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The Cyclical Behavior of Job Creation and Job Destruction: A Sectoral Model

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ABSTRACT

Three key features of the employment process in the U.S. economy are that job creation is procyclical, job destruction is countercyclical, and job creation is less volatile than job destruction. These features are also found at the sectoral (goods and services) level. The paper develops, calibrates, and simulates a two sector general equilibrium model including both aggregate and sectoral shocks. The behavior of the model economy mimics the job creation and destruction facts. Sectoral shocks play a significant role in determining the aggregate level of nonemployment.

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1. INTRODUCTION

In U.S. economy job creation is procyclical, job destruction is countercyclical, and job creation is less volatile than job destruction (Davis and Haltiwanger, 1990). Lilien (1982) has also presented evidence suggesting that cyclical unemployment can be explained by variation in sectoral employment opportunities together with frictions impeding the inter-sector movement of workers. The questions raised by these findings are: Can a multisector dynamic general equilibrium model replicate the pattern of job creation, and destruction that is observed in the U.S. data? Are sectoral shocks important for determining the average rate of unemployment?

The analysis seeks to explain movements in labor market aggregates as the outcome of the interaction of aggregate and sectoral shocks. The model developed to do this is a multi-sector dynamic competitive general equilibrium framework. The model has three key features. First, each market sector gets hit by both aggregate and sectoral shocks. This is similar to the classic Long and Plosser (1983) real business cycle model. Second, it takes time to reallocate labor across sectors. Each sector in the market economy can draw new employees from a pool of unemployed workers seeking a job. This pool is made up of agents who entered it in some earlier period, either because they lost their job in a market sector or left the home sector. This feature

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of the analysis requiring a time cost for job reallocation bears some resemblance to the well-known Lucas and Prescott (1974) equilibrium search model. Third, following Hansen (1985) and Rogerson (1988), it is assumed that labor is indivisible. This assumption ensures that the options of working, searching and staying at home are mutually exclusive.

The model developed reproduces the cyclical pattern of job creation, destruction and reallocation displayed in the U.S. data relatively well. Workers flow between sectors as jobs are created and destroyed in response to both aggregate and sectoral-specific shocks. A main conclusion of the paper is that sectoral shocks are a quantitatively important determinant of aggregate nonemployment.[1] The fact that generally some workers are unemployed, but ready to work, allows sectors to increase their employment more rapidly in reaction to favorable circumstances. In response to Lilien's (1982) sectoral-shift hypothesis Murphy and Topel (1987), studying micro-data (from the CPS), argued that only 2.4 to 4.0 percent of unemployment can be attributed to inter-sectoral labor reallocation. In contrast, Loungani and Rogerson (1989) also using disaggregated data (from the PSID) concluded that inter-industry job reallocation accounts for, on average, 25 percent of unemployment. The findings here are more accord with Loungani and Rogerson (1989) than Murphy and Topel (1987).

The rest of the paper sets out the model in detail and explores its features quantitatively.

2. Model

The multisector dynamic general equilibrium model to be simulated will now be developed.

2.1. Economic Environment.

A continuum of ex ante identical agents is distributed uniformly over the unit interval. In period t an agent can work in one of N productive sectors, search for a job, or stay at home. To describe this, let $\pi_{i,t}$ represent the fraction of agents who are working in sector i at t, and $\pi_{N+1,t}$ denote the fraction of agents who are searching; thus, the fraction of the population currently at home is $1-\sum_{i=1}^{N+1}\pi_{i,t}$ while $1-\sum_{i=1}^{N}\pi_{i,t}$ is the proportion not working. A description of tastes, technology and the stochastic structure of the model follows.

Tastes. Let $c_{it}(l_t)$ represent the consumption of the commodity produced by firms in sector i by an agent whose labor market status is l_t; define $c_t = \{c_{it}(l_t)\}_{i=1}^{i=N}$. Next, normalize an agent's non-sleeping time to one, and assume that work and search require r and s hours respectively. Then, leisure is given by $1-l_t$, with an individual's labor market status described by $l_t \in \{0, s, r\}$. An agent's expected lifetime utility is given by

[1] Andolfatto (1993) studies the equilibrium determination of nonemployment within the context of a matching model (that has both aggregate and idiosyncratic shocks).

$$E\left\{\sum_{t=0}^{\infty}\beta^t U(c_t, 1-l_t)\right\}, \text{ with } \beta \in (0,1). \tag{1}$$

The momentary utility function $U(\cdot)$ has the form

$$U(c_t, 1-l_t) = \begin{cases} A\ln\left[\sum_{i=1}^{N}\theta_i c_{it}^{\rho}(l_t)\right]^{\frac{1}{\rho}} + (1-A)\ln(1-l_t) \text{ for } \rho \neq 0 \\ \\ A\ln\left[\prod_{i=1}^{N}(c_{it}(l_t))^{\theta_i}\right] + (1-A)\ln(1-l_t) \text{ for } \rho = 0; \end{cases} \tag{2}$$

where $-\infty < \rho < 1$, $0 < \theta_i < 1$, and $\sum_{i=1}^{N}\theta_i = 1$.

Technology. Sector i is subject to both aggregate, z_t, and sectoral, $\epsilon_{i,t}$, disturbances. The production technology for sector i is assumed to be:

$$y_{i,t} = z_t\epsilon_{i,t}(h_{i,t})^{\alpha_i}, \tag{3}$$

where α_i is the labor share parameter and $h_{i,t}$ is hours spent working.

Reallocating labor across sectors is costly. One of the costs is assimilating new workers into the production process. This cost is increasing in the number of workers that *join* the sector; specifically, it is assumed that $h_{i,t} = r[\pi_{i,t} - \gamma_i(\max[\pi_{i,t} - \pi_{i,t-1}, 0])^2]$. This is equivalent to saying that when new workers are hired in a period they are less productive than experienced workers. Thus, production is governed by

$$y_{i,t} = z_t\epsilon_{i,t}r^{\alpha_i}[\pi_{i,t} - \gamma_i(\max[\pi_{i,t} - \pi_{i,t-1}, 0])^2]^{\alpha_i}. \tag{4}$$

Also, it is assumed any agent moving across sectors must spend one period in transition.

Stochastic Structure. The aggregate and sectoral disturbances are independent of one another and follow finite-state first-order Markov processes with supports $Z = \{z_1, z_2, ..., z_m\}$ and $\epsilon^{\mathbf{i}} = \{\epsilon_1^i, ..., \epsilon_n^i\}$, respectively.

2.2. Planner's Problem. The planner's dynamic programming problem is shown below.

$$\begin{aligned} V(\pi; z, \epsilon) = & \max_{\{c_i(l)\},\{\pi_i'\}} \left\{ \left(1 - \sum_{i=1}^{N+1} \pi_i'\right) \left[\frac{A}{\rho} \ln \left(\sum_{i=1}^{N} \theta_i \cdot c_i^{\rho}(0)\right)\right] \right. \\ & + \pi_{N+1}' \left[\frac{A}{\rho} \ln \left(\sum_{i=1}^{N} \theta_i \cdot c_i^{\rho}(s)\right) + (1-A) \cdot \ln(1-s)\right] \\ & + \left(\sum_{i=1}^{N} \pi_i'\right) \left[\frac{A}{\rho} \ln \left(\sum_{i=1}^{N} \theta_i \cdot c_i^{\rho}(r)\right) + (1-A) \cdot \ln(1-r)\right] \\ & \left. + \beta E\left[V(\pi'; z', \epsilon') \mid \pi; z, \epsilon\right] \right\} \end{aligned} \tag{P1}$$

subject to

$$\sum_{i=1}^{N+1} \pi_i' \leq 1, \tag{5}$$

$$\sum_{i=1}^{N} \max\{0, \pi_i' - \pi_i\} \leq \pi_{N+1}, \tag{6}$$

and

$$\begin{aligned} & (1 - \sum_{i=1}^{N+1} \pi_i')c_j(0) + (\sum_{i=1}^{N} \pi_i')c_j(r) + \pi_{N+1}' c_j(s) \\ & = z\epsilon_j r^{\alpha_j} \left[\pi_j' - \gamma_j \left(\max\left[\pi_j' - \pi_j, 0\right]\right)^2\right]^{\alpha_j} - I_j(z, \epsilon_j), \end{aligned} \tag{7}$$

for $j = 1, ..., N$. The first constraint limits the amount of labor that can be hired. Equation (6) states that any agent who moves across sectors must first spend one period in transition. Finally, the resource constraint is given by (7). Here the function $I_j(z, \epsilon_j)$ specifies the amount of sector j's output that is used for non-consumption purposes. The planner takes $I_j(z, \epsilon_j)$ as exogenous. This function is discussed in more detail below.

Given the separability of preferences, the planner will select consumption paths that are independent of agents' labor market status.[2] Thus, (P1) can be simplified to

[2]For more detail, see Greenwood and Huffman (1988) or Rogerson and Wright (1988).

$$\begin{aligned} V(\pi;z,\epsilon) &= \max_{\{\pi_i'\}}\left\{\frac{A}{\rho}\ln\left[\sum_{i=1}^{N}\theta_i(z\epsilon_i r^{\alpha_i}[\pi_i'-\gamma_i(\max[\pi_i'-\pi_i,0])^2]^{\alpha_i}-I_i(z,\epsilon_i))^{\rho}\right]\right. \\ &\quad +(1-A)\left[\pi_{N+1}'\ln(1-s)+\left(\sum_{i=1}^{N}\pi_i'\right)\ln(1-r)\right] \\ &\quad \left.+\beta\cdot E\left[V(\pi';z',\epsilon')\mid\pi;z,\epsilon\right]\right\}. \end{aligned} \tag{P2}$$

subject to (5), and (6). Finally, Rogerson (1987), Hornstein (1991) and Prescott and Rios-Rull (1992) discuss how planning problems of the above form can be represented as decentralized competitive equilibrium.

3. Calibration

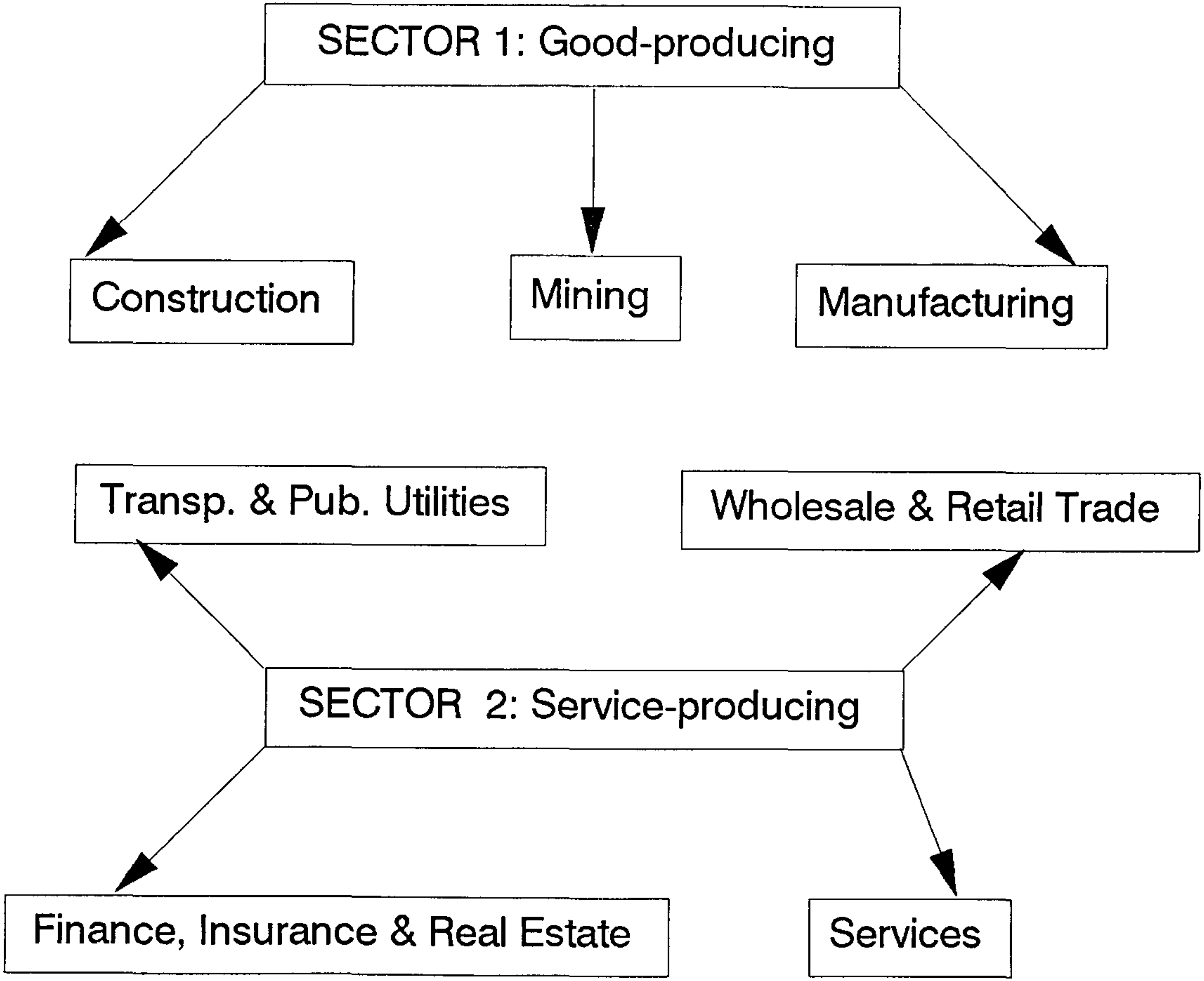
The model is restricted to two sectors, assumed to correspond to the goods and service sectors of the U.S. economy. The industries that make up these sectors are shown in Figure 1; one period is assumed to be one quarter.

3.1. Preference Parameters. The quarterly interest rate is taken to be one percent; thus the discount factor, β, is 0.99. Next, data from the *Monthly Labor Review* shows that, on average, the employed work 39 out of the approximately 100 non sleeping hours per week available to them; consequently, $r = .39$. According to Barron and Mellow (1979), the mean number of hours spent searching per week is approximately 7 which implies $s = .07$. In a similar vein, a value of 0.28 was picked for the coefficient A in the utility function. This results in approximately 25% of *aggregate* non sleeping hours being spent at work. In the U.S. data the goods sector is about 58 percent of the size of the service sector, when measured by employment. This occurs in the model's steady state if $\theta_1 = .41$ ($\theta_2 = .59$). Finally, the parameter $\rho \in (-\infty, 1]$ governs the amount of substitution between goods and services in the utility function. Independent evidence on an appropriate value for ρ is hard to come by. In the subsequent analysis, ρ is assigned a value of 0.55.[3]

3.2. Technology Parameters. The two production function parameters, α_1 and α_2, are set equal to 0.74 and 0.64 respectively. These numbers are labor's share of

[3]The utility function specified in (2) implies that an agent will divide his consumption between goods and services according to the formula $\ln\frac{c_1}{c_2} = \frac{1}{\rho-1}\ln p$, where p is the relative price of goods in terms of services. Estimation of this equation using instrumental variables yielded a value of .55 for ρ. Unfortunately, this point estimate was insignificant at the 95% level of confidence. Still, on the basis of the time series evidence a value of 0.55 is the best guess for ρ.

Fig. 1: Definition of Sectors



income in goods and services sectors for the 1964-1987 period.[4]

3.3. Adjustment Costs. The adjustment cost parameter, γ_i, is set at 3.5 for both sectors. The magnitudes of γ_1 and γ_2 determine the speed of sectoral employment adjustment.

3.4. Shocks. Assume that $\epsilon_1 = 1/\epsilon_2 = \epsilon$ — this amounts to assuming a single relative sectoral shock. Then, using (3) and data for each sector's output and labor input, the aggregate and sectoral Solow residuals are easy to calculate.[5] By doing this it is found that the aggregate shock has a percentage standard deviation of 0.04 and a serial correlation coefficient of 0.93. The numbers for the sectoral shock are 0.015 and 0.93.

The aggregate and sectoral shocks are two-state Markov processes: $z_t \in Z = \{\exp^{\xi}, \exp^{-\xi}\}$ with $\Pr[z' = z_1 | \ z = z_1] = \Pr[z' = z_2 | \ z = z_2]$, and $\epsilon \in E = \{\exp^{\zeta}, \exp^{-\zeta}\}$ with $\Pr[\epsilon' = \epsilon_1 | \ \epsilon = \epsilon_1] = \Pr[\epsilon' = \epsilon_2 | \ \epsilon = \epsilon_2]$. The parameters ξ and ζ are chosen so that the time series properties for the aggregate and sectoral disturbances in the model inherit the time series behavior of the aggregate and sectoral Solow residuals. This implies setting $\xi = .04, \Pr[z' = z_1 | z = z_1] = .965$, $\zeta = .015$ and $\Pr[\epsilon' = \epsilon_1 | \epsilon = \epsilon_1] = .965$.[6]

3.5. Investment. Finally, in the U.S. economy consumption is relatively smooth, and investment is procyclical and highly volatile. This motivates subtracting a certain amount of output, equal to investment, from the right hand side of the resource constraints.[7] The function $I_i(z, \varepsilon_i)$ is intended to capture this. Let the investment

[4]Labor's share of income for sector i, or α_i, was computed from the formula shown below using data from the National Income and Product Accounts:

$$\alpha_i = \frac{COM_i}{NI_i + CCA_i - PI_i},$$

where COM_i is Compensation of Employees for sector i, NI_i is National Income, CCA_i is the Capital Consumption Allowance, and PI_i is Proprietor's Income.

[5]The assumption on the functional form for the sectoral disturbances allows them to be easily identified.

[6]It is straightforward to calculate that the percentage standard deviations of the aggregate and sectoral disturbances are given by ξ and ζ. Likewise, the formulae for autocorrelation coefficients for the shocks are $2\Pr[z' = z_1 | z = z_1] - 1$ and $2\Pr[\epsilon' = \epsilon_1 | \epsilon = \epsilon_1] - 1$, respectively.

[7]The aggregate disturbance will not affect the solution to the model if there is no investment term in the resource constraint (7). This is immediate from problem (P2). Without the $I_i(z, \varepsilon_i)$ term, it is easy to see that z can be factored out of the first term on the righthand side of (P2). Hence it can't affect the maximization.

functions, $I_i(z,\varepsilon_i)$, have the form

$$I_i(z,\varepsilon_i) = \begin{cases} e^{\sigma+\sigma_i} I_i^*, & \text{if } z = e^{\xi}, \text{ and } \varepsilon_i = e^{\zeta} \\ e^{\sigma-\sigma_i} I_i^*, & \text{if } z = e^{\xi}, \text{ and } \varepsilon_i = e^{-\zeta} \\ e^{-\sigma+\sigma_i} I_i^*, & \text{if } z = e^{-\xi}, \text{ and } \varepsilon_i = e^{\zeta} \\ e^{-\sigma-\sigma_i} I_i^*, & \text{if } z = e^{-\xi}, \text{ and } \varepsilon_i = e^{-\zeta}, \end{cases}$$

where the means and standard deviations of $\ln I_i(z,\varepsilon_i)$are given by $\ln I_i^*$and $\sqrt{\sigma^2+\sigma_i^2}$.

In the U.S., aggregate investment is approximately 20 percent of GNP. This implies that in the model steady state $I_1 + pI_2 = .2[y_1 + py_2]$, where p is the relative price of good two. Also, the goods producing sector generates two-thirds as much output as the service sector. If it is assumed that investment spending is spread across sectors proportionally, then the models steady state should display the feature that $I_1/I_2 = y_1/y_2$. Assuming this, along with $I_1 + pI_2 = .2[y_1 + py_2]$, implies $I_1^* = .0445$ and $I_2^* = .0773$. In the U.S. data, investment is four times as volatile as output and the correlation coefficient between aggregate investment and output is 0.95. The percentage standard deviations for the investments were chosen to mimic these observed facts. This involved setting $\sigma = .08$, $\sigma_1 = .06$, and $\sigma_2 = .08$.

4. Findings

The cyclical properties of the above model are developed through simulation. As is now standard, the procedure is to compare a set of stylized facts characterizing the business cycle behavior of the model with a analogous set describing U.S. postwar business cycle behavior over the 1964.1-1987.4 sample period. Appendix A details the computational procedure used to calculate the decision-rules associated with the planner's problem. The procedure used to compute the decision-rules is complicated by the presence of the inequality constraint (6). With these decision-rules in hand, 25 samples of 96 observations (the number of quarters in the U.S. sample period) are simulated. Each simulation run corresponds to a randomly generated sample of 96 realizations of the z and ϵ processes. The data from the simulations is logged and H-P filtered (as is the data for the U.S. economy) and average moments over the 25 samples are computed for each variable of interest.

4.1. Impulse-Response Functions. The dynamic effects that aggregate and sectoral disturbances have on sectoral employment and aggregate nonemployment can be represented in terms of impulse-response functions. Figure 2 plots the impulse response functions associated with an aggregate shock, where the economy is assumed to be in a steady state initially. Employment in both sectors rises, while aggregate nonemployment (or $1 - \pi_1 - \pi_2$) falls. Notice that it takes the economy five periods to move agents out of the searching pool and home sector into work in the two

Fig.2: Impulse Responses: Aggregate Shock

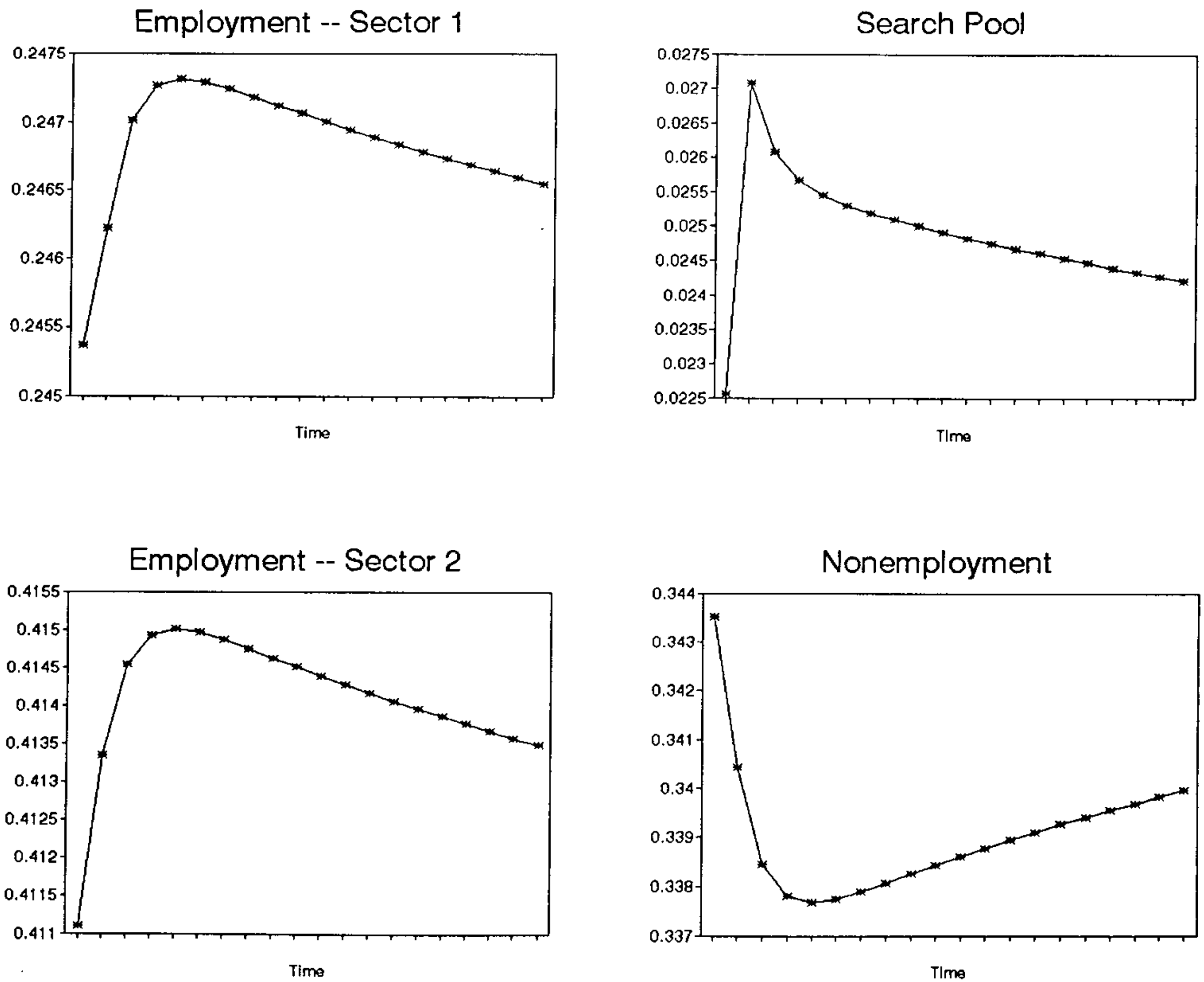
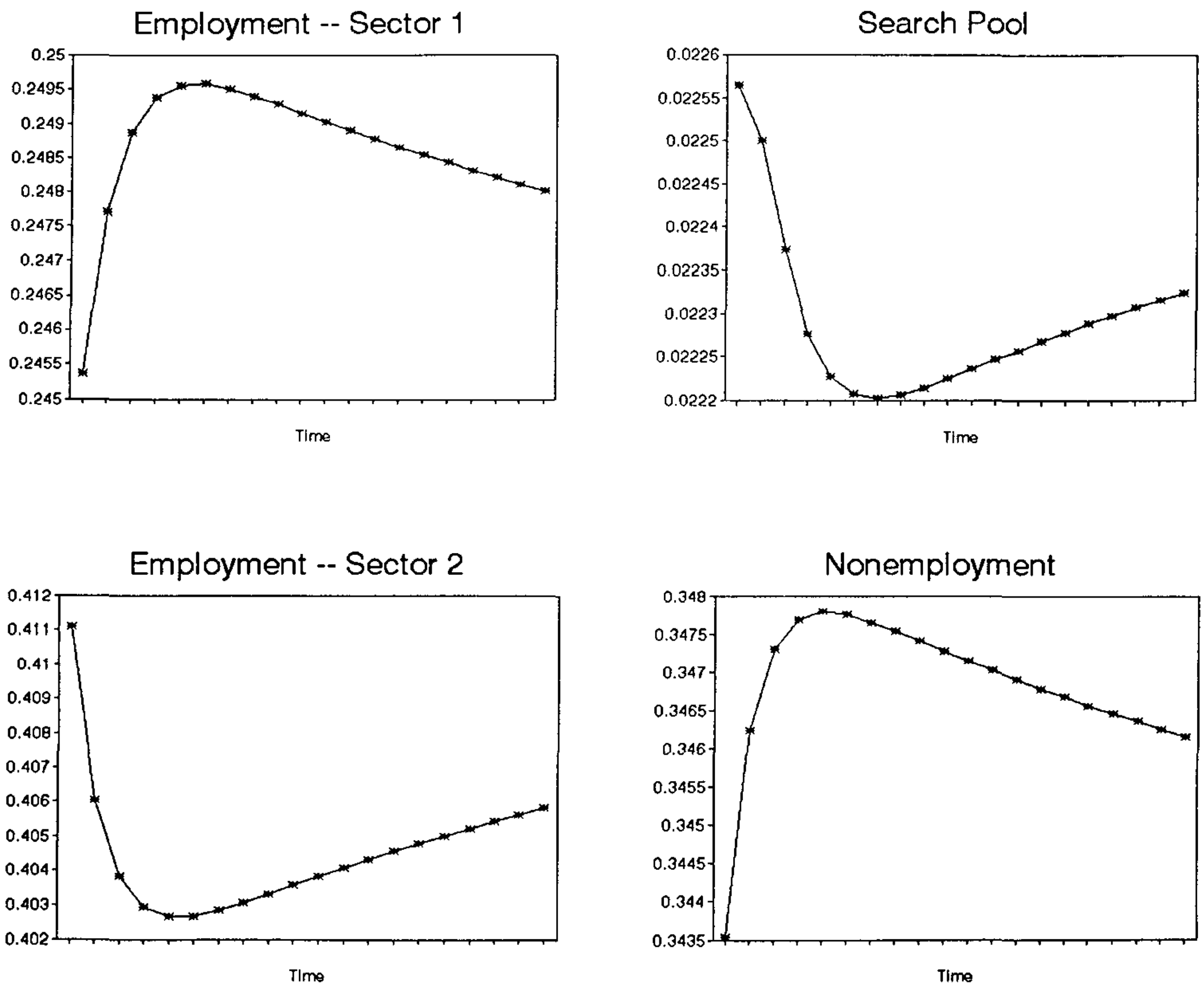


Fig.3: Impulse Responses: Sectoral Shock



market sectors. This illustrates the influence of adding the search pool to the model. The results here are consistent with Jovanovic's (1987) argument that a positive serially correlated aggregate shock will simultaneously increase sectoral employments and search, and decrease aggregate nonemployment. Similarly, Figure 3 shows the impulse response functions for a sectoral shock. A positive sectoral shock increases the productivity of the goods sector relative to services. Consequently, employment in goods (services) production rises (falls). Again, it takes the economy about five to six periods to go through the adjustment process. Observe that nonemployment rises following the sectoral shock. This transpires since sector two is larger than sector one; more workers are withdrawn from sector two in response to the technology shock than are added to sector one with the difference leaving the labor force.

4.2. Aggregate and Sectoral Fluctuations. The amount of job creation in sector i during period t is given by $\max\{0, \pi_{i,t} - \pi_{i,t-1}\}$. Thus, the sector-$i$ job creation rate is defined to be $\max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\pi_{i,t-1}$. Likewise, for sector-$i$ the job destruction rate is $\max\{\pi_{i,t-1} - \pi_{i,t}, 0\}/\pi_{i,t-1}$. The sum of these job creation and destruction rates defines the sector-i job reallocation rate. It follows that the aggregate job creation and destruction rates are $\sum_{i=1}^{2} \max\{0, \pi_{i,t} - \pi_{i,t-1}\}/\sum_{i=1}^{2} \pi_{i,t-1}$ and $\sum_{i=1}^{2} \max\{0, \pi_{i,t-1} - \pi_{i,t}\}/\sum_{i=1}^{2} \pi_{i,t-1}$. The sum of the aggregate job creation and destruction rates defines the aggregate job reallocation rate. These job creation and destruction rates represent the lower bounds on the amount of job creation and destruction in the U.S. economy. This is because they measure net, rather than gross, labor market flows.

Descriptive statistics characterizing the cyclical behavior of U.S. labor market aggregates are presented in Table 1. Table 2 presents the same statistics for the model. The model reproduces the cyclical pattern of job creation, destruction and reallocation displayed in the U.S. data relatively accurately. Specifically,

- In both the model and the data, the job creation rate moves procyclically while the job destruction and reallocation rates are countercyclical.
- The correlations between these variables and GNP are also close to those found in U.S. data.
- In both the model and the data the job destruction rate is more volatile than the either the job creation or reallocation rates. This reflects the importance of the asymmetric nature of the employment process. It is much easier to fire people than to hire them.
- In the data the correlation between hours and productivity is low, as evidenced by the correlation coefficient of 0.20. For the model the number is 0.47, which is also fairly low. The model performs much better than the

Table 1: Cyclical Behavior of U.S. Labor Market Aggregates

Quarterly, 1964.1-1987.4

VARIABLES	S.D.(%)	CORR./OUTPUT	CORR/EMPLOYMENT	AUTOCORR.
OUTPUT	2.50	1.00	0.86	0.85
EMPLOYMENT	1.66	0.86	1.00	0.93
HOURS	1.96	0.91	0.98	0.91
UNEMPLOYMENT		-0.93	-0.92	0.90
JOB CREATION RATE	0.71	0.46	0.12	0.58
JOB DESTRUCTION RATE	1.82	-0.47	-0.20	0.60
JOB REALLOCATION RATE	0.56	-0.14	-0.12	0.35
PRODUCTIVITY	1.06	0.66	0.20	0.63

NOTE: The U.S. economy analyzed in this paper only consists of two sectors. One of them, sector 1, is called the Good-producing sector which includes three 1-digit *SIC(1987) industries: Mining, Construction and Manufacturing. The other, sector 2, is known as the Service-producing sector which* includes following SIC industries: Transportation and Public Utilities, Wholesale Trade and Retail Trade, Finance-Insurance and Real Estate and Services. All industry time series are taken from CITIBASE(1989). Since quarterly GNP by industry is not available, National Income by industry is used as substitute.

The availability of data on average weekly hours worked by industry determines the sample periods starting from 1964.1 to 1987.4.

For productivity, CORR/EMPLOYMENT represents the correlation of productivity and hours.

Table 2: Cyclical Behavior of Labor Market Aggregates
Model: 96 Observations

VARIABLES	S.D.(%)	CORR./OUTPUT	CORR/EMPLOYMENT	AUTOCORR.
OUTPUT	2.09	1.00	0.66	0.66
EMPLOYMENT	0.54	0.66	1.00	0.87
HOURS	0.54	0.66	1.00	0.87
UNEMPLOYMENT		-0.66	-1.00	0.87
JOB CREATION RATE	1.67	0.33	0.01	0.43
JOB DESTRUCTION RATE	1.91	-0.48	-0.20	0.41
JOB REALLOCATION RATE	1.21	-0.12	-0.12	0.33
PRODUCTIVITY	1.77	0.97	0.47	0.64

Note: All time series are detrended by H-P filter. The statistics shown in all tables are the average values after 25 simulations.
For productivity, CORR/EMPLOYMENT represents the correlation of productivity and hours.

Table 3: Cyclical Behavior of Sector Labor Aggregates in U.S. Economy

Quarterly, 1964.1-1987.4

VARIABLES	S.D.(%)		CORR. with OUTPUT		CORR. with EMPL'T		AUTOCORR.	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
OUTPUT	4.12	1.50	1.00	1.00	0.87	0.77	0.83	0.86
EMPLOYMENT	2.94	1.00	0.88	0.77	1.00	1.00	0.92	0.93
HOURS	3.50	1.02	0.93	0.81	0.98	0.98	0.89	0.91
JOB CREATION RATE	1.31	0.56	0.32	0.47	0.03	0.14	0.47	0.60
JOB DESTRUCTION RATE	1.68	2.95	-0.49	-0.40	-0.24	-0.16	0.63	0.38
JOB REALLOCATION RATE	0.96	0.47	-0.37	0.31	-0.24	0.07	0.47	0.43
PRODUCTIVITY	1.54	0.90	0.56	0.75	0.12	0.18	0.54	0.73

NOTE: For productivity, CORR/EMPLOYMENT represents the correlation of productivity and hours.

Table 4: Cyclical Behavior of Sector Labor Aggregates

Model: 25 Simulations with 96 Observations Each

VARIABLES	S.D.(%)		CORR. with OUTPUT		CORR. with EMPL'T		AUTOCORR.	
	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2	Sector 1	Sector 2
OUTPUT	2.34	2.22	1.00	1.00	0.66	0.67	0.73	0.70
EMPLOYMENT	0.81	0.60	0.66	0.67	1.00	1.00	0.87	0.88
HOURS	0.81	0.60	0.66	0.67	1.00	1.00	0.87	0.88
JOB CREATION RATE	1.88	1.82	0.33	0.41	0.03	0.12	0.43	0.51
JOB DESTRUCTION RATE	1.92	2.71	-0.49	-0.50	-0.13	-0.16	0.51	0.35
JOB REALLOCATION RATE	1.23	1.50	-0.05	-0.16	-0.07	-0.08	0.36	0.32
PRODUCTIVITY	1.88	1.86	0.94	0.97	0.39	0.48	0.64	0.63

Table 5: Relation Between Job Creation & Destruction

U.S. Data, Quarterly,1964.1-1987.4

VARIABLES	RJD	RJD1	RJD2
RJC	-0.50	-0.53	-0.36
RJC1	-0.34	-0.37	-0.22
RJC2	-0.56	-0.59	-0.44

NOTE: RJC = Aggregate Job Creation Rate.
RJD = Aggregate Job Destruction Rate.
RJCi = Sector i Job Creation Rate.
RJDi = Sector i Job Destruction Rate.

Table 6: Relation Between Job Creation & Destruction

Model

VARIABLES	RJD	RJD1	RJD2
RJC	-0.09	-0.08	-0.09
RJC1	-0.11	-0.21	-0.03
RJC2	-0.05	0.09	-0.14

standard model on this dimension, and works about as well as models that include government spending or household production — see Hansen and Wright (1992).

Next, some stylized facts describing the behavior of U.S. labor market variables at the sector level are given in Table 3. Table 4 presents the same set of facts for the model. The key findings here are:

- In the data, the job creation, destruction and reallocation rates display the same pattern of cyclical behavior at the sectoral level as they do for the economy as a whole. There is, however, one exception: while the job reallocation moves countercyclically in the goods producing sector it moves procyclically in services.[8] The model replicates fairly closely the correlation structure between these variables and output, except for the procyclical movement of the job reallocation rate in the service sector.
- The model and data share the feature that output and employment are more volatile in goods production than in services.
- The model does a reasonable job matching the hours/productivity correlations at the sectoral level.

Finally, Table 5 reports negative correlations between job creation and destruction rates, at both the aggregate and sectoral levels. Similar findings are reported in Davis and Haltiwanger (1990) and Mortensen (1993). On this,

- The model matches these findings qualitatively, but quantitatively, the magnitudes of the correlations in the model are too small. For instance, in the data, the correlation between job creation and job destruction in the goods producing sector is -0.37 while for the model it is only -0.21.

4.3. The Determination of Aggregate Nonemployment. How much of aggregate unemployment can be accounted for by aggregate and sectoral shocks? In the absence of technology shocks there would be no steady-state search unemployment in the model. Thus, the average value for π_3 is a measure of the amount of unemployment due to aggregate and sectoral disturbances. On this account 1.83 percent of the labor force is unemployed. To break this number down further into the components due to aggregate and sectoral shocks, the aggregate shock can be shut down

[8] The size of the service sector has increased over time while the volume of goods production has declined. Jobs created in the service sector may accelerate during booms and jobs destroyed in the goods sector may speed up in recessions. This hypothesis is consistent with findings in Loungani and Rogerson (1989).

in the model. The average value for of π_3 falls to 1.8; i.e., the sectoral shock has a larger effect on unemployment than the aggregate disturbance does. These findings are more in accord with Loungani and Rogerson (1989) than with Murphy and Topel (1987). The model also predicts that the search is procyclical, that is the correlation between π_3 and output is 0.65.

5. Concluding Remarks

A multisector dynamic general equilibrium model is constructed here to analyze the cyclical pattern of job creation and destruction. The two main ingredients in the model are the Lucas-Prescott (1974) idea that it takes time to find employment and the Rogerson (1988)/Hansen (1985) notion of indivisible labor. It is found that the model can successfully replicate the cyclical patterns of job creation, destruction and reallocation that is observed at both the aggregate and sectoral levels in the U.S. economy. Additionally, in the model aggregate and sectoral job creation and destruction rates are negatively correlated, as they are in the data. Furthermore, the correlation between hours and productivity is relatively low in the model, but not as low as is observed in the U.S. economy. Finally, it is found that both aggregate and sectoral disturbances contribute significantly to aggregate unemployment.

In the model presented here workers were assigned their employment status via a lottery. They were perfectly insured against the possibility of dismissal, in the sense that their consumption in a period was not contingent upon their employment status. One can imagine a world where no such insurance exists. Suppose, instead, that individuals can only insure themselves by saving in the form of a simple asset, such as money or government bonds. Each period those agents currently working in a sector decide whether to stay at work, enter the unemployment pool to search for a new job in another sector, or leave the labor force. Agents in the unemployment pool decide whether to take a job in some sector, remain in the unemployment pool for another period, or leave the labor force. Likewise, those individuals at home must decide whether or not to enter the labor force. Clearly, an individual's decision will be predicated upon both his idiosyncratic circumstance (asset holdings, employment status) and the aggregate situation (distribution of agents and state of technology in each sector). While computationally more complicated, such an analysis will share many of the features of the above model. But it would undoubtedly permit a much richer analysis along some dimensions. For instance, one could study the effect that government policies, such as unemployment insurance, have on intersector mobility and unemployment.[9] The current analysis can be viewed as a first step toward such a model.

[9] This policy experiment could be viewed as embedding the analysis of Hansen and Imrohoroglu (1992) into a multisector general equilibrium model of the form presented here.

6. APPENDIX A: COMPUTATION

Modified Discrete State Space Approach With Value Function Approximation

The neoclassical growth model can be solved using standard discrete state space dynamic programming techniques. In economies with multiple sectors or multiple agents, the standard approach becomes unworkable due to the curse of dimensionality, which limits the practicability of standard discrete state space dynamic programming techniques for large problems.

An alternative treatment of the problem is to store a limited set of coefficients characterizing a parameterized value function and momentary return function.[10] The parameterized objective function can then be maximized using an optimization routine. Two benefits derive from this method: First, computation costs are reduced dramatically; and second, the maximizers are no longer constrained to lie in a discrete subset set of the constraint set.

An obvious candidate in the family of simple functions to use to approximate more complicated functions is the polynomial. However, there are two problems associated with polynomial approximation. First, practical concerns prevent using high order polynomials (even given the Weierstrass theorem). Second, the adequacy of polynomial approximations depends on the differentiability properties of the function that is being approximated. Often, for a smooth function a lower degree polynomial can be used.[11]

The representative agent's optimization problem, characterized by problem (P2) in Section 2, can be simplified to one with only linear constraints by using the following lemma. For this simplified problem, it is easy to check the convexity of the constraint set.

Lemma 1. *The transition constraint*

$$\sum_{i=1}^{2} \max\{0, \pi_i' - \pi_i\} \leq \pi_3 \tag{8}$$

is equivalent to following set of linear inequality constraints:

$$\pi_1' + \pi_2' \leq \pi_1 + \pi_2 + \pi_3, \tag{9}$$

$$\pi_1' \leq \pi_1 + \pi_3, \tag{10}$$

[10] A discussion of numerical techniques used to solve dynamic equilibrium models can be found in Danthine and Donaldson (1993).

[11] Given the assumptions placed on tastes and technology here, the value function will be strictly increasing, strictly concave and continuously differentiable (Stokey et al (1989), Chap 9).

$$\pi_2' \leq \pi_2 + \pi_3. \tag{11}$$

Proof: It is trivial to verify that the set constrained by (8) is same as the one constrained by (9), (10) and (11) . If (8) holds, then the following must be true,

$$(\pi_1' - \pi_1) + (\pi_2' - \pi_2) \leq \pi_3, \tag{12}$$

$$\pi_1' - \pi_1 \leq \pi_3, \tag{13}$$

$$\pi_2' - \pi_2 \leq \pi_3. \tag{14}$$

But this is merely (9)– (11). On the other hand, from (12) – (14) it is easy to derive that

$$\max\{0, \pi_1' - \pi_1\} + \max\{0, \pi_2' - \pi_2\} \leq \max\{0, \pi_3\}, \tag{15}$$

which is equivalent to the transition constraint (8). □

Let $\mathcal{F}$ represent the space of continuous, bounded functions and consider the mapping $T : \mathcal{F} \rightarrow \mathcal{F}$ defined by (P3).

$$\begin{aligned} V^{j+1}(\pi_1, \pi_2, \pi_3; z, \epsilon_1, \epsilon_2) \;=\; & \max_{\{\pi_1', \pi_2', \pi_3'\}} \{ \tfrac{A}{\rho} \ln(\sum_{i=1}^{2} \theta_i (z\epsilon_i r^{\alpha_i} (\pi_i' - \\ & \gamma_i(\max[\pi_i' - \pi_i, 0])^2)^{\alpha_i} - I_i(z, \epsilon_i))^{\rho}) \\ & + (1 - A)[\pi_3' \ln(1 - s) + (\sum_{i=1}^{2} \pi_i') \ln(1 - r)] \\ & + \beta E[V^j(\pi_1', \pi_2', \pi_3'; z', \epsilon_1', \epsilon_2') | \pi_1, \pi_2, \pi_3; z, \epsilon_1, \epsilon_2]\}, \end{aligned} \tag{P3}$$

subject to the constraints (9) – (11) and

$$\sum_{i=1}^{3} \pi_i' \leq 1\,, \quad \pi_i' \geq 0. \tag{16}$$

The mapping T maps V^j to V^{j+1}. This operator is a contraction mapping that has as its unique fixed point the function V defined by (P2).[12] This last observation motivates the computational procedure used here consisting of the following steps:

[12]It is trivial to check that (P3) satisfies Blackwell's sufficiency conditions for a contraction mapping — see Stokey et al (1989).

1. A grid is defined over the model's state space. Specifically, it is assumed that $\pi_1 \in [.14, .32]$, $\pi_2 \in [.34, .52]$, and $\pi_3 \in [0, .225]$.[13] Three grids of 10 equally spaced points are layered over these intervals. These sets of grid points are denoted by Π_1, Π_2, and Π_3, respectively.

2. An initial guess for the 2nd degree polynomial used to approximate the value function over this grid is made.

3. Given the guess for the value function, a maximization routine is used to solve the constrained nonlinear optimization problem (P3) for the optimal decision-rules.[14] This is done for each of the 4,000 points in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$.

4. Using the solution obtained for the optimal decision-rules, a revised guess for the value function is computed. This is done by choosing a new 2nd degree polynomial to approximate the value function. In particular, from (P3) a value for V can be computed for each grid points in the set $\Pi_1 \times \Pi_2 \times Z \times E$. A 2nd degree polynomial is then fitted to these points via least squares. The adequacy of this degree of polynomial can be gauged using the R^2 or χ^2 statistics.

5. The decision-rules are checked for convergence.

Once the decision-rules have been obtained, the model can be simulated and various statistics are generated consequently. Note that function values for the decision-rules will have been computed for each point in the set $\Pi_1 \times \Pi_2 \times \Pi_3 \times Z \times E$. It is then easy to obtain values for the decision-rules at any point in the space $[.14, .32] \times [.34, .52] \times [0, .225] \times Z \times E$ by using multilinear interpolation – see Press et al (1992).

7. Appendix B:The Data Set

As described in Figure1, the goods-producing and service-producing sectors are made up by seven SIC one-digit industries: Mining (1), Construction (2) , Manufacturing (3), Transportation and Public Utilities (4), Wholesale Trade and Retail Trade (5), Finance, Insurance and Real Estate (6) and Services (7). Here the goods-producing sector includes the first three industries while the rest make up the service sector.

All the time series for the postwar U.S. economy are obtained from Citibase. Exceptions are the series for noncorporate capital consumption allowance by industry which came from the National Income and Product Accounts. Output for industry i is measured in 1982 prices. Total hours worked in industry i is the product of

[13] By simulating the model it was determined that system never left these intervals.

[14] This was done using M.J.D. Powell's GETMIN subroutine developed for solving constrained nonlinear optimization problems.

employment and the weekly hours worked per employee in that industry. The output, employment, hours and unemployment series are deflated by the civilian population. Citibase codes are contained in Table 7.

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Table 7: Codes of the Time Series in Citibase

Sample Period: 1964.1-1987.4

VARIABLES	INDUSTRIES						
	1	2	3	4	5	6	7
A: OUTPUT							
National Income	GYWM	GYWC	GYM	GYWTU	GYNRR	GYFIR	GYS
GNP (82)	GA8G14	GA8G15	GA8GM	GA8GTU	GA8GW+GA8GR	GA8GFE	GA8GS
GNP	GAG14	GAG15	GAGM	GAGTU	GAGW+GAGR	GAGFE	GAGS
B: LABOR							
Employment	LPMI	LPCC	LPEM	LPTU	LPT	LPFR	LPS
Weekly Hours Worked Per Employee	LWMI6	LWCC	LPHRM	LWTU	LWTWR	LWFR6	LWS
Unemployment Rate	LURMI	LURC	LURM	LURTPU	LURWR	LURFS	LURFS
C: LABOR SHARE							
Compensation of Total Employees	GAPMI	GAPCC	GAPM	GAPTPU	GAPW+GAPR	GAPFF	GAPS
Proprietor's Income	GAYPMI	GAYPCC	GAYPM	GAYPTU	GAYPTW+GAYPRT	GAYPF	GAYPS
Corporate Capital Consumption Allowance	GACMI	GACCC	GACM	GACTPU	GACW+GACR	GACFF	GACS
D: Population							
Civilian Population				PO16			

[1] The indices of the industries refer to Appendix B.
[2] All time series except ones at annual frequency are seasonally adjusted.
[3] All the time series except GNP (82) in group A and C are nominal.
[4] All the series in group A and C are annual except national income is quarterly and all the series in group B and D are monthly.