On the Economics of Fiscal Populism in an Open Economy

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ABSTRACT

We study a representative agent, open economy in which government-provided services that enter the domestic production function must be financed with distortionary taxes, and focus on the optimal size of government and the associated optimal tax rate. If the government can precommit its actions, it maximizes individual welfare by announcing and implementing a constant tax rate, which we label the "orthodox" tax rate. This tax rate is time inconsistent, and under discretion the government implements a tax that maximizes each period's output. We label this the "populist" tax rate. It may be higher or lower than the "orthodox" rate, depending on whether the elasticity of substitution in production between private and public inputs is below or above one. We also characterize the second-best tax rate that can be sustained through trigger strategies. This best sustainable tax rate is constant and lies between the "orthodox" and "populist" extremes.

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I. Introduction

Over the last two decades, the fiscal policy of both developed and developing countries has become increasingly problematic. Most striking fact is that since 1973 many countries have run unprecedented peacetime deficits and accumulated very high levels of debt.¹ But that is not the only problem. In much of the developing world and arguably also in some developed nations such as the United States, public services have been underprovided --while in others (Scandinavia, some nations in the European Union?) they may have been overprovided at the cost of high distortionary tax rates. In some extreme cases, the label of "fiscal populism" has been applied to countries that run clearly suboptimal and/or unsustainable fiscal policies.²

A large political economy literature has developed to attempt to explain the apparent irrationality of many such fiscal policies. In one strand, models of weak government or segmented policy-making have been developed to explain high levels of spending and the prevalence of pork-barrel projects.³ Another strand of the literature adds political elements to the neoclassical theory of optimal government debt, showing that the existence of inefficient political equilibria can generate debt paths that are reminiscent of recent country experiences.⁴ In all cases, the focus is on the role of heterogeneous agents and groups and on the redistributive aspect of fiscal policy.

By contrast, in this paper we study a representative agent economy ruled by a benevolent government, and ask whether there can be economic reasons for the emergence of suboptimal fiscal policies and fiscal populism. For this question to well posed, we need a role for government. We focus on the provision of public services --infrastructure, law and order, defense, health and education-- which presumably enhance the productivity of privately-owned factors of production.

¹ See Roubini and Sachs (1989).

² Concept is an old one, but it has been brought back into vogue recently by Dornbusch and Edwards (1989) and Sachs (1989) in reference to some high-spending Latin American regimes of the 1970s and 1980s.


⁴ See the recent survey by Alesina and Perotti (1994). See also Velasco (1993).
But financing for these services comes from distortionary taxes, which hinder the efficient allocation of resources and can also detract from the productivity of privately-owned factors of production.

This tradeoff can be particularly acute in the context of an open economy faced with ample international capital mobility. Good infrastructure, an effective court system and timely garbage collection enhance the marginal product of capital and help attract investment funds from abroad, while high tax rates on capital decisively contribute to keep capital away. Both aspects of the tradeoff currently receive much attention in policy analyses. Bill Clinton was elected President on a platform that promised health care and educational reform and a new "information superhighway" as the way to foster capital accumulation in the United States; his Republican opponents countered that the new taxes needed to finance these reforms would keep business investment down. Debate is sharpened even further in capital-scarce developing countries. Government that took power in newly democratic Chile in 1990 charged the outgoing military government with leaving behind a huge "social and infrastructure deficit" and proposed to raise taxes to pay for a large increase in public works and educational spending in order to ensure the country remained attractive to investors; conservative economists argued that such policies would not only fail to ensure sustained growth, but would also lead to a decline in investment and capital flight.

This paper takes a new look, from a positive perspective, at this age-old question. We consider an economy in which government-provided services enter the production function, as in Barro (1990). Private agents can invest at home or abroad, but the government can only tax the factors employed in domestic production. In that context, we ask what is the optimal level of government spending and the associated tax rate, and whether such an optimal policy is implementable as the equilibrium of a dynamic game between the government and the private sector.

We find that if the government can precommit its future actions, it maximizes the welfare

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5 See the manifesto Putting People First: How We Can All Change America, 1992.

6 For a review of this debate and the policies that followed, see Labán and Velasco (1993).
of the representative agent in the economy by announcing and implementing a constant tax rate. Predictably, this rate—which we label the "orthodox" tax rate—represents a compromise between the desire to spend more and provide more services and the constraint given by the fact that anticipated taxes on capital distort investment decisions.

The orthodox tax rate turns out to be time inconsistent --if the government cannot precommit and instead reoptimizes every period, it will not implement the announced optimal tax when the time comes to do so. Intuition for this result is simple. When computing the precommitment optimum, the government acts as a Stackelberg leader, taking into account how investors' demand for domestic capital varies with the level of the tax. But once capital is already in place, this constraint is no longer binding. At that point it is optimal for the government to implement the tax which maximizes domestic output for a given capital stock. Because of the emphasis on myopic output maximization, we term this the "populist" tax rate. In a rational expectations equilibrium agents correctly predict that the populist rate will be implemented, and lower their holdings of domestic capital accordingly. Hence, in the spirit of Kydland and Prescott (1977), the populist equilibrium under discretion is inferior to the equilibrium under precommitment.

A striking feature of the model is that under discretion the tax rate on domestic capital may be higher or lower than the precommitment optimum. If the elasticity between private capital and public services in the production function is above one, once capital has entered the country output is maximized by taxing less than under precommitment and providing a lower level of services. If private capital and public services enter the production function inelastically, on the other hand, output is maximized by taxing more than under precommitment and providing more public services. Hence, populism can be of the standard left-wing variety (discretionary taxes are higher than under orthodoxy) and also of the right-wing variety (discretionary taxes are lower than under orthodoxy).

We also explore whether outcomes better than the populist equilibrium can be sustained through the use of trigger strategies. Precommitment outcome may or may not be supported through government reputation depending on parameter values. When the orthodox tax rate cannot be sustained, other tax paths can indeed be supported in a trigger-strategy equilibrium. In
particular, we are able to characterize the second-best sustainable trajectory of taxes, which involves a constant tax rate that lies between the orthodox and populist extremes.

The paper is organized as follows. Section II presents the basic model. Sections III and IV characterize the cases of precommitment and discretion. Section V analyzes the effects of trigger strategies, while section VI concludes.

II. The Underlying Economy

Consider a small open economy whose representative agent has access to domestic capital $k$ yielding a gross rental rate $R^t_k$ and an internationally traded bond $b$, whose exogenous and constant gross rate of return is denoted by $R$. In addition, domestic capital is rented by a price-taking firm that uses it to produce domestic output. Domestic technology is such that government has a crucial role to play in the provision of public services. As in Barro (1990), domestic output is produced using domestic capital and government-provided services. Profits from the firm, in the form of dividends, also accrue to the agent.

The representative agent's budget constraint is

$$b_{t+1} + k_{t+1} = R b_t + R^t_k k_t + d_t - c_t$$ (1)

where $c$ is consumption and $d$ is the dividend flow.

The sequence of moves is as follows. Agents enter period $t$ with holdings $b_t$ and $k_t$ of foreign and domestic capital carried over from the previous period. At the start of the period, markets open and the agent rents out her domestic capital to the firm. The government then taxes the firm's capital and provides public services. Production takes place, combining government

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7 In what follows we describe and solve the portfolio problem faced by the representative resident of the small open economy. But notice that the portfolio problem (but not the consumption-savings problem) is identical to that faced by a foreign investor considering buying capital in the home economy. Hence, all of our results below concerning investment and international capital movements apply equally to the decisions of domestic residents and foreign investors.
services and the capital still in the hands of the firm. Agents then receive interest income from abroad, rental income on their domestic capital holdings and dividends from the firm. Finally, they consume and allocate their savings between the two assets.

Production takes place according to

$$\mathcal{Y}(k_g) = [a(1-\tau)^\rho k^{-\rho} (1-a) g^{1-\rho}]^{-1/\rho}, \quad -1<\rho \quad \text{and} \quad 0<\gamma<1$$

(2)

where $g$ denotes the flow of government services, $\gamma = [1/(1+\rho)]$ is the elasticity of substitution in production between private capital and public services and $\gamma$ is a returns-to-scale parameter.

A key assumption is that only domestic capital is within the reach of the taxman. The government taxes domestic capital at the rate $\tau$ and uses the revenue to provide services, so that the period-by-period government budget constraint is

$$g_t = \tau_t k_t$$

(3)

The representative individual maximizes the utility function

$$V_0 = \sum_{s=0}^{\infty} \left( \frac{0}{\alpha-1} \right)^{s-1} \beta^s, \quad 0<\beta<1, \quad \alpha>0.$$  

(4)

She is constrained by (1) and also by

$$\lim_{t \to \infty} b_t R^{-\gamma} > 0$$

(5)

which is the usual no-Ponzi-game condition. Control variables are $\epsilon$ and $k$. Acting atomistically, the representative individual takes the sequences $\{r_t\}_{t=0}^\infty$ and $\{d_t\}_{t=0}^\infty$ as given.

First order conditions are

8 The assumption that holdings of the internationally traded bond are untaxed is most natural in cases where $b$ is negative: if the representative agent is a net debtor vis à vis the rest of the world, her interest payments abroad will naturally not be taxed. But even if $b$ is positive this assumption is realistic: many countries (and especially developing countries) find it impossible to tax the assets domestic residents hold abroad.
\[ c_t^{-1/\sigma} - R \beta c_{t+1}^{-1/\sigma} \]  

(6a)

and

\[ R_t^k - R \]  

(6b)

where (6a) is the Euler equation and (6b) the standard requirement that the return on both assets be equalized.

What are the remaining equilibrium conditions? Price-taking, optimal behavior by the firm dictates that the rental rate must equal the marginal product of capital, computed, from the firm’s point of view, taking the sequences \( \{\tau_i\}_{i=0}^\infty \) and \( \{\varepsilon_i\}_{i=0}^\infty \) as given:

\[ R_t^k = k_t^{-1/\gamma} \alpha \gamma \left( \frac{y_t}{k_t^\gamma} \right)^{1/\rho} (1-\rho)^{-\gamma} \]  

(7)

Finally, since domestic capital only lasts one period and therefore the firm accumulates no assets, its dividends must equal its profits:

\[ d_t = y(k_t\varepsilon_t) - R_t^k k_t \]  

(8)

Notice from (2) that factoring and using (3) we can write

\[ y_t = k_t^\gamma \left[ \alpha (1-\tau_i)^{\rho \gamma} (1-\alpha) \varepsilon_t^{\rho \gamma} \right]^{1/\rho} = k_t^\gamma \phi(\tau_i) \]  

(9)

where \( \phi(\tau) \) is a concave function, with a unique maximum at \( \tilde{\tau} \), so that \( \phi(\tilde{\tau}) = 0 \). Using (9) in (7) and rearranging we have

\[ k_t = \left[ R^{-1/\gamma} \phi(\varepsilon_t)^{1/\rho} (1-\alpha)^{\rho \gamma} \right]^{1/\gamma} = k(\tau_t) \]  

(10)

which expresses desired domestic capital holdings as a function only of the expected tax rate. As long as the tax rate is expected to be constant, the stock of capital held at home will be constant as well. Let \( \bar{\tau} \) be given by \( k(\bar{\tau}) = 0 \). It is easy to compute that \( \bar{\tau} \) solves

7
\[
\frac{\phi'(\tau)}{\phi(\tau)} = (\epsilon - 1) \left( \frac{y}{1 - \epsilon} \right) 
\]

(11)

Notice, moreover, that using (9) and (10) domestic output can be written as \( y(\tau_p) k(\tau_p)^{\gamma} \phi(\tau_p) \).

Given the marginal products of both assets are equalized, the constant rate of return facing the representative agent is \( R \). Hence, and as indicated by (6a), consumption grows at a constant rate, which can be rewritten as \( c_{n1}/c_t = (R \beta)\sigma \). Hence, consumption growth is positive (negative) if \( R \) is larger (smaller) than \( \beta^{-1} \).

Substituting (8) into (1) we have

\[
b_{n1} = k_{n1} - Rb_t \cdot y(\tau_p) - c_t 
\]

(12)

Solving (12) forward, imposing solvency condition (5) and using (6a) and (6b) we obtain the following consumption function:

\[
c_t = \lambda \left[ Rb_t \cdot y(\tau_p) \cdot \sum_{n=1}^{\infty} [y(\tau_{n+1}) - Rk(\tau_{n+1})]^R \right] = \lambda \cdot R w_t 
\]

(13)

where \( \lambda = 1 - \beta^{-1} R \sigma \) and were we have also defined wealth as the stock of bonds plus the present value of future net domestic output. Hence, individuals consume a fixed proportion of their anticipated wealth. Updating (13) we can write

\[
\frac{c_{n1}}{c_t} = \frac{w_{n1}}{w_t} \cdot (R \beta)\sigma 
\]

(14)

where the second equality comes from (6a). Hence, wealth grows at the same rate as does consumption, and the economy always finds itself on a balanced steady state growth path. That completes the characterization of equilibrium for a given fiscal policy.
III. The Government's Problem under Precommitment

Under precommitment, the government's problem consists of choosing at time 0 the whole sequence of tax rates that will be in force for the infinitely long planning period. Using (13) we can write individual utility as a function of the tax rates as follows:

\[ V(b_0) = \lambda^{-1/\alpha} \left( \frac{\sigma}{\sigma - 1} \right) \left[ R b_0 \cdot \tau_0^{\lambda} \phi(\tau_0) + \sum_{\delta=1}^{\infty} [\nu(\tau_0) - R_k(\tau_0) R] - \frac{(\sigma - 1)}{\alpha} \right] \]  

(15)

where \( b_0 \) and \( k_0 \) are just given by history. government's first order conditions are

\[ \frac{\partial V(b_0)}{\partial \tau_0} = 0 \Rightarrow \lambda^{-1/\alpha} \cdot c_0 \cdot \phi(\tau_0) = 0 \]  

(16a)

and

\[ \frac{\partial V(b_0)}{\partial \tau_t} = 0 \Rightarrow y(\tau_t) - R_k(\tau_t), \quad \forall \tau > 0 \]  

(16b)

These conditions have a simple interpretation. During the first period, given that the stock of domestic capital is given by history, it is optimal to maximize output \( y_0 \) by maximizing \( \phi(\tau_0) \).

Recall \( \xi \) is given by \( \phi(\xi) = 0 \). It is simple to compute that

\[ \xi = \frac{1}{\left( \frac{\alpha}{1 - \alpha} \right)^{1/\phi}} \]  

(17)

We will refer to \( \xi \) as the "output-maximizing" tax rate. Notice that if \( \rho \rightarrow 0 \), so that the production function becomes Cobb-Douglas, we obtain \( \xi = 1 - \alpha \), which is the "productive efficiency" condition in Barro (1990).

In the following periods, given that the supply of domestic capital is endogenous and depends on anticipated tax rates, the government acts as a Stackelberg leader, maximizing utility subject to the individual policy functions (10) and (13). At the core of the government's problem
is a tradeoff between maximizing domestic output and minimizing the distortion caused by the tax on domestic capital.

After some tedious algebra, and using the definitions of $\epsilon$ and of the $\gamma(\tau_0)$ and $\xi(\tau_0)$ functions, (16b) becomes

$$\frac{\phi'(\tau_0)}{\phi(\tau_0)} = (\epsilon - 1) \left[ \frac{\gamma(1-\psi(\tau_0))}{\gamma[1-\psi(\tau_0)]} - \psi(\tau_0) \right] \left[ \frac{1}{1-\gamma} \right]$$

(18)

where $\psi(\tau) = \frac{\epsilon}{(1-\alpha)\tau^\alpha}$, so that $0 < \psi(\tau_0) < 1$. Let $\tau_*$ be the value of $\tau$ that solves (18).

Comparing (11) and (18) it is easy to see that $\tau_0 > \tau_*$ as $\epsilon > 1$. We will refer to $\tau_*$, for reasons that will become evident shortly, as the "orthodox" tax rate. That is the tax rate that is chosen from period 1 onwards.

Summarizing, we have

**Result 1**: The solution to the government's problem under precommitment is: during the initial period choose the output-maximizing tax rate $\tau_0$; after the initial period and forever choose a constant tax rate given by $\tau_*$.

If the government implements this optimal program, individual utility is given by

$$v^P(b_0) = 1^{1/\sigma} \left[ \frac{\sigma}{\sigma - 1} \right] \left[ Rb_0 + k_0^\gamma \phi(\tau) \cdot \left( \frac{1}{R-1} \right) M(\tau) - Rk(\tau) \right] \left[ \frac{\epsilon}{\sigma} \right]$$

(19)

where the superscript "p" stands for precommitment. This is below the level of utility that could be attained by a benevolent central planner who could dictate domestic investment decisions to private individuals. Inefficiency arises because of the fiscal externality. Since atomistic agents take the provision of public services as given, they do not internalize the effects of public spending on aggregate productivity. Under distortionary capital taxation, that leads them to hold too little domestic capital, and have levels of domestic output and investment that are inefficiently low.
IV. The Government's Problem under Discretion

In this case the government's problem is the same as before, except that it is free to reoptimize at every point in time. At time 0, the solutions to problems with and without precommitment coincides: the government maximizes (15) with respect to \( \tau_0 \), which leads to first order condition (16a) and to the result that \( \tau_0 = \tau \). solutions diverge starting in period 1. Then the government maximizes

\[
V(b_1) - \lambda^{1/\alpha} \left( \frac{\alpha}{\sigma - 1} \right) [Rb_1 + k_1^{\dagger} \phi(\tau_1) + \sum_{t=2}^{\infty} (\gamma^{(t)}) R^{(t)} k(t)]^{(\alpha + 1)/\alpha}
\]

(20)

with respect to \( \tau_1 \). Since, from the perspective of period 1, \( k_1 \) is given, it is optimal once again to set \( \phi'(\tau_1) = 0 \) and therefore \( \tau_1 = \tau \). Because the supply of domestic capital in period 1 has already been decided upon, the public can no longer react to that period's tax rate. Hence, it is optimal for the government to set \( \tau_1 \) to maximize period 1 output. It is easy to see that the same will occur from the vantage point of period 2 and from every successive period.\(^9\) We conclude that the optimal tax path described in Result 1 is time inconsistent. Every time it reoptimizes the government will choose the output-maximizing rate, so that \( \tau_r \neq \tau \). Therefore,

**Result 2:** The optimal tax rate under discretion is constant and given by \( \tau \). Understanding this, the public sets \( \tau_r \leq \tau \) \( \forall \tau \). Hence, the discretionary equilibrium is characterized by an expected and actual tax rate equal to \( \tau \) for all periods.

What are the consequences of discretion? If it is free always to reoptimize the government falls prey to the temptation of "populism": by attempting to maximize output at every point in time and attain the first best, the government reduces the ex-post marginal private return to

\(^9\) This intuition is identical to that found in the classic papers by Kydland and Prescott (1977), Calvo (1979) and Fischer (1980).
domestic investment. Agents with rational expectations understand this, and reduce their holdings of domestic capital. Hence, tax policy induces the "wrong" allocation between foreign and domestic capital. In a rational expectations equilibrium the marginal private return to domestic investment is equal to the international rate of interest, but the level of domestic capital is such that the total value of domestic output is less than it would be under the first best situation.

What is the effect of discretion on the actual level of the tax rate implemented by the government? In most models equilibrium tax rates under discretion are higher than the tax rates that would be chosen under precommitment. Hence, the time inconsistency story is often mentioned as a reason why taxes in the real world may be inefficiently high. That is not necessarily the case in the model of this paper: discretion may lead to tax rates that are inefficiently high or inefficiently low.

To see this recall the tax rate that prevails under orthodoxy is the one that solves (18). Figures 1a and 1b plot the function \( \phi(\tau) \). By definition of \( \hat{\tau} \), this function has a maximum at \( \hat{\tau} \). Equation (18) requires that if \( \varepsilon > 1 \) then \( \phi'(\hat{\tau}) > 0 \), and vice versa. Inspection of Figures 1a and 1b reveals that

\[
\begin{align*}
\text{If } \varepsilon > 1 \text{ and therefore } \phi'(\hat{\tau}) & > 0 \text{ then } \hat{\tau} > \tau_* \\
\text{If } \varepsilon = 1 \text{ and therefore } \phi'(\hat{\tau}) & = 0 \text{ then } \hat{\tau} = \tau_* \\
\text{If } \varepsilon < 1 \text{ and therefore } \phi'(\hat{\tau}) & < 0 \text{ then } \hat{\tau} < \tau_*
\end{align*}
\]

Hence, if the elasticity of substitution is smaller (larger) than unity the populist tax rate is below (above) the orthodox tax rate; the two tax rates coincide whenever the elasticity is one (the Cobb-Douglas case). We therefore have

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10 That result is particularly striking in the literature on the taxation of money balances: Calvo (1978) and Obstfeld (1989). Under precommitment, a government bent on maximizing revenue from inflation typically chooses a finite and constant rate; under discretion, on the other hand, the rate of inflation is typically infinite.

11 That is the special case considered by Barro (1990).

12 For a set of related results, see Leonard (1994).
**Result 3:** Depending on the elasticity of substitution in production between private and public inputs, populism can be of the right-wing or the left-wing variety. Right-wing populism occurs when the elasticity of substitution is below one, so that the tax rate under discretion is above the orthodox rate. Left-wing populism occurs when the elasticity of substitution is above one, so that the tax rate under discretion is below the orthodox rate.

The economics behind this result is straightforward. The expected private after-tax marginal return to domestic capital depends on the tax rate and the volume of government-provided services. Under precommitment, the government takes into account the investor's reaction to anticipated tax rates in deciding upon an announced tax rate. But once capital is in place, one of two things can happen. If private capital and public services enter the production function inelastically ($\epsilon < 1$), output is maximized by taxing less than under precommitment and providing fewer services. That means that if investors had anticipated the orthodox tax rate, they will be confronted with a populist tax rate that is below what they had expected. If private capital and public services enter the production function elastically ($\epsilon > 1$), on the other hand, output is maximized by taxing more than under precommitment and providing more public services. That means that if investors had anticipated the orthodox tax rate, they will be confronted with a populist tax rate that is unexpectedly high. In both cases, however, the after-tax private return to domestic capital will be below the world rate of interest, leading investors to feel "cheated."

The model can therefore encompass a variety of real-life experiences. It is tempting to associate the right-wing populist alternatives with governments that arguably maximized short term capital inflows and output at the expense of an appropriate supply of government services (the United States under Reagan? Chile under Pinochet?) and left-wing populism with the opposite and also common phenomenon (Scandinavian social democracy? Perhaps the United States under Clinton?).

What are the welfare implications of populism? In the discretionary equilibrium individual utility is

\[
V^W(b_o) = \lambda^{-1/\alpha} \left( \frac{\alpha_0}{\alpha - 1} \right) \left[ Rb_o + k_o^T \phi(k) + \left( \frac{1}{R - 1} \right) \left[ \alpha \left( \frac{\epsilon - 1}{\epsilon} \right) \right] \right] \]

(22)
where the superscript "np" stands for no precommitment. Contrast this with (19), which gives individual utility along the precommitment equilibrium. It is easy to check that \( v^P(b_0) > v^{np}(b_0) \).

Hence, discretion — and its offspring populism, whether of the right-wing or left-wing variety — is bad for individual welfare.

**V. Doing Better through Reputation**

Can governments ever escape the populist curse? One possibility is the building of reputation — or in the language of game theory, the utilization by agents in the private sector of history-dependent strategies.\(^{13}\) Consider the case in which the government hopes to sustain through reputation the optimal tax rate under precommitment, \( \tau_* \). Suppose, moreover, that the public forms expectations according to the following rule:

\[
\tau_{e,t} = \tau_* \quad \text{if} \quad \tau_{e,t} = \tau_*, \quad 1 \leq t \\
\tau_{e,t} = \tau_* \quad \text{otherwise}
\]  

(23)

where the superscript "e" denotes an expectation. Notice that, given that under both discretion and commitment \( \tau_{e,t} \), rule (23) does not include the tax rate implemented in period \( t-0 \).

If the government always sets \( \tau_{e,t} \forall t > 0 \), individual welfare starting at any period \( t > 0 \) is

\[
v^P(b_0) = \lambda^{-1/\sigma} \left[ Rb_{t-1} + \frac{\lambda}{\sigma-1} \left[ \frac{\sigma}{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right) \right] \right]^{(\sigma-1)/\sigma}
\]  

(24)

If at the end of period \( t-1 \) agents had expected \( \tau_{e,t} \), (24) specializes to

\[
v^P(b_0) = \lambda^{-1/\sigma} \left[ Rb_{t-1} + \left( \frac{R}{R-1} \right) \left[ \frac{\sigma}{\sigma-1} \right] \right]^{(\sigma-1)/\sigma}
\]  

(25)

\(^{13}\) For the use of such strategies in the context of a related optimal-tax problem, see Chari and Kehoe (1988).
What is the value to the government of deviating from this path? Continue assuming at the end of period \( t-1 \) agents had expected \( \tau_{-t} \). If the government deviates at the start of period \( t \), on the other hand, its best action is to set \( \tau_{-t} \) and maximize domestic output \( y_{t} k(\tau_{t}) \phi(\tau_{t}) / y(\tau_{t}) \).

Given rule (23), agents will expect that \( \tau_{-t} \) starting the next period and forever. What is the government's best response from \( t+1 \) onwards? At any time \( s \) such that \( s \geq t+1 \), the government faces the following problem. Maximize

\[
v(b_{s}) = \lambda^{1/\sigma} \left( \frac{\sigma}{\sigma-1} \left[ R b_{s} \cdot \bar{K} \phi(\tau_{s}) \cdot \left( \frac{1}{R-1} \right) y(\tau_{s}) - R k(\tau_{s}) \right] \right)^{\frac{\sigma-1}{\sigma}}
\]

with respect to \( \tau_{s} \), where value function (26) reflects the fact that private agents will expect \( \tau_{-t} \) forever and invest accordingly. For each \( s \) such that \( s \geq t+1 \), the solution to this problem is to set \( \tau_{-t} \forall s \geq t+1 \). Therefore, after period \( t \) the economy will revert to the stationary discretionary equilibrium, with \( \tau_{t} \) being expected and implemented forever.

During period \( t \), the period of the deviation, forward-looking agents will consume

\[
c_{t} = \lambda \left[ R b_{t} \cdot \bar{K} \phi(\tau_{t}) \cdot \left( \frac{1}{R-1} \right) y(\tau_{t}) - R k(\tau_{t}) \right]
\]

Therefore, individual welfare starting the period of the deviation can be written

\[
v^{d}(b_{t}; \tau_{t}) = \lambda^{1/\sigma} \left( \frac{\sigma}{\sigma-1} \left[ R b_{t} \cdot \bar{K} \phi(\tau_{t}) \cdot \left( \frac{1}{R-1} \right) y(\tau_{t}) - R k(\tau_{t}) \right] \right)^{\frac{\sigma-1}{\sigma}}
\]

where the superscript "d" stands for deviation and the second argument in the \( v^{d}(\cdot) \) function indicates that the deviation occurs from the tax rate \( \tau_{t} \).

The precommitment outcome will be sustainable when the following incentive compatibility constraint is satisfied: \( v^{p}(b_{t}) \leq v^{d}(b_{t}; \tau_{t}) \). Comparing (25) and (28) we see that

\( v^{p}(b_{t}) \geq v^{d}(b_{t}) \) if and only if
\[ k(\tau^*)\psi(\tau^*) - y(\tau^*) \leq \left( \frac{1}{R-1} \right) \left[ (y(\tau^*) - \bar{R}(\tau^*)) - (y(\tau) - \bar{R}(\tau)) \right] \]  \hspace{1cm} (29)

where the L.H.S. is the output gain in period \( t \) resulting from the deviation, while the R.H.S. is the present value of the net output loss associated with a return to the discretionary equilibrium starting in period \( t+1 \).

Notice that the discount rate, \( \beta \), does not enter inequality (29). Unlike other reputation-based models, in this model it is not the case that there a discount rate large enough (close enough to one) to ensure that the precommitment outcome is sustainable. reason is that, given the timing of moves, the current and future output effects of a deviation at time \( t \) are already incorporated into consumption at time \( t \). The deviation shifts up or down the whole profile of consumption starting at \( t \), and the shift is only due to the immediate change in the present value of domestic output. Hence, the rate at which utility from consumption in the future is discounted relative to the utility from consumption in the present is irrelevant.

We conclude:

**Result 4:** The orthodox tax rate and its corresponding equilibrium allocations can be sustained under expectations rule (23) if and only if inequality (29) is satisfied.

Suppose (29) does not hold. What is the best tax rate that can be sustained by reputation?\(^{14}\) The government's problem in this case is to maximize (15) subject to a generalized form of the incentive compatibility constraint: for every level of foreign assets the sequence of expected taxes must be such that the value of continuing along the optimal path is at least as large as the value of deviating from it. Given the structure of (23), this constraint is only relevant from period 1 onwards.

\(^{14}\) For an analysis of a similar problem, see Benhabib and Rustichini (1991).
This problem can be expressed as follows. Maximize

\[
V(b_0) - \lambda \sum_0 \left( \frac{a}{\sigma - 1} \right) \left[ R b_0 + k_0 \phi(t_0) - \sum_{\tau \geq 1} \left( V(\tau) - R \kappa(t_\tau) \right) \right]^{(\sigma - 1)}/\sigma
\]

with respect to the sequence \( \{\tau_\tau\}_{\tau=0}^\infty \), subject to the incentive compatibility constraint

\[
V_r(b_\lambda(\tau_\lambda)) \geq V^d(b_\lambda(\tau_\lambda)) \quad \forall \tau \geq 1
\]

where the superscript "s" stands for sustainable.\(^1\)

Note that maximizing (30) with respect to the sequence \( \{\tau_\tau\}_{\tau=0}^\infty \) implies maximizing the present value of net domestic output \( \kappa(t_\tau) R t_\tau \).

For period zero the solution of this simple: maximize \( \kappa(t_0) R t_0 \phi(t_0) \) by setting \( t_0 = \bar{t} \). For any \( t \geq 1 \) the solution to the government's problem is somewhat more complicated -- but we know the solution set is not empty, for \( t_\tau \bar{t} \) \( \forall \tau \geq 1 \) satisfies the constraints and is feasible by construction.

The following lemma is a first crucial step in characterizing the constrained optimum:

**Lemma 1:** The solution to the problem of maximizing (30) with respect to \( \{t_1, \tau_2, \tau_3, \ldots\} \), subject to (31), involves a constant tax rate \( t_\tau \bar{t} \) \( \forall \tau \geq 1 \).

**Proof:** See Appendix.

We now characterize the constant tax rate \( \bar{t} \). It is clear that \( \bar{t} \) must satisfy:

\[
\begin{align*}
\text{If } &\varepsilon > 1 \quad \text{then } \bar{t} \geq 0 \\
\text{If } &\varepsilon < 1 \quad \text{then } \bar{t} \leq 1
\end{align*}
\]

If such a tax rate is announced and the announcement is believed by agents, individual

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\(^1\)This problem is laid out formally in Appendix II.

\(^2\)Notice that in (30) the future values of \( k \) are written as a function of the contemporaneous tax rate. That is, the private sector capital demand function (10) is already incorporated into (30).
welfare starting at any \( t > 0 \) is:

\[
V^t(b, \tilde{t}) - \lambda^{-1/\sigma} \left( \frac{g}{\sigma-1} \right) \left[ Rb, (\frac{R}{R-1})[v(\tilde{t}) - k(\tilde{t})] \right]^{(\frac{\sigma-1}{\sigma})}
\]

(33)

If at some time \( t > 0 \) the government deviates and implements \( \tilde{t} \) instead, the economy reverts to the discretionary equilibrium starting at time \( t+1 \). Welfare as of period \( t \) is

\[
V^t(b, \tilde{t}) - \lambda^{-1/\sigma} \left( \frac{g}{\sigma-1} \right) \left[ Rb, k(\tilde{t})\phi(\tilde{t}) + \left( \frac{1}{R-1} \right)[v(\tilde{t}) - Rk(\tilde{t})] \right]^{(\frac{\sigma-1}{\sigma})}
\]

(34)

This is, of course, the corresponding deviation value.

If inequality (29) is not satisfied -- so that the value of continuation if never as large as the value of deviation whenever the first best tax rate is implemented-- then the incentive compatibility constraint is always binding after the initial period. That is to say, (31) must be satisfied with equality for \( \tau > 1 \). Inspection of (33) and (34) reveals that this requires

\[
[k(\tilde{t})\phi(\tilde{t}) - y(\tilde{t})] - \left( \frac{1}{R-1} \right) [(v(\tilde{t}) - Rk(\tilde{t})) - (y(\tilde{t}) - Rk(\tilde{t}))]
\]

(35)

which has an interpretation analogous of that of (29), and where \( \tilde{t} \) is defined by (35). Let

\( T(\tau) = [k(\tau)\phi(\tau) - y(\tau)] \)

be the temptation to deviate from the specified path; let

\( E(\tau) = (R-1)^{1} [(v(\tau) - Rk(\tau)) - (y(\tau) - Rk(\tau))] \)

be the enforcement. The following facts about these functions are instructive:

i) Given that we are considering cases in which the first best is not sustainable, \( T(\tau) > E(\tau) \).

ii) \( T(\tau) > E(\tau) \) by construction.

iii) \( E'(\tau) - (R-1)^{1} [v'(\tau) - Rk'(\tau)] \). Therefore, \( E(\tau) \) has a unique maximum at \( \tau \). It is also the case that \( E'(\tilde{\tau}) > 0 \) as \( y'(\tilde{\tau}) - Rk'(\tilde{\tau}) > 0 \).
iv) \( T'(\tau) - \left[ \frac{\phi(\xi)}{\phi(\tau) \psi(\tau)} \right] Rk'(\tau) - y'(\tau) = \left[ \frac{\phi(\xi)}{\phi(\tau) \psi(\tau)} - 1 \right] Rk'(\tau) - [y'(\tau) - Rk'(\tau)]. \) Note for future reference that \( \frac{\phi(\xi)}{\phi(\tau) \psi(\tau)} \rightarrow 1 \) always, since \( \phi(\tau) \) is maximized at \( \tau \) and \( 0 < \phi(\tau) < 1 \) for any tax rate function \( T(\tau) \) has two extrema. It is easy to check that \( T'(\tau) = 0 \), and that \( \tau \) is a minimum of \( T(\tau) \). Other extremum occurs at \( \tau_* \), which is a maximum of \( T'(\tau) \). It is important to note that \( \tau_* > \tau < \tau_* \) as \( \epsilon > 1 \). This is because the derivative \( T'(\tau) \) can only be zero if the terms \( \left[ \frac{\phi(\xi)}{\phi(\tau) \psi(\tau)} - 1 \right] Rk'(\tau) \) and \( [y'(\tau) - Rk'(\tau)] \) have opposite sign. Given that \( k'(\tau - 0), \frac{\phi(\xi)}{\phi(\tau) \psi(\tau)} \rightarrow 1, y'(\tau) \cdot Rk'(\tau) \rightarrow 0, \) and \( \tau < \tau_* \) as \( \epsilon > 1 \), the result follows.

v) Since \( E'(\xi) > T'(\xi) \) as \( y'(\xi) \cdot Rk'(\xi) \rightarrow 0 \) and \( y'(\xi) \cdot Rk'(\xi) \rightarrow 0 \) as \( \epsilon > 1 \), it follows that \( E'(\xi) > T'(\xi) \) as \( \epsilon > 1 \).

vi) If \( \epsilon < 1, E'(\tau) < T'(\tau) \forall \tau > \tau* \); and if \( \epsilon > 1, E'(\tau) > T'(\tau) \forall \tau < \tau* \). Take the case of \( \epsilon < 1 \).

Having \( \tau > \tau* \) implies \( T'(\tau) < 0 \) and therefore \( \left[ \frac{\phi(\xi)}{\phi(\tau) \psi(\tau)} \right] Rk'(\tau) < y'(\tau) \). Using this fact in the expressions for \( E'(\tau) \) and \( T'(\tau) \), recalling that in the range \( \epsilon > \tau* \), we have \( k'(\tau) < 0 \), yields the result. Identical arguments, with reversed signs, hold for the case of \( \epsilon > 1 \).

These observations lead to Figures 2a and 2b, which correspond to the cases of \( \epsilon < 1 \) and \( \epsilon > 1 \). Consider first the case where \( \epsilon < 1 \), so that \( \tau < \tau* \) and \( y'(\xi) \cdot Rk'(\xi) > 0 \). Starting at \( \xi - \tau \) and raising \( \tau \) slightly, so that we come closer to \( \tau* \), both the value of enforcement and temptation rise, but that of enforcement rises more quickly. Hence, in the neighborhood just above \( \tau \), \( E(\xi) > T(\xi) \). At the same time, we know that at \( \tau - \tau* \), \( E(\xi) < T(\xi) \). Hence, in the interval between \( \tau \) and \( \tau* \) there
exists at least one $\bar{\tau}$ such that $E(\tau) - T(\tau)$. This $\bar{\tau}$ is clearly unique in that interval, for $T(\tau)$ only reaches a maximum and bends downward at $\tau_0$. Moreover, there can be no other crossing of the two schedules, for $T(\tau_0) > E(\tau_0)$ and $T(\tau)$ is flatter than $E(\tau)$ everywhere to the right of $\tau_0$. Notice, finally, that the preceding arguments imply that $E'(\bar{\tau}) - T'(\bar{\tau}) < 0$.

The same arguments, but with reversed inequalities, hold in the case of $\epsilon > 1$. The unique $\bar{\tau}$ such that $E(\bar{\tau}) - T(\bar{\tau})$ is to the left of $\tau_0$. Notice also that $E'(\bar{\tau}) - T'(\bar{\tau}) > 0$.

In short, we have:

**Result 5**: Under expectations rule (23), and if the first-best tax rate is not sustainable, the best sustainable tax rate is a constant $\bar{\tau}$ that sets $E(\bar{\tau}) - T(\bar{\tau})$. This constant tax rate lies between the orthodox and the populist rates.

Hence, even if the orthodox tax rate cannot be sustained, reputation can help enforce a constant tax rate that is welfare-superior to the populist tax rate that occurs under discretion. It depends on parameter values, however, how close to or far away from the populist tax rate this best sustainable $\bar{\tau}$ turns out to be. For certain parameter values, we may observe taxes under a reputational equilibrium that are close to or far away from the populist level.

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17 This follows from (iii) and (iv) above.
VI. Conclusions

Who is right about the best policy to maximize individual welfare? Those who emphasize public services at the expense of higher taxes or those who emphasize lower taxes at the expense of less public services? As is often the case in economics, the truth lies somewhere in the middle: there is a best "orthodox" tax rate that optimizes across this tradeoff. But this orthodox tax rate turns out to be time inconsistent, and therefore not necessarily implementable if the government does not have access to a commitment technology. Without the ability to precommit, the government ends up implementing a "populist" tax rate, which may be higher or lower than the orthodox one. Welfare is lowered as a result. Under trigger strategies, a constant tax rate that is welfare-preferred to the populist tax rate may be implementable, but this sustainable tax rate need not coincide with the orthodox one. There is, however, a constant second-best tax rate which lies somewhere the populist and orthodox extremes.
Appendix

I. Proof of Lemma 1

If inequality (29) in the text is not satisfied --so that the value of continuation is never as large as the value of deviation whenever the first best tax rate is implemented-- then the incentive compatibility constraint is always binding after the initial period. That is to say, (31) in the text must be satisfied with equality:

\[ V^*(b, \xi_o) - V^*(b, \xi_i) \quad \forall i \geq 1 \]  \hspace{1cm} (1)

where the sequence \( \{\xi_i\}_{i=1}^{\infty} \) is the sequence of second-best tax rates. What restrictions does (1) imply on the sequence \( \{\xi_i\}_{i=1}^{\infty} \)? The R.H.S. of (1) is

\[ V^*(b, \xi) = \lambda^{1/\sigma} \left( \frac{\alpha}{\rho - 1} \right) \left[ R h \cdot k(\xi)^{\gamma} \Phi(\xi) + \left( \frac{1}{R-1} \right) y(\xi) - R k(\xi) \right] \]  \hspace{1cm} (2)

The L.H.S. is

\[ V^*(b, \xi) = \lambda^{1/\sigma} \left( \frac{\alpha}{\rho - 1} \right) \left[ R h \cdot y(\xi) - \sum_{t=1}^{\infty} [y(\xi_{t+1}) - R k(\xi_{t+1})] R^{-t} \right] \]  \hspace{1cm} (3)

If (1) holds at time \( t \), (2) and (3) reveal that it must be the case that

\[ y(\xi) \cdot \sum_{t=1}^{\infty} [y(\xi_{t+1}) - R k(\xi_{t+1})] R^{-t} = k(\xi)^{\gamma} \Phi(\xi) + \left( \frac{1}{R-1} \right) y(\xi) - R k(\xi) \]  \hspace{1cm} (4)

At time \( t+1 \), an analogous relationship must hold:

\[ y(\xi_{t+1}) \cdot \sum_{t=1}^{\infty} [y(\xi_{t+1}) - R k(\xi_{t+1})] R^{-t} = k(\xi_{t+1})^{\gamma} \Phi(\xi_{t+1}) + \left( \frac{1}{R-1} \right) y(\xi_{t+1}) - R k(\xi_{t+1}) \]  \hspace{1cm} (5)

Subtracting (5) from (4) and rearranging we have
\[ y(\bar{\tau}_{n+1}) - Rk(\bar{\tau}_{n+1}) = y(\bar{\tau}_{n}) - R(\bar{\tau}_{n}) + \phi(\bar{\tau}_{n}) \] (6)

which is the difference equation the sequence \((\bar{\tau}_{n})\) must follow if (1) is to be satisfied.

Recall from the text \(\bar{\tau}\) is the constant second-best tax rate -- which, of course, must correspond to a steady state of (6). As also stated in the text, \(\bar{\tau}\) must satisfy:

\[
\begin{align*}
\text{If } \epsilon > 1 & \quad \text{then } \bar{\tau} \geq 0 \\
\text{If } \epsilon < 1 & \quad \text{then } \bar{\tau} \leq 1
\end{align*}
\] (7)

The text established that within this interval a steady state of (6) exists and is unique. Next we establish that dynamics are unstable around the unique steady state, so that the only allowable trajectory involves a constant tax rate.

Define

\[
\omega(\tau) = \frac{y'(\tau) - Rk'(\tau)}{Rk'(\tau) \left[ 1 - \frac{\phi'(\tau)}{\phi(\tau)\psi(\tau)} \right]}
\] (8)

Along (6) we can easily compute

\[
\frac{d\bar{\tau}_{n+1}}{d\bar{\tau}_{n}} = R \left[ 1 + \omega(\bar{\tau}) \right]
\] (9)

Is \(R[1 + \omega(\bar{\tau})]\) larger or smaller than one? The text establishes that \(y'(\tau) - Rk'(\tau) > 0\) as \(\epsilon > 1\).

Computing those derivatives and rearranging, using the fact that \(y'(\tau) - Rk'(\tau) < 0\) as \(\epsilon < 1\), it is straightforward to show that \(R[1 + \omega(\bar{\tau})] > 1\), regardless of the value of the elasticity. We conclude that around the single steady state the difference equation is unstable: if the system starts above \(\bar{\tau}\) the tax rate will increase over time and if it starts below \(\bar{\tau}\) it will decrease. Figures 3a and 3b depict this situation.

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18 It is not clear whether the graph of the difference equation is always convex or concave. Hence, the cases depicted in Figure 3 are just possibilities. In any case, no arguments presented
Hence, if the system starts at a point other than \( \bar{r} \), there are two possible outcomes: i) if \( \epsilon < 1 \) and the system starts below \( \bar{r} \), or if \( \epsilon > 1 \) and the system starts above \( \bar{r} \), then the tax rate reaches \( \bar{r} \) in finite time; such trajectories cannot be candidates for the second best, for they clearly yield lower utility than a path along which taxes remain constant at \( \bar{r} \); ii) if \( \epsilon < 1 \) and the system starts above \( \bar{r} \), then the tax rate reaches 1 in finite time; if \( \epsilon > 1 \) and the system starts below \( \bar{r} \), then the tax rate reaches 0 in finite time; it is clear that in either case such paths are inadmissible, for eventually the tax rate settles on a constant level that violates (1). The only admissible path, therefore, is one in which the tax rate begins at \( \bar{r} \) in period 1 and remains there forever. Q.E.D.
II. Formal Statement of the Second Best Problem

The Lagrangean that corresponds to the second-best problem is:

$$L = \lambda^{-\alpha} \left( \frac{\sigma}{\sigma - 1} \right) \left[ R b_{o} [k_{o}^{T} \phi(\tau_{o}) - k(\tau_{o})] \right] \cdot \sum_{t=1}^{\tau_e} \left[ \gamma(\tau_{i}) - k(\tau_{i}) \right] \left[ \frac{\sigma_{i}}{\sigma} \right]$$

$$+ \sum_{t=1}^{\tau_e} \mu_{t} (R_{t})^{y} \lambda^{-\alpha} \left( \frac{\sigma}{\sigma - 1} \right) \left[ R b_{t} \sum_{t=0}^{\tau_{e}} [\gamma(\tau_{i}) - k(\tau_{i})] R^{-\alpha} \right] \left[ \frac{\sigma_{i}}{\sigma} \right]$$

$$- \left( \frac{\sigma}{\sigma - 1} \right) \left[ R b_{t} [k(\tau_{e})^{T} \phi(t) - k(t)] (\frac{1}{\gamma(t)}) [\gamma(t) - k(t)] \right] \left[ \frac{\sigma_{i}}{\sigma} \right]$$

(10)

where the $\mu_{t}$'s are the multipliers associated to the sequence of inequality constraints.\(^{19}\)

We can compute, for any $\tau_{e}$, where $T$ is any time after 0:

$$\frac{\partial L}{\partial \tau_{e}} = (\epsilon_{c}^{s})^{-\alpha} \lambda^{-\alpha} R^{-T} [\gamma(\tau_{e}) - Rk(\tau_{e})] + \sum_{t=1}^{\tau_{e} - 1} \mu_{t} (R_{t})^{y} \lambda^{-\alpha} R^{-T} (\epsilon_{c}^{s})^{-\alpha} [\gamma(\tau_{e}) - Rk(\tau_{e})]$$

$$+ \mu_{T} (R_{T})^{y} \lambda^{-\alpha} (\epsilon_{c}^{s})^{-\alpha} \gamma^{y} (\tau_{e}) - \mu_{T} (R_{T})^{y} \lambda^{-\alpha} (\epsilon_{c}^{d})^{-\alpha} \phi(\gamma)(\tau_{e})^{y} - k(\tau_{e})$$

$$+ \sum_{t=1}^{\tau_{e} - 1} \mu_{t} (R_{T})^{y} \lambda^{-\alpha} \left[ \frac{\partial b_{t}}{\partial \tau_{e}} \right] [(\epsilon_{c}^{s})^{-\alpha} (\epsilon_{c}^{d})^{-\alpha}] - 0$$

(11)

where, once again, $\tau_{e}$ denotes the second-best tax rate at time $T$. "s" and "d" superscripts denote values taken by variables along the "sustainable" and "deviation" paths respectively.

Evaluating this derivative at the point where the constraint is binding, so that $\nu(\hat{b}_{e}(\tau_{e})) = 0$ and $\nu(\hat{d}_{e}(\tau_{e}))$ and therefore $c_{e}^{s} - c_{e}^{d}$, for all $t$, expression (11) reduces to

$$\sum_{t=1}^{\tau_{e} - 1} R^{T} \mu_{t} (R_{t})^{y} (\epsilon_{c}^{s})^{-\alpha} [\gamma(\tau_{e}) - Rk(\tau_{e})]$$

$$+ R^{T} \mu_{T} (R_{T})^{y} (\epsilon_{c}^{d})^{-\alpha} [\gamma(\tau_{e}) - \phi(\gamma)(\tau_{e})^{y} - k(\tau_{e})] - 0$$

(12)

\(^{19}\)Multiplying each $\mu_{t}$ by $(R_{t})^{y}$ is simply a convenient normalization.
Notice also that \((e_r)^{1/\alpha}(R_k)^{1/\alpha}\). Using this in (12) we obtain

\[
\left[ \phi(\bar{\tau}_p) - R_k^{\prime}(\bar{\tau}_p) \right] \left[ 1 + \sum_{t=1}^{T} R^{t} \mu_t \right] - R_t^{\prime} \mu_t \left[ \frac{\phi(\bar{\tau}_p)}{\phi(\bar{\tau}_p)\psi(\bar{\tau}_p)} - 1 \right] R_k^{\prime}(\bar{\tau}_p)
\]

(13)

Equation (13) involves \(\bar{\tau}_p\), the sequence of \(\mu\)'s and some parameters. Whenever the constraint is binding, the \(\mu\)'s can be chosen freely to ensure that (13) holds. To show that a constant tax rate satisfies this first order condition, we must show that there is a feasible sequence the \(\mu\)'s can follow so that (13) is satisfied by a constant \(\bar{\tau}\) for any \(T\).

Recall the definition of \(\omega(\bar{\tau})\). With this definition (13) becomes

\[
\omega(\bar{\tau}) = \frac{R_t^{\prime} \mu_t}{1 + \sum_{t=1}^{T} R^{t} \mu_t}
\]

(14)

Computing (14) for a time \(T+1\) and taking differences we see that \(\mu_t\) must follow the difference equation

\[
\mu_t = \left\{ \frac{1}{R[1+\omega(\bar{\tau})]} \right\} \mu_{t-1}
\]

(15)

Having established above that \(R[1+\omega(\bar{\tau})]>1\), we can infer from (15) that all the \(\mu\)'s have the same sign, and they go to zero as time goes to infinity.

Notice that since the L.H.S. of (14) is negative, so must be the R.H.S., and hence the numerator and denominator must be of opposite sign. This requirement is satisfied if all the \(\mu\)'s are negative and if in addition \(\sum_{t=1}^{T} R^{t} \mu_t - \mu_1 \left[ \frac{1 - R^T[1+\omega(\bar{\tau})]^T}{1 - R^T[1+\omega(\bar{\tau})]^T} \right] > -1\) for any \(T\). This can be ensured, once again, by an appropriate choice of \(\mu_1\)—for instance, by picking \(|\mu_1|<1-R^{-1}[1+\omega(\bar{\tau})]^{-1}\)
References


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