Discussion Paper 76

Institute for Empirical Macroeconomics
Federal Reserve Bank of Minneapolis
250 Marquette Avenue
Minneapolis, Minnesota 55480-0291

October 1992

MACROECONOMIC IMPLICATIONS OF INVESTMENT-SPECIFIC TECHNOLOGICAL CHANGE

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ABSTRACT

A quantitative investigation of investment-specific technological change for the U.S. postwar period is undertaken, analyzing both long-term growth and business cycles within the same framework. The premise is that the introduction of new, more efficient capital goods is an important source of productivity change, and an attempt is made to disentangle its effects from the more traditional Hicks-neutral form of technological progress. The balanced growth path for the model is characterized and calibrated to U.S. National Income and Product Account data. The long- and short-run U.S. data are then interpreted through the eyes of this framework. The analysis suggests that investment-specific change accounts for a large part of U.S. growth and is a significant factor in U.S. business cycle fluctuations.

I. INTRODUCTION

Macroeconomics has stressed technological change as a key determinant of economic growth and cycles. The premise to be entertained here is that technological change is largely investment–specific. This form of technological progress is characterized by an increase in the productivity of new vintages of capital relative to their cost in terms of forgone consumption. Examples abound: new and more powerful computers, faster and more efficient means of telecommunications and transportation, robotization of assembly lines, etc.

In contrast, the neoclassical growth model used in the macroeconomic literature stresses "neutral" (or disembodied) technological change — as for example in Solow (1957), Kydland and Prescott (1982), and Long and Plosser (1983). There, technological progress allows all goods to be produced more efficiently. This literature will be extended here by introducing technological change that is specific to (or embodied in) new capital goods. The purpose of the analysis is to examine both the short– and long– run implications of this form of technological change and, more specifically, to measure quantitatively its contribution to both economic growth and cycles.

Two key observations motivate the focus on investment–specific technological change:

(1) Long–run evidence: The relative price of equipment has been declining, and the equipment–to–GNP ratio has been increasing over the postwar period. Both patterns, which are fairly dramatic, are portrayed in Figures 1A and 1B.

(2) Short–run evidence: As Figure 2 shows, there is a negative correlation (–.51) between the detrended relative price of new equipment and new equipment investment.

The standard model incorporating neutral technological change cannot accommodate these observations which point at investment–related developments. Moreover, the

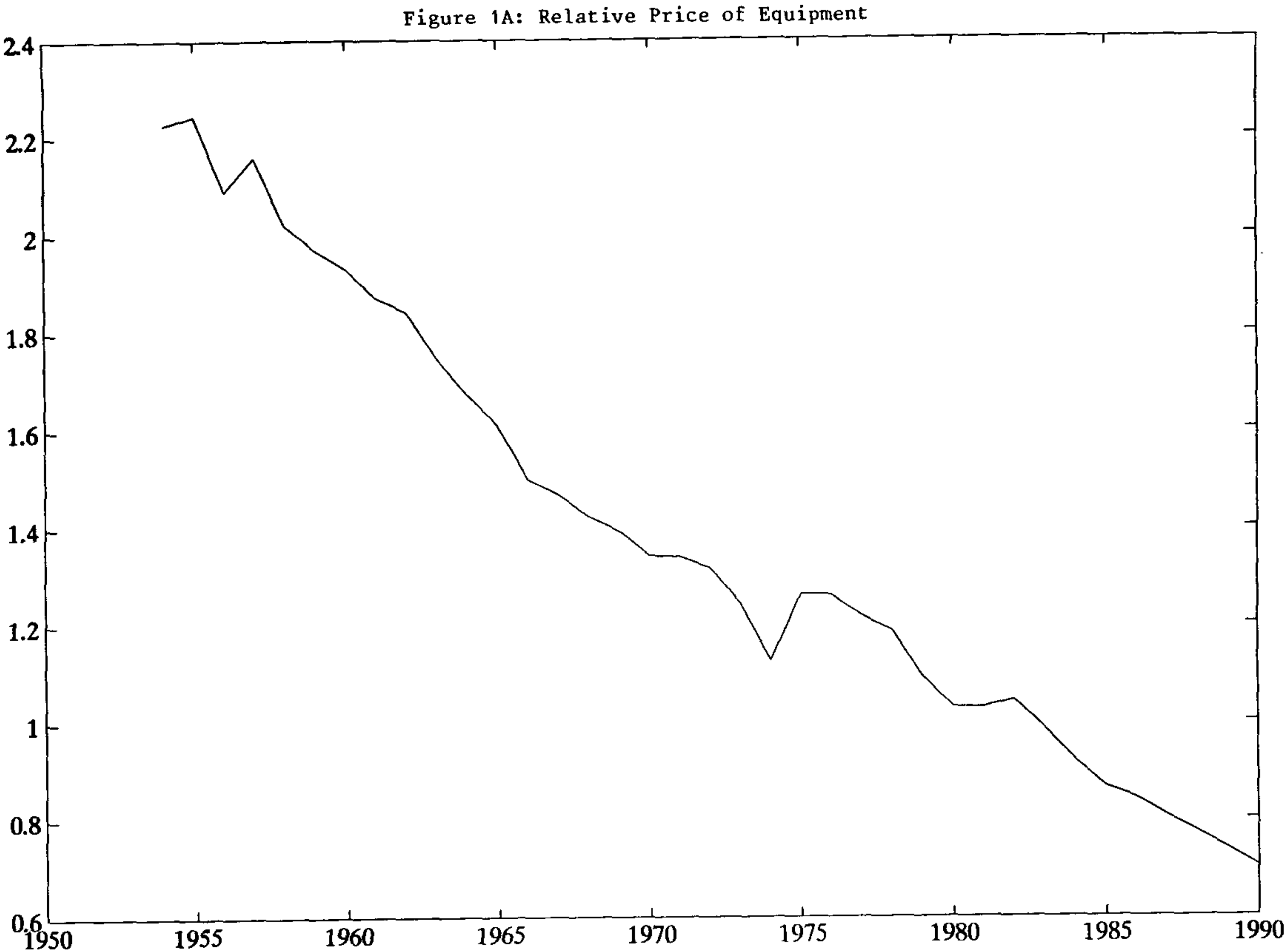


Figure 1A: Relative Price of Equipment
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
1950
1955
1960
1965
1970
1975
1980
1985
1990

Figure 1B: Equipment-to-GNP ratio

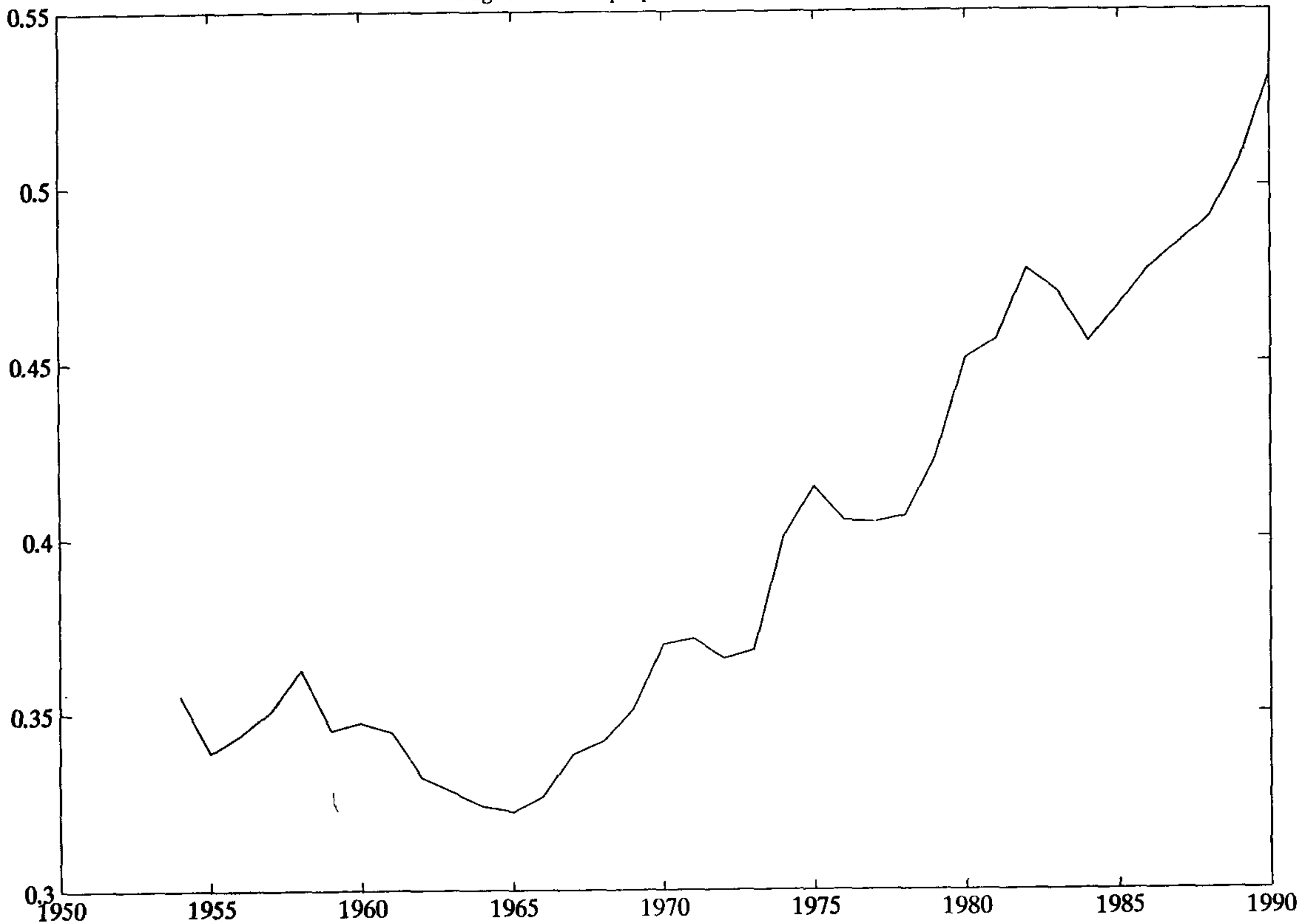
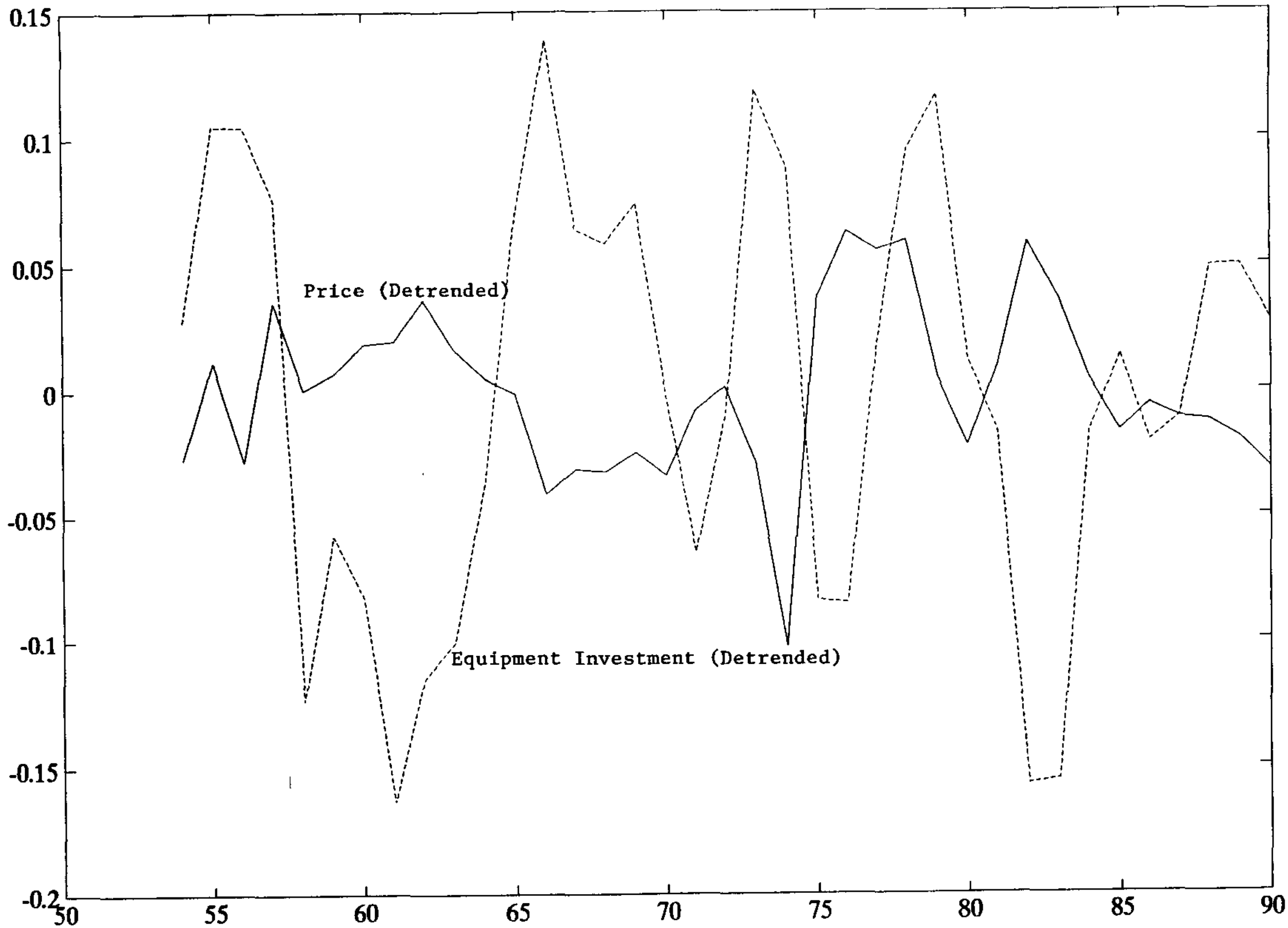


FIGURE 2


0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
50
55
60
65
70
75
80
85
90
Price (Detrended)
Equipment Investment (Detrended)

negative comovement between equipment investment and its relative price both at high and low frequencies suggests that the economic forces involved have been operating on the supply side, rather than on the side of demand. This evidence is important at two levels. First, at the general level, it suggests the story that investment–specific technological change makes new capital less expensive, which in turn stimulates the demand for new capital. The issue investigated here is the extent to which declines in equipment prices trigger both short–run upswings and long–run growth. Second, at a more specific level given the interpretation of the above observations as a shift in supply along a fixed equipment demand curve, the fall in the relative price of equipment can be used as a direct, micro–based measure of investment–specific technological change.

The current analysis is carried out within a simple vintage capital model embedded into a general equilibrium framework. The main feature of the model is that the production of new capital goods becomes increasingly more efficient with the passage of time. Neutral technological change affecting the production of all goods is also allowed for. To sharpen the distinction between the two types of productivity change — investment–specific versus neutral — they are treated in a parallel manner. In particular, both processes evolve exogenously and involve no resource requirements, as is usually assumed in this class of growth models.

Given the quantitative nature of the current analysis, it is important to distinguish between two types of capital goods: equipment and structures. The relative price of structures appears to be stationary over time in the U.S. data, as does the structures–to–GNP ratio — see Figure 3. Casual empiricism also suggests that there is less productivity change in structures than in equipment. This is captured here by adopting the extreme methodological strategy of assuming that investment–specific technological change affects equipment only.

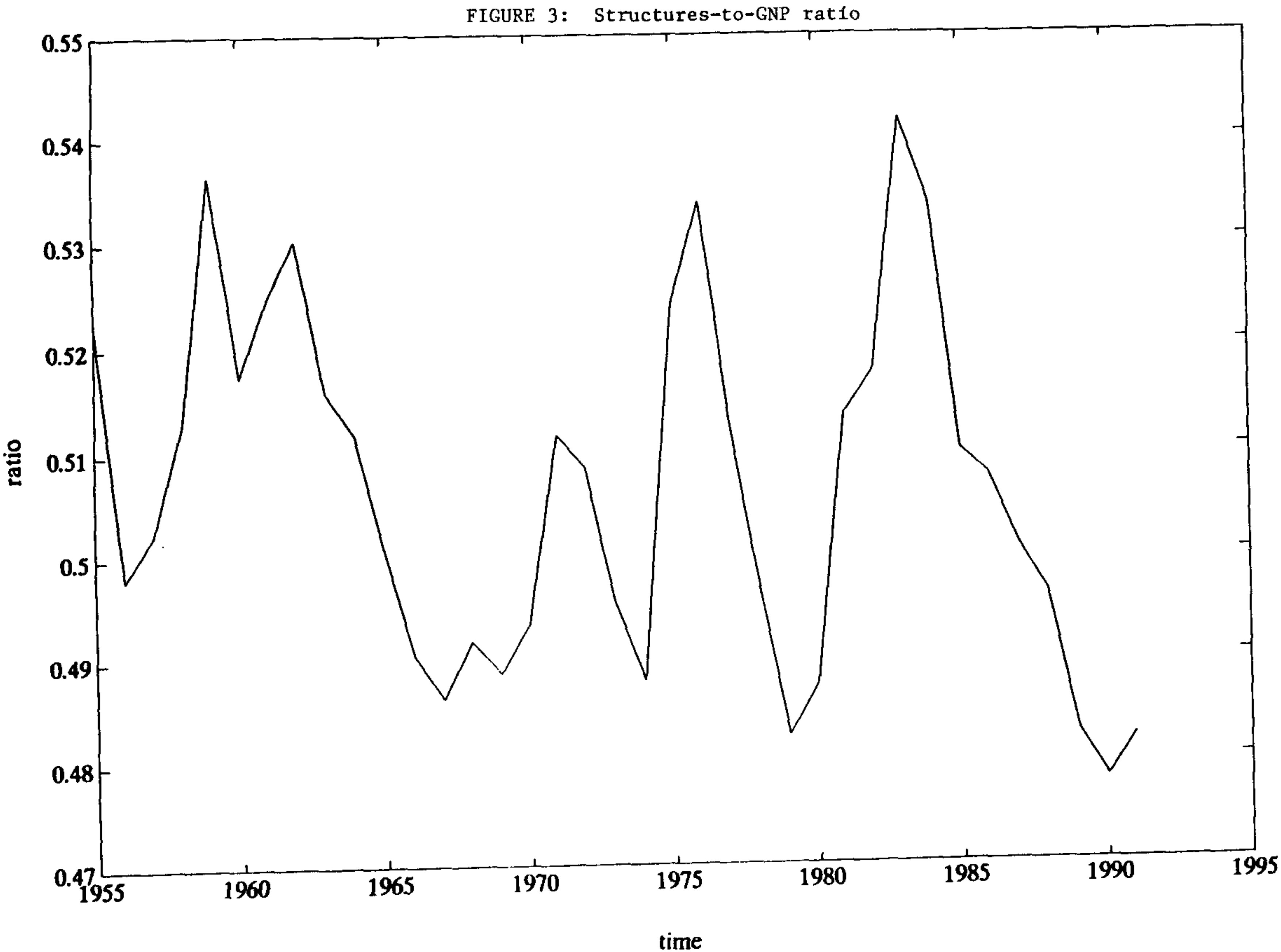


FIGURE 3: Structures-to-GNP ratio
0.55
0.54
0.53
0.52
0.51
0.5
0.49
0.48
0.47
ratio
1955
1960
1965
1970
1975
1980
1985
1990
1995
time

The theoretical model is tailored to meet the observed features of the U.S. postwar data. First, the balanced growth path for the model has the feature that both the stock of equipment and new equipment investment (measured in quality-adjusted units) grow at a higher rate than output. The growth rates are the same, however, if equipment variables are denominated in value terms (i.e., in units of forgone consumption). Second, the model retains the features of balanced growth displayed by the standard model with neutral technological change: a constant real interest rate, constant factor shares of income, etc.

In the decentralized version of the model, a key variable is the equilibrium price for an efficiency unit of newly-produced equipment, using consumption goods as the numeraire. In the framework developed, this price corresponds on the one hand to the inverse of the investment-specific technology shock; on the other it is a direct theoretical counterpart to a price index of quality-adjusted equipment constructed by Gordon (1990). Hence, investment-specific technological change can be identified here with Gordon's price series. The short- and long-run analyses are conducted separately, but carried out within the same framework using the identical measure of investment-specific technological change derived from Gordon's series.

The long-run analysis consists of a growth accounting exercise similar in spirit to earlier work by Solow (1957). He argued that most technological innovations have to be embodied in new vintages of capital in order to contribute to production. Assuming no neutral (or disembodied) technological progress, Solow calculated the rate of change in investment-specific technological progress by matching output with a weighted stream of past investments. The approach followed in the present paper is also related to that in Jorgenson and Griliches (1967) who adjust for the quality of capital (by using CPI data on consumer durables) in order to isolate the residual neutral technological change. The present study has the advantage of being able to employ Gordon's (1990) price index, which was constructed precisely to capture the increased efficiency content in new capital goods.

Investment–specific technological change is found to account for 60 percent of the growth in output per hour worked. Neutral technological change then accounts for the remaining 40 percent. Additionally, a striking result from this growth accounting exercise is that the time series behavior of neutral technological progress is characterized by an actual regress since the early 1970's. This downturn in total factor productivity continues unabated through the end of the sample period. The conclusion from this exercise seems to be that once capital quality is taken into account, the much–discussed productivity slowdown becomes all the more dramatic. Employing a different methodology based on Jorgenson (1966), Hulten (1992) finds a much smaller contribution of investment–specific technological change to output growth, and a correspondingly, higher one for the neutral type. In order to clarify the source of the different results, the distinguishing characteristics of the two methodologies are discussed in detail.

The short–run analysis — which builds on the work of Kydland and Prescott (1982) and Long and Plosser (1983) — addresses a similar question regarding the contribution of investment–specific technological change to business cycle fluctuations. The methodology adopted here for obtaining a measure of the exogenous technology shock is different, however, from that conventionally followed in the business cycle literature. Rather than using output — the very variable whose movements are to be explained — and inputs to measure the Solow residual, an independent measure of the investment–specific technology shock is obtained from the relative price of equipment.

An important feature of the model is that equipment, unlike structures, has a variable rate of utilization.[1] This is due to the more active role it plays in production, which is precisely why equipment is less durable than structures. It is natural, then, to model the depreciation on equipment as an increasing function of its rate of utilization. This formulation has the implication that the faster the rate of improvement in new equipment, the faster old equipment gets depreciated off. This, in turn, translates into higher economic activity.

The model is then solved numerically and simulated given the estimated parameters of the exogenous process for investment–specific technological change. From this analysis emerges the finding that between 22 and 30 percent of the variability in GNP is accounted for by investment–specific technology shocks. Given that investment in new equipment is only 7 percent of GNP, these results indicate an important transmission from investment–specific technology shocks to economic activity.

The paper is organized as follows: Section II presents the model and provides a characterization of its balanced growth path. The model is calibrated to the National Income and Products Accounts (NIPA) in Section III and the long–run analysis carried out. The investigation of the short–run implications is then undertaken in Section IV. Finally, in Section V some concluding remarks are made.

II. THE MODEL

A. The Economic Environment

Consider an economy inhabited by a representative agent who maximizes the expected value of lifetime utility as given by

$$E\left[\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)\right], \tag{1}$$

with

$$U(c,\ell) = \theta \ln c + (1-\theta)\ln \ell, \quad 0 < \theta < 1, \tag{2}$$

where c_t and ℓ_t represent period–t consumption and labor.

The production of final output is governed by the technology:

$$y = F(\tilde{k}_e, k_s, \ell, z), \tag{3}$$

with

$$F(\tilde{k}_e, k_s, \ell, z) = z\, \tilde{k}_e^{\alpha_e}\, k_s^{\alpha_s}\, \ell^{1-\alpha_e-\alpha_s}, \quad 0 < \alpha_e,\ \alpha_s < 1. \tag{4}$$

Production requires the services from labor, ℓ, and two types of capital: equipment, $\tilde{k}_e$, and structures k_s. The variable z is a measure of total factor productivity. It follows the stochastic process $z_{t+1} = \gamma_z^{t+1}\zeta_{t+1}$, where $\gamma_z > 0$ and ζ_{t+1} is governed by the distribution function $Z(\zeta'|\zeta) = \text{prob}\{\zeta_{t+1} = \zeta' | \zeta_t = \zeta\}$. Final goods output can be used for three purposes: consumption, c, for investment in structures, i_s, and for investment in equipment, i_e. Structures can be produced from final output on a one–to–one basis. The stock of structures evolves according to

$$k'_s = (1-\delta_s)k_s + i_s, \quad \text{where } 0 < \delta_s < 1. \tag{5}$$

There are adjustment costs associated with changing the stock of structures. Specifically, changing the capital stock from k_s today to k'_s tomorrow involves incurring adjustment costs in the amount $a_s = A_s(k'_s, k_s)$, where

$$A_s(k'_s, k_s) = \phi_s(k'_s - \kappa_s k_s)^2/k_s, \text{ with } \phi_s,\ \kappa_s > 0 \tag{6}$$

The story is not so simple for equipment, which comes in different efficiency levels, or "types". The treatment of equipment adopted here can be seen as matching Gordon's (1992) framework for quality price adjustment. The model allows for production cost reductions, both for existing types of machines and for new types – – i.e. not produced previously. It is shown how the distribution of machine types – produced and operated – evolves over time under different assumptions regarding the form of these cost reductions. The analysis leads to a simple "reduced form" for the aggregate technology, with an aggregate measure of the stock of equipment in the economy–wide production function. In Appendix A a simpler, but more restricted form of the model is presented in which only one type of equipment is produced in each period. That version shares the same reduced form, and it is provided to the reader as a substitute for the (more technical) discussion up to Subsection D.

A given type of equipment is associated with a level of efficiency indexed by $e \in \mathbb{R}_+$. Furthermore, each type of equipment can be operated at a variable rate $h(e)$. The service flow, $eh(e)$, derived from a unit of equipment in production is the product of its efficiency level, e, and the rate at which it is operated, $h(e)$. A unit of equipment of efficiency level e_1 is worth in production e_1/e_2 units of equipment of type e_2—provided they are operated at the same rate. To calculate the service flow from all equipment used in production requires knowing the number of units of each type. To this end, let $\mathscr{B}$ denote the Borel sigma algebra on $\mathbb{R}_+$ and the measure μ: $\mathscr{B} \to \mathbb{R}_+$ describe the distribution of equipment over efficiency levels. Thus, $\mu(B)$ gives the number of units (or mass) of equipment with an efficiency level lying in the set $B \in \mathscr{B}$. The cumulative service flow accruing from all equipment in production, or $\tilde{k}_e$, reads

$$\tilde{k}_e = \int eh(e)d\mu(e). \tag{7}$$

The next task is to characterize how the distribution of equipment evolves over time. This will depend upon two factors: the production of new equipment of varying efficiency levels and the depreciation of old equipment of different types. Let the measure describing the distribution of new equipment being produced in the current period be represented by ν: $\mathscr{B} \to \mathbb{R}_+$. Also, assume that a unit of old equipment depreciates at the rate $\delta_e(h)$, which is taken to be an increasing convex function of the level at which it is utilized in production. The law of motion of μ can now be written as

$$\mu'(B) = \int_{x \in J(B;h)} d\mu(x) + \nu(B), \quad \text{for } B \in \mathscr{B} \tag{8}$$

with

$$J(B;h) \equiv \{x: x(1-\delta_e(h(x)) \in B\}.$$

Following Greenwood, Hercowitz and Huffman (1988) the 'user cost' function $\delta_e(h)$ is assumed to have the following form:

$$\delta_e(h) = \frac{b}{\omega} h^{\omega}, \quad \omega > 1. \tag{9}$$

The purpose for allowing a variable rate of equipment, and modeling its cost in this way, is to better capture the model's behavior over the business cycle. The long-run analysis does not depend on whether utilization varies.

Unlike structures, equipment is not produced from final output on a one-to-one basis. Specifically, the unit cost of producing a type-e piece of equipment is $G(e;\ \epsilon)$. Here ϵ is a measure of the state of technology in the equipment producing industry with the function $G(e;\ \epsilon)$ postulated to be decreasing in ϵ. The variable ϵ is assumed to follow a stochastic process such that $q_{t+1} \equiv \max_e[e/G(e;\epsilon_{t+1})]$ evolves according to $q_{t+1} = \gamma_q^{t+1}\eta_{t+1}$, where $\gamma_q > 1$ and η_{t+1} is drawn from the distribution function $H(\eta_{t+1}|\eta_t) = \text{prob}\{\eta_{t+1} = \eta' | \eta_t = \eta\}$. The function G is discussed in more detail in Subsection C, where some explicit parametric representations are entertained. Investment expenditure on new equipment (measured in terms of final output), or i_e, therefore is given by

$$i_e = \int G(e;\ \epsilon) d\nu(e). \tag{10}$$

Like structures there are also costs to adjusting the stock of equipment. In current consumption units the value of next period's stock of equipment is $\int G(e;\epsilon)d\mu'(e)$ while the value of today's stock is $\int G(e;\epsilon)d\mu(e)$. The adjustment costs associated with this change in the value of the stock of equipment are given by $a_e = A_e(\int G(e;\epsilon)d\mu'(e), \int G(e;\epsilon)d\mu(e);\eta)$ where

$$A_e(y',y;\eta) = \eta\phi_e(y' - \kappa_e y)^2/y \quad \text{with } \phi_e,\ \kappa_e > 0. \tag{11}$$

Finally, there is a government present in the economy. It levies taxes on the market income earned by labor and capital at the rates τ_ℓ and τ_k. The revenue raised by the government in each period is rebated back to agents in the form of lump-sum transfer payments in the amount τ. The government's budget constraint is

$$\tau = \tau_k(r_e \bar{k}_e + r_s k_s) + \tau_\ell w, \tag{12}$$

where r_e, r_s and w represent the market returns for the services from equipment, structures, and labor. The inclusion of income taxation in the framework is important for the quantitative analysis because of the significant effect that it has on equilibrium capital formation.

B. *Competitive Equilibrium*

The competitive equilibrium for the economy under study will now be formulated. To this end, let the aggregate state-of-the-world be denoted by (s,z,ϵ) where $s \equiv (\mu,k_s)$. Assume that the equilibrium wage and rental rates, w, r_e, and r_s, and per capita transfer payments τ can all be expressed as functions of the aggregate state of the world as follows: $w = W(s,z,\epsilon)$, $r_e = R_e(s,z,\epsilon)$, $r_s = R_s(s,z,\epsilon)$, and $\tau = T(s,z,\epsilon)$. Likewise, let the price for a type-e unit of equipment be given by the function $p(e) = P(e;s,z,\epsilon)$. Finally, suppose that μ and k_s evolve according to the laws of motion $\mu' = M(s,z,\epsilon)$ and $k_s' = K_s(s,z,\epsilon)$. Thus, the movement in s is governed by the transition function $s' = S(s,z,\epsilon) \equiv (M(s,z,\epsilon),\ K_s(s,z,\epsilon))$. The optimization problems facing households and firms can now be formulated. Of course, all agents take the evolution of s, as governed by $s' = S(s,z,\epsilon)$, to be exogenously given.

The Household

The representative household's dynamic programming problem can be cast as:

$$V(\mu,k_s;s,z,\epsilon) = \max_{c,\mu',k_s',\ell,h} \{U(c,\ell) + \beta E[V(\mu',k_s';s',z',\epsilon')]\} \tag{P1}$$

subject to

$$\begin{aligned} c + \int P(e;s,z,\epsilon)d\mu'(e) + k_s' = {} & (1-\tau_k)[R_e(s,z,\epsilon)\int eh(e)d\mu(e) + R_s(s,z,\epsilon)k_s] \\ & + (1-\tau_\ell)W(s,z,\epsilon)\ell + \int P(e;s,z,\epsilon)(1-\delta_e(h(e))d\mu(e) \\ & + (1-\delta_s)k_s + T(s,z,\epsilon) - A_s(k_s',k_s) \end{aligned}$$

$$- A_e\left(\int P(e;s,z,\epsilon)d\mu'(e), \int P(e;s,z,\epsilon)d\mu(e);\eta\right) \tag{13}$$

and

$$s' = S(s,z,\epsilon).$$

The Final–Output Firm

The firm producing final output hires inputs so as to maximize its profits, π_y, in each period. Its maximization problem is

$$\max_{\bar{k}_e, \bar{k}_s, \ell} \quad \pi_y = F(\bar{k}_e,\bar{k}_s,\ell,z) - R_e(s,z,\epsilon)\bar{k}_e - R_s(s,z,\epsilon)\bar{k}_s - W(s,z,\epsilon)\ell. \tag{P2}$$

Due to the constant–returns–to–scale assumption, the firm earns zero profits in each period; i.e., $\pi_y = 0$.

The Equipment–Producing Firm

The firm producing equipment maximizes its profits, π_e, in each period by solving the problem (P3) shown below:

$$\max_{\nu} \ \pi_e = \int [P(e;s,z,\epsilon) - G(e;\epsilon)]d\nu(e). \tag{P3}$$

In competitive equilibrium the profits for this firm will be zero as well. Free–entry into the equipment–producing industry dictates that if a certain type of equipment, e, is being manufactured then its price must equal its unit cost; i.e., $P(e;s,z,\epsilon) = G(e;\epsilon)$ for all type e being produced. Likewise, if a unit of type–e equipment is not being produced then it must be the case that $P(e;s,z,\epsilon) \leq G(e;\epsilon)$.

The competitive equilibrium under study can now be formally defined.

Definition of Equilibrium: A competitive equilibrium is a set of allocation rules $c = C(s,z,\epsilon)$, $\mu' = M(s,z,\epsilon)$, $k'_s = K_s(s,z,\epsilon)$, $\ell = L(s,z,\epsilon)$, $h = H(s,z,\epsilon)$, $\bar{k}_e = K_e(s,z,\epsilon)$ and $\nu = N(s,z,\epsilon)$, a set of pricing and transfer functions $w = W(s,z,\epsilon)$, $r_e = R_e(s,z,\epsilon)$,

$r_s = R_s(s,z,\epsilon)$, $p(e) = P(e;s,z,\epsilon)$ and $\tau = T(s,z,\epsilon)$, and an aggregate law of motion $s' = S(s,z,\epsilon)$ such that

(i) households solve problem (P1), taking as given the aggregate state of the world (s,z,ϵ) and the forms of functions $W(\cdot)$, $R_e(\cdot)$, $R_s(\cdot)$, $P(\cdot)$, $T(\cdot)$ and $S(\cdot)$, with the equilibrium solution to this problem satisfying $c = C(s,z,\epsilon)$, $\mu' = M(s,z,\epsilon)$, $k'_s = K_s(s,z,\epsilon)$, $\ell = L(s,z,\epsilon)$, and $h = H(s,z,\epsilon)$;

(ii) final output producing firms solve problem (P2), given (s,z,ϵ) and the functions $R_e(\cdot)$, $R_s(\cdot)$, and $W(\cdot)$, with the equilibrium solution to this problem satisfying $\tilde{k}_e = k_e H(s,z,\epsilon)$, $\ell = L(s,z,\epsilon)$ and $\tilde{k}_s = k_s$;

(iii) equipment-producing firms solve problem (P3), given (s,z,ϵ) and the function $P(\cdot)$, with the equilibrium solution to this problem satisfying $\nu = N(s,z,\epsilon)$;

and

(iv) the market for final goods and equipment clear each period, implying that

(a) $$c + i_s + i_e = F(\tilde{k}_e,k_s,\ell,z) - a_s - a_e, \tag{14}$$

where

$$i_e = \int G(e;\epsilon)d\nu(e),$$

$$i_s = k'_s - (1-\delta_s)k_s,$$

$$a_s = A_s(k'_s,k_s),$$

$$a_e = A_e\left(\int G(e;\epsilon)d\mu'(e), \int G(e;\epsilon)d\mu(e);\eta\right),$$

(b) $$\tilde{k}_e = \int eh(e)d\mu(e),$$

(c) $$\mu'(B) = \int_{x \in J(B;h)} d\mu(x) + \nu(B), \quad \text{for all } B \in \mathscr{B}$$

C. Characterization of Equilibrium

The equilibrium under study will be partially characterized for simple parameterizations of the unit cost function $G(e;\epsilon)$. First, note that since a unit of type-e_1 equipment can substitute in production perfectly for e_1/e_2 units of type-e_2 equipment, it must transpire that $P(e_1;s,z,\epsilon) = (e_1/e_2)\ P(e_2;s,z,\epsilon)$ for any e_1 and e_2 currently in existence. Second, it is easy to deduce from the perfect substitutability assumption that all types of equipment in existence will be utilized at the same rate. Hence $h(e) = H(e;s,z,\epsilon) = H(s,z,\epsilon) = h$ for all e. Finally, observe that in an efficient allocation only machines of type $e \in \text{argmin}[G(e;\epsilon)/e]$ will be produced in the current period. In this world, all agents care about is the price per *efficiency* unit of equipment.

Now, consider the following forms for $G(e;\epsilon)$:

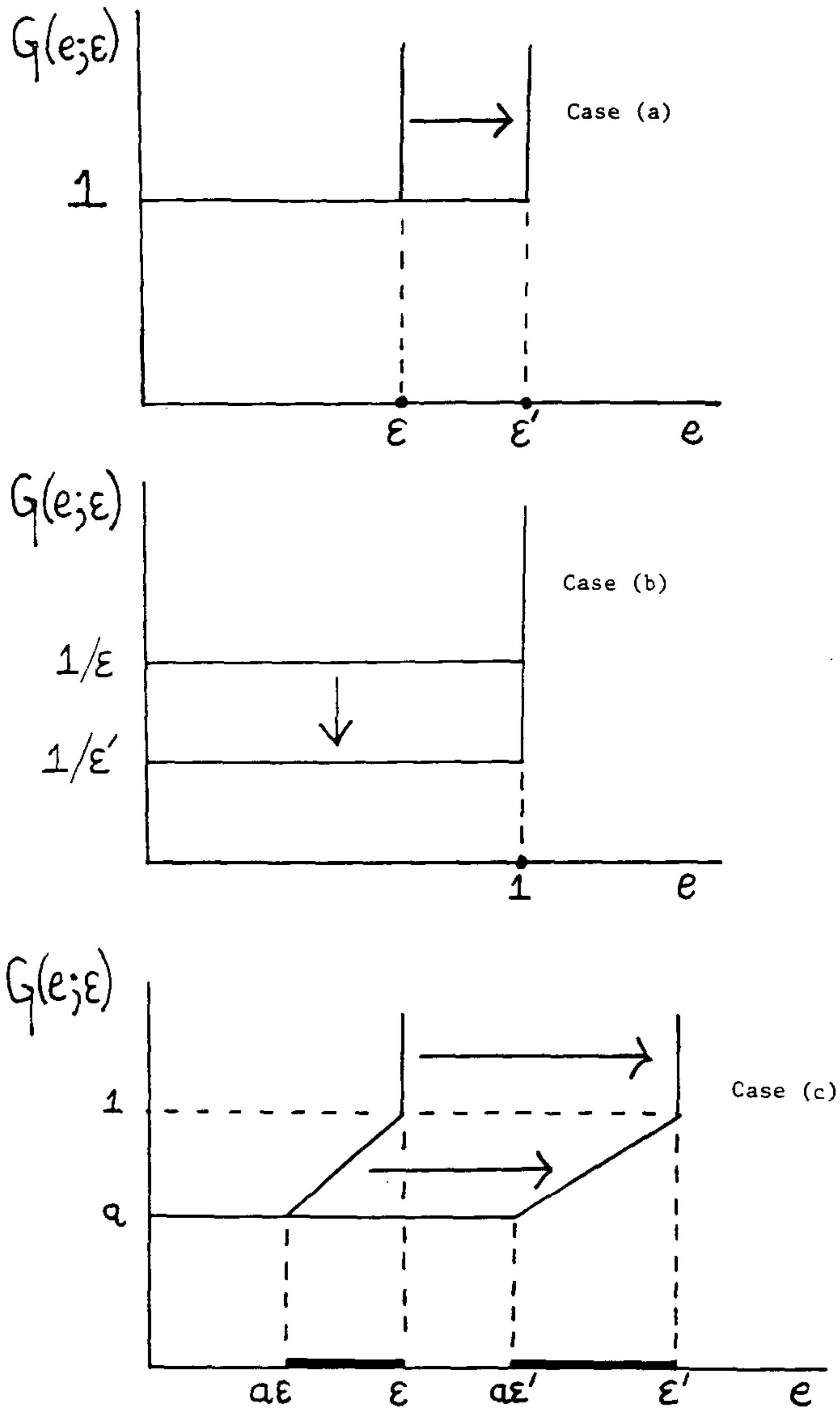
$$\text{(a)}\quad G(e;\epsilon) = \begin{cases} 1, & \text{for } e \leq \epsilon, \\ \infty, & \text{otherwise.} \end{cases}$$

$$\text{(b)}\quad G(e;\epsilon) = \begin{cases} 1/\epsilon, & \text{for } e \leq 1, \\ \infty, & \text{otherwise.} \end{cases} \tag{15}$$

$$\text{(c)}\quad G(e;\epsilon) = \begin{cases} a, & \text{if } e < a\epsilon, \text{where } a < 1, \\ e/\epsilon, & \text{if } a\epsilon < e < \epsilon, \\ \infty, & \text{otherwise.} \end{cases}$$

These three cases are depicted in Figure 4. Assume that ϵ increases over time. Case (a) is an example where in any given period a range of equipment types can be produced, each type at the same cost. Only the latest vintage of equipment $e = \epsilon$ will be produced in the current period, however, since this type has the lowest cost per efficiency unit. This case has the interpretation that as time passes (read as ϵ transits to ϵ') new, more efficient types of equipment may become available at no increase in cost and once a new piece of equipment is invented its cost of production does not change. In Case (b) equipment in the range (0,1] benefits from a continual decline in the cost of production. Only equipment of type $e = 1$ will be produced in any given period, though, since it has the lowest price per efficiency unit. Finally, in Case (c) new types of equipment become available as time progresses. There are also reductions in the cost of production of some old types. Observe that the function $G(e;\epsilon)/e$ attains its minimum over the range $[a\epsilon,\epsilon]$. Thus, in the current period it will be profitable to produce any type of equipment $e \in [a\epsilon,\epsilon]$. This is an example of a

FIGURE 4: Examples of G(·)



situation where technological development leads to new, more efficient capital goods showing up on the market but at the same time older versions continue to be simultaneously produced due to reductions in their production cost.

It now pays to consolidate equipment of different vintages into a single aggregate stock of equipment measured in efficiency units. To begin with, recall that only machines of type $e \in \text{argmin}\,[G(e;\epsilon)/e]$ will be produced in the current period. A natural measure of productivity in the equipment producing sector is therefore given by $q = Q(\epsilon) \equiv \max_e[e/G(e;\epsilon)]$. The variable q represents the amount of equipment measured in efficiency units that can be obtained for a unit of consumption in the current period. Since all machines in existence must sell for the same price per efficiency unit it transpires that $P(e;s,z,\epsilon) = e/Q(\epsilon)$.

Next, note that since all machines are utilized at the same rate the cumulative service flow from all equipment used in production, or $\tilde{k}_{et}$, can be written as

$$\tilde{k}_{et} = h_t \sum_{m=1}^{t-1} \mathrm{v}_{mt}, \tag{16}$$

where v_{mt} is the time-t efficiency unit value of all machines produced at date t–m:

$$\mathrm{v}_{mt} = \begin{cases} [\prod_{s=1}^{m-1}(1-\delta_e(h_{t-s}))]\int e d\nu_{t-m}(e), & \text{for } m > 1 \\ \int e\, d\nu_{t-m}(e), & \text{for } m = 1. \end{cases} \tag{17}$$

Now, let $k_e \equiv \tilde{k}_e/h$ represent the aggregate stock of equipment measured in efficiency units. It transpires from (16) and (17) that k_e evolves according to the law of motion

$$k_e' = (1-\delta_e(h))k_e + \int e d\nu(e).$$

By using (10), in conjunction with the fact $G(e;\epsilon) = e/q$ for machines being currently produced, the above law of motion can be rewritten as

$$k_e' = (1-\delta_e(h))k_e + i_e q. \tag{18}$$

Observe that the stock of equipment in efficiency units can be expressed as a productivity weighted sum of past investment: $k_{et} = \sum_{m=1}^{\infty}[\prod_{s=1}^{m-1}(1-\delta_e(h_{t-s})]i_{et-m}q_{t-m}$.

The household's optimization problem (P1) can now be poised much more simply in terms of choosing k_e' rather than μ'. To do this note from (7) that $k_e h = \int eh d\mu(e)$. This, together with the fact that $P(e;s,z,\epsilon) = e/Q(\epsilon)$, implies that $\int P(e;s,z,\epsilon)d\mu'(e) = k_e'/Q(\epsilon)$. Similarly, $\int P(e;s,z,\epsilon)(1-\delta_e(h(e))d\mu(e) = (1-\delta_e(h))k_e/Q(\epsilon)$. The household's budget constraint (13) can hence be reformulated as

$$c + k_e'/Q(\epsilon) + k_s' = (1-\tau_k)[R_e(s,z,\epsilon)hk_e + R_s(s,z,\epsilon)k_s] + (1-\tau_\ell)W(s,z,\epsilon)\ell \quad (19)$$
$$+ (1-\delta_e(h))k_e/Q(\epsilon) + (1-\delta_s)k_s + T(s,z,\epsilon) - A_s(k',k_s) - A_e(k_e'/Q(\epsilon),k_e/Q(\epsilon);\eta).$$

Observe that the price of an efficiency unit of equipment is $1/Q(\epsilon)$. The rest of the analysis can be expressed in terms of k_e rather than μ without undue alterations.[2]

Discussion: The model presented above can be interpreted in various different ways. For instance, one could imagine a situation where new more efficient capital goods are invented as time progresses. These new capital goods may make obsolete the production of older varieties. Example (a) above illustrates this case. Alternatively, one could think about the capital goods producing sector as a whole becoming more efficient over time. Due to technological change capital goods in some existing set of varieties can be produced for an increasingly lower price in terms of consumption goods as time passes. Example (b) has this flavour to it. Example (c) is a hybrid case. Here new types of capital goods are introduced in production while simultaneously some older varieties continue to be manufactured at lower cost. Now it would be hard to distinguish between technological change due to the introduction of new capital goods or cost reductions in the production of old ones.

In terms of the formal analysis the three examples of investment-specific technological change are identical—they lead to the same reduced form for the model. Hence a key feature of the present analysis is that newly produced equipment, of new or old varieties, tends to have a higher efficiency/cost ratio than in the past. What is important for the analysis is that different vintages of capital can be distinguished between, assigned an efficiency level, and then be aggregated. This allows for an efficiency unit measure of the aggregate capital stock to be constructed as a productivity weighted sum of past investment spending. Since so much of technological change is incarnated in the form of new equipment, it is fundamental in accounting for growth to pay close attention to the characteristics of each capital type. The hedonic pricing methodology employed in Gordon (1990) computes a relative price for new investment by carefully accounting for the characteristics of each capital vintage. As such, it naturally complements the current analysis (where the characteristics space is one–dimensional, namely the efficiency level of each machine).

The form of technological change adopted here is related to earlier work by Solow (1959), Greenwood, Hercowitz and Huffman (1988), and Krusell (1991). These papers present models of vintage capital, and they describe technological change as specific to the sector producing new capital (technological change is 'embodied' in capital). In Solow (1959) investment at time t was assumed to give rise to capital of vintage t, with the associated technology using labor and this type of capital to realize a total–factor productivity of q_t. The optimal allocation of labor across existing vintages implied that aggregate output could be written as a Cobb–Douglas function of total labor and a productivity–weighted sum of the different capital stocks—as in the current analysis. Greenwood, Hercowitz and Huffman (1988) analyze the role that shocks to the efficiency of new capital can play in business cycle fluctuations. The focus of the analysis in Krusell (1991) was on modelling the determination of the technological change process q by explicitly formalizing the R&D decisions as undertaken by profit–maximizing firms. The theorizing there led to a characterization of the q–process similar to that employed here; in particular, it was shown how this kind of R&D could generate sustained, balanced growth.

D. *Balanced Growth*

The balanced growth path for a deterministic version of the above model will now be characterized. In particular, z and q grow at the (gross) rates γ_z and γ_q, respectively. Clearly, along a balanced growth path, output, consumption, investment and the capital stocks will all grow. It is convenient to transform the problem into one which renders all variables constant in the steady state.

To find the appropriate transformation, observe that the resource constraint (14) implies that output, consumption, investment, and adjustment costs all have to grow at the same rate, say g, along a balanced growth path. Then, from the accumulation equation (5) for structures it follows that the stock of structures also have to grow at rate g. Equipment, however, grows faster. From (18) its growth rate, g_e, equals $g\gamma_q$. Finally, the form of the production function (4) implies that $g = \gamma_z g_e^{\alpha_e} g^{\alpha_s}$.[3] Thus, the following restrictions are imposed on balanced growth:

$$g = \gamma_z^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_q^{\frac{\alpha_e}{1-\alpha_e-\alpha_s}}, \tag{20}$$

and

$$g_e = \gamma_z^{\frac{1}{1-\alpha_e-\alpha_s}} \gamma_q^{\frac{1-\alpha_s}{1-\alpha_e-\alpha_s}}. \tag{21}$$

The amount of labor employed and the utilization rate for equipment remain constant along the balanced growth path.

Given a conjectured growth rate for all variables, one can impose a transformation that will render them stationary. Specifically, define first $\hat{x}_t = x_t / g^t$ for $x_t = y_t$, c_t, i_{et} i_{st} k_{st}, a_{st}, and a_{et}, second set $\hat{k}_{et} = k_{et} / g_e^t$, $\hat{q}_t = q_t / \gamma_q^t$, and finally let

$\hat{z}_t = z_t / \gamma_z^t$.[4] The household's and firm's choice problems (P1) and (P2), along with the resource constraint (14), can be rewritten in terms of these transformed variables. A globally stable steady–state exists for the transformed model which corresponds to an unbounded growth path for the original model.[5]

It follows from the analysis above that the stock of equipment grows over time at a higher rate than output, if the relative price of new equipment in terms of output, or $1/q$, is declining secularly. Thus, the model conforms qualitatively with the long–run observation (1) in the introduction. It is also straightforward to check that the properties of the standard neoclassical growth model such as a constant steady–state real interest rate, constant capital and labor share of income, and constant consumption– and investment–to–output ratios are preserved here.

It is interesting to observe that the rental price of a unit of equipment, $F_1(k_e h, k_s, \ell, z) = \alpha_e h^{\alpha_e}(k_s/k_e)^{\alpha_s}(z^{1/(1-\alpha_e-\alpha_s)}\ell/k_e)^{1-\alpha_e-\alpha_s}$, must be continually falling along a balanced growth path since both k_s/k_e and $z^{1/(1-\alpha_e-\alpha_s)}\ell/k_e$ are declining. It is straightforward to calculate that the rental price of equipment falls along a balance growth path at the rate $1/\gamma_q$—assuming that z is constant. How, then, can the real interest rate remain constant? The answer is that the cost of a unit of equipment in terms of consumption goods, or $1/q$, is also declining over time at rate $1/\gamma_q$. Thus, the return from investing a unit of consumption goods in equipment, or $F_1(k_e h, k_s, \ell)q$, remains constant over time.

III. THE ROLE OF INVESTMENT–SPECIFIC TECHNOLOGICAL CHANGE IN ECONOMIC GROWTH

How important quantitatively is investment–specific change for U.S. economic growth? What is the impact of other sources of technological progress? By interpreting U.S.

postwar data through the above framework, the contribution of these different sources of technological change can be quantitatively assessed.

A. *Calibration*

To proceed, values must be assigned to the following parameters:

Preferences: β and θ

Technology: α_e, α_s, δ_s, b, ω, γ_q

Tax rates: τ_k, τ_l

So as to impose a discipline on the quantitative analysis, the calibration procedure advanced by Kydland and Prescott (1982) is adopted. In line with this approach, as many parameters as possible are set in advance based upon either *a priori* information, or so that along the model's balanced growth path values for various economic variables assume their average values for the U.S. data over the sample period.

Care must be taken when matching up the theoretical constructs of the model with their counterparts in the U.S. data. To avoid the problems associated with accounting for quality improvement in new equipment the following general procedure for data construction is adopted. First, the variables in the model's resource constraint, namely y, c, i_e and i_s, are matched up in that data with the corresponding nominal variables from the NIPA divided through by a *common* price deflator. A natural such price in this context is the consumption deflator of nondurable goods and non–housing services—so as to avoid the issue of the accounting of quality improvement in consumer durables. Hence, y, c, i_e and i_s are measured in consumption units. Then, total annual man hours are used for ℓ, and a survey measure of equipment utilization for h (see the data appendix).

The parameters whose values can be fixed upon *a priori* information are:

(i) $\gamma_q = 1.030$. This number corresponds to the average annual rate of decline in the relative price of equipment prices as measured by Gordon's equipment price series and the deflator for consumption nondurables and nonhousing services. The period used for this calculation is 1954–1983, the period covered by Gordon's study.

(ii) $\delta_s = 0.078$. This depreciation rate is obtained using NIPA data as follows: Using the accumulation equation for structures from the model and data on real investment and stocks of capital it is possible to back out a series on the implied depreciation rate by setting $1-\delta_s = \frac{k_{st+1}-i_{st}}{k_{st}}$. The value reported above is an average over the sample. Note that the measures here differ from the NIPA ones in that the latter use a straight-line depreciation method—where capital is being "written off" in equal installments over the given life of the asset—while in the present model it is assumed that capital depreciates at a constant *rate*.

(iii) $\tau_l = 0.4$. In line with Lucas (1990) the marginal tax rate on labor is set at 40%. Picking the effective marginal tax rate on capital income is more difficult. This is a controversial subject with estimates in the literature varying wildly. For instance for the period 1953–1979, Feldstein, Dicks–Mireaux and Poterba (1983, Table 4, Column 1) present annual estimates of the average effective tax rate on capital income that vary from 55% to 85%. Marginal tax rates would presumably be higher still. Also, for purposes of the current analysis the tax rate chosen should also capture the effects of regulation or other hidden taxes that affect investment. This contentious issue is resolved here by backing out an effective marginal tax rate on capital income which results in the model conforming with certain features of the U.S. data.

Picking values for the adjustment cost parameters, ϕ_s, κ_s, ϕ_e, and κ_e, is somewhat problematic. Observe that by setting $\kappa_s = g$ and $\kappa_e = g_e$ no cost of adjustment will be incurred along a balanced growth path [see (6) and (11)]. This assumption will be adopted here. Note that long–run facts cannot be used to determine values ϕ_s and ϕ_e since these parameters do not enter the equations that govern the model's balanced growth path. Hence these parameters will be determined by matching the stochastic version of the model's cyclical behavior with some observations on U.S. business cycle fluctuations. For simplicity, in the subsequent analysis set $\phi_e = (g_e/g)^2\phi_s \equiv \phi$. Thus, only the single adjustment cost parameter ϕ will need to be determined.

Values remain to be chosen for the parameters β, θ, α_e, α_s, b, ω, g and τ_k. These values are set so that the model's balanced growth path displays eight features that are observed in the long-run U.S. data. These features are : (i) an average annual GNP/working-age person growth rate of 1.64%, (ii) an average ratio of total hours worked to nonsleeping hours of the working-age population of 24%, (iii) a capital's share of income of 30%, (iv) an average utilization rate of 82%, (v) a ratio of investment in equipment to GNP of 6.7% and (vi) a ratio of investment in structures to GNP of 3.7%, (vii) an average depreciation rate on equipment of 12.3%, which is calculated in a similar way as δ_s above (note that the depreciation rates on equipment used in the NIPA are assumed to be correct on average), and (viii) an average after-tax return on capital of 7%.

The equations characterizing balanced growth for the model are:[6]

$$\gamma_q = (\beta/g)[(1-\tau_k)\alpha_e\hat{y}/\hat{k}_e + (1-bh^{\omega}/\omega)], \tag{22}$$

$$1 = (\beta/g)[(1-\tau_k)\alpha_s\hat{y}/\hat{k}_s + (1-\delta_s)], \tag{23}$$

$$\tilde{\imath}_e/\hat{y} = (\hat{k}_e/\hat{y})[g\gamma_q - (1-bh^{\omega}/\omega)], \tag{24}$$

$$\tilde{\imath}_s/\hat{y} = (\hat{k}_s/\hat{y})[g - (1-\delta_s)], \tag{25}$$

$$(1-\tau_\ell)(1-\alpha_e-\alpha_s)\frac{\theta(1-\ell)}{(1-\theta)(\hat{c}/\hat{y})} = \ell \ , \tag{26}$$

$$(1-\tau_k)\alpha_e(\hat{y}/\hat{k}_e) = bh^{\omega}, \tag{27}$$

and

$$\hat{c}/\hat{y} + \tilde{\imath}_e/\hat{y} + \tilde{\imath}_s/\hat{y} = 1. \tag{28}$$

Equations (22) and (23) are the Euler equations for equipment and structures. The next two equations, (24) and (25), define the steady-state levels of investment in equipment and structures. The efficiency conditions for labor and utilization are given by (26) and (27). Finally (28) is the resource constraint. The long–run restrictions from the data described above imply the following additional eight equations:

$$g = 1.0164, \tag{29}$$

$$\ell = .24, \tag{30}$$

$$\alpha_e + \alpha_s = .30, \tag{31}$$

$$h = .82, \tag{32}$$

$$\hat{\imath}_e/\hat{y} = .066, \tag{33}$$

$$\hat{\imath}_s/\hat{y} = .037, \tag{34}$$

$$bh^{\omega}/\omega = .124, \tag{35}$$

and

$$(\beta/g) = 1/1.07. \tag{36}$$

Note that (22) to (36) represent a system of fifteen equations in fifteen unknowns, viz $\hat{k}_e/\hat{y}$, $\hat{k}_s/\hat{y}$, $\hat{\imath}_e/\hat{y}$, $\hat{\imath}_s/\hat{y}$, ℓ, h, $\hat{c}/\hat{y}$, g, θ, α_e, α_s, ω, b, τ_k and β. The parameter values obtained are $\theta = .40$, $\alpha_e = .17$, $\alpha_s = .13$, $\omega = 1.83$, $b = .32$, $\tau_k = .49$, and $\beta = .95$.[7] The 49% effective tax rate on gross capital income implies a rate on net capital income lying within the range reported by the Feldstein, Dicks–Mireaux and Poterba (1983) study.

B. *Procedure*

A key objective of the analysis in this section is to quantify the contribution to economic growth from investment–specific technological progress. The general strategy is to use data on equipment prices as measure of investment–specific technological change. Hence, a direct observation on q is available. This series, and other data, are then used to impute a series on neutral, or residual, productivity progress by interpreting the postwar experience through the model outlined above.

More precisely, given time series data on y, k_s, k_e, h and ℓ a time series on neutral technological change z can be constructed using the aggregate production function (4).[8] The key step in this calculation is to obtain a series for the equipment stock using the equipment accumulation technology from (9) and (18):

$$k_e' = (1 - \frac{b}{\omega}h^{\omega})k_e + i_e q.$$

Starting from an initial value the series for k_e is constructed by iterating on this equation using the data on h, i_e and q, and the values for the parameters b and ω described above in Subsection A. The starting k_e was set at its balanced growth level, given the values of y and q at the beginning of the sample. An alternative is to use the standard measure for the equipment stock in that year, which yields very similar results for the measurement of z.

Finally, an additional variable of interest in the analysis is the rate of capacity utilization. As was mentioned above, actual data on h is used to compute the stock of equipment, k_e. A natural question to ask then is: To what extent is a utilization series computed from the model—using the efficiency condition for capacity utilization—consistent with its observed counterpart in the U.S. data? Addressing this question can be taken as a test of the appropriateness of the utilization–depreciation specification for equipment.

C. The Results

The data analysis focuses on two related questions. First, does the postwar picture of total factor productivity growth change when an explicit treatment of investment–specific technological progress is incorporated into the analysis? Second, how much of long–run growth is accounted for by investment–specific technological change?

Figures 5 and 6 plot the q and the computed z series. Two observations are immediate. First, z does not display a strong long–run trend. The average annual growth in neutral productivity change is 0.35%. By comparison, the growth rate in investment–specific productivity is 3.21%.

The second, and most noticeable, feature of Figure 6 is the dramatic downturn in total factor productivity which began in the seventies and continued without interruption until the end of the sample. Two factors in the current analysis contribute to this phenomenon. First, note that investment–specific technological change was high when total net productivity growth was low; i.e., growth in the q series accelerated at the same time as there was a productivity slowdown in the z series. Thus, when changes in q are explicitly accounted for, the slowdown in z tends to be more pronounced. Second, equipment plays an important role, quantitatively, in the analysis. Specifically, had the current analysis treated equipment and structures equally in production, as is implicitly done in conventional analyzes where these two capital stock are simply aggregated together, the magnitude of the downturn would not be as large.

The importance of properly incorporating capital into growth accounting can be illustrated as follows. Suppose that output is produced using only labor according to the constant returns–to–scale production function $y = zl$. Here the Solow residual z corresponds to average labor productivity, y/ℓ, as conventionally measured, which grew at 1.29% per year over the post–war period. Figure 7 plots z for this case. Observe that productivity growth slows down in the 70's, but remains positive. Next, consider the standard one sector growth model. Here output is produced according to $y = zk^{\alpha}\ell^{1-\alpha}$, where k represents standard measure of the *combined* stocks of equipment and

FIGURE 5: q

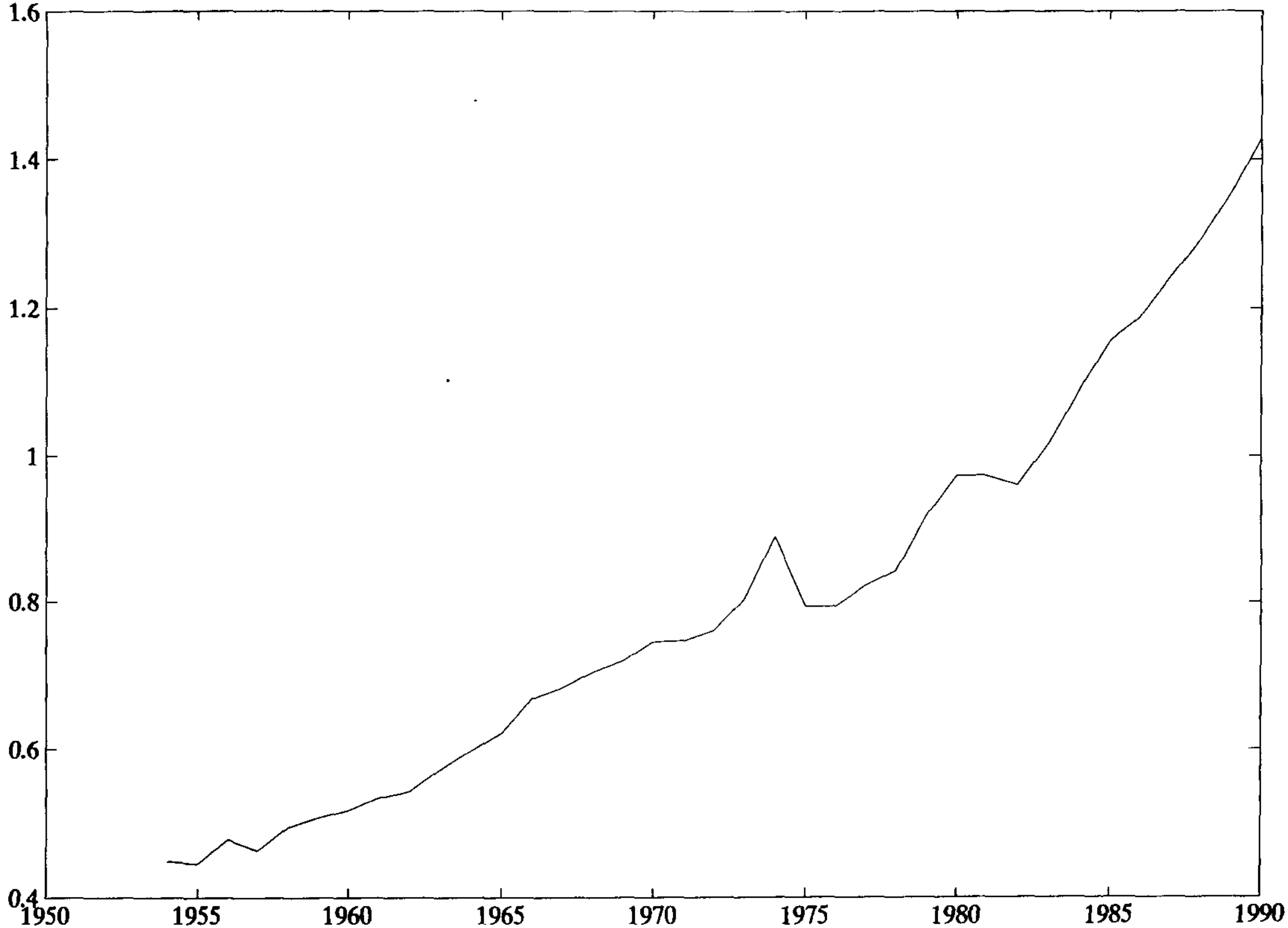


FIGURE 6: z

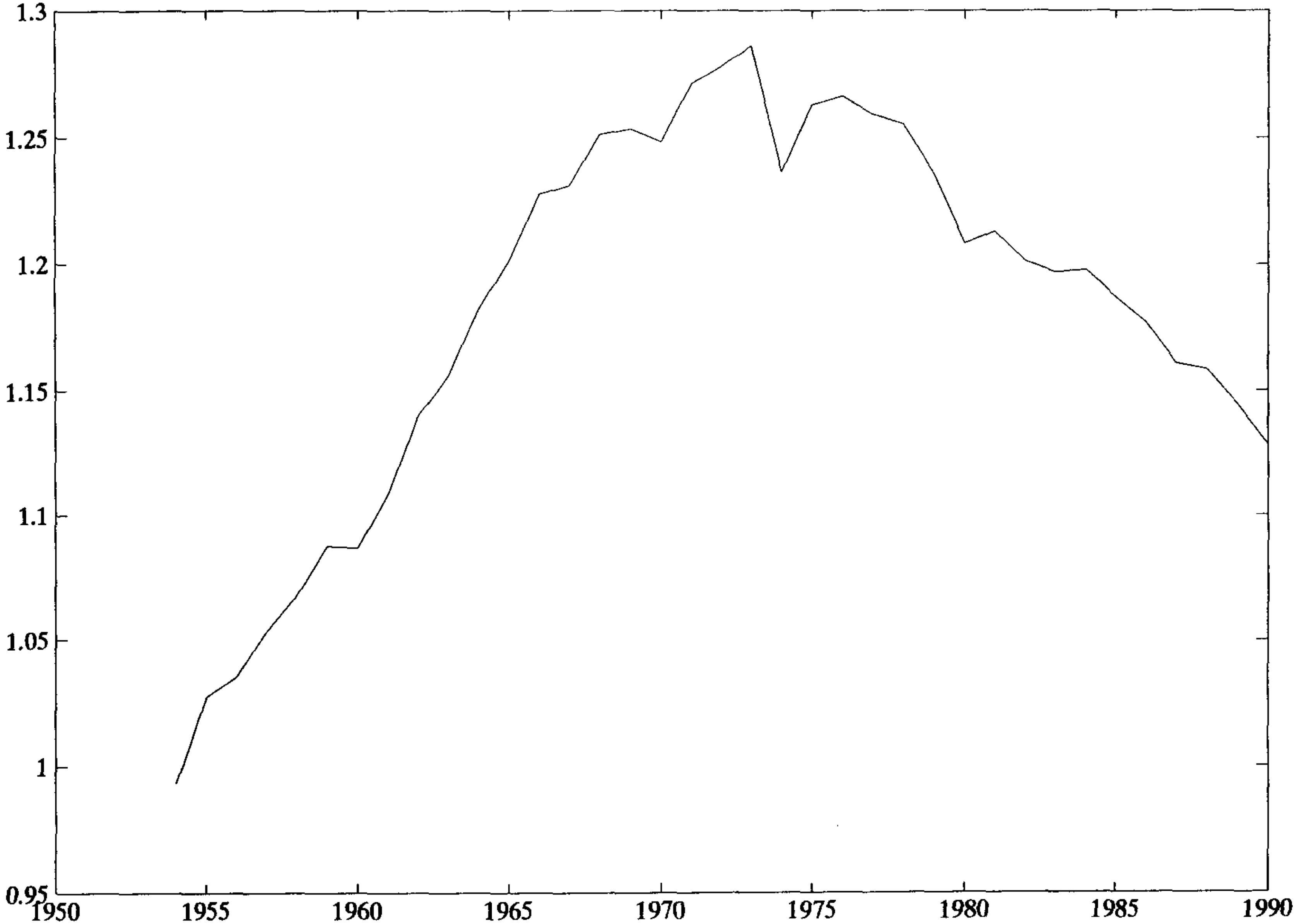
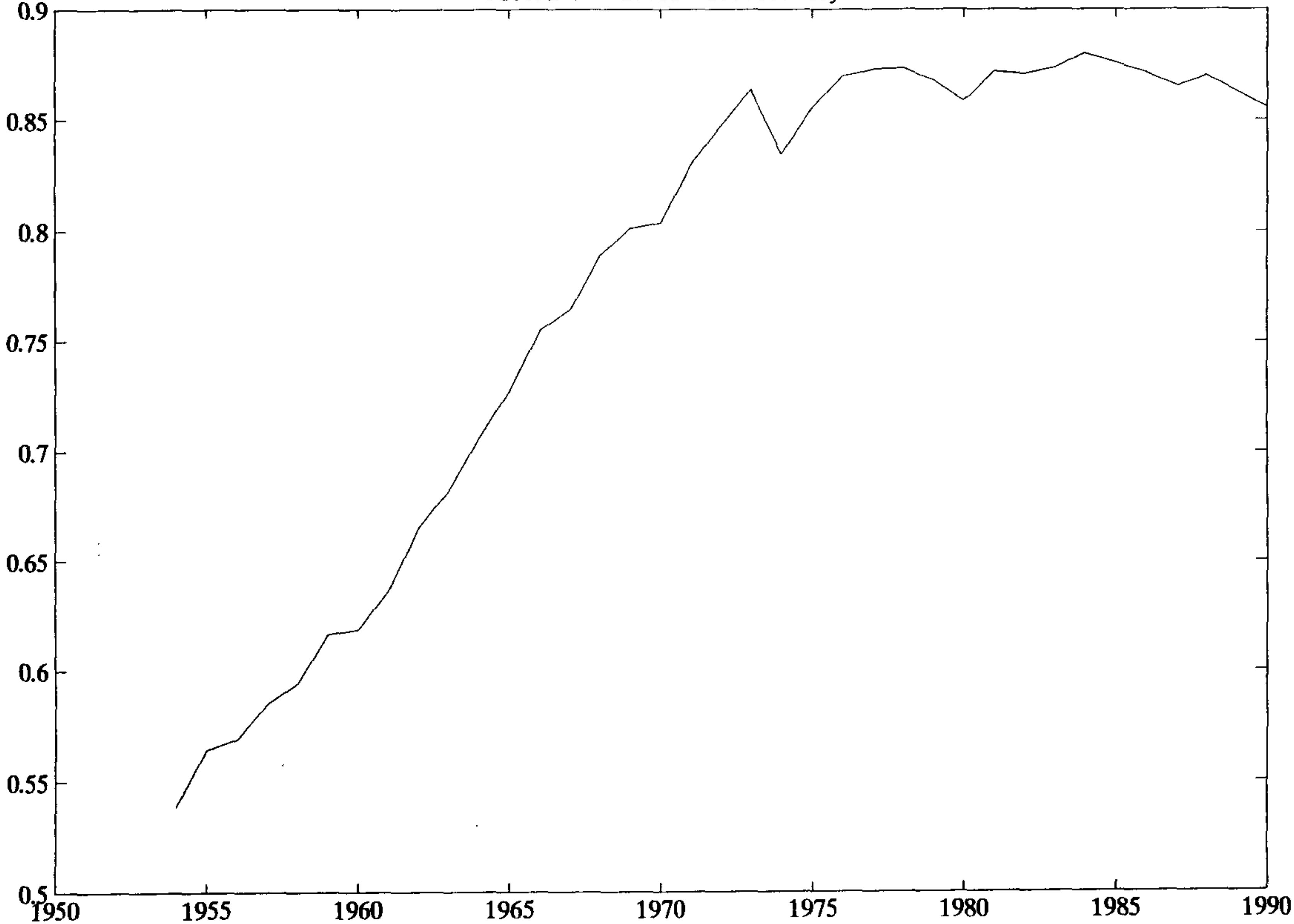

1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
1950
1955
1960
1965
1970
1975
1980
1985
1990

FIGURE 7: Labor Productivity

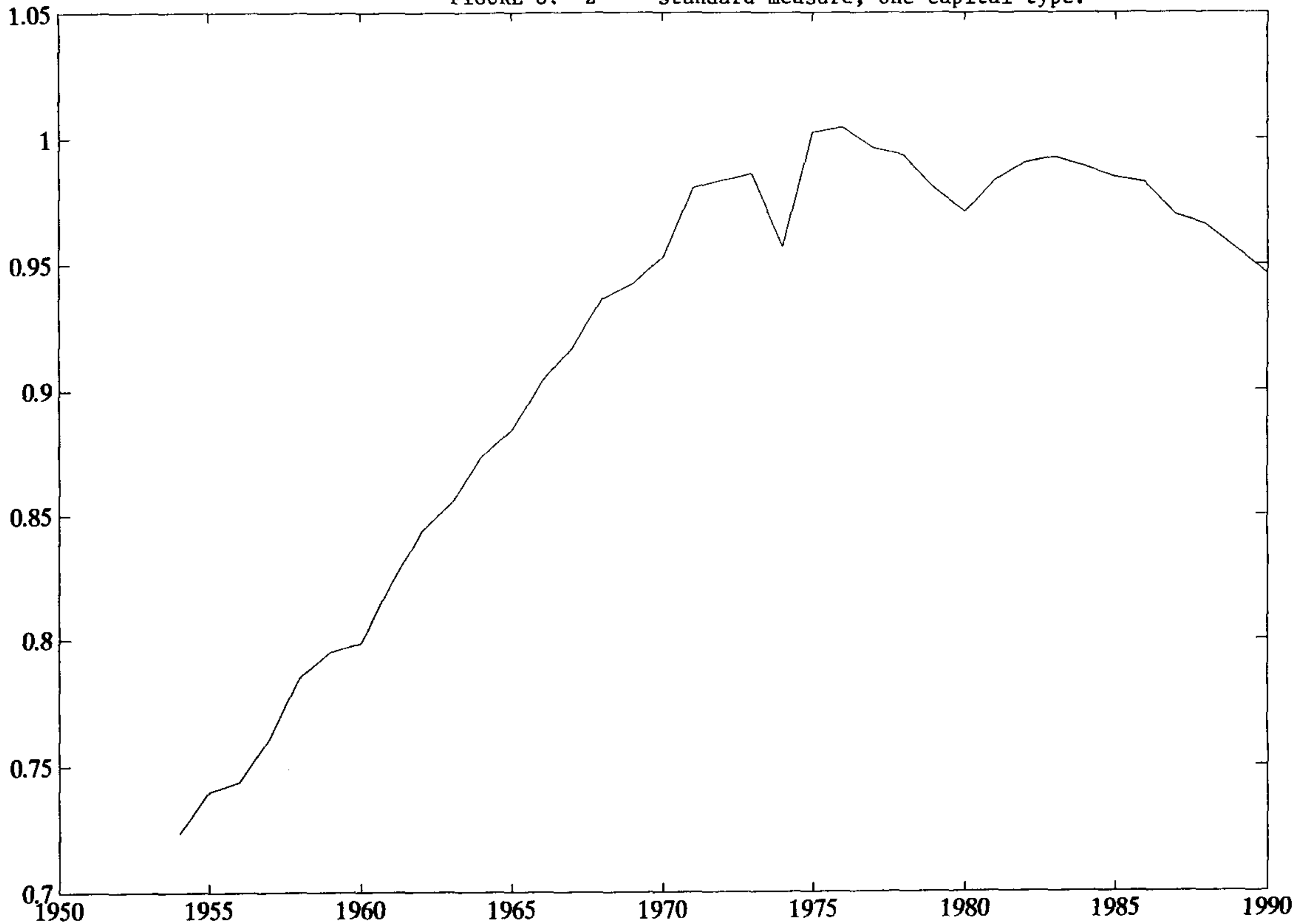


structures. Now, the rate of disembodied technological change is 0.75% per year on average. Figure 8 plots this standard measure of the Solow residual or $z = y/(k^{\alpha}\ell^{1-\alpha})$. The productivity slowdown is now more apparent. Now, disaggregate the capital stock into equipment and structures and assume the aggregate production function is given by $y = zk_e^{\alpha_e}k_s^{\alpha_s}\ell^{1-\alpha_e-\alpha_s}$. If one assumes that the measures of equipment and structures in the NIPA are correct, then the Solow residual grew at 0.72% annually—see Figure 9. Finally, if the stock of equipment is adjusted in line with Gordon's data for investment–specific technological change (and a variable rate of capacity utilization) the growth rate in $z = y\left[(k_e h)^{\alpha_e}k_s^{\alpha_s}\ell^{1-\alpha_e-\alpha_s}\right]$ drops to 0.35%. The difference between the NIPA measure for the stock of equipment and the measure constructed here, which reflects better the improvement in quality of equipment, is shown in Figure 10. The productivity slowdown, as captured by Figure 6, becomes dramatic.

However dramatic the behavior of neutral, or residual, productivity change, the main goal here is not to decompose this variable. The effects of accounting for changes in labor quality, though, as captured by Jorgenson, Gollop and Fraumeni (1987) index will be reported. This index in labor quality shows a slowdown starting in the late sixties: From an average yearly growth of 0.7% during the 1954–1968 period, the series's growth rate drops to 0.25% between 1968–1989. When the labor input measure is adjusted to incorporate the Jorgenson–Gollop–Fraumeni (1987) labor quality index the pattern of z remains the same. Although the average growth of the residual is now zero, there is still a sharp rise prior to the earlier seventies followed by unabated productivity regress.

Using formula (20) and the average growth rates for q and z—3.21 and 0.35—one can obtain estimates of the contributions that the two sources of productivity change made to growth in output per hour worked. These estimates are approximations, given that (20) refers to balanced growth while the technology growth rates are sample

FIGURE 8: z - - standard measure, one capital type.



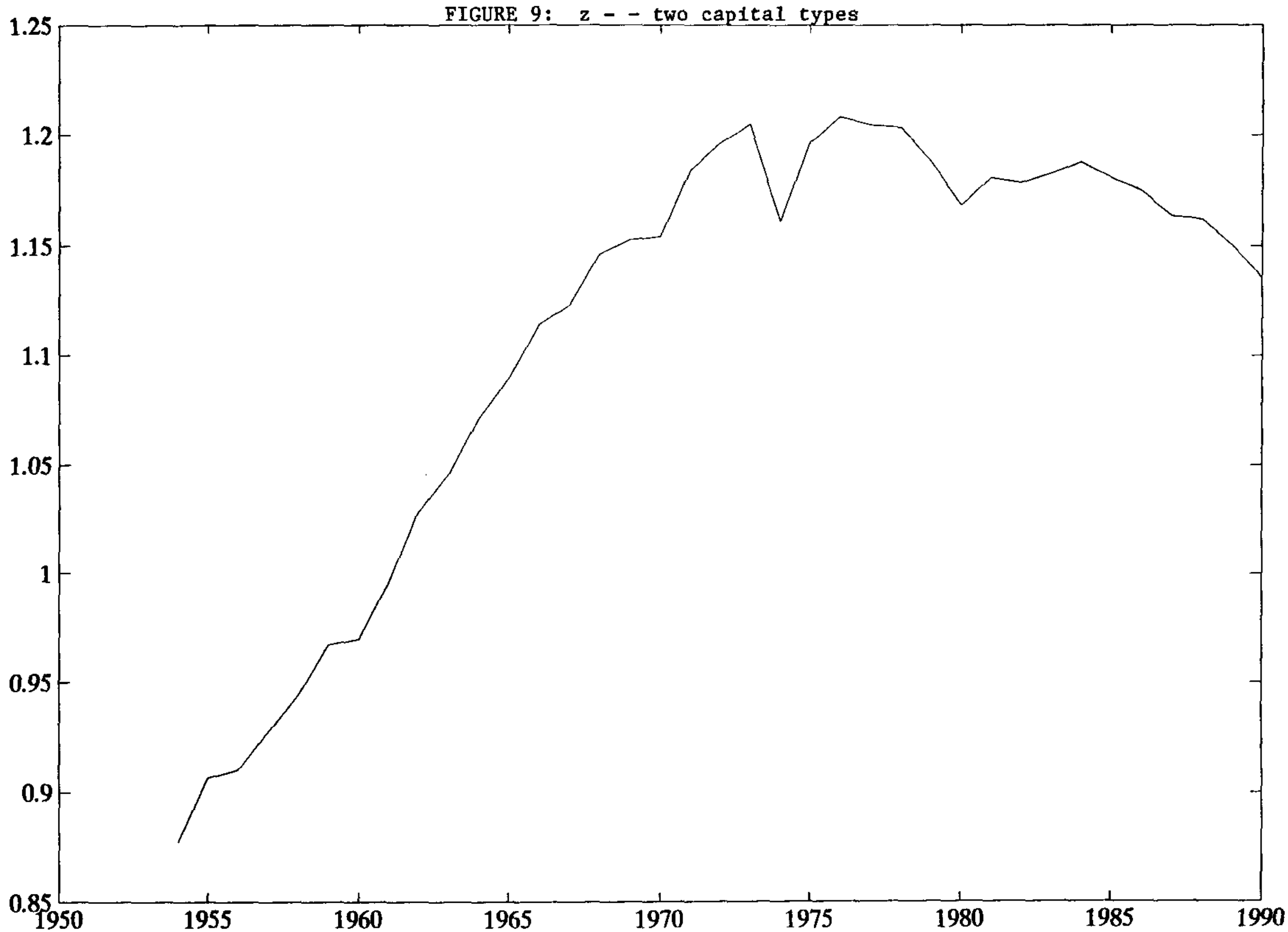
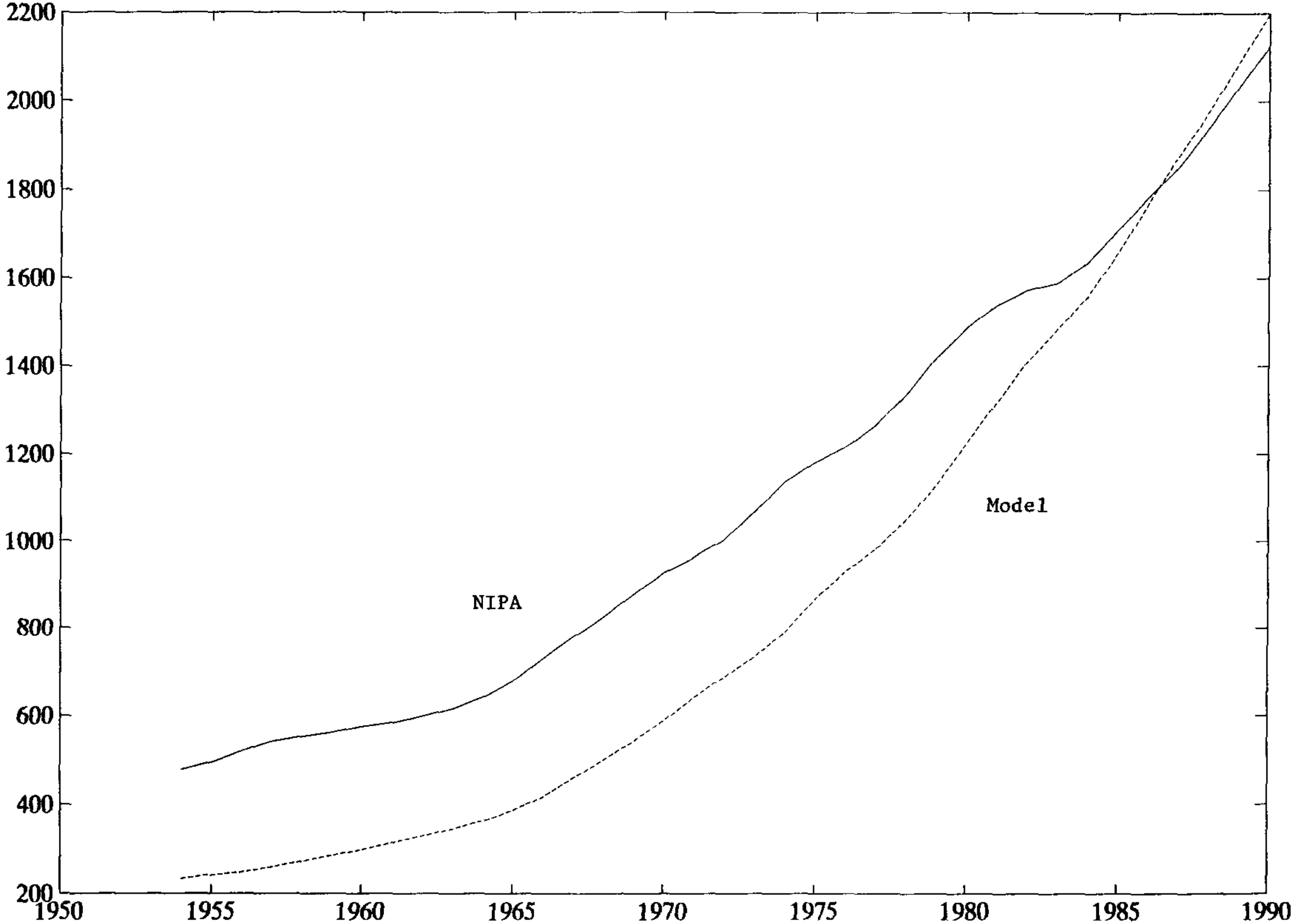


FIGURE 9: z - - two capital types
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
1950
1955
1960
1965
1970
1975
1980
1985
1990

FIGURE 10: Stock of Equipment



averages. The actual average growth rate of output per hour over the 1954–90 sample period is 1.29% per year. With only investment–specific technological change at work [i.e., assume $\gamma_z = 1$ in (20)] output per hour would have grown at 0.8% per year. The corresponding figure for neutral technological change is 0.5%. Hence, investment–specific change technological change contributes about 60% of all output growth with neutral change providing the rest.

The results here differ significantly from the findings in Hulten (1992). He finds that over the 1949–1983 period only 20% of output growth not attributable to inputs is due to investment–specific (or capital–embodied) technological change, and hence that neutral (or disembodied) technological change is quantitatively much more important.[9] The difference between our results and Hulten's lies in the specification of the production technology, as represented by the aggregate resource constraint. Using the notation developed here, Hulten's specification, which follows Jorgenson (1966), can be written as

$$c + i_s + i_e q = F(\cdot), \tag{37}$$

with the law of motion for equipment being given by

$$k'_e = (1-\delta_e)k_e + i_e q.$$

The accumulation equation for equipment used in the current analysis is the same, but the resource constraint, which is consistent with that used by Solow (1959), reads

$$c + i_s + i_e = c + i_s + (1/q)i_e q = F(\cdot). \tag{38}$$

Equations (37) and (38) differ in the aggregation of the individual components of output. In (37) equipment in efficiency units is aggregated *one for one* with real consumption. Hence, Hulten's specification implies setting the marginal rate of transformation between these variables to one, and marginal cost pricing then implies a constant relative price. In contrast, the resource constraint (38) that is used here aggregates efficiency units of equipment investment and the other components of output according to the marginal rate of transformation as measured by the relative price. The factor (1/q) in (38) is precisely the relative price of an efficiency unit of new equipment.

The results from the two procedures differ significantly, given the dramatic decline in the relative price as measured by Gordon and shown in Figure 1. In terms of growth accounting, it is clear why Hulten's procedure captures a much lower share of investment–specific technological change in output growth. Given that the declining trend in the relative price of new equipment is not taken into account on the left–hand side of (37), the measured residual z on the right–hand side has to grow faster – – explaining a larger share of output growth. Put differently, in Hulten's procedure the equipment–specific decline in production costs, as reflected in the relative price, is captured by the measure of neutral technological change.

In terms of the theoretical implications of the specification in (37), note that it can be rewritten as

$$c + i_s + x = F(\cdot),$$

with

$$k'_e = (1-\delta_e)k_e + x,$$

and $x = i_e q$. Given that i_e and q do not enter separately in the model, an optimal allocation for this economy is independent of the behavior of q. Agents choose an optimal path for x, regardless of the behavior of q: i_e is simply chosen as a residual. Hence, if optimality is allowed for, this specification implies that a growth accounting exercise should find a *zero* contribution of investment–specific technological change.

Last, by using the data on k_e, k_s, ℓ, z, and q the rate of capacity utilization predicted by optimality equation (37) can be computed for the postwar period.[10]

$$z a_e (hk_e)^{\alpha_e} k_s^{\alpha_s} \ell^{1-\alpha_e-\alpha_s} = b h^{\omega-1}/q. \qquad (39)$$

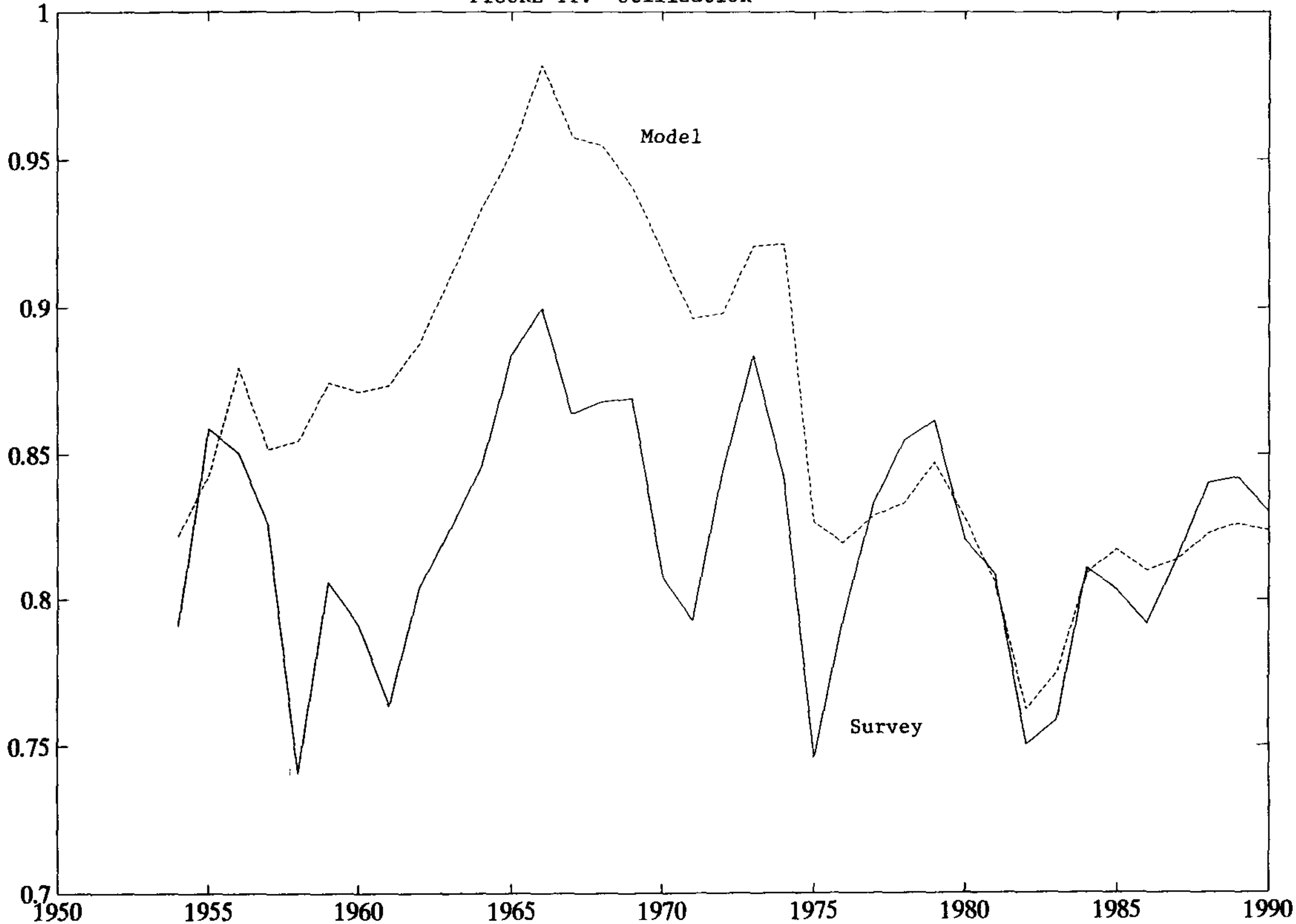
This equation balances the extra output obtained by increasing capacity usage (shown by the lefthand side) against the resulting loss in terms of depreciated equipment (the righthand side). The series can then be compared with the survey–based utilization series. Figure 11 plots the two series. It can be seen that except for a difference in levels during the first half of the sample, the model's h tracks the actual utilization series quite well. This result provides support for the current 'user cost' specification of equipment utilization. Although the variable rate of utilization did not play a critical role in this section as will be seen it is important in the short–run analysis.

IV. THE ROLE OF INVESTMENT–SPECIFIC TECHNOLOGICAL CHANGE IN ECONOMIC FLUCTUATIONS

The quantitative importance of investment–specific technological change will now be gauged by simulating the model that was developed in Sections II and III. To isolate the effects of this form of technological change, only the q–shock will be allowed to operate. The stochastic structure that governs the evolution of this shock is taken from the time series properties that characterize the postwar movement of equipment prices in the U.S. economy.

The analysis consists of comparing a set of summary statistics characterizing the movement of the variables in the model with the corresponding set describing the behavior of U.S. business cycle fluctuations for the 1954–1990 period. All statistics are compiled using data that is logged and Hodrick–Prescott filtered. The model's statistics are generated by simulating the artificial economy developed above for 100 samples of 37 observations—the number of years in the 1954–90 sample period. The statistics reported for the model are the averages of the statistics computed from the individual samples. The main question of interest is: How much of the fluctuations in U.S. GNP can be accounted for by investment–specific technological change?

FIGURE 11: Utilization



Driving a model with investment-specific shocks has some intrinsic difficulties. Unlike the standard real business cycle model, here the technology shock does not directly affect the current production function. Current output is affected only to the extent that the shock can elicit the increased employment of capital and labor in response to changed investment opportunities. A positive shock to the efficiency of new equipment raises the return on equipment investment. This operates to entice equipment investment. Note, though, that equipment investment is only 7 percent of GNP, with only 17 percent of the value of output being derived from the use of equipment in production. Hence, unlike the standard model, the fraction of GNP directly affected by the shock is quite small. Significant movements in GNP may only come at the expense of enormous shifts in investment. Also, the increases in the rate of return on equipment investment that drive business cycle expansions operate to dissuade consumption. There is no guarantee that consumption will even move procyclically in the model. A key ingredient in the model's transmission mechanism is the variable rate of capital utilization. A positive investment shock reduces the cost of utilizing old equipment. This promotes the more intensive utilization, and consequent accelerated depreciation, of old capital which leads to the increased employment of labor and the resulting expansion in output.

To simulate the model, a stochastic process for q must first be specified and estimated. Additionally, a value must be assigned to the one remaining parameter, ϕ, governing adjustment costs. To this end, let q have the following time series representation:

$$\ln q_t = constant + t \cdot \ln \gamma_q + \eta_t$$

and

$$\eta_t = \rho \eta_{t-1} + \xi_t, \text{ with } 0 < \rho < 1 \text{ and } \xi_t \sim N(0,\sigma).$$

Cochrane–Orcutt estimation of the above time series process for the 1954–90 sample period yields the values for γ_q, ρ and σ shown below:

$\gamma_q = .032,$ $\rho = .64$ and $\sigma = .035$, with D.W. = 1.90,
(24.16) (4.94)

where the numbers in parentheses are t–statistics.[11] Note that the stochastic process for investment–specific technological change can be estimated directly using Gordon's equipment price series. This has an advantage. The real business cycle literature has emphasized the 'Solow residual' as the driving force underlying the business cycle. This imputed residual may include — as has been argued by Hall (1989) — other influences, besides technological change, that affect rates of capacity utilization. Government spending, for instance, tends to be positively related with the Solow residual and energy prices negatively so. Finn (1992) has explained these correlations by modelling the effect that such factors have on capacity utilization. Such issues are partially avoided here by using alternative, more direct, evidence on technological change.

There is no good guide available for choosing an appropriate value for ϕ. The model is simulated using two different strategies for picking ϕ. The first strategy chooses ϕ so that the standard deviation for investment in the model equals that found in the data. This results in a value for ϕ of 1.24. Now, in reality, disturbances other than q–shock are also at work—such as the factors underlying the z–shock. Thus, the first strategy will attribute too much of the observed variability in investment to the q–shock. Hence, the model's variation in output can be taken as an upper bound on the contribution of the q–shock to the business cycle.

The second strategy sets ϕ so the correlation between consumption and output in the model mimics that found in the data. On this basis $\phi = 2.97$. This strategy provides a lower bound on the contribution of q–shocks to output fluctuations. With investment–specific shocks the same mechanism which stimulates capital accumulation retards consumption. This substitution away from consumption toward investment in response to a good realization of the q-shock can be weakened by increasing the adjustment parameter, ϕ. Therefore, increasing ϕ should strengthen the procyclicality of consumption. The second strategy, however, is likely to lead to a chosen value for ϕ that is too high. The omitted z-shocks tend to generate strongly procyclical movements in consumption. If these shocks were included, a lower value for ϕ would be called for. The upshot of choosing too high a value for ϕ is too low a value for output volatility. Hence, the results can be interpreted as providing a lower bound on the contribution of q-shocks to the business cycle.

In Table 1 some statistics characterizing the behavior of economic fluctuations for the postwar U.S. economy are presented. The corresponding statistics for the model (when $\phi = 1.24$ and $\phi = 2.97$) are also displayed in the table. Observe that for the U.S. economy, the standard deviation of GNP around its Hodrick–Prescott trend is about 2.4 percent. When $\phi = 1.24$, the figure for the model is about 0.7 percent. By this accounting, approximately 27 percent of business cycle fluctuations can be explained by investment–specific technological change. In the U.S. data, investment and utilization are more volatile than output, and consumption and hours less so. The model exhibits this behavior, but greatly exaggerates the variability in investment spending relative to output. As was discussed above, given the nature of the shock this is to be expected. All of the variables are strongly procyclical in the data.

The correlation between output on the one hand and consumption and utilization on the other is low for the model. As conjectured, a higher value for ϕ increases the correlation between consumption and output.[12] Now investment–specific technological change can be seen to account for about 20 percent of the fluctuations in output. Additionally, hours and utilization vary with output in a manner similar to that exhibited by the U.S. economy.

TABLE 1

Variable	Standard Deviation (%)	Cross–Correlation with Output	Autocorrelation
	U.S. Annual Data, 1954–1990		
Output	2.40	1.00	.43
Consumption	1.39	.89	.61
Investment	5.99	.73	.44
Hours	1.71	.85	.46
Utilization	4.22	.84	.34
	Model (ϕ = 1.24)		
Output	.65	1.00	.31
Consumption	.24	.35	.75
Investment	5.99	.94	.23
Hours	.46	.93	.24
Utilization	2.46	.78	.24
	Model (ϕ = 2.97)		
Output	.49	1.00	.30
Consumption	.21	.89	.50
Investment	3.22	.96	.25
Hours	.24	.95	.25
Utilization	2.16	.89	.25

Note: Data definitions are given in the appendix. The statistics are calculated using data that was logged and Hodrick–Prescott filtered.

A variant of the model incorporating indivisible labor along the lines of Hansen (1985) was also simulated.[13] This specification results in an increased responsiveness of hours worked to the investment shock, which leads to higher output volatility. The results of these simulations, which are reported in Appendix C, are similar to those discussed above. Now, somewhere in the neighborhood of 25 to 31 percent of output variability may be due to investment–specific shocks.

To conclude, it appears that investment–specific technological change may account for a sizable fraction of business cycle fluctuations. By the simple accounting undertaken here anywhere between 20 to 31 percent of output fluctuations can be attributed to this type of shock. This is somewhat surprising in light of the fact that investment in equipment amounts to only 7 percent of GNP.

V. CONCLUSIONS

The analysis in this paper was motivated by two key observations. First, over the long run the relative price of equipment has declined remarkably while the equipment–to–GNP ratio has risen. This suggests that investment–specific technological change may be a factor in economic growth. Second, the short–run data display a negative correlation between the price for equipment on the one hand, and equipment investment or GNP on the other. This hints that investment–specific change may be a source of economic fluctuations.

A simple vintage capital model was constructed here that has the property that the equipment–to–GNP ratio increases over time as the relative price of new capital goods declines. The standard features of the neoclassical growth were otherwise preserved. The balanced growth path for the framework under study was calibrated to the long–run U.S. data. A growth accounting exercise was then conducted with the model. It was found that approximately two–thirds of postwar productivity growth can be attributed to investment–specific technological change. This result may indicate where the highest return on future theorizing about engines of growth lies.[14] Also, a more striking picture emerges of the much discussed productivity slowdown that started in the 1970's. Once the recent rapid improvement in the quality of capital goods is taken into account, the decline in the productivity of other factors is dramatic. Finally, investment–specific technological change may be an important source of economic fluctuations. The accounting exercise undertaken here suggested that about twenty–five percent of economic fluctuations can be attributed to this form of technological change. These findings point to a very specific and potentially important source of economic growth and fluctuations. Although the analysis was undertaken within the context of a framework which assumes constant–return–to–scale technologies and perfect competition, the results should be suggestive to macroeconomists from various perspectives. More elaborate models that allow for endogenous R&D, monopolistic competition, etc., should constitute important robustness tests on the findings.

FOOTNOTES

1. The role of a variable rate of factor utilization in business cycle fluctuations has been studied by Lucas (1967), Greenwood, Hercowitz and Huffman (1988), Kydland and Prescott (1988), Bils and Cho (1991), Burnside, Eichenbaum and Rebelo (1992), and Finn (1992).

2. The required changes are trivial in nature. Denote the aggregate state–of–the–world by (s,z,ϵ) where $s = (k_e,k_s)$. Let the aggregate law of motion for equipment be represented by $k'_e = M(s,z,\epsilon)$. Thus, the law of motion for s reads $s' = S(s,z,\epsilon) \equiv (M(s,z,\epsilon),K_s(s,z,\epsilon))$. Final output producing firms still solve (P2) where $\tilde{k}_e = k_e h$. The market clearing condition becomes

$$c + i_s + i_e = F(k_e h,\ k_s,\ \ell,\ z) - a_s - a_e,$$

where $i_e = [k'_e - k_e(1-\delta_e(h))]/Q(\epsilon)$, $i_s = k'_s - (1-\delta_s)k_s$, $a_s = A_s(k'_s,k_s)$, and $a_e = A_e(k'_e/Q(\epsilon),\ k_e/Q(\epsilon);\eta)$. A competitive equilibrium is a solution to the model such that households solve the transformed version of (P1), firms solve (P2), and the new market clearing condition holds. The transformed model is presented in Appendix A.

3. Recall that the production function reads $F(hk_e,k_s,\ell,z) = z(hk_e)^{\alpha_e}k_s^{\alpha_s}\ell^{1-\alpha_e-\alpha_s}$ once the stock of equipment is consolidated into efficiency units.

4. Again, once the stock of equipment is aggregated into efficiency units the household chooses k'_e instead of μ' in problem (P1) subject to the constraint given by (19). Analogously in (P2) the firm chooses hk_e rather than $\tilde{k}_e$. See footnote (2) for more detail.

5. In the class of CES production functions, the Cobb–Douglas case is the only one permitting a balanced growth path of this sort.

6. This balanced growth path can also be obtained from a more elaborate two sector model. In the model above the first sector produces consumption and structures while the second sector produces just equipment. Here both sectors produce output according to a Cobb–Douglas production function employing services from equipment, structures, and labor. The shock for the first sector is given by z while for the second sector it is zq. When the exponents on equipment, structures and labor are the same across sectors the balanced growth path for this model is given by (22) to (28). It is interesting to note that when the coefficient on equipment is higher in sector two than in sector one the relative price of equipment can decline over time, even when q is constant over time. Practically, however, some simple calculations reveal implausibly high differences in capital's share of income between the two sectors are required to generate the observed price decline in this case. For instance if the weights on structures are the same across sectors, the difference between the weights on equipment must be at least .6 to generate the observed decline in the relative price of equipment.

7. The values for α_e and α_s are close to those found by Duménil and Lévy (1990) who estimated aggregate functions incorporating equipment, structures, and labor over a 100 year period. They found that a Cobb–Douglas production function with time varying coefficients fits the data best. For the sub–period under study here the estimated coefficients did not vary much and consequently the Cobb–Douglas production with constant coefficients is an accurate approximation.

8. Theoretically, the adjustment costs associated with changing the stocks of structures and equipment (when not on a balanced growth path) should be netted out of real income. Quantitatively, these costs are trivial in size for the configurations of the model being studied here and therefore can be safely ignored — see footnote 12.

9. In principle the figures presented here are not directly comparable to Hulten's, since the current ones incorporate the effects of endogenous capital accumulation due to each source of technological advance. In practice, however, the distinction is not important, since the model's balanced growth capital accumulation responds to each type of technological change in proportion to their direct contribution to output growth.

10. This first–order condition arises from (P1) where the problem has been redefined in terms of k_e' rather than μ' so that the constraint (13) takes the form (19). In the absence of adjustment costs the efficiency condition for h is given by $(1-\tau_k)R_e(s,z,\ell) = \delta'(h)/Q(\epsilon)$. But from (P2), $R_e(s,z,\epsilon) = F_1(hk_e,k_s,\ell,z)$. Equation (37) follows immediately.

11. The estimates for the parameters were obtained using data for the sample period 1954 to 1990. The data sample used Gordon's price index for the 1954–83 subperiod and a correction of NIPA price measures for the 1984–90 subperiod. The correction to the NIPA measures involved adjusting downwards the growth rates for the indexes in the producer durable equipment (PDE) categories by 1.5 percent. An exception was the computers category, which already incorporates the quality adjustment used in Gordon (1990). This adjustment to the NIPA numbers was suggested by Robert Gordon. Moreover, the new index for 1984–1990 was constructed by taking an average

of the implicit PDE price deflator (IPD) and the fixed–weight price index (PPI) for PDE. This average reflects the desire to replicate the more elaborate Tornquist index used in Gordon (1990).

12. It is interesting to note that even for this configuration of the model adjustment costs on average amount to no more than .01% of GNP.

13. This line of work draws on the theoretical analysis of Rogerson (1988).

14. See De Long and Summers (1991) for additional evidence supporting the link between equipment investment and growth.

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APPENDIX A: AN ALTERNATIVE TREATMENT OF EQUIPMENT

A simpler version of the model presented in Section II (Subsections A, B and C) is offered here. The simplicity arises from the restriction that only one type of equipment can be produced each period. For convenience, this appendix has the same structure as the relevant subsections in the main text.

A. The Economic Environment

Given that the modelling of preferences, the final output production function and structures remains as in the text [equations (1) – (6)], the discussion here starts directly with equipment. The variable q, which represents the state of the equipment production technology, indicates the amount of equipment, in efficiency units, that can be purchased for one unit of output. We assume that q evolves according to $q_{t+1} = \gamma_q^{t+1}\eta_{t+1}$, where $\gamma_q > 1$ and η_{t+1} is drawn from the distribution function $H(\eta_{t+1}|\eta_t) = \text{prob}\{\eta_{t+1} = \eta' \,|\, \eta_t = \eta\}$.

Only one type of equipment can be produced each period. Hence, the equipment evolution equation can be directly written as:

$$k_e' = (1 - \delta_e(h))k_e + i_e q, \tag{A.1}$$

which corresponds to the type-aggregated law of motion (18) in the text. Here the stock of equipment, k_e, is measured in efficiency units while equipment investment, i_e, is denominated in units of final output. Movements in q can be interpreted in two different ways (compare with the discussion in Subsection II.C). Suppose q increases over time. One could think of this as implying (a) that the efficiency level of a unit of equipment, as indexed by q, increases over time, with the cost of a unit of equipment remaining constant; or (b) that the efficiency level of a unit of equipment remains constant over time with its cost in terms of output, $1/q$, declining.

Installing equipment advolves adjustment costs in line with:

$$a_e = A_e(k'_e/q, k_e/q; \eta) = \eta\, \phi_e\, (k'_e/q - \kappa_e k_e/q)^2/(k_e/q) \text{ with } \phi_e, \kappa_e > 0. \quad \text{(A.2)}$$

As in the text, the production function can be described by:

$$y = F(k_e h, k_s, \ell, z) = z(k_e h)^{\alpha_e} k_s^{\alpha_s} \ell^{1-\alpha_e-\alpha_s}. \quad \text{(A.3)}$$

Final output (net of adjustment costs) can be used for three purposes: consumption, investment in structures, and investment in equipment. Thus, the economy's resource constraint is given by

$$c + i_s + i_e = y - a_s - a_e. \quad \text{(A.4)}$$

Finally, the government budget constraint is, as in the text,

$$\tau = \tau_k(r_e k_e h + r_s k_s) + \tau_\ell w, \quad \text{(A.5)}$$

where r_e, r_s and w are the rental rates for equipment and structures and the real wage, τ_k and τ_ℓ are the tax rates on capital and labor, and τ is a lump-sum transfer payment.

B. Competitive Equilibrium

The aggregate state of the world is described by $\lambda = (s,z,q)$, where $s \equiv (k_e,k_s)$. The equilibrium wage and rental rates w, r_e and r_s, individual transfer payments τ and price of an efficiency unit of equipment p can all be expressed as functions of the state of the world λ: $w = W(\lambda)$, $r_e = R_e(\lambda)$, $r_s = R_s(\lambda)$, $\tau = T(\lambda)$ and $p = P(\lambda)$. Finally, the two capital stocks evolve according to $k'_e = K_e(\lambda)$ and $k'_s = K_s(\lambda)$. Hence, the law of motion for s is $s' = S(\lambda) \equiv (K_e(\lambda),K_s(\lambda))$.

<u>The Household</u>

The dynamic program problem facing the representative household is

$$V(k_e,k_s;s,z,q) = \max_{c,k'_e,\, k'_s,\ell,h} \{U(c,\ell) + \beta\, E[V(k'_e,k'_s;s',z',q')]\} \quad \text{(P1)}$$

subject to

$$c + P(\lambda)k'_e + k'_s = (1-\tau_k)[R_e(\lambda)k_e h + R_s(\lambda)k_s] + (1-\tau_\ell)W(\lambda)\ell +$$

$$P(\lambda)(1-\delta_e(h))k_e + (1-\delta_s)k_s + T(\lambda) - A_s(k_s',k_s) - A_e(P(\lambda)k_e',P(\lambda)k_e;\eta)$$

and $s' = S(\lambda)$.

The Final–Output Firm

The maximization problem of this firm is

$$\max_{\tilde{k}_e h,\, \tilde{k}_s, \ell} \quad \pi_y = F(\tilde{k}_e h, \tilde{k}_s, \ell, z) - R_e(\lambda)\tilde{k}_e h - R_s(\lambda)\tilde{k}_s - W(\lambda)\ell. \tag{P2}$$

The Equipment Producing Firm

This type of firm solves the problem

$$\max_{i_e} \pi_e = P(\lambda)(i_e q) - (1/q)(i_e q). \tag{P3}$$

Since $P(\lambda)$ is the price of one efficiency unit of equipment, the relevant (marginal or average) cost is $1/q$. Obviously, in competitive equilibrium $P(\lambda) = 1/q$.

Definition of Equilibrium

A competitive equilibrium is a set of allocation rules $c = C(\lambda)$, $k_e' = K_e(\lambda)$, $k_s' = K_s(\lambda)$, $\ell = L(\lambda)$ and $h = H(\lambda)$, a set of pricing function $w = W(\lambda)$, $r_e = R_e(\lambda)$, $r_s = R_s(\lambda)$, $p = P(\lambda)$ and $\tau = T(\lambda)$, and an aggregate law of motion for the capital stocks $s' = S(\lambda)$, such that

(i) households solve problem (P1), taking as given the aggregate state of the world $\lambda = (s,z,q)$ and the form of the functions $W(\cdot)$, $R_e(\cdot)$, $R_s(\cdot)$, $P(\cdot)$, $T(\cdot)$ and $S(\cdot)$, with the equilibrium solution to this problem satisfying $c = C(\lambda)$, $k_e' = K_e(\lambda)$, $k_s' = K_s(\lambda)$, $\ell = L(\lambda)$, and $h = H(\lambda)$.

(ii) $k_e H(\lambda)$, $L(\cdot)$, and k_s solve the problem (P2) of the final output producing firms given λ and the functions $R_e(\cdot)$, $R_s(\cdot)$, and $W(\cdot)$.

(iii) equipment–producing firms solve problem (P3) given λ and the function $P(\cdot)$.

(iv) the economy–wide resource constraint holds each period:

$$c + i_e + i_s = F(k_e h, k_s, \ell, z) - a_s - a_e,$$

where

$$i_s = k'_s - (1-\delta_s)k_s,$$
$$i_e = [k'_e - (1-\delta_e(h))k_e]/q,$$
$$a_s = A_s(k'_s, k_s),$$

and

$$a_e = A_e(k'_e/q, k_e/q; \eta).$$

APPENDIX B: DATA

Sample: 1954–1990.

The empirical counterparts of the theoretical variables used in the data calculations are the following:

y: nominal GNP divided by the implicit price deflator for nondurable consumption goods and nonhousing services—base year 1987.

c: nominal consumption expenditure on nondurables and nonhousing services divided by their implicit price deflator—base year 1987.

i_e: nominal investment in producer durable equipment divided by the implicit price deflator for nondurable consumption goods and nonhousing services—base year 1982.

i_s: investment in producer structures in 1987 dollars.

i: total investment in 1987 dollars. Thus, $i = i_e + i_s$.

k_S: net stock of producer structures in 1987 dollars.

l: total hours employed per week—Household Survey data.

q: implicit price deflator for nondurable consumption goods and nonhousing services divided by Robert J. Gordon's (1990, Chp 12, Table 12.4) index of nominal prices for producer durable equipment. Since Gordon's index is only computed through 1983, a correction of the NIPA measures for producer durable equipment was used for the remainder of the sample. See footnote 11 for the details.

h: capacity utilization rates. These are based on surveys conducted by different organizations (the Bureau of Economic Analysis, McGraw Hill, and the Bureau of the Census), and summarized by the monthly Federal Reserve Board Publication, *Federal Reserve Bulletin*).

Notes to Appendix B:

1. Because of the approach taken here to quality improvement in equipment, standard constant price output and equipment data cannot be used in this framework as the corresponding variables y and i_e. These theoretical constructs should be matched with quantities expressed in terms of their cost in consumption units. Correspondingly, y and i_e were computed by deflating nominal GNP and equipment investment by the consumption deflator. For structures this problem does not exist, but for consistency the same procedure was followed.

2. The calculation of the depreciation rates δ_e and δ_s were computed using NIPA's constant price data on both equipment and structures. NIPA equipment figures were used to compute the geometric depreciation rates used for the model that correspond to NIPA's straight–line rates. The latter depreciation rates as constructed by the GNP accountants, are assumed to be correct on average.

APPENDIX C: INDIVISIBLE LABOR

This appendix reports the results for the model when Rogerson–Hansen indivisible labor is used. As is well known, the indivisible labor model can be represented by a version of the standard model where the momentary utility reads $U(c,\ell) = \ln c - \theta \ln \ell$. Recalibrating this model along the lines outlined in Section III resulted in a value for θ of 1.95, the rest of the parameter values remaining the same. The results obtained were:

Variable	**Standard Deviation (%)**	**Cross–Correlation with Output**	**Autocorrelation**
	Indivisible Labor Model ($\phi = 1.41$)		
Output	.75	1.00	.30
Consumption	.27	.68	.66
Investment	5.99	.96	.24
Hours	.59	.95	.24
Utilization	2.48	.82	.25
	Indivisible Labor Model ($\phi = 2.42$)		
Output	.60	1.00	.29
Consumption	.25	.89	.49
Investment	4.00	.96	.24
Hours	.39	.96	.24
Utilization	2.26	.89	.25