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COMMUNICATION, COMMITMENT, AND GROWTH

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Communication, Commitment and Growth

Abstract

We study the effect on the growth of an economy of alternative financing opportunities in a stochastic growth model with incentive constraints. Efficient accumulation mechanisms are designed and computed for economies that differ in their incentive structure. We show that when borrowing is subject to information constraints, there is a computable efficient transfer mechanism that does not affect capital accumulation and investment patterns, even though consumption patterns and the distribution of wealth are affected. In contrast, enforcement constraints can severly reduce the outside financing opportunities and affect investment patterns and economic grwoth. We adapt numerical algorithms for obtaining numerical solutions of these models.

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1 Introduction

Our aim is to study the effect that alternative financing opportunities may have on economic growth. To this purpose, we study different institutional and informational environments, and we analyze the effects on growth, investment, consumption and welfare, of optimal mechanisms under different incentive constraints. We abstract from other factors that can affect growth and that are subject of analysis in the expanding growth literature.

Standard stochastic growth models, in which a single agent must solve an intertemporal optimization problem -see, for example, Brock and Mirman [9]-, can be interpreted as displaying and extreme form of lack of both commitment and communication. Under this interpretation, all investment must be self-financed and smoothing of consumption must take the form of self-insurance through the capital stock, which is the only available asset. Alternatively, an Arrow-Debreu stochastic growth model with many consumers (or countries) and firms (see, for example, Marimon [27]) assumes perfect capital markets based on the equally extreme assumptions of the existence of both complete and costless commitment and communication technologies. As a consequence of these assumptions borrowing and lending by project managers at the market interest rate is unrestricted, idiosyncratic risks are smoothed out through costless pooling and, furthermore, agents never break their promises. In contrast with these assumptions of perfect markets, the historical experience of most economies is full of examples of credit constraints, partial forms of insurance and reneged contracts. For example, investors might not be able to monitor all investments and agents might default on their promises.

The fact that the equilibrium model abstracts from many institutional or contractual elements is, in principle, a welcome simplification. However, some of the predictions of these models are at odds with observed growth patterns. We conjecture that some of these differences between theory and observations can be explained if contractual and organizational elements are incorporated into the growth model[1].

As examples of the differences between the predictions of the standard general equilibrium growth model and the historical experience of both domestic and international economies, consider the following features of standard general equilibrium growth models: *i)* the way contracts (for example, debt contracts) are enforced on agents is the same, regardless of whether these agents are supposed to represent individual countries, or individual households; *ii)* risk-averse agents smooth their consumption and, as a result, individual consumption fluctuations reflect aggregate fluctuations; *iii)* the distribution of wealth has no effects on the process of capital accumulation; *iv)* decreasing return technologies imply that countries (or entrepreneurs) with below average capital labor ratios absorb external financing and, therefore, capital flows from countries that are rich to countries that are poor, and *v)* capital flows to countries (or industries) with positive

[1] Of course we do not claim to be the first ones to think along these lines. For Adam Smith [36], for example, differences in social forms of organization where the central explanation underlying the divergent patterns of growth observed in different economies of his time, such as England and Spain. Similarly, many economic historians, notably North ([30], [31]), have stressed the possible effect of the institutional and the legal environment on economic growth.

idiosyncratic productivity shocks.[2]

In contrast, domestic and international evidence on cross-sectional consumption and investment patterns shows: *i)* patterns of investment and consumption that clearly differ for domestic households and countries, and, for example, significant differences in the way that countries or households can write their debt contracts; *ii)* consumption smoothing between countries is small, and certainly lower than across households in both high and low income areas; *iii)* some negative correlation between income inequality and growth; *iv)* a wide spectrum of borrowing patterns across low and middle income countries, and *v)* finally, a high degree of self-financing in industrialized countries (see, [4], [14], [24], [32], [39], [41], [42], and [43]). Notice that models of sustained growth also have severe difficulties in explaining these facts, in particular, the rich structure of capital flows.

Since standard growth models have difficulties in accounting for the observed patterns of capital flows they do not provide the right framework to address issues such as the effect on economic growth of opening a closed economy to external financing, or the properties of the financial arrangements that should be implemented in this newly opened economies. In contrast, the model analyzed here provides a framework to analyze these issues, both qualitatively and quantitatively, and to account for some observed patterns of international capital flows. For example, in our model, the effect on growth of opening an economy to external financing depends on the country's

[2]Strictly speaking, *iii)* is true when there exists one risk neutral consumer with an unbounded consumption set or when the country is atomless and there is no aggregate uncertainty.

reputation, on the likelihood of its defaulting on its debt, and on the legal structure that determines who are the residual claimants in case of default.

We analyze a model with two agents, one risk neutral (the investor) and the other risk-averse (the manager) who owns the technology and decides how much to invest in physical capital[3]. There is a Cobb-Douglas technology for production and no disutility from labor as in Brock and Mirman' stochastic growth model [9]; the main difference is that investment is converted into new units of capital through a non-linear technology that is affected by stochastic shocks. We analyze the behavior of the model under four different regimes *i)* autarky, where the manager does not use external financing; *ii)* external financing with full information and full enforcement of debt contracts; *iii)* external financing with *partial* information and perfect enforcement of contracts and *iv)* external financing with perfect information but *partial* enforcement of contracts.

We demonstrate that these different communication-commitment technologies have large and very different effects on growth. The results indicate that limited enforcement can severely limit the ability of outside financing to enhance growth, although borrowing is still useful in order to smooth consumption against unforeseen shocks. On the other hand, limited information permits growth levels as high as with perfect information. In our computational experiements we show that the growth rate in the economy with partial information can be one

[3]The principal-agent relationship can be interpreted literally or, following [15], as representing one small risk-averse agent having access to outside perfect capital markets with a riskless security; we return to this point in Section 2.

percentage point higher than in the economy with partial enforcement. In this paper, we always refer to growth as the transitional state from a low initial capital stock to the steady state distribution of capital.

This paper has as a direct predecessor in Marimon [28]. There, it is shown that alternative mechanisms imply different wealth distributions and possibly accumulation paths. Also, that in an economy with limited communication and enforcement, the loss of efficiency due to incentive constraints can be made arbitrarily small if the discount factor is close enough to one. It also relates to the more recent literature that follows Bewley's [6], [7] and Green's [15] analysis of an exchange economy with a continuum of agents with idiosyncratic risks (see [2], [3], [16], [22], [34],[37], and [38]).

There are two main analytical results in this paper. First, once we have characterized the set of efficient contracts for the environment with full information and full enforcement, we show how to construct an efficient mechanism for an environment with limited information and full enforcement recursively using the set of efficient contracts of the full information environment; we call this efficient mechanism the *λ-transfer mechanism*. In the limited information environment, investors can not monitor entrepreneur's investment decisions and can only observe past capital stocks. The *λ-transfer mechanism*, however, preserves the optimal investment policy of the full information environment. That is, in our context -with a risk neutral agent-, the entrepreneur can be induced to follow the optimal investment policy if his stream of consumption is made conditional on output -or capital- realizations, so that it looses the perfect insurance property of the optimal contract with full information and full enforcement.

Second, in the environment with full information and limited enforcement, we assume that it is possible for the manager to take possession of the capital stock and switch to autarky; if he does so, he stays in the autarkic regime forever. Enforcement constraints, then, take the form of participation constraints in which the utility for the risk-averse agent (the manager) of staying in the contract, is always at least as high as the utility from going to the autarkic regime. With these participation constraints, the optimal contacting problem does not have a standard recursive structure suitable for dynamic programming techniques. We show, however, that this problem can be transformed into a saddle-point dynamic programming problem[4]; this is crucial in making the model computable, since it guarantees that the solution is a time-invariant function of a few state variables. In particular, the optimal investment policy depends not only on the current capital stock and autocorrelated shock, as in the full enforcement environment, but also on a new *reputational* state variable that summarizes the credit record of the manager. As a result, when, for example, the initial capital stock is relatively low, as to require an important inflow of outside financing, enforcement constraints are a severe limitation on this flow and the resulting optimal accumulation path can be fairly close to that of an economy with self-financing. Nevertheless, the consumption path is smoother than in the self-financing environment. Therefore, under limited enforcement external financing can be used to smooth out cyclical variations of output, but not to maintain a constant level of consumption along the growth path towards the steady state.

[4]A full treatment of this result is included in [26].

We illustrate, quantify and expand these analytical findings with our numerical analysis and results. We apply the parameterized expectation approach (PEA) developed by Marcet [25]. That is, we parameterize the conditional expectation of the optimality conditions with flexible functional forms, and we iterate on these expectations until they are the best prediction of the future in the series they generate. Some features of our application of PEA are novel. As described in [25], this algorithm is suitable for finding the steady state distribution of a model; however, using similar ideas to the ones found in [29] we can solve for the transitional growth path. Also, the participation constraints take the form of inequality constraints that involve conditional expectations and that are binding in some periods and non-binding in others. To our knowledge this is the first paper where a dynamic model with this type of constraints is solved[5].

The use of computational experiments is crucial in obtaining several of our results. This paper is, therefore, one example of how computations can be used for obtaining results that are in essence theoretical: they illustrate the behavior of the economy, allow us to make quantitative statements, and enrich the analysis. For example, the fact that growth under limited enforcement can be as slow as under autarky is a result from our computational experiments.

The rest of the paper is organized as follows. Section 2 presents the

[5]Phelan and Townsend [34] have computed sequentially efficient mechanisms for stationary economies with no capital accumulation. They follow the approach of linearizing the sequential constraints by means of lotteries over continuation payoffs. With this approach, they can solve for the efficient mechanism by solving a large number of linear programming problems. Our approach differs from theirs in that we do not linearize the problem and, by constructing λ-transfer mechanisms, we can limit most computations to solving maximization problems without information constraints.

model and the theoretical results under autarky and external financing with perfect information and perfect enforcement of contracts; these economies, with no incentive constraints, are used as benchmark for the model with incentive constraints. Section 3 analyzes the model under private information and full enforcement; it defines the λ-*transfer-mechanism* and proves its optimality. Section 4 analyzes the model under partial enforcement and full information; it provides a recursive formulation of this time-inconsistent model. Section 5 describes the computational algorithm in more detail. Section 6 presents some numerical results.

2 Benchmark Economies: Self-financing and External Financing without Incentive Constraints

In all four environments analyzed in this paper some elements -preferences of the agents, exogenous shocks and technologies- remain constant. More precisely, we have two agents: agent one, who is risk averse and decides how much to invest; we refer to him as the manager; agent two is risk neutral and we refer to him as the investor. The technology is described more precisely in equations (1) and (2) below. The main difference with the usual stochastic models of capital accumulation is that the technology that converts units of investment into unit of new capital is non-linear and affected by productivity shocks.

However, financing opportunities differ across environments. In this section we discuss the extreme cases of an autarkic solution and the model of external financing with full information and full enforcement.

With full information and enforcement and given an initial capital stock k_0 and aggregate shock θ_0, efficient transfer mechanisms are obtained as a solution to a dynamic principal-agent problem. An efficient growth mechanism, Γ, specifies state-contingent investment and transfer plans, $\Gamma = \{i_t, \tau_t\}$, and it is indexed by (λ, k_0, θ_0), where $\lambda \in R_+$ is the weight assigned to the risk-averse agent in the following planner's problem:

$$\max \qquad (1-\delta)E_0\left[\sum_{t=0}^{\infty}\delta^t\left[\lambda u(c_t) + (-\tau_t)\right]\right]$$

subject to:

$$c_t + i_t - \tau_t = f(k_t) \tag{1}$$

$$k_t = dk_{t-1} + g(i_{t-1};\theta_t, s_t) \tag{2}$$

$c_t \geq 0, \quad i_t \geq 0, \quad k_0$ given.

Here, $u(\cdot)$ represents the instantaneous utility function of the risk-averse manager, $f(\cdot)$ the production function, and $g(\cdot)$ the function that transforms investment goods into capital goods. The variable c_t represents consumption of the manager; τ_t transfers from the investor to the manager or, alternatively, $-\tau_t$ can be interpreted as the consumption the risk neutral investor obtains for his services. We assume that both agents have the same discount factor. The exogenous stochastic shocks (θ_t, s_t) affect the productivity of investment; where s_t is an idiosyncratic i.i.d. shock, which, in the environment with limited information, is private information, and θ_t is a first order autoregressive individual shock which is public information. Note that the investment technology is such that at the time the investment decision is made the realizations of the shocks are unknown.

Assumptions The following assumptions are made: *i)* the utility function $u(\cdot)$ is strictly concave, twice differentiable and satisfies the Inada conditions: $u'(c) \to +\infty$ as $c \to 0$, $u'(c) \to 0$ as $c \to \infty$; *ii)* f is concave and differentiable; *iii)* the exogenous stochastic processes (θ_t, s_t) are stationary and mutually independent; *iv)* $d \in [0,1]$; *v)* $g(\cdot;\theta,s)$ is differentiable and concave, with fixed range independent of (θ, s); if $i' > i$, then the distribution of $g(i';\cdot,\cdot)$ (second order) stochastically dominates the distribution of $g(i;\cdot,\cdot)$, and $g(\cdot;\theta,s)$ satisfies the Inada conditions described in (i); and *vi)* there exists $\beta > 0$ and $\bar{k}$ such that, for all $k \geq \bar{k}$, $f'(k) \leq \beta$, and, for all θ and i, if $E[g(i;\cdot,\cdot)|\theta] \geq (1-d)k$, then $\delta^{-1} \geq d + \beta E[g'(i;\cdot,\cdot)|\theta]$.

Remark The above assumptions are relatively standard in the stochastic growth literature. The main exception is (v), which is introduced to guarantee that in a private information environment it is not possible *ex-post to* detect investment decisions with probability one from observations on the capital stock and the serially correlated shock, thereby making the problem of monitoring investment in the private information economy interesting. Assumption (vi) guarantees that present discounted values are well defined and, as it can be seen, allows for *long-run* growth of the capital stock[6]. Nevertheless, in this paper we concentrate our attention in the case $\beta < 1$, where there is a stationary state

[6] Jones and Manuelli [18] have similar assumptions in their growth model.

and growth is a transitional process towards the steady state, although our main results generalize to the case described by assumption (vi).

2.1 The environment with self-financing (AU)

The self-financing (autarkic) solution for an economy with lack of communication and commitment is obtained from the above planner's problem by having $\lambda = 1, \tau_t = 0$, for all t, and solving for an optimal investment process $\{i_t\}$. In addition to (1) and (2) the autarkic problem (AU) has the following first order condition [7]:

$$u'(c_t) = \delta E_t[\frac{\partial g_{t+1}}{\partial i_t} \sum_{n=0}^{\infty} (\delta d)^n u'(c_{t+n+1}) f'(k_{t+n+1})] \tag{3}$$

Using standard arguments one can show the existence of a time-invariant investment policy function $i^a(k, \theta)$, a consumption policy function $c^a(k, \theta)$ and a value function $V^a(k, \theta)$.

2.2 The environment with full information and full enforcement (PO)

When both agents observe all shocks and contracts are perfectly enforceable, efficient contracts are solutions to the planner's problem described at the beginning of this section. In addition to (1) and (2), the first order conditions are:

$$1 = \delta E_t \left[\frac{\partial g_{t+1}}{\partial i_t} \sum_{n=0}^{\infty} (\delta d)^n f'(k_{t+n+1}) \right] \tag{4}$$

[7]Throughout the paper $\frac{\partial g_{t+1}}{\partial i_t}$ represents the derivative of the function g with respect to its first argument; notice that this derivative depends on future values of the stochastic shocks.

$$u'(c_t) = \lambda^{-1} \tag{5}$$

It follows that the stationary policy functions $(i^*(\lambda, k, \theta), c^*(\lambda, k, \theta), \tau^*(\lambda, k, \theta))$ have some interesting properties. First, notice that $c^*(\lambda, k, \theta) = c(\lambda)$. That is, the risk-averse agent is fully insured with a constant stream of consumption; which only depends on λ. Second, the investment policy takes the form $i^*(\lambda, k, \theta) = i(k, \theta)$. That is, accumulation paths are independent of the relative weights in the planner's problem. This implies that growth is independent of the wealth distribution. Furthermore, if there is a steady-state distribution of capital stocks, and the initial capital, k_0, is low with respect to the steady-state distribution, then the risk-averse manager borrows heavily in the initial periods in order to finance high investment levels. Finally, since the risk-neutral agent absorbs all the fluctuations, transfers depend on λ and the current values of (k, θ).

The optimal policies define an efficient transfer mechanism for this environment: $\Gamma_\lambda = \{i_t, \tau_{\lambda t}\}$, where $i_t = i(k_t, \theta_t)$ and $\tau_{\lambda t} = \tau^*(\lambda, k_t, \theta_t)$. Given Γ_λ and an initial state (k, θ), the agents' valuation of the contract is given by

$$\begin{aligned} v_1(\lambda, k, \theta) &= (1-\delta)E_0 \sum_{t=0}^{\infty} \delta^t u(c_t) = u(c(\lambda)) \\ v_2(\lambda, k, \theta) &= (1-\delta)E_0 \sum_{t=0}^{\infty} \delta^t (-\tau_{\lambda t}) \end{aligned}$$

Remark (Competitive transfers) Of special interest is the competitive mechanism Γ_{λ^*}, where $\lambda^*(k_0, \theta_0)$ is the only value $\lambda \in R_+$ satisfying $v_2(\lambda^*(k_0, \theta_0), k_0, \theta_0) = 0$. Using standard arguments, it can be shown that, $\lambda^*(k_0, \theta)$ is the inverse of the marginal utility of expenditure in the *competitive equilibrium* in which the agent faces a lifetime budget constraint, has property rights over k_0 and an initial shock θ_0. The existence and uniqueness of $\lambda^*(k_0, \theta_0)$ can easily be derived from our assumptions.

Remark (The continuum of agents formulation) We can now briefly describe an economy with a continuum of agents for which efficient contracts can be characterized by the principal-agent formulation used in this paper. To this end, we adapt the arguments in [15] to an economy with capital accumulation. Assume there is a steady state (see the remark at the end of the previous subsection). That is, the optimal investment policy $i^*(k, \theta)$ and the markovian process $\{\theta\}$ define a transition probability $P(\cdot|(k, \theta)) \in \Delta(K \times \Theta)$ and this transition probability defines an ergodic measure $\mu \in \Delta(K \times \Theta)$; where $(K \times \Theta)$ is a compact subset of R^2_+ and $\Delta(K \times \Theta)$ denotes the set of probability distributions on this subset. Now, assume agents are uniformly distributed in $[0, 1]$ and have independent and equally distributed shocks, $\{(\theta_t, s_t)\}$. Let $\nu \in \Delta([0, 1])$ denote the uniform distribution of agents. Agents have homogeneous preferences. The initial state $(k_0, \theta_0)_a$ for agent a is given by e_a, where the measurable map $e : [0, 1] \rightarrow K \times \Theta$ satisfies $\nu\{a : e_a \in M\} = \mu(M)$, for every (Borel) subset $M \subset K \times \Theta$. Therefore the economy is at the steady state. More generally, one can think that, at the moment that the efficient contract is being designed, a set -of measure one- of agents has reached the steady state. Then,

$$\int [f(k) - i(k, \theta)] d\mu(k, \theta) = \bar{c}$$

for some constant $\bar{c} > 0$. That is, the steady state aggregate consumption is constant. Efficient allocations are now solutions to a planner's problem of the form:

$$\max \quad (1-\delta)\sum_{t=0}^{\infty} \delta^t \int \tilde{\lambda}_a u(c_{a\,t}) d\nu(a)$$

subject to:

$$\int c_{a\,t} d\nu(a) = \bar{c}$$

It follows, that there exists a constant $\alpha > 0$, such that

$$u'(c_{a\,t}) = \alpha \tilde{\lambda}^{-1}$$

Let $\lambda_a = \tilde{\lambda}_a \alpha^{-1}$, then (5) is satisfied, and

$$\begin{aligned} \tau^*(\lambda_a, k, \theta) = \tau^1(\lambda_a) + \tau^2(k, \theta) &= c(\lambda_a) + i(k, \theta) - f(k) \\ \text{satisfies} \int \tau^1(\lambda_a) d\nu(a) = 0 \text{ and} & \int \tau^2(k, \theta) d\mu(k, \theta) = 0 \end{aligned}$$

Notice that the above competitive solution corresponds to a particular choice of social weights; namely: if $e_a = (k_0, \theta_0)$ then $\lambda_a = \lambda^*(k_0, \theta_0)$. The efficient contract is defined for any possible agent and event, not only at the steady state. An individual agent may not be at the steady state, however a set of full measure of agents must be at the steady state for the contract to be feasible. By the law of large numbers, aggregation over this set of agents reduces the contracting problem to the principal-agent problem of our analysis[8].

[8]Here we do not discuss some known technicalities like the right integrability definition for a proper application of the law of large numbers.

3 The environment with limited communication and full enforcement (PI)

We proceed with the study of an economy where investments are not observable and transfer payments can only depend on past transfers and capital stocks. In this environment a transfer mechanism, $\Gamma = \{i_t, \tau_t\}$, makes recommendations for non observable investments and observable transfers as a function of all past public information. We follow Abreu, Pearce and Staccetti [1] who, in turn, follow a dynamic programming approach in characterizing a contract as a prescription for each event of an action and a continuation payoff contingent on the observed consequences (output and capital stock) of the prescribed action.

In general, consider a mechanism Γ that recommends actions $\{x_t\}$ and let z represent the current observable state; z may include all the past history of observables or, in a recursive problem, z can be a finite vector of state variables. Given z, the current action x affects the distribution of tomorrow's state z'. If the agent's current payoff is given by $\pi(x)$ and the present value, at z, of the contract Γ is $v(z)$, then the mechanism (contract) is said to be *sequentially incentive compatible* if for every state z :

$$\begin{aligned} v(z) &= (1-\delta)\pi(x) + \delta E_{(x,z)} v(z') \\ &\geq (1-\delta)\pi(\tilde{x}) + \delta E_{(\tilde{x},z)} v(z') \end{aligned}$$

Γ is said to be a *sequentially efficient mechanism* if it is sequentially incentive compatible and it is not Pareto dominated by any other sequentially incentive compatible mechanism.

The λ-transfer mechanism

In addition to observable transfers, natural candidates as state variables are (k, θ), but these variables are not enough in order to design an incentive compatible mechanism. Recall that in the full information environment $\tau_{\lambda t} = \tau^*(\lambda, k_t, \theta_t)$ guarantees a constant stream of consumption. With private information, if the manager is offered the contract that is optimal under full information (as in section 2), he will under-invest, since $\pi(i, \tau, k) = u(f(k) + \tau - i)$. To create the right incentives, let the state variable be $z = (\lambda, k, \theta)$. Then, ex-post present values can be associated with alternative λ values of the social planner's problem with full communication, and the agent is rewarded or punished along each observed history by changing the weight λ. We call this type of mechanism a λ-transfer mechanism and show that it is a sequentially efficient mechanism.

As we have seen, in the full communication-commitment environment, agents can perfectly smooth their consumption and investment is independent of the weight, λ, given to the representative agent in the planner's problem. The λ-transfer mechanism for an economy with limited communication and perfect enforcement induces a less smooth pattern of consumption. Ex-ante homogeneous agents have an ex-post unequal distribution of wealth. We will show that investment, however, is not affected by the presence of information constraints, and the process of capital accumulation is the same as in the economy with full information.

The λ-transfer mechanism exploits the downward sloping property of the Pareto frontier in the full information and full enforcement (PO) problem. With a downward sloping Pareto frontier, if agent one has weight λ and the state is (k,θ) then for agent $i = 1, 2$, there is a unique value w_i such that $w_i = v_i(\lambda, k, \theta)$. The inverse problem is also well defined. That is, given a value w_i and a state (k,θ) there is a unique λ satisfying $v_i(\lambda, k, \theta) = w_i$.

The λ mechanism is recursively constructed as follows. If the current state is $z = (\lambda, k, \theta)$, then the recommended current actions are $i(k,\theta)$ and $\tau^*(\lambda, k, \theta)$; *i.e.*, the efficient actions of the PO problem with λ. The efficient level of investment, the current shock, and the unobservable shocks, determine tomorrow's capital and observable shock (k', θ'). The λ-transfer mechanism completes the definition of tomorrow's state by means of a function h(.) which determines tomorrow's λ'; we will specify this function in the next paragraph. This implies that, $z' = (h(\lambda, k, \theta, k', \theta'), k', \theta')$. In other words, while in the full information environment, λ is constant and the manager's value of an efficient contract is $\bar{v}_1(\lambda) \equiv v_1(\lambda, k, \theta)$, for all (k,θ), in the private information environment, the λ-transfer mechanism prescribes recursive revisions of λ and, therefore, the manager's present values of his future stream of consumption are also revised -say, from $\bar{v}_1(\lambda) \equiv v_1(\lambda, k, \theta)$ to $\bar{v}_1(\lambda') \equiv v_1(\lambda', k', \theta')$.

We now define $h(\cdot)$. The main idea is that the risk averse manager should bear some of the fluctuations that in the full information environment are solely absorbed by the risk neutral investor. Let $\hat{v}_2(\lambda, k, \theta, k'\theta') = v_2(\lambda, k', \theta') - E_{(i(k,\theta),k,\theta)} v_2(\lambda, k', \theta')$. That is, $\hat{v}_2(\cdot)$ is the investor's deviation of the realized value of utility from the conditional expected value of utility in the full information efficient contract one period before. Define,

$$h(\lambda, k, \theta, k', \theta') = \bar{v}_1^{-1}(\bar{v}_1(\lambda) + \lambda^{-1}.\hat{v}_2(\lambda, k, \theta, k', \theta'))$$

In other words, if the current state is $z = (k, \lambda, \theta)$ and, after the recommendation to follow the optimal action, the observed state is (k', θ') then the manager should suffer (gain) a deviation from $\bar{v}_1(\lambda)$ of $\lambda^{-1}.\hat{v}_2(\lambda, k, \theta, k', \theta')$. That is, agent one is punished or rewarded with the deviation of agent two's utility in the PO problem, properly weighted by λ^{-1}.

The λ-transfer mechanism induces the manager to solve the planner's problem at every state and, therefore, the planner's Bellman's equation becomes the manager's incentive compatibility condition. Furthermore, since, by optimality, the planner's solution can not be improved at any state the λ-transfer solution can not be improved upon. These are the two ideas behind the proof of the following proposition.

Proposition 1. The λ-transfer mechanism is a Sequentially Efficient Mechanism for an economy with limited communication and full enforcement.

Proof. The mechanism is resource feasible since it defines a sequence of feasible actions (from the corresponding PO problems). We now show that it satisfies the incentive compatibility constraints. Let the current state be $z = (\lambda, k, \theta)$, then the λ mechanism is sequentially-incentive compatible if

$$\begin{aligned} v_1(\lambda,k,\theta) &= (1-\delta)\pi(i(k,\theta),\tau(\lambda,k,\theta),k)+ \\ &\quad \delta E_{(i(k,\theta),k,\theta)} v_1(h(\lambda,k,\theta,k',\theta'),k',\theta') \\ &\geq (1-\delta)\pi(\tilde{i},\tau(\lambda,k,\theta),k)+ \\ &\quad \delta E_{(\tilde{i},k,\theta)} v_1(h(\lambda,k,\theta,k',\theta'),k',\theta') \end{aligned}$$

By construction, the last inequality is simply:

$$\begin{aligned} \bar{v}_1(\lambda) &\geq (1-\delta)\pi(\tilde{i},\tau(\lambda,k,\theta),k)+\delta E_{(\tilde{i},k,\theta)}\left[\bar{v}_1(\lambda)+\lambda^{-1}\hat{v}_2(\lambda,k,\theta,k',\theta')\right] \\ &= (1-\delta)\pi(\tilde{i},\tau(\lambda,k,\theta),k)+\delta\bar{v}_1(\lambda)+ \\ &\quad \lambda^{-1}\delta E_{i(\tilde{i},k,\theta)}\left[v_2(\lambda,k',\theta')-E_{(i(k,\theta),k,\theta)}v_2(\lambda,k',\theta')\right] \end{aligned}$$

Using the fact that $v_1(\lambda,k',\theta')=\bar{v}_1(\lambda)$,the last inequality can be expressed as:

$$\begin{aligned} \lambda\left[(1-\delta)\pi(i(k,\theta),\tau(\lambda,k,\theta),k)\right. &+ \left.\delta E_{(i(k,\theta),k,\theta)}v_1(\lambda,k',\theta')\right]+ \\ \left[(1-\delta)(-\tau(\lambda,k,\theta))\right. &+ \left.\delta E_{(i(k,\theta),k,\theta)}v_2(\lambda,k',\theta')\right] \\ \geq \lambda\left[(1-\delta)\pi(\tilde{i},\tau(\lambda,k,\theta),k)\right. &+ \left.\delta E_{(\tilde{i},k,\theta)}v_1(\lambda,k',\theta')\right]+ \\ \left[(1-\delta)(-\tau(\lambda,k,\theta)\right. &+ \left.\delta E_{(\tilde{i},k,\theta)}v_2(\lambda,k',\theta')\right] \end{aligned}$$

This inequality says that in the full information and enforcement (PO) problem with weight λ, at the state (k,θ), $i(k,\theta)$ is the best feasible action. By optimality of $i(k,\theta)$ the inequality is satisfied. This argument, however, not only shows that the λ-transfer mechanism is *sequentially-incentive compatible*, but almost demonstrates its efficiency. We now complete this argument.

Suppose there exists a sequentially incentive compatible mechanism Γ^* that Pareto dominates the λ-transfer mechanism. Let (v_1^*, v_2^*) be the present values attained through Γ^* (for a given state (k_0, θ_0)). Let $\lambda = \bar{v}_1^{-1}(v_1^*)$ and use the initial condition $z = (\lambda, k_0, \theta_0)$ to recursively define the λ-transfer mechanism. Notice that by construction agent one has the same present value for both contracts. Therefore, Pareto dominance requires that $v_2^* > v_2(\lambda, k_0, \theta_0)$. But this is not possible, otherwise we obtain a contradiction with the Pareto optimality of the solution to the full information and enforcement (PO) problem with weight λ.

4 The Environment with Full Information and Limited Enforcement (PC).

Enforcement constraints are very different from the information constraints in the previous section. We now study the case in which society (or the investor) has full commitment and the system of property rights establishes that, when the manager breaches the contract, he can take possession of the existing capital stock, but he will then be prevented from ever re-entering the social mechanism. That is, the current reservation value for the manager is the utility of the autarkic solution given the current capital stock and productivity shock. In this environment, a contract can be enforced only if the utility the manager derives from the contract is, at each point in time, at least as high as the utility from autarky; this means that the manager will have to be compensated so as to make his utility high enough at every period.

With participation constraints, the set of efficient mechanisms for an economy with limited enforcement and full information can be parameterized by (λ, k, M, θ), where the state variable M accounts for the past periods in which the participation constraints have been binding. The participation constraint is a non-standard constraint in dynamic programming. Nevertheless, we show how the problem can be cast in a recursive framework, where the solution is given by a time-invariant function of the natural state variables and the pseudo-state variable M. This approach of making recursive the characterization of the optimal contract has independent interest since it can be applied to other non-recursive problems.

Optimal allocations can be found by maximizing a planner's problem giving different weights to each agent, subject to participation constraints on agent 1, so that we have to solve

Program 1

$$\max_{\{c_t,\tau_t,i_t,k_t\}_{t=0}^{\infty}} (1-\delta)E_0\sum_{t=0}^{\infty}\delta^t[\lambda u(c_t) - \tau_t]$$

$$\text{subject to: } c_t - \tau_t + i_t = f(k_t) \tag{6}$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \quad \textit{and} \tag{7}$$

$$(1-\delta)E_t[\sum_{i=0}^{\infty}\delta^i u(c_{t+i})] \geq V^a(k_t, \theta_t) \tag{8}$$

for all t, where V^a is the value function under autarky. Equations (6) and (7) are the technology constraints, and equation (8) is the participation constraint that makes the utility of the first agent in every period at least as large as the utility he would obtain from switching to an autarkic regime from time t onwards.

The dynamic programming characterization of Program 1 is not trivial, and our treatment is of independent interest to study problems with expectational constraints. To realize the special features of Program 1 let us recall that a standard dynamic program has the following form:

Program 2

$$\max_{\{x_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t r(x_t, x_{t-1}, s_t)$$

$$\text{s.t.} \quad x_t \leq \Gamma(x_{t-1}, s_t), \quad x_{-1} \text{ given.} \tag{9}$$

(see, for example, [23]) where x_t is a vector of *finite* length, s_t a stochastic shock and the functions r and Γ are known and *independent* of the choice for x_t. Unfortunately, the participation constraint (8) is not a special case of (9): even though the conditional expectation in the left side of the participation constraint is a function of past state variables, this function depends on the whole stochastic process $\{c_t\}_{t=0}^{\infty}$. In other words, the function Γ should not depend on the choice of the endogenous variables, but the conditional expectation in equation (8) does.

The next Proposition states that Program 1 can be rewritten in a way that the objective function and the constraints are recursive. A similar approach can be used in most problems with expectational constraints and time inconsistent solutions, for example, [35] applies similar ideas to a model of optimal taxation. The general idea is to introduce expectational constraints in the objective function of the Lagrangean. This will be useful in order to characterize the form of the solution and, perhaps more importantly, to calculate the numerical solution of the problem since, by casting the solution in a recursive framework, we know that the solution is a time invariant function of a small set of state variables.

The following proposition states formally this equivalence.

Proposition 2. The solution to Program 1 is the saddle point of the following Lagrangean:

Program 3

$$\mathcal{L} = (1-\delta)E_0 \sum_{t=0}^{\infty} \delta^t \{(\lambda + M_{t-1})u(c_t) - \tau_t +$$

$$\mu_t[u(c_t) - V^a(k_t, \theta_t)/(1-\delta)]\}$$

subject to , (6) - (7), $\mu_t \geq 0$,

$$M_t = M_{t-1} + \mu_t \quad \text{and} \quad M_{-1} = 0. \tag{10}$$

where μ_t is the Lagrange multiplier of the participation constraint at time t. In Program 3 we minimize with respect to $\{\mu_t\}$ and maximize with respect to $\{c_t, k_t, i_t, \tau_t\}$, .

Proof. By the usual arguments, the solution to Program 1 is the saddle point of

Program 4

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \delta^t \{\lambda u(c_t) - \tau_t +$$

$$\mu_t[E_t[\sum_{i=0}^{\infty} \delta^i u(c_{t+i})] - V^a(k_t, \theta_t)/(1-\delta)]\}$$

subject to the technology constraints (6) - (7) and

$$\mu_t \geq 0$$

Program 4 has conditional expectations in the return function so that it is not yet of the form of program 2. We finally obtain Program 3 by using the law of iterated expectations to eliminate the symbols E_t, rearranging the objective function, and introducing the law of motion for M_t as constraint (10).

Notice that both the return function and the constraints in Program 3 *are* of the form of standard dynamic programs like Program 2 where the feasible set at t is a known function of the past. Arguments adapted from standard dynamic programming, [26] show that the optimal decision for the control variables at time t is a time-invariant function of the state variables (k_t, M_{t-1}, θ_t). Program 3 displays the unusual feature of having Lagrange multipliers in a constraint.[9]

Equation (10) can be viewed as a constraint that the planner imposes on himself in order to follow the optimal path. Given that only k_t and θ_t enter in the return function and in the constraints at time t of the original Program 1, it would be physically possible for the planner to re-set $M_{t-1} = 0$ at any point in time, and this is what the

[9]See [10] for a similar use of these multipliers and [26] for a description of how this approach can be applied to many other models. Kydland and Prescott [21] showed how some time inconsistent models could be placed in a recursive framework in a problem of optimal taxation. They also had past Lagrange multipliers determining future decisions, and they used the expression 'pseudo-state variable' to denote these Lagrange multipliers. Unlike in our case, though, their Lagrange multiplier was the one in the budget restriction of the agent ($u'(c_t)$) and their problem was only recursive after the initial period . In models with uncertainty their approach seems to lose some recursive properties and it is not clear how it would apply to a restriction on the value function as in the PC model.

planner would do if he reoptimized at time t. However, the optimal path set at the initial period calls for a scrupulous observation of (10), and resetting M_t at a later date is suboptimal. This is another version of the time-inconsistency problem of Kydland and Prescott [20].

The first order conditions of the problem are

$$\delta^t[(\lambda + M_t)u'(c_t) - 1] = 0 \tag{11}$$

$$\delta^t[-1 - \delta E_t[\gamma_{t+1}\frac{\partial g_{t+1}}{\partial i_t}]] = 0 \tag{12}$$

$$\delta^t[f'(k_t) - \delta d E_t[\gamma_{t+1}] + \gamma_t - \mu_t\frac{\partial V_t^a}{\partial k_t}/(1-\delta)] = 0 \tag{13}$$

$$\delta^t[E_t[\sum_{j=0}^{\infty}\delta^j u(c_{t+j})] - V^a(k_t, s_t)/(1-\delta)] \geq 0 \tag{14}$$

$$\mu_t[E_t[\sum_{j=0}^{\infty}\delta^j u(c_{t+j})] - V^a(k_t, \theta_t)/(1-\delta)] = 0 \tag{15}$$

the technology constraints (6) and (7), the law of motion for M_t (10), the participation constraint and $\mu_t \geq 0$. In equations (11) to (15), γ_t represents the Lagrange multiplier of constraint (7).

The variable M_t is an accumulation of past multipliers; roughly speaking, if the participation constraint has been binding very often in the recent past, then M_t will be high. The role of M_t in Program 3 is to shift the weight λ given to agent 1 in the objective function of the planner; when the participation constraint is binding, the optimal path calls for augmenting this weight; this increase is maintained for all future periods and consumption is higher forever. Therefore, whenever (8) is binding, the planner compensates agent 1 by increasing his consumption to a certain level and leaving consumption at this level until the participation constraint binds again.

Characterization of Equilibrium in the PC model

In this sub-section we parameterize the function that converts investment into new capital goods as:

$$g(i_t, \theta_{t+1}, s_{t+1}) = a(\theta_{t+1} + s_{t+1})i_t/(1 + i_t) + bs_{t+1}$$

Consumption of agent 1 satisfies

$$u'(c_t) = 1/(\lambda + M_t), \tag{16}$$

so that c_t depends only on $(\lambda + M_t)$. Assuming that the shocks have bounded support, there exists a finite constant $\bar{V}$ such that $\bar{V} \geq V^a(k_t, \theta_t)$ with probability one, so that M_t and c_t will grow until M_t reaches a level such that

$$u(c_t) \geq \bar{V}; \tag{17}$$

this inequality means that the utility of keeping consumption constant for the whole future is higher than the upper bound on autarkic utility. If M_t reaches this level, consumption will not change, since the participation constraint will never be binding again and M_t will be constant from then on.

We can now study the behavior of investment. With the above functional form for g, the first order conditions (12) and (13) reduce to

$$(1 + i_t)^2 = \delta E_t[(\theta_{t+1} + s_{t+1})a \sum_{j=0}^{\infty} (\delta d)^j \alpha k_{t+j+1}^{\alpha-1}] - \tag{18}$$

$$\delta E_t[(\theta_{t+1} + s_{t+1})a \sum_{j=0}^{\infty} (\delta d)^j \mu_{t+j+1} \frac{\partial V_{t+j+1}^a}{\partial k_{t+j+1}} / (1 - \delta)]$$

It is clear that the Euler equation for the case with full enforcement and full information (4) is exactly like (18) without the second conditional expectation that depends on future $\mu's$. We conclude that investment is lower in periods when the participation constraint is likely to be binding in the near future; in this case, the second expectation in equation (18) has a high absolute value, and the left hand side must go down.

We have seen that when M_t reaches a high enough level the participation constraint will never again be binding, so that the second expectation in the right side of (18) vanishes and it becomes (4). Therefore, from this period on investment will always be equal to the level of optimal investment with full enforcement. Hence, the steady state distribution for capital accumulation of the limited enforcement model is the same as in the PO model. In the initial periods, however, when the constraints are binding and M_t is growing, the behavior of investment can not be determined analytically and we will resort to simulations of the model. These are described in section 6.

5 An Algorithm for Solving the Growth Model with Incentive Constraints

We will explain here how to obtain numerical solutions for the various models in this paper. There are four models that we want to solve: autarky equilibrium (AU), Pareto Optimal with full communication and full enforcement (PO), the model with participation constraints (PC) and under private information (PI).

We use the following functional forms:

$$f(k_t) = k_t^\alpha$$

$$g(i_t, \theta_{t+1}, s_{t+1}) = a(\theta_{t+1} + s_{t+1})i_t/(1 + i_t) + bs_{t+1}$$

$$u(c_t) = c_t^{\gamma+1}/(\gamma + 1)$$

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t$$

where ε_t is i.i.d.

5.1 Solving the Autarkic and Pareto Optimal equilibria

The AU model has the following first order conditions:

$$c_t^\gamma = \delta E_t[\frac{\partial g_{t+1}}{\partial i_t} \sum_{j=0}^{\infty} (\delta d)^j c_{t+j+t}^\gamma \alpha k_{t+j+1}^{\alpha-1}] \tag{19}$$

$$c_t + i_t = k_t^\alpha \tag{20}$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \tag{21}$$

To solve this model numerically we use the parameterized expectation approach (PEA). Since there is only one expectation to approximate, the model can be solved quite easily. We substitute the expectation in the right side of (19) by a parameterized function of the state variables $\psi(\beta, k_t, \theta_t)$. We choose ψ in a flexible way so as to approximate the conditional expectation arbitrarily well. In particular, we choose

$$\psi(\beta, k_t, \theta_t) = exp(P_n(\log(k_t), \log(\theta_t))$$

for a given n, where P_n is a polynomial of degree n. The parameters β are the coefficients in the polynomial. We can, in principle, increase the degree of the polynomial until we have a reasonable approximation to the conditional expectation. This functional form is convenient because it is strictly positive, as is the conditional expectation that it intends to approximate.

We want to find the parameter β_f with the following property: if agents use β_f in order to form the expectations of the Euler equation, then $\psi(\beta_f, k_t, \theta_t)$ is the best predictor among functions $\psi(., k_t, \theta_t)$.

The mechanics for finding β are the following:

-Step 1 - fix β. Substitute the conditional expectation in (19) by ψ to obtain:

$$c_t^\gamma(\beta) = \delta\psi(\beta, k_t(\beta), \theta_t). \tag{22}$$

-Step 2 - obtain a long series of the endogenous variables that solves (20), (21) and (22)[10] for this particular β; call this series $\{c_t(\beta), i_t(\beta), k_t(\beta)\}$

-Step 3 - for this series calculate the expressions inside the conditional expectation of (19)[11] and perform a non-linear regression of these variables on $\psi(., k_t(\beta), \theta_t)$; let $S(\beta)$ be the result of this regression.

-Step 4 - finally, use an iterative scheme to find the fixed point of S, and set $\beta_f = S(\beta_f)$ [12].

[10]Note that this is quite simple: $c_t(\beta)$ can be solved directly from (22), $i_t(\beta)$ from (20) and $k_{t+1}(\beta)$ from (21)

[11]Note how the sums $\sum_{j=1}^{\infty}(\delta d)^j c_{t+j}^\gamma \alpha k_{t+j}^{\alpha-1}$ can be calculated very efficiently using backward recursion.

[12]For more detailed description of this approach see [25]. For details on the implementation of the algorithm in a simple growth model see [13].

The solution for consumption, investment and capital is given by

$$\{c_t(\beta_f), i_t(\beta_f), k_t(\beta_f)\}$$

The solution to the PO model does not present any additional difficulties. It can be found by applying Steps 1 to 4 to the corresponding first order conditions of that model.

5.2 Solving for first periods with a low initial capital

The scheme just described can, in principle, approximate the true equilibrium at the steady state distribution arbitrarily well as the length of the simulation and the degree of the polynomial go to infinity. However, if the economy starts at a very low capital stock k_0, the β_f from long run simulations may not be a good approximation to the conditional expectation during the first few periods, as the capital stock grows from k_0 to the steady state distribution. For example, in the first few periods marginal productivity of capital is very high and the long run simulations will not take this into account. This would be a problem in our paper because we are particularly interested in analyzing growth of the economy in the initial periods.

To avoid this problem we find a different policy function (a different β_f) for the initial periods by, instead of running a long realization of the process, finding many realizations of a given (short) length T, starting each realization at k_0. Step 2 is modified as follows:

Step 2b - Obtain a large number N of (independent) realizations of length T, that solve (20), (21) and (22); each initial capital is fixed at k_0 [13].

To obtain arbitrary accuracy in $S(\beta)$, we let $N \to \infty$. Here, T is selected to be long enough for the economy to get in the range of the steady state distribution. In our model, and for the parameters we selected, $T = 50$ was appropriate.

Then we proceed with Step 3 and 4 as before.

One final modification is needed. In the conditional expectation we find discounted sums of future variables, like

$$\sum_{i=0}^{\infty}(\delta d)^i \alpha k_{t+i+1}^{\alpha-1} c_{t+i+1}^{\gamma};$$

these are used in the non-linear regression of Step 4. Since for $t+i > T$ the model is close to the steady state distributions, we could run simulations of length $T+T'$, where T' is large so that the truncated sum is close to the infinite sum, and the k_{t+i}, for witch $t+i > T$ are calculated with the steady state β_f . This is not a good solution, however, because it requires long simulations, as T' may have to be quite large.

Instead, we note that for $t < T$, the expectation in the right side of (19) can be rewritten as

$$E_t\Big(\frac{\partial g_{t+1}}{\partial i_t}[[\sum_{j=0}^{\infty}(\delta d)^j c_{t+j}^{\gamma} \alpha k_{t+j}^{\alpha-1}]+$$

$$(\delta d)^{T-t+1} E_{T+1}[\sum_{j=0}^{\infty}(\delta d)^j c_{T+1+j}^{\gamma} \alpha k_{T+1+j}^{\alpha-1}]]\Big)$$

The expectation conditional on information at $T+1$ involves only variables at the steady state distribution, so we can parameterize it

[13] A similar approach was used in Marshall [29] to solve a model with a non-stationary forcing process.

as a polynomial function of the state variables, and find the parameters in this polynomial by running (only one) regression, with a long simulation at the steady state β_f.

So, the variable predicted in the regression of Step 3 for these periods is

$$\frac{\partial g_{t+1}}{\partial i_t}[[\sum_{j=0}^{T-t}(\delta d)^j c_{t+j+1}^{\gamma}\alpha k_{t+j+1}^{\alpha-1}] + (\delta d)^{T-t+1}\psi^{ss}(\bar{\beta}, k_{T+1}, \theta_{T+1})]$$

where ψ^{ss} is the result of the non linear regression described in the previous paragraph.

5.3 Solving the Problem with Full Information and Limited Enforcement.(PC)

Now we discuss how to solve the model with limited enforcement numerically with PEA, where agent 1 (the manager) is guaranteed at least as much utility as in the autarkic equilibrium in every period, and where both agents observe all the shocks. This model is harder to solve than the previous ones because of the presence of inequality constraints that are binding in some periods and non-binding in others. Further, we now have one additional expectation to parameterize and the additional state variable M_t.

From our discussion of section 4 we see that the following equations have to be satisfied:

$$c_t - \tau_t + i_t = f(k_t) \qquad (23)$$

$$k_{t+1} = dk_t + g(i_t, \theta_{t+1}, s_{t+1}) \qquad (24)$$

$$\mu_t \left[u(c_t) + E_t[\sum_{j=1}^{\infty} \delta^j u(c_{t+j})] - V^a(k_t, \theta_t)/(1-\delta) \right] = 0, \tag{25}$$

$$u'(c_t) = 1/(\lambda + \mu_t + M_{t-1}) \tag{26}$$

$$M_t = M_{t-1} + \mu_t \tag{27}$$

$$(1+i_t)^2 = \delta E_t[(\theta_{t+1} + s_{t+1}) a \sum_{j=0}^{\infty} (\delta d)^j [\alpha k_{t+j+1}^{\alpha-1} - \tag{28}$$

$$\mu_{t+j+1} \frac{\partial V^a_{t+j+1}}{\partial k_{t+j+1}}]];$$

With these equations we can solve the model following steps 1, 2b, 3 and 4 of Subsection 5.2. Only step 2b, which involves solving for the endogenous series, is now more cumbersome. This is because, in addition to the above equations, we need to guarantee that the inequality conditions for μ and the participation constraint (8) are satisfied.

After parameterizing the conditional expectations in equations (25) and (26) the above system provides six equations to solve for $(c_t, \tau_t, k_t, i_t, \mu_t, M_t)$. To solve for c_t and μ_t we proceed as follows: first try the case where the participation constraint (8) is non-binding, so that $\mu_t = 0$, and c_t is given by equation (26). For this solution, we check if the participation constraint is satisfied; if it is, we go on to solve for the remaining variables; otherwise we know that $\mu_t > 0$, so that the large bracket in (25) is equal to zero, which provides an equation to solve for consumption; then we can find μ_t from (26). It is easy to check that μ_t will be positive by construction.

In this model the steady state distribution for investment is the same as in the PO problem with full enforcement, so the only interesting part to solve is in the first few periods as the capital stock and M_t grow to their steady state distributions. Then the scheme we use for the initial periods described in section 5.2 becomes crucial.

Finally, note that the expression inside the conditional expectation of (28) involves the derivative of V^a. Because the productivity of investment is not known at the time investment is realized, the usual formula for the derivative of the value function does not apply (see [23] for this formula). In appendix 1 we find an expression for this derivative that is easy to compute.

5.4 Solving the model with informational incentive compatibility constraints (PI)

In Section 3 we saw that in order to find the equilibrium with the incentive compatible contract at a given period, for a given value of the contract, we have to find the point in the Pareto Optimal frontier that gives the same value for the full information full enforcement model. Then the manager takes the same decision as he would take at that point in the PO frontier, and the continuation payoffs are calculated using the value functions of both agents at that point of the PO frontier. Hence, we need to have the decision functions and value functions readily available at many points of the PO frontier.

We first solve the PO problem for 1000 lambdas between zero and one. At each lambda, we calculate the β_f that corresponds to the expectation involved in the Euler equation and the β_f involved in the conditional expectation of the value function to calculate

$$\hat{v}_2(k_t, k_{t+1}, \lambda, \theta_t, \theta_{t+1}) = v_2(\lambda, k_{t+1}, \theta_{t+1}) - E_t[v_2(\lambda, k_{t+1}, \theta_{t+1})]$$

This is then used to calculate the continuation payoffs as described in Section 3.

6 Characterization of Equilibria and Simulation Results

In this section we characterize the behavior of the four models. We use mainly simulations that are plotted in Figures 1 to 7 at the end of the paper; also, the main results are summarized in Table 2. Those features of the models that we could characterize analytically were described in sections 2, 3 and 4 and we will often refer to them. The series plotted in Figures 1 to 7 correspond to a simulation using the same realization of the exogenous shocks for all series. In order to see results that do not depend on a given realization the reader is referred to Table 2, which reports calculations of some important population moments of the model.

The values of the parameters used in the simulations are described in Table 1.

TABLE 1

marginal productivity of capital	$\alpha = .5$
risk aversion parameter of the manager	$\gamma = -3$
discount factor	$\delta = .95$
autocorrelation parameter of $\log(\theta_t)$	$\rho = .95$
standard deviation of innovation of $log(\theta_t)$	$\sigma_\epsilon = .03$
standard deviation of s	$\sigma_s = .03$
mean of s	$\bar{s} = .2$
undepreciated proportion of capital	$d = .9$
constant in investment function	$a = .6$

Given the choice of d and δ, one period can be interpreted as one year. Most values of the parameters are within the usual range that is used in neoclassical growth models, with the exception of the standard deviations, which are higher than usual. We chose as initial capital $k_0 = 1$, in order to obtain growth rates of around 3 or 4% for the first fifteen periods, which seems reasonable for developing countries.

With our numerical results, we hope to illustrate the behavior of the model and detect the magnitude of the impact on growth and utility of alternative communication and commitment environments.

In Figures 1 to 7, the last two letters identify the environment, so '-au' denotes autarky equilibrium, '-po' Pareto optimal allocation with full information and perfect enforcement, '-pc' participation constraints and '-pi' private information, while the first few letters identify the series that is being plotted. For example, 'kpo' denotes capital in the Pareto Optimal allocation, 'c1pc' consumption of agent 1 (the manager) in the model with participation constraints and so on. For these figures, we plot the first 40 periods as representative of the initial periods, and periods 200 to 240 as representative of the steady state distribution.

TABLE 2

Model	Mean of Growth Rate of Output	Utility of the Manager	Mean of Capital in Steady State	Increase in Consumption
Autarky	2.88%	-0.386	2.57	
PO	3.90%	-0.358	2.53	3.84%
PC	2.91%	-0.383	2.53	0.39%
PI	3.90%	-0.36	2.53	3.55%

Note: "Mean of the growth rate of output" refers to the mean during the first fifteen periods across independent realizations. The utility of the manager is measured at time zero and using many independent replications of the model, conditioning on $k_0 = 1$ but drawing the initial shock θ_0 from the steady state distribution. The "Increase in consumption" refers to the permanent increase in consumption that would equal the present value achieved in the autarkic environment with the present values achieved in the other environments.

6.1 Autarky versus Full Information Full Commitment

We first compare the PO environment with an autarkic environment (AU). We already argued that consumption of the manager is constant in the PO equilibrium, while the investor absorbs all the shocks.

When the initial capital stock is low relative to the steady state distribution, in an economy with external financing the manager can borrow heavily at the beginning to enhance his investments and attain faster growth than he would have attained in an autarkic environment (see Figures 1 and 3). Table 2 tells us that with external financing growth can go from 3% to 4%. However, the mean of the steady state distribution of capital and investment (see Figures 2 and 4 and Table 2) is not significantly different between the two environments, even though the need to use capital as the only asset for self-insurance under autarky causes the mean of capital to be higher in an autarkic regime. Also, we see that investment is more volatile in the Pareto Optimal case; this is, then, an example where an increase in volatility of investment is not undesirable.

Consumption for the manager is constant in the full information - enforcement environment. In contrast, in an autarkic environment with low initial capital stock, consumption grows with the capital stock and fluctuates in response to random shocks.

6.2 Private information (PI) vs. autarky (AU) and vs. full information with full enforcement (PO)

We showed in Section 3 that the λ-transfer mechanism preserves the investment decisions of the full information-enforcement environment when investment decisions are observable. Therefore, capital accumulation paths for the PI coincide with the PO-paths of Figures 1-4. Figures 5 to 7 display the behavior of consumption and utility of the manager (respectively c1- and val1-), and of transfers.

Consumption is affected by the presence of informational incentive constraints, although the manager can smooth his consumption much more than in an autarkic environment and, therefore, attain higher payoff. Also, it is interesting to note that, even though the manager starts out with a very high utility, in the long run he can be worse off under private information than under autarky.

6.3 Limited enforcement with full information (PC) vs. autarky (AU) and vs. full enforcement with full information (PO)

In section 2 we proved that, in the *steady state distribution*, the capital and investment series under participation constraints were equal to the capital accumulation in the PO model and that consumption of agent 1 was constant; thereby, transfers absorbed all the shocks. During the first few periods, however, the behavior of the model PC is quite different from the full optimum. We are reporting the series that correspond to a λ that makes expected discounted transfers at $t = 0$ to be equal to zero, so these series correspond to the equilibrium contract.

In the PC environment the path of capital accumulation (and investment) in the first few periods is very similar to the autarky equilibrium (see Figure 1). This is remarkable since we saw that private information did not have any effect on growth. In fact, in a given realization, the capital stock can even be lower with participation constraints for certain periods (see Figure 1) ; then, it is possible for the utility of the manager to be lower under participation constraints than in autarky in certain periods (see Figure 7). Notice that this does not mean that the participation constraints are violated in these periods: since capital can be smaller under participation constraints, the value for agent 1 of *moving* to autarky after a few periods is lower than if he had started out in autarky.

In the model with partial enforcement, even though borrowing from the investor does not help in growing at a faster rate, it does help the manager smooth out consumption against unforeseen shocks. Figure 5 shows how consumption of the manager grows much more smoothly under participation constraints than under autarky, even though the consumption levels are similar at any point in time. So, in the PO model, borrowing and lending was used for smoothing along the growth path and against unforeseen shocks, but in the PC model it only serves the latter purpose.

The fact that in this model external financing can be used to smooth out consumption against unforeseen shocks makes it possible to have a gain in utility with respect to autarky; the gain is equivalent to an increase in consumption of 0.39% in the first period and leaving consumption constant thereafter, so that the utility gain from external financing under limited enforcement is very small. Cleary, with a higher degree of risk-aversion or increasing the randomness in the economy, it would be possible to increase the utility gain in the PC model relative to the AU model.

Figure 7 tells us that transfers in the PC model are negligible (of the order of 1% the level of total consumption), while transfers under PI have a similar pattern to the optimal transfers.

Appendix 1

Computing the Derivative of the Value Function in Autarky

In order to apply PEA to the model with participation constraints we need to calculate the values inside the conditional expectations of equation 18, so we need to calculate the derivative of V^a. It is convenient to express this derivative in terms only of conditional expectations and functions of variables of the model; we now derive such a formula based on the ideas of Benveniste & Scheinkman. In the rest of this appendix, all variables correspond to the autarky equilibrium so that the superscript 'a' on the variables is suppressed.

The Bellman equation for the autarkic problem is

$$V^a(k_t,\theta_t) = \max_{\{[c_t,i_t]\}} (1-\delta)u(c_t) + \delta E_t V^a(dk_t + g(i_t,\theta_{t+1}),\theta_{t+1})\}$$

subject to the production constraint. The first order conditions of the maximization problem in the right hand side of the Bellman equation

$$u'(c_t) = \delta E_t[V^{a\prime}(k_{t+1},\theta_{t+1}) \\ [\frac{\partial g_{t+1}}{\partial i_t}], \tag{29}$$

where the primes denote derivatives with respect to the first argument of each function involved in this expression.

Letting $f(k_t,\theta_t)$ be the optimal decision function for investment under autarky, we have the following identity

$$V^a(k_t,\theta_t) = (1-\delta)u[k_t^a - f(k_t,\theta_t)] +$$

$$\delta E_t V^a[dk_t + g(f(k_t,\theta_t)), \theta_{t+1}, s_{t+1}].$$

Differentiating both sides with respect to capital we have

$$V^{a'}(k_t,\theta_t) = (1-\delta)u'(c_t)[\alpha k_t^{\alpha-1} - f'(k_t,\theta_t)] + \delta E_t$$

$$[V^{a'}(k_{t+1},\theta_{t+1})[d + g'_{t+1}(i_t)f'(k_t,\theta_t)]].$$

Using 29 this reduces to

$$V^{a'}(k_t,\theta_t) = (1-\delta)u'(c_t)\alpha k_t^{\alpha-1} + \alpha d E_t[V^a(k_{t+1},\theta_{t+1})],$$

and, by recursive substitution we have

$$V^a(k_t,\theta_t) = (1-\delta)E_t[\sum_{j=0}^{\infty}(\delta d)^j u'(c_{t+1})\alpha k_{t+j}^{\alpha-1}],$$

which is the formula that we are seeking. Note that we can approximate this derivative by parameterizing the conditional expectation as a polynomial, and that we can obtain an approximation to this derivative by running one non-linear regression after solving the model with autarky [14] .

[14]Another approach would have been the following two-step procedure: first approximate the *value* function as a conditional expectation of future discounted utilities, and then take the derivative of this approximated value function. The second step here is problematic: if we use a polynomial to approximate the value function there is no reason to believe that the *derivative* of the polynomial will be close to the derivative of the value function. This procedure would be justified only if we used cubic or quadratic splines to approximate the value function.

Figure 1, Capital Stock, initial periods

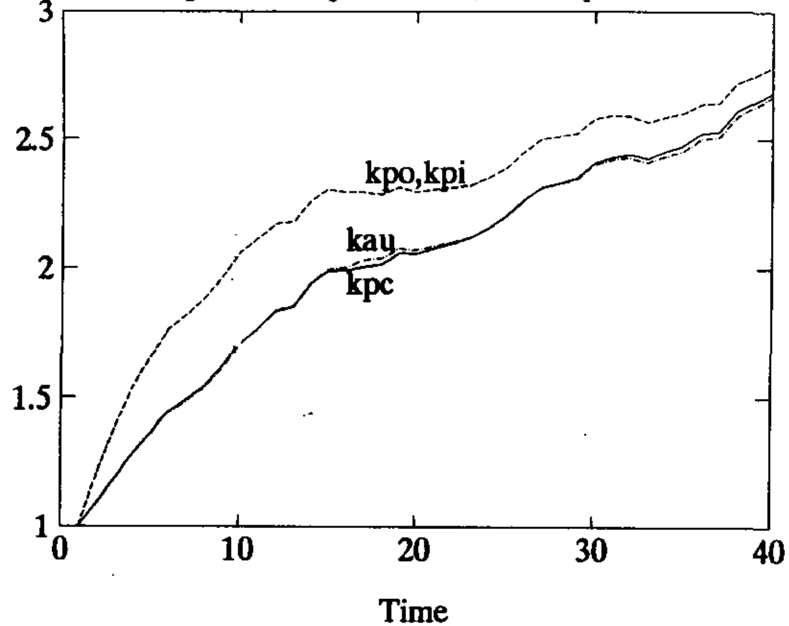


Figure 2, Capital Stock, steady state

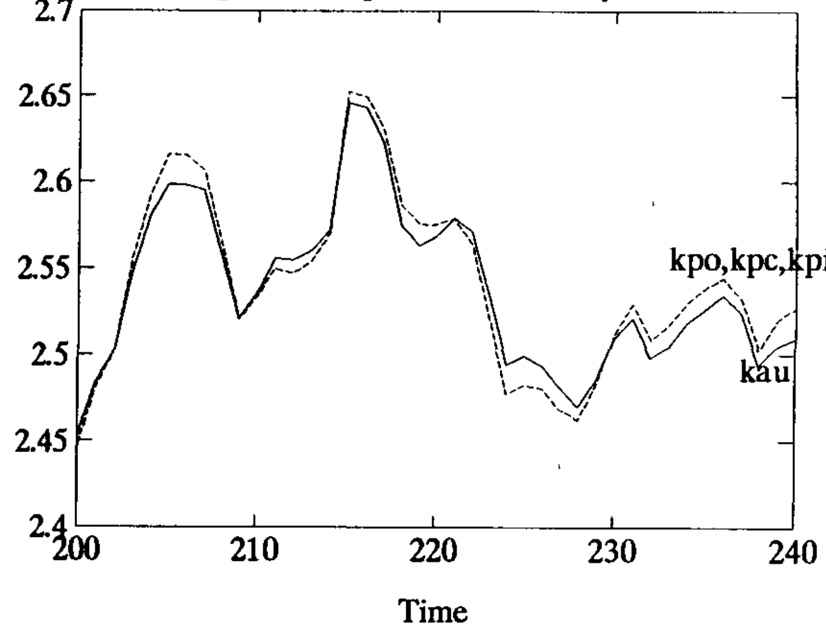


Figure 3, Investment, initial periods

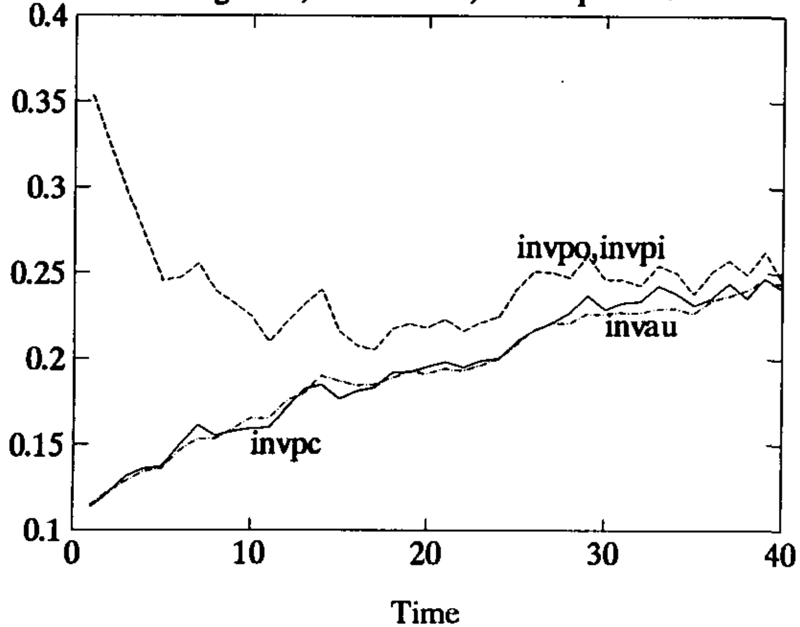
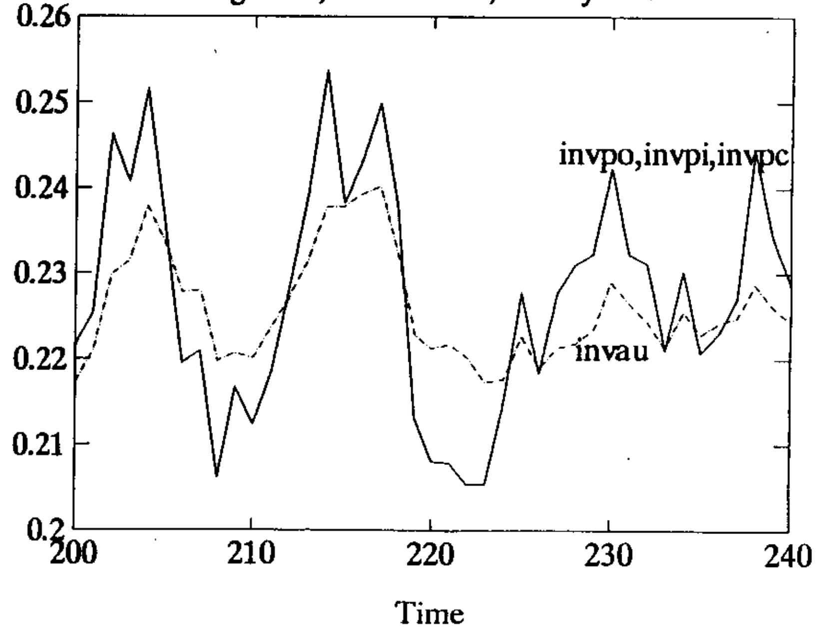


Figure 4, Investment, steady state



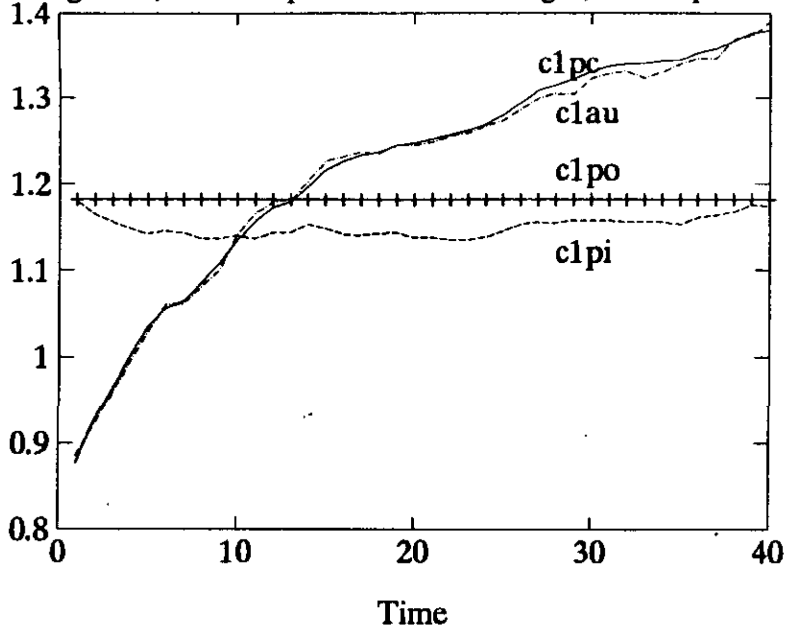


Figure 5, Consumption of the Manager, initial periods

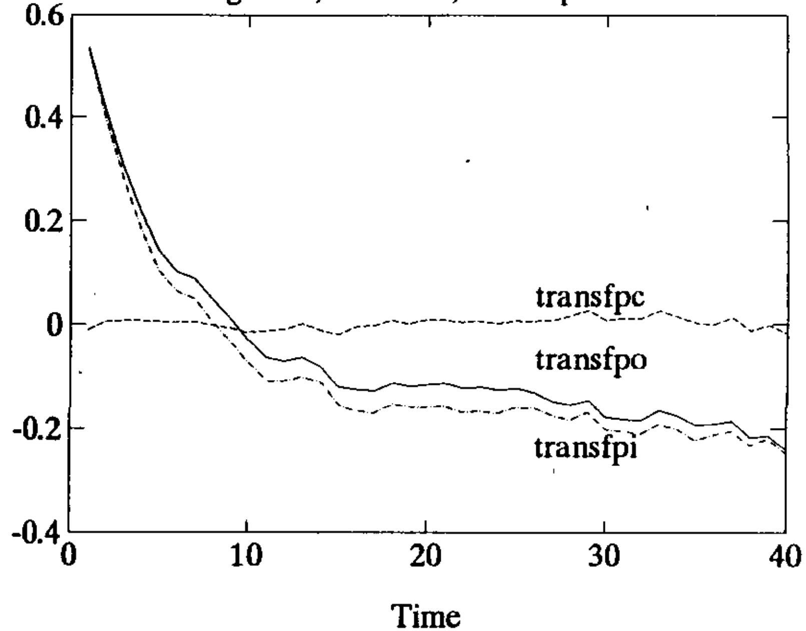


Figure 6, Tranfers, initial periods

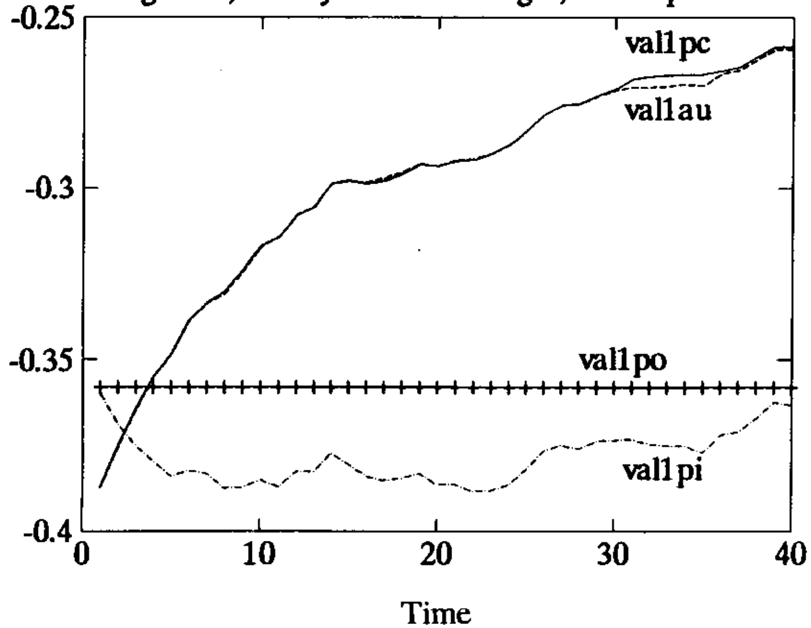


Figure 7, Utility of the Manager, initial periods

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