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LIQUIDITY EFFECTS, MONETARY POLICY, AND THE BUSINESS CYCLE

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ABSTRACT

This paper presents new empirical evidence to support the hypothesis that positive money supply shocks drive short-term interest rates down. We then present a quantitative, general equilibrium model which is consistent with this hypothesis. The two key features of our model are that (i) money shocks have a heterogeneous impact on agents and (ii) ex post inflexibilities in production give rise to a very low short-run interest elasticity of money demand. Together, these imply that, in our model, a positive money supply shock generates a large drop in the interest rate comparable in magnitude to what we find in the data. In sharp contrast to sticky nominal wage models, our model implies that positive money supply shocks lead to increases in the real wage. We report evidence that this is consistent with the U.S. data. Finally, we show that our model can rationalize a version of the Real Bills Doctrine in which the monetary authority accommodates technology shocks, thereby smoothing interest rates.

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1. Introduction.

Economists have long studied the mechanisms by which monetary policy affects aggregate economic activity and interest rates. Much of the recent literature has emphasized the alleged role of "sticky" nominal wages and prices in explaining the expansionary effects of monetary policy. In contrast, this paper studies an alternative channel, one which emphasizes the liquidity effects on interest rates of unanticipated changes in the money supply.

Why emphasize this particular monetary transmission mechanism? The answer is that, in our view, post–war U.S. data support the conclusion that exogenous increases in the supply of money generate substantial, persistent declines in short–term interest rates. This view contrasts sharply with that of the traditional literature on the subject, which has tended to conclude that money supply shocks raise, rather than lower, short–term interest rates (Reichenstein 1987). Section 2 of this paper presents new evidence which, when taken in conjunction with a number of recent papers on the interest rate effects of monetary policy (surveyed in section 2), casts considerable doubt on the basic conclusion reached in the traditional literature.

Surprisingly, existing quantitative models of money are inconsistent with the view that positive money supply shocks drive interest rates down. For example, King (1992) discusses the difficulty of reconciling sticky wage and sticky price models with this view. Modified real business cycle (RBC) models where money is introduced simply via cash–in–advance constraints (as in Greenwood and Huffman (1987), Cooley and Hansen (1989), or Christiano (1991)) or a transactions role for cash (as in Kydland (1989), Marshall (1987) or den Haan (1990)) are also inconsistent with this view. This is because a generic implication of these models is that, if money growth displays positive persistence, then unanticipated shocks to the growth rate of money drive the nominal interest rate *up*, but employment and output *down*. This reflects the fact that, in these models, money shocks affect interest rates exclusively through an anticipated inflation effect. The *only* way for an exogenous shock to the money supply to drive the interest rate down in these models is for the shock to signal a subsequent decline in money growth. Not surprisingly, this requires grossly counterfactual assumptions regarding the law of motion for the money supply.

So, an important challenge is to identify the features of the real world which are missing from existing models and which prevent them from replicating the negative interest rate response to money shocks. In section 3 of this paper, we present a model which allows us to explore the quantitative importance of two features. The first is that money injections have a heterogeneous impact on agents. In stressing this feature, we are following a tradition of theoretical papers which argue that the key to understanding the nonneutralities of money shocks is to understand that they impact differently on different agents (Grossman and Weiss 1983; Rotemberg 1984; Woodford 1987; Baxter, Fischer, King and Rouwenhorst 1990.) This paper follows Lucas (1990) and Fuerst (1992a) in supposing that firms and financial intermediaries are the key subset of agents which absorbs a disproportionately large share of money supply shocks. To generate this result, we suppose, as do Lucas and Fuerst, that households make their nominal consumption–saving decision before the realization of monetary policy. This assumption reflects the view that, in reality, firms and financial intermediaries respond virtually instantaneously to movements in asset prices induced by central bank open market operations, while households' responses are more sluggish. It is well known that whenever a subset of agents is forced to absorb a disproportionate share of a money injection, it is possible that the equilibrium rate of interest will fall.[1] Indeed, in our model, heterogeneity per se guarantees this result.

[1]It is important to emphasize that this is only a possibility. As long as there are anticipated inflation effects associated with a money supply shock, then it is possible that these could swamp, in equilibrium, the liquidity effects associated with heterogeneity. For example, Christiano (1991) shows that this is the case in a plausibly parameterized version of Fuerst's (1992a) model.

But, we find that heterogeneity alone does not generate a large enough interest rate response by comparison with the data. This motivates the second key feature of our model. Specifically, we assume that money shocks occur at a time when firms have already precommited themselves to particular production plans, and that these are difficult to adjust ex post. This is important because we assume firms must finance their variable inputs (i.e., labor) on a pay–as–you–go basis with cash. Since revenues do not accrue until the end of the production period, firms are forced to borrow working capital in advance. The need for money to carry out production gives rise to a well–defined demand for money on the part of firms. The assumption that production plans are difficult to adjust once initiated gives rise to a very small ex post, or short–run, interest elasticity of demand for money on the part of firms. This characteristic of the model conforms well with the view, widely held in the U.S. Federal Reserve System, that the short–run interest elasticity of the demand for total reserves is very close to zero (see Strongin 1992.) This low elasticity greatly amplifies the interest rate impact of a money supply shock in our model.

Section 4 reports the dynamic effects of a money supply shock in a fully parameterized version of our model. We find that the contemporaneous response to an unexpected increase in the growth rate of money is a decline in the nominal interest rate, and an increase in employment, the real wage, consumption, and output. While positive, the contemporaneous rise in the rate of inflation is less than the percentage increase in the growth rate of money. Thereafter, the nominal interest rate and the rate of inflation rise, overshoot and then gradually return to their steady state values. During the overshooting phase, nominal interest rates are higher than they were before the initial increase in the growth rate of money. Consumption, employment and the real wage fall after their initial increase and then also gradually return to their steady state values. Finally, after some delay, investment also increases and then slowly reverts to its steady state level. Taken together, the qualitative response of the system to unanticipated changes in monetary policy is very similar to that described by Friedman (1968) in his 1967 Presidential

Address.

In addition, section 4 presents evidence on a key implication of our model which distinguishes it from an important competing model of the monetary transmission mechanism. In our model, a positive money supply shock leads to a rise in the real wage. Sticky wage models of the sort analyzed by Fischer (1977), Cho and Cooley (1990), and King (1992) and King and Watson (1992) imply the opposite. We show that various empirical measures of the real wage rise in response to a money supply shock. We interpret this evidence as supportive of our model.[2]

Section 5 briefly investigates a subset of our model's policy implications. The same features of our model that generate a liquidity effect also imply that the monetary authority has greater flexibility than households to quickly direct cash to the financial sector when it is needed. Because of this, the model can rationalize a version of the Real Bills Doctrine. According to this doctrine, it is welfare improving for the monetary authority to increase the money supply in response to unanticipated changes in the real production opportunities facing the private sector. Unless the monetary authority stands ready to supply needed working capital in times like this – say, by rediscounting commercial paper – productive opportunities will go unexploited. Interestingly, this perspective on monetary policy is very close to the one that motivated the United States Federal Reserve Act, which begins by stating, among other things, that the central bank should "... furnish an elastic currency, to afford means of rediscounting commercial paper, ..." (Federal Reserve Board, 1988.)

2. Some New Evidence on the Interest Rate Effect of a Money Shock

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[2]This evidence does not distinguish between our model and sticky price models of the sort analyzed in Cho and Cooley (1990), King (1992), and King and Watson (1992). These models imply a rise in the real wage after a money supply shock.

This section presents new empirical evidence to support the hypothesis that positive money supply shocks drive short–term interest rates down. In addition, we reconcile our results with those in the traditional literature.

The results in the traditional literature are based on identifying money supply shocks with unanticipated movements in broad monetary aggregates. When the analysis is redone using the measure of money that is directly affected by open market operations, namely nonborrowed reserves (NBR), the results in the literature are exactly reversed. In particular, innovations to NBR are associated with sharp, persistent declines in short–term interest rates. In addition, innovations in NBR are followed by persistent increases in broader monetary aggregates (Strongin 1992.) A straightforward explanation of these results is that liquidity effects are quantitatively important and that NBR innovations primarily reflect exogenous shocks to the supply of money, while innovations to broader monetary aggregates primarily reflect shocks to demand (say, disturbances to costs of financial intermediation.) Goodfriend (1993), Meulendyke (1989) and Strongin (1992) have sketched models of the Federal Reserve's operating procedures which are consistent with this view. Using a very different style of analysis, Bernanke and Blinder (1990), Gali (1992), King and Watson (1992) and Sims (1986, 1992) also interpret innovations to broad monetary aggregates as primarily reflecting shocks to money demand.

To measure the interest rate response to an exogenous money supply shock, one must first take a stand on an empirical measure of that shock. The traditional literature identifies the money supply shock with the disturbance term in a regression equation of the form,

$$\log M_t = \zeta(\Omega_t) + \epsilon_t. \tag{2.1}$$

Here, Ω_t is a time t information set to be discussed momentarily, ϵ_t is orthogonal to Ω_t, ζ is a linear function and M_t is the money stock. To rationalize interpreting ϵ_t as the exogenous shock to the money supply, (2.1) must be viewed as the monetary authority's decision rule for setting M_t. The set Ω_t includes the set of variables (past, and possibly some current) that the monetary authority looks at when setting the money supply. The fitted residual in this regression, $\hat{\epsilon}_t$, is the empirical measure of the date t money supply shock.[3] The interest rate response to a money shock is measured by the regression coefficients of the interest rate on current and lagged $\hat{\epsilon}_t$'s. These coefficients coincide in population with the impulse response functions emerging from an appropriately specified vector autoregression. We exploit this fact in the calculations reported below.

To proceed, one must specify Ω_t, a measure of M_t and a measure of the short–term interest rate, R_t. In practice, the choice of short–term interest (the three–month Treasury bill rate, the short–term commercial paper rate, or the federal funds rate) does not impact on inference. For simplicity, we work with the federal funds rate. Here we assume that Ω_t is composed of lagged values of the log of real gross national product (GNP), the log of the GNP deflator, log M_t and log R_t.

The solid line in Figure 1 depicts our point estimate of the dynamic response of R_t to an expansionary policy shock for three different measures of M_t. The dashed lines represent a two–standard deviation confidence band about our point estimates. The three measures of M_t underlying Figures 1a – 1c are nonborrowed reserves (NBR), the monetary base (M0) and M1. Seasonally adjusted, quarterly data for the period 1966:1 – 1991:2 were used.[4] Figure 1 reveals that when M_t is measured by either M0 or M1, positive money supply shocks give rise to persistent increases in R_t. This finding reproduces the results

[3]Other papers that adopt this general strategy for measuring money supply shocks include Barro (1977, 1978), Barro and Rush (1980), King (1992), Leeper and Gordon (1992), and Mishkin (1983).

[4]The impulse response functions reported in Figure 1 were based on estimating a four variable VAR, $Z_t = A(L)Z_{t-1} + v_t$, where v_t is iid and $Ev_tv_t' = V$. Here, $Z_t = [\log M_t, \log GNP_t, \log P_t, \log R_t]$, and P_t is the GNP deflator. Also, $A(L) = A_0 + A_1L + ... + A_nL^n$, where L is the lag operator, and $n = 5$ when M is measured by NBR or M0, and $n = 9$ when M is measured by M1. Lag lengths were selected based on the Q–statistics discussed in Doan (1990). The money supply shock is identified as the first element of Dv_t, where D is lower triangular with ones on the diagonal, and $DD' = V$. The confidence intervals were computed using the method described in Doan (1990) using 100 draws from the estimated asymptotic distribution of the VAR coefficients. For further discussion, see Christiano and Eichenbaum (1992b).

underlying claims in the literature that positive money supply shocks drive interest rates up, not down.

In sharp contrast, when M_t is measured by NBR, a positive money supply shock produces a sharp, persistent, statistically significant decline in R_t. Christiano and Eichenbaum (1992b) show that the qualitative features of Figure 1a – 1c are robust to (i) the use of monthly data with industrial production replacing GNP, (ii) splitting the sample at the end of 1979, and (iii) alternative specifications of Ω_t. In particular, they consider four alternatives which involve different specifications of which date t variables enter Ω_t. Specifically, in one case, they include log R_t; in the second, log GNP_t; in the third, log P_t; and in the fourth, log P_t and log GNP_t. The negative dynamic response of R_t to an innovation in NBR_t reflects in part the fact that NBR displays a strong negative correlation with the federal funds rate (Christiano and Eichenbaum 1992b). This is apparent from Figure 2, which displays the detrended federal funds rate and detrended NBR.[5]

The dramatic differences in the results based on NBR and M0 are due to the behavior of borrowed reserves (BR) (i.e., reserves borrowed by banks at the Federal Reserve's discount window.) To see this, consider Figure 1d which reproduces Figure 1b using M0 minus BR as the measure of money. Notice that results based on M0 – BR resemble closely those based on NBR. This finding mirrors the result in Christiano and Eichenbaum (1992b) that M0 – BR and the federal funds rate display a strong, statistically significant negative correlation, while M0 and the federal funds rate display a positive correlation.

These results might seem surprising given the small absolute magnitude of BR.

[5]These correlations correspond to variables which have been logged and then rendered stationary via the Hodrick Prescott (1980) filter. Christiano and Eichenbaum (1992b) show that the negative relation documented in Figure 2 is robust to alternative detrending methods. We emphasize that this filter was used only for the purpose of estimating correlations. It was not used for computing impulse response functions, which are based on the log levels of the data.

Indeed, as column (2) of Table 1 indicates, the average value of the ratio of BR to M0, is only 0.66 percent. But for second moments what matters is not that BR is small, but rather that its changes are typically very large. Column (3) in Table 1 presents evidence on this point. There we report statistics which measure the changes in BR relative to the changes in Total Reserves (TR), M0 and M1. These statistics are calculated as follows. First, define the absolute change in a variable, y_t, relative to TR as

$$v_y = \frac{1}{T} \sum_{t=2}^{T} |y_t - y_{t-1}| / TR_t. \tag{2.2}$$

We normalize by TR in order to ensure that the variable being averaged is stationary. The change in BR relative to the change in another variable, y, is defined as v_{BR}/v_y.

Table 1: Magnitude of Level and Changes in Borrowed Reserves
1966Q1 – 1990Q4

(1) Variable (Y)	(2) mean, BR/Y	(3) v_{BR}/v_Y
Total Reserves	.0250	.64
M0	.0066	.13
M1	.0023	.05

Note: Column 1 — variable analyzed, as indicated, in columns (2) and (3)
Column 2 — sample mean of ratio, borrowed reserves to Y
Column 3 — see equation (2.2) in the text.

As column 3 of Table 1 indicates, changes in BR are on average 64, 13 and 5 percent of changes in TR, M0, and M1, respectively. In light of this, it is not surprising that BR could have such a large impact on the estimated impulse response functions of TR and M0. The sign switch in these functions reflects the well–known fact (documented, for example, in Christiano and Eichenbaum (1992b)) that BR displays a strong positive correlation with the federal funds rate. Goodfriend (1983) and others have argued that this correlation reflects the propensity of banks to increase borrowing at the Federal Reserve's window when the spread between the federal funds rate and the discount rate increases.

In evaluating the model of section 3, it is useful to have a sense of the magnitude of the interest rate response to a one percent, exogenous money supply shock. According to Figure 1a, the response of R_t to a one standard deviation innovation in NBR is $-.001$ to $-.002$, depending on whether one focuses on the first or second quarter response. At the same time, the standard deviation of an innovation to NBR is .015, or 1.5 percent. The average value of the ratio of NBR to M0 over the period 1965:1 – 1990:1 is about 1/4, so that a 1.5 percent innovation in NBR corresponds to a $1.5/4 = .375$ percent innovation in the money supply (as measured by M0). This implies that a one percent jump in the supply of money leads to a $100 \times .001/.00375 = 26.7$, or a $100 \times .002/.00375 = 53.3$ basis points change in the quarterly federal funds rate, depending on whether one uses the first or second period interest rate response.

One source of bias in the previous calculations leads them to understate the interest rate effect of a one percent unexpected increase in the money supply. Federal Reserve discount window lending increases with higher interest rates. Consequently, an exogenous jump in NBR would not show up dollar for dollar in total reserves. Strongin (1992), for example, takes the extreme position that the discount window is operated in such a way that total reserves are completely insulated in the short run from exogenous shocks to NBR. In this case, the proper term to have used for the denominator in the above calculations would have been zero, and we would have reported an infinite liquidity effect! These considerations suggest interpreting our previous calculations as providing a lower bound on the interest rate response to an exogenous shock in the money supply.

To summarize, movements in the federal funds rate are positively associated with movements in broad monetary aggregates, and are negatively associated with NBR.

Algebraically – at least for M0 and NBR – this reflects the role of BR. An important challenge for students of monetary policy is to develop an integrated explanation for these facts. This would require a detailed model of the links between NBR, BR, total reserves, M0, the federal funds rate, other interest rates and the impulses which impact on these variables. Clearly, this is beyond the scope of this paper. Still, our evidence on NBR, in conjunction with the institutional arguments in Goodfriend, Meulendyke, and Strongin, as well as the evidence in Bernanke and Blinder, Gali, King and Watson, and Sims strongly suggest that a first order property of monetary policy is this: exogenous increases in the money supply drive short–term interest rates down, not up.

3. One Way to Think About Liquidity Effects

This section presents a model which is capable of rationalizing the evidence that a positive money supply shock leads to a sharp decline in short–term interest rates.

3.1 The Model

The model economy is populated by three types of perfectly competitive agents: households, goods producing firms and financial intermediaries. We represent each type of agent by a single, representative agent. In addition, there is a monetary authority. At the beginning of time t, the representative household is in possession of the economy's entire beginning–of–period money stock, M_t. By the end of the period, the entire money stock is held by the representative firm. The cash flow pattern from the household, the representative financial intermediary and the monetary authority to the firm is displayed graphically in Chart 1.

At the beginning of time t, the household allocates its cash between two uses: loans to the financial intermediary and purchases of the consumption good. In particular, the household lends N_t dollars, at the gross nominal interest rate R_t, to the financial intermediary, and sets aside $M_t - N_t$ dollars for the purpose of purchasing consumption goods. By assumption nominal consumption must be fully financed with cash. This cash–in–advance constraint can be satisfied using current wage earnings as well as $M_t - N_t$.[6]

In addition to N_t, another source of funds for the financial intermediary is lump sum injections, X_t, of cash by the monetary authority. The financial intermediary lends its cash, $N_t + X_t$, to the firm which requires working capital to finance its production activities. To capture the notion that working capital is required for production we suppose that, while investment is a credit good, labor must be paid in cash on a pay–as–you–go basis.[7] Absent other sources of cash, the firm must therefore borrow enough working capital to cover its labor costs.

Chart 2 provides a graphical description of the way in which money flows back to the household. As owner of the firm, the household receives dividends, F_t, equal to all of the cash which the firm has at the end of the period. Since investment is a credit good, F_t simply equals the firm's nominal revenues from selling consumption goods, net of its interest plus principal payments to the financial intermediary. The financial intermediary passes the cash it receives from the firm on to the household in two forms. First, R_tN_t dollars are sent to the household in payment for the N_t dollars lent to the financial intermediary at the beginning of the period. The remaining cash, which reflects profits from lending the monetary injections to the firm, is sent to the household in the form of dividends, D_t. These payments reflect the fact that the financial intermediary is owned by the household. Finally, the household also receives wage payments from the firm.

We now present a formal description of our model by discussing the objectives and

[6]We allow households to spend their current wage earnings in order to minimize the impact of inflation on average employment in the model. For a further discussion of this, see Christiano (1991).

[7]We make investment a credit good in an effort to minimize the impact of inflation on average employment in the model. For a further discussion of this, see Christiano (1991) and Stockman (1981).

constraints facing the firm, the household, and the financial intermediary.

Firm

The time t technology for producing new goods is given by

(3.1) $$f(K_t, z_tH_t) = K_t^{\alpha}(z_tH_t)^{1-\alpha} + (1-\delta)K_t, \quad 0 < \alpha < 1, 0 < \delta < 1,$$

where

(3.2) $$z_t = \exp(\mu t + \theta_t).$$

Here K_t is the stock of capital at the beginning of time t, H_t represents a weighted average of hours worked over the period, δ is the rate of depreciation on capital and the function $f(\cdot,\cdot)$ denotes new time t output plus the undepreciated part of capital. The variable z_t denotes the time t level of technology which has an unconditional growth rate of μ. As is standard in the RBC literature, we assume that the technology shock θ_t evolves according to

(3.3) $$\theta_t = \rho_\theta \theta_{t-1} + \epsilon_{\theta t},$$

where $0 < \rho_\theta < 1$ and $\epsilon_{\theta t}$ is an iid shock to θ_t with standard deviation $\sigma_{\epsilon\theta}$. The variable $\epsilon_{\theta t}$ is assumed to be orthogonal to all other variables in the model.

There are a variety of ways to capture the sort of ex post inflexibilities in production alluded to in the introduction. A simple way to capture these is to consider a technology in which date t output requires a sustained flow of labor input over the production period. To this end, we suppose that H_t is a function of two discrete sequential labor inputs, H_{1t} and H_{2t}, which are combined via the technology:

$$(3.4) \qquad H_t = [.5(H_{1t})^{(\sigma-1)/\sigma} + .5(H_{2t})^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)},\ 0 < \sigma.$$

Here H_{1t} and H_{2t} denote labor hours in the first and second parts of the production period, respectively. The fact that we split the period into only two parts rather than allowing H_t to depend on a continuous flow of hours worked throughout the period is motivated by considerations of tractability. (One way to interpret (3.4) is that H_{1t} represents time spent producing a nonstorable intermediate input which is later combined with H_{2t} and K_t to generate final output.) In equation (3.4), the parameter σ is the elasticity of substitution between H_{1t} and H_{2t} in production. As σ goes to infinity, the two labor inputs become perfect substitutes. In contrast, as σ goes to zero, (3.4) corresponds to a Leontieff technology in which H_{1t} and H_{2t} are related by fixed coefficients and no substitution is possible. Equation (3.4) characterizes a production process with ex post inflexibility. This is because at the time H_{2t} is selected, the precommited value of H_{1t} imposes a restriction on the firm's production technology.

Given our cash flow assumptions, the firm must borrow working capital from the financial intermediary to cover its labor costs. In particular, it must borrow $W_{1t}H_{1t}$ dollars to finance labor in the first part of the production period and $W_{2t}H_{2t}$ dollars to finance labor in the second part of the production period.[8] Here, W_{it} denotes the time t dollar price for a unit of type i labor, i = 1,2. We denote the gross nominal rate of interest on these two types of loans by R_{it}, i = 1,2. All loans must be repaid to the financial intermediary at the end of period t. Consequently, the total time t costs, inclusive of financing costs, associated with hiring labor equals

[8]We rule out the possibility that firms borrow more than they need to finance production in the first part of the production period. This is a nonbinding restriction on firms since competitive behavior on the part of financial intermediaries implies that firms cannot increase their discounted expected profits by holding extra cash from the first to the second part of the production period.

(3.5) $$R_{1t}W_{1t}H_{1t} + R_{2t}W_{2t}H_{2t}.$$

To capture the notion that open market operations may occur in the midst of ongoing production operations we suppose that H_{1t} is chosen before, and H_{2t} after, the time t realization of monetary policy, X_t.

Each period the firm also invests in capital. Because we assume capital goods are a credit good, the end–of–period cash position of the firm is given by

(3.6) $$F_t = P_t\{f(K_t,z_tH_t) - K_{t+1}\} - R_{1t}W_{1t}H_{1t} - R_{2t}W_{2t}H_{2t},$$

where P_t denotes the time t dollar price of a unit of the consumption good. We assume that F_t is distributed to the firm's owner, the household, at the end of each period after the consumption good market closes.

At this point it is convenient to define the information sets Ω_{t0}, Ω_{t1} and Ω_{t2} where

Ω_{t0} includes aggregate K_t and the values of all model variables dated time t–1 and earlier,

Ω_{t1} includes Ω_{t0} and θ_t,

Ω_{t2} includes Ω_{t1} and X_t.

Acting in the best interests of its owner, the firm maximizes the present discounted value of the dividend flow to the household. Let $U_{C,t+1}$ denote the time t+1 marginal utility of consumption of the household. Then the problem of the firm at time 0 is to choose contingency plans for $\{H_{1t}, H_{2t}, K_{t+1}: t \geq 0\}$ in order to maximize:

(3.7) $$E\,\{\Sigma_{t=0}^{\infty}\beta^{t+1}\frac{U_{C,t+1}}{P_{t+1}}F_t \mid \Omega_{t1}\},$$

subject to the technology for producing new goods, (3.1) – (3.4), and the definition of dividends given by (3.6). The contingency plans for H_{1t} and K_{t+1} are constrained to be functions of the elements of Ω_{t1} while the plan for H_{2t} is constrained to be a function of the elements of Ω_{t2}. In solving its maximization problem, the firm takes $\{R_t, W_{2t}, R_{2t}, P_t, X_t, U_{C,t+1}\}$ to be known functions of Ω_{t2} and takes $\{W_{1t}, R_{1t}, \theta_t\}$ to be known functions of Ω_{t1}. It behaves competitively by taking these objects to be exogenous and beyond its ability to control.

The firm's criterion function, (3.7), reflects our timing assumptions regarding the distribution of dividends to the household. The term $\beta^{t+1}U_{C,t+1}/P_{t+1}$ is the marginal utility to a household of a dollar received at the end of time t. The reason that the subscript t+1 appears in this expression is that time t dividends cannot be spent on consumption until time t+1.

Household

At the beginning of time 0, the household ranks alternative streams of consumption and leisure according to the criterion function:

$$(3.8) \qquad E\,\{\Sigma_{t=0}^{\infty}\beta^t U(C_t, J_t)\,|\,\Omega_{t0}\}.$$

Here β is a subjective discount rate between 0 and 1, C_t denotes consumption at time t, and J_t denotes hours of leisure at time t,

$$(3.9) \qquad J_t = 1 - L_{1t} - L_{2t},$$

where L_{it}, i = 1,2 denotes the number of type i hours worked by the household at time t and the household's time t endowment of hours is normalized at unity. Throughout we assume that the function $U(\cdot,\cdot)$ is given by:

$$(3.10) \qquad U(C_t, J_t) = [C_t^{1-\gamma} J_t^{\gamma}]^{\psi}/\psi \quad \text{for } -\infty < \psi < 1,\ \psi \neq 0,$$
$$= (1-\gamma)\ln(C_t) + \gamma\ln(J_t) \quad \text{for } \psi = 0,$$

where γ is scalar between zero and one.

The household's optimization problem consists of maximizing (3.8) subject to (3.9), (3.10), $N_t < M_t$, its cash constraint,

$$(3.11) \qquad M_t - N_t + W_{1t}L_{1t} + W_{2t}L_{2t} \geq P_tC_t,$$

and its budget constraint,

$$(3.12) \qquad M_{t+1} = R_tN_t + D_t + F_t + (M_t - N_t + W_{1t}L_{1t} + W_{2t}L_{2t} - P_tC_t).$$

In (3.12) D_t denotes time t dividends received from the financial intermediary and is discussed below.

The maximization occurs by choice of contingency plans for setting N_t as a function of the elements of Ω_{t0}, L_{it} as a function of the elements of Ω_{ti}, i = 1,2, and C_t as a function of the elements of Ω_{t2}. In solving its optimization problem, the household behaves competitively by taking $\{R_{1t}, W_{1t}, \theta_t\}$ to be given functions of Ω_{t1} and $\{P_t, R_{2t}, R_t, W_{2t}, F_t, D_t, X_t\}$ to be given functions of Ω_{t2}.

Financial Intermediary

Recall that the financial intermediary has two sources of funds: N_t and cash injections, X_t, from the monetary authority. However, by assumption its supply of loans for financing type 1 labor, N_{1t}, is determined prior to the realization of the time t cash injection. Consequently, the financial intermediary faces the sequence of cash constraints:

$$N_{1t} \leq N_t, \tag{3.13}$$

and

$$N_{1t} + N_{2t} \leq N_t + X_t. \tag{3.14}$$

The variable N_{2t} denotes the supply of loans for financing type 2 labor. Throughout we assume an interior solution for N_{it}, i = 1,2, for which (3.14) holds with equality. This restriction is nonbinding as long as $R_{2t} > 1$.

To display the financial intermediary's problem, we begin by noting that its net cash position at the end of the period, D_t, is given by

$$D_t = R_{1t}N_{1t} + R_{2t}N_{2t} - R_tN_t. \tag{3.15}$$

These are distributed to the household at the end of time t after the consumption good market has closed. Acting in the best interests of its owner, the financial intermediary maximizes:

$$E\,\beta\,\{\frac{U_{C,t+1}}{P_{t+1}} D_t \mid \Omega_{t1}\} \tag{3.16}$$

by choice of contingency plans for $\{N_{1t}, N_{2t}: t \geq 0\}$ subject to (3.13) and (3.14). In addition, the contingency plan for N_{it} is constrained to be a function of the elements of Ω_{ti}, i = 1,2. The financial intermediary is perfectly competitive and takes $\{R_{1t}\}$ to be a known function of Ω_{t1} and $\{R_{2t},R_t\}$ to be known functions of Ω_{t2}.

The interest rate R_t is determined by the condition that the intermediary earns zero profits on funds received from the household. This requires that

$$R_t = [N_{1t}R_{1t} + (N_{2t} - X_t)R_{2t}]/N_t. \tag{3.17}$$

The market structure which we have imposed allows the firm and financial intermediary to interact only in sequential spot markets for loans. Other arrangements are of course possible. In considering these alternatives, it is important to bear in mind that there is no way for agents to diversify away from the risk arising from aggregate shocks to the money supply. For example, the financial intermediary might promise to deliver a noncontingent level of N_{2t} at a noncontingent rate of interest prior to seeing the realization of X_t. In this case, the financial intermediary would have to enter into a spot market for funds if the cash injection turned out to be lower than anticipated. It would end up borrowing funds from the firm at a premium. In effect the financial intermediary would be paying a state–contingent cancellation fee on the previously negotiated N_{2t} loans. Feasibility would require this, since it would still be the case that total loans cannot exceed $N_t + X_t$. Although the distribution of dividends between the firm and financial intermediary would differ in this market structure from the one used in this paper, we suspect that the associated equilibrium allocations and the liquidity effects of unanticipated money shocks would not be different.

Market Clearing and Equilibrium

In addition to optimizing behavior on the part of the different agents in the model we also require that, in equilibrium, markets clear. For the loan market, this condition is given by $N_{it} = W_{it}H_{it}$, $i = 1,2$. The condition that labor markets clear is given by $L_{it} =$

H_{it}, i = 1,2, while the condition that the goods market clears is given by $C_t + K_{t+1} - (1-\delta)K_t = K_t^{\alpha}(z_tH_t)^{1-\alpha}$. Finally, we require that the aggregate demand and supply of money are equated. This requires that the value of M_{t+1} in (3.12) equals the money supply.

To complete our specification of the model, we specify the following law of motion for the growth rate of money, $x_t \equiv X_t/M_t = (M_{t+1} - M_t)/M_t$:

$$(3.18) \qquad x_t = (1-\rho_x)x + \rho_x x_{t-1} + \epsilon_{xt} + \nu\epsilon_{\theta t}.$$

This law of motion is a slightly modified version of the specification used in most monetized RBC models. See, for example, Cooley and Hansen (1989), den Haan (1990), Kydland (1989), Cho and Cooley (1990), Hodrick, Kocherlakota and Lucas (1991), Marshall (1987), King (1992), and King and Watson (1992). In (3.18), ϵ_{xt} is an iid money supply shock that is orthogonal to all variables dated t–1 and earlier, as well as to $\epsilon_{\theta s}$ for all s. We denote the standard deviation of ϵ_{xt} by $\sigma_{\epsilon x}$. The standard assumption in the literature is that $\nu = 0$. In section 5 we will also analyze policies in which the monetary authority accommodates technology shocks, that is, $\nu > 0$.

A rational expectations equilibrium consists of functions $\{C_t, N_{2t}, H_{2t}, L_{2t}, P_t, W_{2t}, R_{2t}, R_t\}$ of Ω_{t2}, functions $\{N_{1t}, H_{1t}, L_{1t}, K_{t+1}, W_{1t}, R_{1t}\}$ of Ω_{t1} and a function N_t of Ω_{t0} such that agents optimize and markets clear. Obtaining these functions exactly is not possible. Instead we follow Christiano (1991) in constructing approximations. Details are provided in an appendix to this paper, Christiano and Eichenbaum (1992a), which is available on request. In addition we discuss the existence and uniqueness of the linear approximate equilibrium in that appendix.

We conclude the presentation of our model by summarizing the timing conventions and their interpretation. The first decision made during a period is N_t, which is a function of Ω_{t0}. Then, K_{t+1}, H_{1t} are decided based on Ω_{t1} and finally, C_t, H_{2t} are determined as a function of Ω_{t2}. We interpret these timing assumptions as capturing in an analytically convenient way the notion that, in reality, different decisions are made by different agents at different frequencies in time relative to the frequency with which open market operations are carried out and with which shocks to technology occur. Thus, in effect we assume that household portfolio decisions, as captured by N_t, are revised most infrequently. Firm investment decisions and initial production commitments (i.e., H_{1t}) are revised more frequently, but still at a lower frequency than that at which open market operations are carried out. Finally, household consumption and ongoing production decisions (i.e., H_{2t}) are assumed to be made at the same frequency as open market operations. The impact of these assumptions on our analysis is discussed below.

3.2 The Role of Heterogeneity.

The two key distinguishing features of our analysis are that (i) monetary shocks have a heterogeneous impact on agents, and (ii) production is inflexible ex post. In this subsection we discuss the impact of the first feature. To highlight the role that heterogeneity plays, we consider a special case of our model in which the ex post inflexibility feature is not present. We refer to this as the *sluggish saving model*, which is defined by the condition that the money shock is known at the time that L_{1t}, R_{1t} and W_{1t} are determined. The model is identical to our model in all other respects. In particular, we retain the assumption that N_t must be chosen prior to the realization of the money shock. As Chart I makes clear, this implies that money shocks have a heterogeneous impact on agents, since firms must absorb a disproportionate share (100 percent) of money injections.

According to the following Proposition, in the sluggish savings model, interest rates drop, while employment and the real wage rise in response to a positive money supply shock.

Proposition 1: Suppose that

(i) the household and financial intermediary cash constraints, (3.11) and (3.14), are satisfied as a strict equality in date t,

(ii) the household and firm first order conditions are satisfied as a strict equality at date t,

(iii) $M_t + X_t > 0$.

Then, in the sluggish saving model,

$$R^*_{x,t} < 0,\ L^*_{x,t} > 0,\ \omega^*_{x,t} > 0.$$

Here, $L^*_{x,t} = d\log(L_{1t}+L_{2t})/dx_t$, $R^*_{x,t} = dR_t/dx_t$, $\omega^*_{x,t} = d(W_{it}/P_t)/dx_t$, $i = 1,2$, where dx_t is an unanticipated shock to money.[9]

In Appendix A, we prove the portion of Proposition 1 pertaining to $L^*_{x,t}$. Here, we sketch the proof of the remainder of the proposition. We do so in a way that emphasizes the crucial role that firm labor demand plays in determining $R^*_{x,t}$. The basic idea is that the interest rate must drop by an amount sufficient to induce firms to voluntarily hire the increase in equilibrium employment.

In Appendix A, we show that, in the sluggish saving model, $W_{1t} = W_{2t} \equiv W_t$. Also, $L_{1t} = L_{2t} \equiv L_t$ and $R_t = R_{1t} = R_{2t}$. Under these circumstances, the firm's Euler equation for H_{it}, $i = 1,2$, is

$$\frac{W_t}{P_t} = \frac{1}{2} f_{H_t}/R_t. \tag{3.20}$$

[9]It is difficult to establish conditions under which the assumptions of Proposition 1 hold with probability one. However, one can establish that (i) holds in the nonstochastic steady state version of the model as long as $R_t > 1$. This is equivalent to the restriction $(1+x)\exp[-\mu(1-\gamma)\psi]/\beta > 1$. In addition, it is easy to determine whether (ii) and (iii) are nonbinding in nonstochastic steady–state. We assume that if model parameters are such that (i) – (iii) hold in nonstochastic steady state, then they will hold with arbitrarily high probability in the stochastic version of the model, for sufficiently small shocks.

Here, f_{H_t} denotes the partial derivative of the function f with respect to H_t.[10] Relation (3.20) can be expressed as a static labor demand schedule in real wage, employment space. This schedule is depicted by the downward–sloped solid line in Figure 3. As in standard RBC models, increases in the capital stock or positive technology shocks shift labor demand to the right, exerting expansionary pressure on aggregate employment and output. Unlike in standard RBC models, a fall in R_t also shifts the labor demand curve to the right. This is because the firm equates the marginal product of labor to the real cost of hiring labor, taking the cost of working capital into account.

Now consider labor supply. Conditional on a given level of consumption, the household Euler equations for L_{it}, i = 1,2, define a static, upward–sloped labor supply schedule:

$$\frac{W_t}{P_t} = \frac{\gamma}{1-\gamma} \frac{C_t}{1 - 2L_t}. \tag{3.21}$$

This schedule is depicted by the upward–sloped solid line in Figure 3.[11]

Since a positive money shock increases employment, it also increases output. By assumption, investment cannot respond to a money shock. It follows that equilibrium C_t must rise in response to a positive money shock, so that the labor supply curve shifts to the left. This is depicted by the upward–sloped dashed line in Figure 3. Here, C′ denotes the new level of consumption. The only way, then, for equilibrium employment to increase – as required by Proposition 1 – is for the labor demand curve to shift to the right. But, this requires that R_t drop, thus establishing that $R^*_{x,t} < 0$. In Figure 3, we denote the new,

[10]In (3.20), $(1/2)f_{Ht}$ is $f_{H_{1t}}$ or $f_{H_{2t}}$, taking into account that in the sluggish savings model $H_{1t} = H_{2t}$.

[11]Apart from the case $\psi = 0$, our "consumption constant" concept of labor supply differs from the "λ constant" concept used in the empirical labor literature (λ is the marginal utility of wealth.) For proving our results, the consumption constant concept turns out to be more convenient.

lower value of R by R′. With the labor demand curve shifting to the right, and the labor supply curve shifting to the left, the real wage must rise, thus establishing that $\omega^{*}_{x,t} > 0$.

3.3 The Role of Ex Post Inflexibilities.

In this subsection we discuss the impact of ex post inflexibilities in production. These arise because of our assumptions that (i) L_{1t} is chosen prior to the realization of X_t and (ii) L_{1t} and L_{2t} are imperfect substitutes technologically as long as $\sigma < \infty$. The two propositions discussed in this subsection establish that the role of ex post inflexibilities in production is to magnify the quantitative response of the system to money supply shocks. To convey this in the simplest way possible, we evaluate the impact effects in nonstochastic steady state. To this end, let L^{*}_{x} and R^{*}_{x} denote the value of $L^{*}_{x,t}$ and $R^{*}_{x,t}$ in nonstochastic steady state. In Appendix A we prove the following proposition:

Proposition 2: Suppose the conditions of Proposition 1 hold. Then, in our model,

$$L_x > L^{*}_{x},\ \omega_x > 0,\ \frac{dL_x}{d\sigma} = 0.$$

Here, $L_x = d\log(L_{1t}+L_{2t})/dx_t$, and $\omega_x = d(W_{2t}/P_t)/dx_t$, evaluated in nonstochastic steady–state. According to Proposition 2, the employment response to a money shock in our model exceeds that in the sluggish saving model. As in the sluggish saving model, the real wage rises in response to a money supply shock. Finally, the proposition indicates L_x is not a function of σ.

In Appendix A we prove the following proposition:

Proposition 3: Suppose the conditions of Proposition 1 hold. Then, in our model,

(i) R_x is differentiable and monotone in σ,

(ii) $\frac{dR_x}{d\sigma} > 0$

(iii) there exists a $\bar{\sigma} > 0$ such that for all $\sigma < \bar{\sigma}$, $R_x < R_x^*$.

Here, R_x denotes dR_t/dx_t, evaluated in nonstochastic steady state. So, when L_{1t} and L_{2t} are sufficiently imperfectly substitutable, the interest impact of a money shock exceeds that in the sluggish saving model.

To gain intuition into the fact that R_x falls as σ goes to zero, recall that the firm is the marginal agent who must absorb unanticipated cash injections. Consequently, the interest elasticity of its demand for real balances plays a primary role in determining R_x. The interest elasticity that is relevant is an ex post elasticity, η, which takes into account the fact that firms have already initiated production plans (by setting L_{1t}) at the time a cash injection occurs. When evaluated in the nonstochastic steady state of the model,[12]

$$\eta = \frac{1}{\alpha + 1/\sigma}. \tag{3.22}$$

Note that η is a strictly increasing function of σ. Not surprisingly, as η goes to zero, R_x becomes more negative (see Appendix A). Consequently, R_x is increasing in σ.

The intuition behind the fact that η is increasing in σ can be obtained by considering the extreme case when $\sigma = 0$, when the firm's ex post money demand elasticity

[12] To obtain (3.22), note that the firm's first order condition for H_{2t} implies $W_{2t}R_{2t} = P_t f_{H_{2,t}}$. Differentiating this, holding market prices, the state of technology, H_{1t} and the capital stock fixed, we get $dH_{2t}/dR_{2t} = \exp(-\mu t)(W_{2t}/P_t)/[\exp(-\mu t)f_{H_2H_{2,t}}] = \exp(-\mu t)(f_{H_{2t}}/R_{2t})/[\exp(-\mu t)f_{H_2H_{2,t}}]$. Here, $f_{H_2H_{2,t}}$ is the second derivative of (3.1) with respect to H_{2t}. In nonstochastic steady state, $H_{1t} = H_{2t} = H_t = H$, where H_t is defined in (3.4). Then, it is easily verified that in steady–state, $\exp(-\mu t)f_{H_2H_{2,t}} = -f_H[\alpha + 1/\sigma]/(4H)$, where f_H is the derivative of the product of (3.1) and $\exp(-\mu t)$ with respect to H_t, evaluated in steady–state. Equation (3.22) follows by substitution, by using the fact that in steady–state, $\exp(-\mu t)f_{H_{2t}} = .5f_H$, $W_{1t} = W_{2t} = W$, and $R_{2t} = R_{1t} = R$, and from the definition, $\eta \equiv -d\log(W_{1t}H_{1t}+W_{2t}H_{2t})/d\log(R_{2t})$.

is zero. In this case, deviations of L_{2t} from the level planned when L_{1t} was chosen, generate no extra output. As a result, there is no reduction in R_{2t} sufficiently great to induce firms to voluntarily absorb extra working capital, since the derived marginal product of that working capital equals zero.

4. Empirical Results

In this section we analyze the quantitative properties of our model. First, we describe how we assigned values to the model parameters. Second, we compute the quantitative response of the model variables to a money supply shock. Apart from the fact that interest rates go down, an important distinguishing feature of our model is that real wages are predicted to rise after a money supply shock. In the final subsection, we report evidence on the empirical plausibility of this implication.

4.1 Parameter Values

Our model has 12 free parameters: β, ψ, θ, α, γ, δ, μ, ρ_θ, $\sigma_{\epsilon\theta}$, x, ρ_x, $\sigma_{\epsilon x}$. Throughout, β was set a priori to $(1.03)^{-.25}$ and Ψ was set to zero. The parameters x, ρ_x and $\sigma_{\epsilon x}$ were set to the values discussed in Christiano (1991). He reports, using data on M0 covering the period 1959Q1 – 1984Q1, sample estimates for these objects equal to .0119, .80 and .004. Estimates of ρ_x based on NBR and M1 are lower, and so we also consider a value of $\rho_x = .32$. For ρ_θ and $\sigma_{\epsilon\theta}$ we use the point estimates obtained by Burnside, Eichenbaum and Rebelo (1992): $\rho_\theta = .9857$ and $\sigma_{\epsilon\theta} = .01369$. The remaining parameters were estimated using aggregate U.S. time series data.

The data for Y_t, C_t, L_t, K_t and I_t correspond to the series discussed in Christiano (1988), and cover the period 1959Q1 – 1984Q1. The per capita consumption measure is the sum of private sector consumption of nondurables and services, the imputed rental value of the stock of consumer durables, and government consumption. The per capita hours–worked data consist of Hansen's (1984) hours worked data. The per capita stock of capital was measured as the sum of the stock of consumer durables, producer structures and equipment, government and private residential capital, and government nonresidential capital. Data on per capita investment, I_t, are the flow data that match the capital stock concept.

The parameter δ was equated to the sample average rate of depreciation on capital, i.e., the sample average of $1 - (K_{t+1} - I_t)/K_t$. This yields a value for δ equal to .0212. In our model, the average growth rate of equilibrium output equals μ, the growth rate of per capita output. In light of this, we set μ equal to .0041, the sample average growth rate of per capita GNP.

Our point estimates of α and γ were designed to equate the model's implications for the means of $L_{1t} + L_{2t}$, and K_t/Y_t with the sample averages of our empirical measures of these variables. We approximate the model's mean implications for $L_{1t} + L_{2t}$ and K_t/Y_t by their nonstochastic steady–state values. Using the assumption that the representative household has a time endowment of 1460 hours per quarter, we obtain point estimates of α and γ equal to .357 and .76, respectively.[13]

4.2 Quantitative Properties of the Model

[13]Denote the steady state values of L_{1t}, L_{2t} and H_t by H, and the steady state value of K_t/Y_t by K/Y. The sample average of hours worked per person is 320, which translates into an estimate of 320/1460 = .219 for the average fraction of available time worked. The sample mean of the capital–output ratio is 10.59. Thus, our estimation strategy involves choosing values of α and γ to ensure: H = .219/2, K/Y = 10.59. The firm's Euler equation for investment implies that, in nonstochastic steady state, $\exp(\mu)/\beta = \alpha(Y/K) + (1-\delta)$, where we have used the fact that, in nonstochastic steady–state, $C_{t+1}/C_t = \exp(\mu)$. Setting Y/K, μ and δ to the values specified in the text and solving for α, we obtain $\alpha = .347$. The household and firm Euler equation for L_{2t} imply that in steady state $\gamma = \{(C/Y)[2H/(1-2H)]R_2/(1-\alpha) + 1\}^{-1}$,where $R_2 = R = (1+x)/\beta$. In steady–state, $K_{t+1}/K_t = \exp(\mu)$. It follows that $C/Y = 1-\{\exp(\mu)-(1-\delta)]K/Y$. Using the previously assigned values of 2H, x, α, μ, δ and K/Y and solving or γ, we obtain $\gamma = .76$. The value of C/Y implied by our point estimates is .73, after rounding. The average of the ratio of consumption to output is .7246. All sample averages used in this footnote were taken from Christiano (1988, Table 1).

Table 2 presents the contemporaneous percent change in quarterly hours worked, $L_{1t} + L_{2t}$, and the percentage point change in the nominal interest rate, R_t, in response to a one percentage point shock to the growth rate of money. We denote these magnitudes, when evaluated in nonstochastic steady–state, by L_x and R_x.

It is useful to compare the properties of our model with a version in which all decisions are made after the realization of θ_t and x_t (i.e., one in which the information in Ω_{t2} is contained in Ω_{t0} and Ω_{t1}). We call this version of our model the *basic cash–in–advance model.* Its properties are also reported in Table 2. As row 1 indicates, when $\rho_x = 0$, (i.e., the monetary shock is purely transitory), then $L_x = R_x = 0$ in the basic cash–in–advance model. This is not surprising. A purely transitory shock to the growth rate of money corresponds to a permanent increase in the level of the money stock. It is well known that this kind of disturbance is neutral in the basic cash–in–advance model, i.e., it has no effects on either quantities or relative prices. The only effect is a proportional jump in the price level which leaves both the rate of inflation and the nominal interest rate unaffected.

Rows 2 and 4 indicate that if $\rho_x > 0$, then $R_x > 0$ and $L_x < 0$. Note that the larger ρ_x is, the larger is the rise in the nominal interest rate and the larger is the fall in hours worked. To understand this result, it is useful to think of a persistent increase in x_t as a combination of a purely transitory increase in x_t and an anticipated increase in the future growth rate of money. The larger ρ_x is, the larger the magnitude of the anticipated increase in the future growth rate of money. Transitory increases in x_t do not impact on the nominal interest rate, the inflation rate or hours worked. However, the anticipated increase in x_t exerts upward pressure on the rate of inflation. This in turn induces a rise in the nominal interest rate. With the cost of working capital up, the net cost of hiring labor increases, inducing firms to reduce their demand for labor. Not surprisingly, this drives equilibrium hours worked down. Consistent with row 3, the only way for a positive innovation in x_t to generate a fall in the nominal interest rate and an increase in hours worked is for the increase in x_t to signal a substantial fall in the future growth rate of money (and inflation), i.e., $\rho_x < 0$. This is grossly counterfactual.

Table 2 indicates, that in the sluggish savings model, a one percentage point shock in the growth rate of money drives hours worked up by .11 of a percent and drives R_t down by about 17 basis points.[14] This appears to be substantially less than what is observed in the data (see section 2.) So, while heterogeneity can induce a fall in interest rates following a money supply shock, the magnitude of that fall seems small.

Consider now the results for our model. (These appear in the columns labelled "sluggish saving and inflexible production".) With $\sigma = 10$, a one percentage point shock to x_t drives hours worked up by .56 of a percent and drives R_t down by about .45 percent (i.e., 45 basis points). Consistent with Proposition 3, when σ falls from 10 to .5, the interest rate effect becomes larger so that now a one percentage shock to x_t drives R_t down by nearly one percent.

Figure 4 displays the dynamic impulse response functions of the basic cash–in–advance model (solid line) and our model ($\sigma = .5$, dashed line) to a one standard deviation shock in the growth rate of money. These response functions were generated under the assumption that ρ_x equals .8. Consider first the basic cash–in–advance model. Notice that in the impact period of the shock, the interest rate R_t rises. At the same time, investment I_t rises while consumption C_t falls. This is because the rise in R_t acts like a tax on the cash good (consumption) and a subsidy on the credit good (investment). Notice also that the fraction of time worked (L_t) falls. This effect can be viewed as reflecting a leftward shift in the labor demand curve and a rightward shift in the labor supply curve. The former is induced by the rise in R_t, while the latter is induced by the fall in C_t. Both shifts contribute to a fall in the real wage rate W_t/P_t. That L_t falls reflects that the shift in the labor demand curve dominates the shift in the supply curve. Given our assumption

[14]Note that for both models, the magnitude of the employment and interest rate responses to a money shock are independent of ρ_x. This is consistent with the results in Appendix A.

of diminishing marginal labor productivity, the marginal cost of hiring labor, R_tW_t/P_t, must rise since L_t falls. Finally, since L_t has fallen and the stock of capital is unchanged, current output must also fall. With output down and the stock of money up, prices rise by more than the percentage change in the money supply.

Since $\rho_x > 0$, monetary growth continues to be high relative to its steady–state level after the shock. With the growth rate of money declining over time, the inflation rate also declines toward its steady–state value. Consequently, R_t is also high relative to its steady–state value, but declining over time. Since R_t is declining, consumption slowly rises to its steady–state level, while investment declines to its steady–state level. Since a high value of R_t depresses labor demand, as long as R_t is high, hours worked and the real wage stay low, and the marginal cost of hiring labor stays high.

In sharp contrast to the basic cash–in–advance model, our model implies that the contemporaneous values of R_t falls while C_t and L_t rise in response to a positive money shock. With L_t up and with diminishing marginal labor productivity, the marginal cost of hiring labor, R_tW_t/P_t, falls. The contemporaneous increase in the price level is muted by the increase in aggregate output. As a result, the initial rise in the inflation rate is less than proportional to the initial percentage increase in the money supply.

The intuition regarding the dynamic response of the system thereafter is similar to that for the basic cash–in–advance model. With $\rho_x > 0$, the growth rate of M_t continues to be high, only slowly reverting to its steady state level. Consequently inflation is also high relative to its steady state, but declining over time. Following the impact period, R_t actually rises above its initial value, reflecting anticipated inflation effects. Thereafter, R_t declines to its steady state value. Investment, consumption and hours worked respond in the expected manner as the nominal interest rate (the relative price of cash goods) first rises and then falls to its steady state level. The cash goods – hours worked and consumption – first fall and then rise to their steady state levels, while the credit good – investment – first rises and then gradually falls to its steady state level. On the basis of

Figure 4, we conclude that our model rationalizes, at least at a qualitative level, the description of the effects of expansionary monetary policy given by Friedman (1968).

Although Figure 4 indicates that our model can account for the contemporaneous component of the interest rate response to a money supply shock, the model clearly does not account for the persistence of that effect. Using the sluggish saving model, Christiano and Eichenbaum (1992c) show that persistence can be introduced by assuming costs of adjusting $Q_t \equiv M_t - N_t$. The effect of this is clear by inspecting Chart 1. With Q_t slow to adjust, when $M_t + X_t$ returns to the household at the end of period t, much of a period t money shock is automatically passed on to financial intermediaries and from there on to firms in period t+1. Thus, by making assumptions that cause Q_t to adjust slowly, firms are, in effect, forced to absorb a disproportionate share of a money injection for several periods. By spreading out, over time, the heterogeneous impact of a money shock the liquidity effect also is spread out over time. Presumably, incorporating costs of adjusting Q_t into our model would also cause liquidity effects to be persistent. We have not done so on this paper in order to keep the analysis focused as sharply as possible on the heterogeneity and ex post inflexibility features of our model.

4.3 Why Take Our Model of Liquidity Effects Seriously?

An important distinguishing feature of our model is its implication that the equilibrium real wage rises after a positive money shock. In this section we discuss the empirical plausibility of this implication.

Figure 5 displays the impulse response function of several measures of the real wage to an innovation in NBR, using the methodology described in section 2. The estimation period is 1966:1 – 1991:2. Each of the three columns of graphs represents results based on different measures of the real wage. In the first column the real wage is measured by average hourly earnings in the total, private, nonagricultural sector (Citibase data mnemonic LEH77). In the second and third columns, the real wage is measured by real compensation in the nonagricultural sector (LBCPU7) and real average hourly earnings in manufacturing (LEHM, deflated by the consumer price index, PUNEW). Each row corresponds to a different way of constructing Ω_t in the monetary policy function, (2.1). In each case, lagged values of output, the price level, the rate of interest, and the real wage are included in Ω_t. What differentiates the alternative specifications of Ω_t is the list of variables whose contemporaneous value is included in Ω_t. The first row corresponds to a specification of Ω_t in which the contemporaneous values of the price level, the real wage and the level of output are included. The second row corresponds to a specification in which the contemporaneous value of no variable is included. The third row corresponds to the case in which output, price, the interest rate and the real wage are included. Finally, the fourth row corresponds to a specification of Ω_t in which the contemporaneous values of output and the price level are included. The solid line in each graph represents our point estimate of the real wage response to a money supply shock. The dashed lines represent plus and minus one standard deviation lines.[15]

The striking result in Figure 5 is that for all specifications of Ω_t and for all three measures of the real wage, the real wage responds positively to a positive money supply shock. In several cases, the positive sign of the response is statistically quite significant. In one sense, these findings are clearly supportive of the monetary transmission embedded in our model. At the same time, after some lag, the real wage response is so large that it

[15]The impulse functions and confidence intervals were computed using the same methodology as the one used for Figure 1, and described in an earlier footnote. In particular, the results in the first row of Figure 4 are based on estimating a five variable VAR, $Z_t = A(L)Z_{t-1} + v_t$, where v_t is iid and $Ev_tv_t' = V$ and $Z_t = [\log GNP_t, \log P_t, \log (w/p)_t, \log M_t, \log R_t]$, where M_t is measured by nonborrowed reserves, R_t is measured using the federal funds rate, P_t is the GNP deflator, and w/p is the real wage, measured as indicated in the text. Also, $A(L) = A_0 + A_1L + ... + A_nL^n$, where L is the lag operator, and n = 5. The money supply shock is identified as the fourth element of Dv_t, where D is lower triangular with ones on the diagonal, and $DD' = V$. For the results in the second row, $Z_t = [\log M_t, \log GNP_t, \log P_t, \log R_t, \log(w/p)_t]$, and the money supply shock is the first element of Dv_t; in the third row, $Z_t = [\log GNP_t, \log P_t, \log R_t, \log(w/p)_t, \log M_t]$, and the money supply shock is the fifth element of Dv_t; in the fourth row, $Z_t = [\log GNP_t, \log P_t, \log M_t, \log R_t, \log (w/p)_t]$ and the money supply shock is the third element of Dv_t.

dominates the interest rate response (compare Figures 1a and 5.) That is, the marginal cost of hiring labor first falls, but then rises. The initial response is consistent with our model. However, the lagged response would undoubtedly be a problem from the point of view of a modified version of the model which implies persistence in the liquidity effect.

5. Policy Implications.

In this section we briefly discuss the fact that in our model, fixed k–percent money growth rules of the type advocated by Milton Friedman are not optimal. Among other things, this discussion serves to highlight the basic frictions in our model economy. In our setup, private agents cannot quickly direct cash to the financial sector in response to unanticipated technology shocks. Because of this, favorable production opportunities go unexploited, at least in the short run. Specifically, according to the following proposition, the contemporaneous employment response to a technology shock is zero.

Proposition 4: Suppose the conditions of Proposition 1 hold and $\nu = 0$. Then, in the sluggish saving model and in our model:

$$L_\theta = L_\theta^* = 0.$$

Proof: See Appendix A.

Here, $L_\theta = L_\theta^*$ denote the derivative of equilibrium employment with respect to an unanticipated technology shock, evaluated in nonstochastic steady state, for our model and the sluggish savings model, respectively.

Taken together, Propositions 1, 2 and 4 suggest that it may be welfare improving for the monetary authority to increase the money supply in response to unanticipated technology shocks, i.e., to set $\nu > 0$ in (3.18). We interpret such a policy as embodying a version of the Real Bills Doctrine. A simple way to see this is to focus on the sluggish saving model. Figure 6 displays the response of the sluggish saving model economy to a one–standard deviation shock in technology, $\epsilon_{\theta t}$. The solid line corresponds to $\nu = 0$, the case of nonaccommodative monetary policy. Consistent with Proposition 4, employment does not respond during the impact period of the shock. At the same time, there is a substantial rise in the interest rate, due in part to the surge in investment stimulated by the technology shock.[16]

The dashed line in Figure 6 displays the response of our model economy to a one standard deviation technology shock when $\nu = .3$, so that monetary policy is accommodative. Note that now equilibrium employment increases in response to the technology shock. Moreover, because of the liquidity effect in our model, this policy response has the effect of smoothing the interest rate response to a technology shock. Not surprisingly, we found that moving from $\nu = 0$ to $\nu = .3$ leads to a small increase in the representative agent's utility function, (3.10). The previous results are consistent with related findings reported in Fuerst (1992a,b). In future research we plan to pursue, in greater detail, the nature of optimal policy in models of the sort described in this paper.

6. Conclusion

This paper presents a model in which heterogeneity and ex post production inflexibilities are required to account quantitatively for the observed interest rate response

[16] To see the role of investment, substitute out for the real wage in the firm's and household's first order condition for labor to get, $R_t = .5[(1-\gamma)/\gamma](1-2L_t)f_{H,t}/(Y_t+(1-\delta)K_t-K_{t+1})$, where $Y_t = K_t^{\alpha}(z_tH_t)^{(1-\alpha)}$, $f_{H,t} = (K_t/H_t)^{\alpha}z_t^{(1-\alpha)}$, and z_t is given in (3.2). Given that L_t does not respond to a shock in θ_t, the only way for R_t to change in response to a technology shock is via its impact on z_t and K_t+1. Other things the same a jump in K_{t+1} drives up the rate of interest. It follows that, because K_{t+1} is positively related to θ_t, the equilibrium jump in the interest rate in Figure 6 would have been smaller, had we specified that K_{t+1} is chosen prior to the realization of θ_t.

to a money supply shock. We conclude by highlighting some of the model's shortcomings and important areas for future research. First, our model cannot address the empirical links between nonborrowed reserves, higher monetary aggregates and short–term interest rates. For example, it cannot simultaneously account for the fact that short–term interest rates comove negatively with nonborrowed reserves, but positively with broader monetary aggregates like the monetary base and M1. In our view, formally accounting for these features of the data will require explicitly modeling the Federal Reserve discount window and the money multiplier. The latter task will certainly involve a more interesting model of financial intermediaries and a distinction between inside and outside money. We view this as an important area of future research. Second, a key assumption of the monetary transmission mechanism in this paper is that the household's nominal consumption–saving decision is sluggish over a significant interval of time. In current work with Charles Evans, we are investigating the empirical plausibility of this assumption using flow of funds data.

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Appendix A: Proof of Propositions 1 – 4

1. Proof of Proposition 1.

Given the discussion in section 3.2, all that remains to be proved is the result, $L^{*}_{x,t} > 0$. We do that here. In the sluggish saving model, the household's first order conditions for L_{1t} and L_{2t} imply $W_{1t} = W_{2t}$ and the financial intermediary's first order condition implies $R_{1t} = R_{2t}$. The firm's first order conditions for type 1 and type 2 labor then imply $L_{1t} = L_{2t}$. Finally, loan market clearing and (3.17) imply $R_t = R_{1t} = R_{2t}$. Without loss of generality, we impose these conditions as a constraint on the model. In addition, we denote $L_t \equiv L_{it}$, $W_t = W_{it}$, $i = 1, 2$.

Combining the financial intermediary's cash constraint, (3.14), and the loan market clearing condition

$$\text{(A.1)} \qquad 2W_tL_t = (n_t + x_t)M_t,$$

where $n_t \equiv N_t/M_t$. Differentiate (A.1), and take into account $dn_t/dx_t = 0$, to get

$$\text{(A.2)} \qquad L^{*}_{x,t} \equiv \frac{1}{L_t}\frac{dL_t}{dx_t} = \frac{1}{n_t + x_t}[1 - \frac{2L_t}{M_t}\frac{dW_t}{dx_t}].$$

Equation (3.11) and (A.1) imply

$$\text{(A.3)} \qquad P_tC_t = (1 + x_t)M_t.$$

Substituting (A.3) and (A.1) into the labor supply equation, (3.21), and differentiating while taking into account $dn_t/dx_t = 0$, we get $dW_t/dx_t = M_t/(1-\gamma)$. Substituting this into (A.2),

$$\text{(A.4)} \qquad L^{*}_{x,t} = \frac{1}{n_t + x_t}[1 - \frac{2L_t}{1-\gamma}].$$

According to (A.4), $L^{*}_{x,t}$ is positive if, and only if, $2L_t/(1-\gamma)$ is less than one. To see that this condition is satisfied in our model, divide (A.1) by (A.3), then substitute out for W_t/P_t using the labor supply function, (3.21), and rearrange to get,

$$\text{(A.5)} \qquad \frac{2L_t}{1-\gamma} = \frac{1}{1 + \gamma(M_t-N_t)/(M_t+X_t)},$$

which is positive and less than one by the facts that $M_t + X_t > 0$, $M_t - N_t > 0$.

2. Proof of Proposition 2.

We begin by considering the analog of (A.1) in our model:

$$\text{(A.6)} \qquad W_{1t}L_{1t} + W_{2t}L_{2t} = (n_t + x_t)M_t.$$

Totally differentiating (A.6) and evaluating the result in nonstochastic steady–state, we obtain:

$$\text{(A.7)} \qquad L_{x,t} \equiv \frac{1}{L_{1t}+L_{2t}}\frac{dL_{2t}}{dx_t} = \frac{1}{n_t+x_t}[1 - \frac{L_{2t}}{M_t}\frac{dW_{2t}}{dx_t}].$$

Here, we have used the fact that L_{1t}, n_t and W_{1t} cannot respond to an unanticipated change in x_t, and that $W_{1t} = W_{2t}$ in nonstochastic steady state. Equations (3.11) and (A.6) imply (A.3). The household's first order condition for L_{2t} is $W_{2t}/P_t = (\gamma/(1-\gamma))C_t/(1-L_{1t}-L_{2t})$. Substituting (A.3) and (A.6) into the last equation, we find that, in equilibrium, $(1/M_t)dW_{2t}/dx_t = 1/[(1-\gamma)(1-L_{1t})]$. Substituting this into (A.7),

$$\text{(A.8)} \qquad L_{x,t} = \frac{1}{n_t+x_t}[1 - \frac{L_{2t}}{(1-\gamma)(1-L_{1t})}].$$

The value of (A.8) in nonstochastic steady–state is denoted L_x. In nonstochastic steady–state, L_{1t} and L_{2t} in our model coincide with L_t in the sluggish saving model. Moreover, the nonstochastic steady–state values of n_t and x_t also coincide in the two models. Using these facts and $0 < (1-\gamma) < 1$, $0 < (1-L) < 1$, a comparison of (A.4) and (A.8) establishes the result that $L_x > L_x^*$.

The part of the proposition pertaining to ω_x can be obtained by using the household's first order condition for L_{2t} and the fact that C_t and L_{2t} respond positively to an unexpected jump in x_t. To see that L_x does not depend on σ evaluate (A.8) in nonstochastic steady state and take into account that L, n and x are independent of the value of σ.

3. Proof of Proposition 3.

Combining the household's and firm's first order condition for L_{2t}, we get $R_{2t} = [(1-\gamma)/\gamma][(1-L_{1t}-L_{2t})/C_t]f_{H_{2,t}}$. Then, $R_{2x,t} \equiv dR_{2t}/dx_t = [(1-\gamma)/\gamma]\{-f_{H_{2t}}/C_t - [\gamma/(1-\gamma)]R_{2t}(f_{H_{2t}}/C_t) + (1-L_{1t}-L_{2t})f_{H_2H_{2,t}}/C_t\}L_{2x,t}$, where $L_{2x,t} \equiv dL_{2t}/dx_t$. Let $c_t \equiv \exp(-\mu t)C_t$, $f_{H,t}$ denote the derivative of the product of (3.1) and $\exp(-\mu t)$ with respect to x_t, and let c and f_H denote the values of these variables in nonstochastic steady state. Then $f_{H_{2t}}/C_t = (1/2)f_H/c$ in steady state. Also, $\exp(-\mu t)f_{H_2H_{2,t}} = -f_H[\alpha + 1/\sigma]/(4H)$. Thus, $R_{2x} = -(1/2)[(1-\gamma)/\gamma](f_H/c)\{1 + [\gamma/(1-\gamma)]R + [(1-2H)/(2H)](\alpha+1/\sigma)\}L_{2x}$. Now, $L_{2x}=2LL_x$. Then,

$$\text{(A.9)} \qquad R_{2x} = -\frac{1-\gamma}{\gamma}\frac{f_H L}{c}\{1 + \frac{\gamma}{1-\gamma}R + \frac{1-2H}{2H}/\eta\}L_x,$$

where $\eta = (\alpha + 1/\sigma)^{-1}$ is the ex post interest elasticity of money demand. To get R_x, the value of dR_t/dx_t in nonstochastic steady state, scale and rearrange (3.17), and the loan market clearing conditions, to get $R_t = [w_{1t}L_{1t}R_{1t} + (n_t-w_{1t}L_{1t})R_{2t}]/n_t$. Then, $R_x = [(n-w_1L_1)/n]R_{2x}$. But, in steady-state, $w_1L_1 = w_2L_2$, so that $w_1L_1 = (1/2)(n+x)$. Thus,

$$\text{(A.10)} \qquad R_x = \frac{n-x}{2n}R_{2x}.$$

The only way σ enters R_x is via η in R_{2x}. Part (i) follows from the fact that R_{2x} is differentiable and monotone in η and η is differentiable and monotone in σ. Part (ii) follows from the fact that $d\eta/d\sigma >$ and $dR_{2x}/d\eta > 0$. To get part (iii) note that

$$\text{(A.11)} \qquad R_x^* = -\frac{1-\gamma}{\gamma}\frac{f_H L}{c}\{1 + \frac{\gamma}{1-\gamma}R + \frac{1-2H}{2H}\alpha\}L_x^*.$$

Comparing (A.9) and (A.11), and taking into account $\sigma > 0$ and $L_x^* < L_x$, it follows that $R_{2x} < R_x^*$. Note, in general it is not true that $R_x < R_x^*$ since $(n-x)/(2n)$ is less than one. For example, when $\sigma = 20$, $x = .2$, $\delta = .0212$, $\beta = 1.03^{-.25}$, $\mu = .0041$, $\alpha = .346$, $\gamma = .761$, then $(n-x)/(2n) = .35$ and $R_x = -.415$, $R_x^* = -.426$. Part (iii) of the proposition holds because $R_x \to -\infty$ as $\sigma \to 0$.

4. Proof of Proposition 4.

The proof can be carried out in the context of the sluggish saving model, since our assumption of ex post inflexibility is irrelevant when $\nu = 0$, given that the technology shock is realized at the time L_{1t} and L_{2t} are chosen. Thus, we impose $L_{1t} = L_{2t} = L_t$. Equations (3.21), (A.1) and (A.3) imply $[\gamma/(1-\gamma)](1+x_t) + n_t + x_t = w_t$. Since θ_t appears nowhere on the left of the equality, it follows that $dw_t/d\theta_t = 0$, where $d\theta_t$ is an unexpected change in θ_t. But, since $[\gamma/(1-\gamma)](1+x_t) = w_t(1-2L_t)$, it follows that $dL_t/d\theta_t = 0$ too.

Table 2

The Contemporaneous Impact of a Money Growth Shock in Three Models

Percentage Change in Hours Worked (L_x) and
Percentage Point Change in the Nominal Interest Rate (R_x)
in the Period of a One-Percentage-Point Surprise Increase in Money Growth†

Parameters‡		Models							
		Basic Cash-in-Advance		Sluggish Savings		Sluggish Savings and Inflexible Production $\sigma = 10$		Sluggish Savings and Inflexible Production $\sigma = .5$	
	Money Growth Persistence ρ_x	L_x	R_x	L_x	R_x	L_x	R_x	L_x	R_x
(1)	0	0	0	.110	−.170	.558	−.452	.558	−.984
(2)	.81	−.197	.816	.110	−.170	.558	−.452	.558	−.984
(3)	−.81	.026	−.816	.110	−.170	.558	−.452	.558	−.984
(4)	.32	−.026	.322	.110	−.170	.558	−.452	.558	−.984

†The derivatives, $L_x = d \log L/d\epsilon_x$ and $R_x = dR/d\epsilon_x$, are evaluated in nonstochastic steady state. Regarding the sluggish savings model, L_x and R_x correspond to the L_x^* and R_x^* in the paper.

‡The parameter $\psi = 0$ is a curvature parameter on the utility function, $u(c,L) = [c^{(1-\gamma)}(1-L)^{\gamma}]^{\psi}/\psi$, $L = L_1 + L_2$. ρ_x is the autocorrelation of money growth; and δ is the rate of depreciation on capital. The other parameters are set at $\beta^* = 1.03^{-0.25}$, $\mu = 0.0041$, $\theta = 0$, $x = 0.0119$, $\rho_\theta = 0.9857$, $\alpha = 0.346$, $\delta = 0.0212$, and $\gamma = 0.761$.

Chart 1 and 2

Cash Flow in the Model Economies

Chart 1: Cash flow to firms

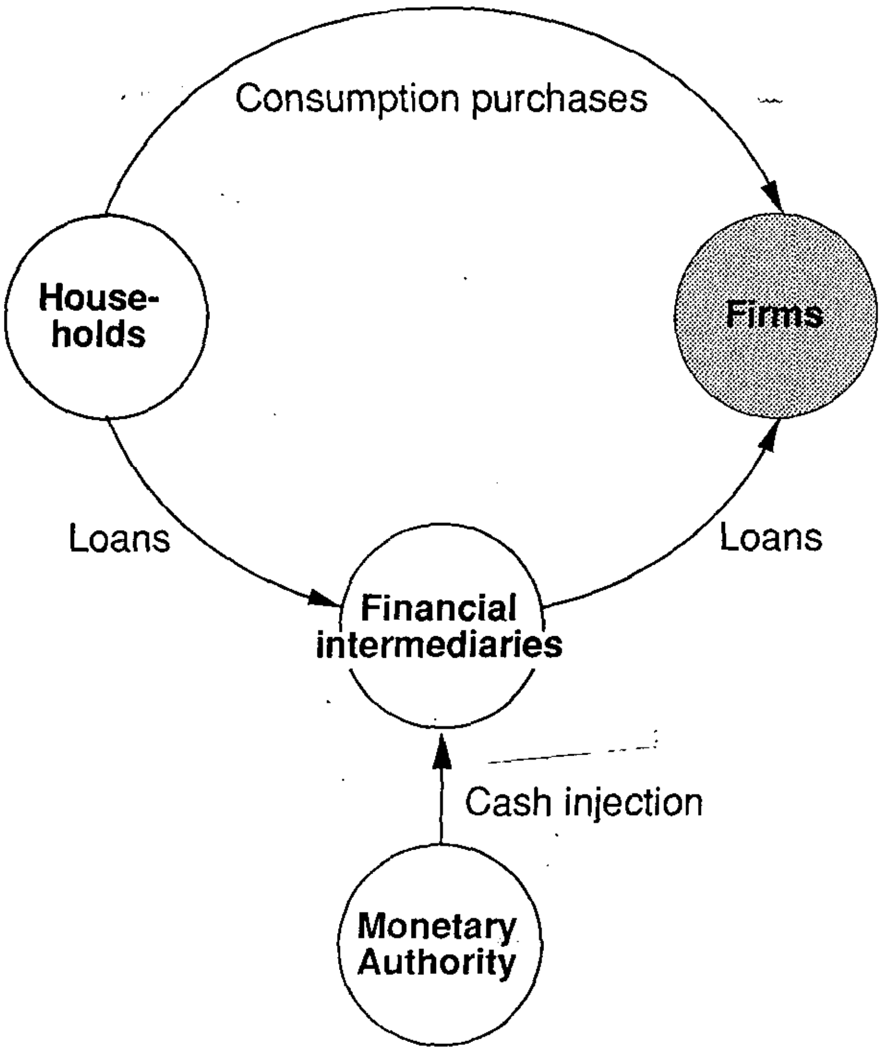


Chart 2: Cash flow back to households

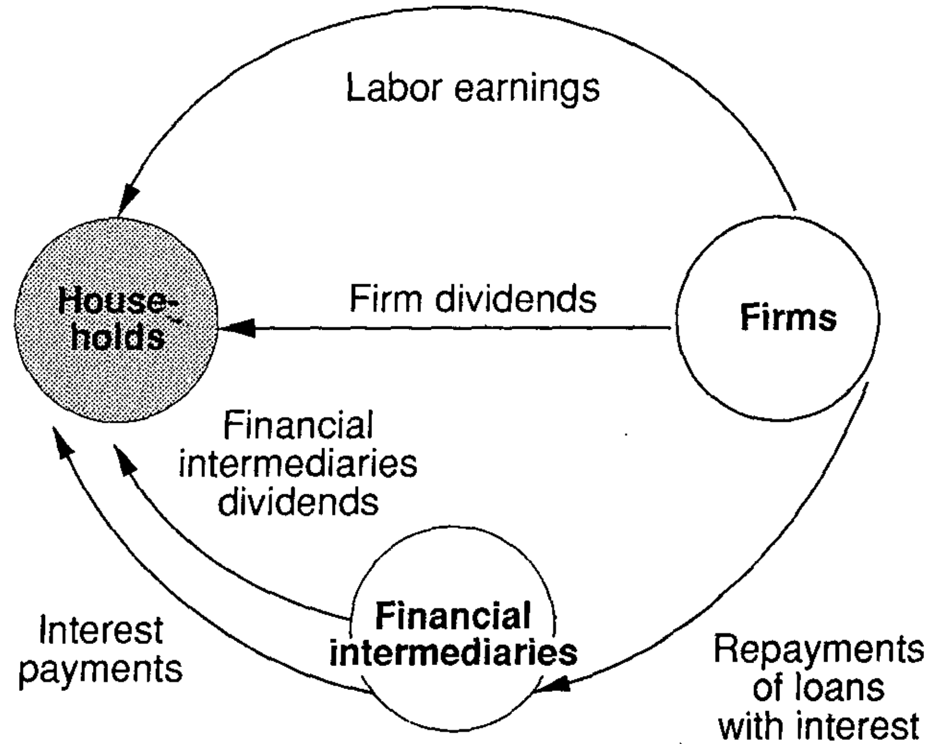


Figure 1A: Response of R to NBR

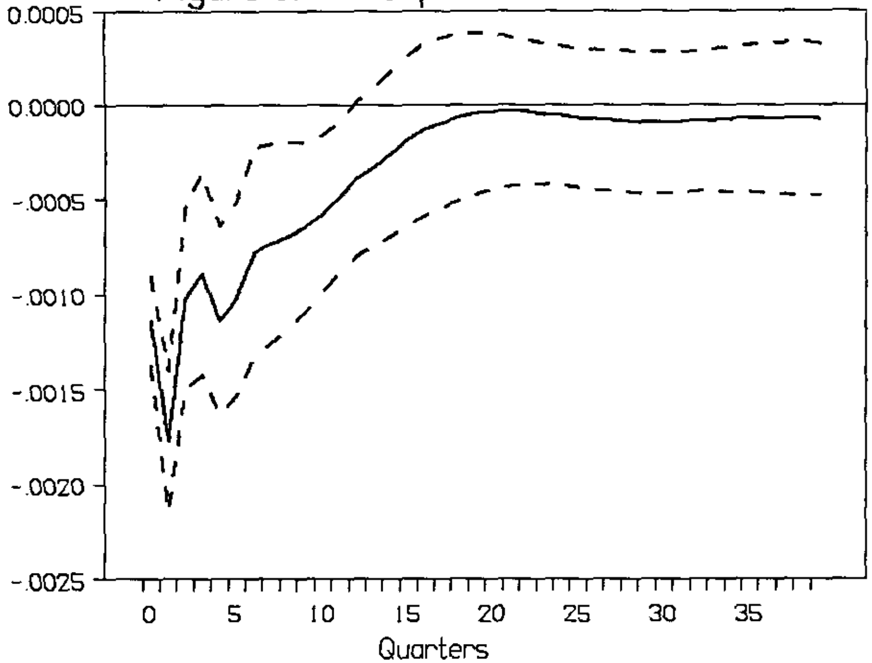


Figure 1C: Response of R to M1

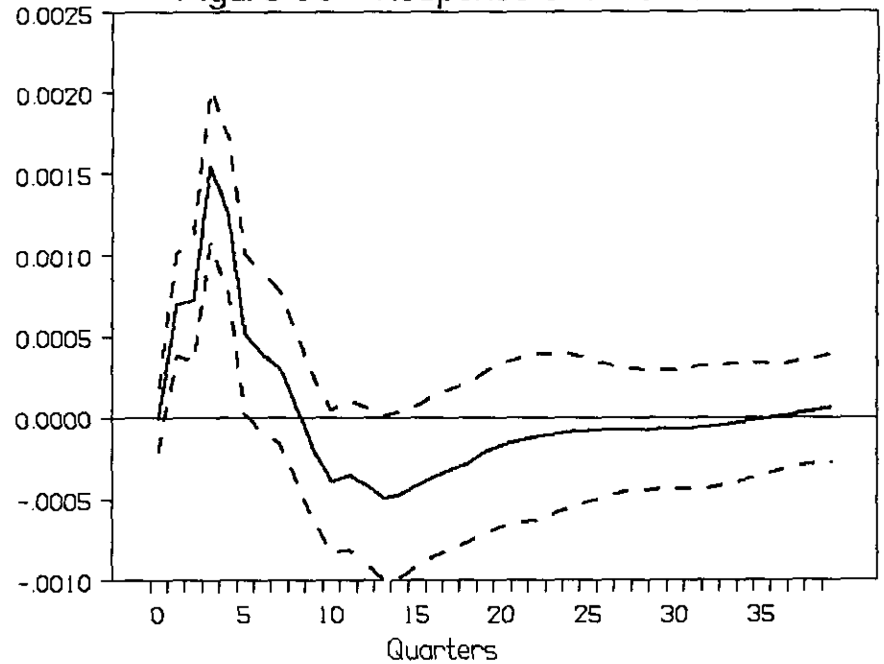


Figure 1B: Response of R to M0

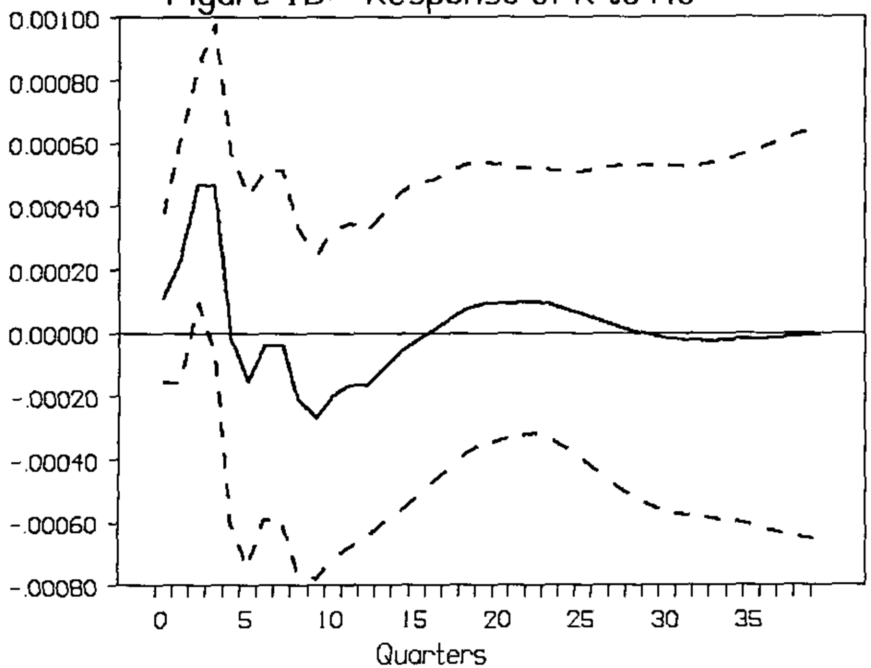


Figure 1D: Response of R to M0-BR

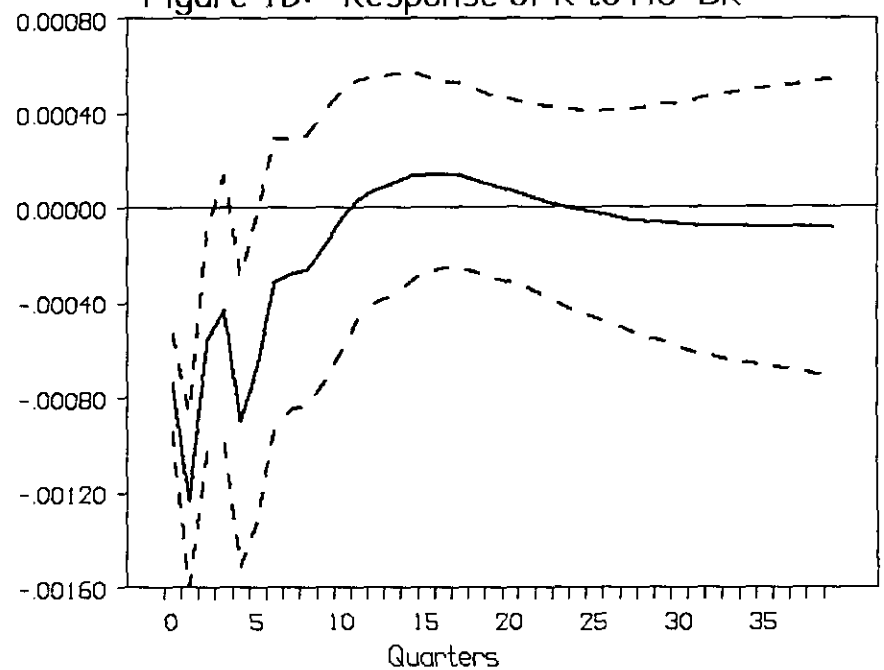
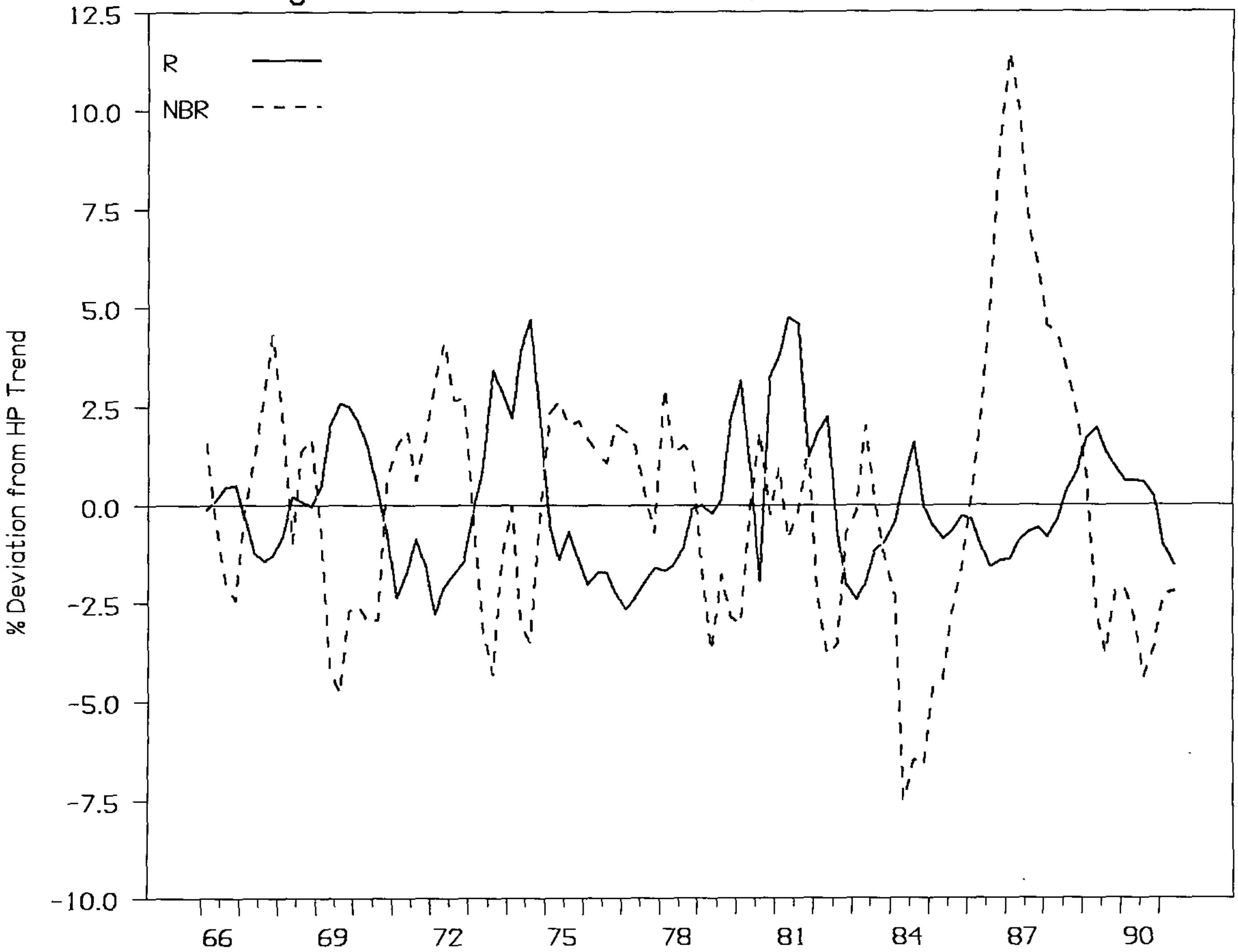


Figure 2: Detrended R and NBR, 1966:1-1991:2



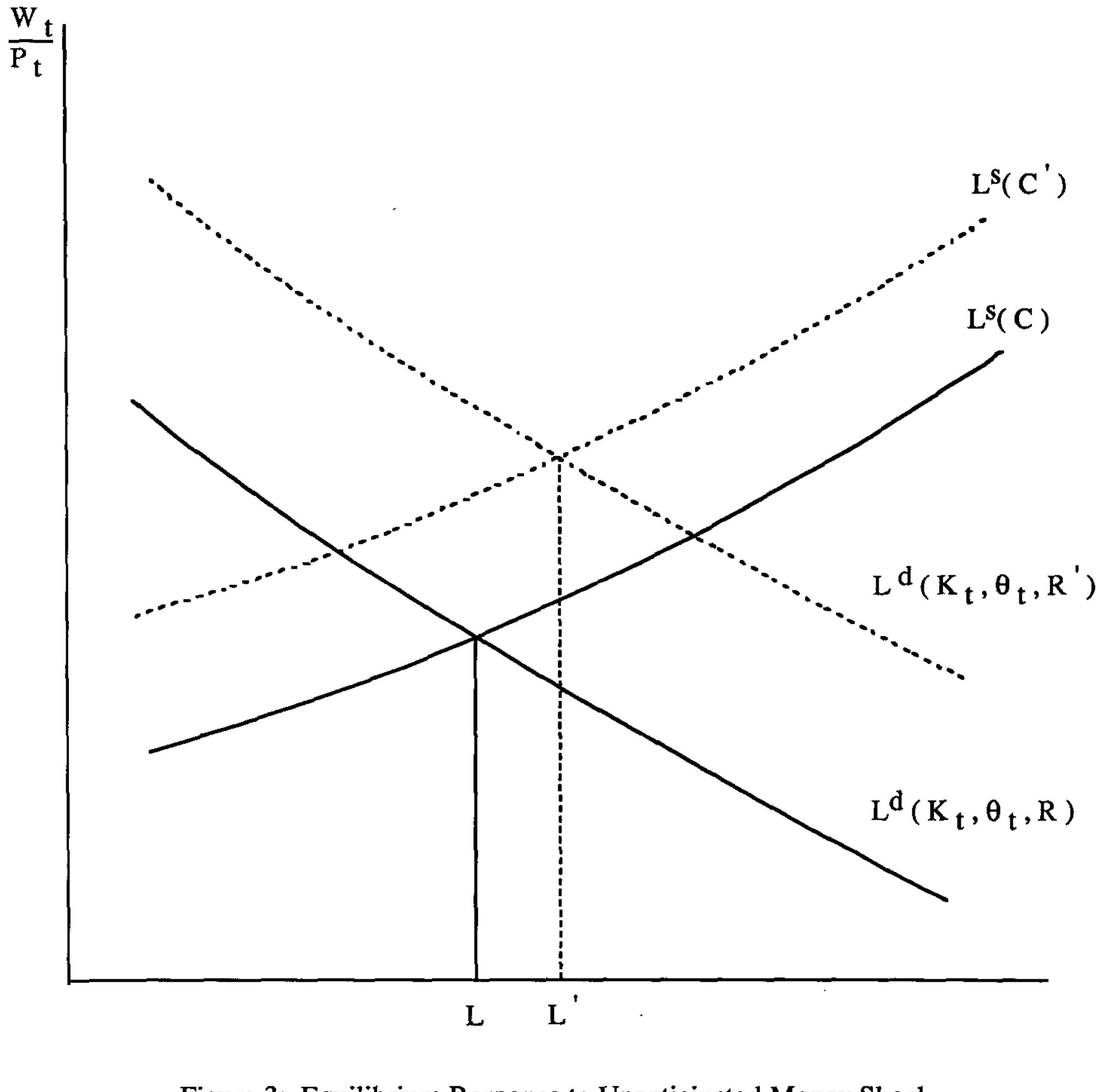


Figure 3: Equilibrium Response to Unanticipated Money Shock

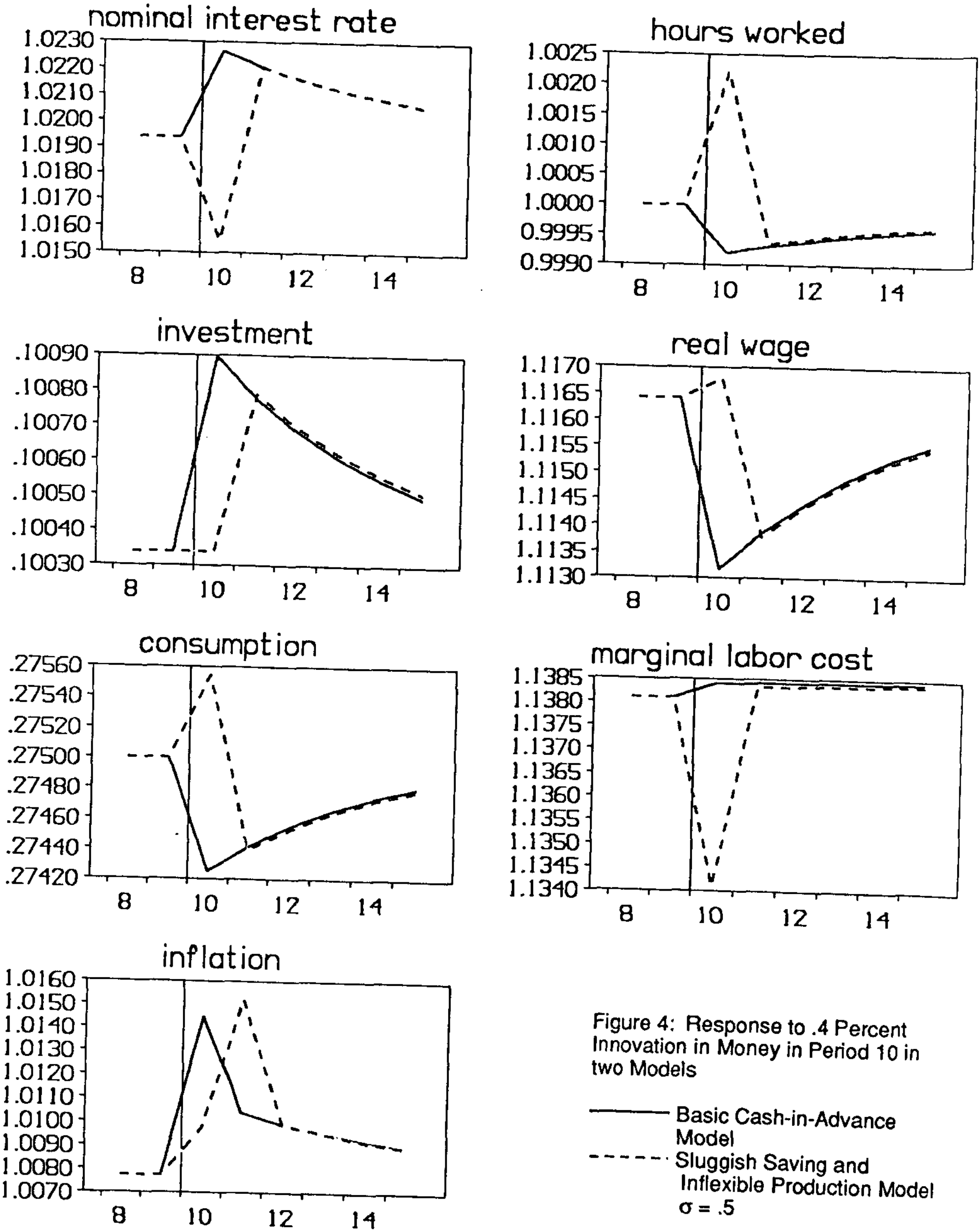


Figure 4: Response to .4 Percent Innovation in Money in Period 10 in two Models

Basic Cash-in-Advance Model

Sluggish Saving and Inflexible Production Model $\sigma = .5$

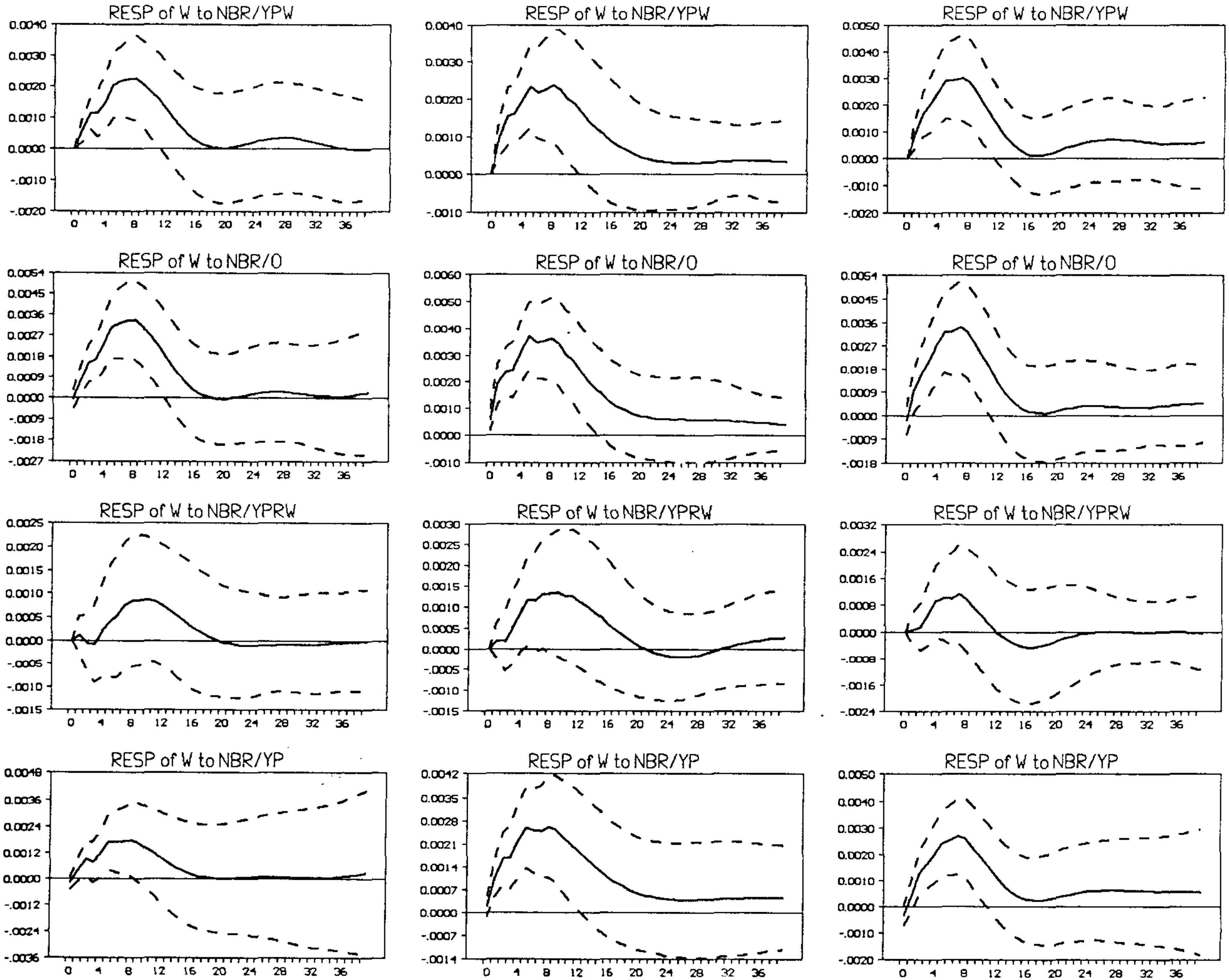
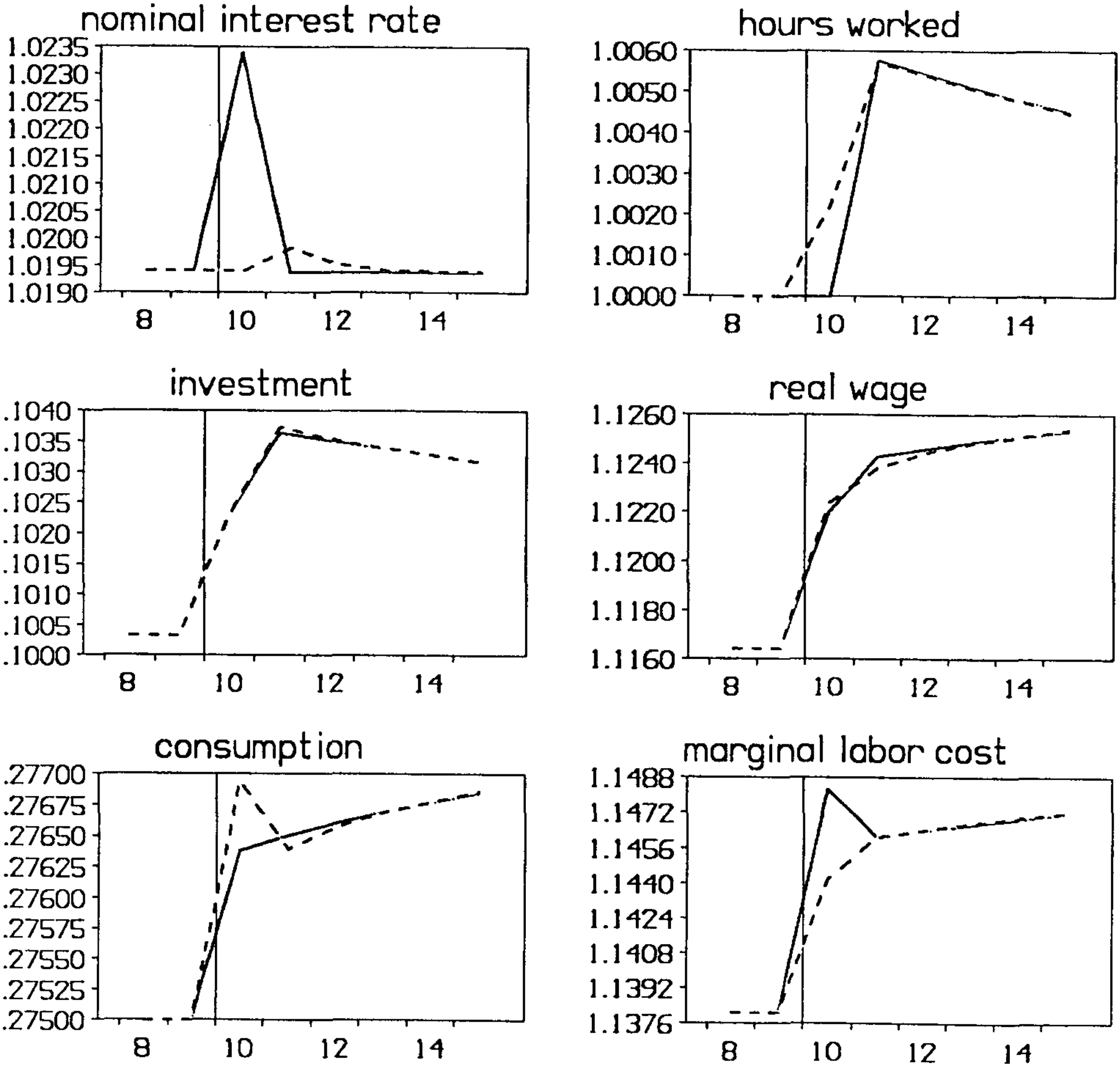


FIGURE 5: REAL WAGE RESPONSE TO MONEY SUPPLY SHOCK - U.S. DATA



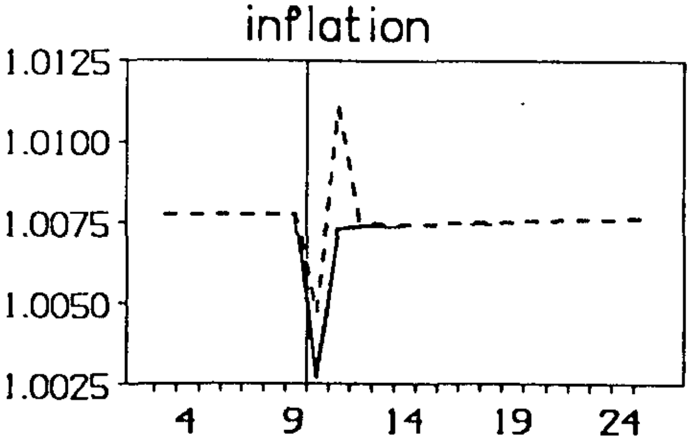


Figure 6: Response of Sluggish Saving and Inflexible Production economy to a technology shock

—— v = 0 (no accommodation)

- - - - v = .3 (accommodation)