Firm Heterogeneity, Export Participation, and Trade Reform Dynamics*

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Abstract

We develop a model of establishment export dynamics consistent with the enormous establishment level heterogeneity in exporting and productivity. Specifically, we assume that firms face an up-front, sunk cost of entering foreign markets and a smaller period-by-period continuation cost. In response to persistent firm-specific productivity shocks, firms start and stop exporting. The model generates exporter hysteresis in that the productivity threshold to start exporting exceeds the threshold to stop exporting. The model is calibrated to match the characteristics and dynamics of U.S. exporters as well as the growth in U.S. trade from 1970 to 1992. We find a model with sunk costs of exporting best captures U.S. trade dynamics over this period. We then use the calibrated model to quantify the welfare gains to a world-wide elimination of tariffs. We find that the welfare gains to trade reform depend importantly on transition dynamics. Welfare measures that include transition dynamics exceed steady state welfare measures by 1.8 percent of lifetime consumption. We find a 5 percent cut in tariffs generates a persistent 0.7 percent increase in TFP.

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1. Introduction

Recent evidence documenting substantial differences between exporters and non-exporters has lead Eaton and Kortum (2002) to develop a new theory of international trade that emphasizes productive heterogeneity across firms. This theory predicts that the most productive firms do the bulk of all exporting. Reducing tariffs reallocates production away from relatively unproductive non-exporters to relatively productive exporters. In calibrated GE versions of these models, Eaton and Kortum (2002) and Alvarez and Lucas (2006) find that the welfare gains to tariff cuts are larger than in standard models without firm heterogeneity.

In this paper, we reconsider the welfare gains to trade reform in a two country model of establishment dynamics.[1] The model shares with previous work an emphasis on firm-level productive heterogeneity. In contrast with previous work, we emphasize the dynamic features of export participation. Two features generate exporter dynamics. First, consistent with the evidence of Das, Roberts and Tybout (2006), we assume individual firms face a large, up-front sunk cost of entering a foreign market and a smaller, period-by-period cost of continuing in that market. Second, in the presence of idiosyncratic technology shocks, nonexporting firms start exporting only when the expected value of exporting covers the entry costs. Exporters continue to export as long as the value of doing so exceeds the continuation cost. This generates what Krugman and Baldwin (1991) call exporter hysteresis in that firms continue to export even after their production costs have risen far above the levels which led them to start exporting. Exporter hysteresis implies that some relatively unproductive firms export and some relatively productive firms only sell at home, and is important in matching the dispersion among exporters and non-exporters.

One aim of the paper is to develop a model consistent with both the characteristics of ex-

[1] The basic model is a version of Alessandria and Choi (2005) extended to allow for persistent idiosyncratic productivity shocks and tariffs.

porters and non-exporters emphasized in previous work along with properties of exporter and firm dynamics. A key advantage to modelling entry and exit to exporting is that it captures the behaviour of marginal exporters and non-exporters. These are the firms that are most affected by trade policy. The model we develop in the next section generates a stationary distribution of firms across productivity and exporting participation. We calibrate the structure of idiosyncratic shocks and the fixed costs to match the transitions into and out of exporting. We also target more standard moments relating to firm creation and labor reallocation. To match U.S. firm dynamics and characteristics we find that sunk costs of starting to export exceed the costs of continuing to export by only about 5.6 percent. In section 4, we explore the relationship between firm characteristics, exporter dynamics and tariffs.

A second goal of the paper is to study the dynamics of trade growth and export participation in response to changes in trade policy. In section 3, we calibrate the model to match the increase in U.S. trade flows from 1970 to 1992 given the change in tariffs over the same period. We then compare the timing of trade expansion in the model with the data. In this period, U.S. trade is non-linear in tariff rates. That is, trade increases proportionally more when tariffs are already low. Yi (2003) has argued that this is a puzzle for standard models.

Our model of sunk costs of exporting generates a similar trade-tariff relation for two reasons. First, with the sunk cost, the mass of exporters is a state variable and the result of past investments in exporting. Consumption smoothing motives thus slow the expansion in export capacity. Second, the distribution of exporters matters for the pattern of trade. Cuts in tariffs encourage increased entry into foreign markets and decrease exit. In the short run, we find that increases in exporter participation increases occur more through a reduction in the exit margin. Keeping relatively unproductive exporters around longer discourages more productive firms from entering and leads to higher prices, which generate slower gains in trade. Thus, cuts in tariffs take a while to have their

full impact and so some of the increases in trade when tariffs were already at low are a result of past tariff cuts.

Our third goal is to quantify the impact on productivity of the reallocative effect of a tariff cut emphasized in Eaton and Kortum and Alvarez and Lucas. In contrast to these papers, in our model the productivity distribution over firms is a state variable. It is the result of the past entry decisions and idiosyncratic shocks of firms and thus evolves quite slowly. In section 5 we consider the move from 5 percent world tariffs to free trade. We find that on impact, this tariff reduction generates a drop in aggregate productivity as firms divert resources from producing to building export capacity. In subsequent periods, productivity booms as more productive firms begin exporting. However, over time productivity decays as more relatively unproductive firms begin exporting. The dynamics of TFP following this 5 percentage point tariff reduction look surprisingly like an aggregate TFP shock. In period 2, productivity rises about 0.9 percent and over the subsequent 100 periods has an autocorrelation of 0.95.

Our final goal is to use the quantitative theory to estimate the benefits to different trade reforms going forward. In section 5 we consider the move from 5 percent world tariffs to free trade. Surprisingly, comparing steady states, we find that steady state consumption actually declines by 1.22 percent. However, once one considers the transition to this new steady state, we find that removing tariffs generates a 0.54 percent increase in lifetime consumption. The difference between the transition and steady state results bear explaining. In our model, capital is produced using an aggregate of all goods available in a country, while firms and exporters are produced using just labor to be consistent with balanced growth. A tariff then lowers the price of creating firms relative to capital and so with tariffs, the economy accumulates too many firms and not enough capital. The higher steady state consumption with tariffs is only a result of greater past sacrifice in creating firms. Removing tariffs leads to a benefit along the transition because the existing firms can be

reallocated across countries more efficiently and fewer resources need to be expended in creating new firms.

This paper is related to three lines of research. First, our focus on the welfare gains to trade liberalization is similar to the work by Eaton and Kortum (2002), Bernard, Eaton, Jensen and Kortum (2003) and Alvarez and Lucas (2006). These papers evaluate the welfare gains to trade liberalization in static, multicountry Ricardian models with productivity heterogeneity, tariffs, and transportation costs. Unlike these papers, we consider a dynamic model with entry and exit subject to fixed costs and allow for capital accumulation. Melitz (2003) also considers the gains to trade in a dynamic model with firm heterogeneity, but does not quantify these gains.[2] Chaney (2005) discusses the dynamics of trade and firm dynamics following trade liberalization. The second line of research uses quite similar models with fixed costs of trade, but is more focused on understanding international business cycle fluctuations (see Ruhl (2003), Alessandria and Choi (2005), and Ghironi and Melitz (2005)). Finally, there is a third, partial equilibrium literature that studies the export decisions of firms. Baldwin and Krugman (1991) and Dixit (1991) develop models of export decisions with an exogenous exchange rate process. Das, Roberts and Tybout (2001) develops these models further and use them to estimate the sunk costs of exporting. As partial equilibrium studies these papers can not evaluate welfare.

The paper is organized as follows. The next section develops a two-country dynamic general equilibrium model with export penetration and continuation costs. Section 3 discusses the calibration of the model. Section 4 discusses the relationship between tariffs, exporter characteristics, trade and welfare in the steady state of the model. In Section 5 we examine the transition dynamics following the world-wide elimination of tariffs. Section 6 concludes.

[2] Baldwin and Forslind (2006) discuss the welfare gains to trade reform in the Melitz model. They point out that trade reform may result in a reduction in the number of varieties available.

2. The Model

In this section we develop a model of establishment dynamics and export participation. Each period there is a mass of existing establishments distributed over sectors, productivity, countries, and export status. Productivity is stochastic and generates movements of establishments into and out of exporting. Unproductive establishments also shut-down. New establishments are created by incurring a sunk cost.

There are two countries, *home* and *foreign*. Each country is populated by a continuum of identical, infinitely lived consumers with mass of one. Each period, consumers are endowed with fixed L units of labor and supply them inelastically in the labor market.

In each period of time, the economy experiences an event s_t. Let $s^t = (s_0, \cdots, s_t)$ denote the history of events from period 0 up to and including period t. The probability of a history s^t, conditional on the information available at period 0, is defined as $\pi(s^t|s_0)$. The initial realization at period 0, s_0, is given.

In this economy, there exists a complete set of one-period state-contingent nominal bonds, $B(s^t)$, denominated in the home currency. Let $B\left(s^{t+1}, s^t\right)$ denote the home consumer's holding of a bond purchased in state s^t with payoff in state s^{t+1}. Let $B^*\left(s^{t+1}, s^t\right)$ denote the foreign consumer's holding of this bond. The state-contingent bond pays 1 unit of home currency if s^t occurs, and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the nominal price of the state-contingent bond $B(s^{t+1})$ given s^t.

In each country there are two intermediate good sectors, tradable and non-tradable sectors. In each sector, there is a large number of monopolistically competitive firms each producing a differentiated goods. The mass of varieties in the tradable and non-tradable goods sectors are $N_T\left(s^t\right)$ and $N_N\left(s^t\right)$, respectively. An intermediate good producer uses capital and labor inputs to produce its variety. In each sector, firms differ in terms of total factor productivity, capital, and the markets they serve. All firms sell their product in their own country, but only some firms in the

tradable good sector export their good abroad. When a firm in the tradable good sector exports goods abroad, the firm incurs some international trading cost. The firm has to pay tariffs to the government of the destination country with an ad valorem tariff rate of τ. Additionally, the firm has to pay some fixed costs to export its goods abroad. The size of the cost depends on the producer's export status in the previous period. There is a (relatively) high up-front sunk cost f_0 that must be borne to gain entry into the export market. In subsequent periods, to continue exporting, firms incur a lower but nonzero period-by-period fixed continuation cost f_1. If a firm does not pay this continuation cost, then it ceases to export. In future periods, the firm can only begin exporting by incurring the entry cost f_0 again. These costs are valued in units of labor in the domestic country. The cost of exporting implies that the set of goods available to consumers and firms differs across countries and is changing over time. We assume that the fixed costs must be incurred in the period prior to exporting. This implies that the set of foreign varieties is fixed at the start of each period. All the firms are owned by domestic consumers and the goods are used for both consumption and investment. Any potential firm can enter either the tradable or non-tradable sector by paying sunk costs of f_E. New entrants can actively produce goods and sell their products from the following period.

Firms are heterogenous by their technology, export status, sector, and nationality. The measure of home country tradeable firms with technology z and export status, $m \in \{0, 1\}$, equals $\varphi_T\left(z, m, s^t\right)$. The measure of home country non-tradeable firms with technology z equals $\varphi_{NT}\left(z, s^t\right)$. The distribution of firms over technology, exporting status and sector are part of the aggregate state variable. We find the evolution of this distribution is central to the quantitative results.

A. Consumers

Home consumers choose consumption, investment, and bond holdings to maximize their utility:

$$(1) \quad V_{C,0} = \max \sum_{t=0}^{\infty} \beta^t U(C_t),$$

subject to the sequence of budget constraints,

$$(2) \quad P_t C_t + P_t K_{t+1} + Q_t B_t \leq P_t W_t L_t + P_t R_t K_t + (1-\delta) P_t K_t + B_{t-1} + P_t \Pi_t + P_t T_t,$$

where β is the subjective time discount factor with $0 < \beta < 1$; P_t is the price of final good; C_t is the consumption of final goods; K_t is the capital available in period t; Q_t and B_t are the price of bonds and the bond holdings; W_t and R_t denote the real wage rate and the rental rate of capital; δ is the depreciation rate of capital; Π_t is the sum of real dividends from the home country's producers; and T_t is the real lump-sum transfer from the home government.

The problem of foreign consumers is analogous to this problem. Prices and allocations in the foreign country are represented with an asterisk. To be clear, money has no role in this economy. However, the local currency is used as a unit of account so that the foreign budget constraint is expressed as

$$(3) \quad P_t^* C_t^* + P_t^* K_{t+1}^* + \frac{Q_t}{e_t} B_t^* \leq P_t^* W_t^* L_t^* + P_t^* R_t^* K_t^* + (1-\delta) P_t^* K_t^* + \frac{B_{t-1}^*}{e_t} + P_t^* \Pi_t^* + P_t^* T_t^*,$$

where * denotes the foreign variables and e_t is the nominal exchange rate with home currency as numeraire.

The first order conditions for home consumers' utility maximization problems are

$$(4) \quad Q_t = \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}},$$

where $U_{C,t}$ denotes the derivative of the utility function with respect to its argument. The price of the state-contingent bond is standard. With arbitrage, the complete asset markets assumption implies that the real exchange rate, q_t, is proportional to the ratio of marginal utility of consumption across countries

$$(5) \quad q_t \equiv \frac{e_t P_t^*}{P_t} = \kappa \frac{U_{C,t}^*}{U_{C,t}},$$

where $\kappa = q_0 U_{C,0} / U_{C,0}^*$.[3]

B. Final Good Producers

In the home country, final goods are produced using only home and foreign intermediate goods. A final good producer can purchase from any of the home intermediate good producers but can purchase only from those foreign tradable good producers that are actively selling in the home market.

The production technology of the firm is given by a Cobb-Douglas function for tradable and non-tradable aggregate inputs, $Y_T\left(s^t\right)$ and $Y_N\left(s^t\right)$, with the tradable share γ

$$(6) \quad D\left(s^t\right) = Y_T^{\gamma}\left(s^t\right) Y_N^{1-\gamma}\left(s^t\right),$$

where $D(s^t)$ is the output of final goods, $Y_T\left(s^t\right)$ and $Y_N\left(s^t\right)$ are the aggregates of tradable and non-tradable goods, respectively. The aggregation technology of the firm is given by a constant

[3] In the simulation exercises, κ is normalized to be 1.

elasticity of substitution (henceforth CES) function

$$(7) \qquad Y_T(s^t) = \left(\int_{z\times m} y_H^d\left(z,m,s^t\right)^{\frac{\theta-1}{\theta}} \varphi_T\left(z,m,s^t\right) + \int_z y_F^d(z,1,s^t)^{\frac{\theta-1}{\theta}} \varphi_T^*\left(z,1,s^t\right) \right)^{\frac{\theta}{\theta-1}},$$

$$(8) \qquad Y_N(s^t) = \left(\int_z y_N^d(z,s^t)^{\frac{\theta-1}{\theta}} \varphi_{NT}\left(z,s^t\right) \right)^{\frac{\theta}{\theta-1}},$$

where $y_H^d\left(z,m,s^t\right)$, $y_F^d(z,1,s^t)$, and $y_N^d\left(z,s^t\right)$ are inputs of intermediate goods purchased from home tradable good producer with technology z and export status m, foreign tradable exporter with technology z, and home non-tradable good producer with technology z, respectively. The elasticity of substitution between intermediate goods within a sector is θ.

The final goods market is competitive. Given the final good price at home $P(s^t)$, the prices charged by each type of tradeable and nontradeable good, the final good producer solves the following problem

$$(9) \qquad \max \Pi_F\left(s^t\right) = P\left(s^t\right) D\left(s^t\right) - \int_{z\times m} P_H\left(z,m,s^t\right) y_H^d\left(z,m,s^t\right) \varphi_T\left(z,m,s^t\right)$$
$$- \int_z y_F^d(z,1,s^t) P_F\left(z,1,s^t\right)(1+\tau)\varphi_T^*\left(z,1,s^t\right) - P_N\left(s^t\right) Y_N\left(s^t\right),$$

subject to the production technology (6), (7), (8).[4] Solving the problem in (**??**) gives the input demand functions,

$$(10) \qquad y_H^d\left(z,m,s^t\right) = \gamma \left[\frac{P_H\left(z,m,s^t\right)}{P(s^t)}\right]^{-\theta} \left[\frac{P_T(s^t)}{P(s^t)}\right]^{\theta-1} P\left(s^t\right) D(s^t),$$

$$(11) \qquad y_F^d\left(z,1,s^t\right) = \gamma \left[\frac{(1+\tau)\, P_F\left(z,1,s^t\right)}{P(s^t)}\right]^{-\theta} \left[\frac{P_T(s^t)}{P(s^t)}\right]^{\theta-1} P\left(s^t\right) D(s^t),$$

$$(12) \qquad y_N^d\left(z,m,s^t\right) = (1-\gamma) \left[\frac{P_N\left(z,m,s^t\right)}{P(s^t)}\right]^{-\theta} \left[\frac{P_N(s^t)}{P(s^t)}\right]^{\theta-1} P\left(s^t\right) D(s^t),$$

[4] Notice that the production function is only defined over the available products. It is equivalent to define the production function over all possible varieties but constrain purchases of some varieties to be zero.

where the prices indices are defined as

$$(13) \quad P_T\left(s^t\right) = \left\{\int_{z\times m} P_H\left(z,m,s^t\right)^{1-\theta}\varphi_T\left(z,m,s^t\right) + \int_z [(1+\tau)\,P_F(z,1,s^t)]^{1-\theta}\varphi_T^*\left(z,1\right)\right\}^{\frac{1}{1-\theta}},$$

$$(14) \quad P_N\left(s^t\right) = \left\{\int_z P_N(z,1,s^t)^{1-\theta}\varphi_N\left(z,1,s^t\right)\right\}^{\frac{1}{1-\theta}},$$

$$(15) \quad P\left(s^t\right) = \left[\frac{P_T\left(s^t\right)}{\gamma}\right]^{\gamma}\left[\frac{P_N\left(s^t\right)}{1-\gamma}\right]^{1-\gamma}.$$

C. Intermediate Good Producers

All the intermediate good producers produce their differentiated good using capital and labor inputs. Specifically, an intermediate good producer with technology, z, can produce has a Cobb-Douglas production technology,

$$(16) \quad y = Ak^{\alpha}l^{1-\alpha},$$

Non-Tradable Good Producers

Consider the problem of a non-tradable good producer from the home country in period t with technology $z = \ln A$. The producer chooses the current price P_N, inputs of labor l_N and capital k_N to solve

$$(17) \quad V_N\left(z,s^t\right) = \max \Pi_N\left(z\right) + n_s\left(z\right)Q_t E\left\{V_N\left(z',s^{t+1}|s^t,z\right)\right\},$$

$$(18) \quad \Pi_N\left(z,s^t\right) = \left[\frac{P_N\left(z,s^t\right)}{P\left(s^t\right)}\right]y_N\left(z,s^t\right) - W\left(s^t\right)l_N\left(z,s^t\right) - R\left(s^t\right)k_N\left(z,s^t\right)$$

subject to the production technology (16), and the constraints that supplies to the non-tradable goods market $y_N\left(z,s^t\right)$ are equal to demands by final good producers $y_N^d\left(z,s^t\right)$ and from (12). Here, $\phi\left(z'|z\right)$ denotes the pdf of z' conditional on z provided that the firm survived.

Tradable Good Producers

A producer in the tradable good sector is described by its technology and export status, (z, m). Each period, it chooses current prices $P_H\left(z, m, s^t\right)$ and $P_H^*\left(z, m, s^t\right)$, and inputs of labor $l_T\left(z, m, s^t\right)$, capital $k_T\left(z, m, s^t\right)$ and next period's export status, m', to solve

$$V_T\left(z, m, s^t\right) = \max \Pi_P\left(z, m, s^t\right) - m'W\left[f_1 m + (1-m) f_0\right] + n_s(z) Q_t EV\left(z', m', s^{t+1} | s^t, z\right)$$

$$\Pi_P\left(z, m, s^t\right) = \left(\frac{P_H\left(z, m, s^t\right)}{P_t}\right) y_H\left(z, m, s^t\right) + m\left(\frac{e\left(s^t\right) P_H^*\left(z, m, s^t\right)}{P_t}\right) y_H^*\left(z, m, s^t\right)$$
$$- W\left(s^t\right) l_T\left(z, m, s^t\right) - R\left(s^t\right) k_T\left(z, m, s^t\right),$$

subject to the production technology (16) and the constraints that supplies to home and foreign tradable goods markets are equal to demands by final good producers from (10) and the foreign analogue of (11);

$$(19) \quad y_{H,t}^{d*}\left(z, m, s^t\right) = \gamma\left[\frac{(1+\tau) P_H^*\left(z, m, s^t\right)}{P^*\left(s^t\right)}\right]^{-\theta}\left(\frac{P_T^*\left(s^t\right)}{P^*\left(s^t\right)}\right)^{\theta-1} P^*\left(s^t\right) Y^*\left(s^t\right).$$

Let the value of the producer with $\ln z_{T,t}(i) = z_t(i)$ if it decides to export in period $t+1$ be

$$V_T^1\left(z, m, s^t\right) = \max \Pi_P\left(z, m, s^t\right) - W\left(s^t\right)\left[f_1 m + (1-m) f_0\right] + n_s(z) Q\left(s^t\right) EV\left(z', 1, s^{t+1} | s^t, z\right),$$

and let the value if it does not export in period t be

$$V_T^0\left(z, m, s^t\right) = \max \Pi_{P,t}\left(z, m, s^t\right) + n_{s^t}(z) Q\left(s^t\right) EV_T\left(z', 1, s^{t+1} | s^t, z\right).$$

Then, the actual value of the producer can be defined as

$$
\begin{aligned}
(20) \quad V_T\left(z,m,s^t\right) &= \max\left\{V_T^1\left(z,m,s^t\right), V_T^0\left(z,m,s^t\right)\right\} \\
&= \max \Pi_{P,t}\left(z,m,s^t\right) + \max\left\{-W\left(s^t\right)\left[f_1 m + (1-m) f_0\right]\right. \\
&\quad + n_s(z)\, Q\left(s^t\right) EV\left(z', 1, s^{t+1} | s^t, z\right), \\
&\quad \left. n_s(z)\, Q\left(s^t\right) EV\left(z', 0, s^{t+1} | s^t, z\right)\right\}.
\end{aligned}
$$

Clearly the value of a producer depends on its export status and is monotonically increasing and continuous in z given m, and the states of the world. Moreover V_T^1 intersects V_T^0 from below as long as there are some firm who do not export.[5] Hence, it is possible to solve for the firm productivity at which a firm is indifferent between exporting or not exporting; that is, the increase in firm value from exporting equals the cost of exporting. This level of firm technology differs by the firm's current export status. The critical level of technology for exporters and non-exporters, $z_1\left(s^t\right)$ and $z_0\left(s^t\right)$, satisfy

$$
\begin{aligned}
(21) \quad V^1\left(z_1\left(s^t\right), 1, s^t\right) &= V^0\left(z_1\left(s^t\right), 1, s^t\right), \\
(22) \quad V^1\left(z_0\left(s^t\right), 0, s^t\right) &= V^0\left(z_0\left(s^t\right), 0, s^t\right).
\end{aligned}
$$

D. Free Entry

Each period, a new firm can be created by hiring f_E workers. New firms can enter either the tradeable or nontradeable sector. Firms incur these entry costs prior in the period prior to production. Once the entry cost is incurred firms receive an idiosyncratic productivity shock from the distribution $\phi_E(z)$. Entrants are also subject to a death shock with probability η_s. New entrants

[5] If the difference between f_0 and f_1 is relatively large, the economy may have $V^1 > V^0$ for all $z_t(i) \in (-\infty, \infty)$ for some periods.

into the tradeable sector can not export in their first productive period.

$$(23) \quad V_T^E\left(s^t\right) = \max -W_t f_E + n_{s^t} Q_t E V_T^0\left(z', 0, s^t\right) \phi_E\left(z'\right) \leq 0$$

$$V_N^E\left(s^t\right) = \max -W_t f_E + n_s Q_t E V_N\left(z', 0, s^t\right) \phi_E\left(z'\right) \leq 0$$

Let the mass of entrants in the tradable and non-tradable good sectors who pay the entry cost in period t be $N_{TE}\left(s^t\right)$ and $N_{NE}\left(s^t\right)$, while the mass of incumbents in the tradable and non-tradable good sector be $N_T\left(s^t\right)$ and $N_N\left(s^t\right)$.

The mass of exporters and non-exporters are given by

$$(24) \quad N_1\left(s^t\right) = \int_z \varphi_T\left(z, 1, s^t\right),$$

$$(25) \quad N_0\left(s^t\right) = \int_z \varphi_T\left(z, s^t\right),$$

and the mass of firms in the tradable good sector is given as

$$N_T\left(s^t\right) = N_1\left(s^t\right) + N_0\left(s^t\right).$$

These trade costs imply that only a fraction $n_x\left(s^t\right) = N_1\left(s^t\right) / N_T\left(s^t\right)$ of home tradable goods are available in foreign country in period t.

The mass of firms in the non-tradable sector is written as

$$N_N\left(s^t\right) = \int_z \varphi_N\left(z, s^t\right)$$

Let $N_{NE}\left(s^t\right)$ and $N_{TE}\left(s^t\right)$ respectively measure the measure of firms entering in the nontradeable and tradable sectors.

Given a policy rule for exporting, $m'\left(z, m, s^t\right)$, we can measure the fraction of firms that

start exporting among non-exporters as

$$n_0\left(s^t\right) = \frac{\int_z n_s\left(z\right) \cdot m'\left(z, 0, s^t\right) \cdot \varphi_T\left(z, 0, s^t\right)}{\int_z n_s\left(z\right) \cdot \varphi_T\left(z, 0, s^t\right)} = \frac{\int_{z_{0,t}}^{\infty} n_s\left(z\right) \varphi_T\left(z, 0, s^t\right) dz}{\int_{-\infty}^{\infty} n_s\left(z\right) \varphi_T\left(z, 0, s^t\right) dz}$$

Similarly, we can measure the stopper ratio, the fraction of exporters who stop exporting among surviving firms, as

$$n_1\left(s^t\right) = \frac{\int_z n_s\left(z\right) \cdot m'\left(z, 1, s^t\right) \cdot \varphi_T\left(z, 1, s^t\right)}{\int_z n_s\left(z\right) \cdot \varphi_T\left(z, 1, s^t\right)} = \frac{\int_{-\infty}^{z_{1,t}} n_s\left(z\right) \varphi_T\left(z, 1, s^t\right) dz}{\int_{-\infty}^{\infty} n_s\left(z\right) \varphi_T\left(z, 1, s^t\right) dz}$$

E. Government

The government collects tariffs from importers, final good producers, and equally distribute the tariff revenue to domestic consumers each period. The government's budget constraint is given as

$$(26) \quad T\left(s^t\right) = \tau \int_z \left(\frac{P_F\left(z, 1, s^t\right)}{P\left(s^t\right)}\right) y_F\left(z, 1, s^t\right).$$

F. Aggregate Variables

The investment, $X\left(s^t\right)$, is given by the law of motion for capital

$$X\left(s^t\right) = K'\left(s^t\right) - \left(1 - \delta\right) K\left(s^t\right)$$

The nominal exports and imports are given as

$$
\begin{aligned}
(27) \quad EX^N\left(s^t\right) &= \int_z e\left(s^t\right) P_H^*\left(z,1,s^t\right) y_H^*\left(z,1,s^t\right) \\
&= \gamma\left(1+\tau\right)^{-1} q\left(s^t\right)\left(\frac{P_T^*\left(s^t\right)}{P^*\left(s^t\right)}\right)^{\theta-1} P\left(s^t\right) Y^*\left(s^t\right), \\
(28) \quad IM_t^N &= \int_z P_F\left(z,1,s^t\right) y_F\left(z,1,s^t\right) \\
&= \gamma\left(1+\tau\right)^{-1}\left(\frac{P_T\left(s^t\right)}{P\left(s^t\right)}\right)^{\theta-1} P\left(s^t\right) Y\left(s^t\right),
\end{aligned}
$$

respectively. The nominal GDP of home country is defined as the sum of value added from non-tradable, tradable and final goods producers,

$$
Y^N\left(s^t\right) = P\left(s^t\right) Y\left(s^t\right) + EX^N\left(s^t\right) - IM^N\left(s^t\right).
$$

The trade to GDP ratio is given as

$$
TR\left(s^t\right) = \frac{EX^N\left(s^t\right) + IM^N\left(s^t\right)}{2Y^N\left(s^t\right)}.
$$

The total labor used for production, $L_P\left(s^t\right)$, is given by

$$
L_P\left(s^t\right) = \int_{z\times m} l_T\left(z,m,s^t\right) + \int_z l_N\left(z,s^t\right).
$$

The domestic labor[6] hired by exporters, $L_X\left(s^t\right)$, is given by

$$
(29) \quad L_X\left(s^t\right) = f_0 \int_z m'\left(z,0,s^t\right)\varphi_T\left(z,0\right) + f_1 \int_z m'\left(z,1,s^t\right)\varphi_T\left(z,1\right).
$$

[6] Entry costs are measured in units of labor to ensure a balanced growth path. In reality, firms incur costs at home and abroad in entering foreign markets. Some of these costs reflect the purchase of services while others are fees collected by governments which are rebated to consumers. We find that the aggregate properties of the model do not depend much on the form of these costs.

From (29), we see that the trade cost, measured in units of domestic labor, depends on the exporter status from the previous period.

Aggregate profits are measured as the difference between profits and fixed costs and equal

$$\Pi_t = \Pi_{F,t} + \int_{z\times m} \left\{\Pi_T(z,m,s^t) - m'\left(z,m,s^t\right) W_t \left[f_1 m + (1-m) f_0\right]\right\} \\ + \int_z \Pi_N(z,s^t) - f_E W_t \left(N_{TE,t} + N_{NE,t}\right)$$

For each type of good, there is a distribution of firms in each country. For the sake of exposition we have written these distributions separately by country and type of firm. It is also possible to rewrite the world distribution of firms over types as $\varphi : R^+ \times \{0,1\} \times \{H,F\} \times \{T,NT\}$, where now we have indexed firms by their origin and their sector. The exogenous evolution of firm technology as well as the endogenous export participation and entry decisions determine the evolution of this distribution. The law of motion for this distribution is summarized by the operator T which maps the world distribution of firms and entrants into the next period's distribution of firms,

$$\varphi' = T\left(\left\{\varphi, N_{TE}, N_{NE}, N_{TE}, N_{NE}\right\}, s^t\right).$$

G. Equilibrium Definition

In an equilibrium, variables satisfy several resource constraints. The final goods market clearing conditions are given by $Y_t = C_t + X_t$, and $Y_t^* = C_t^* + X_t^*$. Each individual goods market clears; The labor market clearing conditions are $L = L_{P,t} + L_{X,t} + f_E\left(N_{TE,t} + N_{NE,t}\right)$, and the foreign analogue, where labor hired by exporters, $L_{X,t}$, is given by (29) and the foreign analogue; The capital market clearing conditions are $K_t = \int_{z\times m} k_T\left(z,m,s^t\right) + \int_z k_N\left(z,s^t\right)$, and the foreign analogue. The government budget constraint is given by (26) and the foreign analogue. The

profits of firms are distributed to the shareholders, Π_t, and the foreign analogue; The international bond market clearing condition is given by $B_t + B_t^* = 0$; Finally, our decision to write the budget constraints in each country in units of the local currency permits us to normalize the price of consumption in each country as $P_t = P_t^* = 1$.

An equilibrium of the economy is a collection of allocations for home consumers C_t, B_t, K_{t+1}; allocations for foreign consumers C_t^*, B_t^*, K_{t+1}^*; allocations for home final good producers; allocations for foreign final good producers; allocations and prices for home non-tradable good producer; allocations and prices for foreign non-tradable good producer; allocations, prices, and export policies for home tradable good producer; allocations, prices and export decisions for foreign tradable good producer; labor used for exporting costs at home and foreign; labor used for entry costs; transfers T_t, T_t^* by home and foreign governments; real wages W_t, W_t^*, real rental rates of capital R_t, R_t^*, real and nominal exchange rates q_t and e_t; and bond prices Q_t that satisfy the following conditions: (i) the consumer allocations solve the consumer's problem; (ii) the final good producers' allocations solve their profit maximization problems; (iii) the non-tradable good producers' allocations, prices solve their profit maximization problems; (iv) the tradable good producers' allocations, prices, and export statuses solve their profit maximization problems; (v) the entry conditions for tradable and non-tradable sectors hold; (vi) the market clearing conditions hold; and (vii) the transfers satisfy the government budget constraint.

3. Calibration

We now describe the functional forms and parameter values of our benchmark economy. To highlight the role of export participation decisions, we also calibrate an alternate model with no fixed costs of exporting which we call the *No Cost* model.[7] The parameter values used in the simulation

[7] In some of the figures we also plot the response of a *Fixed cost* model in which startup and continuation costs are identical, i.e. $f_0 = f_1$.

exercises are reported in Table 1.

The instantaneous utility function is given as

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

where $1/\sigma$ is the inter-temporal elasticity of substitution. The choice of the discount factor, β, the rate of depreciation, δ, risk-aversion, σ, and capital share, α, are standard in the literature. The labor supply of consumers is normalized to be 1, $L = 1$.

The firm size distribution is largely determined by the underlying structure of shocks. We assume that entrants draw productivity based on

$$\ln A_{it} = \mu_E + \varepsilon_{it}, \qquad \varepsilon_{it} \tilde{} N(0, \sigma_\varepsilon^2),$$

while an an incumbent's productivity follows

$$\ln A_{it} = \rho \ln A_{it-1} + \varepsilon_{it}, \qquad \varepsilon_{it} \tilde{} N(0, \sigma_\varepsilon^2).$$

The assumption that firm technology follows an AR(1) with shocks drawn from an iid normal distribution implies that this conditional distribution follows a normal distribution with mean ρz and variance σ_ε^2. We assume that $\mu_E < 0$ so that entrants start out small relative to incumbents. We also assume that firms receive an exogenous death shock which depends on a firm's last period productivity, A_{it}, so that the probability of death is given as

$$n_d = 1 - n_s = \lambda e^{-\lambda A_{it}}.$$

In order to quantify the gains to trade reform in a dynamic environment, we need a model

that can generate reasonable trade growth and captures the entry and exit decisions of both new and exporting firms. For this reason, we target changes in aggregate trade flows as well as dynamic moment of exporters and non-exporters. We calibrate the model to match trade flows in two separate years, 1970 and 1992. In each year, we assume the economy is in its steady state but with a different tariff rates. From Yi (2003) the tariff rate in 1970 is $\tau_{1970} = 0.1038$ and in 1992 it is $\tau_{1992} = 0.045$. Overall, we have 9 parameters, θ, γ, ρ, σ_ε, μ_E, λ, f_0, f_1, f_E, which we choose to match the following nine observations:

1. Trade to gdp ratio in 1970 of 5.6 percent.
2. Trade to gdp ratio in 1992 10.3 percent.
3. A starter rate of 10 percent in 1992, based on Annual Survey of Manufactures 85 to 92.
4. A stopper rate of 17 percent in 1992, based on ASM.
5. Exporter output premia of 1.126 in logs based on 1992 Census of Manufactures
6. Job creation and destruction rate of 9.6 percent based on U.S. Census Bureaus' Longitudinal Research Database from 1973 to 1993, Baldwin, Dunne and Haltiwanger (1998).
7. Five-year exit rate of entrants of 36.2 percent based on plants that first began producing in 1967, 1972, and 1977 Census of Manufactures, estimated by Dunne et al. (1989).
8. A 3.1 percent share of aggregate employment accounted for by firms in the first and second year of existence SBA (2001).
9. The mass of firms in 1992 is normalized to 2.

The first two observations about trade shares are based on NIPA and are measured in nominal terms. The next three observations about exporter dynamics and exporter premia are based on work by Bernard and Jensen (1999) using the Annual Survey of Manufactures from 1985 to 1992 and 1992 Census of Manufactures. The next three moments about job reallocation, survival rates, and new

firm size, help to pin down both firm creation, destruction, and the evolution of establishment productivity. The moments along with those generated in the models are reported in table 2.

The parameter θ determines both the producer's markup as well as the elasticity of substitution across varieties. Our calibration generates an elasticity of 3.25. This is on the high side of industry estimates. For instance, using the U.S. quarterly data of 163 industries at the 3-digit SIC level from 1980:1 to 1988:4, Gallaway, McDaniel and Rivera (2003) estimate that the elasticities range from 0.14 to 3.49. However, as Ruhl (2003) shows in a similar model to ours, estimates of the elasticity of substitution over the business cycle which do not take into account the extensive margin are biased downwards. Our calibration yields a markup over marginal cost of 44.4 percent, however with our free entry conditions the profits accruing to firms offset the fixed costs of exporting and entering so that the model does not have any pure profits. The parameter γ determines the expenditure share of tradable goods and our calibration yields $\gamma = 0.366$.

The parameters for the model with no fixed costs of exporting are reported in the bottom of the table in the row titled *No Cost*. In this model, all firms export always and so the moments we match are about firm dynamics and aggregate trade flows. With no extensive margin, for the change in tariffs to match the trade growth we observe from 1970 to 1992, the no cost model requires goods to be much more substitutable, $\theta = 15$, and the tradable sector to be smaller, $\gamma = 0.3066$. Because goods are quite substitutable the model requires technology shocks to be much smaller and more persistent to match the evidence on job reallocation.

Given our calibrated models, we now compare the amount of trade they predict against actual data for the period 1970 to 2000. To construct the trade share in the models, we feed through the series of annual tariffs from Yi (2003). Trade shares and tariff rates are plotted in Figure 1. Each new tariff rate is assumed to be completely unanticipated and permanent. Panel a shows the predicted path of trade shares in the model and the data. The no cost model matches the trade share in 1970

and 1992 exactly, while the sunk cost model underpredicts the 1992 trade share, even though we have calibrated the model to the 1992 share. The reason for the differences is that our calibration assumed that the economy had reached a new steady state[8] by 1992. The lower trade share suggests that the converging to the new steady state is a long process. The sunk cost model is closer to the data in the 70s and early 80s. Both models substantially underpredict trade growth in the post 1992 period.

Panel b shows the relationship between tariffs and trade in the models and the data. The sunk cost model appears to provide the best fit with the data over this period as it leads to slower trade growth in response to lower tariffs. The sunk cost model also better matches the data in that it generates increases in trade integration even in periods when tariffs are constant. For instance, we see that only the sunk cost model generates increased trade integration in the period with tariffs constant at 4.5 percent.

4. Tariffs and Steady State

We now study how the structure of the steady state economy depends on tariffs. We first explore the impact of tariffs on the characteristics of exporters vs. non-exporters. We then study how tariffs affect aggregates such as trade, export participation, consumption, and investment.

A. Exporter characteristics and dynamics

Figure 3 plots the productivity distribution of establishments in the tradable sector in the stationary steady state with a tariff rate of $\tau = 0.045$. We also plot the distribution of exporters, $\varphi_{T,t}(z,1)$, and non-exportrers, $\varphi_{T,t}(z,0)$. The sunk costs along with the lags in exporting imply that there is a larger range of productivities for which some firms export and some firms just sell

[8] We have also calibrated a version of the model which approximately hits the 1992 trade share along the transition. This version of the model generates more trade growth in the 70 to 92 period and afterwards. We do not present these results because the calibration procedure is much more time consuming and so we do not get as tight a match on some other moments. For completeness though, we present these results in panels c and d of Figure 2.

at home. Notice that the distribution of productivities is not quite normally distributed as there is more mass of low productivity firms. This is a result of the productivity disadvantage of entering firms.

Figure 4 depicts the relationship between tariffs and some exporter characteristics. Panel a plots the productivity of the marginal starter, z_1, and the marginal stopper, z_0, against tariffs ranging from 0 to 30 percent. Both measures are increasing in the tariff rate as the higher tariff lowers the value of exporting.

Panel b plots the exporter output and productivity premia in the stationary distribution of firms against the tariff rate. For low tariff rates, the exporter productivity premia is largely independent of the tariff rate. For higher tariffs, the productivity premium is increasing with the tariff rate. We get this non-linear relationship between tariffs and the exporter productivity premium because the tariff alters the type of firms that export. As we increase the tariff, we increase the productivity cutoff to start and stop exporting. For low tariffs, when the pool of exporters is large relative to the pool of non-exporters, an increase in tariffs adds a lot of productive firms to the non-exporting pool, without changing the average productivity of the exporting pool. The non-linear productivity-tariff relation contributes to a U-shaped output premium in the tariff rate. Absent tariffs, the output premium is increasing in the productivity premium. However, holding the productivity premium constant, the output premium is decreasing with tariffs as higher tariffs reduce each firms exports. For tariffs less than about 13 percent, the direct effect of tariffs on output dominates the indirect effect working through the productivity premium. For tariffs greater than 13 percent, the effect of tariffs on productivity dominates.

Panel c plots the equilibrium starter and stopper rate for each tariff level. As tariffs increase, we find that nonexporters start exporting less frequently and those firms that do export, exit fairly frequently. The duration of exporting is inversely proportional to the stopper rate. With 10 percent

tariffs, the model predicts that each export spell lasts about 4.2 years. Under free trade, the duration of each export spell rises to about 8.4 years.

B. Aggregates

We start our analysis of the aggregate effect of tariffs by considering trade related variables. Panel a in Figure 5 plots the nominal trade to gdp ratio against tariffs in both the sunk cost and no cost models. To a first approximation the models generate a similar relationship between trade and tariffs. This is in part due to our calibration, which constrains the two models to generate the same amount of trade with tariffs of 10 percent and 4.5 percents. Outside this range there are some noticeable differences. For instance, for tariff rates, below 4.5 percent the no cost model predicts a larger increase in trade as tariffs are decreased. Similarly as tariffs increase beyond 10 percent, the sunk cost model predicts a larger drop off in trade. That the sunk cost model matches up so closely with the no cost model even though goods in the no cost model are about 5 times more substitutable than in the sunk cost model is due to the extensive margin. Panel b plots the share of firms in the tradeable sector that are exporters. Moving from 10 percent tariffs to free trade increases the export participation from 17.8 percent of the firms to 48.2 percent. Similarly, as we increase tariffs above 10 percent, exporters exit foreign markets in droves.

We now consider some non-trade related aggregates. For exposition, all series are measured relative to the level under free trade in natural logs. We first consider how tariffs affect firm creation and capital accumulation. Panel c shows the mass of tradable and non-tradable firms is increasing in tariffs. It also shows that the capital stock is increasing in tariffs. Notice that the capital stock expands by less than the number of firms so that the capital intensity of each firm is declining. Panel d shows that the mass of available tradable varieties, measured as export and domestic tradables, is mostly increasing with tariffs although at a much smaller rate than the mass of tradable firms.

Moving from free trade to 30 percent tariffs increases the mass of tradables by about 43 percent, but the net effect is to increase the mass of available varieties by about 2.8 percent. The pro-variety of tariffs is also found in the work by Baldwin and Forslind (2005). Thus, tariffs encourage firm creation over capital accumulation and investing in export capacity.

There are two reasons why increasing tariffs encourage firm creation. First, tariffs raise the relative price of physical capital to firms as physical capital is produced using labor and capital while firms and export capacity are produced just using labor. Second, tariffs lower the benefits of investing in export capacity. When we cut tariffs, we make it easier to produce additional varieties of goods by incurring the smaller cost of exporting. We see that tariffs encourage savings through investment in firms and capital, while discouraging saving through export capacity.

The bottom panel plots consumption against tariffs. Steady state consumption is increasing in tariffs so that free trade leads to lower steady state welfare. This result is not so perverse as it appears at first glance. As we have explained, free trade leads to less accumulation of firms. Thus, to reach the free trade steady state, the economy gave up substantially less consumption in the past. To assess the costs or benefits of reducing tariffs it is necessary then to consider the transition dynamics.

5. Transition Dynamics following Tariff Reductions

In this section, we consider a move from a world in which both countries charge 5 percent tariffs to free trade. We assume this policy is completely unanticipated.[9] We first discuss the impact on trade and exporters. We then discuss the aggregate implications. The evolution of the firm distribution gives rise to a long, drawn out transition to the new steady state. To simplify the presentation we plot the first 150 years which contain the most interesting dynamics. The long-run

[9] This distinction is quite important in the sunk cost model, as an anticipated trade reform will generate a change in trade prior to the reform.

changes are reported in Table 3.

A. Exporters and Trade

Figure 6 plots the evolution of trade related variables along the transition to the new steady state. The trade to GDP ratio and the export ratio are in panels a and b. From panel a, in both the no costs and sunk cost model trade expands immediately with the cut in tariffs as existing varieties become less expensive. In the no cost model, the trade share jumps to 5.6 percentage points to its new long-run level in one period. In the sunk cost model, the immediate increase is smaller and the total trade expansion is much more drawn out. In the first period trade increases from 9.9 percent to 11.2 percent as existing varieties are less expensive. In the second period, trade expands to 12.6 percent as a bunch of new firms start exporting. From then on, trade grows more gradually to its long-run value of 13.8 percent. The trade dynamics reflect the participation decisions of exporters depicted in the bottom panel. The exporter ratio increases by about 8 percentage points in the first period after the tariff cut and then gradually expands another 8 percentage points as it converges to the new steady state.

From panel c we see that the expansion in exporter participation occurs through a decrease in the productivity thresholds to start or stop exporting. The decline in these thresholds is largest in the initial period and then the declines are more gradual over time. The thresholds decline for two reasons. First, as we will see, output grows gradually in response to the tariff decline. As the market expands more firms can profitably incur the fixed costs. Second, over time the mass of domestic tradeable firms declines increasing the share of the market available for exporters. These declining thresholds generate the gradual increases in the exporter ratio.

Another way of understanding the dynamics of the exporter ratio is to consider the stopper and starter rates depicted in panel d. The cut in the tariff leads to an immediate increase in the

starter rate and a decrease in the stopper rate. The stopper rates increases somewhat in the second period to a level to above its long-run level. Likewise, the starter rate declines to a level below their long-run levels. These dynamics are a result of the firm distributions. The bounceback in the stopper rate occurs because a lot of the unproductive firms that would have exited under the old tariff regime decide to wait it out another period. Likewise the cut in the previous period's starter rate means that a lot of marginal firms under the old regime are now exporting. Thus, the distribution of exporters is concentrated in a range close to the exit threshold leading to the relatively high exit rate. The low starter rate also reflects the high entry rate in the previous period and the low exit rate as the mass of firms in the range around the new threshold is reduced.

B. Aggregates

Table 3 shows that making welfare statements based on steady states can be quite misleading. Just comparing steady states in the sunk cost model, we find a drop in welfare equal to 1.22 percent of lifetime consumption. However, including the transition period, we find that this policy generates an increase in welfare of about 0.54 percent of lifetime consumption. Including transition dynamics lowers the welfare gain in the no cost model from 0.33 percent to 0.19 percent. The large difference in gains in the sunk cost model derive from being able to use the firms accumulated in the old steady state more efficiently along the transition to the new steady state. Thus, along the transition the model generates a sustained economic expansion with consumption, output, productivity, and investment booming.

Panel a, b, and c of Figure 7 plot consumption, output and investment following the cut in tariffs. Generally speaking, the tariff cut leads to a consumption boom followed by an investment and output boom. Consumption increases in the first period after the tariff cut, and continues to build for the next 15 years at which point it is 0.8 percent above old steady state level. After year

15, consumption declines as it transits it to its new steady state level 1.22 percent below its initial level. The transition is quite drawn out as it takes nearly 70 years for consumption to go below the old steady state level.

Panel d shows that the amount of labor used in production drops 0.48 percent in the year of the tariff cut. Labor rebounds in the following year to only 0.23 percent below the old steady state but then deteriorates slowly to the new steady state 0.79 percent its initial level. The decline in the amount of labor used for production reflects a substantial increase in labor used to build and maintain export capacity and a smaller decline in the labor used to create firms. The particularly large drop in labor in year 1 reflects the large expansion in exporter participation. The increase in export participation and consumption are financed out of the capital stock and so we see a decline in investment in year 1 which drags down output in the first year. In the following year, investment recovers and output goes positive. Output continues to expand until year 12 until it is about 0.98 percent above the old steady state. From then on, output declines very slowly to its new steady state 1.12 percent below the old state.

Panel e depicts the evolution of productivity. We measure productivity as a Solow residual. Productivity changes because the tariff ends up reallocating production across firms. The tariff also affects productivity through its effect on firm investment in export capacity. This is clearest from the 0.2 percent fall in productivity in the first period. To build export capacity, firms shift workers from producing goods. These investments do not show up in our measure of output and hence give the appearance of a decline in productivity. Moreover, we do not control for export capacity resulting from these investments and so our measures may not accurately reflect TFP. This is clear in the second period, as productivity increases by 0.9 percent as relatively productive exporters begin to produce. After the rebound in productivity, productivity remains high and gradually decays. The autocorrelation of the Solow residual is 0.95, about what one finds in the aggregate data. Thus, the

model generates TFP shocks that look like the aggregate data out of changes in tariffs.

Panels g and h depict the change in the mass of tradable and non-tradable firms. The non-tradable sector gradually declines in both models. The tradable sector expands in the no cost model and declines in the sunk cost model. These differences arise because the no cost model can only expand the available tradable varieties by investing in new firms, while the sunk cost model can create more varieties by investing in export capacity. The decline in variety in the sunk cost model contributes to the decline in productivity over time as it leads to a decrease in the productivity of the marginal exporter. In the long-run the reallocative effect emphasized in the literature disappears.

6. Conclusion

Recent research has emphasized the gains to trade reform in models with firm level heterogeneity. We reconsider these gains in a model of firm heterogeneity and endogenous export participation. In contrast to previous research, our model captures the key features of exporter dynamics and allows for capital accumulation. We find that a model with these export dynamics captures the key features of U.S. trade growth from 1970 to 2000. In particular, the model generates lags in trade expansion in response to cuts in tariffs as establishment entry into foreign markets takes time.

Our model of endogenous export participation generates lags in export participation that are both too short and too long. In particular, following a cut in tariffs the model predicts a large increase in export participation. This burst of exporting is then followed by a long drawn out expansion in export participation. More empirical work is necessary to know to what extent the dynamics of export participation following a trade liberalization follow this pattern.

We find that the gains to trade reform are larger once one takes into account firm heterogeneity. We also find that steady state comparisons of welfare give quite misleading predictions about

the benefits to removing tariffs. The reason is that with endogenous export participation, tariffs discourage investment in export capacity and encourage investment in firm creation. With tariffs, economies overaccumulate firms. Removing tariffs allows existing firms to produce more efficiently and generates a sustained boom in productivity.

Appendix

A1. Equations for the Model Solution

The following equations describe the equations for the model solution. For simplicity, we normalize final good prices to be 1 in two countries, $P_t = P_t^* = 1$. Also, we apply the symmetry of the model.

Consumers

$$
\begin{aligned}
Q_t &= \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma}, \\
q_t &= 1 = \left(\frac{C_t}{C_t^*} \right)^{\sigma}, \\
1 &= \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(R_{t+1} + 1 - \delta \right), \\
X_t &= K_t - (1-\delta) K_{t-1}, \\
C_t + X_t + Q_t B_t &= W_t L_t + R_t K_{t-1} + B_{t-1} + \Pi_t + T_t.
\end{aligned}
$$

Final Good Producers

$$
\begin{aligned}
P_{N,t} &= \left(\int_{\mathcal{I}_{N,t}} p_{N,i,t}^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \\
P_{T,t} &= \left\{\int_{\mathcal{I}_{T,t}} p_{H,i,t}^{1-\theta} di + \int_{\mathcal{E}_t^*} \left[(1+\tau)\, p_{F,i,t}\right]^{1-\theta} di\right\}^{\frac{1}{1-\theta}}, \\
P_t &= 1 = \left(\frac{P_{T,t}}{\gamma}\right)^{\gamma} \left(\frac{P_{N,t}}{1-\gamma}\right)^{1-\gamma}, \\
y_{N,i,t} &= (1-\gamma)\, p_{N,i,t}^{-\theta} P_{N,t}^{\theta-1} Y_t, \\
y_{H,i,t} &= \gamma p_{H,i,t}^{-\theta} P_{T,t}^{\theta-1} Y_t, \\
y_{F,i,t} &= \left[(1+\tau)\, p_{F,i,t}\right]^{-\theta} P_{T,t}^{\theta-1} Y_t, \\
Y_t &= C_t + X_t.
\end{aligned}
$$

Non-Tradable Good Producers

$$
\begin{aligned}
MC_t &= \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}, \\
p_{N,i,t} &= \left(\frac{\theta}{\theta-1}\right) \frac{MC_t}{A_{N,i,t}}, \\
(1-\alpha)\, R_t k_{N,i,t} &= \alpha W_t l_{N,i,t}, \\
y_{N,i,t} &= A_{N,i,t} k_{N,i,t}^{\alpha} l_{N,i,t}^{1-\alpha} = \frac{A_{N,i,t}}{MC_t} \left(\frac{W_t}{1-\alpha}\right) l_{N,i,t}, \\
\pi_{N,i,t} &= p_{N,i,t} y_{N,i,t} - R_t k_{N,i,t} - W_t l_{N,i,t} = \left(\frac{1-\gamma}{\theta}\right) A_{N,i,t}^{\theta-1} I_{N,t}^{-1} Y_t, \\
P_{N,t} &= \left(\frac{\theta}{\theta-1}\right) MC_t I_{N,t}^{\frac{1}{1-\theta}},
\end{aligned}
$$

where $I_{N,t} = \int_{\mathcal{I}_{N,t}} A_{N,i,t}^{\theta-1} di$. In aggregates we have

$$\begin{aligned} L_{N,t} &= \int_{\mathcal{I}_{N,t}} l_{N,i,t} di = (1-\gamma)\left(\frac{\theta-1}{\theta}\right)\left(\frac{1-\alpha}{W_t}\right) Y_t, \\ K_{N,t} &= \int_{\mathcal{I}_{N,t}} k_{N,i,t} di = (1-\gamma)\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R_t}\right) Y_t. \end{aligned}$$

The value of the firm is given as

$$V_{N,t}(A) = \left(\frac{1-\gamma}{\theta}\right) A^{\theta-1} I_{N,t}^{-1} Y_t + \int_{-\infty}^{\infty} n_s(A) Q_t V_{N,t+1}(A') \phi_N(A'|A) dA',$$

where $\phi_N(A\prime|A)$ is the conditional probability of $A\prime$ given A provided that the firm survives the next period. The entrant's value is given as

$$V_{NE,t} = -W_t f_E + Q_t \int_{-\infty}^{\infty} V_{N,t}(A) \phi_{NE}(A) dA.$$

The density of non-tradable good firms for productivity, A', evolves as

$$D_{N,t}(A') = \int_{-\infty}^{\infty} \phi_N(A'|A) n_s(A) D_{N,t-1}(A) dA + N_{NE,t-1} \phi_{NE}(A').$$

The mass of non-tradable good producers is given as

$$N_{N,t} = \int_{-\infty}^{\infty} D_{N,t}(A) dA.$$

Tradable Good Producers

$$
\begin{aligned}
p_{H,i,t} &= P^*_{H,i,t} = P_{F,i,t} = \left(\frac{\theta}{\theta-1}\right)\frac{MC_t}{A_{T,i,t}}, \\
(1-\alpha)\, R_t k_{T,i,t} &= \alpha W_t l_{T,i,t}, \\
y_{T,i,t} &= A_{T,i,t} k^{\alpha}_{T,i,t} l^{1-\alpha}_{T,i,t} = \frac{A_{T,i,t}}{MC_t}\left(\frac{W_t}{1-\alpha}\right) l_{T,i,t}, \\
\pi_{T,i,t} &= p_{H,i,t} y_{H,i,t} + m_{i,t-1} p^*_{H,i,t} y^*_{H,i,t} - R_t k_{T,i,t} - W_t l_{T,i,t} - m_{i,t} f_m W_t \\
&= \left(\frac{\gamma}{\theta}\right)\left[1 + m_{i,t-1}(1+\tau)^{-\theta}\right] A^{\theta-1}_{T,i,t} I^{-1}_{T,t} Y_t - m_{i,t} f_m W_t, \\
P_{T,t} &= \left(\frac{\theta}{\theta-1}\right) MC_t \left[I_{T,t} + (1+\tau)^{1-\theta} I^*_{X,t}\right]^{\frac{1}{1-\theta}},
\end{aligned}
$$

where $I_{T,t} = \int_{\mathcal{I}_{T,t}} A^{\theta-1}_{T,i,t} di$ and $I^*_{X,t} = \int_{\mathcal{E}^*_t} A^{*\theta-1}_{T,i,t} di$, $f_m = f_0$ if $m_{i,t-1} = 0$, otherwise $f_m = f_1$. In aggregates we have

$$
\begin{aligned}
L_{T,t} &= \int_{\mathcal{I}_{T,t}} l_{T,i,t} di = \gamma\left(\frac{\theta-1}{\theta}\right)\left(\frac{1-\alpha}{W_t}\right) Y_t \left[I_{T,t} + (1+\tau)^{-\theta} I_{X,t}\right]\left[I_{T,t} + (1+\tau)^{1-\theta} I^*_{X,t}\right]^{-1}, \\
K_{T,t} &= \int_{\mathcal{I}_{T,t}} k_{T,i,t} di = \gamma\left(\frac{\theta-1}{\theta}\right)\left(\frac{\alpha}{R_t}\right) Y_t \left[I_{T,t} + (1+\tau)^{-\theta} I_{X,t}\right]\left[I_{T,t} + (1+\tau)^{1-\theta} I^*_{X,t}\right]^{-1}.
\end{aligned}
$$

The value of the firm is given as

$$
\begin{aligned}
V_{m,t}(A) &= \left(\frac{\gamma}{\theta}\right) A^{\theta-1} Y_t \left[1 + m(1+\theta)^{-\theta}\right]\left[I_{T,t} + (1+\tau)^{1-\theta} I^*_{X,t}\right]^{-1} \\
&\quad + \max\left\{\begin{array}{c} \int_{-\infty}^{\infty} n_s(A)\, Q_t V_{0,t+1}(A')\, \phi_T(A\prime|A)\, dA\prime, \\ \int_{-\infty}^{\infty} n_s(A)\, Q_t V_{1,t+1}(A_{t+1})\, \phi_T(A'|A)\, dA' - W_t f_m \end{array}\right\},
\end{aligned}
$$

where $\phi_T(A'|A)$ is the conditional probability of $A\prime$ given A provided that the firm survives the next period. The entrant's value is given as

$$
V_{TE,t} = -W_t f_E + Q_t \int_{-\infty}^{\infty} V_{0,t}(A_t)\, \phi_{TE}(A_t)\, dA_t.
$$

The density of tradable good firms for productivity, A', evolves as

$$\begin{aligned} D_{0,t}\left(A'\right) &= \int_{-\infty}^{A_{0,t-1}} \phi_T\left(A'|A\right) n_s\left(A\right) D_{0,t-1}\left(A\right) dA + \int_{-\infty}^{A_{1,t-1}} \phi_T\left(A'|A\right) n_s\left(A\right) D_{1,t-1}\left(A\right) dA + N_{TE,t-1}\phi \\ D_{1,t}\left(A'\right) &= \int_{A_{0,t-1}}^{\infty} \phi_T\left(A'|A\right) n_s\left(A\right) D_{0,t-1}\left(A\right) dA + \int_{A_{1,t-1}}^{\infty} \phi_T\left(A'|A\right) n_s\left(A\right) D_{1,t-1}\left(A\right) dA. \end{aligned}$$

The mass of starters and continuing exporters before the death shocks are given as

$$\begin{aligned} N_{01,t} &= \int_{A_{0,t-1}}^{\infty} D_{0,t-1}\left(A\right) dA, \\ N_{11,t} &= \int_{A_{1,t-1}}^{\infty} D_{1,t-1}\left(A\right) dA. \end{aligned}$$

The mass of tradable producers is given as

$$N_{T,t} = \int_{-\infty}^{\infty} \left[D_{0,t}\left(A\right) + D_{1,t}\left(A\right)\right] dA.$$

A1. Steady State

$$\begin{aligned} 1 &= \beta\left(R + 1 - \delta\right), \\ Q &= \beta, \\ X &= \delta K \\ Y &= C + X. \end{aligned}$$

$$\begin{aligned} \text{(A1)} \quad K &= K_T + K_N \\ &= \left(\frac{\theta - 1}{\theta}\right)\left(\frac{\alpha}{R}\right)\left(C + \delta K\right)\left\{\gamma\left[I_T + \left(1+\tau\right)^{-\theta} I_X\right]\left[I_T + \left(1+\tau\right)^{1-\theta} I_X\right]^{-1} + \left(1-\gamma\right)\right\}. \end{aligned}$$

From the producers' problem, we have

$$\begin{aligned}\Pi &= \left[\frac{\theta}{(\theta-1)(1-\alpha)}\right] WL_P - RK - WL_P - Wf_E\left(N_{TE}+N_{NE}\right) - Wf_0N_{01} - Wf_1N_{11} \\ &= \left[\frac{\theta}{(\theta-1)(1-\alpha)}\right] WL_P - RK - WL.\end{aligned}$$

The tariff revenue is given as

$$T = \tau\gamma\left(1+\tau\right)^{-\theta} I_X \left[I_T + \left(1+\tau\right)^{1-\theta} I_X\right]^{-1} Y.$$

So, the budget constraint of consumers can be rewritten as

$$\text{(A2)}\ \ C + \delta K = \left[\frac{\theta}{(\theta-1)(1-\alpha)}\right] WL_P + \tau\gamma\left(1+\tau\right)^{-\theta} I_X \left[I_T + \left(1+\tau\right)^{1-\theta} I_X\right]^{-1} \left(C+\delta K\right).$$

The labor employed for production is given as

$$\begin{aligned}L_P &= \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{R}{W}\right) K \\ &= L - f_E\left(N_{TE}+N_{NE}\right) - f_0N_{01} - f_1N_{11}.\end{aligned}$$

For other variables, we have

$$\begin{aligned}MC &= \left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}, \\ P_N &= \left(\frac{\theta}{\theta-1}\right) MCI_N^{\frac{1}{1-\theta}}, \\ P_T &= \left(\frac{\theta}{\theta-1}\right) MC\left[I_T + \left(1+\tau\right)^{1-\theta} I_X\right]^{\frac{1}{1-\theta}}, \\ P &= 1 = \left(\frac{P_T}{\gamma}\right)^{\gamma}\left(\frac{P_N}{1-\gamma}\right)^{1-\gamma}.\end{aligned}$$

The values of firms are given as

$$
\begin{aligned}
V_N(A) &= \left(\frac{1-\gamma}{\theta}\right) A^{\theta-1} I_N^{-1} Y + \int_{-\infty}^{\infty} n_s(A)\, Q V_N\left(A'\right) \phi_N\left(A'|A\right) dA', \\
V_{NE} &= 0 = -W f_E + Q \int_{-\infty}^{\infty} V_N(A)\, \phi_{NE}(A)\, dA, \\
V_0(A) &= \left(\frac{\gamma}{\theta}\right) A^{\theta-1} Y \left[I_T + (1+\tau)^{1-\theta} I_X\right]^{-1} \\
&\quad + \max \left\{ \begin{array}{c} \int_{-\infty}^{\infty} n_s(A)\, Q V_0(A')\, \phi_T(A\prime|A)\, dA', \\ \int_{-\infty}^{\infty} n_s(A)\, Q V_1(A')\, \phi_T(A\prime|A)\, dA' - W_t f_0 \end{array} \right\}, \\
V_1(A) &= \left(\frac{\gamma}{\theta}\right) A^{\theta-1} Y \left[1 + (1+\theta)^{-\theta}\right] \left[I_T + (1+\tau)^{1-\theta} I_X\right]^{-1} \\
&\quad + \max \left\{ \begin{array}{c} \int_{-\infty}^{\infty} n_s(A)\, Q V_0(A')\, \phi_T(A\prime|A)\, dA', \\ \int_{-\infty}^{\infty} n_s(A)\, Q V_1(A')\, \phi_T(A\prime|A)\, dA' - W_t f_1 \end{array} \right\}, \\
V_{TE} &= 0 = -W f_E + Q \int_{-\infty}^{\infty} V_0(A)\, \phi_{TE}(A)\, dA.
\end{aligned}
$$

The density of firm productivity is given as

$$
\begin{aligned}
D_N\left(A'\right) &= \int_{-\infty}^{\infty} \phi_N\left(A'|A\right) n_s(A)\, D_N(A)\, dA + N_{NE} \phi_{NE}\left(A'\right), \\
D_0\left(A'\right) &= \int_{-\infty}^{A_0} \phi_T\left(A'|A\right) n_s(A)\, D_0(A)\, dA + \int_{-\infty}^{A_1} \phi_T\left(A'|A\right) n_s(A)\, D_1(A)\, dA + N_{TE} \phi_{TE}\left(A'\right), \\
D_1\left(A'\right) &= \int_{A_0}^{\infty} \phi_T\left(A'|A\right) n_s(A)\, D_0(A)\, dA + \int_{A_1}^{\infty} \phi_T\left(A'|A\right) n_s(A)\, D_1(A)\, dA.
\end{aligned}
$$

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Table 1: Parameter Values

Benchmark Model	
Preferences	$\beta = 0.96, \quad \sigma = 2, \quad \theta = 3.23, \quad \gamma = 0.3664$
Production	$\alpha = 0.36, \quad \delta = 0.10,$ $Var(\eta) = 0.069^2, \rho = 0.84, \lambda = 5.55$
Entry costs	$f_E = 2.478$
Trade costs	$f_0 = 0.228, \quad f_1 = 0.216$
Tariffs	$\tau_{1970} = 1.1038, \tau_{1992} = 1.045$
Variations	
No Costs	$f_E = 0.324, f_1 = f_0 = 0, \theta = 15,$ $\gamma = 0.3066, Var(\eta) = 0.0098^2, \mu_E = 0.159, \lambda_D = 4.468, \rho = 0.91$
Fixed cost	$f_1 = f_0 = 0.362, f_E = 3.423, \theta = 2.43, \gamma = 0.492$ $Var\,(\eta) = 0.103^2;\ \rho = 0.861;\ \mu_E = 0.445;\ \lambda_D = 6.118;$

Table 2: Data and Model Moments

	Data	Sunk	No Cost
TR/Y (1970), NIPA	0.055	0.055	0.056
TR/Y (1992), NIPA	0.103	0.103	0.103
Starter Rate 90-92, ASM	0.10	0.10	-
Stopper Rate 90-92, ASM	0.17	0.17	-
Exporter Output Premium, CM 92	1.126	1.126	-
Annual Job Reallocation	0.096	0.096	0.0967
Entrant 5-year exit rate	0.362	0.373	0.374
Share of employment in 0 to 1 yr firms (2001 SBA)	0.031	0.031	0.031

	Sunk Cost	No Cost	Fixed Export
C1	-1.22	0.33	-3.53
EX/Y*	3.92	5.53	4.60
Y	-1.11	0.48	-3.44
Capital Stock	-0.62	1.31	-2.97
Z	-0.90	0.13	-2.42
LP1	-0.79	-	-1.72
Non-Tradable Varieties	-1.28	-0.49	-2.20
Domestic Tradeable Varieties	-10.53	1.13	-10.63
Total Tradable Varieties	-0.11	1.13	-2.32
Exporter Ratio*	15.46	-	11.15
Exporter Output Premia	8.41	-	3.63
Exporter Productivity Premia	0.38	-	-1.48
Transition Welfare	0.54	0.19	0.74
SS Welfare	-1.22	0.33	-3.53

* Measured as percentage points

Table 3: Percent Change in Steady State and Transition Changes from eliminating 5% Tariff

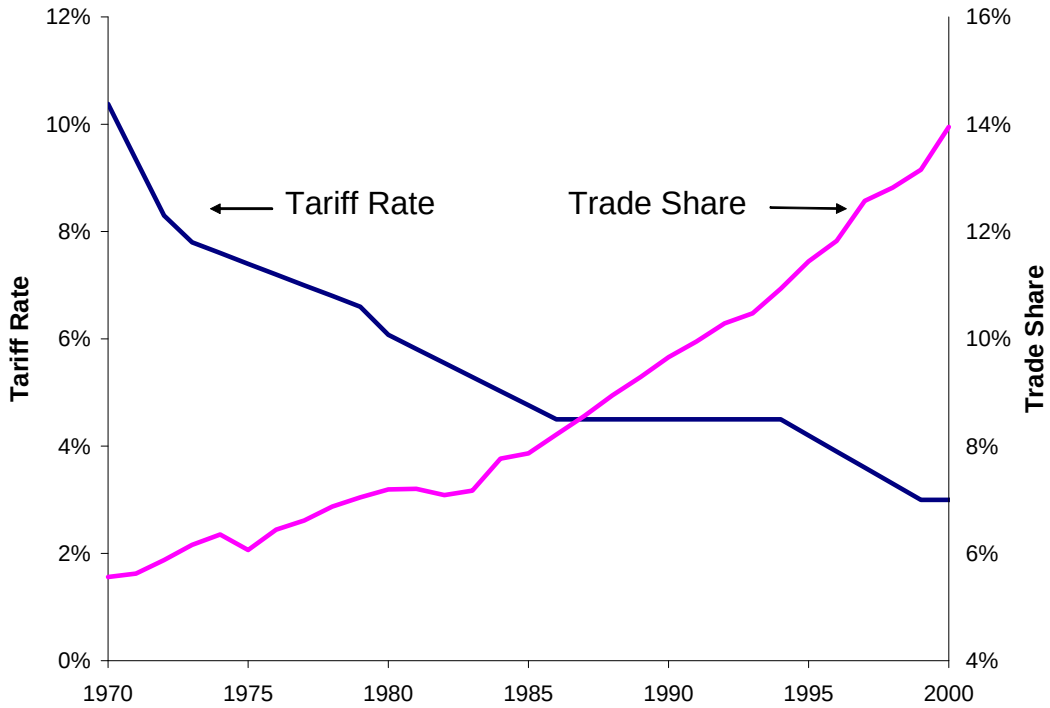


Figure 1: Trade and Tariff Rates

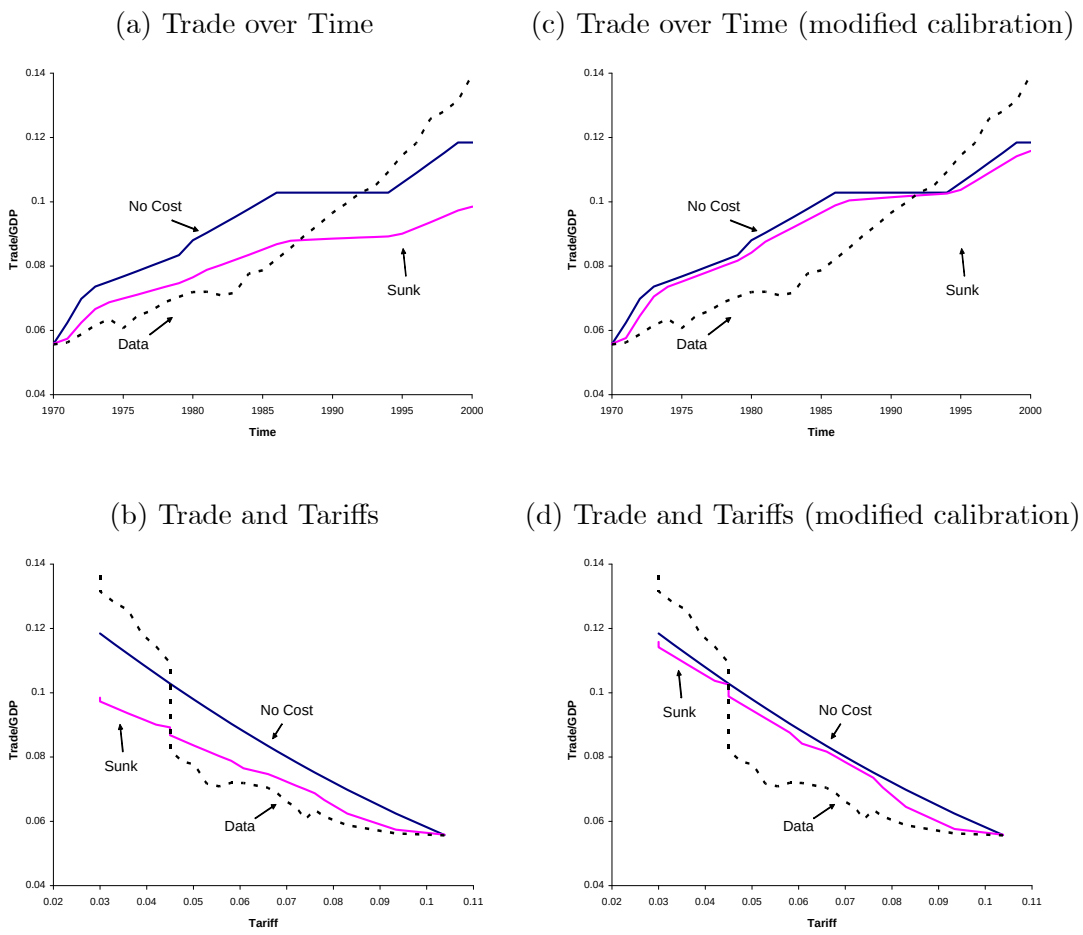


Figure 2: Trade and Tariffs, Data and Models

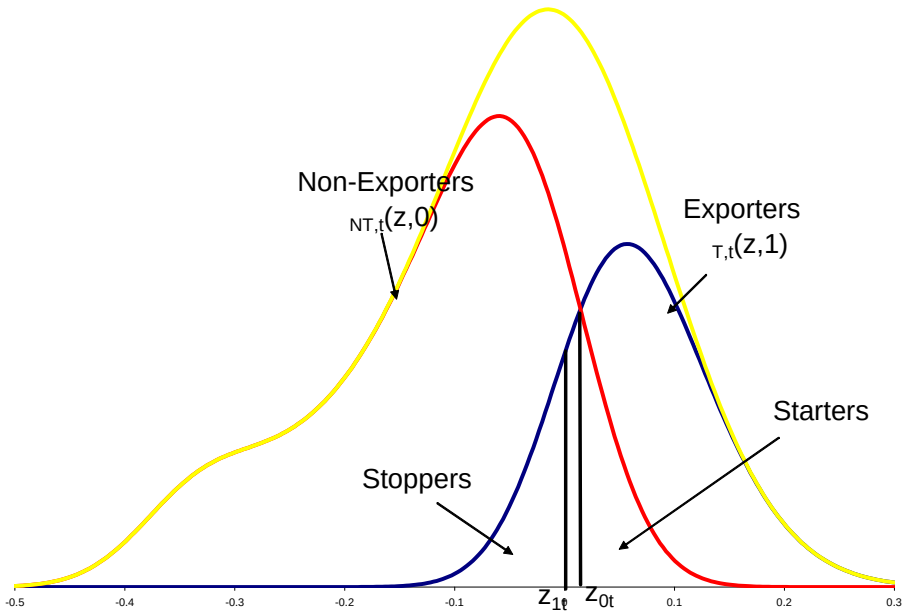
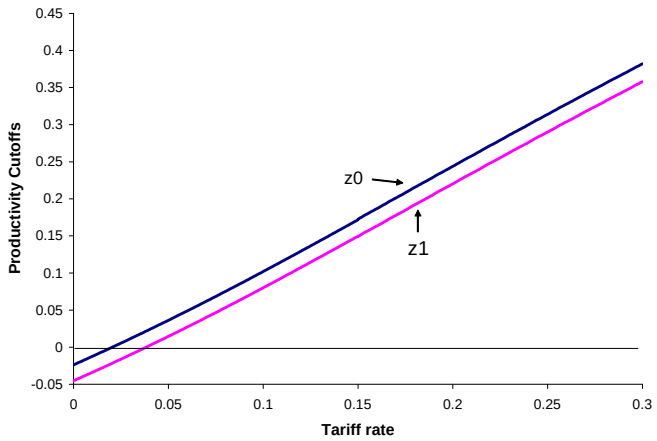
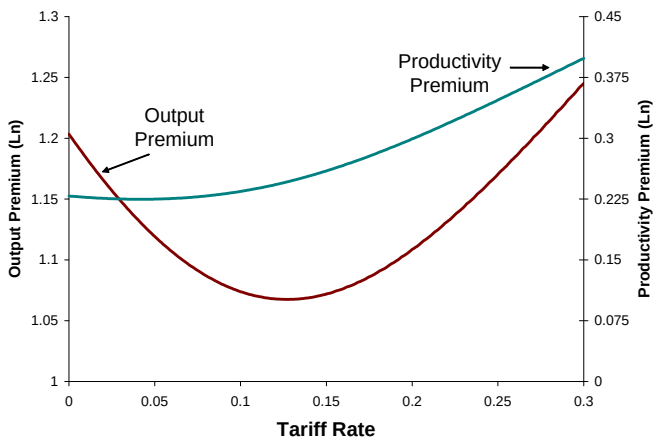


Figure 3: Steady State Aggregates and Tariffs

(a) Entry and Exit Thresholds



(b) Exporter Output and Productivity Premia



(b) Starter and Stopper Rates

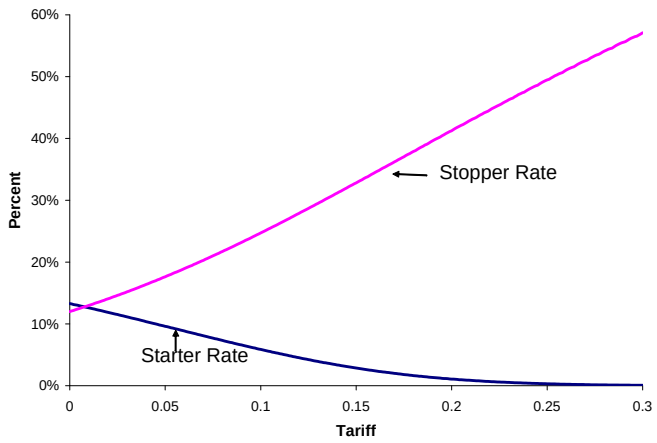
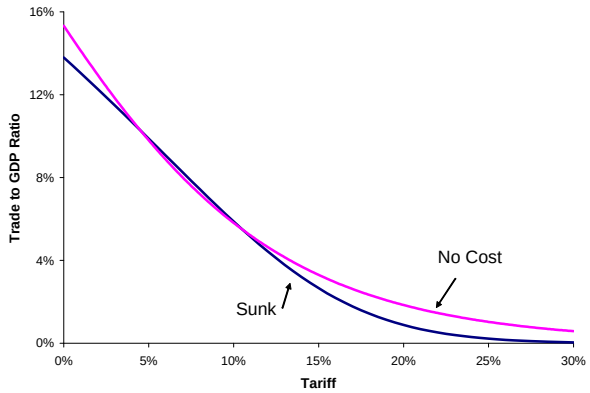
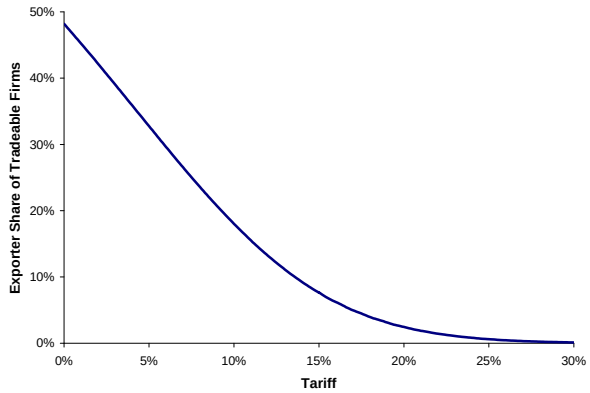


Figure 4: Steady State Firm Characteristics and tariffs

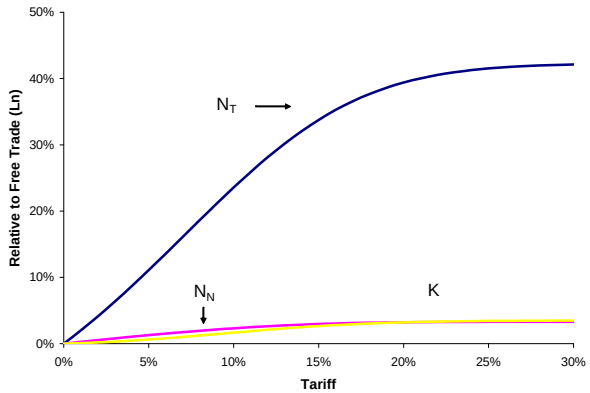
(a) Trade Share



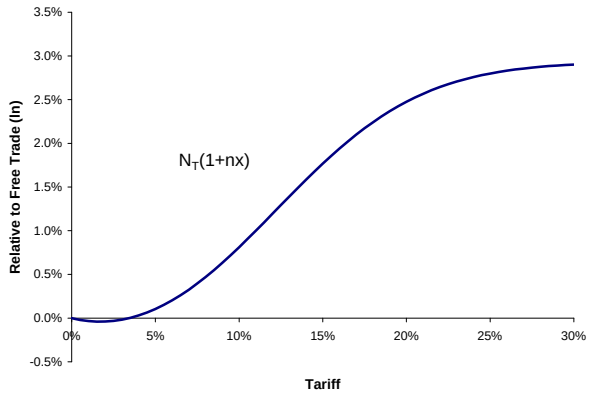
(b) Exporter Ratio



(c) Tradable & Nontradable Firms plus Capital



(d) Export + Domestic varieties



(e) Consumption

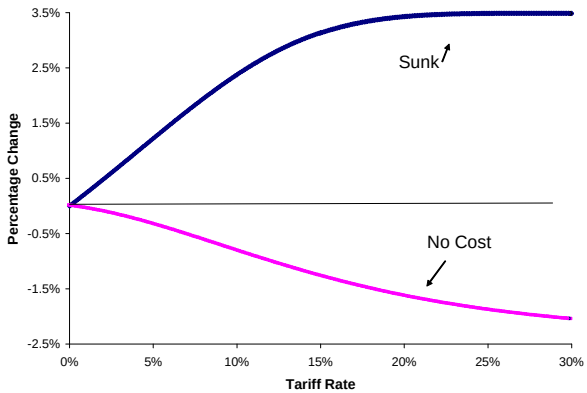


Figure 5: Steady State Aggregates and Tariffs

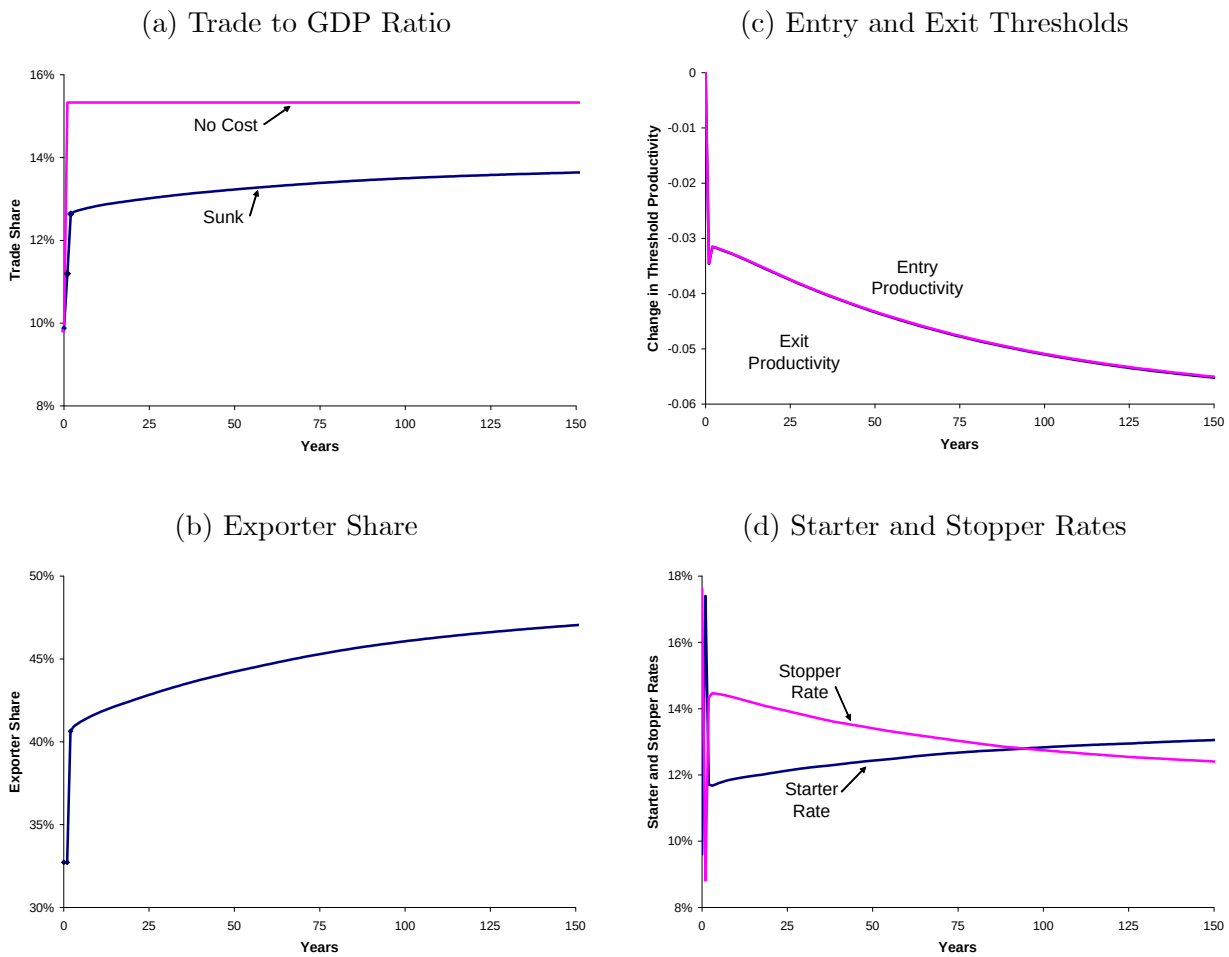


Figure 6: Trade Dynamics from 5 percent Tariff to Free Trade

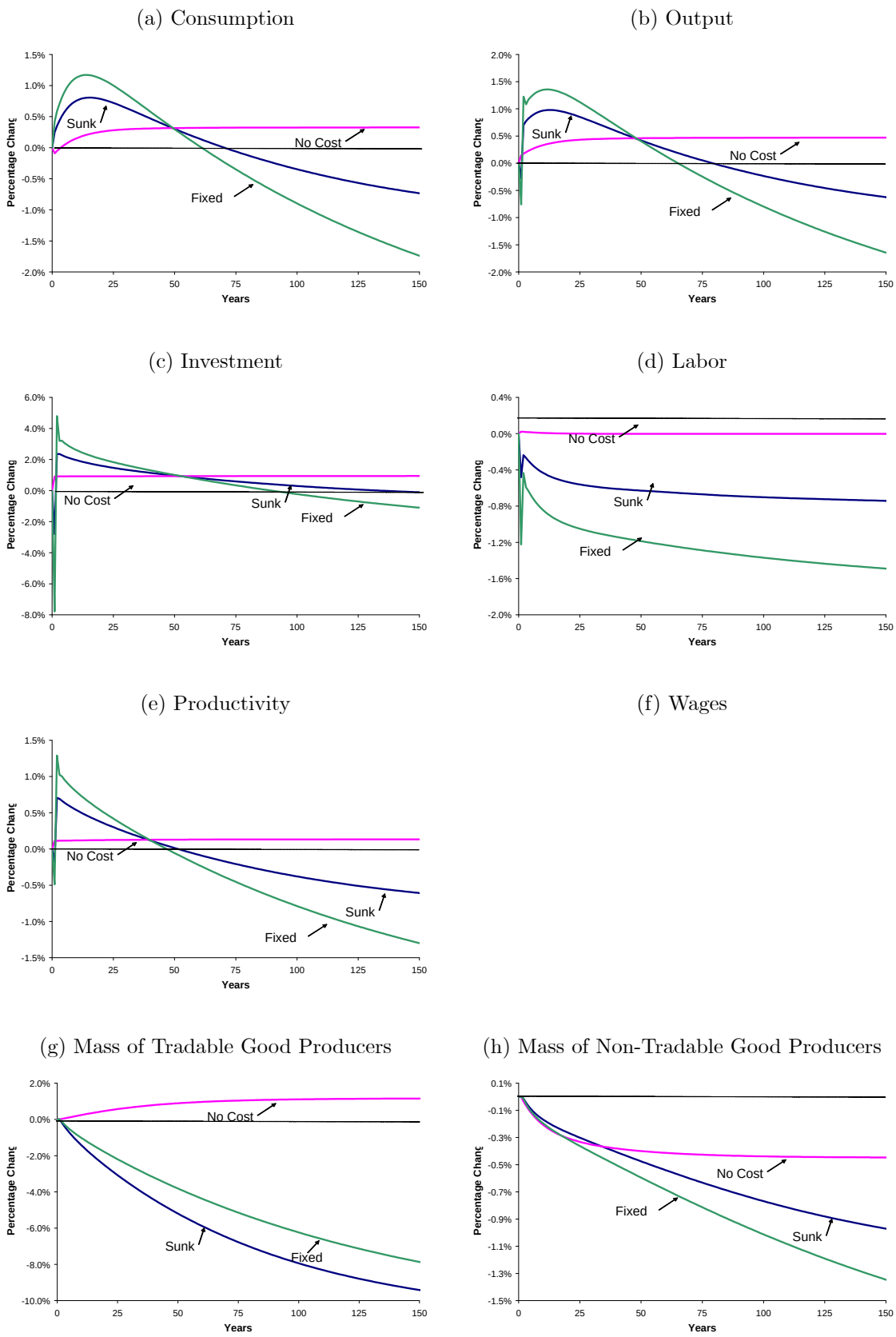


Figure 7: Aggregate Dynamics from 5 percent tariff to Free Trade

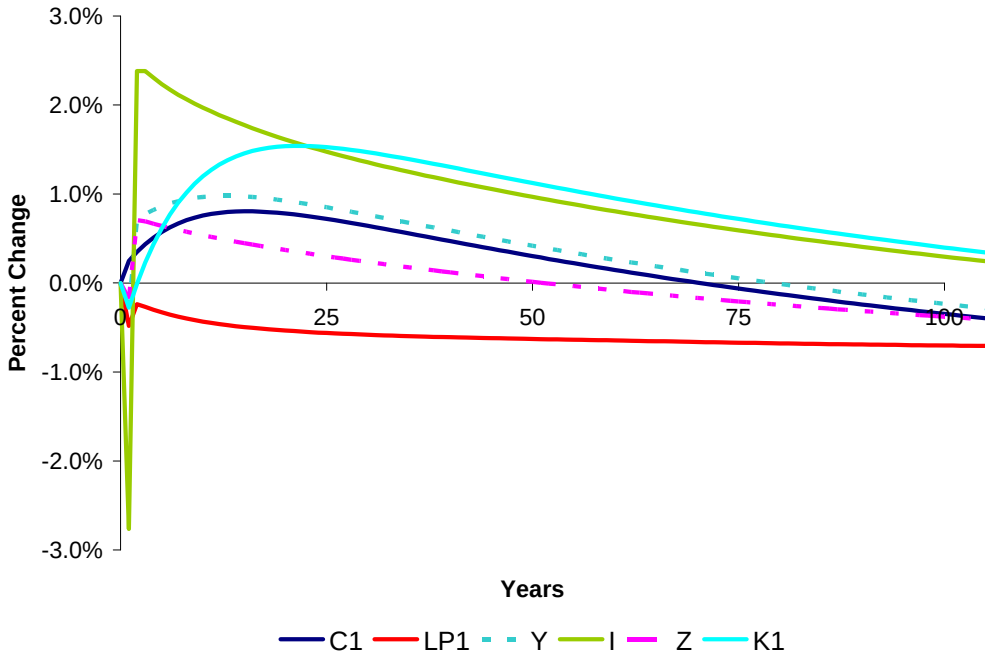


Figure 8: Response to move to free trade in Sunk Cost model