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The Efficiency and Welfare Effects of Tax Reform: Are Fewer Tax Brackets Better Than More?

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ABSTRACT

Using the well-known dynamic fiscal policy framework pioneered by Auerbach and Kotlikoff, we examine the efficiency and welfare implications of shifting from a linear marginal tax rate structure to a discrete rate structure characterized by two regions of flat tax rates of 15 and 28 percent. For a wide range of parameter values, we find that there is no sequence of lump-sum transfers that the (model) government can feasibly implement to make the shift from the linear to the discrete structure Pareto-improving. We conclude that the worldwide trend toward replacing rate structures having many small steps between tax rates with structures characterized by just a few large jumps is not easily accounted for by efficiency arguments. In the process of our analysis, we introduce a simple algorithm for solving dynamic fiscal policy models that include "kinks" in individual budget surfaces due to discrete tax codes. In addition to providing a relatively straightforward way of extending Auerbach-Kotlikoff-type models to this class of problems, our approach has the side benefit of facilitating the interpretation of our results.

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1. Introduction

The 1980s was the decade of tax reform. The American economy alone experienced two major changes in federal personal income-tax legislation, the Economic Recovery Tax Act of 1981 (ERTA) and the Tax Reform Act of 1986 (TRA86). But significant change was not limited to the United States. By 1989, tax legislation had been passed in Australia, Canada, Denmark, New Zealand, Japan, Sweden, and the United Kingdom, with proposals for reform pending in many other nations (see Tanzi [1987], Boskin and McLure [1990], and Whalley [1990b]).

Although actual and proposed tax legislation within each of these countries was multifaceted, sometimes with substantial variance in details, reform proposals shared certain broad characteristics across countries. Most striking among these was the uniform tendency toward lower top marginal tax rates, fewer rate brackets, and "base broadening." For example, in the latest rounds of reform, top statutory marginal rates in the federal personal tax codes fell from 34 to 29 percent in Canada, 83 to 40 percent in the United Kingdom, and 50 to 31 percent in the United States.[1] Corresponding to these changes were reductions in the number of rate brackets from 10 to 3 (Canada), 11 to 2 (U.K.), and 12 to 3 (U.S.). These examples and others are summarized in table 1.

The motivation for these changes was clearly the growing perception that the distortionary effects of high marginal tax rates had resulted in substantial inefficiencies. Consequently, an essential impulse for tax reform was, and is, the desire to create more-efficient income tax systems by substituting base-broadening measures for high marginal tax rates. Reductions in the number of rate brackets are presumably meant to reinforce this goal by simplifying the tax code and minimizing distortions through the creation of

[1] Effective marginal tax rates can differ from statutory rates due to special treatment of credits, deductions, and exemptions at certain threshold income levels. An obvious example is the TRA86 provision for phasing out personal exemptions for high-income taxpayers.

broad classes of income over which marginal tax rates are essentially flat. Although often implicit, this motivation for reducing the number of rate brackets is sometimes explicit in discussions of specific tax reform proposals. For example, in discussing the Takeshita reforms in Japan, Noguchi (1990, page 118) describes the U.K. and U.S. changes in rate structures as "developments ... toward flat-rate income taxes," while Ishi (1989) refers to the rate structure implemented in Japan as a "modified flat-tax" system.

However, a brief glance at figures 1A-1C, which depict various vintages of Canadian, Japanese, and U.S. personal income-tax rate structures, suggests the problematic nature of concluding that a smaller number of rate brackets is less distortionary than a larger number. Although it is true that recent rate structures have wider bands of income over which the marginal tax rate is flat, it is also true that jumps in the marginal rate are much more significant for some taxpayers. It is unclear, *a priori*, which structure will, on net, most significantly distort household consumption and work-effort decisions. Given the almost universal tendency toward reforms of this nature, it is surprising that these issues have not been given more attention.

That, then, is the goal of this paper. Using the well-known dynamic fiscal-policy framework pioneered by Auerbach and Kotlikoff (1987, henceforth AK), we examine the welfare and efficiency implications of shifting from linear to discrete marginal tax-rate structures. In other words, we consider the pure distortionary effects of replacing a tax structure with many (infinitely small) steps between marginal tax rates with one defined by two large bands of flat tax rates connected by a single, large, discrete jump.

We find that our hypothetical two-bracket code, which is roughly patterned after the rate structure in the 1989 U.S. personal income tax code, is *less* efficient than alternative linear tax codes with similar average-tax progressivity and present-value revenue implications. Specifically, following the general procedures outlined in AK, we find that there is no sequence of lump-sum transfers the government could feasibly implement that would make the shift from the linear to the discrete rate structure Pareto-improving. This finding is generally robust to parameter assumptions, to the chosen method for equalizing revenues, and to the degree to which the change is anticipated or unanticipated.

In the process of our analysis, we introduce a simple algorithm for solving AK models with discrete tax codes. The key to our strategy lies in noting that there exists a continuous tax code that replicates the necessary conditions for utility maximization of an individual facing the hypothesized discrete tax structure. Because we consider only compensated income tax systems, this equivalence, along with our standard preference assumptions, implies that the two rate structures will yield the same individual consumption and leisure plans.

In addition to providing a relatively straightforward method of solving the discrete tax problem, our approach has the side benefit of facilitating the interpretation of our results. When individuals facing a discrete jump in the marginal tax rate choose to be at the "kink" in their budget surfaces, they act *as if* they are in a marginal tax bracket that is higher than the actual statutory bracket. The government, however, collects revenue only at the lower statutory rate. This discrepancy reduces the efficiency of the discrete rate structure. In the pure life-cycle framework that we consider, the inefficiencies associated with this sort of bunching weigh most heavily during relatively productive periods of a taxpayer's life. Hence, for the cases we examine, these inefficiencies typically outweigh the gains of flattening the rate structure over most income ranges.

2. The Simulation Model

A. Households and Preferences

Our model economy is populated by a sequence of distinct cohorts that are, with the exception of size, identical in every respect. Each generation lives, with perfect certainty, for 55 periods (interpreted as adult years) and is $1+n$ times larger than its predecessor.

Individuals "born" at calendar date b choose perfect-foresight consumption (c) and leisure (l) paths to maximize a time-separable utility function of the form

$$U_b = \sum_{t=1}^{55} \beta^{t-1} u(c_{t,b+t-1}, l_{t,b+t-1}), \tag{1}$$

where $u_i > 0$, $u_{ii} < 0$, $\lim_{i\to\infty} u_i = 0$, $\lim_{i\to 0} u_i = \infty$, and u_i is the partial derivative of the function $u(\cdot)$ with respect to argument i. The preference parameter β is the individual's subjective time-discount factor. We assume that β>0, but do not strictly require β<1.

Letting $a_{t,s}$ equal the sum of capital and government debt holdings for age t individuals at time $s = b+t-1$, maximization of equation (1) is subject to a sequence of budget constraints given, at each time s, by

$$a_{t,s} = (1+r_s)a_{t-1,s-1} + \varepsilon_t w_s (1-l_{t,s}) + v'_{t,s} - T(y^*_{t,s}) - z_{t,s} - c_{t,s}, \tag{2}$$

where w_s is the real pre-tax market wage at time s, r_s is the real return to assets held from time s-1 to s,[2] ε_t is an exogenous labor-efficiency endowment in the tth period of life, and $v'_{t,s}$ ($z_{t,s}$) refers to lump-sum transfers (taxes) received (paid) by age t individuals at time s.

The function $T(y^*_{t,s})$ defines the amount of income tax paid, which depends on the tax base given by $y^*_{t,s} = r_s a_{t-1,s-1} + \varepsilon_t w_s (1-l_{t,s}) - d$. The constant d represents a fixed level of deductions and exemptions used to convert gross income to taxable income. In the linear tax case, the function $T(\cdot)$ is defined as

$$T^{Linear}_{t,s} = \int_{y=d}^{y^*_{t,s}} \tau(y)dy, \tag{3}$$

where $\tau(y)$ defines the marginal tax rate as a linear function of taxable income. In the discrete tax case, the function is defined as

$$T^{Discrete}_{t,s} = \begin{cases} \tau^L y^*_{t,s} & \text{if } y^*_t \le \bar{y}, \\ \tau^L \bar{y} + \tau^H (y^*_{t,s} - \bar{y}) & \text{if } y^*_t > \bar{y}. \end{cases} \tag{4}$$

2 Capital and government debt are assumed to be perfect substitutes in households' portfolios.

Note that at any time s, there are three distinct possibilities with respect to the budget constraint in the discrete tax case, corresponding to the cases where $y^*_{t,s} < \tilde{y}$, $y^*_{t,s} > \tilde{y}$, and $y^*_{t,s} = \tilde{y}$. The latter applies when individuals are at the kink in the budget constraint.

In addition to equation (2), we impose the initial condition that all individuals are born with zero wealth, and the terminal condition that the present value of lifetime resources cannot exceed the present value of lifetime consumption plus tax payments. In the absence of a bequest motive and lifetime uncertainty, this wealth constraint implies that $a_{55,s} = 0$.

B. The Government

The government in our model raises revenues through a combination of distortionary income taxes, debt issues, and lump-sum taxes. Government purchases of output equal zero at all times, and all government revenues are eventually redistributed to households in the form of lump-sum transfers. We specifically require that revenues raised from the income tax be rebated in the form of lump-sum payments to the individuals from whom they are collected.

Initially, we assume that no lump-sum transfers or taxes exist, except those necessary to compensate for income taxation, and that D_0, the amount of government debt at the beginning of time, is zero. Thus, $z_{t,s} = 0$ and $v'_{t,s}$ equals the amount of income tax revenue collected for an age t individual at time s. These assumptions, which we relax to calculate efficiency measures in section 6, imply that debt issues are zero for all s.

C. Firms and Technology

Output in the model is produced by competitive firms that combine capital (K) and labor (L) using a neoclassical, constant-returns-to-scale production technology. Aggregate capital and labor supplies (in per capita terms) are obtained from individual supplies as

$$K_s = \sum_{t=1}^{55} \frac{a_{t,s-1}}{(1+n)^{t-54}} - \frac{D_{s-1}}{1+n} \tag{5}$$

and

$$L_s = \sum_{t=1}^{55} \frac{\varepsilon_t (1 - l_{t,s})}{(1+n)^{t-55}}. \tag{6}$$

Note that the capital stock at time s is given by private and public saving decisions at time s-1. Also, recall that we initially assume $D_s = 0$ for all s.

The production function is written in terms of the capital-labor ratio κ as

$$q_s = f(\kappa_s), \tag{7}$$

where q_s is per capita output and $f(\cdot)$ is defined such that $f' > 0, f'' < 0$, $\lim_{\kappa \to \infty} f' = 0$, and $\lim_{\kappa \to 0} f' = \infty$. The competitive wage rate and (gross) interest rate are given by

$$w_s = q_s - \kappa f'(\cdot) \tag{8}$$

and

$$r_s = f'(\cdot) - \delta, \tag{9}$$

where δ is the depreciation rate on physical capital.

3. A Simple Computational Method for Solving the Discrete Tax Problem

We are fundamentally interested in the following question: What are the welfare and efficiency implications of shifting from a linear tax code to one that can be represented by a step function? Our algorithm for solving the linear case is similar to that described in detail in AK (chapter 4), but a brief description here will help to motivate our discussion of the discrete case. For simplicity, we will focus our attention on the steady states. Although computationally more complex, the technique for obtaining solutions for the transition path from one steady state to another is analogous.

A. Solution Procedure for the Linear Tax Code

Given the tax code of equation (3), the following steps are employed to obtain steady-state solutions:

(i) Conjecture values for K and L (and hence for r and w).

(ii) Conjecture a sequence of marginal tax rates, τ_t, for $t = 1$ through 55.

(iii) Let $u_{i,t}$, $i=c,l$ denote the age t marginal utility of consumption and leisure, respectively, and let λ_t denote the LaGrange multiplier associated with the time t budget constraint in equation (2). Given the conjectured net prices, use equation (2) and the first-order conditions

$$u_{c,t} - \lambda_t = 0, \tag{10}$$

$$u_{l,t} - \lambda_t \varepsilon_t w(1-\tau_t) = 0, \quad \text{and} \tag{11}$$

$$-\lambda_{t-1} + \lambda_t [1 + r(1-\tau_t)] = 0 \tag{12}$$

to solve for the optimal consumption and leisure plans for individual members of each generation.

(iv) Apply the implied path of wage and asset income to the tax code and update the path for marginal tax rates.[3]

(v) Repeat steps (iii) and (iv) until the optimal paths of consumption and leisure are consistent with the marginal tax rates they imply.[4]

(vi) Aggregate individual labor and asset supplies to obtain updates for K and L.

(vii) Repeat steps (ii) through (vi) until aggregate labor and asset supplies are consistent with individual consumption and leisure decisions.

Because the utility function given in equation (1) is concave and the budget constraints in equation (2) are convex, the arguments in Stokey and Lucas (1989, chapter 4) will guarantee that these procedures determine the optimal consumption and leisure plans given r, w, and the linear tax code.

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[3] Updates are obtained using the Gauss-Seidel method.

[4] For some ages, individuals may be at a kink where taxable income equals zero. This is the case when an individual who faces a marginal tax rate of zero has taxable income greater than zero and would be in the 15 percent marginal tax bracket. However, if the individual faced a marginal tax rate of 15 percent, the household would have taxable income less than zero. In this situation, the above procedure does not work, necessitating the solution procedure we develop for the discrete tax code.

B. Solution Procedure for the Discrete Tax Code

Now, consider the two-bracket, discrete tax code given by equation (3). The application of steps (ii) through (v) in this case is complicated by the need to ensure that $y_t^* \leq \bar{y}$ when $\tau_t = \tau^L$ and $y_t^* > \bar{y}$ when $\tau_t = \tau^H$. In general, a straightforward application of the algorithm described for the linear case need not converge, because the procedure does not rule out consumption and leisure paths that imply, for some ages, that $y_t^* > \bar{y}$ when $\tau_t = \tau^L$ and $y_t^* < \bar{y}$ when $\tau_t = \tau^H$. That is, when faced with a 15 percent tax rate, the individual will work hard enough to be in the 28 percent marginal tax bracket. However, a person facing a marginal tax rate of 28 percent would work only hard enough to be in the 15 percent tax bracket. Such paths, of course, are not feasible.

More formally, the discrete tax case differs from the linear case due to the necessary addition of the constraints

$$(\tau_t - \tau^H)(y_t^* - \bar{y}) \geq 0. \tag{13}$$

If $y_t^* < \bar{y}$, so that $\tau_t = \tau^L$, or $y_t^* > \bar{y}$, so that $\tau_t = \tau^H$, then the constraint in equation (13) is not binding at time t. Thus, the first-order conditions (10)-(12) remain valid when $y_t^* \neq \bar{y}$. When $y_t^* = \bar{y}$, equations (11) and (12) become

$$u_{l,t} - \lambda_t \varepsilon_t w(1-\tau^L) + \mu_t(\tau^L - \tau^H)\varepsilon_t w = 0 \tag{11'}$$

and

$$-\lambda_{t-1} + \lambda_t \beta[1 + r(1-\tau_t)] - r\beta\mu_t(\tau^L - \tau^H) = 0, \tag{12'}$$

where μ_t is the LaGrange multiplier associated with the constraint in equation (13).

Fortunately, the algorithm described for the linear tax code can be simply amended to incorporate the changes implied by equations (11′) and (12′). It is straightforward to verify that there is some tax rate given by

$$\tilde{\tau} = \tau^L + \frac{\mu_t(\tau^H - \tau^L)}{\lambda_t}, \quad \tau^L \leq \tilde{\tau} \leq \tau^H \tag{14}$$

that also satisfies necessary conditions (10), (13), (11′), and (12′). This equivalence suggests a simple modification of the algorithm described above in steps (ii) through (v): First, replace the discrete structure in equation (3) with a hypothetical structure that allows a continuum of marginal tax rates between τ^L and τ^H. Second, replace step (v) above with

(v′) Repeat steps (iii) and (iv) until, for each t, (a) $\tau_t = \tau^L$ and $y_t^* < \bar{y}$, (b) $\tau_t = \tau^H$ and $y_t^* > \bar{y}$, or (c) $\tau_t = \tilde{\tau}_t$ and $y_t^* = \bar{y}$.

It remains only to verify that the sequence of consumption and leisure choices obtained from this procedure does in fact maximize utility. Because the utility function is concave and the budget set is convex, to prove sufficiency we must prove that the implied value function is continuously differentiable. We sketch the general proof in appendix 1.

To illustrate the nature of the individual choice problem under the discrete code, we devise a simple two-period model with given net-of-tax prices and preferences defined by

$$U(c,l) = (lnc_1 + lnc_2) + (lnl_1 + lnl_2). \tag{15}$$

We also assume $\tau^L = 0.15$ and $\tau^H = 0.28$, first- and second-period effective wages equal to 25 and 27, a real interest rate equal to 0.03, and $\bar{y} = 10$. In figure 2, we plot the values of $\tilde{\tau}_2$ implied by the optimal choices of consumption and leisure given various (exogenous) values of initial assets a_0. For this example, high values of a_0 result in consumption and leisure choices such that $\tau_1 = \tau_2 = \tau^L$ and first- and second-period income is less than $\bar{y}$. Conversely, very low values of a_0 are associated with choices that yield income greater than $\bar{y}$ in both periods, and hence $\tau_1 = \tau_2 = \tau^H$.

For a wide range of initial asset values, equilibrium outcomes for the consumer are such that utility is maximized at kinks in the budget surface. In these cases, individuals make consumption and leisure choices as if they face the effective tax rate $\tilde{\tau}_2 = \tau^L + \frac{\mu_2(\tau^H - \tau^L)}{\lambda_2} > \tau^L$. For example, a person born with initial assets of approximately seven acts as if he faces a marginal tax rate of 20 percent, although his statutory tax rate is 15 percent. A 20 percent statutory rate would, by construction, induce the individual to choose his taxable income to equal $\tilde{y}$.

It is this wedge between the marginal tax rate applied by the fiscal authority and the effective rate on which private decisions are made that suggests a potential inefficiency in the discrete tax code that does not exist in the linear case: For individuals at tax-induced kinks in their budget constraints, distortions arise from the effective rate $\tilde{\tau}_t$, while revenues are based on the lower rate τ^L. In the example depicted by figure 2, the discrepancy between τ_2' and τ^L rises rapidly as the level of initial assets falls (and hence the endogenous level of income rises).

Further insight is obtained by defining the transformed multiplier $\mu_t' = \mu_t(\tau^H - \tau^L)$, which has the usual interpretation as the utility price of constraining income to $\tilde{y}$. Thus, by rearranging equation (14), we see that $\tilde{\tau}_t = \tau^H$ when $\lambda_t(\tau^H - \tau^L) = \mu_t'$; that is, when the utility loss (in terms of consumption) from being in the higher tax bracket just equals the utility loss from constraining income to $\tilde{y}$.

4. Model Calibration

A. Technology

The simulation exercises reported in section 5 assume an aggregate production technology given by

$$q_s = Ak_s^{\theta}, \tag{16}$$

where θ is capital's share in production and A is an arbitrary scale factor. Our benchmark value for θ is 0.36, following Kydland and Prescott (1982). The value of A is chosen to scale steady-state cohort incomes to values consistent with average household income in 1989, the year for which the tax code is calibrated. We discuss this choice in more detail below.

In the benchmark model, we assume that the depreciation rate of physical capital is 10 percent per period, a choice that, again, is motivated by the arguments in Kydland and

Prescott. The population growth rate is set to the postwar U.S. average of 1.3 percent per year, and the life-cycle labor efficiency profile $\{\varepsilon_t\}_{t=1}^{55}$ is calculated by interpolating estimates in Hansen (1986). A description of this profile is given in appendix 2.

B. Preferences

We assume that preferences are isoelastic, specializing equation (1) to

$$U_b = \sum_{t=1}^{55} \beta^{t-1} \left(\frac{c_{t,b+t-1}^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} + \alpha \frac{l_{t,b+t-1}^{1-\frac{1}{\sigma_l}}}{1-\frac{1}{\sigma_l}} \right), \tag{17}$$

where the preference parameters σ_c, σ_l, and α represent the intertemporal elasticities of substitution in consumption and leisure and the utility weight of leisure, respectively. In our benchmark model, we assume $\sigma_c = 1$, so that equation (17) becomes

$$U_b = \sum_{t=1}^{55} \beta^{t-1} \left(ln(c_{t,b+t-1}) + \alpha \frac{l_{t,b+t-1}^{1-\frac{1}{\sigma_l}}}{1-\frac{1}{\sigma_l}} \right). \tag{17'}$$

This form has the special property, not generally exhibited by specification (17), that the capital-labor ratio is invariant to the scale factor A in equation (16).[5] Also, evidence from state-level data reported by Beaudry and van Wincoop (1992) suggests preferences that are logarithmic in consumption.[6]

MaCurdy's (1981) study of men's labor supply suggests σ_l values in the range of 0.1 to 0.45, a result that is largely confirmed in related studies (see Pencavel [1986]). However, Rogerson and Rupert (1991) argue that, because of corner conditions, estimates of the degree of intertemporal substitution obtained from conventional analyses

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[5] Scale invariance follows from the fact that changes in the level of wages have offsetting wealth and substitution effects on individual labor supply decisions. This property is also used to justify incorporating preferences similar in form to equation (17') in real business-cycle models with exogenous rates of labor-augmenting technical progress (see King, Plosser, and Rebelo [1988]).

[6] Beaudry and van Wincoop also claim (footnote 10) that they found no evidence supporting either non-separabilities between consumption and leisure or the absence of time-separability in consumption, results that generally support the specification in equation (17). However, their maintained model does include "rule-of-thumb" consumers, or individuals who do not behave according to the pure life-cycle/permanent-income hypothesis that we assume.

of male labor supply are likely to be understated. Furthermore, despite greater disparity in estimates obtained from studies of female labor supply, there is broad agreement that the elasticity is higher for women (see Killingsworth and Heckman [1986]). Based on this evidence, in our benchmark model we set $\sigma_l = 0.25$ and choose the parameter α so that steady-state hours worked by an individual at peak productivity is slightly greater than one-third of total time endowment, which we take to be 16 hours per day.

Most empirical studies find values for the subjective discount factor β in the neighborhood of 1.0, sometimes slightly lower (Hansen and Singleton [1982]), sometimes slightly higher (Eichenbaum and Hansen [1990]). We choose a benchmark value of 0.99. Together with the other parameter choices, this value results in a steady-state real pre-tax interest rate of about 3.7 percent (which corresponds closely to the [apparent] historical average of real pre-tax returns on long-maturity riskless bonds in the United States[7]) and a steady-state capital output ratio of 2.63 (which corresponds closely to the ratio of total capital to GDP in the United States over the 1959-1990 period[8]).

C. The Tax Code

The benchmark tax code is patterned after the statutory U.S. personal tax code for 1989. Over the income region that is relevant in our simulations, the 1989 schedule was given by

$$\tau_{t,s}^{Discrete} = \begin{cases} 0.15 & \text{if} \quad y_{t,s}^{*} \leq \$30,950, \\ 0.28 & \text{if} \quad y_{t,s}^{*} > \$30,950. \end{cases} \tag{18}$$

We refer to this tax code as the "tax-reform" case.

The income levels obtained from the model are matched to the tax code as follows: First, we define y^H as the highest income level obtained from an initial calibration

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[7] See Siegel (1992), which reports average rates for the 1800-1990 period. We note, for the record, that average real rates appear to differ significantly across particular subperiods. Specifically, real returns to long-term bonds averaged 1.46 percent over the period 1889-1978, but 5.76 percent outside that interval.

[8] The measure used to construct the U.S. capital stock is the constant-cost net stock of fixed reproducible tangible wealth reported in the January 1992 *Survey of Current Business*. This measure includes consumer durables and government capital.

simulation. This variable is scaled to match the average income level for the cohort aged 45-54 in 1988, which we calculate to be \$44,217 in 1989 dollars.[9] In all subsequent simulations, income levels obtained from the model are converted by taking their ratio relative to y^H and multiplying by \$44,217. To obtain taxable income, we then subtract exemptions and deductions of \$11,206.[10] Given that the intertemporal elasticity of substitution for consumption is assumed to be unity, this is equivalent to scaling the model so that gross income matches the data, and then normalizing by

$$A = \frac{\$44,217}{y^H}.$$

5. The Welfare Effects of Shifting from a Linear to a Discrete Tax Code

In this section, we examine the effects of shifting to the tax-reform code from the linear code under the maintained assumption of revenue neutrality. Holding the structure of the discrete code constant, two natural approaches to achieving this are 1) choosing the intercept of the linear code to equalize revenues, and 2) adjusting deductions to equalize revenues. We focus on the intercept-adjusted approach, a choice motivated by the fact that equalizing revenues in this way yields similar degrees of average-tax progressivity in both the linear-tax and tax-reform steady states.

Thus, we parameterize the function $\tau(y)$ in equation (3) as

$$\tau_{t,s}^{Linear}(y) = \psi + 0.0000024 y_{t,s} \tag{19}$$

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9 The data used in constructing this variable were taken from *Current Population Reports*, series P-60, No. 166. The cohort mean is obtained by multiplying the median income of families with household heads aged 45-54 by the ratio of average to median family income for the entire population. All money values in this paper are quoted in 1989 dollars.

10 This total is obtained by adding personal exemptions of \$5260 to deductions of \$5946. The exemption total is obtained by multiplying the per person exemption of \$2000 specified in the 1989 tax code by 2.63, the average household size in 1989. The deduction level is calculated as a weighted average of the standard deduction and the average level of itemized deductions for taxpayers with adjusted gross incomes between \$0 and \$50,000. Preliminary data from 1989 tax returns, reported in the Spring 1991 issue of the *Statistics of Income Bulletin*, indicate that 19 percent of all returns in the relevant income range included itemized deductions, with an average value of about \$9124. The standard deduction in 1989 was \$5200.

and iterate over the intercept ψ until the present value of income tax revenues generated by the linear code is within 0.001 percent of the present value of revenues generated by the tax-reform transition path and steady state.[11] Throughout this section we will focus on simulations conducted with the benchmark parameterization.

It is useful to first examine the incidence of the income tax in the linear-tax and tax-reform steady states. Figure 3 shows marginal tax rates faced by age cohorts in each tax regime. In the tax-reform case, we plot both the statutory marginal rates and the "effective" tax rate, τ', that determine the choices of cohorts at kinks in their budget constraints.

Approximately 35 percent of the population, accounting for 47 percent of steady-state income, face lower marginal tax rates under the linear system.[12] The rate reductions are concentrated -- and especially pronounced -- at high income levels. The highest marginal tax rate in the linear case is just over 22 percent, as opposed 28 percent in the tax-reform regime.

Table 2 provides information on average tax-rate progressivity. Although no more than an informal summary of the nature of a particular tax code, this measure does provide a sense of how average tax liabilities are related to income, highlighting the sort of comparisons often invoked in discussions of alternative tax regimes. Thus, as claimed above, the results in table 2 do suggest that in the long run, the linear and tax-reform codes we are considering exhibit similar degrees of progressivity, subject to the usual caveats about the validity of the average tax measure.

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[11] Equation (19) was obtained by fitting a regression line to the 1965 statutory tax code. The regression equation is estimated over the income range $0 - $54,000, which covers the incomes generated by the model. Present values are calculated at the interest rates realized under tax reform, that is, along the transition path and in the new steady state. Measuring revenue neutrality under a fixed assumption about interest rates, while not strictly consistent with ex post neutrality, seems consistent with the fashion in which tax legislation is actually contemplated. We choose to use transition-path and final steady-state interest rates, as opposed to initial steady-state interest rates, because the final, tax-reform steady state is the same in all our simulations.

[12] These percentages are higher yet if we include individuals at kinks, who behave as if they face higher-than-statutory rates.

Armed with these observations, we turn next to examining the welfare implications of shifting from the linear-tax regime to the tax-reform regime. Figure 4 illustrates calculations, obtained from the benchmark model and two alternative preference specifications (specifically, two alternative choices for the intertemporal elasticity of substitution in leisure), of welfare gains arising from an unanticipated change in tax regime. Welfare gains are calculated as the percentage increase in full wealth that must be *taken away* from an individual in the tax-reform regime in order to generate the same utility he would have enjoyed if the linear code had stayed in effect. Negative numbers therefore represent welfare losses.

Cohorts are identified in figure 4 by year of death. Thus, the welfare number for period 1 of the transition path represents the gain by an individual age 55 at the time the tax-reform regime becomes effective. All cohorts alive in the initial (linear-tax) steady state have died by period 55 of the transition path.

In the long run, tax reform generates welfare losses, with the magnitude of the loss positively related to the willingness of individuals to shift leisure intertemporally. The intuition for this relationship between welfare costs and σ_l can be appreciated by recalling that, because heterogeneity in the steady state is due strictly to life-cycle characteristics, the highest incomes in the model are earned by individuals who are at their peak levels of labor productivity. As shown in figure 3, this is exactly the period of the life cycle for which tax reform implies higher marginal tax rates relative to the linear regime. The distortions on labor supply created by this fact are magnified for higher degrees of willingness to substitute leisure across periods of life. Thus, an important factor in the relative efficiency of the linear versus discrete tax structure is that for roughly the same degree of progressivity, the marginal tax rate faced by the highest-income individuals need not be as high in the linear case as in the tax-reform case.

The welfare effects apparent in figure 4 arise primarily from the direct distortions of the tax-reform code vis-à-vis the hypothesized initial linear code, not from general equilibrium effects associated with changes in interest rates and wages.[13] In figure 5, we compare the welfare effects for the benchmark model with the effects obtained when the entire path of interest rates and wages is held fixed at the initial steady-state values. Although general equilibrium effects mitigate the welfare losses somewhat, the picture that emerges is little changed by the partial equilibrium assumption, especially in the long run.

Furthermore, losses to cohorts alive at the time of the change in tax structure are not due to the unanticipated nature of the regime change. In figure 6, we plot welfare gains along the transition path for the polar case of a change in the tax code that is completely anticipated. In particular, we assume that the tax code changes at year 55 of the transition path, so that all individuals know the code that will prevail over their life cycle with perfect certainty. For comparability, we designate year 1 as the first period of the tax-regime change for both the anticipated and unanticipated cases. As figure 6 clearly demonstrates, the pattern of welfare gains is essentially the same in each.

Finally, we consider the previously discussed deduction-based method for equalizing the present value of revenues in the two tax regimes. Specifically, we set the intercept ψ in equation (19) equal to 0.146 and iterate over deductions in the initial steady state until, as before, the present value of income tax revenues generated by the linear code is the same as the present value of revenues generated by the tax-reform code.[14] For the benchmark model, this procedure yields deductions of $14,642 in the initial steady state. In this sense, the shift to the tax-reform code, which assumes a deduction level of $11,260, also involves a form of base-broadening.

The welfare calculations for these experiments are shown in figure 7 for the same parameter choices used to construct figure 4. The long-run welfare losses of tax reform

[13] Recall that for the simulations in this section, we assume that lump-sum taxes and transfers maintain zero net tax payments for every cohort at every point in time. Therefore, wealth effects arise only as a result of changes in the aggregate levels of capital and labor, which are in turn reflected in interest rates and wages.

[14] The choice of $\psi = 0.146$ is motivated by the same regressions used to determine the slope of the linear code. See footnote 11.

are somewhat lower when revenues are equalized by adjusting deductions in the linear code than in the intercept-adjusted experiments. However, as reported in table 2, equalizing revenues by deduction adjustments results in greater average-tax progressivity than does the intercept-adjusted linear code or the tax-reform code.[15] Essentially, the increase in marginal rates on high-productivity/high-asset cohorts associated with tax reform is smaller when taxes are equalized by increasing deductions in the linear code, resulting in the smaller long-run welfare losses.

This last observation underscores a critical point that bears reemphasizing. The relative welfare effects of each of the tax structures we consider are dependent on the relative levels of marginal tax rates necessary to preserve revenue neutrality. The discrete code examined here generates welfare losses because a linear code with similar average-tax progressivity (or less progressivity, for that matter) allows the application of lower rates to the critical high-income cohorts.

6. The Efficiency Effects of Shifting from a Linear to a Discrete Tax Code

The pattern of welfare effects in figures 4-7 clearly indicates that the shift from our hypothesized linear-tax regimes to the tax-reform regime is not Pareto-improving. However, the welfare calculations presented do not provide a simple measure that summarizes the economic cost of the change. Furthermore, as shown in figure 8, there are long-run welfare *gains* for some plausible alternatives to the benchmark model. For these cases, the question is open as to whether there exists a set of transfers that preserves some of these long-run gains, while eliminating all welfare losses of cohorts alive along the transition path. In other words, is the shift to the tax-reform regime Pareto-improving for some plausible alternative parameterizations of the model?

---

[15] Note, from table 2, that the marginal tax rate reported for the lowest income cohort is zero. This reflects the fact that, for this cohort, deductions exceed steady-state income. Rather than allow a negative tax, we set the tax rate to zero. This introduces a kink at zero taxable income in the linear tax-code case.

To address these issues, we calculate an efficiency measure in the spirit of the one introduced in Auerbach, Kotlikoff, and Skinner (1983). To obtain this measure, we assume that the government implements a lump-sum transfer scheme that maintains status quo utility levels for all cohorts alive in the initial steady state. These transfers are financed by government borrowing or lending, which is ultimately paid for by lump-sum taxes on, or subsidies to, future generations. The efficiency gain is measured as the constant wealth-equivalent amount of utility that each of these generations realizes when the general equilibrium effects of the government transfer scheme are implemented in the economy.[16]

To this end, we note that when the government sector is extended in this fashion, the per capita level of debt evolves according to the relationship

$$D_s = (1+r_s)\frac{D_{s-1}}{1+n} + V_s - Z_s, \tag{20}$$

where

$$V_s = \sum_{t=1}^{55}(1+n)^{55-t} v_{t,s} \tag{21}$$

and

$$Z_s = \sum_{t=1}^{55}(1+n)^{55-t} z_{t,s}. \tag{22}$$

The transfer $v_{t,s}$ in equation (21) (a transfer to an age t individual at time s) differs from $v'_{t,s}$ in equation (2) by an amount equal to the distribution of lump-sum transfers that compensate for revenues raised from the income tax. Letting s=1 be the first period of the transition path and normalizing the population at s=1 to unity, intertemporal budget balance for the government requires that

$$D_1 + V_1 + \sum_{s=2}^{\infty}\frac{V_s(1+n)^{s-1}}{\prod_{i=2}^{s}(1+r_i)} = Z_1 + \sum_{s=2}^{\infty}\frac{Z_s(1+n)^{s-1}}{\prod_{i=2}^{s}(1+r_i)}. \tag{23}$$

The algorithm for obtaining our efficiency measure proceeds in the following steps.

[16] Auerbach, Kotlikoff, and Skinner refer to the hypothetical government agency that implements these policies as the "Lump Sum Redistribution Authority."

(i) Conjecture a sequence of interest rates for the transition path and the new (tax-reform) steady state.

(ii) Calculate the present value of lump-sum taxes, net of lump-sum transfers, that would be needed to maintain all cohorts at the initial steady-state level of utility. Refer to the resulting number as the "utility-compensation surplus," or UCS. If positive, the UCS determines the present value of transfers that can redistributed by the government while maintaining long-run budget balance. If negative, the UCS determines the present value of taxes that must be raised to maintain budget balance.

(iii) Maintain the utility level of all cohorts alive at the time of the tax regime change, so that the government budget balance is satisfied by solving for the constant tax or transfer, as a percentage of each cohort's full wealth, that can be applied to all subsequent cohorts while just exhausting the UCS.[17]

(iv) Use the path of taxes and transfers from steps (ii) and (iii), along with the associated path of government debt implied by equation (20), to recalculate the entire problem, as described in section 3.

(v) Update interest rates and the UCS until the procedures converge to an equilibrium that satisfies public and private budget constraints, all market-clearing conditions, and the first-order conditions governing individual consumption and leisure choices. Once the problem has converged, the efficiency gain is the percentage of full wealth that is redistributed to (or taken from) all cohorts born after the change in tax regime, as calculated in step (iii).

---

[17] Full wealth, Ω, is defined as the present value of wage income when the entire time endowment is allocated to labor. Thus,

$$\Omega = \varepsilon_1 w_b + \sum_{t=2}^{55} \frac{\varepsilon_t w_{b+t-1}}{\prod_{i=2}^{t} (1 + r_{b+i-1})}.$$

The efficiency gains due to a shift from the linear-tax structure to the tax-reform structure are reported in table 3 for alternative parameterizations of the model. Losses are associated with all the cases considered, even those in which there is a long-run welfare gain from shifting to tax reform, as in figure 8. Thus, the short-run welfare losses that occur in figure 8 dominate the long-run welfare gains.

When revenues are equalized by adjusting the intercept of the linear code in the benchmark model, the shift to the tax-reform code results in an efficiency loss of 0.23 percent of full wealth. More generally, calculated losses range from 0.12 percent to 0.35 percent, depending on the chosen parameters. When revenues are equalized by adjusting deductions, the efficiency losses are uniformly smaller, but still range from 0.05 percent to 0.17 percent of full wealth. As shown, losses increase with individuals' willingness to shift resources intertemporally, again reflecting the fact that high-tax periods correspond to periods of high relative saving rates and high labor productivity.

Again, the efficiency losses represent the percentage increases in full lifetime wealth that would be needed to compensate every cohort born after the regime change, given that those born before the tax code change have already received lump-sum transfers (taxes) and are thus indifferent between the two regimes. As a point of comparison with similar exercises, Auerbach and Kotlikoff (1987, chapter 5) report efficiency losses associated with switching from a 15 percent income tax to an equal-revenue wage tax that fall in a range from approximately zero to 0.7 percent.[18] To put some perspective on these magnitudes, the full wealth of each cohort in the tax-reform steady state is about 63 percent of total output. Thus, a reduction in full wealth of 0.23 percent represents an annual loss equal to about 0.14 percent of output in the model. Converting full wealth in

[18] Auerbach and Kotlikoff's calculations use the initial, rather than final, steady state as the basis for comparison. Furthermore, our numbers are not strictly comparable to theirs due to differences in parameterization. However, we feel these differences are small enough to make comparisons of the results informative.

the model to 1989 dollars implies an efficiency loss equivalent to roughly $2,330 per person born (or reaching working age) after the regime change.

7. Concluding Remarks

Significant reductions in the number of marginal tax-rate brackets -- that is, a trend toward structuring systems of personal income taxation such that there exists wide bands of income over which marginal tax rates are flat -- has been a striking characteristic of worldwide tax reform over the past decade. In this paper, we have argued that this trend cannot be easily accounted for by appealing to the efficiency gains inherent in tax codes with just a few brackets separated by discrete rate jumps. Relative to revenue-neutral linear tax codes, changing to a simple two-bracket discrete rate structure creates efficiency losses in all the numerical experiments we conduct. Furthermore, in most cases welfare gains are negative, even in the long run.

Two explanations come immediately to mind for the discrepancy between the reality of recent tax reforms and the message of our analysis. First, our analysis is conducted in a purely life-cycle framework. Hence, in steady-state equilibria, all cohorts face exactly the same life-cycle profile of relatively high taxes during periods of peak productivity and saving. The inefficiency of the discrete code we consider follows in important ways from the fact that, holding average-tax progressivity constant, shifting from an equal-revenue linear code requires marginal tax-rate increases during this phase of the life cycle. This result is in turn related to distortions in leisure and consumption decisions at kinks in each cohort's budget constraint that do not increase income tax revenues to the government.

These effects would likely be mitigated in a more general framework that included intracohort heterogeneity. For instance, suppose that there existed two types of agents, "rich folks" and "poor folks." It is conceivable that the two-bracket tax code could be structured so that the shift from the linear tax would result in poor folks facing only the lower rate and rich folks facing only the higher rate over their entire lives. In this event, the discrete tax code would be equivalent to a flat-tax regime, which would almost certainly create welfare and efficiency gains. In a slightly less extreme case, some portion of each cohort would face the life-cycle pattern of rates on which we have focused, while for others, the poor-folk/rich-folk scenario would be relevant. It is an open question, then, as to what effects would dominate.

The second explanation for the widespread adoption of rate-bracket reductions is that, perhaps for administrative or political reasons, they are a necessary concomitant to lowering the level of tax rates and the various base-broadening measures that also characterized tax reform in the 1980s. In this case, the institutional approach advocated by Slemrod (1990) may ultimately be necessary to fully understand the consequences of the income tax systems that have undeniably come to dominate industrialized economies.

FIGURE 1A:
MARGINAL TAX RATES IN CANADA

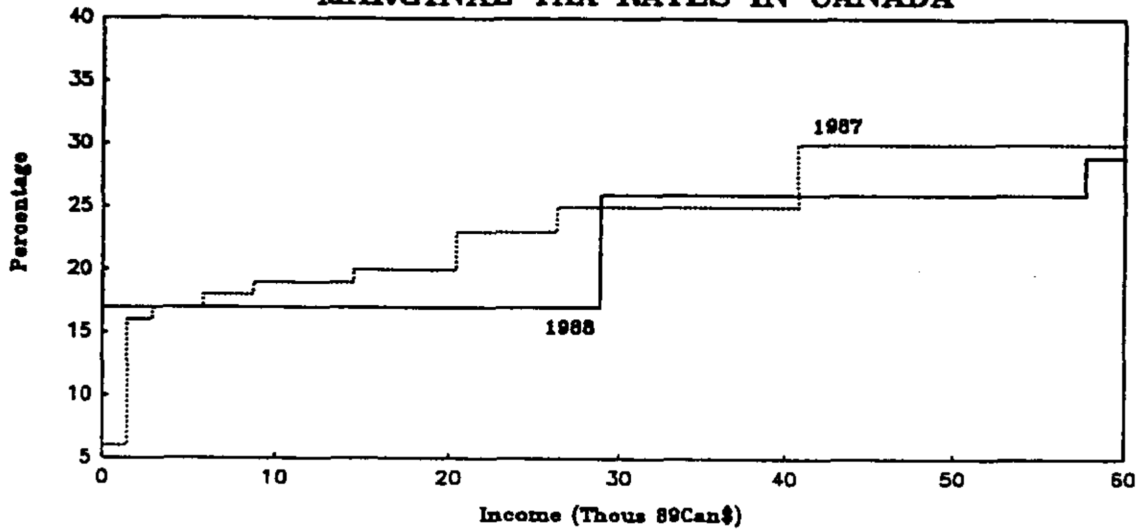


FIGURE 1B:
MARGINAL TAX RATES IN JAPAN

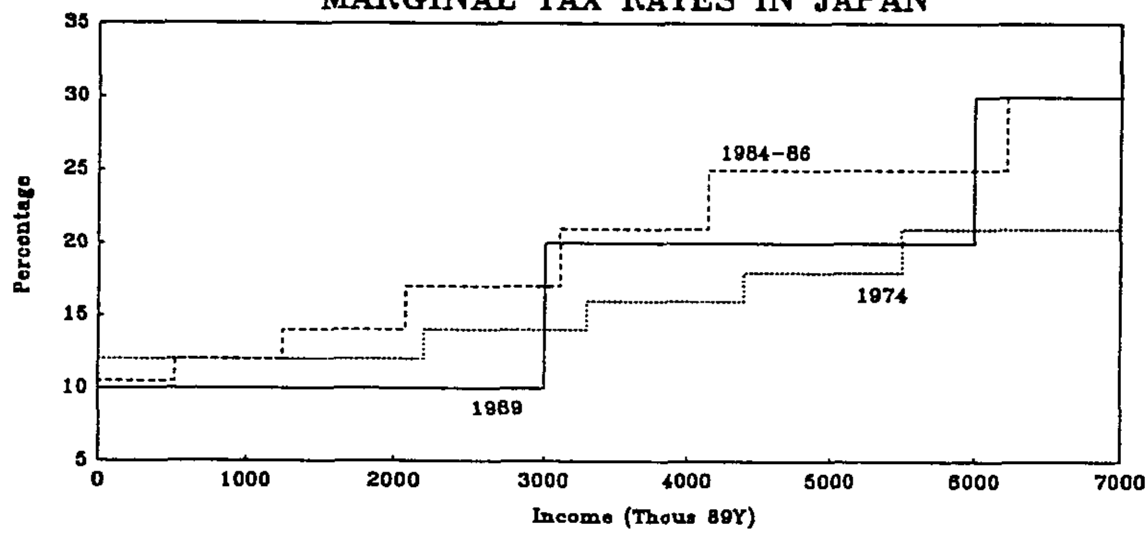
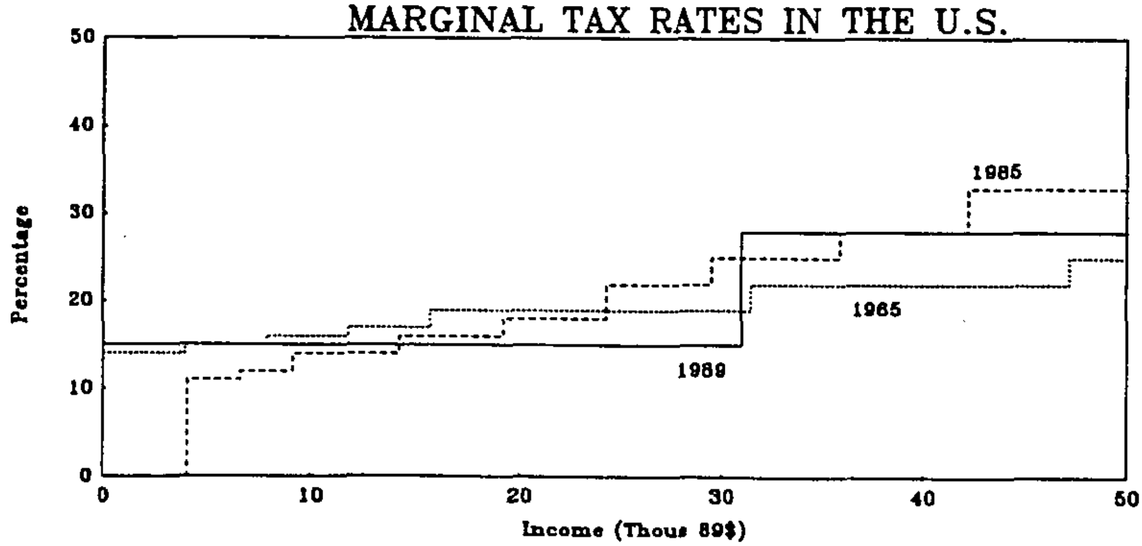
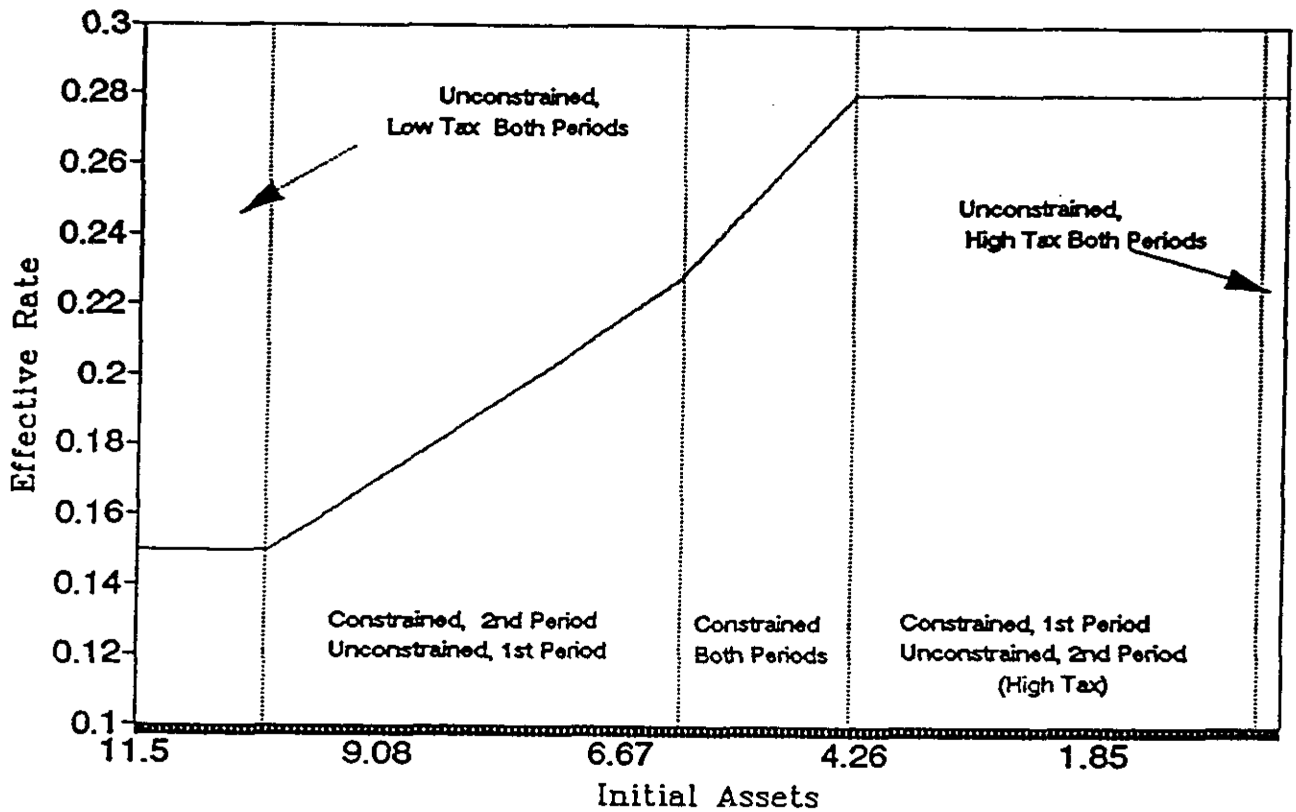


FIGURE 1C:
MARGINAL TAX RATES IN THE U.S.



NOTE: Figures are scaled to a maximum of $50,000 equivalent U.S. dollars.
SOURCES: Whalley (1990b), Ishi (1989), Statistics of Income (1965–89), and the IMF (July 1992).

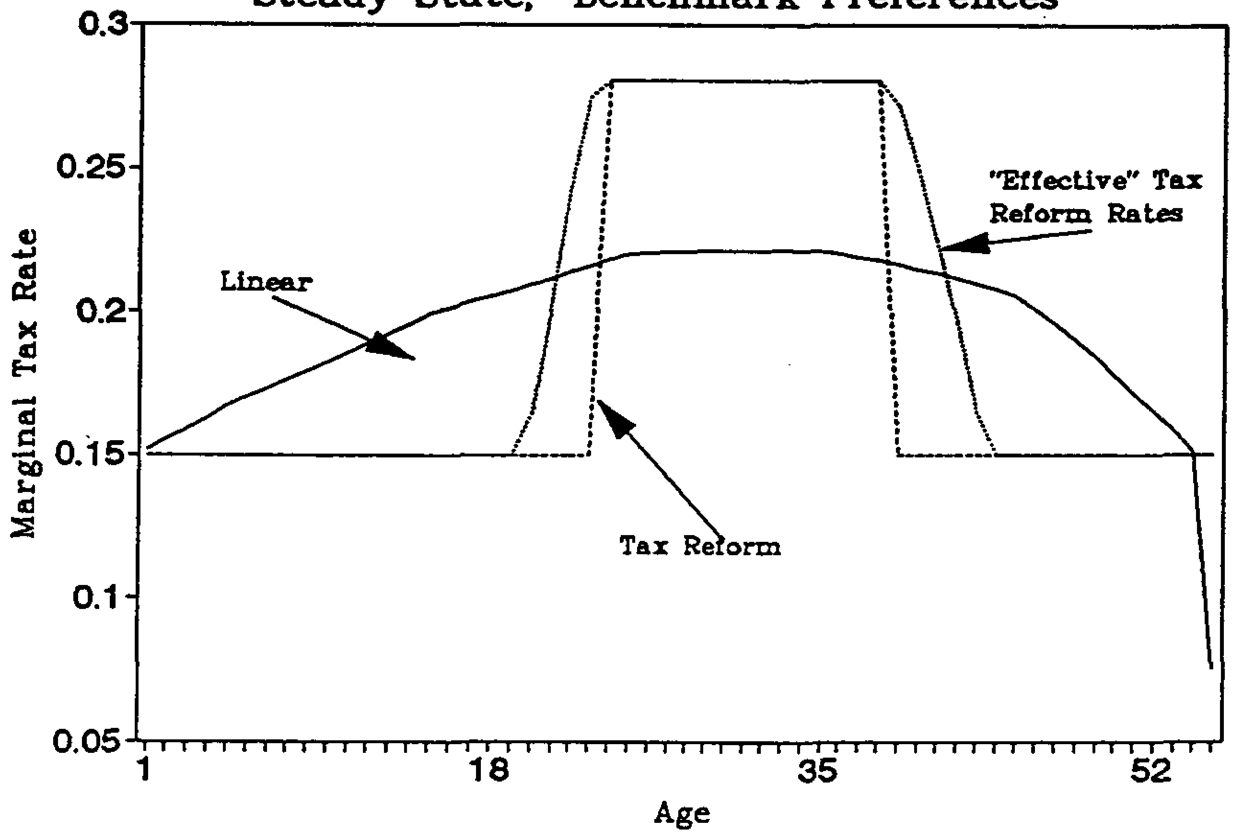
Figure 2: Effective Marginal Rate
2nd Period, Two–Period Model



Source: Authors' calculations.

Figure 3: Marginal Tax Rates

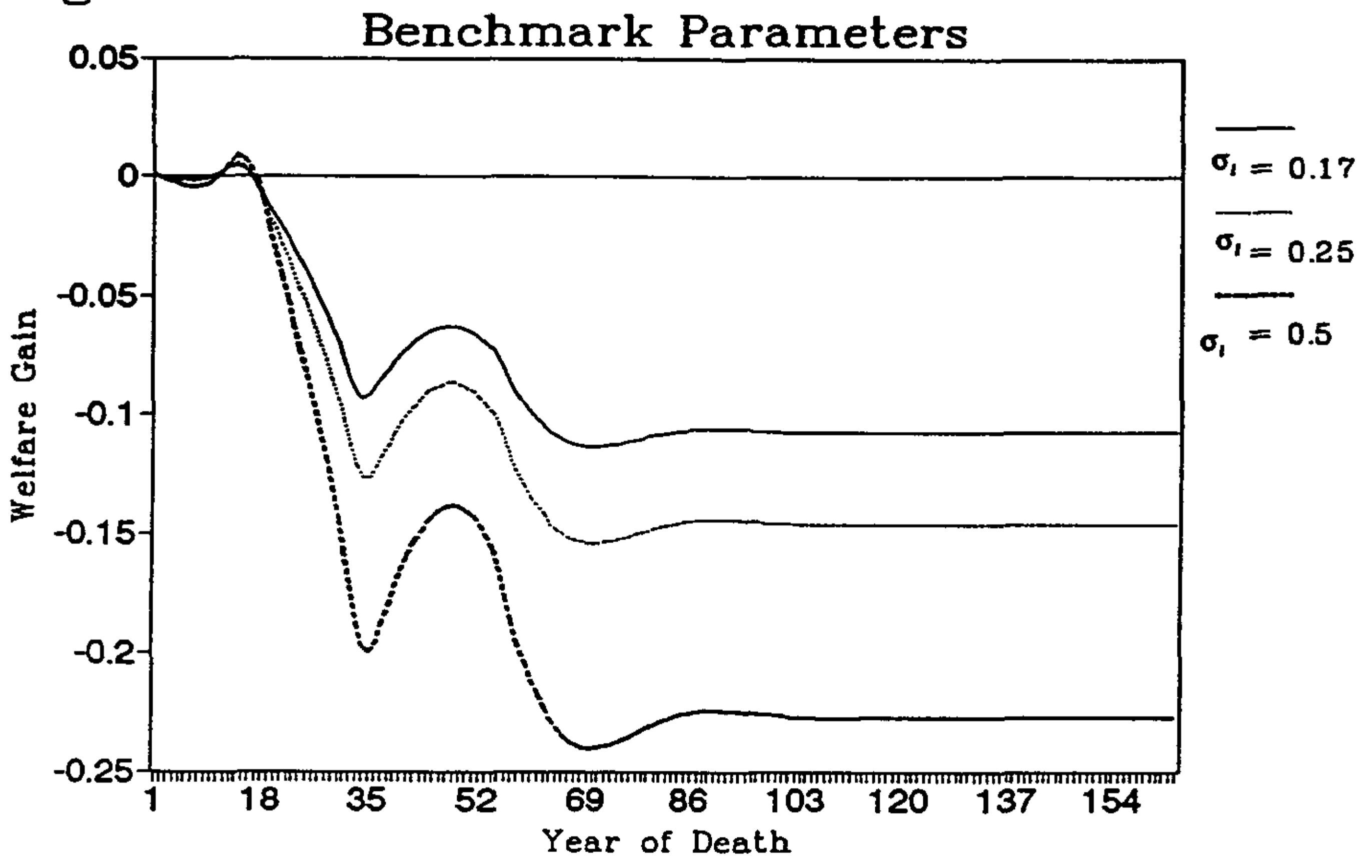
Steady State, Benchmark Preferences



Note: Revenues are equalized by adjusting the intercept of the linear tax code.

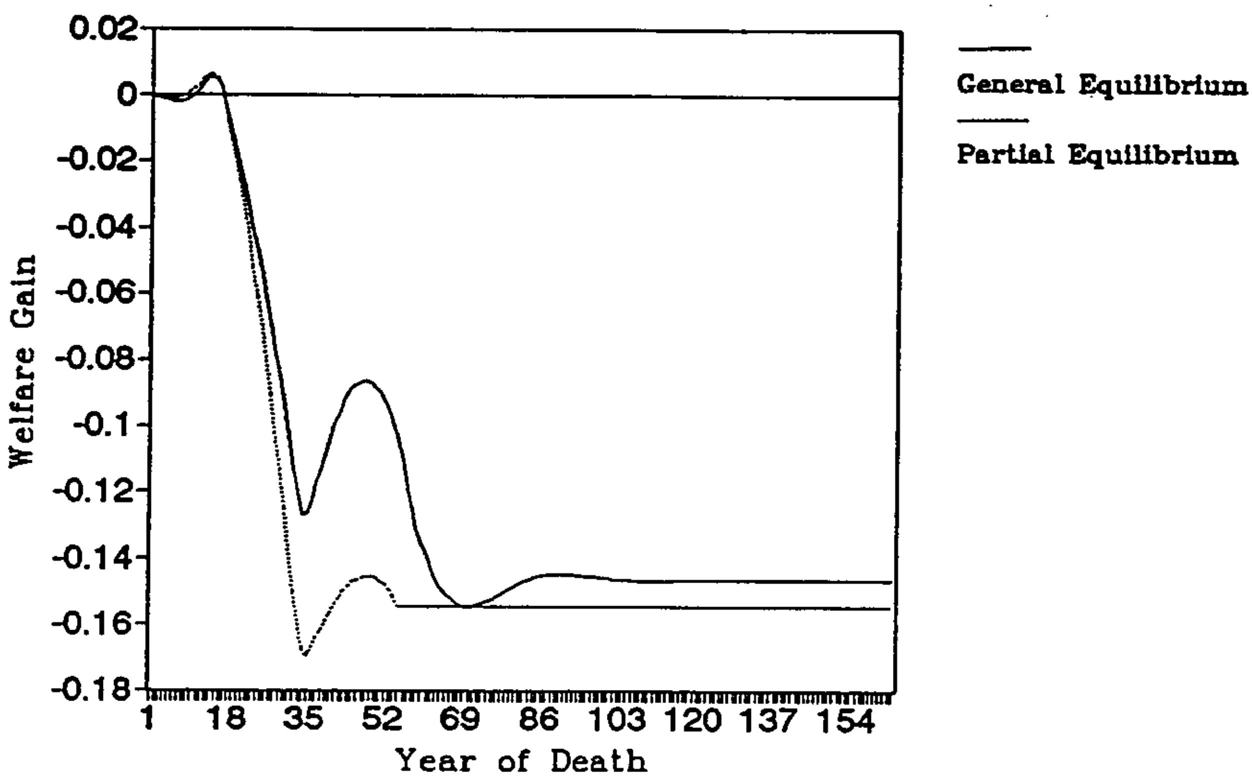
Source: Authors' calculations.

Figure 4:Welfare Gain Due to Tax Reform



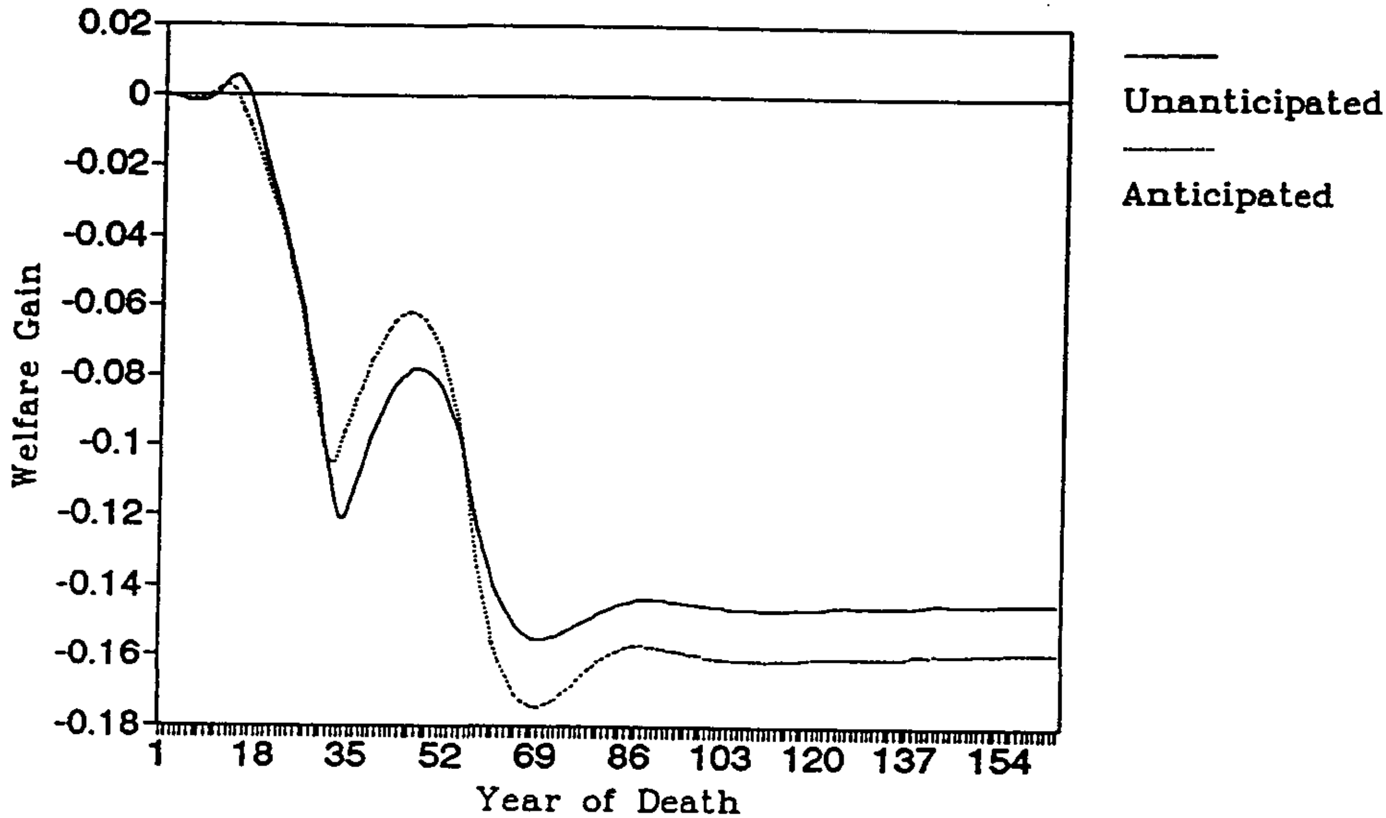
Note: Revenues are equalized by adjusting the intercept of the linear tax code.
Source: Authors' calculations.

Fig. 5: Welfare Gain Due to Tax Reform
Benchmark Parameters



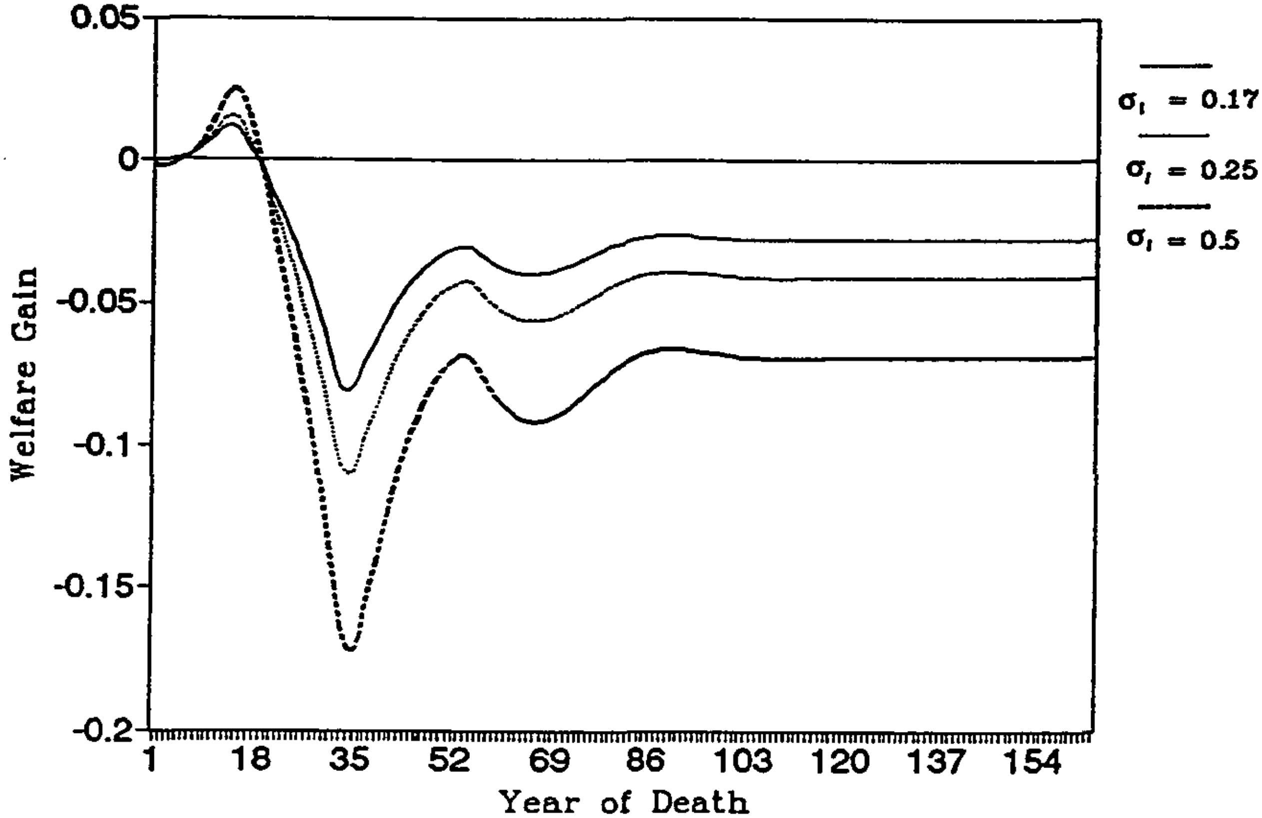
Note: Revenues are equalized by adjusting the intercept in the linear tax code.
Source: Authors' calculations.

Fig. 6: Welfare Gain Due to Tax Reform
Anticipated vs. Unanticipated



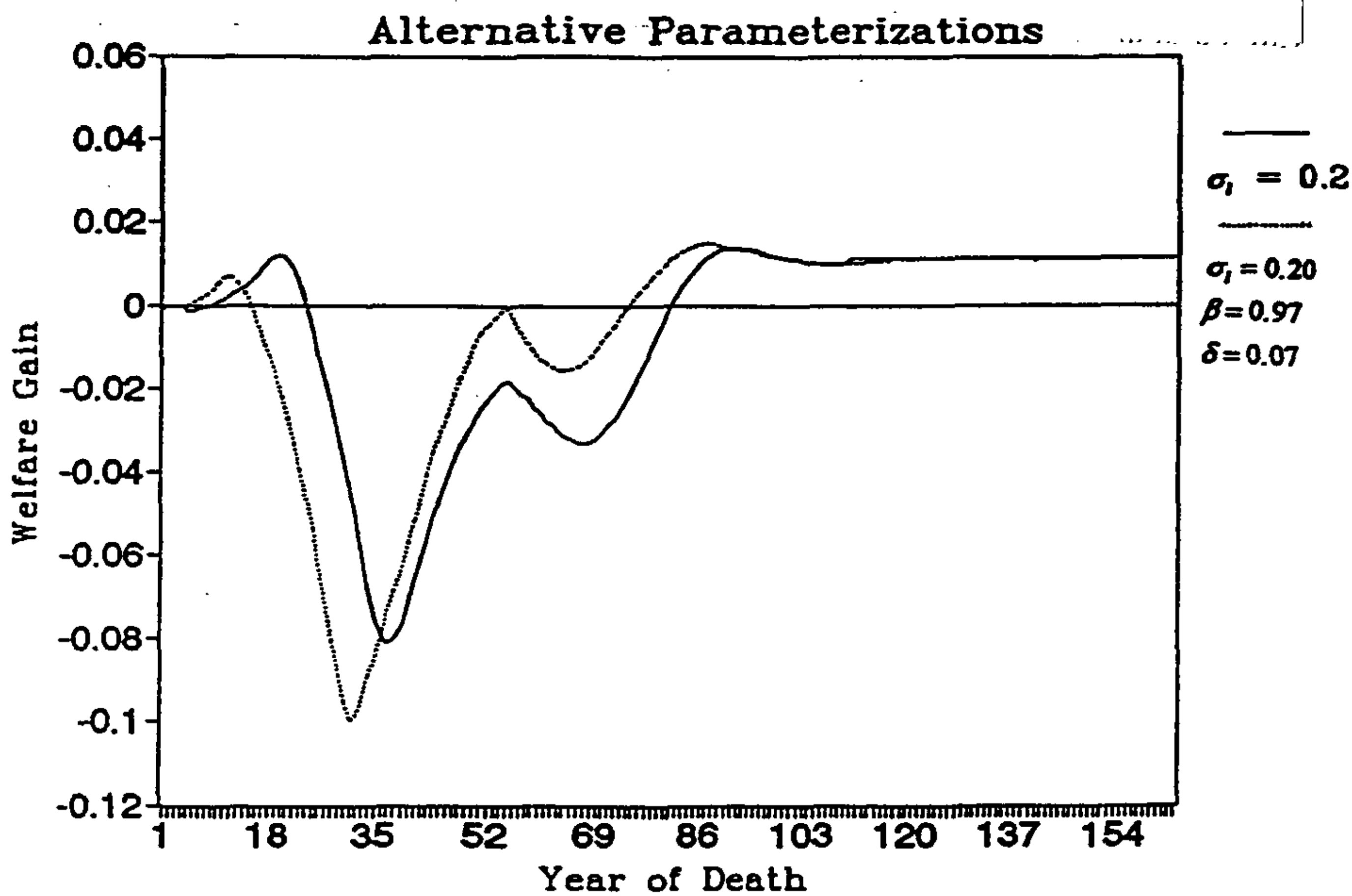
Note: Revenues are equalized by adjusting the intercept in the linear tax code.
Source: Authors' calculations.

Fig. 7: Welfare Gain Due to Tax Reform
Benchmark Parameters



Note: Revenues are equalized by adjusting deductions in the linear tax code.
Source: Authors' calculations.

Fig. 8: Welfare Gains Due to Tax Reform



Note: Revenues are equalized by adjusting deductions in the linear tax code.
Source: Authors' calculations.

Table 1: Specific Elements of World Tax Reform

Country	Top Marginal Tax Rate, Pre-Reform	Year	# of Pre-Reform Brackets	Top Marginal Tax Rate, Post-Reform	Year	# of Post-Reform Brackets
Australia	60%	1980-86	5	49%	1987-88	4
				47%	1992	5
Austria	62%	1982-88*	10**	50%	1989	5
Belgium	72%	1983-88	13**	50%	1989-92	7
Canada	34%	1987*	10	29%	1988-92	3
Italy	65%	1983-87	9	56%	1988	8
				51%	1992	7
Japan	70%	1984-86	15	60%	1987	12
				50%	1988-92	5
Netherlands	72%	1982-86*	9	66%	1987-88	5
				60%	1990-92	4
New Zealand	66%	1979-85	5	48%	1986	3
				33%	1988-92	2
Sweden	80%	1985*	11	72%	1986	4
				50%	1991-92***	4
United Kingdom	83%	1978*	11	60%	1979	6
				40%	1988-92	2
United States	50%	1983-85	15	33%	1986	3
				31%	1992	3

Notes: * Rate may have been in effect prior to earliest date indicated.
** Figures refer to number of rate brackets in 1988.
*** From 0 to SEK 186,600, the national tax is a flat SEK 100. For incomes in excess of SEK 186,600, the tax is SEK 100 plus 20 percent of the excess.

Sources: Platt (1985), Tanzi (1987), Boskin and McLure (1990), Whalley (1990a,b), various issues of the OECD Economic Survey, and the 1992 and 1982 editions of Price Waterhouse's *Individual Taxes: A Worldwide Summary*.

Table 2: Average Tax-Rate Comparisons: Steady-State, Benchmark Parameters

	Low Income	Median Income	High Income
Tax Reform Code	2.3	10.4	11.8
Linear Code, Intercept Adjusted to Equalize Revenues	2.1	9.9	11.9
Linear Code, Deductions Adjusted to Equalize Revenues	0.0	10.0	12.6

Source: Authors' calculations.

Table 3: Efficiency Gains Due to Tax Reform

	Revenues equalized by adjusting intercept in the linear code	Revenues equalized by adjusting deductions in the linear code
Benchmark	-0.225	-0.097
$\sigma_l = 0.17$	-0.166	-0.069
$= 0.50$	-0.346	-0.153
$\beta = 1.005$	-0.121	-0.049
$= 0.976$	-0.347	-0.164
$\sigma_c = 0.2$	-0.158	-0.078
$= 0.33$	-0.192	-0.086
$\delta = 0.07$	-0.256	-0.113
$\sigma_l = 0.20$ $\beta = 0.971$ $\delta = 0.07$	-0.362	-0.165

Source: Authors' calculations.

Appendix 1: More on the Computational Method for Solving the Discrete-Tax-Code Problem

Our algorithm for solving individual consumption and leisure paths for the tax code in equation (13) relies on the validity of replacing the discrete structure with an equivalent continuous structure. Because this hypothetical tax code is, by construction, identical to the actual tax code when the conditions $\tau_t = \tau^L$ and $y_t^* < \bar{y}$ or $\tau_t = \tau^H$ and $y_t^* > \bar{y}$ are satisfied, we need only consider the case when the constraint $(\tau_t - \tau^H)(y_t^* - \bar{y}) \geq 0$ is binding. As in the text, we will focus on the steady state, recognizing that transition-path solutions are directly analogous under our perfect certainty assumption.

Let

$$W(a) = u[c(a), l(a)] + \beta V[G(a^*)], \tag{A1}$$

where $G(a)$ denotes the transition equations defined by the budget constraints in equation (2), a^* is the asset choice that solves

$$V(a) \equiv \max_{c,l,a'} \{u(c,l) + \beta V(a')\}, \tag{A2}$$

and a' represents next-period's asset choice.

Because $u(\cdot)$ is concave, $W(a)$ is concave. Furthermore, $W(a)$ is continuously differentiable if its derivative, $W'(a)$, exists and is continuous. If $W'(a)$ is continuous, then $V'(a)$ is continuous by Benveniste and Scheinkman (1979). To demonstrate the continuity of $W'(a)$, we need to consider the points at which $y^* = \bar{y}$. That is, we must show that $W'_{uc}(a^*) = W'_c(a^*)$ (where c indicates the constraint is binding and uc indicates it is not) at the indifference points where $\tilde{\tau} = \tau^L$ and $\tilde{\tau} = \tau^H$.

By definition, $\bar{y} = \varepsilon w[1 - l(a^*)] + ra^*$ when the income constraint binds. Thus, differentiating (A1) and substituting from this constraint and the first-order conditions gives

$$\begin{aligned} W_c'(a^*) &= u_c \frac{dc}{da} + u_l \frac{dl}{da} \\ &= \frac{1+r(1-\tilde{\tau})}{w(1-\tilde{\tau})} u_l. \end{aligned} \tag{A3}$$

For simplicity, we assume that the labor-efficiency variable, ε, is equal to one.

Similarly, by exploiting the first-order conditions for the unconstrained case, we obtain

$$\begin{aligned} W_{uc}'(a^*) &= u_l \left[\frac{1}{w(1-\tau)} + \frac{w(1-\tau)u_{cc}}{u_{ll}} \right] \frac{dc}{da} \\ &= \frac{u_l[1+r(1-\tau)]}{w(1-\tau)} \left[\frac{u_{ll} + w^2(1-\tau)^2 u_{cc}}{u_{ll}[1+r(1-\tau)]} \right] \frac{dc}{da} \\ &= \frac{u_l[1+r(1-\tau)]}{w(1-\tau)}. \end{aligned} \tag{A4}$$

But recall that, by construction, $\tilde{\tau} = \tau + \frac{\mu(\tau^H - \tau^L)}{\lambda}$. Therefore, because $\mu=0$ when the income constraint no longer binds, from equations (A3) and (A4) we have the desired result.

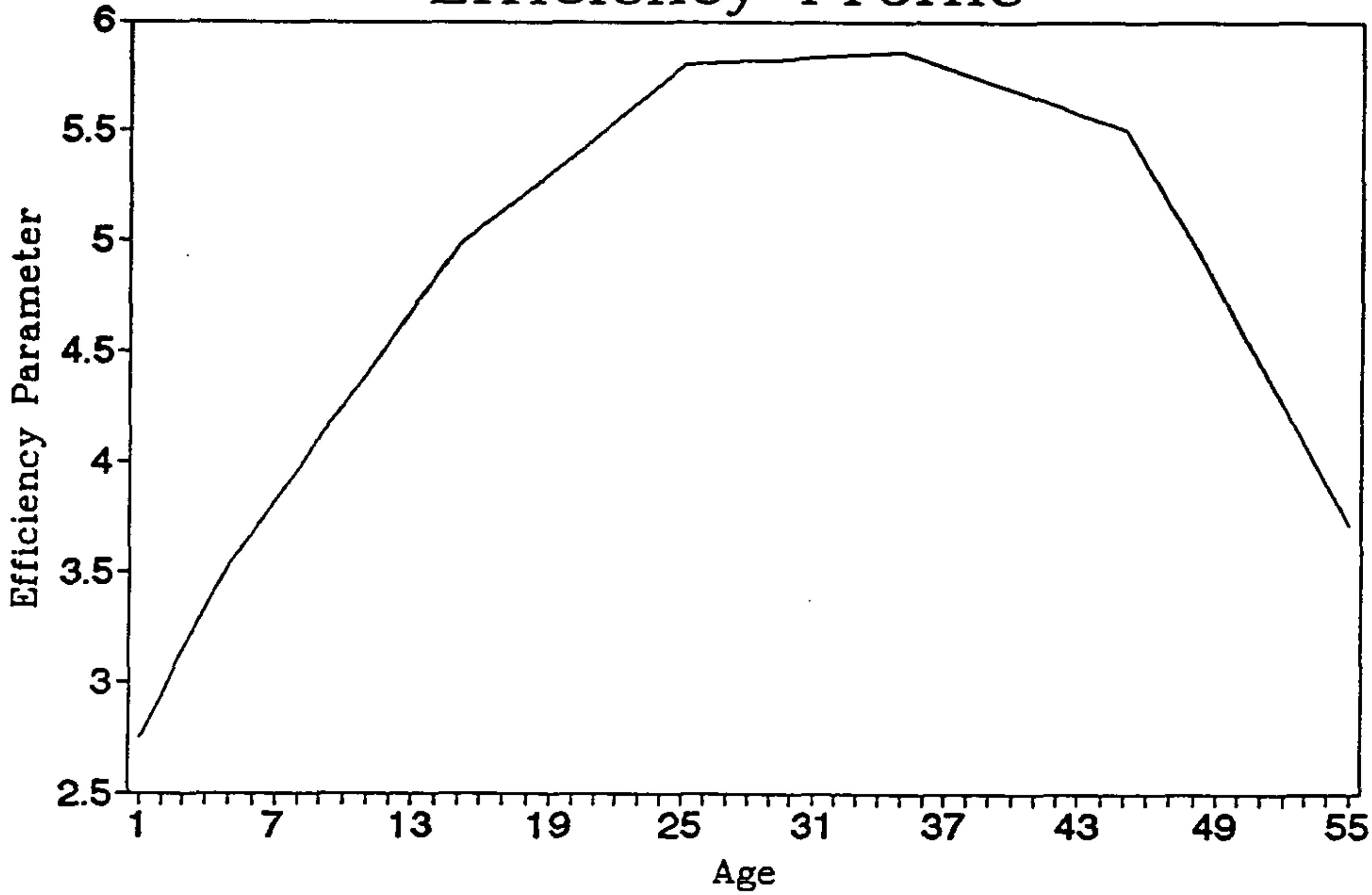
Appendix 2: The Labor Efficiency Profile

The efficiency profile in section 4A is calculated by interpolating the estimates in the data appendix to Part III, "Fluctuations in Total Hours Worked: A Study Using Efficiency Units," in Hansen (1986). The piecewise linear function used in defining this profile is given by

$$\varepsilon_t = \begin{cases} 5.8*(0.44+0.034t) & \textit{for } t=1 \textit{ to } 5 \\ 5.8*(0.485+0.025t) & \textit{for } t=6 \textit{ to } 15 \\ 5.8*(0.65+0.014t) & \textit{for } t=16 \textit{ to } 25 \\ 5.8*(0.975+0.001t) & \textit{for } t=26 \textit{ to } 35 \\ 5.8*(1.22-0.006t) & \textit{for } t=36 \textit{ to } 45 \\ 5.8*(2.345-0.031t) & \textit{for } t=46 \textit{ to } 55. \end{cases}$$

With this function, ε_t peaks at t=35, at which point its value is 113 percent higher than the lowest value, at t=1. From t=35, ε_t declines to the final period of life, t=55. At t=55, ε_t is approximately 37 percent lower than its peak value. The full efficiency profile is shown in figure A2.1.

Figure A2.1: Life-Cycle Labor-Efficiency Profile



Sources: Hansen (1986) and authors' calculations.

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