Bank of Portugal Lectures Summer 2006

Time-Varying Risk, Interest Rates,

and Exchange Rates

Based on work by Alvarez, Atkeson, and Kehoe

Data and Model

- Data
  - Interest rate differentials across countries variable and persistent
  - Exchange rates approximately random walks

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- Model
  - General equilibrium monetary model
  - Interest differentials variable and persistent
  - Exchange rates approximately random walks
  - Interest differentials all driven by risk

- Overall, the new view of finance amounts to a profound change. We have to get used to the fact that most returns and price variation comes from variation in *risk premia* (Cochrane 2001).

Interest rate differentials and exchange rates

- i_t, i_t^* nominal interest rates in dollars and euros (compounded)

- Log dollar return on euro bonds

$$i_t^* + E_t \log e_{t+1} - \log e_t$$

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- Excess return of euro bonds over dollar bonds

$$p_t \equiv i_t^* + E_t \log e_{t+1} - \log e_t - i_t$$

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- If e_t a random walk, then the interest differentials driven by variations in excess returns

$$i_t - i_t^* = \underbrace{\left[E_t \log e_{t+1} - \log e_t\right]}_{\text{constant}} - p_t$$

Our view: Interest rate differentials all driven by risk _____

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- Log dollar return on euro bonds

$$i_t^* + E_t \log e_{t+1} - \log e_t$$

- Excess return is a *risk premium*

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- If e_t a random walk, then the *interest differentials are risk premia*

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- Excess return is a *risk premium*

$$p_t \equiv i_t^* + E_t \log e_{t+1} - \log e_t - i_t$$

- For intuition

$$i_t^* - i_t = -\underbrace{\left[E_t \log e_{t+1} - \log e_t\right]}_{\text{constant}} + p_t$$

Risk varies even more than interest differentials

- High interest rate currencies tend to appreciate

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$$\mathrm{var}(p_t) \geq \mathrm{var}(i_t - i_t^*)$$

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- Intuition

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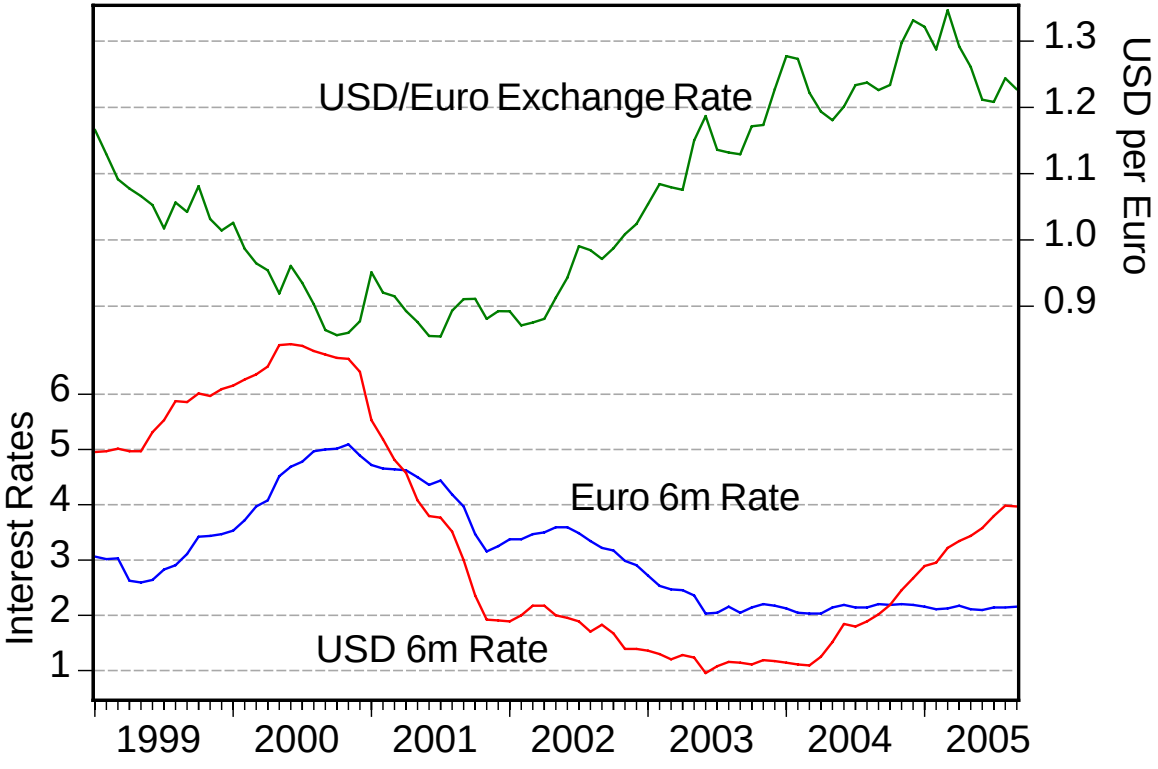
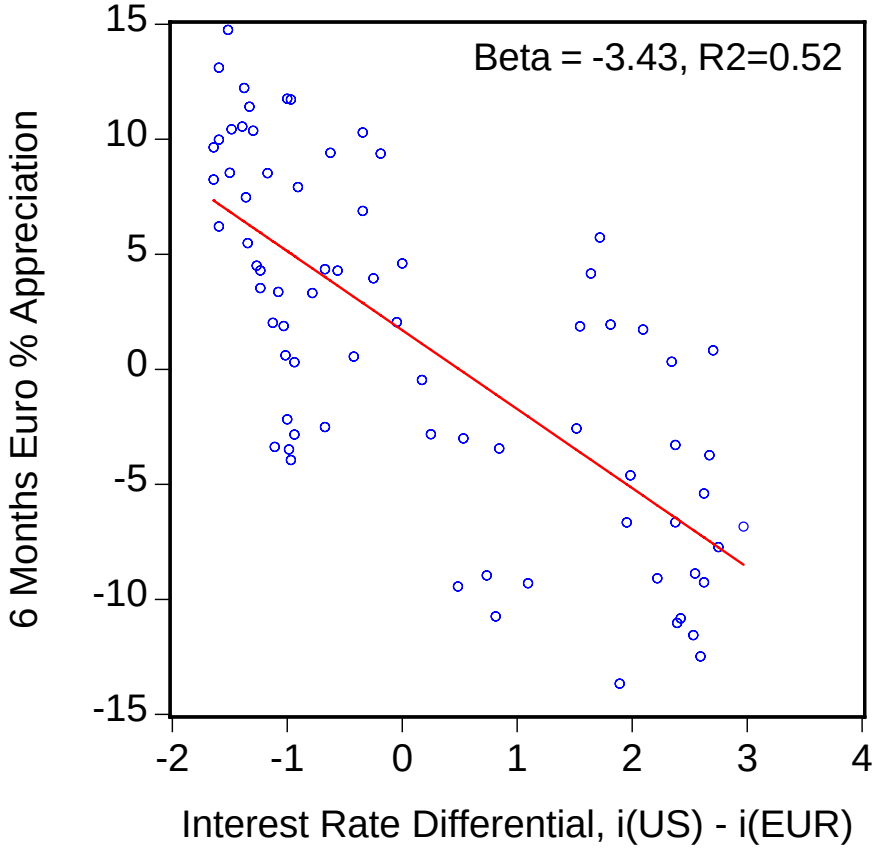
$$\mathrm{var}(p_t) \geq \mathrm{var}(i_t - i_t^*)$$

- Data: In Fama regression typical estimate $b < 0$

$$\log e_{t+1} - \log e_t = a + b(i_t - i_t^*) + u_{t+1}$$


USD/Euro Exchange Rate
Euro 6m Rate
USD 6m Rate
Interest Rates
USD per Euro
1
2
3
4
5
6
0.9
1.0
1.1
1.2
1.3
1999
2000
2001
2002
2003
2004
2005


Beta = -3.43, R2=0.52
6 Months Euro % Appreciation
15
10
5
0
-5
-10
-15
-2
-1
0
1
2
3
4
Interest Rate Differential, i(US) - i(EUR)

Model

- General equilibrium model
  - generates time-varying risk
  - intuition from pen and paper
  - direction for research, not quantitative solution to puzzle

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Model

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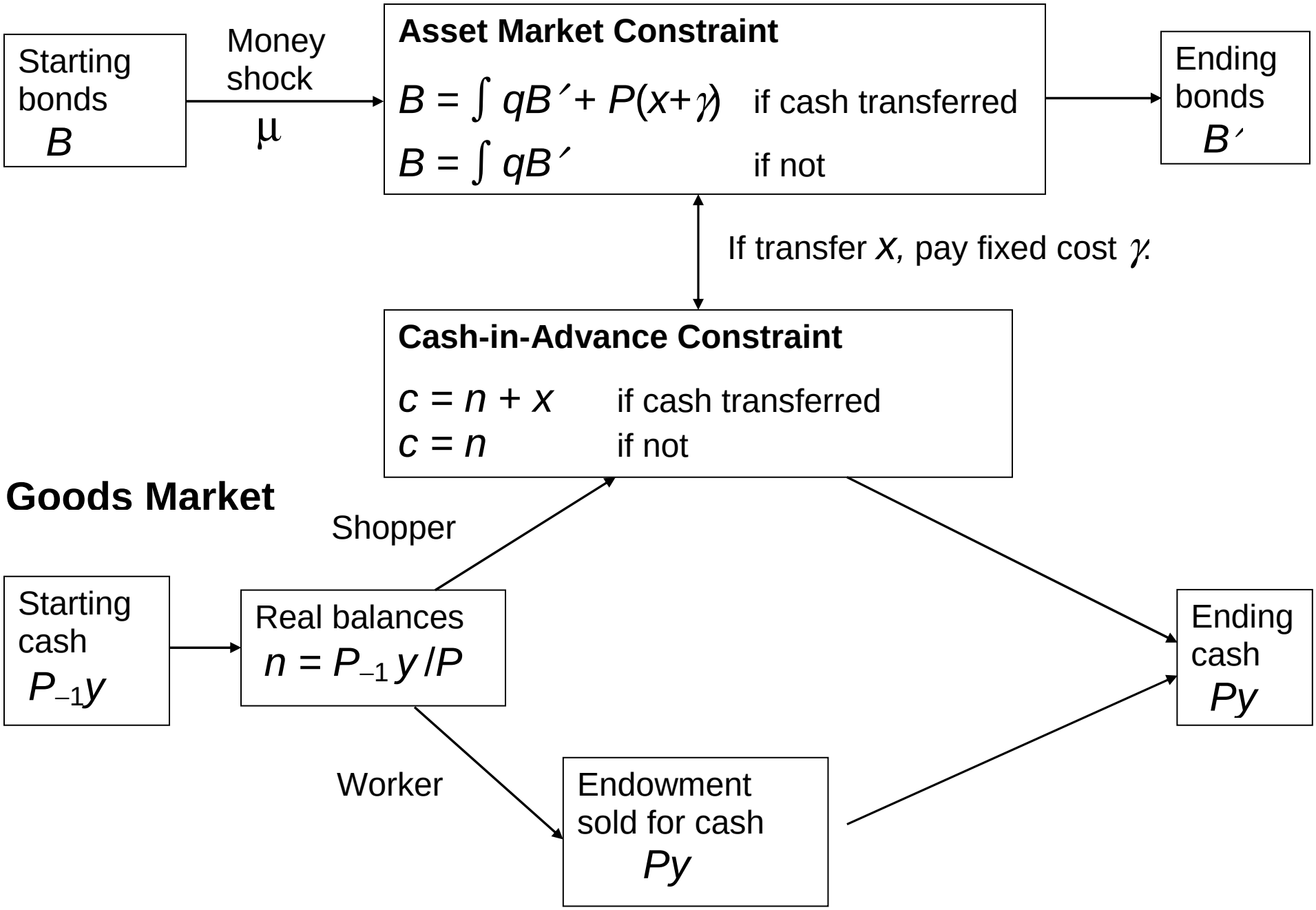
- No trade in goods
- Real fixed cost to transfer cash b/w asset and goods markets
  - each household has a constant fixed cost γ over time
  - $\gamma \sim f(\gamma)$ across households
- Only shocks are money growth shocks
  - $\mu(s^t)$ home, $\mu^*(s^t)$ foreign

Asset Market



Characterizing equilibrium

- CIA binds: $P(s^t) = M(s^t)/y$

- Cutoff rule for paying fixed cost

  pay if $\gamma \leq \overline{\gamma}(s^t)$

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$$\frac{P(s^{t-1})y}{P(s^t)} = \frac{y}{\mu(s^t)} \quad \text{where} \quad \mu(s^t) \equiv \frac{M(s^t)}{M(s^{t-1})}$$

  - consumption of active (pay γ)

  $c_A(s^t)$ regardless of γ

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  - Micro evidence

Equilibrium solves planning problem

Choose $c_A(\mu)$, $\overline{\gamma}(\mu)$ to solve

$$\max\ U(c_A(\mu))F(\overline{\gamma}(\mu)) + U(y/\mu)[1 - F(\overline{\gamma}(\mu))]$$

s.t.

$$c_A(\mu)F(\overline{\gamma}(\mu)) + \int_0^{\overline{\gamma}(\mu)} \gamma f(\gamma)d\gamma + \left(\frac{y}{\mu}\right)[1 - F(\overline{\gamma}(\mu))] = y$$

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- *Proposition* 2

  As μ increases, more households become active ($\overline{\gamma}'(\mu) > 0$)

Plan: Given allocations, now asset prices _____

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  - Formulas for interest rates, exchange rates, risk premia

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  - Second order approximation to pricing kernel
- How model can qualitatively account for data

Now

- Pricing kernel for nominal assets
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Pricing kernels and formula for interest rates

- Dollar assets, Euro assets

$$m_{t+1} = \frac{\beta U'(c_A(\mu_{t+1}))}{U'(c_A(\mu_t))} \frac{1}{\mu_{t+1}}, \quad m^*_{t+1} = \frac{\beta U'(c_A(\mu^*_{t+1}))}{U'(c_A(\mu^*_t))} \frac{1}{\mu^*_{t+1}}$$

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- Any asset with dollar return R_{t+1} at $t + 1$

$$1 = E_t \, m_{t+1} \, R_{t+1}$$

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- Euro bond

$$i^*_t = -\log E_t \; m^*_{t+1}$$

Formulas for expected depreciation and risk premium

- State-by-state arbitrage

$$m^*_{t+1} = \frac{e_{t+1}}{e_t} m_{t+1}$$

- Exchange rate depreciation

$$\log e_{t+1} - \log e_t = \log m^*_{t+1} - \log m_{t+1}$$

- Risk premium

$$p_t = E_t \log m^*_{t+1} - E_t \log m_{t+1} - (\log E_t m^*_{t+1} - \log E_t m_{t+1})$$

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Risk from Jensen inequality terms

Recap

- Pricing kernel for nominal assets
  - Formulas for interest rates, exchange rates, risk premia

Now

- How money shocks effect pricing kernel
  - Second order approximation to pricing kernel

How money shocks effect marginal utility

- How does marginal utility change with money growth?

$$\phi(\mu) \equiv \frac{-d \log U'(c_A(\mu))}{d \log \mu} = \sigma \frac{d \log c_A(\mu)}{d \log \mu} \quad \text{when} \quad U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

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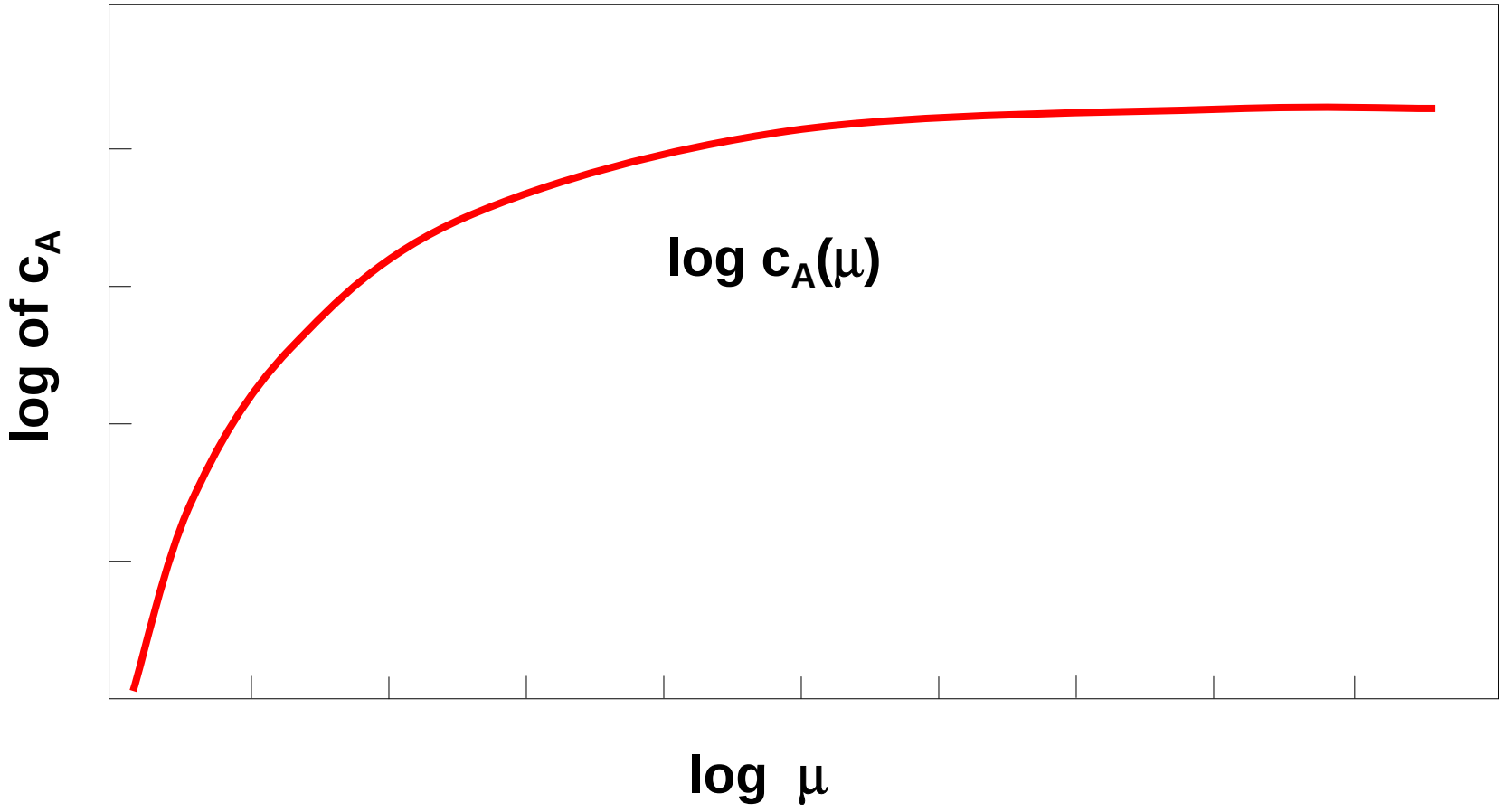
- Key to generating variable risk premia
  - $\text{Log}\, c_A(\mu)$ concave in $\log \mu$
  - Marginal utility of active households more sensitive to money growth at low levels of money growth
  - Variability of pricing kernel decreases with μ

The Log of the Consumption of Active Households



Second-order approximation to pricing kernel

- Approximate marginal utility

$$\log U'(c_A(\mu_t)) = \log U'(c_A(\bar{\mu})) - \phi\hat{\mu}_t + \frac{\eta}{2}\hat{\mu}_t^2$$

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where

$$\phi \equiv -\frac{d\log U'(c_A(\bar{\mu}))}{d\log\mu} = \sigma\frac{d\log c_A(\bar{\mu})}{d\log\mu}$$

$$\eta \equiv \frac{d^2\log U'(c_A(\bar{\mu}))}{(d\log\mu)^2} = -\sigma\frac{d^2\log c_A(\bar{\mu})}{(d\log\mu)^2}$$

Second-order approximation to pricing kernel

- Approximate marginal utility

$$\log U'(c_A(\mu_t)) = \log U'(c_A(\bar{\mu})) - \phi\hat{\mu}_t + \frac{\eta}{2}\hat{\mu}_t^2$$

- Implied pricing kernel

$$\log m_{t+1} = \log\left(\frac{\beta}{\mu}\right) - \phi\hat{\mu}_{t+1} + \frac{\eta}{2}\hat{\mu}_{t+1}^2 + \phi\hat{\mu}_t - \frac{\eta}{2}\hat{\mu}_t^2 - \hat{\mu}_{t+1}$$

Shock processes

- Shock processes

$$\hat{\mu}_{t+1} = E_t \hat{\mu}_{t+1} + \varepsilon_{t+1}, \qquad \hat{\mu}^*_{t+1} = E_t \hat{\mu}^*_{t+1} + \varepsilon^*_{t+1}$$

key: $\varepsilon_t, \varepsilon^*_t$ **constant conditional variances** (also normal and independent)

Recap

- Pricing kernel for nominal assets
- How money shocks effect pricing kernel

Now

- How model can qualitatively account for data
  - Proposition with sufficient conditions
  - Numerical example

Recap

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- How money shocks effect pricing kernel

Now

- How model can qualitatively account for data
  - Proposition with sufficient conditions
  - Numerical example
  - Cross country implications
  - Long-term forward premium

How model can qualitatively account for data

- 2 steps
  - money growth $\mu \uparrow \Rightarrow$ risk premium falls (a lot)
  - money growth $\mu \uparrow \Rightarrow$ expected depreciation falls (a little)

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    expect an appreciation

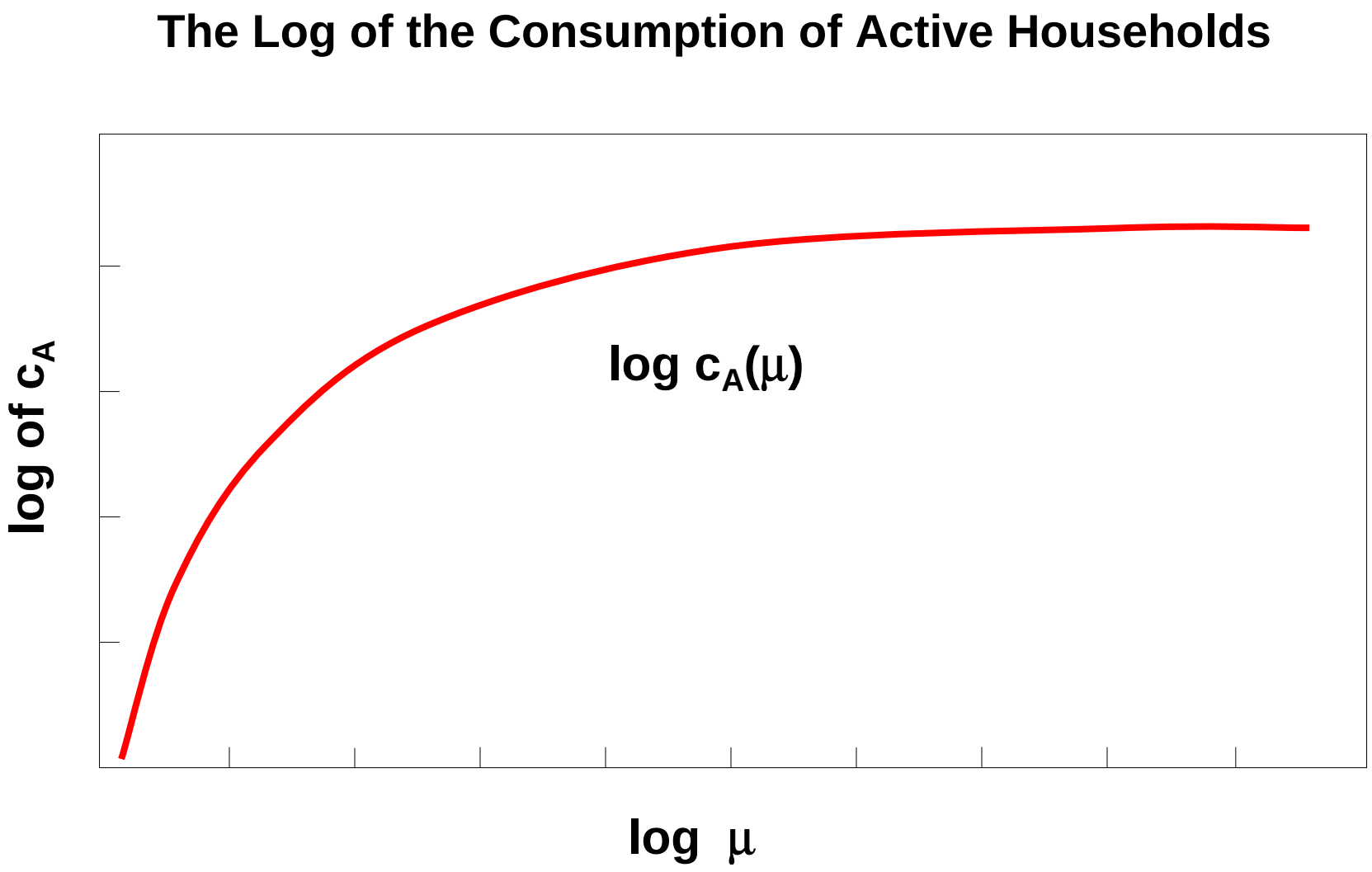
How model can qualitatively account for data

- 2 steps
  - money growth $\mu \uparrow \Rightarrow$ risk premium falls (a lot)
  - money growth $\mu \uparrow \Rightarrow$ expected depreciation falls (a little)

- Then money growth $\mu \uparrow \Rightarrow$ interest differential rises

$$i_t - i_t^* = [E_t \log e_{t+1} - e_t] - p_t$$

Step 1: Risk premium falls with money growth _____



Step 1: Risk premium falls with money growth

- *Proposition* 4. Under second-order approximation

$$p_t = \frac{1}{2}\,\frac{1}{1-\eta\sigma_\varepsilon^2}[\mathrm{var}_t \log m_{t+1} - \mathrm{var}_t \log m_{t+1}^*]$$

$$\mathrm{var}_t(\log m_{t+1}) = [-(1+\phi)+\eta E_t \hat{\mu}_{t+1}]^2 \sigma_\varepsilon^2 + \text{constant}$$

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- Risk premium falls with money growth

$$\frac{dp_t}{d\hat{\mu}_t} \propto -\eta\,\frac{dE_t\hat{\mu}_{t+1}}{d\hat{\mu}_t} < 0$$

  - when $\eta > 0$ ($\log c_A(\mu)$ concave in $\log\mu$)
  - and money growth persistent ($dE_t\hat{\mu}_{t+1} / d\hat{\mu}_t > 0$)

Step 2: Expected nominal depreciation falls with money growth _____

Nominal Depreciation		Real Depreciation		Inflation Differential
$\log\left(\frac{e_{t+1}}{e_t}\right)$	=	$\log\left(\frac{v_{t+1}}{v_t}\right)$	+	$\log\left(\frac{\mu_{t+1}}{\mu^*_{t+1}}\right)$

- money growth $\mu \uparrow$ implies
  - an expected real appreciation
  - inflation differential $\uparrow$
  - need expected real appreciation to dominate

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- money growth $\mu \uparrow$ implies
  - an expected real appreciation (expected real depreciation $\downarrow$)
  - inflation differential $\uparrow$
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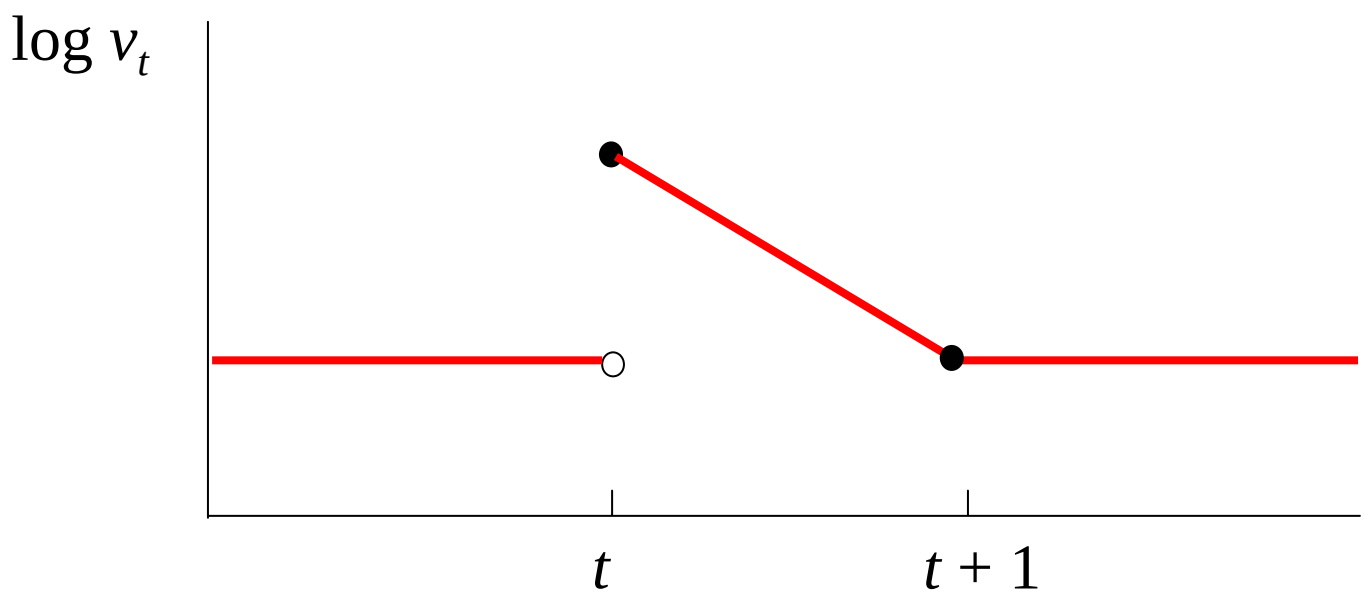
Step 2: Increase in money growth leads to expected appreciation ___

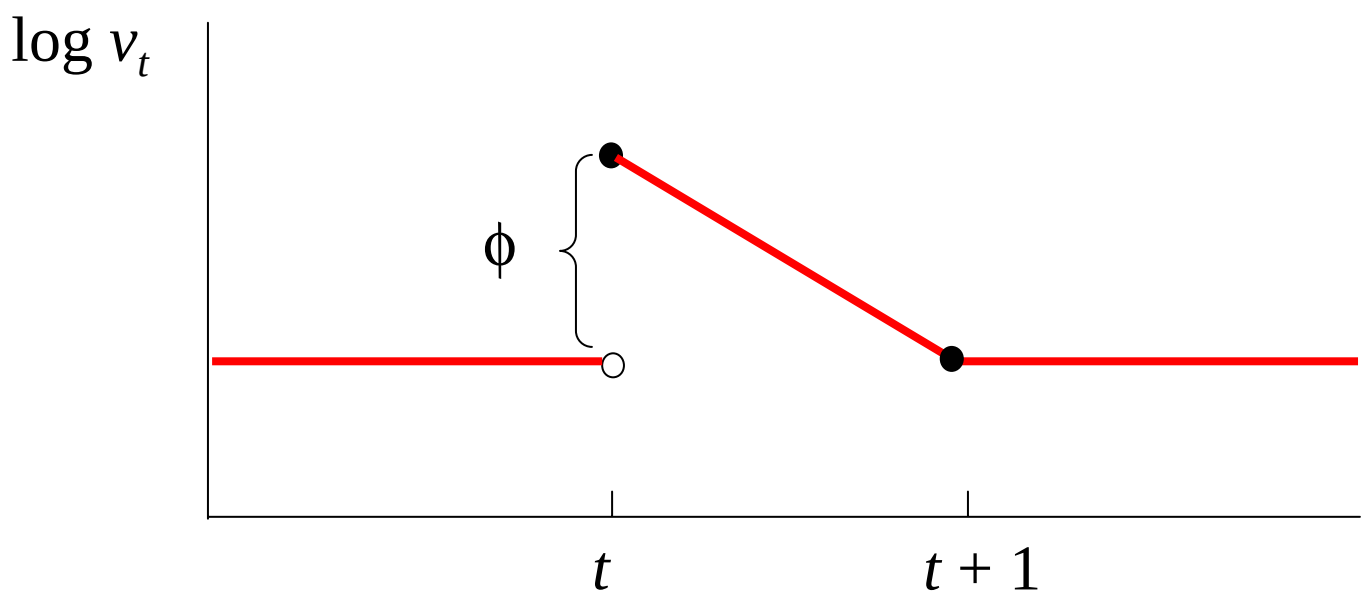
- Real exchange rate $v_t = U'(c_A^*(\mu_t^*))/U'(c_A(\mu_t))$
- For intuition let μ_t be i.i.d. and suppose $\mu_t \uparrow$



Step 2: Increase in money growth leads to expected appreciation ___

- Real exchange rate $v_t = U'(c_A^*(\mu_t^*))/U'(c_A(\mu_t))$

- So $d\log v_t / d\log \mu_t = -d\log U'(c_{At}) / d\log \mu_t = \phi$



Step 2: Need large enough expected real appreciation _____

- When μ_t is persistent

$$\frac{d}{d\hat{\mu}_t} E_t \log\left(\frac{v_{t+1}}{v_t}\right) = \phi\left[\frac{dE_t\hat{\mu}_{t+1}}{d\hat{\mu}_t} - 1\right] < 0$$

Conditions for high interest rate currencies to appreciate _____

- Want increase in money growth to decrease $E_t \log(e_{t+1} / e_t)$

- Want increase in money growth to increase $i_t - i_t^*$

Conditions for high interest rate currencies to appreciate _____

- Want increase in money growth to decrease $E_t \log(e_{t+1} / e_t)$
  - Need segmentation parameter ϕ large
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  - Need risk to fall enough with μ

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- Want increase in money growth to increase $i_t - i_t^*$
  - Need curvature parameter η large
- *Proposition* 5: If $1-\eta\sigma_\varepsilon^2 < \frac{1+\phi}{\phi}\frac{dE_t\hat{\mu}_{t+1}}{d\hat{\mu}_t} < 1$ then
  - $\mu_t \uparrow$ implies $i_t - i_t^* \uparrow$ (first inequality)
  - $\mu_t \uparrow$ implies $E_t \log e_{t+1} - \log e_t \downarrow$ (second inequality)

Numerical Example

- Money growth

$$\hat{\mu}_{t+1} = g(\hat{\mu}_t) + \varepsilon_{t+1} \qquad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2), \quad \mathrm{corr}(\varepsilon_t, \varepsilon_t^*) = \rho_\varepsilon$$

- Function g chosen so that exchange rate a random walk

$$E_t \log e_{t+1} = \log e_t$$

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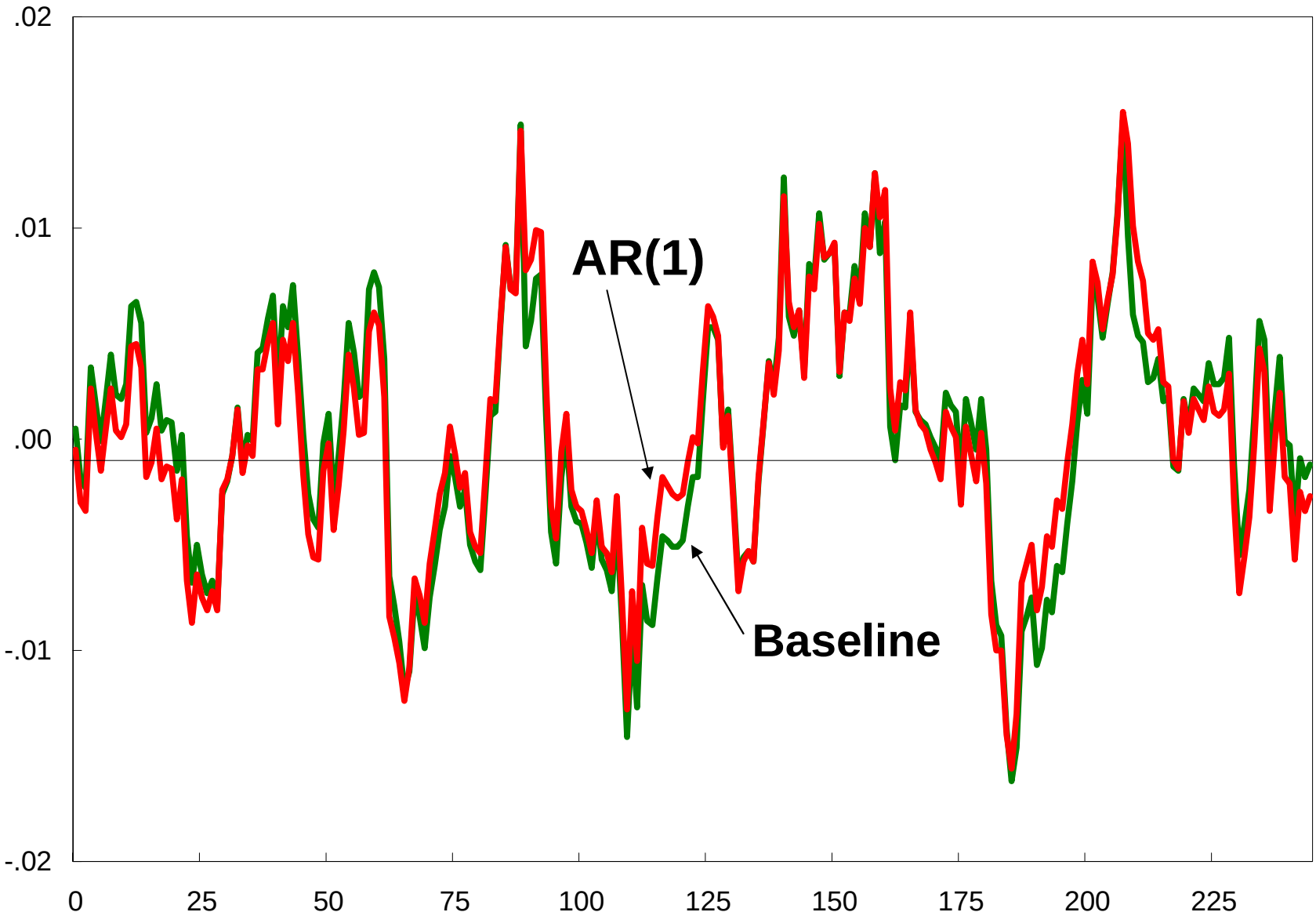
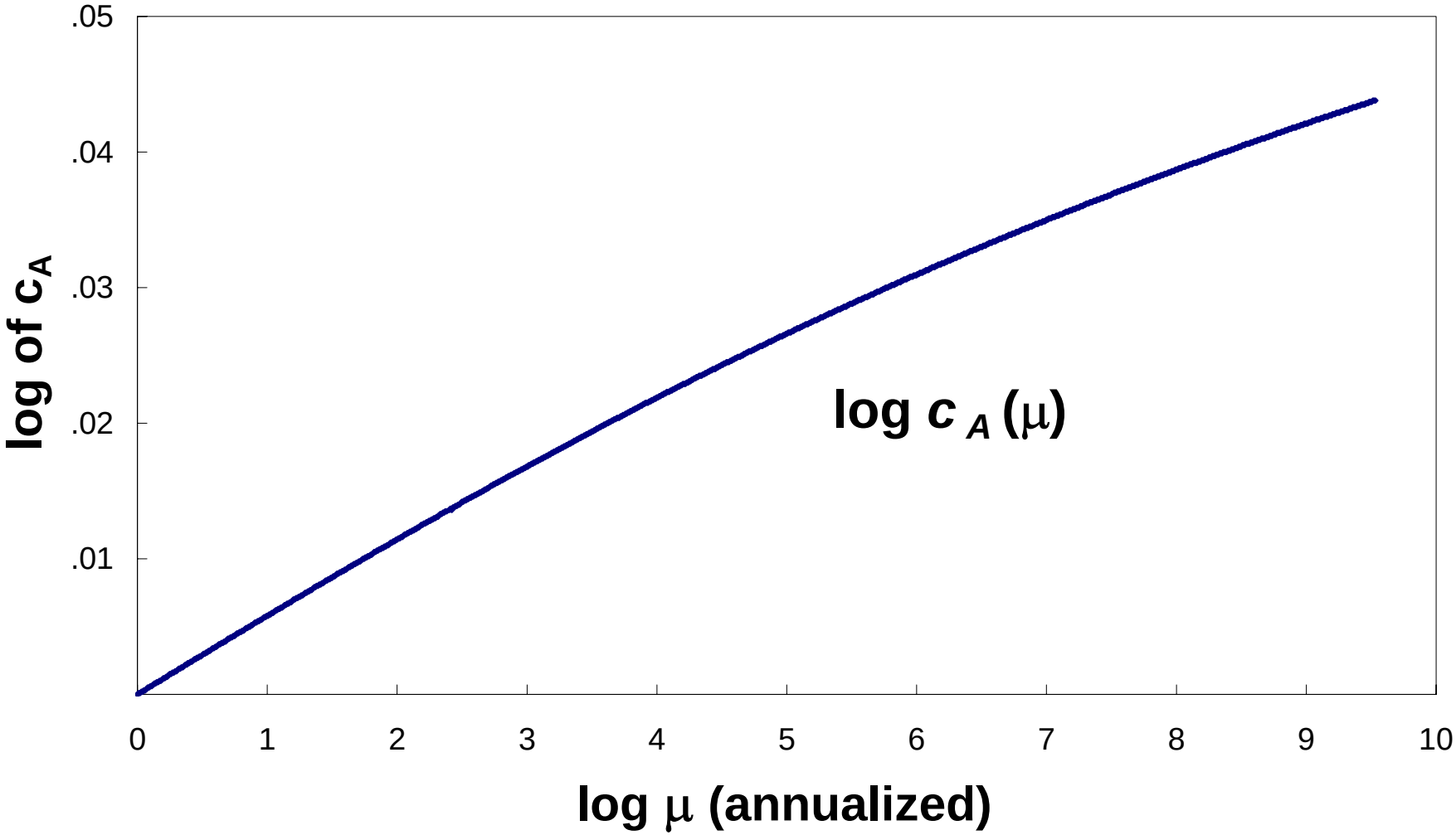
- Other parameters
  - $\sigma = 2$, $\bar{\mu} = 5\%$ annual
  - Money shocks: $\sigma_\varepsilon = .35\%$, $\mathrm{corr}(\varepsilon_t, \varepsilon_t^*) = 1/2$
  - Fixed costs: 12.5% have zero costs, rest have uniform costs on [0, 10%]
  - Gives $\phi = 10.9$, $\eta = 1007$

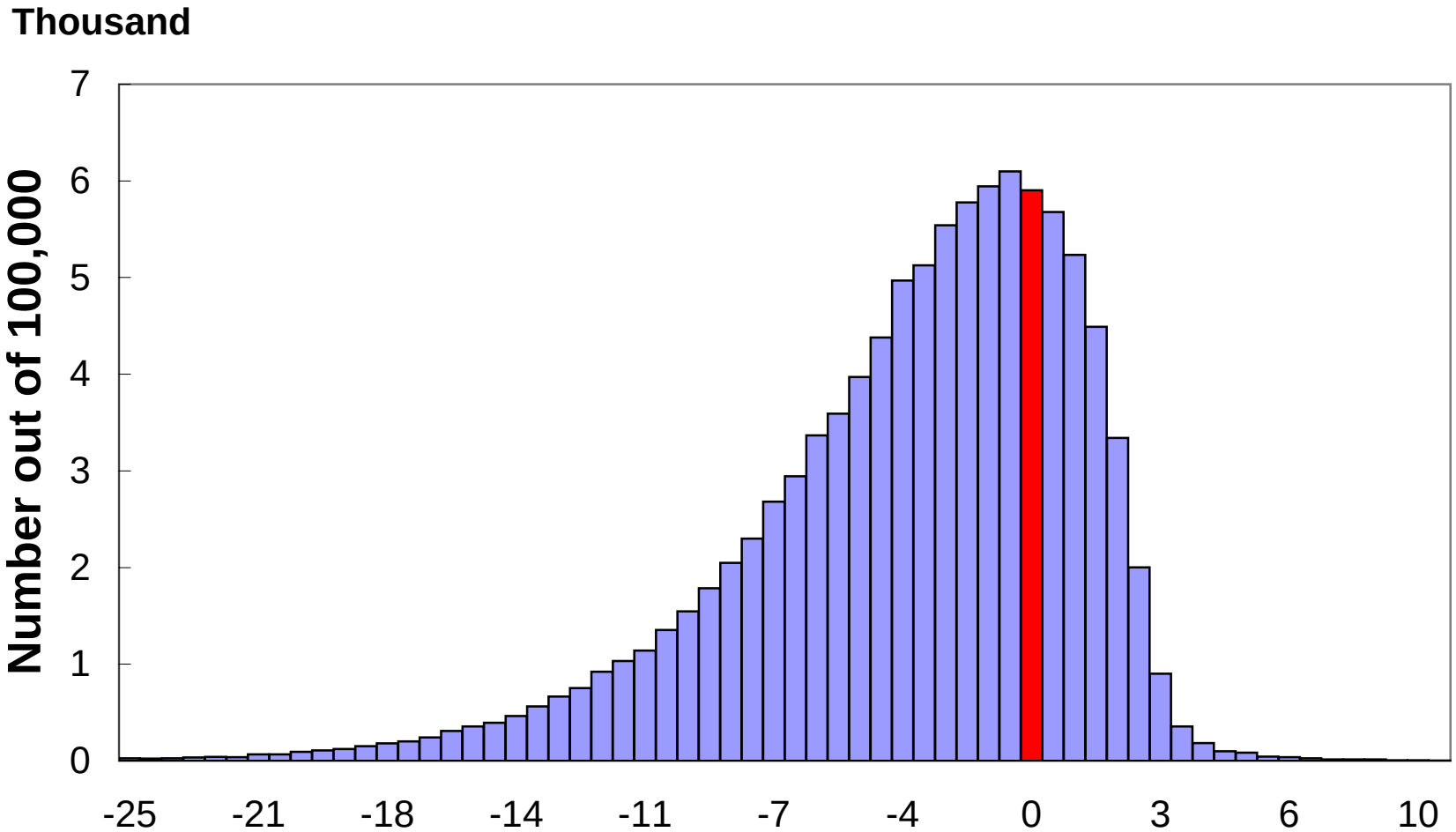
Realizations of Money Growth Using
Our Baseline Process and an AR(1) Process



The Log of the Consumption of Active Households



What the Model Implies for the Slope Coefficients in the Fama Regression



Slope Coefficient *b* of the Fama Regression
(Mean = –3.69, simulation length 245)

Properties of Exchange Rates and Interest Rates

Statistics/Variable	Data
Standard Deviations	
Exchange Rates $(\log e_{t+1} - \log e_t)$	36
Interest Rates $(i_t - i_t^*)$	3.5
Autocorrelations	
Exchange Rates $(\log e_{t+1} - \log e_t)$	.04
Interest Rates $(i_t - i_t^*)$	.83

Sources of data: Backus, Foresi, and Telmer (2001)

Properties of Exchange Rates and Interest Rates

Statistics/Variable	Data	Model
Standard Deviations		
Exchange Rates $(\log e_{t+1} - \log e_t)$	36	58
Interest Rates $(i_t - i_t^*)$	3.5	1.3
Autocorrelations		
Exchange Rates $(\log e_{t+1} - \log e_t)$	.04	0
Interest Rates $(i_t - i_t^*)$	.83	.92

Sources of data: Backus, Foresi, and Telmer (2001)

Cross Country Implications

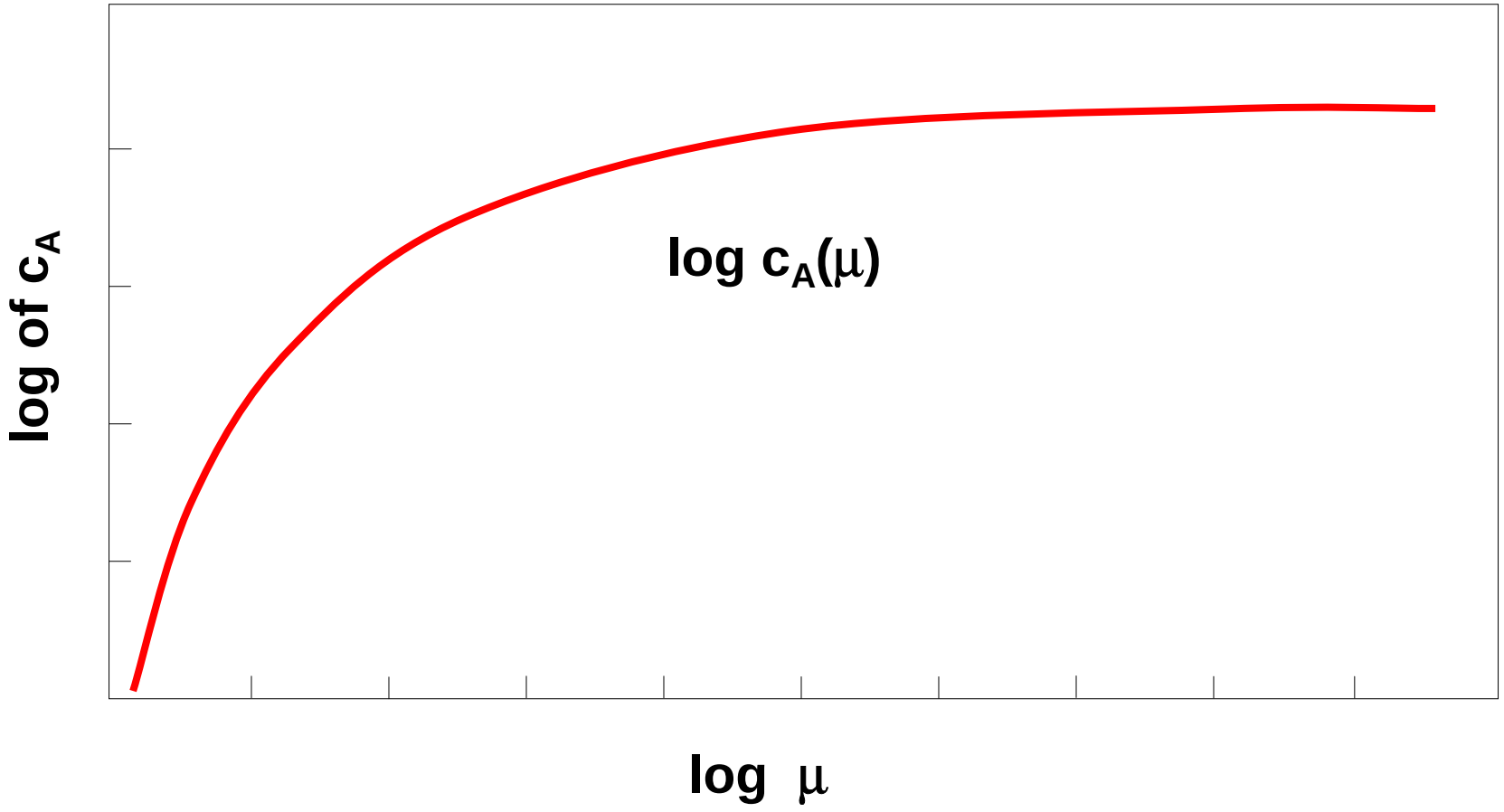
- Empirical studies: Countries with high inflation
  - slope coefficient *b* in Fama regression close to 1

- Model's implications similar
  - slope coefficient *b* in Fama regression is one
  - reason
    - high inflation implies market not segmented

Long-Term Forward Premium

- Data
  - forward premium disappears with long-term bonds
- Model
  - disappears in model also
  - reason
    - real risk doesn’t grow with horizon
    - long-term risk is same as in model with no segmentation

The Log of the Consumption of Active Households



Conclusion

- Time-varying risk essential for interest differentials
- Showed segmented markets give time-varying risk
- Highlighted key forces at work

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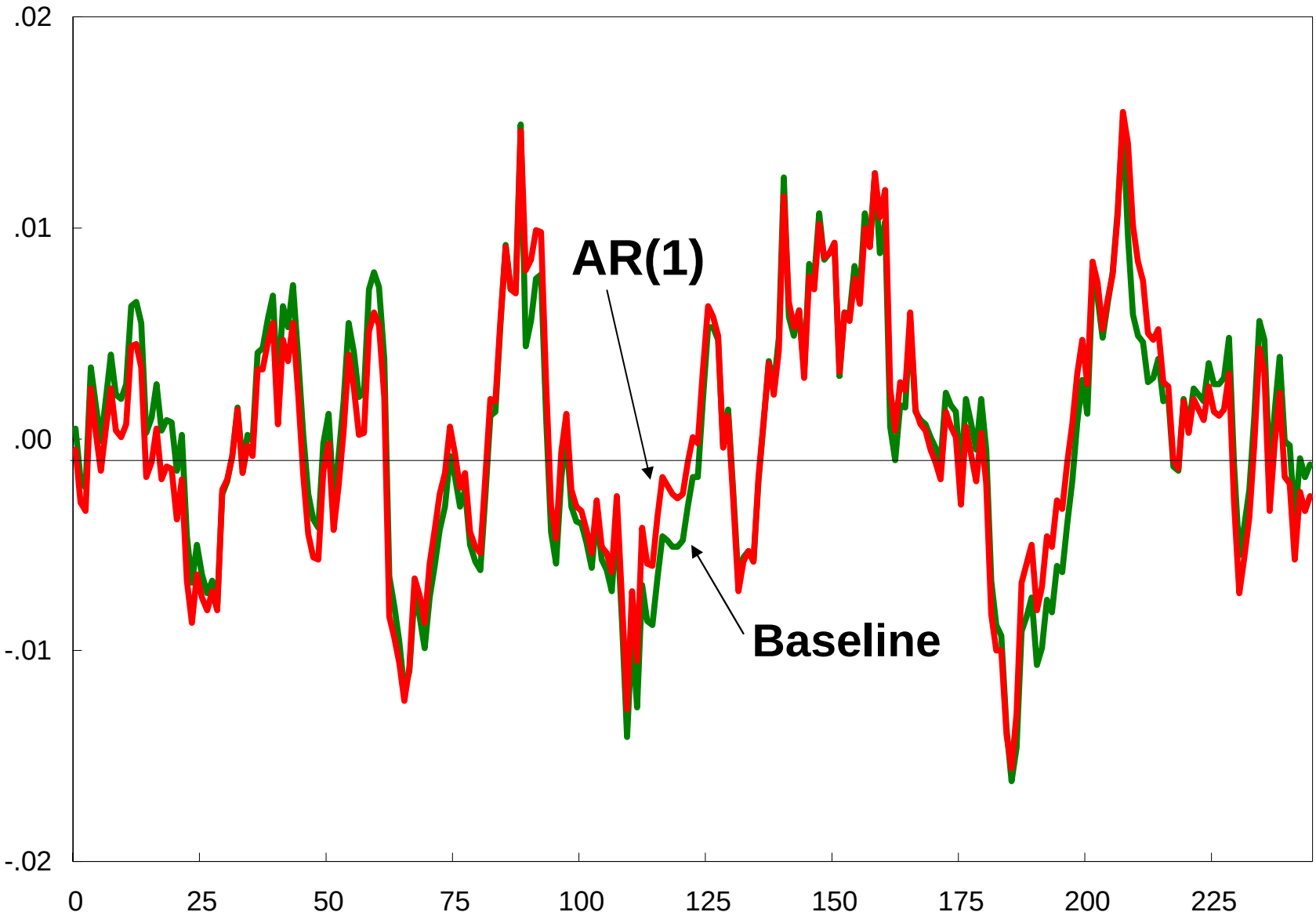

Concavity of log $c_A(\mu)$

- *Proposition* 3

  The log of consumption of active households is strictly increasing and strictly concave in $\log \mu$, around $\mu = 1$

- Numerical Example