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RISK-TAKING, GLOBAL DIVERSIFICATION, AND GROWTH

Maurice Obstfeld

University of California at Berkeley,

National Bureau of Economic Research,

and Centre for Economic Policy Research

ABSTRACT

This paper develops a dynamic continuous-time model in which international risk sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth is an attendant world portfolio shift from safe, but low-yield, capital into riskier, high-yield capital. The presence of these two types of capital is meant to capture the idea that growth depends on the availability of an ever-increasing array of specialized, hence inherently risky, production inputs. A partial calibration exercise based on Penn World Table consumption data implies steady-state welfare gains from global financial integration that for some regions amount to several times initial wealth.

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Introduction

Standard models of international asset trade lack mechanisms linking an economy's long-run output growth rate to its financial openness. Within such models, the gains from asset trade, at least between industrial economies, typically are estimated to be quite modest under common specifications of individuals' preferences.[1] The contribution of this paper is a simple model of global portfolio diversification in which a link between growth and financial openness emerges very naturally. Within this model, an economy that opens its asset markets to trade may experience an increase in expected consumption growth and a substantial rise in national welfare.

Recent analyses of economic growth due to Romer (1986, 1990), Lucas (1988), and others explore mechanisms through which growth rates are endogenously determined by technological parameters, intertemporal preferences, market structures, and government policies. Extensions of these mechanisms to multi-economy frameworks, notably those contained in the treatise by Grossman and Helpman (1991), show that international trade in goods may accelerate or slow growth by shifting resources among alternative productive uses. The model set out below pursues this line of approach, showing that a pure expansion of opportunities for trade across states of nature may itself promote resource reallocations favorable to long-term economic growth.

The paper's model supposes that each country can invest current resources in two linear projects, one safe and one risky. This setup is a stylized rendition of the idea, developed by

[1] The theoretical literature is exemplified by models of Lucas (1982), Stulz (1987), and Svensson (1988). Recent attempts to quantify the gains from international asset trade within general-equilibrium models include Cole and Obstfeld (1991), Mendoza (1991), Backus, Kehoe, and Kydland (1989), and van Wincoop (1991).

Romer (1990) and by Grossman and Helpman (1991), that ongoing growth depends on the willingness to invest in supplying specialized, hence inherently risky, production inputs. Because risky technologies in my model have higher expected returns than safe ones, international asset trade, which allows each country to hold a globally diversified portfolio risky investments, encourages all countries simultaneously to shift away from low-return, safe investments to high-return, risky investments. Thus, provided risky returns are imperfectly correlated across countries, and provided some risk-free assets are initially held, a small increase in diversification opportunities always raises expected growth as well as national welfare.[2]

Several earlier papers have explored ideas closely related to those illustrated below.

Greenwood and Jovanovic (1990) develop a model in which financial intermediaries encourage high-yield investments and growth by performing dual roles, pooling idiosyncratic investment risks and eliminating *ex ante* downside uncertainty about rates of return. The analysis below, however, shows that even the former role of financial diversification can be an important spur to growth. The much simpler framework I choose allows closed-form solutions for an unrestrictive class of isoelastic preferences. As a result, quantitative welfare comparisons become simple and links between preferences and growth are clarified.[3]

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[2]With its linear technologies, this paper's model is a special case of the continuous-time stochastic model of Cox, Ingersoll, and Ross (1985). Their focus, however, is on asset pricing rather than on growth; and their assumptions on preferences are more restrictive than those entertained below. Explicit production externalities of the type first posited by Arrow (1962), and featured in much of the literature on endogenous growth, are not modeled below (although such externalities could be incorporated into a broadly similar framework). Instead, endogenous growth springs from non-diminishing private returns to capital, as in work of Uzawa (1965), King, Plosser, and Rebelo (1998), Barro (1990), Jones and Manuelli (1990), and Rebelo (1991).

[3]The Greenwood-Jovanovic assumption of a sunk cost of entering the financial intermediation network leads, however, to much richer dynamics than those that emerge from my model.

Bencivenga and Smith (1991) provide another analysis in which financial intermediaries, which in their setting provide liquidity, encourage savings to flow into relatively unproductive uses. The random element in their model is not the productivity of investment, as below, but a preference shock that creates a demand for liquid assets.

Finally, Devereux and Smith (1991) examine an explicitly multi-economy model of diversification and growth, and illustrate how the risk reduction implied by diversification may promote or retard growth, with the outcome depending on assumptions about intertemporal consumption substitutability and the nature of uncertainty. Their analysis, however, does not allow for aggregate shifts in the global portfolio of risky assets. A special case of this paper's model, one in which countries initially hold no riskless assets and asset returns are symmetrically distributed, yields some of the main conclusions reached by Devereux and Smith.[4]

The paper is organized as follows. Section I describes a closed economy in which technological uncertainty is follows a continuous-time diffusion process. Even within so simple a setting, it is possible to show how a reduction in uncertainty can spur economic growth. This and related results about growth are derived in section II, which studies the closed economy's competitive equilibrium. Section III extends the prior discussion to encompass intertemporal

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[4]The foregoing capsule review lists only a few papers that are especially relevant to the approach taken below to model the effects of uncertainty and financial markets on growth. A number of other related studies have appeared. For example, Dellas (1991) suggests models of growth and uncertainty that also are driven by the effects of uncertainty on the allocation of investment. Cooley and Smith (1991) model the process through which financial markets can foster early investment in schooling, which in turn allows later participation in specialized activities subject to learning-by-doing effects. Bertola's (1991) model shows how, in the presence of labor-mobility costs, firm-specific uncertainty may reduce productive efficiency and growth. Aghion and Saint-Paul (1991) argue that a greater amplitude of aggregate fluctuations actually can encourage growth. In their model, recessions are particularly favorable times for productivity-enhancing investment.

preferences of the nonexpected utility variety. Such preferences allow an inquiry into the distinct effects of risk aversion and intertemporal substitutability on growth.

The impact of global financial integration in a multi-economy world is studied in section IV. This section contains the paper's central results concerning international diversification, growth, and real interest rates. Section V presents a pair of simple two-country examples to illustrate how some structural assumptions may lead to large welfare gains from financial integration, while others may result in smaller gains of the type often found in contexts where long-run growth rates are exogenously determined.

A more realistic example, partially calibrated to global consumption data provided by Summers and Heston (1991), is contained in section VI. Here, the estimated gains from moving to a regime of perfect global financial markets differ by region and can be as large as several times initial wealth (under some admittedly unrealistic assumptions concerning technologies and the costs of capital relocation). The calibration exercise of section VI is unsatisfactory in some other respects. One is the model's failure, given the means and variances of countries' consumption-growth rates, to replicate observed average rates of return to equity and real interest rates without unrealistically high levels of individual risk aversion and intertemporal substitutability. Another problem, from an empirical point of view, is the model's assumption that all income risk can be traded. Section VII summarizes what has been learned.

I. Individual Choice in a Closed Economy with Uncertainty

The closed economy is populated by identical infinitely-lived individuals who face the choice between consuming or investing a single good. A representative household is assumed for now to maximize the intertemporal objective

$$(1) \quad U(0) = \mathrm{E}_0 \int_0^{\infty} u[C(t)]e^{-\delta t}dt,$$

where E_t is a mathematical expectation conditional on time-t information, $C(t)$ is time-t consumption, and $\delta > 0$ is the subjective rate of time preference. The period utility function in (1) takes the form

$$(2) \quad u(C) = C^{1-R}/(1 - R),$$

where $R > 0$.[5]

The foregoing preference formulation has the sole virtue of familiarity. In section III, I will extend the model to a class of preferences broader than the isoelastic von Neumann-Morgenstern class described by (1) and (2).

Individuals save by accumulating capital. As in Solow's (1956) growth model, one unit of consumption can be transformed into one unit of capital, or vice versa, at zero cost. Capital comes in two varieties, however, riskless capital offering a sure instantaneous yield of r and risky capital offering a random instantaneous yield with *expected value* $\alpha > r$. So individuals face a portfolio decision--how to allocate their wealth between the two assets--as well as a saving decision.

Let $V^B(t)$ denote the cumulative time-t value of a unit of output invested in safe capital

[5]When $R = 1$, $u(C) = \log(C)$.

at time 0 and $V^K(t)$ the cumulative time-t value of a unit of output invested in risky capital at time 0. Clearly $V^B(0) = V^K(0) = 1$. With all payouts reinvested and continuously compounded, $V^B(t)$ obeys the ordinary differential equation

(3) $dV^B(t) = rV^B(t)dt.$

The stochastic law of motion for $V^K(t)$ is motivated by first examining an approximating economy where time passes in discrete increments of length h. Over the interval from t to $t + h$, the percent increase in $V^K(t)$ is equal to αh plus an i.i.d. shock with variance $\sigma^2 h > 0$,

$$V^K(t+h) - V^K(t) = \alpha V^K(t)h + \sigma V^K(t)[z(t+h) - z(t)],$$

where $z(t+h) - z(t) \sim N(0,h)$ for all t. The limit of the process above, as $h \to 0$, is the geometric diffusion process

(4) $dV^K(t)/V^K(t) = \alpha dt + \sigma dz(t).$

In (4), $dz(t)$ is a standard Wiener process, such that $z(t) = z(0) + \int_0^t dz(s)$.[6]

[6]Equation (4) implies that $V^K(t)$ is lognormally distributed: by Itô's Lemma, $V^K(t) = V^K(0)\exp\{(\alpha - \frac{1}{2}\sigma^2)t + \sigma[z(t) - z(0)]\}$. Since $\mathrm{var}[z(t) - z(0)] = t$, the expected growth rate of $V^K(t)$ is α, that is, $E_0 V^K(t)/V^K(0) = e^{\alpha t}$. The assumption of i.i.d. uncertainty is analytically convenient, but it compromises the model's empirical fit. Log U.S. consumption, for example, does not follow an exact random walk, as the model will imply. I.i.d. uncertainty is in part responsible for the extreme equity-premium and risk-free rate puzzles noted below (section VI).

Per capita wealth $W(t)$ is the sum of per capita holdings of the safe asset, $B(t)$, and per capita holdings of the risky asset, $K(t)$:

$$(5) \quad W(t) = B(t) + K(t).$$

To derive the stochastic differential equation governing the evolution of wealth, I return to the temporary assumption of a discrete time interval of length h. Asset holdings for period $t + h$ depend on asset holdings for period t, on the increase in asset values between dates t and $t + h$, and on the rate of consumption over period $t + h$:

$$B(t+h) + K(t+h) = [V^B(t+h)/V^B(t)]B(t) + [V^K(t+h)/V^K(t)]K(t) - C(t+h)h.$$

Subtracting $B(t)$ and $K(t)$ from both sides above and taking the limit as $h \rightarrow 0$, we get

$$dB(t) + dK(t) = [dV^B(t)/V^B(t)]B(t) + [dV^K(t)/V^K(t)]K(t) - C(t)dt.$$

Equations (3), (4), and (5) therefore imply that

$$(6) \quad dW(t) = rB(t)dt + \alpha K(t)dt + \sigma K(t)dz(t) - C(t)dt.$$

Let $\omega(t)$ denote the fraction of wealth invested in the risky asset. An alternative way to write (6) is as follows:

$$(7)\quad dW(t) = \{\omega(t)\alpha + [1-\omega(t)]r\}W(t)dt + \omega(t)\sigma W(t)dz(t) - C(t)dt.$$

Merton (1971) derives closed-form solutions for the problem of maximizing (1) subject to (2), (7), and an initial wealth endowment $W(0) = W_0$. Optimal consumption is a constant proportion μ of wealth,

$$(8)\quad C(t) = \mu W(t),$$

where

$$(9)\quad \mu = (1/R)\{\delta - (1 - R)[r + (\alpha - r)^2/2R\sigma^2]\}.$$

The fraction of wealth ω optimally invested in the risky asset is also a constant, namely

$$(10)\quad \omega = (\alpha - r)/R\sigma^2.$$

II. Closed-Economy Equilibrium

Equilibrium growth in this closed economy can now be described. Because the two capital goods can be interchanged in a one-to-one ratio, instantaneous asset-supply changes always accommodate the equilibrium asset demand given by (10). Notice, however, that in the aggregate a closed economy can not go short in either asset. Thus, if $\omega \geq 1$, only risky capital is held. My earlier assumption that $\alpha > r$ rules out the possibility that only safe capital is held.

To understand how the economy's equilibrium looks when $\omega \geq 1$, we must explicitly consider the bond market, which was kept in the background until now. The representative-agent assumption implies that we can always take an individual's net bond holdings to be zero in equilibrium. When some risk-free capital is demanded, the real interest rate on bonds, i, is simply equal to r, the sure marginal product of that capital. But when there is an *incipient* excess demand for risky capital, the real interest rate adjusts so that the portfolio share individuals wish to devote to risky capital is unity. In this case the real interest rate i exceeds r; and i becomes the risk-free rate relevant to individual decisions. The equilibrium interest rate forcing $\omega = 1$ is $i = \alpha - R\sigma^2$, by (10). With i in place of r, equation (9) still gives the equilibrium consumption-wealth ratio, i.e., $\mu = (1/R)\{\delta - (1 - R)[i + (\alpha - i)^2/2R\sigma^2]\}$.

Together, equations (7) and (8) imply the equilibrium wealth-accumulation equation

$$(11)\quad dW(t) = [\omega\alpha + (1-\omega)r - \mu]W(t)dt + \omega\sigma W(t)dz(t),$$

where it is understood that ω can never exceed 1 in a closed-economy equilibrium. By (8) and (11), equilibrium consumption follows the stochastic process

$$(12)\quad dC(t) = [\omega\alpha + (1-\omega)r - \mu]C(t)dt + \omega\sigma C(t)dz(t).$$

Define g as the instantaneous expected growth rate of consumption:

$$g \equiv \frac{1}{C(t)} \frac{\mathrm{E}_t \, dC(t)}{dt}.$$

Equation (12) shows that g is endogenously determined as the average expected return on wealth, $\omega\alpha + (1-\omega)r$, less the consumption-wealth ratio, μ. Itô's Lemma, applied to (12), reveals the time-t consumption level to be

$$(13)\quad C(t) = C(0)\exp\{(g - \tfrac{1}{2}\omega^2\sigma^2)t + \omega\sigma[z(t) - z(0)]\}.$$

Note that for any $t > 0$, $\mathrm{E}_0\, C(t)/C(0) = e^{gt}$.

By (9) and (10),

$$(14)\quad g = \frac{(1 + R)(\alpha - r)^2}{2R^2\sigma^2} - \frac{(\delta - r)}{R}$$

provided $\omega < 1$. If $\omega = 1$, the economy's expected growth rate can be expressed as

$$(14')\quad g = \frac{\alpha - \delta}{R} - \frac{(1 - R)\sigma^2}{2},$$

which follows from (14) upon substitution of $i = \alpha - R\sigma^2$ for r.

To gain some preliminary insight into the determinants of growth, consider the effects of a fall in σ. If the economy holds some risk-free capital, so that (14) is valid, the growth rate will rise unambiguously. Equation (9) discloses that the effect of the change on the consumption-wealth ratio is ambiguous. The dominant effect on mean consumption growth, however, is that of the portfolio shift from risk-free to risky capital [equation (10)], which increases the average return to saving sufficiently to swamp any increase in the propensity to consume out of wealth. (It should be recognized that the dominance of the portfolio-shift effect results from the specific isoelastic class of preferences assumed above.)

When all of the economy's capital is already in risky form, there can be no equilibrium portfolio shift for a closed economy. In this case equation (14′) is relevant; it shows that a fall in σ raises growth when $R < 1$ but lowers it when $R > 1$. This is the result found by Devereux and Smith (1991). With the economy's production side held fixed, a fall in σ raises growth if and only if it lowers the consumption-wealth ratio. But a fall in σ now affects consumption by pushing up the real interest rate, $\alpha - R\sigma^2$. Since $1/R$ is the elasticity of intertemporal substitution, a rise in the real interest rate lowers C/W (and raises growth) when $R < 1$, but raises C/W (lowering growth) when $R > 1$.

In either case economic growth is decreasing in the impatience parameter δ and increasing in α. The effect of a rise in the safe rate r is ambiguous, however, because a rise in r diverts investment away from more productive risky capital.

When the economy holds both types of capital, the technological parameters α and σ influence the individual's lifetime utility only through their effect on the growth rate, g. This property of the model turns out to be useful in evaluating the growth effects of international asset-market integration. To prove it, I calculate $J(W_0)$, the maximized value of the intertemporal objective (1).

Taking powers of (13) and conditional expectations leads to

$$E_0C(t)^{1-R} = C(0)^{1-R}\exp[(1 - R)(g - \tfrac{1}{2}R\omega^2\sigma^2)t].$$

The maximized value of lifetime utility, equation (1), therefore is

$$(15)\quad J(W_0) = \frac{\mu^{1-R}W_0^{1-R}}{(1 - R)[\delta - (1 - R)(g - \tfrac{1}{2}R\omega^2\sigma^2)]}.$$

Application of (9), (10), and (14) (which is valid when $\omega < 1$) reduces (15) to

$$(16)\quad J(W_0) = (1 + R)^R W_0^{1-R}[2\delta - (1 - R)(g + r)]^{-R}/(1 - R).$$

Clearly an increase in g due to a rise in α or a fall in σ raises lifetime utility.[7]

For an economy specialized in risky capital, however, (14′) leads to

$$J(W_0) = W_0^{1-R}(\alpha - g)^{-R}/(1 - R).$$

Given α and the preference parameters, changes in g can come about only through changes in σ when no riskless capital is held [see (14′)]. As we have seen, a fall in σ may stimulate or depress growth in this case, despite its unambiguously positive welfare effect.

[7]Notice that $[2\delta - (1 - R)(g + r)] > 0$ is required for the existence of an individual optimum.

III. Distinguishing Risk Aversion from Intertemporal Substitutability

Before going further with this model, it is necessary to extend it to a wider class of preferences than the one described by (1) and (2). As is well known, the expected-utility preferences assumed in the last two sections do not allow separate analysis of risk aversion and consumption substitutability over time: above, the former characteristic is measured by R, the latter by $1/R$. Dynamic welfare comparisons that confuse these two concepts can be seriously misleading, however.[8] Furthermore, we would like to address the positive question of how preference parameters influence growth. The effects of intertemporal substitutability on growth have been analyzed extensively (for example, by Romer 1990, Grossman and Helpman 1991, and Rebelo 1991). The effects of attitudes toward risk have not.

Epstein and Zin (1989) and Weil (1989, 1990) present a class of nonexpected-utility preferences that encompasses the preferences used above. These papers assume that time is discrete, but continuous-time extensions by Svensson (1989) and by Duffie and Epstein (1992) provide formulations that are readily applied to the model at hand.

With an arbitrary discrete time interval of length h, lifetime utility from time t onward, $U(t)$, can be defined recursively by the difference equation

$$(17)\quad U(t) = (1 - R)^{-1}\left\{C(t+h)^{1-(1/\varepsilon)}h + e^{-\delta h}[(1 - R)\mathrm{E}_t U(t+h)]^{[1-(1/\varepsilon)]/(1-R)}\right\}^{(1-R)/[1-(1/\varepsilon)]}.$$

Above, $R > 0$ is the (constant) coefficient of relative risk aversion and $\varepsilon > 0$ the (constant)

[8]For further discussion, see Obstfeld (1991).

elasticity of intertemporal substitution.[9] When these parameters are related by $\varepsilon = 1/R$, (17) reduces to the discrete-time analogue of (1) and (2).

The form of equation (17) suggests two conjectures. The first is that maximized lifetime utility $U(0)$ is given by $J(W_0) = (aW_0)^{1-R}/(1 - R)$ for some constant a. The second is that optimal consumption is given by $C(t+h) = \mu W(t)$ for some constant μ. Following Svensson (1989), it can be shown that in continuous time, the stochastic Bellman equation for the problem of maximizing $U(0)$ in (17) subject to (7) and $W(0) = W_0$ is:

$$(18)\quad 0 = \max_{\mu,\omega}\{[1 - (1/\varepsilon)]^{-1}[(\mu/a)^{1-(1/\varepsilon)} - \delta] + [\omega\alpha + (1-\omega)r - \mu - \tfrac{1}{2}R\omega^2\sigma^2]\}.$$

The first-order conditions for this maximization imply the consumption-wealth ratio

$$(19)\quad \mu = \varepsilon\{\delta - [1 - (1/\varepsilon)][r + (\alpha - r)^2/2R\sigma^2]\}$$

and the portfolio share $\omega = (\alpha - r)/R\sigma^2$. The latter asset-demand function is the same as in the expected-utility case [equation (10)], implying that portfolio behavior depends on risk aversion alone. But (19) reduces to (9) only when $R = 1/\varepsilon$. Consumption behavior thus depends on attitudes toward intertemporal substitution as well as toward risk. A final implication of the first-order conditions for (18) is

$$(20)\quad a = \mu^{1/(1-\varepsilon)}.$$

[9]The special cases in which $R = 1$ or $\varepsilon = 1$ can be described separately.

Because equations (8) and (11) remain valid, the economy's growth rate g can be calculated as before, but with (19) in place of (9). In equilibrium, the answer depends again on whether there is a positive demand for safe capital. If some is demanded ($\omega < 1$), the growth rate is a natural generalization of (14),

$$(21)\quad g = (1 + \varepsilon)(\alpha - r)^2/2R\sigma^2 - \varepsilon(\delta - r);$$

otherwise ($\omega = 1$), growth is given by the analogue of (14′),

$$(21')\quad g = \varepsilon(\alpha - \delta) + (1 - \varepsilon)R\sigma^2/2.$$

Notice that the earlier conclusions about the effect of variability on growth are unchanged. When some risk-free capital is held, a reduction in σ always stimulates growth; but when all capital is risky, a reduction in that risk stimulates growth only if the intertemporal substitution elasticity ε exceeds unity.

One can now sensibly ask, however, about the separate impacts of intertemporal substitution and risk aversion on growth.

Consider intertemporal substitution first. In deterministic growth models, the rate of growth is determined by

$$g = \frac{1}{C(t)}\frac{dC(t)}{dt} = \varepsilon(r - \delta).$$

Thus, a rise in the intertemporal substitution elasticity ε raises growth provided r, the private rate of return to investment, exceeds δ, the rate of time preference.

In the present model with uncertainty, however, equations (21) and (21′) both can be written as

$$(22)\quad g - \tfrac{1}{2}R\omega^2\sigma^2 = \varepsilon[\omega\alpha + (1-\omega)r - \tfrac{1}{2}R\omega^2\sigma^2 - \delta].$$

The left-hand side of (22) is the *risk-adjusted* expected growth rate: the negative risk adjustment, $-\tfrac{1}{2}R\omega^2\sigma^2$, is proportional to the degree of risk aversion and to the instantaneous variance of growth. The right-hand side is the difference between the risk-adjusted expected rate of return to investment and the time-preference rate. We therefore have a result analogous to the certainty case. Since the portfolio weight ω is independent of intertemporal substitutability, a rise in the elasticity ε raises the growth rate whenever the risk-adjusted expected return on the optimal portfolio exceeds δ.

Equations (21) and (21′) also reveal how the degree of risk aversion influences growth. The effect of lower R parallels that of lower σ. If some riskless capital is held, lower risk aversion is associated with higher growth. But if only risky capital is held, the effect of R on g is proportional to $1 - \varepsilon$. This last difference can be of either sign.

As in the last section, expected growth is decreasing in δ and increasing in α; but changes in r have an ambiguous effect on growth.

I conclude this section by presenting the expressions for maximized lifetime utility corresponding to (16). These expressions follow from equation (20). Let i again denote the risk-free interest rate, which equals r if $\omega < 1$ but equals $\alpha - R\sigma^2$ if $\omega = 1$. Then for any equilibrium $\omega \in [0,1]$, it can be shown that

$$(23)\quad J(W_0) = W_0^{\,1-R}\{(1+\varepsilon)^{-1}[2\varepsilon\delta + (1-\varepsilon)(g+i)]\}^{(1-R)/(1-\varepsilon)}/(1-R).$$

As before [see equation (16)], i is constant at r when $\omega < 1$, so in this case the technology parameters α and σ influence lifetime utility only through their effects on g.[10] When $\omega = 1$, however, we have $J(W_0) = W_0^{1-R}(\alpha - g)^{(1-R)/(1-\varepsilon)}/(1-R)$.

IV. Growth Effects of International Economic Integration

All of the results above can be extended to describe a multi-country world economy. This analysis will yield predictions about the effect of economic openness on growth.

Let there be N countries, indexed by $j = 1, 2, \ldots, N$. Each country has a representative resident with preferences of the form specified in (17). Preferences may be country specific, however. Country j's representative individual has a relative risk aversion coefficient R_j, an intertemporal substitution elasticity ε_j, and a rate of time preference δ_j.

The safe rate of return r is common to all countries (a condition relaxed in section VI). The cumulative value of a unit investment in country j's capital follows the geometric diffusion

$$(24)\quad dV_j^K(t)/V_j^K(t) = \alpha_j\,dt + \sigma_j\,dz_j(t), \quad j = 1, 2, \ldots, N.$$

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[10]Notice that $[2\varepsilon\delta + (1-\varepsilon)(g+r)] > 0$ is required for the existence of an individual optimum. Equation (23) follows from the observation that μ can be written as a function of g, i, δ, and ε only.

Country-specific technology shocks in (24) display the instantaneous correlation structure

(25) $dz_j dz_k = \rho_{jk} dt.$

The symmetric $N \times N$ covariance matrix $\boldsymbol{\Omega} \equiv [\sigma_j \sigma_k \rho_{jk}]$ is assumed to be invertible.

Our goal is to characterize a global equilibrium with free asset trade. The first step is to describe individuals' decision rules when they can invest in the N risky technologies described by (24) and (25) as well as the safe technology.

Let $\mathbf{1}$ denote the $N \times 1$ column vector with all entries equal to 1, let $\boldsymbol{\alpha}$ denote the $N \times 1$ column vector whose kth entry is α_k, and let $\boldsymbol{\omega}_j$ denote the $N \times 1$ column vector whose kth entry is the demand for country k's risky capital by a resident of country j. A generalization of the last section's argument (as in Svensson 1989) shows that an individual from country j has the following vector of portfolio weights for the N risky assets:

(26) $\boldsymbol{\omega}_j = \boldsymbol{\Omega}^{-1}(\boldsymbol{\alpha} - r\mathbf{1})/R_j.$

The task of describing individual decision rules is simplified by the availability of a mutual-fund theorem identical to the one proved by Merton (1971) in a similar setting. Asset demands of the form (26) imply that every individual will wish to hold the *same* mutual fund of risky assets. The ratio of risk-free wealth to wealth invested in the mutual fund is, however, an increasing function of investor risk aversion. What is convenient about the mutual-fund theorem is its implication that (10) and (19) remain valid, with α replaced by the weighted expected return on the mutual fund, and with σ^2 replaced by the variance of this weighted return.

Equation (26), as noted above, implies that the proportions in which individuals wish to hold the risky assets are independent of nationality. The $N \times 1$ vector of portfolio weights for the resulting mutual fund will be

$$(27)\quad \boldsymbol{\theta} \equiv \boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})/\mathbf{1}'\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1}),$$

where a "prime" (′) denotes a matrix transpose. Since the portfolio weights in (27) are constants, the analysis can proceed as if there is a single risky asset in the world with mean return $\alpha^* = \boldsymbol{\theta}'\alpha$ and with return variance $\sigma^{*2} = \boldsymbol{\theta}'\boldsymbol{\Omega}\boldsymbol{\theta}$.[11]

To envision equilibrium, imagine that N autarkic economies are opened up to free multilateral trade. Since all types of capital may be freely transformed into each other, there can be no changes in the relative prices of assets, which are fixed at 1. Instead, available *quantities* adjust to balance demands, given the world real interest rate, i^*, and the technological parameters in $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$. For example, there may be an initial global excess demand for country 61's risky capital, in which case risky capital resident in country 61, K_{61}, expands under foreign

[11]For example, let the scalar quantity ω_j^* denote $\mathbf{1}'\omega_j$, the share of country j's wealth invested in risky assets. Then by (26),

$$\begin{aligned}
\omega_j^* &= \mathbf{1}'\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})/R_j \\
&= [\mathbf{1}'\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})/R_j][(\alpha - r\mathbf{1})'\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})/(\alpha - r\mathbf{1})'\boldsymbol{\Omega}^{-1}\boldsymbol{\Omega}\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})] \\
&= (\alpha - r\mathbf{1})'[\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})/\mathbf{1}'\boldsymbol{\Omega}^{-1}(\alpha - r\mathbf{1})]/R_j\boldsymbol{\theta}'\boldsymbol{\Omega}\boldsymbol{\theta} \\
&= (\boldsymbol{\theta}'\alpha - r)/R_j\boldsymbol{\theta}'\boldsymbol{\Omega}\boldsymbol{\theta} = (\alpha^* - r)/R_j\sigma^{*2}.
\end{aligned}$$

ownership, while other countries' capital stocks shrink.

It will generally turn out that world investors desire to go short in some countries' risky capital stocks. Since this is not possible in the aggregate, these capitals will be swapped into other forms and the associated activities will simply shut down. In equilibrium, the remaining $M \leq N$ risky capital stocks make up a market portfolio whose proportions are specified by the mutual-fund theorem.

Notice that individual countries can now go short in risk-free capital, that is, can invest a share of wealth greater than 1 in the global mutual fund of risky assets. They do this by net issues of risk-free bonds to foreigners. It may happen, however, as in the closed economy analysis above, that there is an *ex ante* global excess demand for the mutual fund. In this case, the world real interest rate, i^*, rises above r until the global excess demand for risky capital disappears.

More formally, assume that $M \leq N$ risky capital stocks remain in operation after trade is opened and that they are available in the positive quantities $K_1, K_2, \ldots, K_M$. To conserve on notation, let $\boldsymbol{\alpha}$ now denote the $M \times 1$ subvector of mean returns and $\boldsymbol{\Omega}$ the associated $M \times M$ covariance matrix of returns. Define the $M \times 1$ vector of mutual-fund weights $\boldsymbol{\theta}$ by

$$\boldsymbol{\theta} \equiv \boldsymbol{\Omega}^{-1}(\alpha - i^*\mathbf{1})/\mathbf{1}'\boldsymbol{\Omega}^{-1}(\alpha - i^*\mathbf{1}),$$

[equation (27) with the real interest rate i^* in place of r] and denote the fund return's weighted mean and variance by α^* and σ^{*2}, respectively. Then an equilibrium must satisfy the conditions

$$K_j / \sum_{j=1}^{M} K_j = \theta_j > 0 \qquad \text{(for all } j = 1, 2, \ldots, M\text{)},$$

$$\sum_{j=1}^{M} K_j = \sum_{j=1}^{N} (\alpha^* - i^*) W_j / R_j \sigma^{*2},$$

where θ_j is the jth component of $\boldsymbol{\theta}$ and W_j is country j's wealth.

In an integrated world equilibrium, national consumption levels can grow at different rates on average despite the single risk-free interest rate i^* prevailing in all countries. Country j's mean growth rate is

$$(28) \quad g_j^* = (1 + \varepsilon_j)(\alpha^* - i^*)^2 / 2R_j\sigma^{*2} - \varepsilon_j(\delta_j - i^*).$$

Given the world interest rate, country j grows more quickly the greater its tolerance for risk and the lower its degree of impatience. Subject to the condition discussed in the last section, an increase in willingness to substitute intertemporally also is associated with higher growth. Provided any risk-free capital is held in the world, $i^* = r$; but if not, a decrease in all countries' risk aversion implies a higher world interest rate and an ambiguous effect on growth.

Consider next the impact of economic integration on growth. The most straightforward case is that in which all countries hold riskless capital before integration and some continue to hold it afterward. In this case countries share a common risk-free interest rate, r, both before *and after* integration. Equation (23) shows that the expected growth rate must rise in all countries. Economic integration does not change any country's wealth because of the assumption that different types of capital are costlessly interchangeable. But in the present distortion-free setting, trade must raise welfare; and equation (23) shows that at an unchanged interest rate, welfare rises if and only if growth rises. The intuition behind this result follows from the discussion in section II. International portfolio diversification encourages a global shift from (relatively) low-return, low-risk investments into high-return, riskier investments.

A similar argument, again based on equation (23), shows that any country whose risk-free interest rate falls upon integration with the rest of the world must experience an increase in expected growth. Growth can fall only in a country whose real interest rate rises. For such a country, however, the risk-reducing benefits of diversification necessarily outweigh the adverse welfare effect of lower expected growth. (As earlier, the specific parametric class of preferences assumed here is responsible for the strong predictions about growth described above.)

V. Two Simple Examples

This section works out two numerical examples to show how the growth effects of international diversification can imply a large welfare payoff from financial integration. A number of applied studies, for example Lucas (1987), Cole and Obstfeld (1991), and van Wincoop (1991), take consumption growth to be exogenous in their evaluations of the costs of income variability. By comparing the welfare effects in the examples to the numbers a researcher would find if consumption growth were assumed to be exogenous, I can quantify the difference that endogenous consumption growth makes.

Example 1. Imagine a symmetric two-country world ($N = 2$) in which $r = 0.02$, $\alpha_1 = \alpha_2 = 0.05$, $\sigma_1 = \sigma_2 = 0.1$, and returns to capital are uncorrelated, $\rho_{12} = 0$. Preferences are the same in the countries, with $\varepsilon = 0.5$, $R = 4$, and $\delta = 0.02$. Under financial autarky, residents of each country hold a fraction of wealth $\omega = (\alpha - r)/R\sigma^2 = 0.75$ in the domestic risky asset. Equation (21) implies a mean consumption growth rate of $g = \frac{1}{2}(1 + \varepsilon)(\alpha - r)\omega - \varepsilon(\delta - r) = 1.6875$ percent. In both countries the risk-free real rate of interest, i, is equal to r, i.e., $i = 0.02$.

Now let the two countries engage in asset trade. The optimal global mutual fund is divided equally between the two countries' risky capitals. This portfolio's mean rate of return is $\alpha^* = 0.05$ with instantaneous return variance $\sigma^{*2} = (0.1)^2/2 = 0.005$. Each country's total demand for risky assets will now be $\omega^* = (\alpha^* - i^*)/R\sigma^{*2}$; it is simple to check that at a world real interest rate of $i^* = 0.03$, $\omega^* = 1$. Thus, financial integration leads to a rise in the real interest rate, from 0.02 to 0.03, and an equilibrium in which risk-free assets are no longer held. The increase in the world real interest rate reflects lower precautionary saving due to a reduction in the variability of wealth.[12]

From (28) we can calculate the expected consumption growth rate g^* in the integrated equilibrium. Equilibrium growth averages 2 percent per period, as compared with the rate of 1.6875 percent per period characterizing the pre-trade situation.

The welfare gain from economic integration can be calculated as a *compensating variation*: by what percentage λ must wealth be increased under financial autarky so that people

[12]The instantaneous variability of wealth falls from $(0.75)^2(0.1)^2 = 0.005625$ to $(0.1)^2/2 = 0.005$.

enjoy the same level of utility as under financial integration? Using (23) and (28), one finds that $\lambda = 0.371$, or 37.1 percent of initial wealth. This is a very large welfare gain. It is derived from two sources: the opportunity to trade consumption risks *given* the stochastic process governing consumption growth, and the endogenous effect of this risk sharing on the consumption-growth process itself.

Notice that this example assumes an instantaneous reallocation of capital from risk-free to risky uses. Such speedy adjustment would not be observed in practice. Instead, the shift in relative capital stocks would be spread out over time; the post-trade portfolio proportions just described would be reached eventually, but not in the short run. The welfare gain just calculated thus is more realistically viewed as the steady-state increase in annual income due to diversification; it provides no more than an upper bound on the short-run income effect.

Example 2. Let's look at an example in which (1) the induced growth effects of financial integration are essentially zero, and (2) the variance of consumption is closer to the type of number characteristic of the richer industrialized economies. In this case, the welfare effects of financial integration will turn out to be much smaller than above. Let all parameters be as in the previous example, with the exception that now $\sigma_1 = \sigma_2 = 0.02$. Given this change, both countries will hold risky capital only in the pre-trade equilibrium, and their real interest rates will coincide at $i = 0.0484$. In each country, therefore, equation (21) or (21′) gives $g = 0.0154$ as the expected growth rate of consumption.

Under financial integration people will hold a risky asset, the equal-shares mutual fund, whose variance is half that of either country's capital and whose mean rate of return is 5 percent. In the pooled equilibrium the real interest rate is $i^* = 0.0492$, slightly above its level under autarky, and the growth rate of consumption declines very slightly, to $g^* = 0.0152$ percent. The compensating variation measure of the welfare gain from financial integration is now $\lambda = 0.0116$, or 1.16 percent of initial wealth.

The only difference between examples 1 and 2 is that the variance of the risky-capital shock is 25 times larger in the first case than in the second. This leads to a welfare gain from financial integration that is about 32 times larger in the first case. Without knowing about endogenous growth, we might have guessed naively that the welfare gains would be 25 times as great in the first example, not 32 times as great. The resulting underestimate of the gains from financial integration is economically substantial.

A more rigorous way to assess the contribution of endogenous growth is to ask what conclusion a researcher would reach in the examples above if he took the observed consumption processes to be exogenous. The assumption that reducing economic variability does not greatly affect the growth rate of consumption has been typical in recent applied studies of the cost of consumption variability.

Equation (13) implies that under financial autarky, a researcher using annual data would observe the per capita consumption process

$$\log C(t) - \log C(t-1) = 0.0141 + v(t), \quad \sigma_v^2 = 0.00563,$$

given the assumptions of example 1, and the process

$$\log C(t) - \log C(t-1) = 0.0152 + v(t), \quad \sigma_v^2 = 0.00040,$$

given those of example 2. Taking these consumption processes as exogenous, the researcher might suppose that international diversification would halve each of the two variances above, leaving expected growth--which equals the regression constant plus $\frac{1}{2}\sigma_v^2$--unchanged. It is easy to compute the implied compensating-variation measures of welfare gain,[13] which are reported in table I (left-hand column) beside the true gains calculated earlier (right-hand column).

Table I Endogenous Growth and Welfare Comparisons

	Welfare gain assuming exogenous growth (percentage of wealth)	*Welfare gain assuming endogenous growth* (percentage of wealth)
Example 1	21.5	37.1
Example 2	1.2	1.1

In example 2 the growth effects of international financial diversification are minimal. Thus, assuming exogenous consumption growth makes little difference to the answer. But when larger growth effects are present, analyses that fail to account for them can be misleading. Under the parameters of example 1, the true gain from financial integration is 73 percent higher than the number one finds ignoring the endogeneity of growth.

[13]See Obstfeld (1991).

VI. An Example Based on Global Consumption Data

This section is devoted to a final example of the gains from international financial integration. The example is based on actual consumption-growth data, as reported by Summers and Heston (1991) in the Penn World Table (Mark 5). The welfare effects reported below should not be taken as a literal prediction about reality; they simply indicate that, when matched to some realistic parameters, the preceding model could imply very large gains from asset trade.

The example considers an eight-region world, consisting of North America, South America, Central America, East Asia, Noneast Asia, Northern Europe, Southern Europe, and Africa. Within each region, real per capita consumption is a population-weighted average of national per capita consumptions. I use data spanning the period 1960-1987. Only countries with data available over this entire period, and with data quality of at least C− according to Summers and Heston, are included.[14]

Equation (13) implies that the logarithm of per capita consumption follows the random walk with drift

$$\log C(t) - \log C(t-1) = g - \tfrac{1}{2}\sigma_v^2 + v(t),$$

where $v(t) = \omega\sigma[z(t) - z(t-1)]$ and $\sigma_v = \omega\sigma$.[15] Table II reports the information one extracts

[14]National consumption per capita is measured at 1985 international prices as PWT variable 3 times PWT variable 6 (see Summers and Heston 1991, p. 362, for exact definitions). Consumption of nondurables and services only would be a superior consumption measure for the purpose at hand, but data are unavailable for most countries.

[15]Recall that investments in a country's risky capital have cumulative payoffs that follow (4), and that ω denotes the share of risky capital in the optimal portfolio.

Table II Global Regions and Their Consumption Processes, 1960-1987

Mean and standard deviation of annual consumption growth rate (percent)

	NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
g	2.35	3.11	1.68	3.64	0.91	2.87	3.13	1.31
σ_v	1.76	4.57	2.96	2.12	3.02	1.31	3.03	3.59

Correlation coefficients of regional consumption growth rates

	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
NAm	-0.248	-0.113	0.393	0.117	0.366	0.118	-0.415
SAm		0.147	0.134	-0.467	0.440	0.391	0.139
CAm			0.365	-0.136	0.289	0.115	0.525
EAsia				-0.048	0.753	0.369	0.074
NAsia					-0.299	-0.166	-0.299
NEur						0.474	-0.035
SEur							0.321

Regional groupings

North America (NAm): Canada, United States. South America (SAm): Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Paraguay, Peru, Uruguay, Venezuela. Central America (CAm): Costa Rica, Dominican Republic, El Salvador, Guatemala, Honduras, Jamaica, Mexico, Trinidad. East Asia (EAsia): Hong Kong, Japan, South Korea, Malaysia, Philippines, Thailand, Australia, New Zealand. Noneast Asia (NAsia): India, Israel, Pakistan, Sri Lanka, Syria. Northern Europe (NEur): Austria, Belgium, Denmark, Finland, France, West Germany, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Sweden, Switzerland, United Kingdom. Southern Europe (SEur): Cyprus, Greece, Malta, Portugal, Spain, Turkey, Yugoslavia. Africa (Afr): Cameroon, Côte d'Ivoire, Kenya, Morocco, Senegal, South Africa, Tanzania, Tunisia, Zimbabwe.

by fitting this equation to the data: estimates of g and σ_v for the eight regions, as well as an estimate of the correlation matrix of regional consumption shocks. Because production shocks are the only source of consumption uncertainty in the model, the matrix in table II is also the correlation matrix of regional productivity shocks, $z(t) - z(t-1)$. With perfect risk pooling, all entries in this correlation matrix would be 1.

The moments in table II provide a basis for calibrating the model empirically. Any such attempt runs immediately, however, into two well-known problems: the equity-premium puzzle of Mehra and Prescott (1985) and the risk-free rate puzzle of Weil (1989).

To appreciate the equity-premium puzzle, let i again be a country's risk-free rate.. By (10), the equity premium can be expressed in terms of the consumption variance σ_v^2 as

$$(29) \quad \alpha - i = R\sigma_v^2/\omega.$$

Table II shows that in most countries, the variability of consumption growth is too small to generate equity premia on the plausible order of 5 percent per year without some combination extremely high risk aversion and a very low portfolio share for risky assets. In an attempt to meet the data half way, I will assume that $R = 18$ and that the equity premium is 4 percent per year in all regions. Under these assumptions, (29) leads to estimates of ω that are reported in table III. With the exceptions of South America and Africa, where the variability of annual consumption growth is exceptionally high (standard deviations of 4.57 and 3.59 percent, respectively), these portfolio shares for risky assets seem implausibly low. I nonetheless use them to infer estimates of $\sigma = \sigma_v/\omega$, the standard deviation of the underlying annual production shock. These, too, are reported in table III.[16]

Table III highlights a counter-intuitive empirical implication of the model. Equation (10) implies that for given values of the equity premium and R, there is an *inverse* relation between the observed variability of consumption growth, σ_v , and the variability of the underlying

Table III Pre-Trade Portfolio Share of Risky Assets (ω), Expressed as a Fraction, and Standard Deviation of the Annual Return to Risky Investment (σ), in Percent

	NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
ω	0.140	0.939	0.395	0.203	0.412	0.077	0.412	0.579
σ	12.60	4.86	7.50	10.47	7.35	16.98	7.34	6.19

technology shock, σ: $\sigma = (\alpha - i)/R\sigma_v$. Thus, table III suggests that in those regions where the variability of consumption growth is lowest, the variability of technology shocks is greatest. In Northern Europe, for example, the standard deviation of the annual consumption growth rate is only 1.3 percent (table II), yet that of the return to risky capital is reckoned at 17 percent. Conversely, the corresponding standard deviation for risky capital held by South Americans is estimated to be only 4.9 percent. The result could be overturned if the equity premium had a sufficiently strong positive correlation with consumption variability; but the empirical basis for

[16]A value of $R = 18$ would be regarded as unrealistically high by many. Kandel and Stambaugh (1991) marshall arguments to the contrary. To refine my calibration procedure, one could bring additional data to bear, for example, average equity premia for individual countries. Such an attempt would force one to restrict the sample of countries studied.

such an assumption has not been established. Risky nontradable income, which is present in reality, would also break the tight link between consumption variability and the riskiness of capital investments.

Consider next the implications of the risk-free rate puzzle. Equation (21) can be rewritten in the general form

$$(30)\quad g = \tfrac{1}{2}(1 + \varepsilon)R\sigma_v^2 - \varepsilon(\delta - i).$$

Given the low values for σ_v^2 suggested by table II, however, the mean growth rates g in the table cannot be matched unless some combination of the following is true: R is very large, ε is very large, δ is negative, or i is high. Maintaining the assumption that $R = 18$ and setting $\delta = 0.02$ and $\varepsilon = 1.1$, I compute region-specific risk-free interest rates that generate, through (30), the mean consumption growth rates reported in table II.

Table IV reports these rates. Even though an unrealistically high intertemporal substitution elasticity ($\varepsilon = 1.1$) was assumed, the interest rates in the table are still on the high side for some of the regions, in line with the risk-free rate puzzle. Notice that the risk-free rate

Table IV Riskless and Risky Rates of Return, in Percent per Year

	NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
i	3.60	1.25	2.02	4.54	1.26	4.32	3.28	0.98
α	7.60	5.25	6.02	8.54	5.26	8.32	7.28	4.98

is calculated to be relatively low in countries where consumption variability is relatively high. This pattern results mainly from the low risk tolerance assumed earlier, and reflects the precautionary motive for saving.[17] Mean national rates of return to risky capital are calculated as $\alpha = 0.04 + i$. For convenience, I report these rates in the second row of table IV.

The numbers reported in tables II-IV allow computation of the covariance matrix of risky capital returns, and hence of the global equilibrium that would obtain after financial integration

Table V Characterizing Equilibrium under Global Financial Integration

*Equilibrium shares in the risky mutual fund**

NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
0.105	0.225	0.098	0.101	0.205	0.000	0.207	0.058

*Shares sum to 0.999 because of rounding.

Other characteristics of the post-trade equilibrium

Expected annual return on the risky mutual fund (α^*), percent	6.31
Standard deviation of mutual-fund annual return (σ^*), percent	3.41
Share of mutual fund in total wealth (ω^*), fraction	0.847
World annual real rate of interest (i^*), percent	4.54
Expected annual growth rate of consumption (g^*), percent	4.37

[17]Consumption variability is highest in less-developed regions of the Western Hemisphere and Asia. The low real interest-rate levels that these regions therefore display are consistent with the "financial repression" hypothesis of the economic development literature.

(recall section IV). Table V reports the equilibrium portfolio shares in the optimal global mutual fund of risky assets, along with the mean and standard deviation of the fund's annual return (α^* and σ^*), the share of the fund in global wealth (ω^*), the prevailing world interest rate (i^*), and the common new world growth rate (g^*). Notice that Northern European capital disappears entirely from the world portfolio, essentially because it is highly correlated with East Asian capital (the correlation coefficient is 0.753 according to table II) but has a slightly lower expected return (table IV). Equilibrium holdings of risk-free capital are located exclusively in East Asia.

A note of interpretation is in order at this point. The 1960-1987 data sample already is based on some international risk sharing. For example, the high correlation between East Asian and Northern European log-consumption innovations probably reflects some cross-holding of capital. The non-appearance of Northern Europe in the optimal global portfolio therefore does not really mean that no Northern European capital is held in equilibrium. Prior to *full* market pooling, East Asians already hold a portfolio that includes some Northern European capital; after pooling, it is this portfolio, rather than the one Northern Europeans hold, for which demand is positive. Nothing in the calculations requires literal autarky in the pre-integration equilibrium.

Although the expected return on the global portfolio is significantly below that on East Asian capital, for example, global pooling does lead to a substantial reduction in risk (refer back to the second row of table III). In addition, the consumption growth rate rises everywhere. At 4.37 percent per year, equilibrium growth is substantially above even East Asia's pre-trade high of 3.64 percent (table II). This sharp increase comes partly from a drop in the consumption-to-wealth ratio, but primarily from the shift of world wealth into riskier high-yield capital.

The gains from asset trade, reported in table VI, are impressively large, ranging from

Table VI Gains from International Financial Integration, as a Percentage of Wealth

NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
124.5	238.0	299.1	22.6	478.4	61.1	98.8	463.4

478.4 percent of wealth for Noneast Asia to "only" 22.6 percent for East Asia. The uneven regional distribution of trade gains is easy to understand. Areas where returns initially are low gain disproportionately from access to more productive investment technologies. (These gains are especially large because of the assumed absence of diminishing returns to investment.) Naturally, the gains in table VI also reflect the advantages of worldwide risk sharing.

As an alternative experiment, consider a world where equity premia in some developing regions exceed those in the industrialized world. To model this possibility, while keeping the average equity premium constant in some sense, I continue to assume $R = 18$, $\varepsilon = 1.1$, and $\delta = 0.02$, but take $\alpha - i$ to be 0.05 in South and Central America and in Noneast Asia; 0.04 in East Asia and Southern Europe; and 0.03 in North America, Northern Europe, and Africa.

Now full financial integration causes regional growth rates to rise from the values in table II to 5.03 percent annually. The share of risky assets in the global portfolio is unity ($\omega^* = 1$) and the world interest rate $i^* = 4.71$ percent per year. Without going into further detail, I report the concomitant welfare gains in table VII. These gains are even larger than those shown in table VI, and continue to accrue disproportionately to developing regions.

The results in tables VI and VII must be applied cautiously. As already noted, the model lacks realism along several dimensions. In particular, I have assumed away capital relocation

Table VII Gains from Financial Integration, as a Percentage of Wealth, under an Alternative Equity-Premium Assumption

NAm	SAm	CAm	EAsia	NAsia	NEur	SEur	Afr
185.7	330.0	407.8	56.0	635.8	105.0	152.9	616.7

costs; and by assuming a technology that fixes the relative price of different types of capital at unity, I have abstracted from the initial *asset-price effects* of financial integration that would, in reality, modify the preceding welfare gains substantially. (Clearly, the biggest winners above are the same regions whose capital stocks would be worth least in an integrated equilibrium.) Also, the calibration underlying the tables assumes preference parameters that may seriously overstate both risk aversion and willingness to substitute consumption over time. Welfare gains even a tenth as large as those in the tables would be significant, however, particularly for countries in the developing world.

VII. Conclusion

This paper has demonstrated that international risk sharing can yield substantial welfare gains through its positive effect on expected consumption growth. The mechanism linking global diversification to growth was the attendant world portfolio shift from safe, but low-yield, capital into riskier, high-yield capital.

The model makes this theoretical point cleanly, but its empirical applicability is limited by several factors. One set of factors revolves around the equity-premium and risk-free rate puzzles familiar from United States data. Another, not entirely separate, issue is the probable importance of nontradable income risk. The model assumes a single consumption good and ignores the presence of goods that do not enter international trade at all. Finally, the absence of capital-adjustment costs and of capital-gains effects are drawbacks, except, perhaps, for analyzing comparative steady states. Further empirical and theoretical work is needed before reliable welfare evaluations can be made using models such as the one presented here.[18]

Underlying the model's stylized assumptions was the idea that growth requires an ever-increasing array of specialized inputs, the development of which is inherently risky. An extension of the analysis would model explicitly the mechanisms leading to the creation of special-purpose versus general-purpose production inputs. Grossman and Shapiro (1982) study a static model of the decision to specialize. A dynamic model incorporating similar decisions would be likely to have implications for growth similar to those derived above.

[18]Nontradable income risk might well enhance the gains from international risk sharing, as in van Wincoop's (1991) model.

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