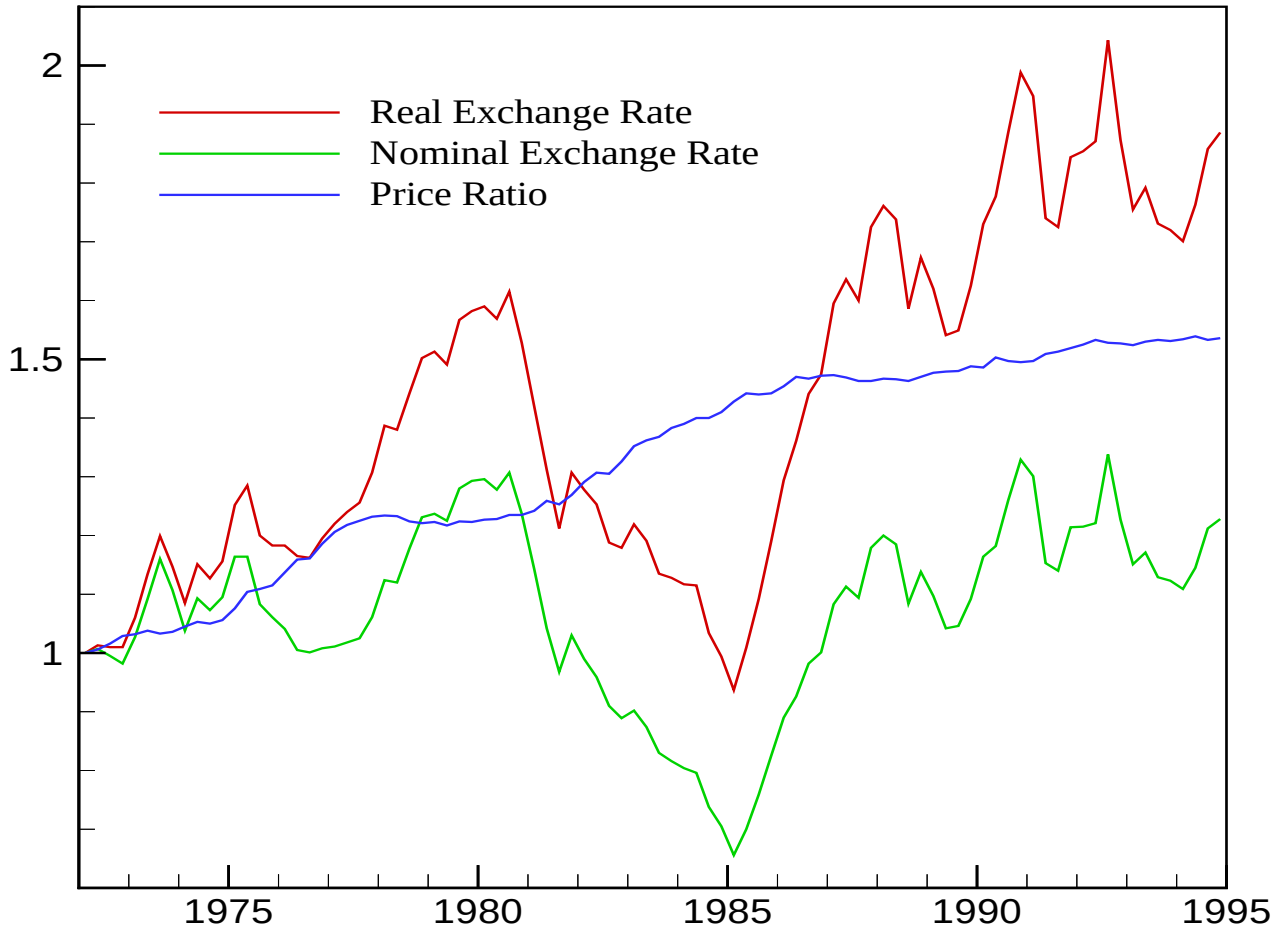
Bank of Portugal Lectures Summer 2006

Can Sticky Price Models Generate Volatile
and Persistent Real Exchange Rates

Based on work by Chari, Kehoe, and McGrattan

Exchange Rates and Prices

Figure 1
Exchange Rates and Price Ratio



$$e = \$/\text{euro}$$

$$q = eP^*/P$$

Motivation

- **Real exchange rate ($q = eP^*/P$):**
  - **volatile**
  - **persistent**
  - **highly correlated with nominal (e)**

Why?

- **Conventional wisdom:**
  - **Monetary shocks**
  - **Sticky prices**

    Mussa, Dornbusch, and many others
  - **Deviations from LOP**

    Engel, Marston, Knetter, others

Conventional Wisdom

- **Qualitatively: sounds reasonable**
- **Quantitatively?**

Punchline

- **For sticky price story to work:**
  1. **Pricing to market**
  2. **High risk aversion**
  3. **Long stickiness**

- **Why?**
  1. **For real x-rate variation**
  2. **For volatility**
  3. **For persistence**

Main discrepancy

- **Consumption-real exchange rate anomaly:**

  corr (q, c-c*)

  - **high in model**
  - **no obvious pattern in data**

Other Work

- **Svensson (1986)**
- **Ohanian-Stockman (1993)**
- **Obstfeld-Rogoff (1995)**
- **Kollmann (1996)**
- **Betts-Devereux (1996)**

Outline

- **Source of real exchange rate variation**
  - **nontraded-traded: tiny**
  - **deviations from LOP: large**
- **Sticky price model**
  - **deviations from LOP**
- **Quantitative implications**
  - **volatility and persistence?**
  - **anomalies**
- **Future work**
  - **weaken asset market linkages**

Source of Real Xrate Variation

$q = eP^*/P$ real exchange rate

$q_T = eP_T^*/P_T$ traded goods relative price

$v = q/q_T$ due to nontraded $-$ traded prices

$$\Rightarrow \quad \log q = \log q_T + \log v$$

- **If PPP holds for traded goods**

$$\mathrm{var}(\log q) = \mathrm{var}(\log v), \quad \mathrm{var}(\log q_T) = 0$$

- **We find the opposite**

$$\mathrm{var}(\log q) \simeq \mathrm{var}(\log q_T), \quad \mathrm{var}(\log v) \simeq 0$$

Interpretation of v

Approximately

$$P = P_T^{1-\alpha} P_N^{\alpha}$$

$$P^* = {P_T^*}^{1-\gamma} {P_N^*}^{\gamma}$$

$$\Rightarrow \quad v = \left(\frac{P_N^*}{P_T^*}\right)^{\gamma} / \left(\frac{P_N}{P_T}\right)^{\alpha}$$

ratio of relative nontraded-traded goods prices

Var v small

- **Prices within country: high correlation**
  - P_T **and** P_N **move together**
  - P_T^* **and** P_N^* **move together**

- **Prices across countries: low correlation**
  - P_T **and** eP_T^* **don't**
  - P_N **and** eP_N^* **don't**

CPI-based decomposition

- **OECD categories:**
  - **Food**
  - **All goods less food**
  - **Rent**
  - **Services less rent**

- **Traded = Food + All goods less food**

CPI-based decomposition

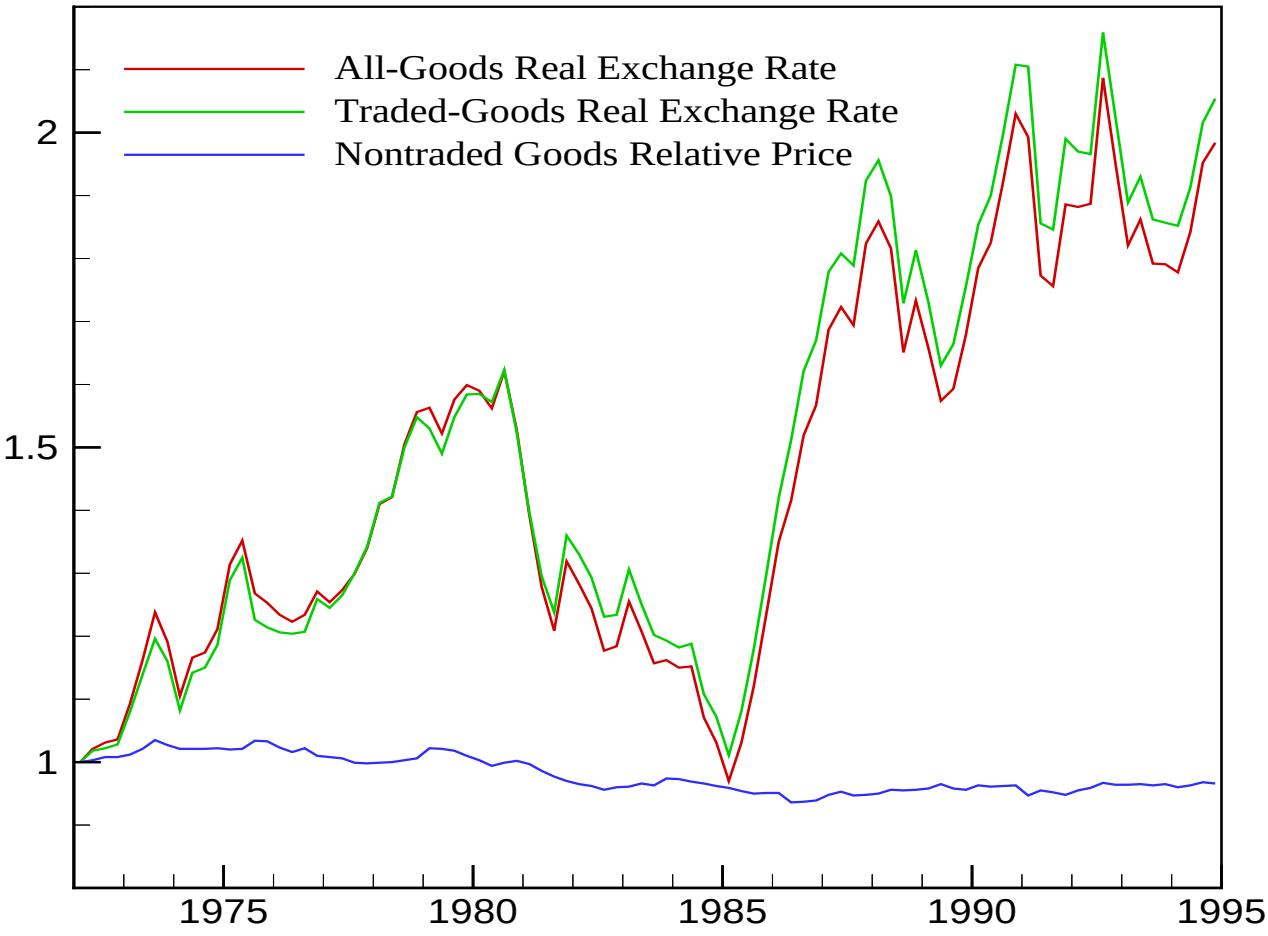
- **Variance decomposition**

$$\begin{aligned} \mathrm{var}(\log q) &= \mathrm{var}(\log q_T) + \mathrm{var}(\log v) + 2\mathrm{cov}(\log q_T, \log v) \\ (4.29) &= \quad (4.89) \quad + \quad (0.08) \quad + \quad (-0.68) \end{aligned}$$

$$\frac{4.89 - 0.68}{4.29} \times 100 = 98 \text{ percent}$$

CPI-based decomposition

Figure 2
Real Exchange Rate Components Using CPIs



$$\mathrm{var}(\log\ q) = 4.29$$

$$\mathrm{var}(\log\ q_T) = 4.89$$

Other checks

- **Deflator-based decomposition**
  - **nontraded = services**
  - **97% from q_T**
- **Distribution costs?**
  - **Std. dev. of WPI real x-rate = 7.61**
  - **Std. dev. of CPI real x-rate = 7.81**

Deflator-based decomposition

- **OECD categories:**
  - **Durable goods**
  - **Semi-durable goods**
  - **Nondurable goods**
  - **Services**

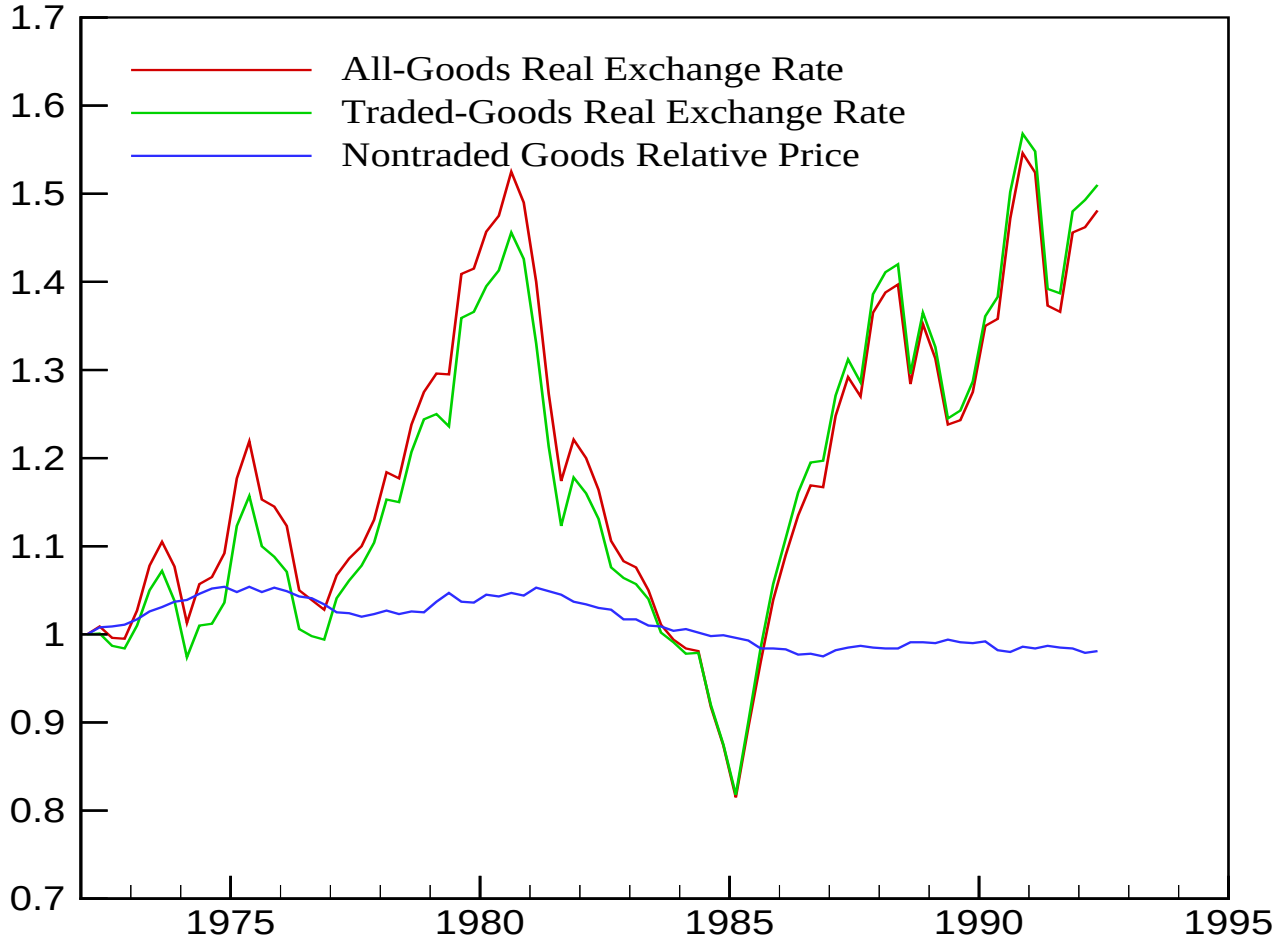
- **Nontraded = Services**

$$\begin{aligned} \mathrm{var}(\log q) &= \mathrm{var}(\log q_T) + \mathrm{var}(\log v) + 2\mathrm{cov}(\log q_T, \log v) \\ (2.17) &= \quad (2.3) \quad + \quad (0.07) + \quad (-0.20) \end{aligned}$$

$$\frac{2.3 - 0.2}{2.17} \times 100 = 97 \text{ percent}$$

Deflator-based decomposition

Figure 3
Real Exchange Rate Components Using Deflators



$$\text{var}(\log\ q) = 2.17$$

$$\text{var}(\log\ q_T) = 2.3$$

Overview of Economy

- **Final goods producers (competitive)**

- **Intermediate goods producers (monopolistically competitive and sticky)**

- **Consumers**

Final goods

- **Technology**

$$y = [\,\omega_1\, y_H^{\rho} \;\; + \;\; \omega_2\, y_F^{\rho}\,]^{\frac{1}{\rho}}$$

$$y_H = \left(\int_0^1 y_H(i)^{\theta}\, di \right)^{\frac{1}{\theta}}$$

$$y_F = \left(\int_0^1 y_F(i)^{\theta}\, di \right)^{\frac{1}{\theta}}$$

- **Producers**

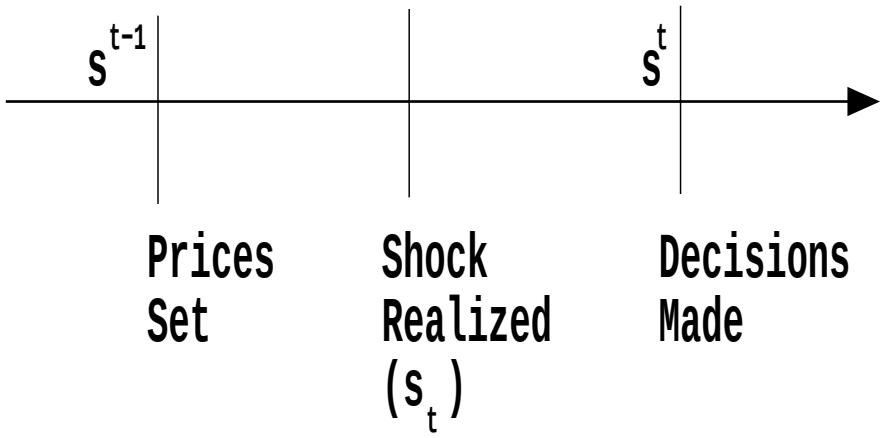
$$\max_{\substack{y_H(i) \\ y_F(i)}} \; Py - \int_0^1 P_H(i) y_H(i)\, di - \int_0^1 P_F(i) y_F(i)\, di$$

Timing of Events

- s_t = **stochastic event (money shocks)**

- $s^t = \{s_0, \ldots, s_t\}$

- **Time line:**



Monopolist in U.S.

- **Price discriminates**
  - **dollar price for U.S. market**
  - **euro price for European market**
- **Prices fixed for one year**
  - **4 cohorts, staggered**
- **Maximize present value of profits**
- **In steady state, $P_H = eP_H^*$, not otherwise**

Monopolist i at Home

- **Monopolist's problem: Drop i and s^t**

$$\max \sum_{t=0}^{\infty} Q_t \left[P_{Ht} y_{Ht} + e_t P^*_{Ht} y^*_{Ht} - W_t l_t - P_t x_t \right]$$

subject to

$$y_{Ht} + y^*_{Ht} = F(k_t, l_t)$$

capital

$$k_{t+1} = (1-\delta) k_t + x_t - \phi(x_t/k_t) k_t$$

demand

$$y_{Ht} = y_H(P_H, \bar{P}_H, P, Y)$$

and fixed prices

$$P_{H,0} = \ldots = P_{H,N-1}$$
$$P_{H,N} = \ldots = P_{H,2N-1}$$

Optimal prices

- **Markup over expected marginal costs**

  ex: Price setter at $t = 1$

$$P_H(i, s^0) = \frac{\sum_{t=1}^{N} \sum_{s^t} Q(s^t) A(s^t) MC(s^t)}{\theta \sum_{t=1}^{N} \sum_{s^t} Q(s^t) A(s^t)}$$

$$P_H^*(i, s^0) = \frac{\sum_{t=1}^{N} \sum_{s^t} Q(s^t) A^*(s^t) MC(s^\tau)}{\theta \sum_{t=1}^{N} \sum_{s^t} Q(s^t) A^*(s^t) e(s^t)}$$

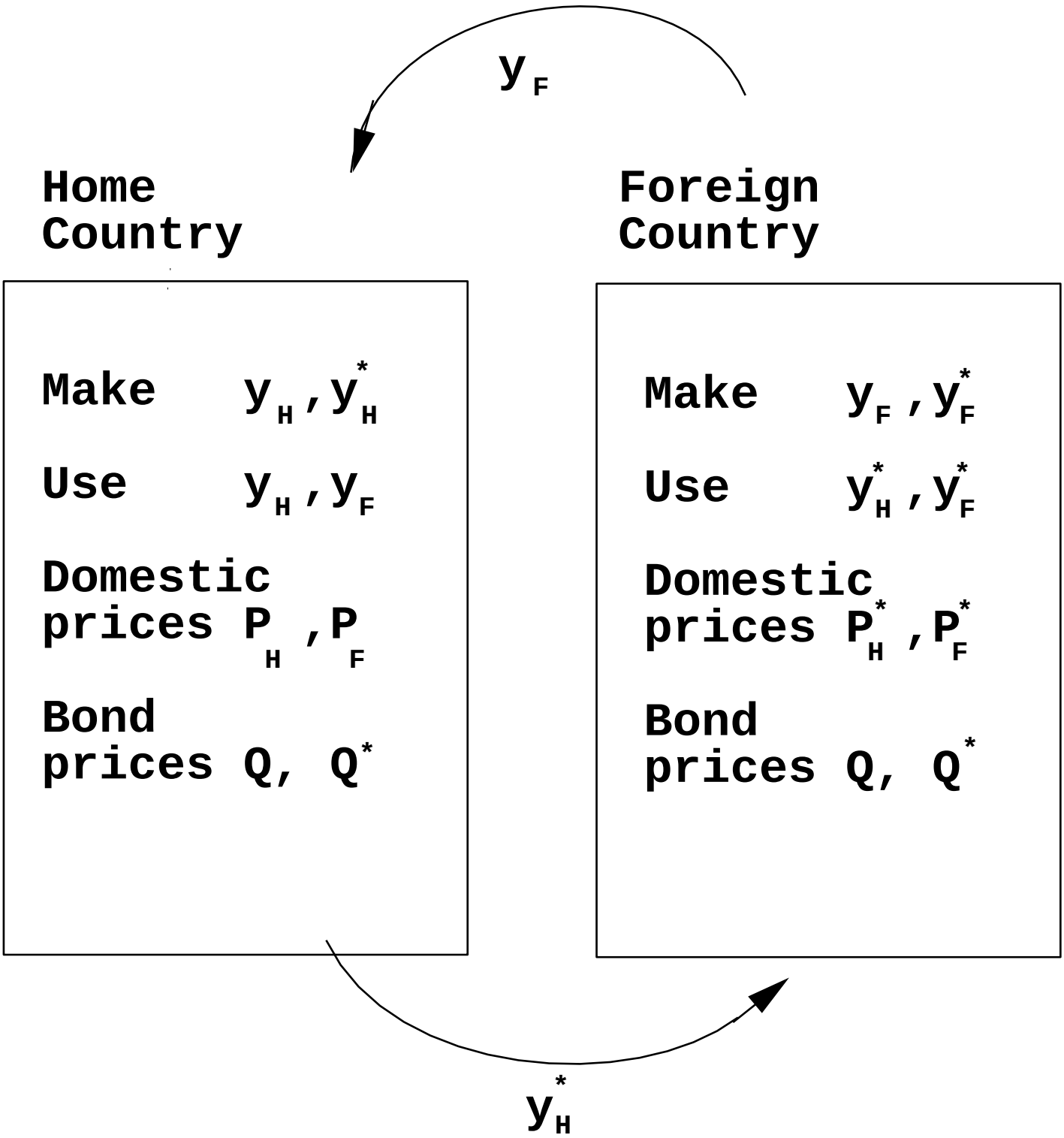
- **Steady state**

  $P_H = eP_H^* = MC/\theta$

  deviations from LOP due to shocks

  keep economy out of steady state

Overview of Economy



Consumers at home

- **Consumers solve**

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\ U\left(c(s^t), l(s^t), \frac{M(s^t)}{P(s^t)}\right)$$

subject to budget constraints:

$$P(s^t)c(s^t) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1})$$

$$\leq W(s^t)l(s^t) + M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t)$$

- **Complete markets**

  - $B(s^t, s_{t+1})$= **bonds**

  - $Q(s^{t+1}|s^t)$= **price**

Key implication from asset market

- **Bond market first order condition**
  - **U.S. consumers:**

$$Q_t = \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}$$

  - **European consumers:**

$$Q_t = \beta \frac{U^*_{c,t+1}}{U^*_{c,t}} \frac{P^*_t}{P^*_{t+1}} \frac{e_t}{e_{t+1}}$$

- **Equate and iterate:**

$$q_t = \frac{U^*_{c,t}}{U_{c,t}}$$

Key implication from asset market

$$q_t = \kappa \, \frac{U^*_{c,t}}{U_{c,t}}$$

Equilibrium

- **Processes for money**

$$M(s^t) = \mu(s^t)\, M(s^{t-1})$$
$$M^*(s^t) = \mu^*(s^t)\, M^*(s^{t-1})$$

- **Market clearing**

$$y(s^t) = c(s^t) + \int_0^1 x(i, s^t)\, di$$

$$l(s^t) = \int_0^1 l(i, s^t)\, di$$

$$0 = B_H(s^t) + B_H^*(s^t)$$

- **Equilibrium: allocations and prices st**

  i) **consumers and producers maximize**

  ii) **market clearing**

Parameterization

- **Utility**

$$\frac{\left[(ac^{\frac{\eta-1}{\eta}} + (1-a)m^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}}\right]^{1-\sigma}}{1-\sigma} + \psi\frac{(1-\ell)^{1-\xi}}{1-\xi}$$

- ψ **and** ξ **so**
  - $\bar{\ell} = 1/4$
  - **labor supply elasticity = 2**

- η **and** a **from money demand**

$$\log\left(\frac{M}{P}\right) = \text{const.} + \log c - \eta \log\left(\frac{i}{1+i}\right)$$

  - **consumption elasticity** $= 1$
  - **interest elasticity** $(\eta) = .39$
  - **average real balances** $\Rightarrow a$

Parameterization

Final goods

$$y = [\,\omega_1\, y_H^{\rho} \;\; + \;\; \omega_2\, y_F^{\rho}\,]^{\frac{1}{\rho}}$$

$$y_H = \left(\int_0^1 y_H(i)^{\theta}\, di \right)^{\frac{1}{\theta}}$$

- **11% markup gives θ**
- **Elasticity of substitution**
  - $1/(1-\rho)$ **= 1.5**
- **US export share to Europe**
  - **1.6% GDP gives** ω_1/ω_2

Parameterization

Intermediate goods

- **Production**

$$F(k_t, l_t) = k_t^{\alpha} l_t^{1-\alpha}$$

  - $\alpha = 1/3$

- **Adjustment costs**

$$\phi\left(\frac{x_t}{k_t}\right) = \frac{b}{2}\left(\frac{x_t}{k_t} - \delta\right)^2$$

  - b **so that** σ_c/σ_y **= 0.79**

Shocks

- **Money growth process**

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \epsilon_{\mu,t}$$

estimate

- $\rho_\mu = 0.57$

choose

- **std. dev. to match output volatility**
- **corr**$(\epsilon_\mu, \epsilon_{\mu^*})$ **to match corr**(y, y^*)

Preferences and real exchange rates

- **Taylor expansion of $q = U_c^*/U_c$:**

$$\hat{q} = A(\hat{c} - \hat{c}^*) + B(\hat{m} - \hat{m}^*) + D(\hat{\ell} - \hat{\ell}^*)$$

$$A = -\frac{cU_{cc}}{U_c}$$

$$B = -\frac{mU_{cm}}{U_c} \quad \text{irrelevant}$$

$$D = -\frac{lU_{cl}}{U_c} = 0$$

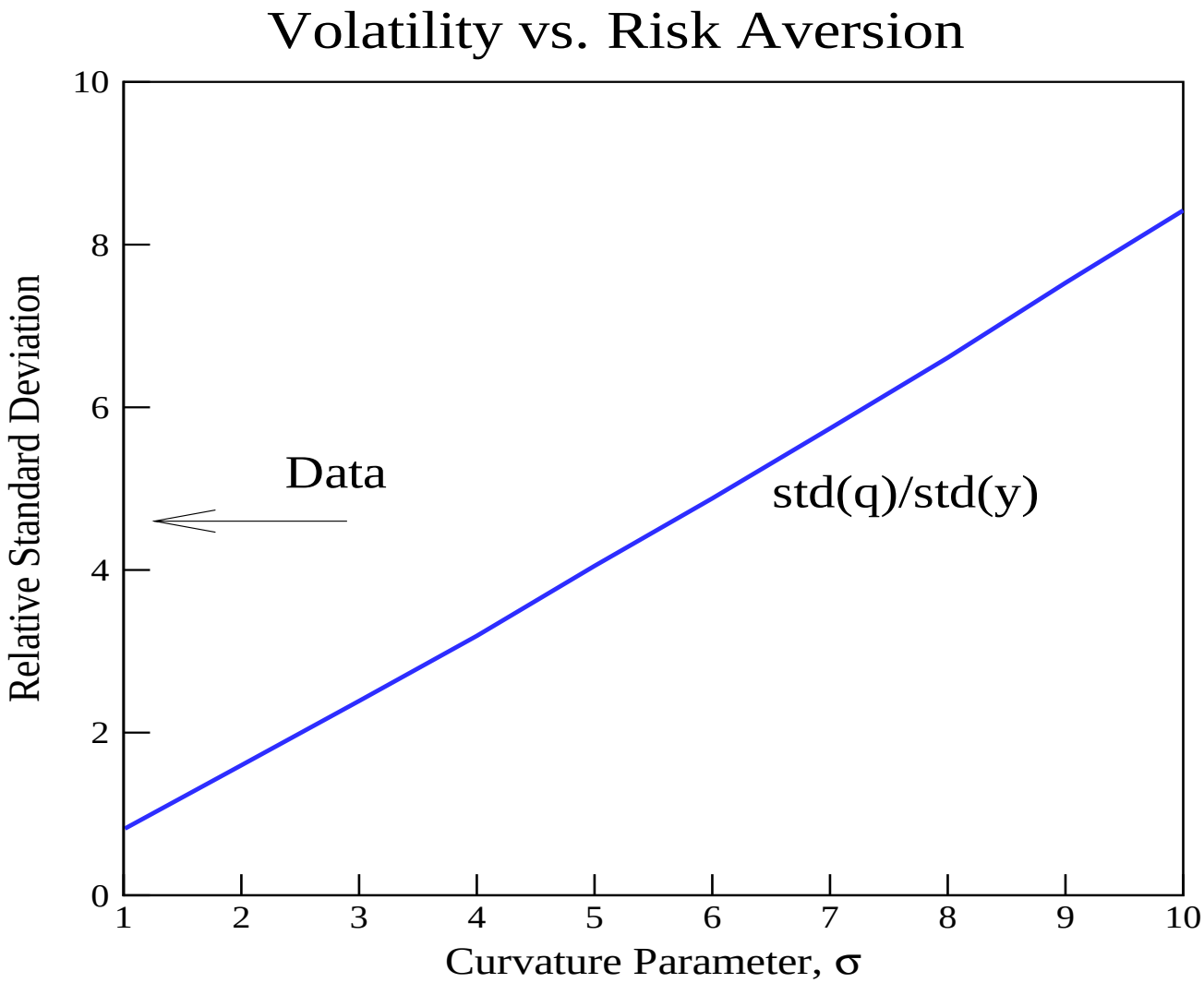
- **Separable preferences:** $U(c, m) + V(l)$

  $\Rightarrow$ **volatility if A big**

Findings

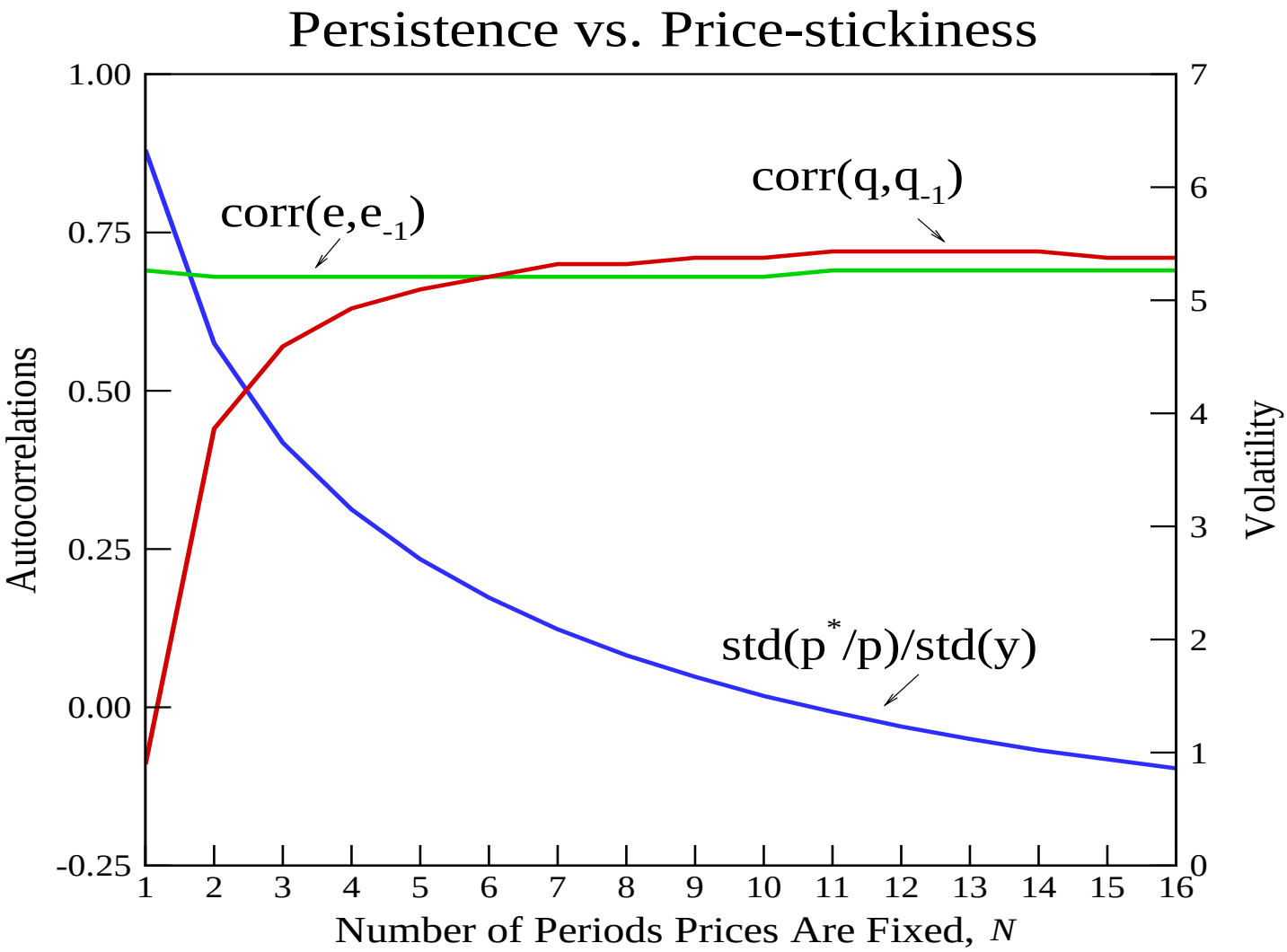
- **Generate volatility?**
  - **with high risk aversion**
- **Generate persistence?**
  - **with long stickiness**

Volatility vs. Risk Aversion



$$\hat{q} \approx -\frac{cU_{cc}}{U_c}(\hat{c} - \hat{c}^*)$$

Persistence vs. Price Stickiness



data:

$$\mathbf{corr}(q, q_{-1}) = .86$$

$$\mathbf{corr}(e, e_{-1}) = .83$$

$$\mathbf{std}(P^*/P)/\mathbf{std}(y) = .74$$

Anomalies

- **Persistence anomaly**

	data	model
$\mathbf{corr}(q, q_{-1})$	**.83**	**.62**

- **Consumption–Real exchange rate anomaly**

	data	model
$\mathbf{corr}(q, c - c^*)$	**−.35**	**1**

Problem with Separable Preferences

- **Unbalanced growth**
  - **If** c, m, w **grow at 2%**
  - $U_\ell / U_c = w$
  - **In 4 years, labor supply = 0**

- **Add technical change in nonmarket**

$$U = \frac{(ac_t^\nu + (1-a)m_t^\nu)^{\frac{1-\sigma}{\nu}}}{1-\sigma} + \psi \frac{(z^t(1-\ell_t))^{1-\xi}}{1-\xi}$$

- U_ℓ / U_c **grows at wage rate**
  - $\sigma = \xi$ **knife-edge**

Sensitivity: Taylor Rule

- **Nominal interest rate:**

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left[aE_t\pi_{t+1} + b\log GDP_t\right] + \epsilon_{r,t}$$

- **Estimates from Clarida, Gali, Gertler:**

$\rho_r = 0.79$, $a = 2.15$, $b = 0.9$

- **Persistence anomaly remains**

	data	**model**
corr(q, q_{-1})	**.83**	**.48**

Addressing persistence anomaly

- **Problem**
  - **wages rise immediately**
  - **producers raise prices asap**
  - **little endogenous stickiness**

- **Solve with sticky wages?**

	data	bench	sticky w
$\mathbf{corr}(q, q_{-1})$	**.83**	**.62**	**.69**

Addressing Consumption–Xrate Anomaly

- **corr**$(q, c - c*)$ **varies across countries**
- **Addressing anomaly**
  - **Nonseparable utility**

    labor

    habit persistence
  - **Asset market frictions**

    exogenous incomplete markets

    Baumol-Tobin fixed costs

    enforcement constraints

Nonseparable Labor

$$U = \left\{ [ac^{\nu} + (1-a)(M/P)^{\nu}]^{\frac{\gamma}{\nu}} (1-l)^{1-\gamma} \right\}^{1-\sigma} / (1-\sigma)$$

- **Real exchange rate not volatile**

$$\hat{q} = A(\hat{c} - \hat{c}^*) + B(\hat{m} - \hat{m}^*) + D(\hat{\ell} - \hat{\ell}^*)$$

  - c **and** ℓ **move together**
  - **raising** σ **raises** A **and** D

	data	**bench**	**nonsep**
$\text{std}\,\hat{q}/\text{std}\,\hat{y}$	**4.36**	**4.27**	**.05**

Habit persistence

- **Adds other terms in U_c**

$$V = \max \sum \beta^t U(c_t - dc_{t-1})$$

$$\frac{\partial V}{\partial c_t} = \beta^t [U_{ct} - d\beta U_{c,t+1}]$$

$$\hat{q} = \underbrace{\frac{A}{1-d}[(\hat{c}_t - \hat{c}_t^*) - d(\hat{c}_{t-1} - \hat{c}_{t-1}^*)]}_{\text{predicted } q}$$

corr(q, predicted q) $\leq -.19$

Incomplete Asset Market

- **Only noncontingent dollar bonds**

- **Drop:**

$$q_t = \frac{U^*_{c,t}}{U_{c,t}}$$

- **More like:**

$$q_t = E_{t-1} \frac{U^*_{c,t}}{U_{c,t}}$$

- **Quantitatively: almost no change**

Endogenously segmented asset markets

- **Fixed cost to swap assets for money**
  - **Random visits to asset markets**
  - **Money has distributional effects**
  - **Money moves real exchange rates**

- **Break link between q, $c - c^*$:**

$$q = \frac{U_c(c^*_{\text{active}})}{U_c(c_{\text{active}})}$$

  - $c_{\text{active}} \neq$ **aggregate consumption**

Enforcement constraints

- **Incomplete enforcement of contracts**

$$E_t \sum_{s=t} \beta^{s-t} U(c_t, l_t) \geq \text{Value of financial autarky}$$

- ○ **Effective welfare weights random**

$$q = zU_c^* / U_c$$

- z **ratio of sum of enforcement multipliers**

Conclusions

- **Big puzzle: movements in real x-rates**
- **Main story: sticky prices**
- **Weaknesses**
  - **persistence anomaly**
  - **consumption-real x-rate anomaly**
- **Future: asset market frictions**