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MONEY AND GROWTH REVISITED

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ABSTRACT

Results in Lucas (1987) suggest that if public policy can affect the growth rate of the economy, the welfare implications of alternative policies will be large. In this paper, a stochastic, dynamic general equilibrium model with endogenous growth and money is examined. In this setting, inflation lowers growth through its effect on the return to work. However, the welfare costs of higher inflation are modest.

Keywords: inflation, endogenous growth, monetary policy, quantitative theory, real business cycle theory

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1. Introduction

A widely held belief in economics is that if public policy can affect the economy's underlying growth rate, then alternative public policies will have large welfare implications. For example, Lucas (1987) estimates that consumers would be willing to part with up to 17% of their consumption (forever) to raise its growth rate from 3% to 4% per annum. King and Rebelo (1990) have used a real business cycle model with endogenous growth to analyze the effects of changes in the income tax rate. Raising this tax rate from 20% to 30%, roughly from the average Japanese tax rate from 1965 to 1975 to the average U.S. tax rate over the same period, results in a welfare loss in excess of 60% of consumption. Almost all of this welfare cost can be traced to the effects of the tax on growth. These numbers are large when compared with estimates of the losses arising from business cycle fluctuations. For example, Lucas (1987) calculates the gains from eliminating the cyclical variability in consumption to amount to no more than 0.1% of consumption, while Greenwood and Huffman (1991) place the potential benefits of business cycle stabilization at 0.5% of GNP. The question asked in this paper is whether large welfare costs result from higher rates of inflation. Below, an endogenous growth model is presented in which higher long run inflation lowers growth, yet the welfare costs of inflation are very small.

International time series data provides some insight into the relationship, if any, between inflation and real growth. In Table 1, 62 of 82 countries exhibit a negative correlation between inflation and per capita real output growth.[1] This evidence is consistent with results in Backus and Kehoe (1989) showing that inflation is counter-

[1] Data for Table 1 was obtained from the International Financial Statistics (IFS) tape. Countries were included in Table 1 according to availability of the following data: real output, nominal output, the consumer price index, and population. Some countries were dropped due to very short time series. Grenada was removed due to a recorded population of zero; this is a limitation of the IFS data available on tape.

cyclical in the post-World War II period for the ten countries they consider.[2] While correlations do not imply causality, theories of inflation and real growth must at some point address the predominantly negative correlation seen in the international data.

There is a large literature incorporating money into the neoclassical growth model. In Tobin (1965), money competes with capital for a place in the portfolios of households. One prediction from Tobin's model is that money growth and capital are *positively* correlated. Sidrauski (1967), using a model with money in the utility function, develops long run superneutrality results. Stockman (1981) presents a model in which money growth and capital are *negatively* related when a cash-in-advance constraint applies to both consumption and investment. Money is superneutral in Stockman (1981) when consumption alone is subject to the cash-in-advance constraint.

In real business cycle models, as advanced by Kydland and Prescott (1982) and Long and Plosser (1983), money typically plays no role. An exception is Cooley and Hansen (1989a) who introduce money through a cash-in-advance constraint on consumption in an effort to assess the welfare costs of inflation. Higher inflation has the effects typically associated with a cash-in-advance constraint—see, for example, Aschaeur and Greenwood (1983) and Carmichael (1989). Specifically, higher inflation reduces the effective return to working since income earned in the current period cannot be spent until the next. This leads households to substitute leisure for labour, consequently reducing output and consumption.[3]

In steady state, Cooley and Hansen (1989a) report that a 10% inflation rate

[2] Backus and Kehoe (1989) examine business cycle behaviour of ten countries for which at least a century of data is available. The primary focus of their paper is on moments from Hodrick-Prescott filtered data, although results for growth rate filtering are included. Inflation is predominantly countercyclical in the pre-World War I and Interwar periods.

[3] These effects subsume the taxation effect of inflation emphasized by, for example, Stockman (1981).

results in a welfare cost of about 0.4% of income relative to an optimal monetary policy. This is somewhat smaller than the 0.8% and 0.5% figures calculated by Fischer (1981) and Lucas (1981), respectively, using the "traditional" welfare triangle analysis associated with Bailey (1956).[4] Imrohoroglu (1990), using a model in which optimizing households hold money to insure against unemployment, suggests that welfare triangles may underestimate the true costs of inflation by a factor of three or more. Along the steady state, balanced growth path of the endogenous growth model analyzed below, a welfare cost of less than 0.03% of income results from a 10% money growth rate (8.7% inflation rate)—an order of magnitude smaller than Cooley and Hansen (1989a)!

Growth theory typically assumes that long run growth occurs at some exogenous rate. For many issues, this supposition is likely innocuous. However, when considering public policy questions this may be a poor assumption, as King and Rebelo (1990) have shown in the context of income taxation. While Howitt (1990) considers a model in which money can affect the economy's long run growth rate, he does not quantify this effect, nor the implications for welfare.

In the model developed here, endogenous growth arises through human capital accumulation as suggested by Lucas (1988). Rebelo (1990) has examined some of the theoretical properties of such models, and King and Rebelo (1990) have used such a model to analyze the welfare effects of income taxation. There are two productive activities in the model: market or physical output production, and new human capital production. While each activity is constant-returns-to-scale in physical capital and human capital-augmented labour effort, there are increasing-returns-to-scale at the economy level to the three inputs, physical capital, labour effort and human capital.

[4] The experiment considered by Fischer (1981) and Lucas (1981) is to lower the inflation rate from 10% to 0%.

It is in this way that perpetual growth is feasible.

Money enters the model via a cash-in-advance constraint. As in Cooley and Hansen (1989a), higher money growth-cum-inflation reduces the return to working. However, here there are two channels of effect since there are two productive activities. In equilibrium, the wage rate must be equalized across the two sectors since labour is freely mobile . As a result, not only does market output fall, but so does human capital production. It is this latter avenue through which inflation affects long run growth in this economy.

Since households fundamentally care only about the paths of consumption and leisure, it is sufficient to consider what happens with these variables to understand the welfare results presented below. Through its effect on the path of consumption, households also care about the real growth rate of the economy. Higher money growth lowers the real growth rate, making households worse off. There is, however, an offsetting effect: lower growth means that less output needs to be devoted to capital accumulation (to maintain the new, lower real growth rate) and more can be allocated to consumption. In the steady state, balanced growth calculations below, consumption relative to human capital falls only slightly for moderate money growth rates.

As mentioned above, labour effort in both the market sector and human capital production activity fall in the face of higher money growth. This serves to make leisure *more* responsive to changes in the money growth rate. This also helps to ameliorate the negative effects which lower real growth has on household welfare. Rather than increasing the costs of inflation, endogenous growth, through its effects on consumption and leisure, serves to reduce the welfare costs of moderate inflations.

The remainder of the paper is organized as follows. In Section 2, the physical environment is presented, household and firm problems cast, competitive equilibrium defined, and the balanced growth path transformation performed. The model is parameterized, calibrated and simulated in Section 3. Welfare results are found in Section 4 for both the steady state, balanced growth path and the stochastic version of the model. Section 5 concludes.

2. The Model

2.1 The Economic Environment

The representative household maximizes the expected value of a discounted stream of utility given by:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t), \quad 0 < \beta < 1 \tag{2.1}$$

where: c_t is consumption at date t, and ℓ_t is leisure in period t. The household's time endowment is normalized to one so that ℓ_t is the fraction of time the household allocates to leisure. In addition to the usual properties, it is assumed that the utility function can be written as $U(c, \ell) = u(c)v(\ell)$ where $u(c)$ is homogeneous of degree $1 - \sigma$. This assumption is essentially the same as that made in King, Plosser and Rebelo (1988), and similar to that in Greenwood and Hercowitz (1991). As in King, Plosser and Rebelo, such a specification for utility simultaneously allows for (positive) growth of consumption and zero growth in leisure (the household's time endowment is fixed at unity).

The timing of transactions within a period proceeds as follows. The typical household enters period t with physical capital, k_t, human capital, h_t, and nominal cash balances, m_t. At the start of the period, the state-of-the-world is revealed; in particular, the current period market sector productivity shock, z_t, and gross per capita growth rate of money, g_t, are revealed. The government makes a transfer to the household, v_t, in the form of nominal balances. Taking as given the rental price of physical capital, r_t, and the wage rate paid on human capital-augmented labour effort, w_t, the household chooses ϕ_t, the fraction of physical capital allocated to the market sector, and n_t, the fraction of time devoted to the market sector. Time and physical capital not allocated to the market are used to produce new human capital as described below.

The representative household finances its purchases of the consumption good through beginning-of-period cash balances which are the sum of balances from from the previous period, m_t, and transfers from government, v_t. That is, the typical household faces a cash-in-advance constraint of the form,

$$p_t c_t \leq m_t + v_t \tag{2.2}$$

where p_t is the price level in period t. Investment can be thought of as a credit good while consumption is a cash good.

At the end of the period, the household receives from firms factor payments for capital and labour. These payments, in nominal terms, are $p_t r_t \phi_t k_t$ and $p_t w_t n_t h_t$, respectively. Along with any unspent cash balances, the household allocates its earnings between purchases of the physical investment good, $p_t i_t$ in nominal terms, and the accumulation of nominal cash balances to take into next period, m_{t+1}. The household's budget constraint can now be written as:

$$c_t + i_t + \frac{m_{t+1}}{p_t} \leq r_t \phi_t k_t + w_t n_t h_t + \frac{m_t + v_t}{p_t} \tag{2.3}$$

A quantity of physical capital, $(1 - \phi_t)k_t$, and human capital-augmented labour effort, $(1 - \ell_t - n_t)h_t$, were not allocated to the market sector and are used instead to produce new human capital. The evolution equation for human capital is given by:

$$h_{t+1} = F^h\Big[(1 - \phi_t)k_t, (1 - n_t - \ell_t)h_t\Big] + (1 - \delta_h)h_t \tag{2.4}$$

where $F^h(\cdot)$ is homogeneous of degree one in physical capital and human capital-augmented labour effort, and δ_h is the depreciation rate of human capital. Notice that an allocation of time to market or human capital production implies an allocation of human capital to these activities as well.

A number of institutional arrangements can support the real allocations analyzed below. Here, it is convenient to think of human capital accumulation as a "household" activity. Alternatively, human capital could be produced in an "education" sector with a price attached to human capital, as in King and Rebelo (1990). Here, the price of human capital is a shadow price.

The law of motion for physical capital is:

$$k_{t+1} = (1 - \delta_k)k_t + i_t \tag{2.5}$$

where δ_k is the depreciation rate of physical capital.

Firms have access to a constant-returns-to-scale production function which produces output, y_t, according to:

$$y_t = F^m(\phi_t k_t, n_t h_t; z_t) \tag{2.6}$$

where z_t is a productivity shock, assumed to evolve as:

$$z_t = \rho z_{t-1} + \epsilon_t \tag{2.7}$$

Output can be divided between consumption and physical investment,

$$y_t = c_t + i_t \tag{2.8}$$

Finally, government's actions are taken to be exogenous. Government finances its transfer to households through the creation of money, facing the per capita budget constraint,

$$v_t = (g_t - 1)m_t \tag{2.9}$$

where the gross growth rate of money, g_t, evolves according to:

$$\ell n g_t = \psi \ell n g_{t-1} + (1-\psi)\ell n \overline{g} + \xi_t \tag{2.10}$$

where $\overline{g}$ is the long run, average rate of money growth and ξ_t is a random shock.

2.2 Competitive Equilibrium

Denote the state by $s = (k, h, m, z, g)$ where time subscripts have been dropped in the usual fashion. Suppose that prices and the government transfer can be written as functions of the state; viz, $p = P(s)$, $r = R(s)$, $w = W(s)$ and $v = \Upsilon(s)$. Further suppose that the laws of motion for k, h and m are described by $K(s)$, $H(s)$ and $M(s)$, respectively. Write the law of motion for the productivity shock as $z' = Z(s,\epsilon) \equiv \rho z + \epsilon'$, and for money growth as $g' = G(s,\xi') \equiv \exp\{\psi g + (1-\psi)\overline{g} + \xi'\}$. Now, s evolves according to $s' = S(s,\epsilon',\xi'; K, H, M) \equiv S\Big(K(s), H(s), M(s), Z(s,\epsilon'), G(s,\xi')\Big)$.

The problem faced by the representative household is to choose consumption, $\tilde{c}$, an allocation of time to leisure and market activity, $\tilde{\ell}$ and $\tilde{n}$, stocks of physical capital, human capital and cash balances, $\tilde{k}'$, $\tilde{h}'$ and $\tilde{m}'$, and a division of physical capital between the market sector and human capital production, $\tilde{\phi}$, which solve the following dynamic programming problem:

$$V(\tilde{k}, \tilde{h}, \tilde{m}; s) = \max_{\tilde{c}, \tilde{\ell}, \tilde{n}, \tilde{\phi}, \tilde{k}', \tilde{h}', \tilde{m}'} \left\{ U(\tilde{c}, \tilde{\ell}) + \beta \mathrm{E} V(\tilde{k}', \tilde{h}', \tilde{m}'; s') \right\} \tag{P1}$$

subject to

$$\tilde{c} + \tilde{k}' + \frac{\tilde{m}'}{P(s)} \leq R(s)\tilde{\phi}\tilde{k} + W(s)\tilde{n}\tilde{h} + (1-\delta_k)\tilde{k} + \frac{\tilde{m} + \Upsilon(s)}{P(s)} \tag{2.11}$$

$$P(s)\tilde{c} \leq \tilde{m} + \Upsilon(s) \tag{2.12}$$

$$\tilde{h}' = F^h\Big[(1-\tilde{\phi})\tilde{k}, (1-\tilde{n}-\tilde{\ell})\tilde{h}\Big] + (1-\delta_h)\tilde{h} \tag{2.13}$$

and

$$s' = S(s, \epsilon', \xi'; K, H, M) \tag{2.14}$$

The problem of a typical firm is to maximize period profits through its choice of $\check{\phi}\check{k}$ and $\check{n}\check{h}$:

$$\max_{\check{\phi}\check{k},\, \check{n}\check{h}} \left\{F^m(\check{\phi}\check{k}, \check{n}\check{h}; z) - R(s)\check{\phi}\check{k} - W(s)\check{n}\check{h}\right\} \tag{P2}$$

Since $F^k(\cdot)$ is constant-returns-to-scale, in equilibrium zero profits are earned and it is not necessary to account for distributed profit income in the household's problem.

Definition: A competitive equilibrium consists of policy functions, $c = C(s)$, $\ell = L(s)$, $n = N(s)$, $\phi = \Phi(s)$, $h' = H(s)$, $k' = K(s)$ and $m' = M(s)$, pricing functions $p = P(s)$, $r = R(s)$ and $w = w(S)$, and a transfer function $v = \Upsilon(s)$ such that:

(i) Households solve (P1) taking as given the state-of-the-world, $s = (k, h, m, z, g)$ and the functions $R(s)$, $W(s)$, $P(s)$, $K(s)$, $H(s)$, $M(s)$ and $\Upsilon(s)$, with the solution to this problem being $\tilde{c} = C(s)$, $\tilde{\phi} = \Phi(s)$, $\tilde{\ell} = L(s)$, $\tilde{n} = N(s)$, $\tilde{k}' = K(s)$, $\tilde{h}' = H(s)$, and $\tilde{m}' = M(s)$.

(ii) Firms solve (P2), given s and the functions $R(s)$ and $W(s)$, with the solution having the form $\check{\phi}\check{k} = \Phi(s)k$ and $\check{n}\check{h} = N(s)h$.

(iii) Goods and money markets clear:

$$c + k' = F^m(\phi k, nh; z) + (1 - \delta_k)k \tag{2.15}$$

and

$$m' = m + v \tag{2.16}$$

Assuming that the household's constraints hold with equality (the budget constraint will hold with equality due to non-satiation while the cash-in-advance constraint will hold with equality for sufficiently rapid money growth), the definition of a competitive equilibrium implies that the allocation rules for c, ϕ, ℓ, n, k', h', m' and the pricing function p are implicitly defined by the market clearing conditions, (2.15) and (2.16), and the following:

$$\frac{U_2(c,\ell)}{hF_2^m(\phi k, nh; z)} = \beta \mathrm{E}\left\{\frac{U_1(c',\ell')}{p'/p}\right\} \tag{2.17}$$

$$\frac{U_2(c,\ell)}{hF_2^m(\phi k, nh; z)} = \beta \mathrm{E}\left\{\frac{U_2(c',\ell')}{h'F_2^m(\phi' k', n'h'; z)}\Big[F_1^m(\phi' k', n'h'; z) + 1 - \delta_k\Big]\right\} \tag{2.18}$$

$$\begin{aligned}\frac{F_2^m(\phi k, nh; z)}{F_2^h\Big[(1-\phi)k,(1-n-\ell)h\Big]} \times \frac{U_2(c,\ell)}{hF_2^m(\phi k, nh; z)} = \\ \beta\mathrm{E}\Bigg\{\frac{U_2(c',\ell')}{h'F_2^m(\phi' k', n'h'; z)}\Bigg[(1-\ell')F_2^m(\phi' k', n'h'; z) \\ + \frac{F_2^m(\phi' k', n'h'; z)}{F_2^h\Big[(1-\phi')k',(1-n'-\ell')h'\Big]}(1-\delta_h)\Bigg]\Bigg\}\end{aligned} \tag{2.19}$$

$$\frac{F_1^m(\phi k, nh; z)}{F_2^m(\phi k, nh; z)} = \frac{F_1^h\Big[(1-\phi)k,(1-n-\ell)h\Big]}{F_2^h\Big[(1-\phi)k,(1-n-\ell)h\Big]} \tag{2.20}$$

$$h' = F^h\Big[(1-\phi)k,(1-n-\ell)h\Big] + (1-\delta_h)h \tag{2.21}$$

$$pc = m + v \tag{2.22}$$

Equation (2.17) illustrates how money distorts decisions in this environment. Under an optimal monetary policy (in which case the cash-in-advance constraint does not bind), (2.17) would look, instead, like:

$$U_2(c,\ell) = hF_2^m(\phi k, nh; z)U_1(c,\ell) \tag{2.23}$$

In (2.23), as in (2.17), the marginal utility of leisure is equated to the marginal return to working, evaluated in terms of utility. However, the cash-in-advance constraint introduces a wedge of inefficiency in (2.17) since money earned in the current period cannot be spent until the next. Consequently, the left-hand side of (2.17) represents the utility cost of accumulating the last unit of nominal cash balances while the right-hand side gives the return, evaluated in terms of current-period utility. The gross inflation rate, p'/p, is the return earned on money. Thus, even if perfectly anticipated, inflation erodes the value of cash balances and so affects real variables in the model economy. This last effect is the taxation aspect of inflation emphasized by Stockman (1981).

Equation (2.18) governs the accumulation of physical capital.[5] The term in square brackets on the right-hand side is the return, in consumption units, earned by holding the last unit of capital acquired for one period. Since capital is mobile within a period, the rental price of capital in market and human capital production must be the same in equilibrium. In terms of current period utility gain, this return must just equal the cost of acquiring that last unit of physical capital, which is given by the left-hand side of (2.18).

Human capital accumulation is governed by (2.19). Since labour can costlessly and instantaneously be switched between the market sector and production of human capital, it follows that the return earned by labour must be equalized across the two sectors. Since $(1-\ell)$ is the fraction of time allocated to working in a period, the term in square brackets in (2.19) is the return, in consumption units, to the last unit of human capital accumulated. Notice that $F_2^m(\cdot)/F_2^h(\cdot)$ is the shadow price of human capital (in units of consumption). On the margin, the last unit of human capital acquired must generate a benefit which just equals its cost, given by the left-hand side of (2.19).

(2.20) is an efficiency condition which arises since both labour effort and physical capital are freely mobile across sectors within a period. Equations (2.20) and (2.21) can be thought of as determining the allocation of physical capital and non-leisure time between the market sector and human capital production.

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[5] Lucas (1990) provides an alternative method to interpret accumulation equations like (2.18) and (2.19).

Finally, (2.22) reproduces the cash-in-advance constraint.

2.3 Balanced Growth

To facilitate the use of computational techniques, it is convenient to consider the balanced growth path for the economy. Recalling that $U(c,\ell) = u(c)v(\ell)$ where $u(c)$ is homogeneous of degree $1-\sigma$, it follows that $U_1(c,\ell)$ is homogeneous of degree $-\sigma$ in c while $U_2(c,\ell)$ is homogeneous of degree $1-\sigma$ in c. Since $F^m(\cdot)$ and $F^h(\cdot)$ are each homogeneous of degree one in their two arguments, their partial derivatives are homogeneous of degree zero. Consequently, from the system of equations implicitly defining the allocation functions and pricing function, (2.15)–(2.22):

(a) the allocation functions $C(s)$, $L(s)$, $N(s)$, $\Phi(s)$, $K(s)$ and $H(s)$ are each homogeneous of degree one in (k,h) and homogeneous of degree zero in m;

(b) the function governing money accumulation, $M(s)$, is homogeneous of degree zero in k and h, and homogeneous of degree one in m; and

(c) the pricing function, $P(s)$, is homogeneous of degree zero in k, homogeneous of degree -1 in h, and homogeneous of degree one in m.

Now, define $\hat{c} = c/h$, $\hat{k} = k/h$, $\hat{p} = ph/m$ and $\hat{s} = (\hat{k},1,1;z,g)$. Then the functions $\hat{c} = C(\hat{s})$, $\ell = L(\hat{s})$, $n = N(\hat{s})$, $\phi = \Phi(\hat{s})$, $\hat{k}' = K(\hat{s})$, $h'/h = H(\hat{s})$, $\hat{m}' = M(\hat{s})$ and $\hat{p} = P(\hat{s})$ are implicitly defined by:

$$\hat{c} + \left(\frac{h'}{h}\right)\hat{k}' = F^m(\phi\hat{k},n;z) + (1-\delta_k)\hat{k} \tag{2.24}$$

$$\hat{m}' = g \tag{2.25}$$

$$\frac{gU_2(\hat{c},\ell)}{\hat{p}F_2^m(\phi\hat{k},n;z)} = \beta\left(\frac{h'}{h}\right)^{1-\sigma}\mathrm{E}\left\{\frac{U_1(\hat{c}',\ell')}{\hat{p}'}\right\} \tag{2.26}$$

$$\frac{U_2(\hat{c},\ell)}{F_2^m(\phi\hat{k},n;z)} = \beta\left(\frac{h'}{h}\right)^{-\sigma}\mathrm{E}\left\{\frac{U_2(\hat{c}',\ell')}{F_2^m(\phi'\hat{k}',n';z)}\left[F_1^m(\phi'\hat{k}',n';z) + 1 - \delta_k\right]\right\} \tag{2.27}$$

$$\frac{U_2(\hat{c},\ell)}{F_2^h\Big[(1-\phi)\hat{k},1-n-\ell\Big]} = \beta\left(\frac{h'}{h}\right)^{-\sigma} \mathrm{E}\left\{U_2(\hat{c}',\ell')\left[1-\ell'+\frac{1-\delta_h}{F_2^h\Big[(1-\phi')\hat{k}',1-n'-\ell'\Big]}\right]\right\} \tag{2.28}$$

$$\frac{F_1^m(\phi\hat{k},n;z)}{F_2^m(\phi\hat{k},n;z)} = \frac{F_1^h\Big[(1-\phi)\hat{k},1-n-\ell\Big]}{F_2^h\Big[(1-\phi)\hat{k},1-n-\ell\Big]} \tag{2.29}$$

$$\hat{p}\hat{c} = g \tag{2.30}$$

$$\frac{h'}{h} = F^h\Big[(1-\phi)\hat{k},1-n-\ell\Big] + (1-\delta_h) \tag{2.31}$$

3. Model Parameterization and Calibration

There are two tasks undertaken in this section. The first is to provide specific forms for the utility and production functions used and assign values to the various parameters in the model. The second is to compare the model against the U.S. economy.

3.1 Model Parameterization

The period utility function is parameterized as:

$$U(c,\ell) = \frac{[c^{\omega}\ell^{1-\omega}]^{1-\gamma}}{1-\gamma}, \quad 0<\gamma<1,\ \gamma>1 \tag{3.1}$$

The production functions are specified as:

$$F^m(\phi k, nh; z) = A_m e^z (\phi k)^{\alpha}(nh)^{1-\alpha} \tag{3.2}$$

and

$$F^h\Big[(1-\phi)k,(1-n-\ell)h\Big] = A_h\Big[(1-\phi)k\Big]^{\theta}\Big[(1-\ell-n)h\Big]^{1-\theta} \tag{3.3}$$

Innovations to the market productivity shock, ϵ_t, are assumed to lie in a two point set,

$$\epsilon_t \in \{-\varphi, \varphi\} \tag{3.4}$$

These innovations are assumed to be equally likely:

$$\text{prob}\,\{\epsilon_t = -\varphi\} = \text{prob}\,\{\epsilon_t = \varphi\} = \frac{1}{2} \tag{3.5}$$

Likewise, the innovations to money growth, ξ_t, are assumed to lie in a two point set,

$$\xi_t \in \{-\zeta, \zeta\} \tag{3.6}$$

and

$$\text{prob}\,\{\xi_t = -\zeta\} = \text{prob}\,\{\xi_t = \zeta\} = \frac{1}{2} \tag{3.7}$$

The innovations to productivity and money growth are assumed to be independent.

To solve and simulate the model, the following parameters must be assigned values:

Preferences:	$\omega,\ \gamma,\ \beta,\ \sigma$
Market Production:	$A_m,\ \alpha,\ \delta_k,\ \rho,\ \varphi$
Human Capital Production:	$A_h,\ \theta,\ \delta_h$
Government:	$\phi,\ \zeta,\ \overline{g}$

As in the seminal work of Kydland and Prescott (1982), as much discipline as possible is imposed by choosing parameter values based on either micro evidence, or to obtain long run averages observed in the data.

As noted by Davies and Whalley (1989) and King and Rebelo (1990), there is little evidence to guide the choice of parameters for the human capital production function. To minimize discretion, the market production function and physical capital are used as guides in the choice of human capital parameters. The capital share parameters, α and θ, are set equal to 0.36, capital's average share of GNP for the U.S. economy in the post-Korean War period.[6] The scale parameters, A_m and A_h, also share the same value, 0.105, which is chosen to achieve a steady state growth rate of 0.3542, the average quarterly growth rate of per capita U.S. GNP over the period 1954Q1–1989Q4. From the homogeneity results in Section 2.3, the model's results are insensitive to normalizing A_m to unity and allowing A_h to adjust to achieve the target growth rate. Conceptually, this would be equivalent to changing the units in which h, the stock of human capital, is measured.

The model is compared with quarterly data. Kydland and Prescott (1982) suggest an annual depreciation rate for capital of 10%. Restricting the depreciation rates, δ_k and δ_h, to have a common value, this corresponds to setting each to 0.025. The discount factor, β, is chosen such that in steady state, a real return of 1% is earned on physical capital. Evaluating (2.27) in steady state, β is chosen to satisfy:

$$1 = \beta \left(\frac{h'}{h} \right)^{-\sigma} \times 1.01 \tag{3.8}$$

where h'/h is the steady state real growth rate. This implies a value of 0.9954 in the benchmark economy.

[6] King and Rebelo (1990) consider a smaller capital share parameter for the human capital sector since this reduces the sensitivity of growth to changes in the income tax rate in their model.

The key parameters governing the stochastic process of the productivity shock are its autocorrelation coefficient, ρ, and its variability which is governed by φ. The value for ρ is 0.95 as suggested by Prescott's (1986) analysis of the properties of Solow residuals for the U.S. economy. However, since human capital plays no role Prescott's work, it would be inconsistent to use his estimate of the variance of the Solow residuals to fix the variance of the productivity shock in this model. Instead, the value of φ was chosen such that the standard deviation of the growth rate of output from the model matches that of U.S. GNP.[7] This implies a value of 3.6952×10^{-4} for φ.

Mehra and Prescott (1985) cite micro evidence on the coefficient of relative risk aversion, and suggest that it has a value between 1 and 2. Recalling that $U(c,\ell) = u(c)v(\ell)$ where $u(c)$ is homogeneous of degree $1-\sigma$, the evidence in Mehra and Prescott guides the choice of the parameter σ. For the purposes of the benchmark model, setting σ to 1.5 seems reasonable.

The parameter ω, which governs the importance of consumption relative to leisure in the period utility function, is chosen such that in steady state, households allocate 24% of their time to market production. This fraction corresponds to the per capita fraction of time spent working by the U.S. working age population. The value of ω is thus 0.2281. Notice that, with σ, this leads to a value of 3.1922 for γ.

Finally, parameters describing government's actions must be chosen. $\overline{g}$, the average quarterly growth rate of money, is 1.014%, the observed quarterly growth rate of per capita U.S. M1 over 1959Q2–1989Q4.[8] The autoregressive coefficient of money growth, ψ, and the variance of its innovations are obtained by estimating a first-order autoregressive process to money growth. The resulting values are 0.5814 and

[7] Hansen (1985) and Greenwood, Hercowitz and Huffman (1988) perform similar exercises.

[8] The database used has U.S. M1 starting in 1959Q1; one quarter is lost in calculating the growth rate.

8.2357×10^{-3}, respectively, the last of which is also the value of ζ.

3.2 Model Results

Two sets of tables are presented for the U.S. economy and the model. The first set consists of moments based on quarterly growth rates (first difference in logs), while the second consists of moments for Hodrick-Prescott filtered data. In typical real business cycle exercises—see, for example, Kydland and Prescott (1982) and Hansen (1985)—the model abstracts from growth and Hodrick-Prescott filtering is used as an "agnostic" means of detrending the data. Since the model presented above explicitly incorporates endogenous growth, it seems appropriate to base the comparison of the model with the U.S. data on growth rate filtered data. Moments for Hodrick-Prescott filtered data are provided, however, to facilitate comparisons with studies of other real business cycle models. Emphasis in the presentation will, however, be placed on the moments for the growth rate filtered data. Table 2 presents selected growth rate filtered moments for the U.S. economy while Table 4 provides the same moments for data logged and Hodrick-Prescott filtered.

In matching the model up with U.S. macroaggregates, the following assumptions have been made. First, consumption in the model is associated with consumption of non-durables and services in the U.S. National Accounts. Second, investment is taken to be measured by fixed investment. Finally, as noted above, M1 is the monetary aggregate chosen to match up with money in the model, although summary statistics for other aggregates are provided.

The balanced growth version of the model is solved using a procedure suggested by Coleman (1989). Essentially, this algorithm seeks policy and pricing functions which satisfy the Euler equations and constraints. For details on implementing the algorithm, see Coleman (1989) or Gomme and Greenwood (1990). A key feature of this algorithm, exploited here, is that it can be used to seek non-pareto optimal equilibria.[9]

Moments for the model are obtained by simulating the functions thus obtained, taking care to transform variables from their balanced growth values. Since the number of observations affects the degree of smoothing achieved by the Hodrick-Prescott filter, 50 sets of 144 observations, the number of quarters from 1954Q1 to 1989Q4, were generated. The averages of the moments across the 50 sets are presented in Table 3 for growth rate filtered data, and Table 5 for Hodrick-Prescott filtered data.[10]

Concentrating on the growth rate filtered data (Tables 2 and 3), it can be seen that the model does well in replicating the core U.S. business cycle facts that consumption varies less than output while investment varies more, although in the model investment varies too much relative to the U.S. economy. The model has problems capturing the magnitude of the correlation between consumption and output exhibited by the U.S. data, and generates a negative correlation between productivity and output where this is positive in the data.

For the real variables (that is, excluding money and the price level), the model uniformly delivers negative first-order autocorrelations which stands in contrast with the positive correlations seen in the U.S. data. This is likely due to the assumption that in the model both labour and physical capital are perfectly mobile within a period.[11] One means to improve the behaviour of the model is to introduce adjust-

[9] Cooley and Hansen (1989a) use a modified linear-quadratic procedure; see Hansen and Prescott (1991) for details. King and Rebelo (1990) use an alternative linear-quadratic technique; see King, Plosser and Rebelo (1988) for particulars.

[10] With the exception of currency, the moments reported for the monetary aggregates for the U.S. economy are based on data from 1959Q1. No attempt was made to shorten the simulated samples for money in the moments reported for the model.

[11] One side effect is that the model responds fully to a shock in one period

ment costs to the physical capital stock.[12] The household's budget constraint is now written:

$$c_t + i_t + \frac{1}{2}\mu \frac{\left(k_{t+1} - \frac{h_{t+1}}{h_t} k_t\right)^2}{k_t} + \frac{m_{t+1}}{p_t} \leq r_t \phi_t k_t + w_t n_t h_t + \frac{m_t + v_t}{p_t} \tag{3.9}$$

where higher values of μ reflect higher costs of adjustment. Notice that the gross growth rate of human capital multiplies current period capital in the numerator of the adjustment cost term. This has the effect of penalizing physical capital growth which is differs from than that of human captial. As a result, the steady state, balanced growth behaviour of the model with adjustment costs is identical to that of the benchmark economy. Adjustment costs of this form have been used by Mendoza (1991) and Cardia (1991).

Moments for the adjustment cost version of the model are presented in Tables 6 (growth rate filter) and 7 (Hodrick-Prescott filter) for $\mu = 1.0$. This value for μ is about fifty times larger than used by Mendoza (1991).[13] Such an extreme value for μ was chosen to emphasize that adjustment costs alone cannot "fix" the benchmark economy, although they do help.

As above, attention is focused on moments for growth rate filtered data. Notice first that large adjustment costs are unable to produce positive autocorrelation in *all* of the real variables. In particular, the autocorrelation of output is now positive (0.21), but not as high as in the U.S. economy (0.33). While still negative, the autocorrelations of investment, hours and productivity rise from −0.53, −0.49 and −0.64, respectively, to −0.27, −0.15 and −0.35. These compare with figures of 0.49,

[12] Separate laws of motion for the physical capital stocks used in the market sector and human capital production have no effect on the rapid adjustment of the model to shocks. King and Rebelo (1990) consider adjustment costs to human capital. Another alternative would be to consider time-to-build à la Kydland and Prescott (1982).

[13] Comparing the adjustment costs parameters is complicated since in (3.9) above the adjustment costs are expressed releative to the capital stock while Mendoza (1991) does not. The formulation above is appropriate for models with growth.

0.52 and 0.03 for the U.S. economy. Clearly, adjustment costs help on this dimension, but not enough.

Turning now to the standard deviations of macroaggregates, it can be seen that the standard deviation of investment falls sharply from 6.17 under the benchmark economy to 3.34. This should come as no surprise since adjustment costs serve to penalize large changes in the growth rate of physical capital. Indeed, Mendoza (1991) and Cardia (1991) introduced adjustment costs to their open economy models specifically to reduce the volatility of investment.[14] While the variability of investment is still too high, it is much more in line with what is seen in the U.S. economy. Also, the standard deviation of hours conforms much better to the U.S. data, and the standard deviation of productivity actually falls below that seen in the U.S. economy—setting $\mu = 0.1$ just about matches this moment.

Finally, consider the correlations with output. With adjustment costs, the correlation with output of consumption and productivity are very close to their U.S. counterparts (0.47 and 0.37, respectively, versus 0.46 and 0.47 in the U.S. data), a large improvement over the benchmark economy. However, the correlations with output of investment and hours, already higher than seen in the U.S. data, are raised even higher.

Overall, the contribution of adjustment costs is to bring generally greater conformity between the moments of the model and the U.S. economy. However, even the relatively larger adjustment costs imposed above are unable to provide deliver correlations for all of the real variables.

Turning to the behaviour of the nominal variables in the benchmark economy, it

[14] In Mendoza (1991) and Cardia (1991), the world interest rate is taken as given. Consequently, changes in productivity induce large changes in the capital stock as agents in the economy attempt to equate the real return to capital to the world real interest rate. High investment variability results when there are no adjustment costs.

should not be surprising that the behaviour of money in the model closely matches that observed in the U.S. economy—the parameters governing money growth were chosen such that this should be true. However, the inflation rate is not high enough in the model; using a broader definition of money would help since the broad aggregates grow faster than M1 in the U.S. economy.[15] Also, inflation is too variable and not as highly autocorrelated as observed in the U.S. data; as seen in Table 6, adjustment costs do not help on this dimension.

As in King, Plosser and Rebelo (1988), the model makes strong predictions regarding growth rates. For example, the model restricts the growth rate of hours worked to be zero since the household's time endowment is fixed while the U.S. data shows a modest decline.[16] Some likely explanations for the decline in hours in the U.S. are: average full-time hours of employees has been declining, there are more part-time workers, and the unemployment rate has an upward trend. These effects are partially offset by the increased participation rate.

The model also restricts output, consumption and investment to grow at a common rate while the U.S. data shows that consumption and investment have grown faster than output.

The effects money has on the benchmark economy are explored in Table 8. Here, standard deviations for macroaggregates (growth rate filtered) are provided for the U.S. economy, the benchmark economy, and a non-monetary version of the model.[17]

[15] In the model, the inflation rate is given by the growth rate of money less the real growth rate of the economy. Calibrating to a higher long run money growth rate would, then, lead to a higher average inflation rate.

[16] In Tables 2 and 4, hours are measured by hours of all persons in the business sector. If, instead, hours are measured either by hours of all employees in the business sector or hours of all persons in the non-farm business sector, the growth in hours is close to zero, although still negative. The growth rate of hours is -0.04% per quarter using household data rather than -0.08% as reported in Table 2

[17] For the non-monetary version, the cash-in-advance constraint is removed. This is equivalent to an optimal monetary policy, in which case the cash-in-advance constraint does not bind.

The most striking result is the effect money has on consumption. In the benchmark economy, the standard deviation of consumption almost matches that found in the U.S. economy while in the non-monetary economy, the standard deviation of consumption is virtually zero. It would seem that money is important in generating plausible consumption variability for this model.

The benchmark economy is compared with more standard (stationary) real business cycle models in Table 9. Results in Hansen (1985) for the divisible labour case are taken to be representative of standard real business cycle models.[18] Results from Cooley and Hansen (1989a) are included to provide an assessment of the importance of endogenous growth. All moments are based on Hodrick-Prescott filtered data. The benchmark economy performs well with respect to the standard deviation (relative to output) of consumption and hours. The relative standard deviation of productivity is too high in the benchmark economy while the indivisible labour model (Cooley and Hansen (1989a)) produces numbers which are too low. The Cooley and Hansen model out-performs the benchmark economy with respect to the behaviour of the price level, although both exaggerate its variability and are not sufficiently negatively correlated with output, vis-à-vis the U.S. economy. The benchmark economy does poorly on the dimensions of the correlation of consumption with output and the correlation of the capital stock with output. Arguably, the benchmark economy performs better with respect to the correlation between productivity and output: it delivers a small negative correlation where a small positive correlation is seen in the U.S. data while the other models produce large positive correlations.

Finally, Hansen (1985) introduced indivisible labour to a real business cycle model to account for the ratio of the standard deviation of hours to the standard deviation

[18] The results for Hansen's (1985) indivisible labour model are qualitatively similar to those reported in Cooley and Hansen (1989a)

of productivity. Standard (divisible labour) models deliver a ratio of about one while the U.S. data exhibits a ratio of about two.[19] Hansen's indivisible labour model actually goes too far: it raises this ratio to 2.7. From Table 5, it can be seen that the benchmark economy gives a ratio of about 1.9, very close to that seen in the U.S. data, but *without the indivisible labour assumption*.

4. Welfare Results

The task at hand is to provide a measure of the welfare costs of money growth-cum-inflation for the environment described above. Throughout, the functional forms and parameter values for the benchmark model have been used.

At some minor abuse of notation, the $t = 0$ value function for a household can be written as:

$$V^a(k_0, h_0, m_0; s, \lambda) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^a + \lambda y_t^a, \ell_t^a) \tag{4.1}$$

where the a superscript denotes equilibrium allocation rules obtained for monetary regime a (assumed not to depend on λ), and $\lambda \hat{y}_t^a$ is a lump-sum equivalent variation payment made to households. Using the properties of $U(\cdot)$, (4.1) can be rewritten as:

$$V^a(k_0, h_0, m_0; s, \lambda) = h_0^{1-\sigma} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(\hat{c}_t^a + \lambda \hat{y}_t^a, \ell_t^a) \left[\prod_{\tau=0}^{t} \left(\frac{h_{\tau+1}^a}{h_\tau^a} \right) \right]^{1-\sigma} \tag{4.2}$$

This transformation is computationally convenient. The welfare cost of operating monetary regime a rather than regime b is measured by the unique value of λ satisfying $V^a(k_0, h_0, m_0; s, \lambda) = V^b(k_0, h_0, m_0; s, \lambda = 0)$.

In generating measures of the welfare costs of alternative monetary regimes, the experiment being considered is to face the representative household with a choice between two regimes, but the *same* initial conditions—that is, the same initial physical

[19] This ratio is higher than reported in Hansen (1985), but is in line with the results reported in Kydland and Prescott (1990) for either the household or establishment surveys for hours, and to the figures presented in Cooley and Hansen (1989a)

and human capital stocks and nominal cash balances. Consequently, transition costs from one regime to another have been ignored.

Conceptually, this method of calculating welfare gains/losses is the same as the exercises conducted by, for example, Cooley and Hansen (1989a), Greenwood and Huffman (1991) and Lucas (1990).

4.1 Steady State Results

These experiments can be thought of as setting the variance of the innovations to the productivity shock and money growth equal to zero. There are two motives for starting with steady state, balanced growth path calculations of the welfare costs of inflation: first, they are directly comparable with the welfare costs reported in Cooley and Hansen (1989a); and second, the calculations are simple enough that they can be verified with a calculator.

In steady state, (4.2) can be written as:

$$V^a(k_0, h_0, m_0; s, \lambda) = \frac{h_0^{1-\sigma} U(\hat{c}^a + \lambda \hat{y}^a, \ell^a)}{1 - \beta \left[\left(\frac{h'}{h} \right)^a \right]^{1-\sigma}} \tag{4.3}$$

Denote the optimal monetary regime—in which case the cash-in-advance constraint does not bind—by an asterisk superscript. Then the cost of operating monetary policy a relative to the optimal policy is the unique, positive value of λ satisfying:

$$\frac{U(\hat{c}^*, \ell^*)}{1 - \beta \left[\left(\frac{h'}{h} \right)^* \right]^{1-\sigma}} = \frac{U(\hat{c}^a + \lambda \hat{y}^a, \ell^a)}{1 - \beta \left[\left(\frac{h'}{h} \right)^a \right]^{1-\sigma}} \tag{4.4}$$

Given the functional form for $U(\cdot)$, (4.4) can be solved directly for λ.

The behaviour of the model and welfare costs of alternative monetary policies are summarized in Table 10. Note, in particular, that the welfare costs of a 10% money growth rate (8.7% inflation rate) is less than 0.03% of income while Cooley and Hansen (1989a) report a cost of 0.4%. Some insight as to why the welfare costs of inflation are so modest can be culled from Table 10.

Note that while period utility, defined over consumption normalized by human capital and leisure, $U(\hat{c}, \ell)$, is *increasing* for moderate money growth rates, lifetime utility, $V(\hat{s}, \lambda)$, is monotonically decreasing.

Higher money growth has the expected effects: it lowers (normalized) consumption and real growth, and raises leisure. However, the decline in consumption is slight. The goods market clearing condition, reproduced below along the steady state balanced growth path, sheds some light on why this is so.

$$\hat{c} + \left(\frac{h'}{h}\right)\hat{k} = F^m(\phi\hat{k}, n, z = 0) + (1 - \delta)\hat{k} \tag{4.5}$$

The term $\hat{k}h'/h$ is the amount of capital households must take from a period to stay on the balanced growth path. Noticing from Table 10 that $\hat{k}$ is unaffected by the money growth rate, the fall in the real growth rate induced by increased money growth allows a reallocation of output from capital accumulation to consumption. That normalized consumption falls results from the negative effect money growth has on output.

As mentioned previously, increases in the growth of money lowers the return to working. Since labour is perfectly mobile within a period between the market sector and human capital production, in equilibrium the return to working in the sectors will be equalized. As a consequence, there are two productive activities from which labour is drawn into leisure rather than just one as in models which abstract from growth, like Cooley and Hansen (1989a). If households do not value leisure, it can be shown that along the steady state balanced growth path, changes in money growth have no real effects—a result similar to that of Stockman (1981)—and consequently no welfare effects. It is the augmented response of leisure to increases in the money growth rate, relative to that found in Cooley and Hansen (1989a), which helps to compensate households for the fall in the real growth rate, and the slight decline in consumption.

Results for hyperinflations are included in Table 10 not in the belief that the model can be used to assess the effects of such high inflation rates on the U.S. economy—clearly, agents would change their behaviour in the face of such inflation rates and the transactions technology would be expected to change—but rather to verify that such monetary policies have drastic effects on the model economy. Surely enough, this is exactly what happens: consumption falls sharply, and for sufficiently high money growth, negative real growth is registered. As well, large welfare costs are associated with such policies.

In a sense, it may not be too surprising that the benchmark economy generates such small costs of inflation: the cash-in-advance constraint applies only to the consumption good so that the effect of inflation on the engine of growth, human capital accumulation, is minimal. Alternative cash-in-advance constraints are now explored.

The welfare costs in Table 11 are organized in two groups: stationary environment, and endogenous growth environment.[20] The stationary economy is essentially Cooley and Hansen (1989) except that the indivisible labour assumption has been dropped, and the parameters are calibrated in the same way as the benchmark economy. For each environment, welfare costs are presented for the cases in which the cash-in-advance constraint applies to consumption alone, and to consumption and physical investment. For the endogenous growth environment, welfare costs are also calculated for a cash-in-advance constraint applied to consumption, physical investment and human capital investment.[21]

Consider, first, the effect of different cash-in-advance constraints across the same

[20] Only welfare costs are reported in Table 11. Full results of these experiments are available from the author upon request.

[21] Adding human capital investment to the cash-in-advance constraint requires a relatively straightforward modification of the envirnonment presented above; see King and Rebelo (1990).

environment. For the stationary environment, adding investment to the cash-in-advance constraint raises the costs of inflation by up to a factor of five. Qualitatively similar effects are seen for the endogenous growth environment. However, the costs increase by a larger percentage.[22] Somewhat curious is the fact that for low money growth rates, adding human capital investment to the cash-in-advance constraint results in *lower* costs of inflation relative to the cash-in-advance constraint applied to only consumption and physical investment. On the one hand, adding human capital investment to the cash-in-advance constraint lowers growth and so welfare. On the other hand, second best considerations suggest that since the economy is already distorted, adding another distortion may be welfare-improving.[23] For low money growth rates, second best considerations dominate while for higher money growth rates growth rate effects prevail.

Turning now to cross-environment comparisons, consider first the case in which the cash-in-advance constraint applies only to purchases of the consumption good. As mentioned earlier, a 10% money growth rate results in a welfare cost about ten times higher in the stationary model than the endogenous growth environment. The percentage difference diminishes as higher money growth rates are considered.

For historically relevant money growth rates—up to, say, ten percent annually—adding investment to the cash-in-advance constraint still results in higher costs of inflation in the stationary environment. However, the difference in the welfare costs between the two environments is now smaller in percentage terms. As well, if one is willing to consider money growth rates far outside the U.S. historical experience, it

[22] Notice that in Table Z that as the money growth rate increases, some welfare costs cannot be calculated for the endogenous growth model. This is because the real growth rate falls sufficiently that the welfare costs calculation would require taking the root of a negative number.

[23] Such an effect is seen in Greenwood and Huffman (1991) in which output smoothing is welfare-improving given that the economy is already distorted by taxes.

can be seen that the endogenous growth environment can now generate welfare costs of inflation much larger than the stationary environment

The basic message from this section is that endogenous growth actually serves to lower the welfare costs of inflation. This effect arises from the two opposing effects associated with money growth. The first, direct, effect is that lower real growth is welfare-reducing. The second operates through the additional choice variables of the individual household. This latter effect is reflected through the capital accumulation process and its effect on consumption, and the attenuation of the response of leisure to changes in money growth. The net effect is ambiguous, but is seen above to lower the welfare costs of inflation.

4.2 Welfare Results for the Stochastic Economy

The basic task here is the same as in the previous section: find the welfare costs of alternative monetary regimes. The primary difference between the two sections is that here the variance of the innovations to the productivity shock and money growth are set equal to their values in the benchmark economy. Endogenous growth complicates the task of calculating welfare costs somewhat. To start, define

$$J^a(\hat{s}_0, \lambda) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(\hat{c}_t^a + \lambda \hat{y}_t^a, \ell_t^a) \left[\sum_{\tau=0}^{t} \left(\frac{h_{\tau+1}}{h_\tau} \right)^a \right]^{1-\sigma} \tag{4.6}$$

Clearly, $V(k_0, h_0, m_0; s_0, \lambda) = h_0^{1-\sigma} J^a(\hat{s}_0, \lambda)$. (4.6) can be rewritten in the form of a Bellman equation:

$$J^a(\hat{s}^a, \lambda) = U(\hat{c}^a + \lambda \hat{y}^a, \ell^a) + \beta \left[\left(\frac{h'}{h} \right)^a \right]^{1-\sigma} E J^a(\hat{s^a}', \lambda) \tag{4.7}$$

In comparing the welfare cost of monetary regime a relative to regime b, the task is to find the value of λ satisfying

$$\int J^a(\hat{s}^a; \lambda) \mathrm{d}\Gamma(\hat{s}^a) = \int J^b(\hat{s}^b; \lambda = 0) \mathrm{d}\Gamma(\hat{s}^b) \tag{4.8}$$

where $\Gamma(\hat{s})$ is the distribution function for $\hat{s}$.

$J(\hat{s};\lambda)$ is obtained by iterating on the Bellman equation, (4.7). The integrals above are approximated by averaging observed values of $J(\hat{s};\lambda)$ over arbitrarily long simulations. Letting T denote the length of the simulation,

$$\int J(\hat{s};\lambda)\mathrm{d}\Gamma(\hat{s}) \approx \frac{1}{T}\sum_{j=1}^{T} J(\hat{s}_j;\lambda) \tag{4.9}$$

Equation (4.8) is now effectively a single equation in the unknown, λ.

The first set of welfare results are based on deviations from the benchmark economy. These experiments consist of, first, increases in the variability of money growth, and, second, increases in the average, quarterly growth rate of money.

Results for increased money variability are summarized in Table 12. The experiment conducted is to increase the standard deviation of ξ, the innovation to the money growth process, by some known factor. This leads to an equiproportional increase in the standard deviation of the money growth rate. In addition to welfare costs, Table 12 also includes the standard deviations of macroaggregates for the model economy based on growth rate filtered data—other moments are little affected by these experiments, and moments for Hodrick-Prescott filtered data shows qualitatively similar behaviour. One striking result in this table is that of the real variables, consumption is the only one which responds substantially to changes in the variance of money growth. This fits well with the results in Table 8 which compared the performance of the benchmark economy with a version of the model without the cash-in-advance constraint. In Table 8, it was seen that money plays an important role in generating plausible consumption variability.

Table 13 summarizes the results of the growth rate experiments. The columns denote the percentage point increase in the quarterly money growth rate. Of interest is the fact that the variability of macroaggregates increases only slightly in the face of higher money growth, with this effect noticeable for money growth rates which are outside the U.S. historical experience.

Turning now to the welfare costs in Tables 12 and 13, notice that the welfare costs of increased money growth variability are small when compared to the costs of increases in the growth rate of money: doubling the standard deviation of money growth is less costly in welfare terms than increasing the money growth rate by 0.25 percentage points per quarter.

The second set of welfare results duplicate the experiments conducted for the steady state, balanced growth path of the model economy; welfare results for both are included in Table 14 to facilitate comparison of the two. Qualitatively, the costs of inflation are similar to those seen in steady state. While the costs of money growth are higher for the stochastic version of the model, the increase in the cost is relatively small. The results in this table fit well with the conclusion above indicating that money growth per se is more important in welfare terms than variability of money growth.

5. Conclusions

The usual intuition, as exemplified by Lucas (1987), is that if public policy can affect an economy's real growth rate, then large welfare costs will result. This suggested that a model of endogenous growth would deliver large welfare costs of money growth-cum-inflation—certainly larger than found in the stationary environment of Cooley and Hansen (1989a). In the endogenous growth model examined above, increased money growth-cum-inflation has the expected effects of lowering consumption, real growth and labour effort, yet the welfare costs are smaller than obtained by Cooley and Hansen (1989a). Lower real growth lowers household welfare, but the human capital sector introduces new choice variables which allows the household to avoid some of these costs. This latter effect is reflected in the capital accumulation decision, and consequently a small decline in consumption, and an increased response of leisure to changes in the money growth rate.

The analysis above compares welfare *across* different monetary regimes. Lucas (1990) and King and Rebelo (1990) have pointed out the importance of transitional dynamics in considering policy changes. Accounting for transitional dynamics should *lower* the welfare costs calculated above. On the other hand, results in Imrohoroglu (1990) suggest that introducing heterogeneity would *increase* the costs of higher money growth.

Finally, government revenue requirements have been ignored in the computation of welfare results above. It may be interesting to think about the mix of government taxes by introducing labour and capital taxation. In considering a change from the U.S. tax structure to an optimal mix of labour and capital taxes, Lucas (1990) calculates the benefit to be about one percent of consumption. In a stationary environment, Cooley and Hansen (1989b) find the inflation tax to be an efficient means of raising government revenue relative to labour and capital taxes. The large welfare costs of income taxation computed by King and Rebelo (1990) in an endogenous growth model similar to the one analyzed above indicate that the results of Cooley and Hansen (1989b) may be strengthened by considering an endogenous growth model. However, the analysis above suggests that intuition cannot always be trusted.

Table 1: Inflation–Per Capita Real Growth Rate Correlations International Time Series Evidence

Country	Period	Correlation
Argentina	1959–1989	-0.05275
Australia	1950–1989	-0.35413
Austria	1976–1988	0.12366
Bahrain	1976–1988	0.11883
Bangladesh	1974–1988	0.34065
Belgium	1954–1988	-0.27764
Bolivia	1961–1984	-0.38552
Botswana	1975–1989	-0.48713
Brazil	1964–1988	-0.33534
Burundi	1971–1989	-0.15403
Cameroon	1970–1985	0.04952
Canada	1949–1988	-0.17897
Chile	1964–1989	-0.44053
Columbia	1969–1988	-0.28983
Costa Rica	1961–1989	-0.55981
Cyprus	1961–1988	-0.25148
Denmark	1951–1989	-0.45390
Dominican Republic	1964–1988	-0.13504
Ecuador	1966–1989	-0.23851
El Salvador	1952–1989	-0.42507
Fiji	1970–1988	0.16429
Finland	1961–1987	-0.43244
France	1951–1989	-0.38174
Germany	1961–1989	-0.31514
Ghana	1965–1988	-0.13037
Greece	1950–1988	-0.64105
Guatemala	1952–1989	-0.18438
Guyana	1961–1988	-0.20401
Haiti	1967–1987	0.39248
Honduras	1951–1989	-0.06100
Hungary	1973–1988	-0.36113
Iceland	1951–1988	-0.25263
India	1961–1988	0.06931
Indonesia	1965–1989	-0.36907
Iran	1965–1987	-0.35374
Ireland	1949–1988	-0.09517
Israel	1965–1988	-0.14150
Italy	1961–1989	-0.33788
Japan	1961–1988	-0.32264
Jamaica	1953–1988	-0.47148
Jordan	1970–1988	0.04949
Kenya	1967–1988	-0.26887
Korea	1967–1986	-0.57646
Kuwait	1973–1988	-0.39039

Table 1 (continued)		
Country	Period	Correlation
Liberia	1966–1986	-0.23082
Luxembourg	1951–1986	-0.04201
Malasia	1981–1989	0.39854
Malta	1971–1988	0.38392
Malawi	1955–1988	0.18086
Mauritius	1964–1987	-0.02026
Mexico	1949–1986	-0.65904
Morocco	1965–1988	0.24233
Myanmar	1968–1988	-0.34465
Nepal	1965–1989	0.04275
Netherlands	1957–1989	0.10029
New Zealand	1973–1987	-0.22765
Nicaragua	1974–1988	-0.08691
Nigeria	1962–1989	-0.48619
Norway	1955–1989	-0.24359
Panama	1957–1989	-0.00449
Pakistan	1951–1989	-0.03473
Paraguay	1960–1989	0.13211
Philippines	1950–1989	-0.53678
Portugal	1967–1986	-0.49788
Saudi Arabia	1968–1988	0.30833
Singapore	1961–1989	0.05194
Spain	1955–1989	-0.52442
St. Vincent	1977–1985	-0.30949
Sweden	1978–1986	-0.60975
Swaziland	1951–1989	0.08122
Switzerland	1949–1989	-0.24902
Syrian Arab Republic	1961–1988	-0.20626
Tanzania	1966–1988	-0.55888
Togo	1971–1986	-0.29603
Trinidad and Tobago	1967–1987	0.19769
Tunisia	1969–1988	-0.41738
Turkey	1958–1988	-0.43260
United Kingdom	1949–1989	-0.46028
United States	1956–1989	-0.22931
Uruguay	1949–1989	0.18278
Venezuela	1958–1989	-0.56122
Yugoslavia	1969–1988	-0.67549

Source: International Financial Statistics tape. Inflation is measured by the percentage change in the consumer price index. Real output is typically measured by real GDP (gross domestic product) or real GNP (gross national product).

Table 2: United States, 1954Q1–1989Q4
Growth Rate Filtered

	Growth Rate	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	0.35	1.01	0.33	1.00
Consumption	0.40	0.52	0.25	0.46
Investment	0.43	2.61	0.49	0.70
Hours	-0.09	0.93	0.52	0.70
Productivity	0.44	0.75	0.03	0.47
Currency	1.05	0.80	0.90	0.08
Base	1.11	0.64	0.78	0.10
M1	1.01	0.99	0.58	0.13
M2	1.56	0.77	0.61	0.15
M3	1.74	0.75	0.78	0.15
L	1.69	0.64	0.80	0.14
GNP Deflator	1.11	0.67	0.74	-0.26
CPI	1.07	0.85	0.86	-0.27

All variables except the price indexes have been deflated by the 16+ population. All variables except the monetary aggregates and price indexes are expressed in constant 1982 dollars. Output is measured by gross national product; consumption by consumption of non-durables and services; investment by gross fixed investment; and hours by total hours of persons in the business sector (establishment survey). Productivity is defined by output divided by hours. Moments for the monetary aggregates (base, M1, M2, M3 and L) are based on data over 1959Q2–1989Q4. CPI denotes the consumer price index (1982–1984=100). The GNP deflator is the implicit GNP deflator (1982 base).

Table 3: Selected Model Moments
Growth Rate Filtered

	Growth Rate	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	0.35	1.01	-0.12	1.00
Consumption	0.35	0.48	-0.22	0.16
Investment	0.35	6.17	-0.53	0.74
Labour: Market	0.00	1.73	-0.49	0.82
Labour: Human Capital	0.00	1.95	-0.49	-0.82
Leisure	0.00	0.04	-0.21	-0.29
Productivity	0.35	1.07	-0.65	-0.38
ϕ	0.00	1.71	-0.49	0.82
Capital Stock	0.35	0.11	-0.20	0.93
Money	0.98	0.95	0.54	-0.06
Price Level	0.63	1.25	0.22	-0.11

Table 4: United States, 1954Q1–1989Q4 Hodrick-Prescott Filtered			
	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	1.70	0.85	1.00
Consumption	0.85	0.84	0.75
Investment	5.35	0.89	0.89
Hours	1.77	0.88	0.88
Productivity	0.85	0.67	0.16
Currency	0.71	0.88	0.24
Base	0.84	0.88	0.41
M1	1.63	0.87	0.31
M2	1.48	0.89	0.46
M3	1.50	0.92	0.48
L	1.09	0.91	0.58
GNP Deflator	0.89	0.91	-0.55
CPI	1.41	0.94	-0.57

All variables except the price indexes have been deflated by the 16+ population. All variables except the monetary aggregates and price indexes are expressed in constant 1982 dollars. Output is measured by gross national product; consumption by consumption of non-durables and services; investment by gross fixed investment; and hours by total hours of persons in the business sector (establishment survey). Productivity is defined by output divided by hours. Moments for the monetary aggregates (base, M1, M2, M3 and L) are based on data over 1959Q2–1989Q4. CPI denotes the consumer price index (1982–1984=100). The GNP deflator is the implicit GNP deflator (1982 base).

Table 5: Selected Model Moments Hodrick-Prescott Filtered			
	Standard Deviation	Autocorrelation	Correlation with Output
Output	0.88	0.35	1.00
Consumption	0.45	0.42	0.24
Investment	3.93	-0.23	0.71
Labour: Market	1.18	-0.06	0.85
Labour: Human Capital	1.33	-0.07	-0.84
Leisure	0.04	0.44	-0.34
Productivity	0.63	-0.44	-0.20
ϕ	1.17	-0.07	0.85
Capital Stock	0.11	0.49	0.78
Money	1.81	0.89	-0.05
Price Level	1.96	0.81	-0.10

Table 6: Adjustment Cost Version
Growth Rate Filtered

	Growth Rate	Standard Deviation	First-order Autocorrelation	Correlation with Output
Output	0.35	1.01	0.21	1.00
Consumption	0.35	0.50	-0.18	0.40
Investment	0.35	3.34	-0.27	0.80
Labour: Market	0.00	1.18	-0.15	0.90
Labour: Human Capital	0.00	1.00	-0.15	-0.89
Leisure	0.00	0.09	-0.18	-0.51
Productivity	0.35	0.53	-0.35	-0.09
ϕ	0.00	1.14	-0.15	0.89
Capital Stock	0.35	0.09	0.47	0.77
Money	1.00	0.99	0.55	-0.08
Price Level	0.63	1.29	0.24	-0.21

Table 7: Adjustment Cost Version
Hodrick-Prescott Filtered

	Standard Deviation	Autocorrelation	Correlation with Output
Output	1.33	0.71	1.00
Consumption	0.50	0.49	0.47
Investment	2.91	0.34	0.83
Labour: Market	1.24	0.54	0.94
Labour: Human Capital	1.04	0.54	-0.92
Leisure	0.09	0.49	-0.59
Productivity	0.46	0.35	0.37
ϕ	1.19	0.54	0.93
Capital Stock	0.17	0.87	0.59
Money	1.89	0.88	-0.05
Price Level	2.04	0.81	-0.16

Table 8: Contribution of Money to the Model Standard Deviations, Growth Rate Filtered			
	U.S.	Benchmark	Non-monetary
Output	1.01	1.01	1.00
Consumption	0.52	0.48	0.03
Investment	4.00	5.03	4.99
Labour: Market	0.93	1.75	1.73
Labour: Human Capital		1.59	1.58
Leisure		0.08	0.01
Productivity	0.75	1.09	1.08

Table 9: Comparisons with Other Models Hodrick-Prescott Filtered								
	U.S.		Hansen		Cooley-Hansen		Benchmark	
	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
Consumption	0.50	0.75	0.31	0.89	0.36	0.72	0.56	0.24
Investment	5.25	0.89	3.14	0.99	3.29	0.97	4.46	0.71
Capital Stock	0.22†	0.28†	0.27	0.06	0.28	0.06	0.13	0.78
Hours	1.04	0.88	0.52	0.98	0.77	0.98	1.34	0.85
Productivity	0.50	0.16	0.50	0.98	0.29	0.87	0.72	-0.20
Price Level					0.98	-0.27	2.23	-0.10
CPI	0.83	-0.57						
GNP deflator	0.52	-0.55						

Results for Hansen (1985) are for the divisible labour case; the indivisible case is similar to the results for Cooley-Hansen.

Results for the Cooley and Hansen (1989a) model are for the autoregressive growth rate ($\overline{g} = 1.015$) case.

Column (a): standard deviation relative to the standard deviation of output.

Column (b): correlation with output.

†From Cooley and Hansen (1989a).

Table 10: Steady State Welfare Results Alternate Annual Money Growth Rates				
	optimal	0%	5%	10%
$\hat{k}$	0.5625	0.5625	0.5625	0.5625
ϕ	0.4785	0.4783	0.4780	0.4777
ℓ	0.4947	0.4960	0.4984	0.5007
n	0.2418	0.2411	0.2398	0.2385
$\hat{c}$	0.0102	0.0101	0.0101	0.0100
$U(\hat{c},\ell)$	-14.8842	-14.8414	-14.7616	-14.6869
$V(\hat{s},\lambda=0)$	-2310.5359	-2310.5685	-2310.8070	-2311.2498
$\hat{y}$	0.0263	0.0262	0.0261	0.0260
quarterly growth rate	0.3639	0.3602	0.3530	0.3463
annual inflation rate	-3.9532	-1.4329	3.5428	8.5198
welfare cost (%)	0.0000	0.0011	0.0091	0.0238
	100%	1,000%	10,000%	100,000%
$\hat{k}$	0.5625	0.5625	0.5625	0.5625
ϕ	0.4739	0.4617	0.4436	0.4240
ℓ	0.5296	0.6060	0.6873	0.7487
n	0.2229	0.1819	0.1387	0.1065
$\hat{c}$	0.0093	0.0074	0.0053	0.0035
$U(\hat{c},\ell)$	-13.8384	-12.3412	-11.8418	-12.5634
$V(\hat{s},\lambda=0)$	-2335.6150	-2603.3832	-3508.2076	-5578.1379
$\hat{y}$	0.0249	0.0216	0.0179	0.0149
quarterly growth rate	0.2596	0.0211	-0.2527	-0.4772
annual inflation rate	98.2595	999.4913	10032.2465	100340.1116
welfare cost (%)	0.8198	9.2787	38.4535	113.8397

Table 11: Alternative Cash-in-advance Constraints					
Money Growth	(a)	(b)	(c)	(d)	(e)
0%	0.0813	0.2760	0.0011	0.1009	0.0397
5%	0.1869	0.6313	0.0091	0.3375	0.3109
10%	0.2947	0.9909	0.0238	0.6249	0.8169
100%	2.2899	7.4712	0.8198	10.7877	42.4822
1,000%	14.3696	48.1123	9.2787	297.0850	n/a
10,000%	47.0701	178.7911	38.4535	n/a	n/a
100,000%	111.1804	509.9434	113.8397	n/a	n/a

(a) Stationary economy, cash-in-advance on consumption.
(b) Stationary economy, cash-in-advance on consumption and investment.
(c) Endogenous growth economy, cash-in-advance on consumption.
(d) Endogenous growth economy, cash-in-advance on consumption and physical investment.
(e) Endogenous growth economy, cash-in-advance on consumption, physical investment and human capital investment

Table 12: Money Variance Experiments
Stochastic Economy Welfare Results

	benchmark	+5%	+10%	+25%	+50%	+100%
Standard Deviation:						
Output	1.01	1.01	1.01	1.01	1.01	1.02
Consumption	0.48	0.50	0.52	0.60	0.71	0.95
Investment	5.03	5.03	5.03	5.03	5.03	5.03
Labour: market	1.75	1.75	1.75	1.76	1.76	1.78
Labour: human capital	1.59	1.59	1.59	1.59	1.59	1.59
Leisure	0.08	0.08	0.09	0.10	0.12	0.16
Productivity	1.09	1.09	1.09	1.09	1.09	1.10
Money	0.99	1.04	1.09	1.23	1.48	2.81
Price Level	1.28	1.35	1.41	1.60	1.92	1.97
Average Real Growth:	0.35	0.35	0.35	0.35	0.35	0.35
Welfare Cost (%, $\times 10^{-3}$):		0.05	0.11	0.29	0.64	1.54

Table 13: Money Growth Experiments
Stochastic Economy Welfare Results

	benchmark	+0.25	+0.50	+1.00	+2.50	+10.00
Standard Deviation:						
Output	1.01	1.01	1.01	1.01	1.02	1.05
Consumption	0.48	0.48	0.48	0.48	0.48	0.48
Investment	5.03	5.03	5.03	5.04	5.07	5.20
Labour: market	1.59	1.75	1.76	1.76	1.77	1.83
Labour: human capital	1.75	1.59	1.59	1.59	1.60	1.64
Leisure	0.08	0.08	0.08	0.08	0.08	0.08
Productivity	1.09	1.09	1.09	1.09	1.10	1.14
Money	0.99	0.99	0.99	0.99	0.99	0.99
Price Level	1.28	1.28	1.28	1.28	1.28	1.28
Average Real Growth:	0.35	0.35	0.35	0.35	0.34	0.30
Welfare Cost (%, $\times 10^{-3}$):		2.45	5.21	11.63	38.04	314.38

Table 14: Alternate Annual Money Growth Rates
Stochastic Economy Welfare Results

Annual Money Growth Rate	Welfare Cost: Steady State	Welfare Cost: Stochastic
0%	0.0011	0.0542
5%	0.0091	0.0628
10%	0.0238	0.0782
100%	0.8198	0.8838
1,000%	9.2787	9.4230
10,000%	38.4535	39.3500
100,000%	113.8397	119.3031

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