compensation with an associated estimate of η equal to 1.53, and (ii) total nondefense purchases, implying an estimate of η equal to 1.10. In all three instrumental variables regressions there is only minor explanatory power in the first stage regression.

Caballero and Lyons [1989] estimate aggregate and industry-level equations that are similar to ours, and conclude that there are significant economies of scale that are external from the industry point of view, but internal to the U.S. as a whole. They experienced similar problems in obtaining good instruments, and also experimented with a variety of methods of estimation. Using our notation, the externality parameter preferred by Caballero and Lyons is η=1.3. Based on our results and those of Caballero and Lyons, we set η=1.3 (corresponding to a value of ϵ of about .23) in studying the quantitative implications of the increasing returns model in section 4 below.

3.2 Measuring preference shocks

This paper focuses on the effects on the macroeconomy of shocks to preferences, in contrast to the supply shifts (technology shocks) normally stressed in real business cycle theory. Although technology shocks are not directly observable in the macroeconomic data, "observations" on the technology variable are routinely constructed as residuals from a specified production function. That is, conditional on a particular model, unobservable technology shocks become measurable. In a similar spirit, Hall [1986] and Parkin [1988] have suggested a method of isolating preference shocks as residuals from Euler equations. Using this procedure, the marginal conditions from the utility function relate the unobserved preference shift to observable variables in a manner analogous to that employed by Solow to measure shifts to technology.[10]

In our model, the requirement that the marginal rate of substitution between leisure and consumption equals the real wage provides a straightforward method of identifying preference shifts, as follows:

$$\frac{\Delta_t}{C} \cong \log(C_t) - \log(w_t) + \left[\frac{N}{1-N}\right]\left[\frac{N_t - N}{N}\right] \tag{13}$$

where C and N denote the steady state levels of consumption and labor supply.[11]

To construct this measure of preference shocks, we need empirical measures of consumption, labor supply, and the real wage rate. The details of the data we used can be found in the Appendix. Our measure of consumption includes expenditure on services and nondurable consumption goods, and excludes purchases of consumer durables. Our labor supply measure is computed as per capita hours multiplied by the labor force. We measure the returns to labor as compensation per employee hour (see the Appendix for the details of the data used).

Table 2 reports the statistical properties of our measure of the stochastic process for the preference shock. We found that a first-order autoregression (in logarithms), including a constant and a time trend, described the preference shock quite well. The preference shock is highly persistent, with an autoregressive coefficient of .97. We will use this estimated persistence parameter and the estimated innovation variance in computing the responses of our model to preference shocks.

3.3 Other parameters

In addition to the returns-to-scale parameter and the stochastic process for the demand shocks, we must specify values for the standard parameters of preferences and technology. Many of the values we have assigned are standard in the real business cycle literature. Three that are somewhat less standard are as follows. First, we have set the depreciation rate of capital at 6%

per year. Second, the combined rate of exogenous growth due to population growth and exogenous technical change is equal to 3.6% per year. Third, we have set the steady state income tax rate at its current average level of 30%, and steady state government expenditure at its post-war average level of 20% of GNP. With these modifications, the model generates a realistic steady state investment-to-output ratio (about 15%). Table 3 presents a complete list of the parameter values, and also provides a convenient review of notation.

4. Dynamic Properties of the Model

In this section, we investigate the properties of macroeconomic time series generated by our model economy with productive externalities and increasing returns when driven by solely preference shocks. By comparing the responses of this model to the standard, constant returns to scale model, we can learn about the role of increasing returns and productive externalities in determining the response of the economy to this type of shock. As shorthand, we denote by IR the model with increasing returns and productive externalities, and we use CR to denote the model with constant returns to scale and no externalities.

Two related methods are used to evaluate the models' responses to preference shocks. First, we generate a sequence of preference shocks from the data using equation (13). Next, we conduct stochastic simulations of the models, by feeding this sequence of shocks through the model and computing the response of output, consumption, etc. We then examine whether the two models' responses to these shocks resembles U.S. business cycles, by looking of plots of actual and simulated time series. The second approach to model evaluation is familiar from the real business cycle literature, and involves informal comparison of selected moments generated from the model to the corresponding moments from U.S. time series.

4.1 Stochastic simulation

Figure 2 plots the time series for the preference shift variable, Δ_t, computed from U.S. time series using equation (13), and our compensation-based measure of the real wage. For comparison, we have also plotted real output output (real GNP) over the same period. Both time series have been filtered using the Hodrick-Prescott [1980] (HP) filter in order to render them stationary.[12] We see from this figure the persistence of the preference shocks, and their tendency to covary positively with movements in output (the correlation between these two series is .61).

Figure 3 plots the response of the increasing returns economy when driven by the preference shock time series computed from the data. For comparison, actual U.S. time series are plotted as well (again, both the model-generated time series and the data have been HP-filtered). For all variables—output, consumption, investment, and labor input—we find that the model's responses move closely with the data, but with a tendency toward lower amplitude. Thus the IR model does produce a characteristic business cycle response when driven by demand shocks, if a business cycle is defined in the following, fairly conventional way. First, there is positive comovement of output, investment, consumption, and labor input over the cycle. Second, each of these variables exhibits persistent deviations from their trend values in response to shocks (i.e., business cycles are protracted events). Third, there is a characteristic pattern of relative volatility in the macro aggregates: consumption is less volatile than output; and investment is substantially more volatile than output. More statistical detail is provided in Table 4, discussed below.

We seek to isolate the role played by productive externalities and increasing returns in generating these business cycle phenomena. To do this, we feed the same series of demand shocks into the CR model (η=1) that were used to construct Figure 3. The resulting series are plotted in Figure 4. Beginning with output we see that cyclic volatility is lower in the CR model compared with the IR model (see Figure 3), although the CR model does generate an output time series that is highly correlated with the data. Second, looking at consumption and labor input, we see that the CR model predicts behavior that roughly resembles the data in terms of volatility. However, when we turn to investment, we see that the CR model driven by preference shocks fails to reproduce two of the central features of actual business cycles: the high volatility of investment and its strong positive comovement with output and consumption.

Table 4 presents the statistical moments of the U.S. data and the corresponding model moments. (In all cases, the moments are for HP-filtered time series.) For each of the models, three sets of statistics are reported: first, population moments which summarize the large-sample implications of the model; second, sample moments for the stochastic simulations plotted in Figures 3 and 4; and third, correlations between the simulated time series plotted in Figures 3 and 4 and the corresponding U.S. time series.

Beginning with cyclic volatility, we find that the (population) standard deviation of output in the IR model is about 54% as large as the standard deviation of output in the data. By comparison, the volatility in the CR model is substantially smaller. In terms of the standard deviation of output, the CR model generates about one-half the volatility of the IR model.

Both the IR model and the CR model, when driven by demand shocks alone, generate consumption volatility statistics that match well with the data. However, because these models generate output volatility that is low relative to the data, the relative volatility statistics for consumption are too large. In fact, the CR model predicts that consumption is about one-and-one-half times as volatile as output, which is clearly inconsistent with the data.

As noted above, it is investment behavior which most sharply differentiates the response of the two models. The IR model predicts investment movements which are about 2.75 times as volatile as output movements, compared with 3.15 in the data. The level of investment volatility is low, as is the level of output volatility. But recall that we are not trying to explain all the movement in macro aggregates with demand shocks. Rather, we are investigating whether demand shocks produce a characteristic business cycle response in the increasing-returns economy. Our interpretation of these statistics and simulations is that the IR economy does generate business cycles in response to preference shocks. The CR model, on the other hand, does very badly in terms of the investment response. First, the investment response is very weak—investment is only .01 times as volatile as output in the model. But even worse: *investment is strongly negatively correlated with output!* (The reasons for this are developed more fully in the next section.)

Finally, with respect to labor market variables, we observe that the volatility of labor input is higher in the IR model than in the CR model, and is not too far from the level of volatility found in the data. That is, the IR model can explain nearly all of the cyclic fluctuation in labor as a response to preference shocks alone. This phenomenon is discussed more fully in Section 5 below.

Both the CR model and the IR model predict strong, negative correlations between wages and output when driven by preference shocks alone (because our production function is Cobb-Douglas, the wage rate, which equals the marginal product of labor, also equals the average product of labor, reported as y/N in the Table). Further, both models predict strong negative comovement of wages and labor input. Both set of predictions are strongly at variance with the facts.

4.2 Impulse responses to an innovation in demand

Additional insight into the dynamic properties of the IR model is provided by tracing out the impulse response to a demand shock. We consider a shock to preferences that would raise consumption by one percent of its steady state level on impact if we held fixed all prices faced by the representative consumer. As noted above, our estimates are that shifts in demand are assumed to be highly persistent: with ρ_{Δ}=.97, slightly over one half of the original demand shift will be present after twenty quarters. Figure 5 shows the dynamic response of prices and quantities to the demand shift in the IR model; for comparison, we also plot the responses of the CR model.

Impact Effects: At date t=1, when the innovation to demand takes place, the effects of the demand shock on output are much larger in the IR model than in the CR model: output increases by .64% of its steady state level in the former, and only .33% in the latter. This increased response can be traced to two sources. First, a given increase in labor input simply yields more output under increasing returns (with η=1.3, the impact output response via this channel is $1.3 * .33\% = .43\%$). Second, labor input is much more responsive to demand shocks in the presence of increasing returns.

This just pushes the question back one stage however—*why* does labor input respond more elastically when there are productive externalities and increasing returns? One way to think about the difference between the responses of the CR and IR models is to notice that the external effect operates on the individual's production function, $Y_t = AF(K_t, N_t)\underline{Y}_t^{\epsilon}$, much like a technology shock. The external effect temporarily raises the position of the private production function, inducing additional labor input. The magnitude of this "production function shift" is $\epsilon\hat{Y}_t$, which is one-third of the output response displayed in Figure 5 for the IR model.

In a decentralized market system, individuals are induced to alter their behavior by changes in relative prices such as the real interest rate and the real wage rate. Compared with the CR model, the IR model displays larger labor supply responses because (i) the real interest rate displays a larger positive response to demand shocks; and (ii) the real wage displays smaller negative response to demand shocks. The second of these simply reflects the fact that the marginal product of labor declines less sharply with labor input in the increasing returns model. The elasticity of the real wage rate with respect to labor input is $\eta\theta_N - 1$. Under constant returns, this magnitude is $-.42$, compared with a value of $-.25$ under increasing returns with η=1.3.

The larger response of the real interest rate in the IR model stems from the fact that the demand shock induces increased investment, leading to a higher equilibrium rate of return. This higher return induces intertemporal substitution in consumption and labor supply. Consumption thus increases less on impact in response to the demand shock with η=1.3 than it does with η=1; labor supply increases by more.

A notable feature of the impact response of output is that output rises about one-for-one with the preference shock.[13] By comparison, the output effect of the demand shift in the CR model shown in Figure 5 is only .50. These relative magnitudes are consistent with the analyses of Aiyagari, Christiano and Eichenbaum [1991] and Baxter and King [1991] who found that that large effects of demand disturbances required strong supply-side responses of capital and labor. In the present context, these strong supply-side responses arise in the IR model, but not in the CR model.

Persistence and Comovement: The predictions of the IR model for the persistence and comovement of macroeconomic time series differ markedly from the predictions of the CR model. First, the transition path dynamics of the IR model involve positive comovements of labor input and gross investment with the capital stock, while these comovements are negative in the CR model. This characteristic means that the IR model contains inherently stronger propagation mechanisms than the CR model. That is, a positive innovation to the capital stock in the IR model leads to increased labor supply, which in turn leads to an increased marginal product of capital and an incentive to invest further. Yet this propagation mechanism is not, by itself, sufficient to generate business cycles, given an arbitrary stochastic process for the demand shocks. In fact, high serial correlation in the demand shocks is necessary if this model is to generate business cycles with realistic amplitude, comovement, and persistence. If the demand shocks were purely temporary, the contemporaneous outcomes generated by the model would bear little resemblance to initial phases of U.S. business cycle expansions or contractions. In particular, with $\rho_\Delta = 0$ (or, indeed, with $\rho_\Delta < .93$), investment would respond negatively to shocks to consumption demand, with its role as a buffer dominating the input demand linkages highlighted above. Thus without persistence in demand shocks, there would not be important serial correlation in output and labor input. But our estimated process for the preference shock is indeed persistent, with ρ_Δ = .97.

Second, we find that the IR model proceeds somewhat more slowly than the CR model along the transition path.[14] In the CR model, the transitional dynamics coefficient μ_1 is .964, which implies that a 1% drop in the capital stock will be half rebuilt in 19 quarters. In the IR model, μ_1=.972, which corresponds to a half life of 24 quarters. To sum up: while the IR model has stronger internal propagation mechanisms than the standard CR model, these propagation mechanisms are still relatively weak. For either model to produce realistic cyclic behavior, it is necessary for the shocks driving the economy to be highly persistent.

5. Combining Preference Shocks with Productivity Shocks

In the preceding sections we found that preference shocks generate a business cycle response in all of the quantity variables if the economy is characterized by increasing returns to scale, but do not under constant returns to scale. However, both the IR and CR models did very badly in their predictions concerning aspects of the labor market. Productivity was predicted to be strongly countercyclical by both models, with a related prediction of strong negative correlation between hours and wages. The standard equilibrium business cycle model driven by procyclical productivity has exactly the opposite problem: this model predicts correlations that are too high relative to the data. This raises the intriguing possibility that combining preference shocks with productivity shocks would lead to empirically reasonable predictions for business cycle behavior of these variables. This section is therefore devoted to a comparison of the cyclic behavior of the IR and CR models when these models are subject to combined shocks to preferences and to productivity.

Following traditional practice, we measure productivity shocks as "Solow residuals" from the production function, equation (2). This measure of productivity is not invariant to the existence of increasing returns. Specifically, the productivity shift variable, A_t, is measured as follows :

$$\text{CR model:} \qquad \log(A_t) = \log(Y_t) - \theta_K \log(K_t) - \theta_N \log(N_t) \tag{14a}$$

$$\text{IR model:} \qquad \log(A_t) = (1/\eta)\log(Y_t) - \theta_K \log(K_t) - \theta_N \log(N_t). \tag{14b}$$

We found that both measures of the productivity shift variable were well-approximated by a low-order autoregression; in fact, both measures of productivity appear to follow a random walk. The details of the estimation are reported in Table 5. For the purpose of model simulation, we imposed the unit root in technology in both the CR and IR models. The innovation variances were computed from restricted regressions reported in Table 5. Because the IR measure of the productivity shock scales output by the factor $1/\eta$, the productivity shock has smaller variance in the IR model. The estimated standard deviation of the shock is .83% per quarter in the CR model, compared with .65% per quarter in the IR model.[15] Finally, we assume that there is zero correlation between the technology shocks and the productivity shocks.

The results of incorporating stochastic movements in productivity into our model are reported in Table 6. For both the CR model and the IR model, we report results for the combined productivity and preference shocks, as well as for productivity shocks alone. Looking first at the results for productivity shocks alone, we see that the differences between the CR model and the IR model are relatively minor. The main difference is that relative (and absolute) investment volatility is substantially higher the IR model than in the CR model. Both models generate consumption volatility that is too high, relative to the data, although the CR model is worse than the IR model.

When driven by productivity shocks alone, both models predict far too little volatility in labor input. This finding differs from those of standard real business cycle (RBC) studies. It is due to the interaction of two dimensions along which our model is not standard, but which makes the model more accurate empirically in terms of producing realistic steady state shares of consumption, investment, government expenditure, and tax revenue. These alterations are as follows. First, we have set steady state government expenditure and taxation at their current average levels: $G/Y = .20$, $\tau = .30$. (In the early RBC literature, these parameters were typically set to zero.) Second, we have specified a random walk process for productivity, whereas much previous research has specified a process in which shocks to productivity were highly persistent, but not permanent. Each of these modifications has the effect of scaling back the labor response to a productivity shock; the combination means that labor moves very little in response to productivity shocks.

The two models generate similar predictions for the average product of labor (labor productivity) when driven by productivity shocks alone; both models predict that the average product of labor will be more volatile and more highly correlated with output than the data suggest is reasonable. Finally, as is typically the case with models driven by productivity shocks alone, the predicted correlations between wages and hours (or labor productivity and hours) are much too high, relative to the data. Comparing the responses of the IR and CR model to preference shocks alone versus productivity shocks alone, we see that both models predict strong labor supply movements in response to preference shocks, but a weak response to productivity shocks.

When we combine productivity shocks with preference shocks, we find that both the IR model and the CR model perform reasonably well, but that the IR model does better along three dimensions. First, the IR model explains nearly all of the variance in output when driven by the two shocks together; the CR model explains somewhat less (94% for the IR model, compared with 71% for the CR model). Second, the IR model does better in terms of matching the relative volatilities of consumption, investment, labor input, and labor productivity than does the CR model, although both models still overpredict somewhat the level of consumption volatility. Third, the IR model comes very close to matching the cyclic behavior of labor input, wages, and labor productivity.

This last point is an important one, as the cyclic behavior of these variables has come to be an important metric for the evaluation of business cycle models. In fact, Christiano and Eichenbaum [1992] have gone so far as to argue that "The ability to account for the observed correlation between the return to working and hours worked is a traditional litmus test by which aggregate models are judged."[16] Since the critiques of Classical and Keynesian models put forth by Dunlop [1938] and Tarshis [1939], single-shock theories of the business cycle have repeatedly drawn fire for their strongly counterfactual implications for the cyclic behavior of labor input, wages and labor productivity. In fact, as Table 6 shows, in the absence of preference shocks both wages and labor productivity are perfectly positively correlated with output. Further, both wages and labor productivity are strongly positively correlated with labor input. Looking back at Table 4, we see that the both models predict near-perfect negative correlation between wages or labor productivity and output when the models are subject to preference shocks alone, and a corresponding near-perfect negative correlation between labor input and the returns to labor, whether measured as wages or as labor productivity. When the two shocks are combined, however, both models continue to predict positive correlations between (i) labor productivity and output and (ii) between wages and output. But the correlations are now much less than 1.0; the IR model comes closest to matching the moments generated from the data.

Further, when the two shocks are combined, the IR model predicts a correlation of –.13 between hours and wages, compared with –.05 in the data. The CR model predicts a correlation of –.19. For productivity, the correlation with hours is –.04 in the data (recall that the model's predictions for labor productivity are perfectly correlated with its predictions for wages since our production function is Cobb–Douglas at the individual level). Thus, the Dunlop–Tarshis labor market facts are not a puzzle from the standpoint of our model.

6. Summary and Conclusions

In this paper, we have constructed an equilibrium model with an increasing–returns–to–scale technology which is driven by shocks to preferences and productivity. When driven by preference shocks alone, the increasing returns model produces the characteristic business cycle behavior of consumption, investment, output, and labor input. However, a constant returns to scale model model driven by preference shocks alone does not produce realistic cyclic behavior: most notably, the volatility of investment is approximately zero, and investment is negatively correlated with movements in output.

We also study the behavior of the two models when driven solely by shocks to total factor productivity. We find that, in both models, the cyclic variation in labor input is much smaller than that found in the data. This finding conflicts with the predictions of standard RBC models, and is due to two dimensions along which our model differs from prior RBC models First, based on empirical analyses of the statistical properties of the "Solow residual," we specify that total factor productivity followed a random walk. Second, we parameterize our model to deliver realistic steady state shares of consumption, investment, government expenditure, and tax revenues. These two factors are equally important in reducing labor's response to productivity shocks.

When we combine productivity and preference shocks, we find that the IR model produces responses that, compared with the CR model, more closely mimic the characteristic cyclic behavior of U.S. macroeconomic aggregates. Both models generate reasonable predictions for the cyclic volatility of labor input but, as mentioned earlier, the response of labor input is due almost entirely to responses to preference shocks. Finally, when preference shocks are combined with productivity shocks, both models are able to mimic the salient business cycle attributes of labor input, wages, and labor productivity. Specifically, both models predict a positive (but not perfect) correlation between labor productivity and output. Further, both models predict a weak correlation between hours worked and wages (or labor productivity). This so-called "Dunlop-Tarshis" observation has proved exceedingly difficult to reproduce in previous equilibrium business cycle models, but it falls out quite easily in both the CR and IR versions of our two–shock model.

Based on these results, our conclusions are as follows. First, along many dimensions the IR model performs better than the CR model in terms of generating accurate predictions for the cyclic behavior of macroeconomic aggregates. However, in many cases, notably the implications for labor market variables, the differences between the IR model and the CR model are small. Second, we find that a two shock model which combines productivity shocks with preference shocks is necessary to replicate empirical measures of (i) the cyclic volatility of labor input, and (ii) the aforementioned labor–market regularities. These predictions cannot be obtained from either model when driven by a single shock, whether to preferences or technology.

Endnotes

1. Two alternative research paths have been explored which retain the central features of Solow's [1957] analysis, namely (i) use of the aggregate production function as an organizing device for aggregate time series; and (ii) competitive analysis as an organizing device for studying market interactions. First, one branch of research on growth and business cycles has treated the gap between output and input growth a measure of "technical progress" and explored the implications of this hypothesis for the dynamic evolution of the economy, as in the work of Solow [1956] and Prescott [1986]. A second approach path has viewed the input series as imperfectly measured. In the literature on economic growth, this has motivated new measurements designed at improving series on labor and capital input (see e.g. Denison [1962] and Jorgenson, Gollop and Fraumeni [1987]). In the literature on business cycles, this idea has motivated both additional measurement (Kydland and Prescott [1989]) and theory (Burnside, Eichenbaum and Rebelo [1990] and Rotemberg and Summers [1990]).

2. Contemporaneously with our research, some parallel work has been carried out by Klenow [1990], in which the shocks to the economy consist of technology shocks and shocks to government expenditure.

3. However, other authors such as Hall [1988] and Bernanke and Parkinson [1990] propose alternative explanations for the same empirical phenomena. The empirical analysis of Bernanke and Parkinson uses industry data to attempt to distinguish between increasing returns versus labor hoarding as an explanation for procyclical productivity. Their results are mixed, with some industries supporting the increasing returns hypothesis.

4. Recently, however, other sources of shocks have been considered. For example, government expenditure and tax shocks have been studied by Baxter and King [1991], Aiyagari, Christiano, and Eichenbaum [1991], McGrattan [1990].

5. Bencivenga [1992] conducts an econometric analysis of an equilibrium business cycle model driven by preference shocks.

6. Expositions of this model have been provided by Barro [1984] and King, Plosser, and Rebelo [1988a,b].

7. See, for example, Deaton and Muellbauer [1980].

8. Baxter [1991] provides a general description of the Euler-equation approach to computing suboptimal equilibria and provides a discrete state space approach that is capable of handling problems that are less well behaved than ours. Taylor and Uhlig [1990] summarizes this and several other strategies for computing equilibria, several of which are useful in the context of distorted economies.

9. There has been relatively little work on the accuracy of log-linear approximations in the context of distorted economies. However, the preliminary results of Dotsey and Mao [1990] give us confidence in our results, since they show that the King, Plosser and Rebelo [1987] methods are highly accurate in economies with tax distortions that are much larger than the external effects studied here. Their work uses Baxter's [1991] discrete state space Euler equation approach with fine grids to yield "exact solutions" and shows that the KPR log-linear approximations are remarkably accurate.

10. It is often argued that technology shocks are "observable" while preference shocks are not. But neither type of shock is <u>directly</u> observable; measurement of both types of shocks is conditional on a particular model. For example, in Section 5 the measure of technology shocks that we obtain under increasing returns to scale is different from the measure of technology shocks under the maintained hypothesis that production function is constant returns to scale.

11. In deriving this expression from the first-order necessary conditions for the consumer's problems, we have used the fact that $\theta_L C = wL$.

12. Stationarity is not required at this point, but it will become important later when we wish to compute sample and population moments for these series.

13. To convert the percentage responses of output to commodity units, the percentage responses must be divided by the steady state share of consumption (.6501 in this model).

14. This can be understood as follows. Suppose that capital is below its steady state level. In both models, this implies that the rate of return is above its steady state level. The magnitude of this increase in the rate of return is governed by the elasticity of the marginal product of capital with respect to capital: this elasticity is $\eta\theta_K - 1$, which is $-.58$ with $\eta=1$ and is $-.25$ with $\eta=1.3$. Thus the rate of return is higher in the CR model than in the IR model. It is the interest rate which signals that consumption should be postponed to undertake the net investment necessary to restore the capital stock to its steady level. Hence consumption will be growing faster in the CR model than in the IR model. The effect is quantitatively important in terms of the transitional dynamics of the capital stock.

15. Thus one attractive feature of the IR model is that it does not require productivity shocks that are as large as those needed by the CR model. This should please some critics of the RBC literature who have found the necessary magnitude of the productivity shocks to be unreasonably large.

16. Christiano and Eichenbaum [1992], page 1. These authors also construct a multi-shock model of fluctuations; their model is subject to shocks in technology and to government spending. They also introduce the possibility of measurement error in labor input. The combination of these two features also results in empirically-reasonable values for the correlation between labor input and the returns to labor, although neither feature is sufficient in itself.

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APPENDIX

This Appendix provides detailed information on the data we used in this project. With the exception of capital stock data, all data is from the Citibase database. The time period covered is 1955:1–1990:3. Where appropriate, Citibase mnemonics are given in parentheses.

Output:	Real GNP divided by noninstitutional population including resident armed forces per capita (GNP82/POP)
Consumption:	Consumption of services and nondurables, per capita ((GCS82+GCN82)/POP)
Investment:	Gross private domestic investment, per capita (GIF82/POP)
Labor input:	Average hours of work per week, (household data) multiplied by employment of the civilian labor force, per capita: (LHCH*EMPL/POP)
Compensation:	Compensation per manhour, deflated using the GNP deflator ((GCOMP/GD)/(AWH*EMPL))
Productivity:	Output per manhour (GNP82 ÷ (LHCH*EMPL/POP))
Capital stock:	Our aggregate capital stock measure is producers' durable equipment plus structures. This data was provided by the Board of Governors of the Federal Reserve System.

Table 1

Estimates of Returns to Scale Parameter
Annual U.S. Data, 1953–1985
Total Private Industry*

estimation method:	η	s.e. (η)	D–W	R^2: stage 1	R^2: stage 2
OLS	1.45	.155	1.32	—	.73
IV–total defense purchases –civilian defense comp –military defense comp	1.81	.582	1.18	.08	.68
IV–civilian defense comp –military defense comp	1.53	.562	1.27	.08	.73
IV–total nondefense purchases	1.10	2.13	1.60	.01	.69

* Data on output (value added), manhours, capital and labor compensation taken from a larger data base constructed by Shapiro [1987] for his analysis of sectoral Solow residuals. The growth of total input was calculated by the formula $\log(Z_t/Z_{t-1}) = (1-\theta_N)\log(K_t/K_{t-1}) + \theta_N \log(N_t/N_{t-1})$, where $\theta_N = .54$ is the sample average value of labor's share in total private industry.

** Instrumental variables estimates are constructed using variables from the National Income and Product Accounts, table 3.7B. The basic series (CITIBASE mnemonic) are: Federal National Defense Purchases (GGFEN); Compensation of Defense Employees, Military (GGFNCM); Compensation of Defense Employees, Civilian (GGFNCC); and Federal Nondefense Purchases of Goods and Services (GGFEO). Real purchases were created by deflating by the implicit deflator for gross national product. Continuously compounded growth rates of these real quantities were used in the regressions reported above.

Table 2

Estimate of Stochastic Process
for Preference Shocks

As discussed in the text, the wage rate was measured as compensation per manhour. Given this measure of the real wage, the preference shock time series was computed from equation (13) in the text. Next, we estimated a time series process for the log-level of the shock. The data are sampled at the quarterly interval from 1955:1–1990:3. Coefficient estimates and other statistics are as follows (standard errors are in parentheses):

$$\log(\Delta_t) = \underset{(0.0799)}{0.0976} + \underset{(2.1e-5)}{(3.1e-5)*t} + \underset{(0.0210)}{0.9739}\log(\Delta_{t-1}) + d_t$$

Standard error of regression: 0.0097

R^2: 0.945

Durbin-Watson statistic: 2.22

Table 3

Notation and Parameter Values

A. Preferences

momentary utility function: $u(C,L) = \log(C_t - \Delta_t) + \theta_L \log(L_t)$

θ_L chosen so that L = .8 and N = 1-L = .2

lifetime utility function: $U = \sum_t \beta^t u(C_t, L_t)$

β chosen so that steady state real rate is .065

B. Production Function

production function: $Y_t = [A_t K_t^{\theta_K} N_t^{\theta_N}]^\eta$

θ_N, θ_K chosen to match U.S. factor share data: $\theta_N = .58$, $\theta_K = .42$

η estimated in Table 1

accumulation of private capital: $K_{t+1} - K_t = I_t - \delta_K K_t$

$\delta_K = .10$

Table 4

Selected Moments for the Models and the Data

	standard deviation					relative standard deviation				
		IR model		CR model			IR model		CR model	
	data	sim.	pop.	sim.	pop.	data	sim.	pop.	sim.	pop.
y	1.71	0.75	0.93	0.43	0.54	1.00	1.00	1.00	1.00	1.00
c	0.89	0.68	0.85	0.67	0.83	0.52	0.91	0.91	1.54	1.54
i	5.39	2.07	2.57	0.01	0.01	3.15	2.76	2.75	0.02	0.02
N	1.44	0.99	1.23	0.75	0.93	0.84	1.32	1.32	1.72	1.72
w	0.78	0.25	0.31	0.31	0.39	0.45	0.33	0.33	0.72	0.72
y/N	0.97	0.25	0.31	0.31	0.39	0.57	0.33	0.33	0.72	0.72

	persistence (AR(1) coeff.)					zero-order cross-corr. with y				
		IR model		CR model			IR model		CR model	
	data	sim.	pop.	sim.	pop.	data	sim.	pop.	sim.	pop.
y	0.85	0.67	0.79	0.67	0.78	1.00	1.00	1.00	1.00	1.00
c	0.86	0.68	0.79	0.67	0.78	0.75	1.00	1.00	1.00	1.00
i	0.89	0.67	0.78	0.67	0.78	0.88	1.00	1.00	–0.99	–0.99
N	0.83	0.67	0.78	0.67	0.78	0.83	1.00	1.00	1.00	1.00
w	0.57	0.67	0.78	0.67	0.78	0.31	–0.98	–0.97	–1.00	–1.00
y/N	0.58	0.67	0.78	0.67	0.78	0.54	–0.98	–0.97	–1.00	–1.00
corr(w,N)						–0.05	–0.99	–0.98	–1.00	–1.00
corr(y/N,N)						–0.04	–0.99	–0.98	–1.00	–1.00

continued...

Table 4, cont'd.

correlation between model-generated time series and U.S. time series

	y	c	i	N	w	y/N
IR	0.63	0.71	0.65	0.77	0.37	0.04
CR	0.62	0.71	−0.64	0.78	0.33	0.06

Notes:

1. All data and model output have been Hodrick-Prescott filtered.
2. "sim" refers to moments computed for the stochastic simulations; "pop" refers to population moments.
3. See the Appendix for details of the dataset employed.

Table 5

Estimate of Stochastic Process
for Productivity Shocks

The productivity variable was measured using equations (14a,b). Results are reported for two specifications: (i) an AR(1) in the log-levels of productivity, and (ii) imposition of a unit root in the log of productivity.

(i) $A_t = \beta_0 + \beta_1 A_{t-1} + u_t$

Coefficient estimates and other statistics are as follows (standard errors are in parentheses):

	β_0	β_1	se(u)	R-sq.	D-W
IR model:	-0.0056 (0.0722)	0.9994 (0.0088)	.0065	.989	2.01
CR model:	-0.1420 (0.1452)	0.9844 (0.0159)	.0083	.964	1.84

(ii) $A_t - A_{t-1} = \beta_0 + u_t$

Coefficient estimates and other statistics are as follows (standard errors are in parentheses):

	β_0	se(u)	R-sq.	D-W
IR model:	-0.0011 (0.0005)	.0065	.000	2.01
CR model:	-0.0003 (0.0007)	.0083	.000	1.86

Table 6

Comparing IR and CR models with
Preference Shocks and Productivity Shocks

	standard deviation					relative standard deviation				
		IR model		CR model			IR model		CR model	
	data	both shocks	prod only	both shocks	prod only	data	both shocks	prod only	both shocks	prod only
y	1.71	1.66	1.38	1.44	1.33	1.00	1.00	1.00	1.00	1.00
c	0.89	1.25	0.92	1.45	1.19	0.52	0.75	0.67	1.01	0.89
i	5.39	5.85	5.26	3.84	3.84	3.15	3.52	3.82	2.68	2.88
N	1.44	1.24	0.15	0.94	0.17	0.84	0.75	0.11	0.66	0.13
w	0.78	1.28	1.24	1.28	1.22	0.45	0.77	0.90	0.89	0.91
y/N	0.97	1.28	1.24	1.28	1.22	0.57	0.77	0.90	0.89	0.91

	persistence (AR(1) coeff.)					zero-order cross-corr. with y				
		IR model		CR model			IR model		CR model	
	data	both shocks	prod only	both shocks	prod only	data	both shocks	prod only	both shocks	prod only
y	0.85	0.79	0.79	0.79	0.79	1.00	1.00	1.00	1.00	1.00
c	0.86	0.80	0.81	0.80	0.81	0.75	0.98	0.99	0.99	0.99
i	0.89	0.79	0.79	0.78	0.78	0.88	0.99	1.00	0.91	0.99
N	0.83	0.78	0.79	0.79	0.83	0.83	0.65	0.90	0.49	0.72
w	0.57	0.80	0.80	0.80	0.81	0.31	0.67	1.00	0.76	1.00
y/N	0.58	0.80	0.80	0.80	0.81	0.54	0.67	1.00	0.76	1.00
corr(w,N)						–0.05	–0.13	0.87	–0.19	0.65
corr(y/N,N)						–0.04	–0.13	0.87	–0.19	0.65

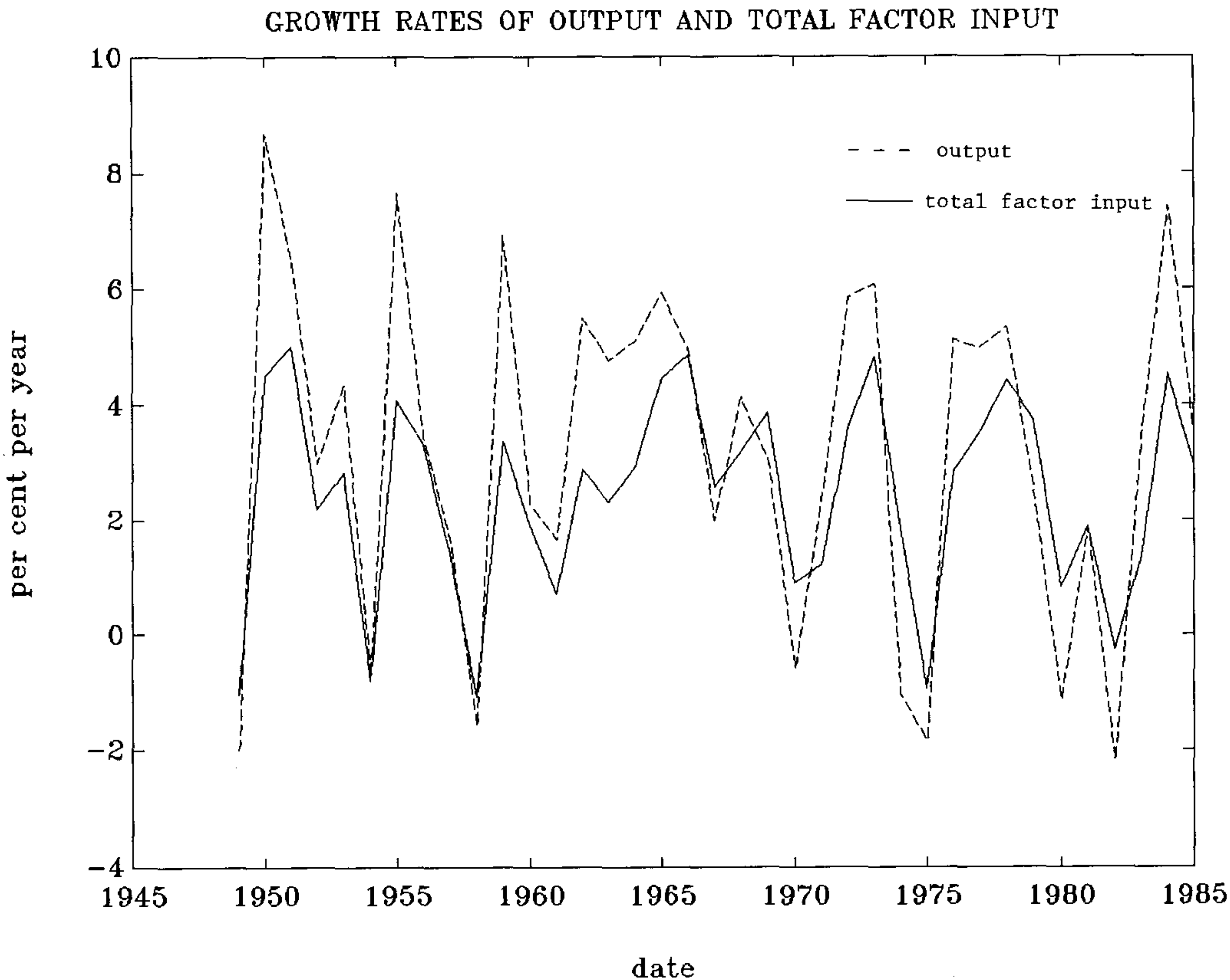
GROWTH RATES OF OUTPUT AND TOTAL FACTOR INPUT
output
total factor input
per cent per year
date
10
8
6
4
2
0
−2
−4
1945
1950
1955
1960
1965
1970
1975
1980
1985

FIGURE 1

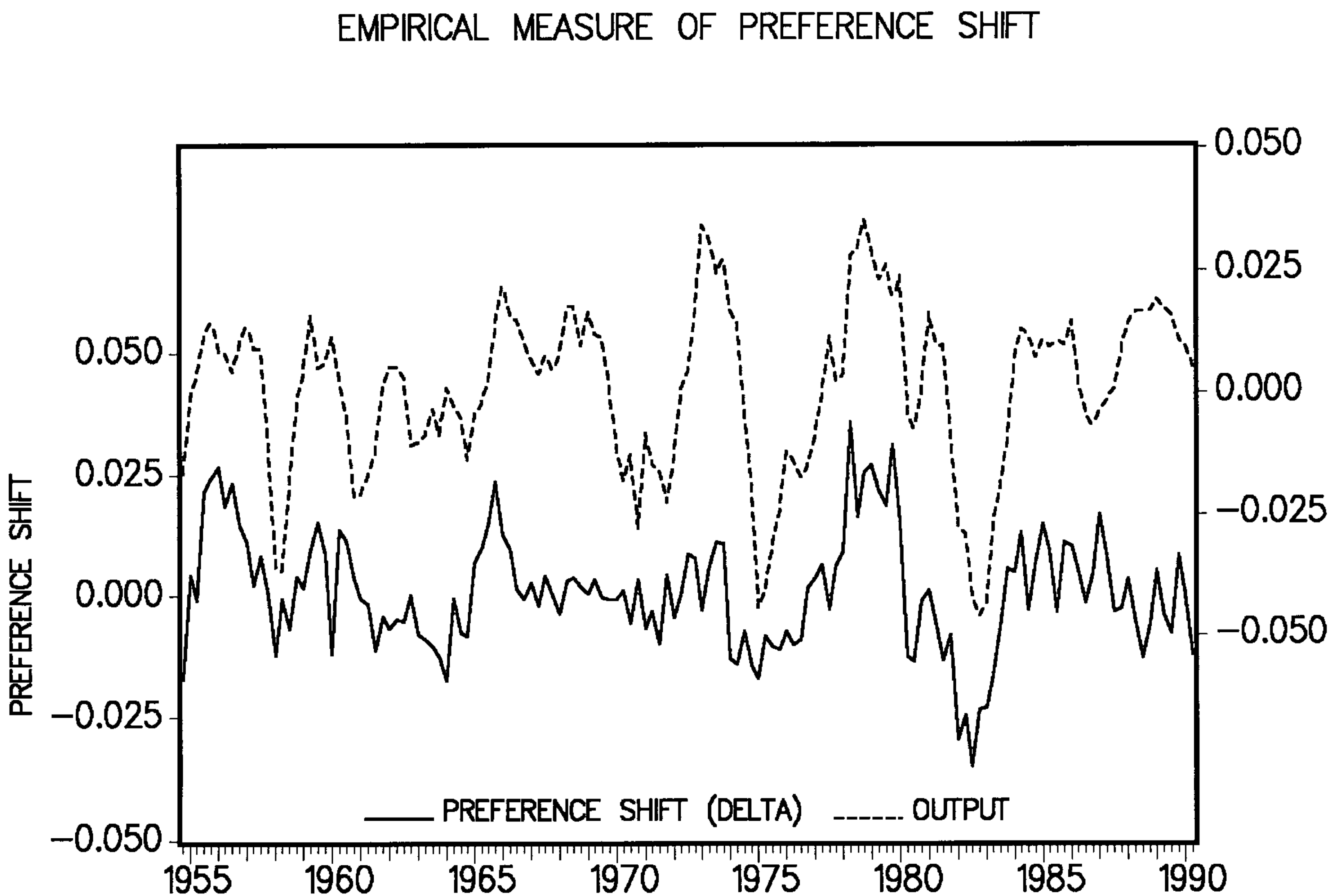
EMPIRICAL MEASURE OF PREFERENCE SHIFT
PREFERENCE SHIFT
OUTPUT
0.050
0.025
0.000
−0.025
−0.050
1955
1960
1965
1970
1975
1980
1985
1990
PREFERENCE SHIFT (DELTA)
OUTPUT

FIGURE 2

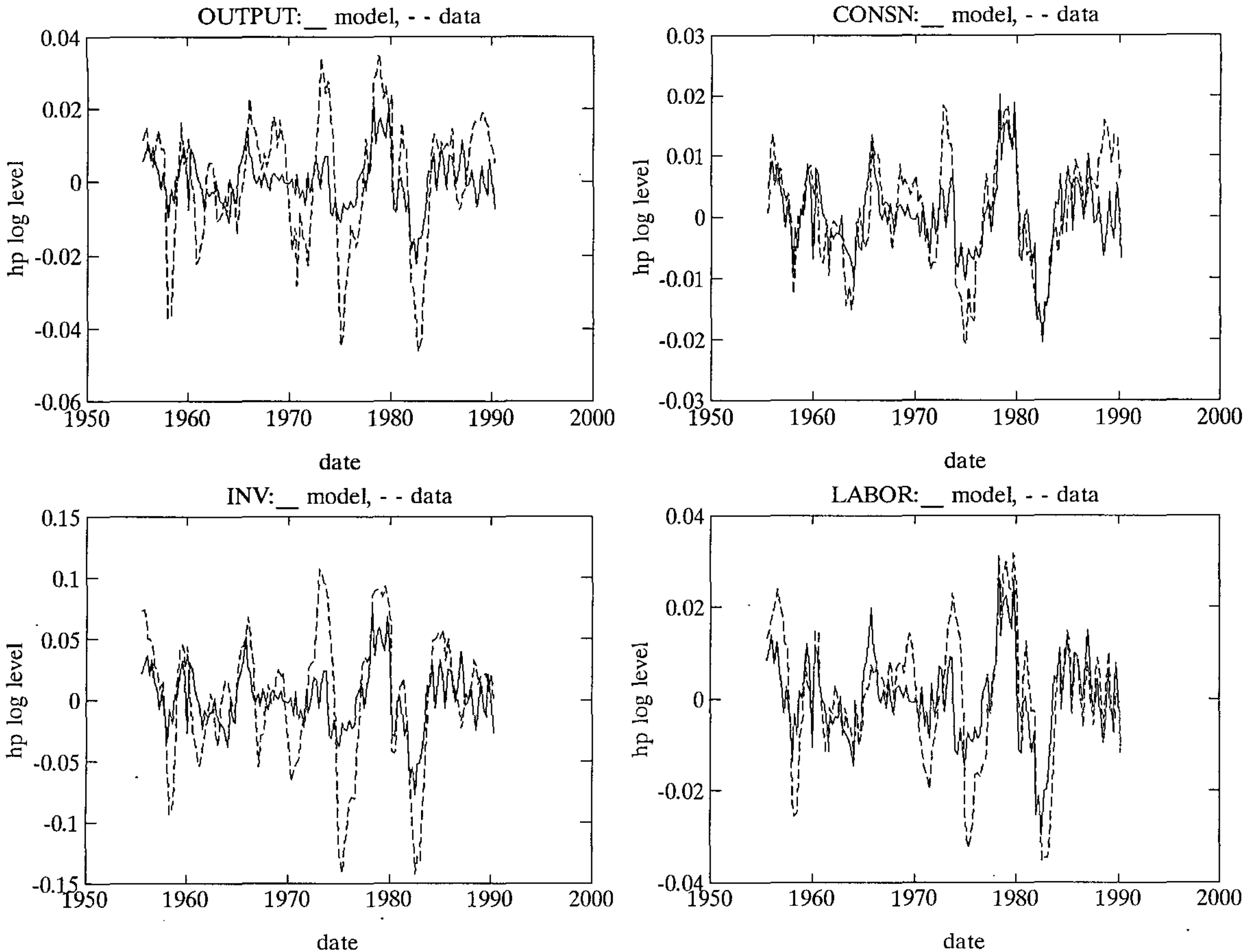
OUTPUT:__ model, - - data
CONSN:__ model, - - data
INV:__ model, - - data
LABOR:__ model, - - data
hp log level
date
0.04
0.02
0
-0.02
-0.04
-0.06
0.03
0.01
-0.01
-0.03
0.15
0.1
0.05
-0.05
-0.1
-0.15
1950
1960
1970
1980
1990
2000

FIGURE 3

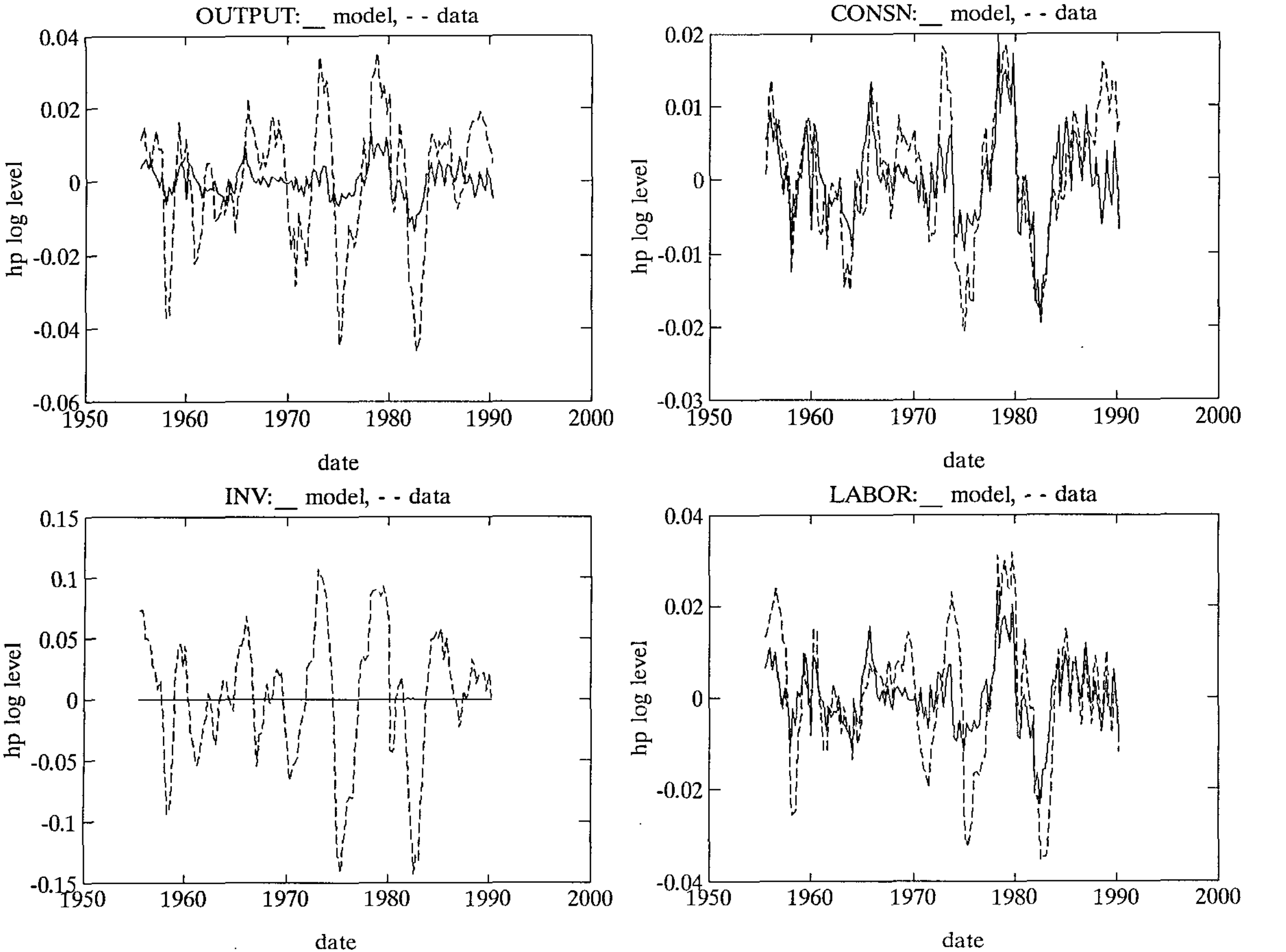
OUTPUT:__ model, - - data
CONSN:__ model, - - data
INV:__ model, - - data
LABOR:__ model, - - data
hp log level
date
0.04
0.02
0
-0.02
-0.04
-0.06
0.01
-0.01
-0.03
0.15
0.1
0.05
-0.05
-0.1
-0.15
1950
1960
1970
1980
1990
2000

FIGURE 4

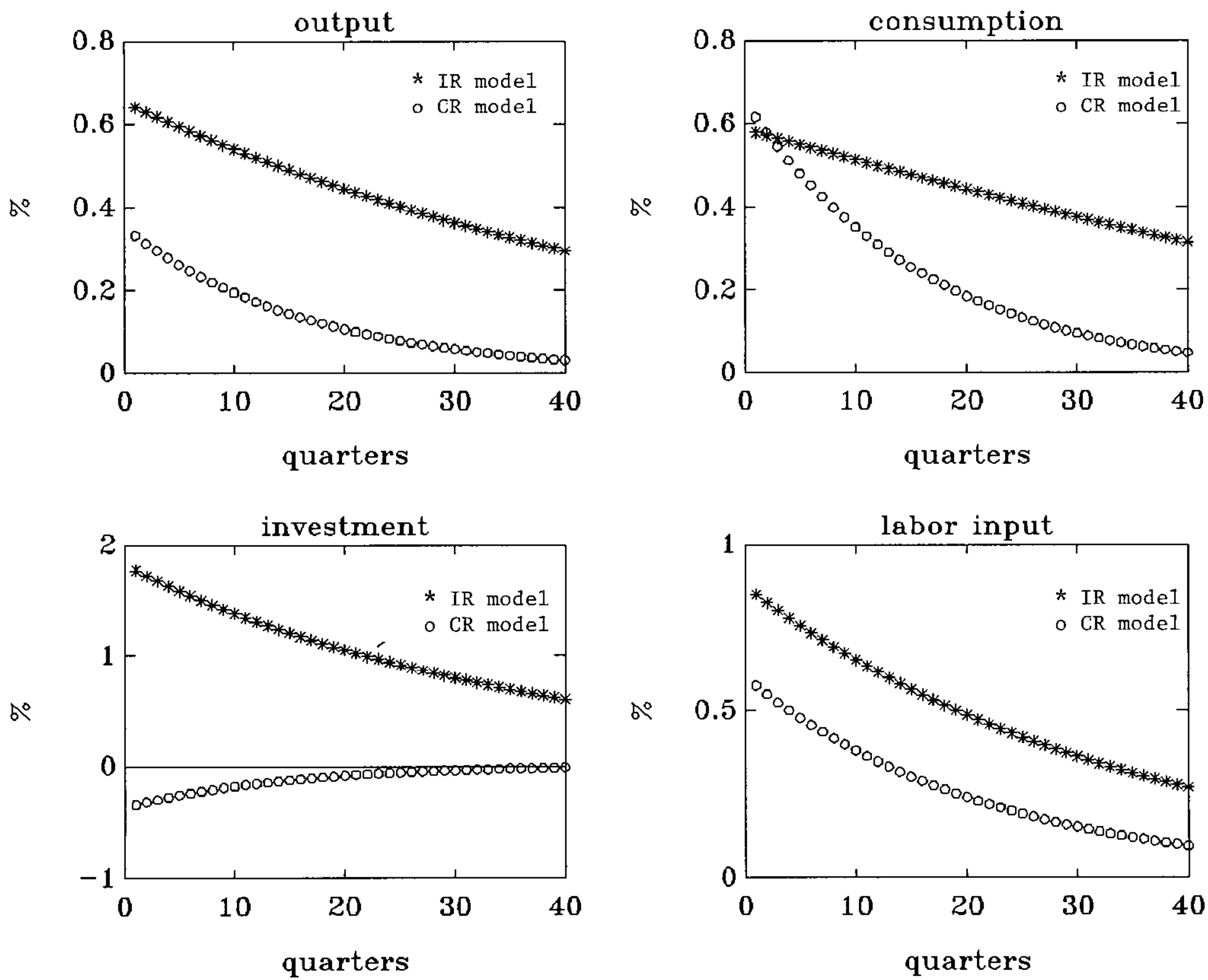

Figure 5-A: Impulse responses to preference shock -- quantity variables

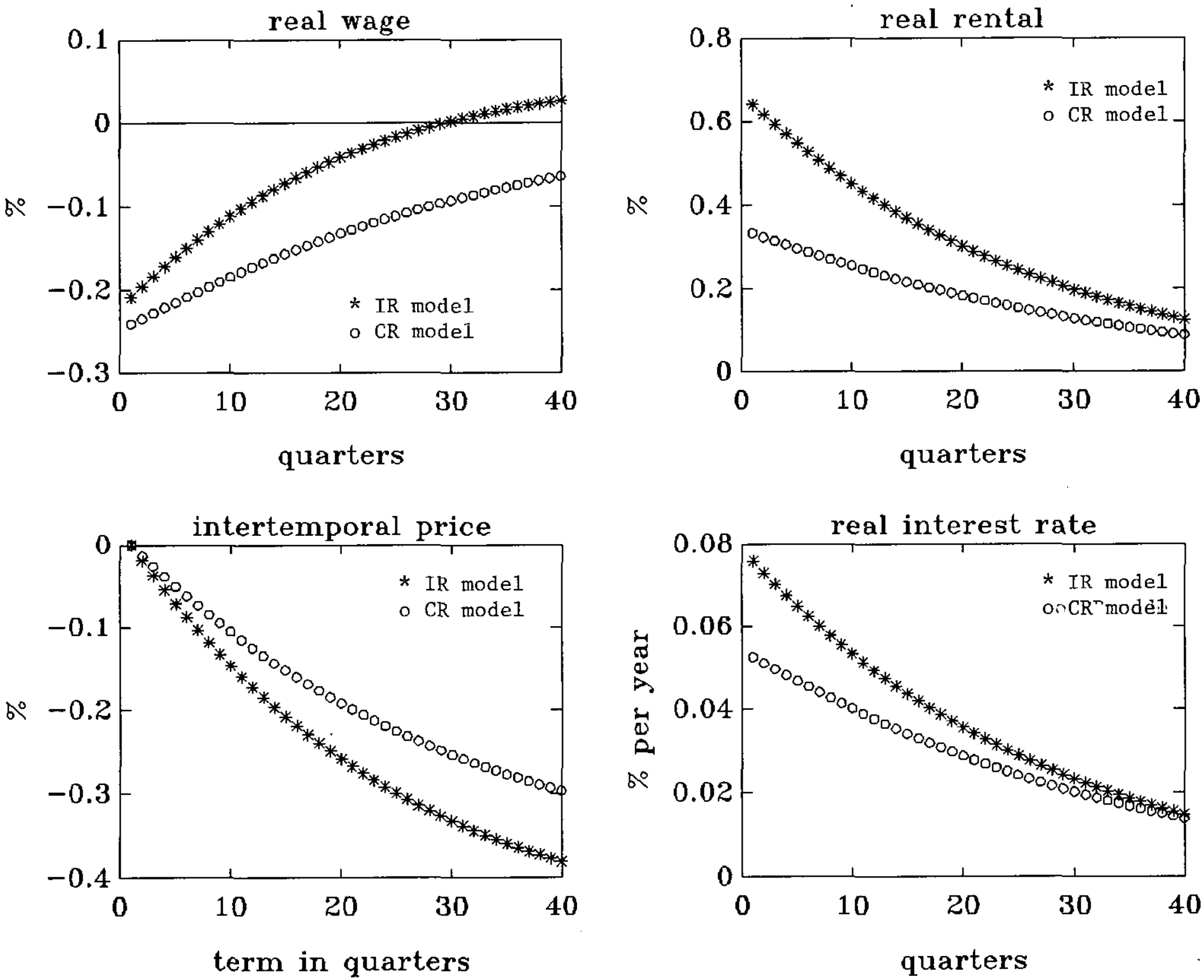

Figure 5-B: Impulse responses to preference shock -- Prices and interest rates