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Modeling the Liquidity Effect of a Money Shock

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To choose a monetary policy, officials at the Federal Reserve need to determine and compare the likely economic effects of all their alternatives. Obviously, they can't do that using the real world as their laboratory. Their only practical option is to experiment with artificial economies, or *models*. Often policymakers work out these experiments in their heads, using a simple, intuitive version of the model of a favored economic adviser or a former professor. Sometimes they complement this approach by computer experiments, using elaborate, mathematically explicit models. Either way, though, the wisdom of the policies that are ultimately selected depends critically on the quality of the models used to select them.

Economists are searching for a good model that can help monetary policymakers make wise choices. For a while, back in the 1960s, economists thought they had such a model—or, at least, they seemed likely to have one soon. Their hopes rested with the large 200-plus equation macroeconometric models of the time, which were based on the ideas of John Maynard Keynes. Beginning in the 1970s, however, those models were seriously challenged by economic theory and experience. This led academic economists to explore in other directions. Although the search has not yet led to a good model, I think one path looks particularly promising. In this paper, I describe and evaluate the work of some researchers now on that path and attempt to move a little further down it myself.

An important characteristic for a good model to have is the ability to reproduce the real world's response to simple monetary policy experiments. Many economists agree, for instance, that the evidence supports the following view: when the Fed surprises financial markets by suddenly increasing the rate of growth of the money supply, the nominal interest rate falls, and employment and output rise, at least in the short run.[1] The presumption that this is what happens is a basic premise guiding the implementation of monetary policy: when the Fed wants to get the interest rate on federal funds down, reserves are injected into the financial system, not withdrawn from it.

The effect of such a surprise change in money growth (a positive *money shock*) is thought to be the result of two opposing forces. One is known among economists as the *liquidity effect:* The extra money in the economy pushes down interest rates, which stimulates economic activity. The other force, which pushes interest rates up and may depress economic activity, is known as the *anticipated inflation effect.* That occurs if, as seems plausible based on the data, a surprise increase in money growth leads people to expect more such increases in the future, and so more inflation. That leads borrowers and lenders to add an inflation premium to interest rates. This may lead to a reduction in employment

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*The author thanks Martin Eichenbaum for drawing his attention to the Fuerst (1990) and Lucas (1990) work on which this paper builds and for many extensive discussions. The author is collaborating with Eichenbaum on further research on the topic of this paper. The author has also benefited from discussions with Gary Hansen and Finn Kydland.

[1] For a discussion of the empirical evidence for this proposition, see the work by Friedman and Schwartz (1963), Cagan and Gandolfi (1969), Barro (1978), Darby (1979), Barro and Rush (1980), Melvin (1983), Mishkin (1983), Sims (1986), Cochrane (1989), Cook and Hahn (1989), Romer and Romer (1989), and King (1990).

and output if, for example, interest charges are an important component of firms' operating costs. The widespread view among economists is that the liquidity effect of a money shock is stronger than the anticipated inflation effect, at least in the short run.[2] Milton Friedman's 1967 presidential address to the American Economic Association contains the classic statement of this view (Friedman 1968). He suggests that the liquidity effect dominates for one or two years after a money shock.

This view about the dominant liquidity effect is missing from one otherwise very promising class of models. They are monetary versions of the *real business cycle models* pioneered by Kydland and Prescott (1982) and Long and Plosser (1983). Real business cycle models have been surprisingly successful at accounting for several nonmonetary features of U.S. business cycles (Prescott 1986). They have been less successful, however, once money is involved. Existing versions of these models that include a role for money imply that the immediate response to a positive money shock is a rise in interest rates and a fall in employment and output. This reflects that these models display only the anticipated inflation effect; they miss the liquidity effect altogether.

Why that is so I explain here. I do it by working with a prototype version of a monetary-real business cycle model, one that introduces money by requiring that transactions in the economy be financed with previously accumulated cash. This type of model is known as a *cash-in-advance model*.[3]

Then I go on to describe a modification to this type of model that was recently proposed by Lucas (1990) and analyzed further by Fuerst (1990) as a way to introduce the liquidity effect.[4] The modification is to assume that households cannot continuously revise their consumption and saving decisions. Thus, after a money shock, they cannot immediately adjust to the changed financial market circumstances. The *Fuerst-Lucas model* preserves the cash-in-advance model's assumption that the Fed conducts its open market operations directly with financial institutions. Those institutions are assumed to be in continuous contact with the firms which borrow from them in order to finance their operations. It is easy to see that the modification introduces a liquidity effect. With households out of the picture in the short run, firms have to absorb a disproportionately large share of a money injection, which creates a downward pressure on the nominal interest rate. By lowering firms' costs, a lower interest rate encourages them to borrow more and expand the scale of their operations, which creates an upward pressure on economic activity.

Of course, merely introducing a liquidity effect into the model may not be enough. The liquidity effect must be sufficiently strong to dominate the anticipated inflation effect. Whether or not that is true in this model depends on the precise relationship among its variables, or the values of its parameters. Fuerst (1990) shows that there exist feasible parameter values for which the liquidity effect dominates. However, I find that for plausible parameter values it doesn't.

I then investigate a natural modification to the Fuerst-Lucas model. I assume that firms' investment decisions, like households' consumption and saving decisions, are not revised continuously and so do not respond instantaneously to a money shock. In a plausibly parameterized version of the model (what I call the *sluggish capital model*), my change produces a liquidity effect stronger than the anticipated inflation effect. Unfortunately, though, the model then breaks down in another way: It no longer accounts very well for some nonmonetary features of U.S. business cycles.

Despite this mixed result, I see monetary versions of real business cycle models as potentially good models for the Fed to experiment with. My study shows where further work on this type of model is required and suggests what direction that work should take.

The Model Economies

Similarities and Differences

□ *Cash Flow*

All the model economies I work with here share the same pattern of cash flow among their three types of economic agents: *households,* goods-producing *firms,* and *financial intermediaries*. Before discussing the differences between the model economies, I will emphasize their similarities by discussing this cash-flow pattern.

In the models, time evolves in discrete units, called *periods* (which are specified to be one quarter long in the quantitative results reported later). At the beginning of a period, households are in possession of the economy's entire stock of money, which they have accumulated from labor, interest, and dividend earnings in the previous period. During the first part of a period, households circulate all their money to firms by consumption purchases and loans to the financial intermediaries, which then relend the money to firms. New money enters the economy by an injection from the monetary authority into the financial intermediaries, and this is also lent to firms. This flow of money from households and the monetary authority to firms is diagrammed in Chart 1.

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[2] My use of the terms *liquidity effect* and *anticipated inflation effect* depart very slightly from convention. In the literature, they refer exclusively to a money shock's impact on the interest rate, while I include its impact on employment and output.

[3] I work with models in which wages and prices are perfectly flexible. See King 1990 for a discussion of the difficulties in accounting for the dominant liquidity effect in models in which prices or wages are inflexible.

[4] Fuerst and Lucas build on previous work by Grossman and Weiss (1983) and Rotemberg (1984). For another model that displays a liquidity effect, see Baxter, Fisher, King, and Rouwenhorst 1990.

Charts 1 and 2

Cash Flow in the Model Economies

Chart 1 To Firms

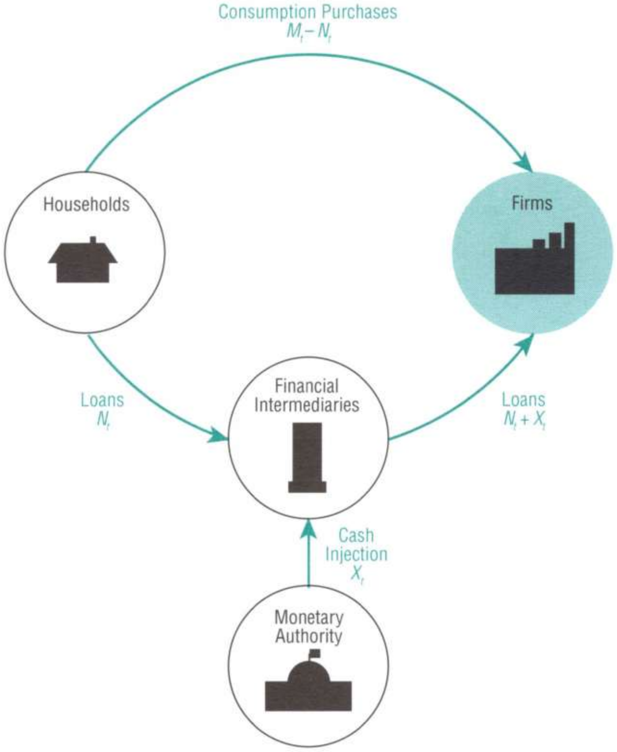
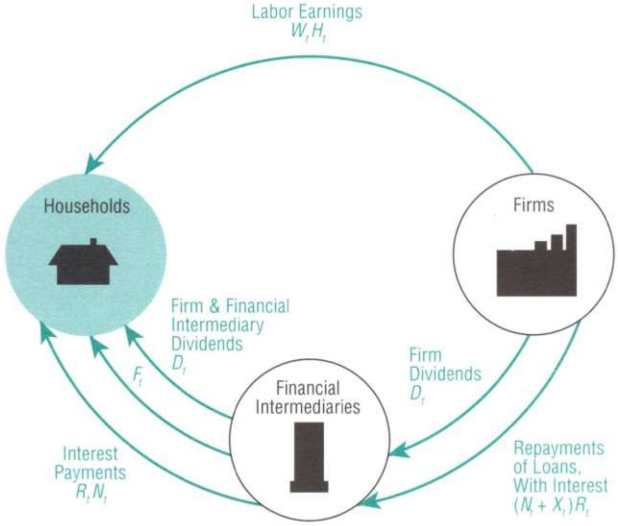


Chart 2 Back to Households



The models' financial intermediaries are abstractions intended to capture the great variety of real-world institutions and markets that let households supply funds, directly or indirectly, to the ultimate users: firms that use physical capital and workers to produce the economy's output. The stock exchange is an example of a direct channel, since it lets households give funds directly to firms. Examples of indirect channels are banks and money market funds, since these institutions accept money from households and relend it to firms. Thus, the funds moving from households to financial intermediaries in the models include such things as money used to acquire stock, money deposited in banks to increase a household's saving account balance, and money sent to money market funds. They also represent the retained earnings of corporations, these being thought of as paid to households, which then channel the funds back to firms through financial intermediaries.

Money is circulated back to households at the end of the period by the channels diagrammed in Chart 2.

Two sources of cash to households are their paychecks, which they get directly from the firms, and firm dividend payments, which are funneled to them by the financial intermediaries. Firm dividend income is simply all the cash firms have on hand at the end of the period: their sales receipts net of expenses; households get this income because they own the firms.

Two other sources of cash to households originate with firms' repayment of loans received from the financial intermediaries. One component of these repayments reflects loans to firms of money the intermediaries borrowed from households at the beginning of the period. This is returned to households in the form of interest and principal payments. Another component reflects loans to firms of the cash injections received by the intermediaries from the monetary authority. Firms' repayments of these loans are transmitted to households in the form of financial intermediary dividend payments. Households get these dividend payments because they also own the financial intermediaries.

This cash-flow pattern is clearly a simplification of what happens in the real world. There, the pattern is not nearly as synchronized as it is in the models. For example, never in the real world is all the economy's cash concentrated in the hands of households. Also, unlike in the models, the decision intervals of real-world households are not fixed at one period. Rather, these intervals are unsynchronized, are of various lengths, and are chosen independently by each individual household.

□ *Timing*

The models are distinguished by the flexibility agents have in responding to surprise changes in the economy, or *shocks*. Agents in the models experience two types of shocks: a cash injection by the monetary authority, which I will call a *money growth shock* (or just a *money shock*), and a change in the state of technology of firms, which I will call a *technology shock*. The flexibility of an agent's response to either of these types of shocks is determined by the timing of the agent's decision in relation to the timing of the shock.

The *basic cash-in-advance model* assumes agents have perfect flexibility in responding to a shock: all decisions are made after—and therefore completely reflect—the current-period surprise change in money growth or technology. These decisions include those of households deciding how to split their money holdings between consumption and loans—their *portfolio* decisions—and how to split their time between labor and leisure. The decisions also include those of firms deciding how much labor to hire and how much to expand their plant and equipment, that is, to *invest*.

The other two versions of the model assume agents have less-than-perfect flexibility in responding to a shock. The *Fuerst-Lucas model* assumes households make their portfolio decisions before they know the current-period value of a surprise change. The rationale for this assumption is not modeled, but presumably it reflects costs that make continually altering the quantity of cash spent on consumption, with every new bit of information, less-than-optimal behavior. All other decisions in the Fuerst-Lucas model are assumed to be perfectly flexible and so to fully reflect the current-period value of a shock.

The third model, the *sluggish capital model,* assumes that both the household portfolio decisions and the firm investment decisions must be made before the current value of a shock is known. The assumption that investment cannot respond instantly to a shock is intended to capture the real-world fact that investment decisions require at least some advance planning. All other decisions in the sluggish capital model are assumed to respond perfectly flexibly to a shock.

A Closer Look

I now describe in more detail the circumstances faced by the agents in the models.

□ *Households*

Households supply labor and purchase the output of the firms, which in the models is limited to one type of good. Households also own the firms and the financial intermediaries.

At the beginning of each period t, the models assume households have the economy's entire money stock, M_t, at their disposal. They have two uses for these funds: N_t dollars are loaned to the financial intermediaries, while $M_t - N_t$ dollars are set aside to purchase the consumption good during the current period. In particular, if P_t is the period t price level for the one good and C_t the households' period t consumption of that good, then households face a *cash-in-advance constraint:*

$$(1) \quad P_tC_t = M_t - N_t.$$

The assumption captured here—that all consumption purchases must be paid for with previously accumulated cash—is obviously extreme; it ignores the real-world pervasiveness of credit.[5]

To better understand equation (1), compare it with the less extreme assumption that households simply find it convenient to use some cash in making purchases. This convenience may arise from a reduction in costs (*transaction costs*) that comes from using cash. For example, gasoline station owners offer a lower price to cash customers because doing business with them costs the owners less than doing business with credit card customers. These considerations suggest an alternative to equation (1) that captures its spirit without being so extreme. In particular, one could specify that households do not require cash for purchases, but that, for a given level of purchases, they suffer a loss of some kind that can be decreased by holding cash balances.[6] This alternative to the cash-in-advance approach to money demand that is implicit in (1) is re-

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[5] The conventional way to express the cash-in-advance constraint is as a weak inequality: $P_tC_t \le M_t - N_t$. This version of the constraint allows the possibility that households could choose not to spend all the cash set aside for consumption purchases. I work with equation (1) because I only consider parameter values for which households make their cash constraint hold as an equality. That this is nonbinding can be verified ex post in simulations of the model by verifying that the marginal utility of money in each period is no less than the discounted expected marginal utility of money in the next period.

[6] Suggested losses from getting by with low cash balances include lost leisure time (McCallum 1983, Kydland 1989, Den Haan 1990) and lost real resources (Marshall 1987, Sims 1989, Huh 1990).

ferred to as the *transaction cost approach* to money demand.[7]

As noted earlier, in the models here, households have four sources of money at the beginning of each period. These are labor income in the previous period, $W_{t-1}L_{t-1}$; interest earnings on cash loaned to financial intermediaries in the previous period, $R_{t-1}N_{t-1}$; dividend payments from financial intermediaries, F_{t-1}; and dividend payments from firms, D_{t-1}. Here, W_{t-1} denotes the period $t-1$ wage rate, L_{t-1} denotes the hours of work supplied by households during period $t-1$, and R_{t-1} denotes the gross return at the end of period $t-1$ on one dollar loaned at the beginning of period $t-1$. Each household's *budget constraint* specifies that its uses and sources of money be equated:[8]

$$(2)\qquad (M_t - N_t) + N_t = R_{t-1}N_{t-1} + F_{t-1} + D_{t-1} + W_{t-1}L_{t-1}.$$

One feature of equations (1) and (2) is worth stressing. As emphasized above, according to (1), households can only buy the consumption good with the cash balances that are available to them at the beginning of a period. An implication of this is that they cannot apply labor earnings from the current period to current-period consumption.

One way to visualize this aspect of the model is to think of each household as containing two members: a worker and a shopper. At the beginning of a period, the shopper is given the sum $M_t - N_t$ to purchase C_t. The worker goes to work and receives a paycheck for labor services just a bit too late to pass it on to the shopper in the current period; the soonest the worker can make the paycheck available for spending is the next period. The intent of this abstraction is to capture the fact that receipts and payments of real-world households are not fully synchronized. This fact—together with a desire to use at least some cash in transactions, captured rather bluntly by (1), and an unwillingness to spend a lot of effort juggling between cash and interest-bearing accounts, captured by the fixed planning interval assumption—may be a major reason real-world households do not hold smaller amounts of money.

The situation in which the households find themselves in period t depends on the version of the model under consideration.

Consider first the basic cash-in-advance model. In this model, in period t, each household takes M_t, R_t, P_t, W_t, F_t, and D_t as given. In addition, it knows what values these variables $\{R_{t+j}, P_{t+j}, W_{t+j}, F_{t+j}, D_{t+j}; j = 1,2,3,...\}$ will take on, depending on the future (as yet unknown) realizations of the technology and money growth shocks. Taking as given these things and the fact that (1) and (2) must be satisfied in all periods, the household selects values for C_t, L_t, and N_t and contingency plans for its future decisions. These plans relate future decisions, C_{t+j}, L_{t+j}, and N_{t+j}, to the values taken on by the shocks in periods $t+j$ and earlier, for $j = 1, 2, 3, ...$ These objects are selected to maximize expected discounted utility:

$$(3)\qquad E_t \sum_{j=0}^{\infty} (\beta^*)^j\, u(C_{t+j}, L_{t+j}).$$

In (3), E_t is the expectation operator, conditional on all information available in periods t and earlier; β^* is the discount rate; and u is the period utility function.

The following utility function (which nests as a special case standard ones used in the real business cycle literature) will be used in the quantitative analysis reported later:

$$(4)\qquad u(C_t, L_t) = \begin{cases} [C_t^{(1-\gamma)}(1-L_t)^{\gamma}]^{\psi}/\psi & \text{for } \psi \neq 0 \\ (1-\gamma)\log(C_t) + \gamma\log(1-L_t) & \text{for } \psi = 0. \end{cases}$$

Here, $1 - L_t$ denotes the quantity of leisure time, and the total time available for work—the time endowment—is set at 1. This fixes the units in which L_t is measured in terms of fractions of the time endowment. If, for example, one prefers to think instead in terms of hours worked per quarter, and the time endowment is 16 hours per day (hence, 1,460 hours per quarter), then $L_t = 0.5$ signifies 730 hours of work per quarter. When the curvature parameter, ψ, is set to zero, (4) is the utility function used in two real business cycle models: Hansen's (1985) divisible labor model and the model in Long and Plosser 1983. When $\psi \neq 0$, this is the utility function used in the real business cycle model in Kydland and Prescott 1982.

Although the household's problem was just posed as having to choose values for three variables (C, L, and N), the fact that (1) must always hold implies that we can ignore one of those decisions. In particular, I think of the household's problem as having to choose a set of plans for only $\{L_{t+j}, N_{t+j}; j \geq 0\}$. Plans for these variables then automatically imply plans for C_{t+j} via (1) for $j \geq 0$.

Now consider a household's situation in the Fuerst-Lucas and sluggish capital models. Here, the funds households decide to lend to financial intermediaries, N_{t+j}, must be contingent on realizations of the shocks in periods $t+j-1$ and earlier, for $j \geq 0$. This is because these models assume

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[7] The cash-in-advance approach has been pursued by Lucas (1984,1990), Svensson (1985), Greenwood and Huffman (1987), and Lucas and Stokey (1987). The transaction cost approach has been pursued, in models closely related to those in this paper, by Marshall (1987), Kydland (1989), and Sims (1989). The pioneering studies of the transaction cost motive for holding money balances include those by Baumol (1952), Tobin (1956), and Miller and Orr (1966, 1968). An approach closely related to the transaction cost approach is the *cash-credit good* model studied by Lucas and Stokey (1983).

[8] Note that I assume the number of shares in the firms and the financial intermediaries is fixed and the shares are not traded. Allowing shares to be traded would only complicate the notation without altering the substance of the analysis.

that the portfolio decision must be made before the current-period realization of the shocks. The other decision, how many hours to work, L_{t+j}, is contingent on the period $t + j$ realized values of the shocks for $j \geq 0$.

□ *Firms*

In all three models, firms possess the economy's capital stock, K_t, and its production technology. They use these and the hours of labor they hire, H_t, to produce output.

Output is related to the inputs through this production technology:

$$(5) \quad f^*(K_t, z_t H_t) \equiv K_t^{\alpha}(z_t H_t)^{(1-\alpha)} + (1-\delta^*)K_t$$

where $0 < \alpha < 1$; z_t is the state of technological knowledge, which is determined outside the model; δ^* is the fixed rate of depreciation on a unit of capital; and $f^*(K_t, z_t H_t) - (1-\delta^*)K_t$ is gross output, Y_t. The state of technology has two parts: a deterministic trend, $\exp(\mu t)$, and random deviations about that trend, $\exp(\theta_t)$. That is,

$$(6) \quad z_t = \exp(\mu t + \theta_t).$$

More about the law of motion of θ_t will be said later.

Each period, besides hiring labor, firms invest in capital. Before hiring workers or investing, though, firms must borrow cash from financial intermediaries. This is because they start the period with no cash: all cash accumulated in the previous period is assumed to have been distributed through dividends. In particular, firms must borrow an amount of cash equal to

$$(7) \quad W_t H_t + P_t I_t$$

where I_t denotes gross purchases of investment goods:

$$(8) \quad I_t \equiv K_{t+1} - (1-\delta^*)K_t.$$

In (7), P_t is the price of a unit of investment goods. This is identical to the price of the consumption good since the economy's single output good can be transformed one-for-one into consumption or investment goods. The dividends a firm pays out at the end of period t, D_t, equal its total cash receipts, $P_t Y_t$, minus its total cash outlays:

$$(9) \quad D_t = P_t Y_t - R_t(W_t H_t + P_t I_t).$$

The expression after the minus sign in (9) represents the cash firms need to repay the financial intermediaries at the end of the period in return for borrowing $W_t H_t + P_t I_t$ at the beginning of the period.

Through firms' control over investment, they confront a trade-off between current and future dividends. For example, by setting I_t at a high level, a firm raises future dividends at the cost of lower current dividends. Because firms face this trade-off, something has to be assumed about how they weigh current and future dividends when they make employment and investment decisions. Since firms are owned by households, a natural assumption is that each firm behaves in the best interests of its shareholders. Thus, I assume a firm values a dividend dollar in a particular period t by the marginal utility to the households of a dollar at the end of period t. So the firm seeks to maximize

$$(10) \quad E_t \sum_{j=0}^{\infty} [(\beta^*)^{j+1} u_{c,t+j+1}/P_{t+j+1}] D_{t+j}.$$

Here the bracketed term is the marginal utility to a shareholder of a dollar received at the end of period $t + j$. The reason the subscript $t + j + 1$ appears here is that a dollar at the end of period $t + j$ cannot be spent until the following period. Firms take $u_{c,t+j}$, P_{t+j}, W_{t+j}, and R_{t+j} as given functions of the realizations of the shocks at and before period $t + j$, for $j \geq 0$.

In the basic cash-in-advance model and in the Fuerst-Lucas model, the firms seek to maximize (10) by choice of contingency plans which relate I_{t+j}, H_{t+j} to the period $t + j$ and earlier values of the shocks, for $j \geq 0$. In the sluggish capital model, I_{t+j} is a function of the shocks in period $t + j - 1$ and earlier, while H_{t+j} continues to be a function of the shocks in period $t + j$ and earlier.

□ *Financial Intermediaries*

In the models, financial intermediaries accept loans, N_t, from households, which are repaid at the end of the period at a gross rate of interest, R_t. Financial intermediaries loan this money to firms at the same rate of interest. Firms' loans must be repaid at the end of the period, in time for the financial intermediaries to use the proceeds to repay households.

Financial intermediaries also receive new cash injections, X_t, from the economy's monetary authority. This money is also loaned to firms, which repay $R_t X_t$ at the end of the period. This is distributed to households in the form of dividends F_t. Thus, financial intermediary dividend payments are

$$(11) \quad F_t = R_t X_t.$$

Finally, the financial intermediaries act as a conduit for sending firms' dividends, D_t, to households.

□ *Shocks and Equilibrium*

The shocks in the model economies are disturbances to the random part of technology, θ_t, and to the rate of growth of money, x_t. Here, $x_t \equiv X_t/M_t$, where X_t, again, is cash injections from the monetary authority to the financial intermediaries. I assume that the two shocks enter this way:

(12) $\theta_t = (1-\rho_\theta)\theta + \rho_\theta\theta_{t-1} + \varepsilon_{\theta,t}$

(13) $x_t = (1-\rho_x)x + \rho_x x_{t-1} + \varepsilon_{x,t}$.

The shocks to technology and money growth, $\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$, are mutually uncorrelated at all leads and lags and are uncorrelated with θ_{t-j}, x_{t-j}, $j > 0$. They are the part of θ_t and x_t that cannot be predicted based on past values of the variables in the model. For this reason, $\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$ are referred to as the *unexpected* components of θ_t and x_t. The parameters ρ_θ and ρ_x in equations (12)–(13) control the autocorrelation properties of θ_t and x_t. In particular, the correlation between θ_t and θ_{t-j} is just ρ_θ^j for $j > 0$. A similar interpretation for ρ_x also exists. Finally, θ and x are the unconditional means of θ_t and x_t, and I denote the standard deviation of $\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$ by $\sigma_{\varepsilon,\theta}$ and $\sigma_{\varepsilon,x}$.

In general equilibrium in these models, firms and financial intermediaries maximize the value of dividends, households maximize utility, and markets clear. Clearing in the loan market requires that the demand for cash loans, $W_tH_t + P_tI_t$, and the supply of cash loans, $N_t + X_t$, be equated:

(14) $W_tH_t + P_tI_t = N_t + X_t$.

Similarly, clearing in the labor market requires that labor demand, H_t, and labor supply, L_t, be equated:

(15) $H_t = L_t$.

Goods market-clearing requires that demand, $C_t + I_t$, equal supply, Y_t:

(16) $C_t + I_t = Y_t$.

The cash-in-advance constraint and loan market-clearing imply that the demand for and the supply of money be equated. Period t demand for money is the sum of household demand, P_tC_t, and firm demand, $W_tH_t + P_tI_t$. Period t supply of money is M_{t+1}. The money market-clearing condition is, then,

(17) $P_tC_t + W_tH_t + P_tI_t = M_t + X_t = M_{t+1}$.

The first equality follows from (1) and (14), and the second follows by definition of M_{t+1}.

Solving the Basic Cash-in-Advance Model

To quantify the impact of the shocks in the three models requires *solving* the models. That means determining, given an arbitrary pattern of realizations of the shocks, how much firms will invest and how much households will work, save, and consume in equilibrium. It also means determining the equilibrium rate of inflation and nominal rate of interest. Here I use a particular strategy to solve the basic cash-in-advance model.

My solution strategy focuses on several *efficiency conditions* that must be satisfied given that markets clear and agents optimize. In addition to enabling me to solve the models, these conditions play other important roles in the analysis. First, they are used to gain intuition about the dynamic impact (the impact over time) of money growth shocks on interest rates, employment, and output. This intuition is useful as a guide for interpreting the quantitative results. Second, the efficiency conditions are used to derive the models' implications for money demand regressions of the kind reported in the money demand literature. Third, the efficiency conditions are used to define econometric estimators for the models' parameters.

Employment Decisions

□ *Households*

To start solving the basic cash-in-advance model, I derive an efficiency condition associated with households' decision about how many hours to work, L_t. The condition is obtained by positing that households have set L_t optimally and then working out the implication of the fact that no change in that decision can increase expected discounted utility, (3).

Suppose, for example, that a household were to increase labor supply by one unit. The utility cost of this is $-u_{L,t}$, where $-u_{L,t} \equiv -\partial u(C_t,L_t)/\partial L_t$ evaluated at the optimal choices of C_t and L_t. The benefit is that the household earns a wage, W_t, which can be spent next period on W_t/P_{t+1} units of the economy's consumption good. The discounted value of this to the household is $\beta^* u_{c,t+1}W_t/P_{t+1}$, where $u_{c,t+1} \equiv \partial u(C_{t+1},L_{t+1})/\partial C_{t+1}$. Here, the derivative is evaluated at the optimal plan for C_{t+1} and L_{t+1}. From the household's perspective in period t, this benefit is a random variable since, under its optimal plan, C_{t+1} and L_{t+1} are functions of θ_{t+1} and x_{t+1}, which are not known in period t. The household's concern is with expected utility, (3), so it evaluates the benefit as $E_t\beta^* u_{c,t+1}W_t/P_{t+1}$ or, equivalently, $(W_t/P_t)E_t\beta^* u_{c,t+1}(P_t/P_{t+1})$.

If the household's undisturbed plan were, indeed, optimal, as we suppose, then the costs and benefits of the above one-unit deviation from the optimal plan must exactly match. Thus,

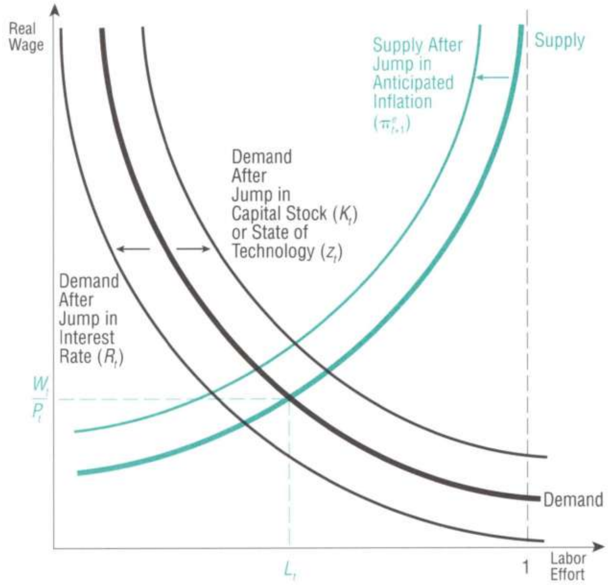
(18) $-u_{L,t} = (W_t/P_t)\, E_t\beta^* u_{c,t+1}(P_t/P_{t+1})$.

Some intuition about (18) may be obtained by graphing it in the standard, static real wage/labor effort diagram, as in

Chart 3. There, $-u_{L,t}(E_t\beta^* u_{c,t+1}P_t/P_{t+1})^{-1}$ is graphed conditional on the specification of the utility function (4) and on fixed values of C_t and $\pi^e_{t+1} \equiv (E_t u_{c,t+1}P_t/P_{t+1})^{-1}$. From (4), it is easy to see that labor supply is independent of C_t when $\psi = 0$ and shifts left with a decrease in C_t when $\psi < 0$. This is because the marginal utility of leisure is not a function of C_t when $\psi = 0$, but increases with a decrease in C_t when $\psi < 0$. Similarly, when $\psi > 0$, labor supply shifts right with a decrease in C_t. Now consider the dependence of labor supply on π^e_{t+1}, which roughly is an increasing function of the expected gross change in the price level from one period to the next. Labor supply shifts left with an increase in this variable. This is because a given real wage, W_t/P_t, is worth less to a household the higher inflation is since the household cannot spend it until the next period.

Chart 3

Labor Market Equilibrium



□ *Firms*

Now consider the decisions of firms to hire hours of labor, H_t. Suppose a firm considers the following change from its optimal employment plan. It borrows one dollar in period t, at a cost of owing R_t at the end of the period. It uses the dollar to hire $1/W_t$ units of labor time, which increases the firm's revenue by $P_t f^*_{H,t}/W_t$, where $f^*_{H,t} = \partial f^*(K_t, z_t H_t)/\partial H_t$ evaluated at the optimal choices of K_t and H_t. In equilibrium, these costs and benefits must cancel:

(19) $W_t R_t / P_t = f^*_{H,t}$.

Failure of (19) to hold would contradict the assumption in (10) that firms maximize the present value of dividends. For example, if the left side of (19) exceeded the right, then firms could increase dividends simply by decreasing the hours of labor employed.

Expression (19), labor demand, is also graphed in Chart 3. There, it is conditional on a given value of the capital stock, K_t, the state of technology, z_t, and the interest rate, R_t. An increase in R_t shifts labor demand left because this increases the cost of hiring labor, which can only be covered by raising labor's marginal productivity. For a given value of the capital stock and the state of technology, this requires reducing the amount of labor hired. Increases in the capital stock or technological knowledge shift labor demand right, since both increase labor productivity.

Saving/Investment Decisions

□ *Households*

I now derive an efficiency condition associated with households' decision on how much money, N_t, to lend to financial intermediaries, or their saving decision. This, too, is done by studying the costs and benefits associated with a particular deviation from a household's optimal decision.

In particular, suppose the household increases N_t by one dollar. On the cost side, this would decrease consumption spending by one dollar, which would decrease period t real consumption by $1/P_t$, which would decrease utility by $u_{c,t}/P_t$. On the benefit side, the extra dollar invested with the financial intermediaries would generate R_t dollars at the end of the period. These can be used to buy R_t/P_{t+1} units of the consumption good next period. The discounted expected value of those goods is $E_t R_t \beta^* u_{c,t+1}/P_{t+1}$. If the original plan is optimal, as we suppose, then these costs and benefits must be equal:

(20) $u_{c,t}/P_t = \beta^* R_t E_t(u_{c,t+1}/P_{t+1})$.

Equation (20) can be used to gain insight into the model's implication for the link between interest rates and inflation and, hence, money growth. To do this, it is convenient to rewrite (20) in terms of R_t:

(21) $R_t = (u_{c,t}/P_t)/(\beta^* E_t u_{c,t+1}/P_{t+1})$

$= \{E_t[(\beta^* u_{c,t+1}/u_{c,t})(P_t/P_{t+1})]\}^{-1}$.

In the expression to the right of the first equality, the numerator is the utility value of a dollar in period t and the denominator is the discounted expected value of the utility value of a dollar in period $t+1$. According to (20)–(21), households select N_t so as to equate the relative utility value of a dollar in period t and period $t+1$ to the one-period nominal rate of interest.

The relationship between the nominal rate of interest and other variables in (20)–(21) is closely related to the classic theory of interest determination outlined by Irving Fisher (1930). According to that theory, the interest rate is the sum of the real rate of interest and expected inflation. These are referred to as the nominal interest rate's *Fisherian fundamentals,* or just its *fundamentals.*

The real rate of interest, r_t, is the amount of consumption goods the household is willing to give up next period in exchange for receiving a unit of consumption in the current period. It is straightforward to show that

$$(22)\quad r_t = \{E_t(\beta^* u_{c,t+1}/u_{c,t})\}^{-1}.$$

Now rewrite (21) using the formula $E_t x_t y_t = E_t x_t E_t y_t + \mathrm{cov}_t(x_t, y_t)$, where E_t and cov_t denote the expectations and covariance operators, conditional on information available at time t. Use this and (22) to get

$$(23)\quad R_t = \{r_t^{-1} E_t(1/\pi_{t+1}) + \mathrm{cov}_t(\beta^* u_{c,t+1}/u_{c,t}, P_t/P_{t+1})\}^{-1}$$

where $\pi_{t+1} \equiv P_{t+1}/P_t$, the gross rate of inflation. According to (23), when there is no uncertainty, $R_t = r_t \pi_{t+1}$. This is approximately the sum of the real rate of interest and the rate of inflation.[9] In addition, if the covariance term in (23) is small enough to ignore, then $R_t = r_t[E_t(1/\pi_{t+1})]^{-1}$, which to a first approximation is also the sum of the real interest rate and the expected inflation rate.

This suggests that, in the basic cash-in-advance model, R_t is determined by its Fisherian fundamentals. This determination rests sensitively on households' willingness and ability to quickly adjust their lending, N_t, so as to ensure that (20)–(23) hold in each period. Interestingly, many studies have documented that (20) is not supported by the data (for example, Hansen and Singleton 1982). One interpretation of that fact is that household borrowing is not as flexible as the basic cash-in-advance model assumes.

□ *Firms*

Now consider the firms' decisions to invest.

Suppose a firm borrows P_t dollars to buy one unit of extra capital, so that $P_t R_t$ has to be repaid at the end of period t. The extra capital does not generate extra revenue until period $t+1$. At the end of that period, the firm's investment generates $P_{t+1} f_K^*(K_{t+1}, z_{t+1} H_{t+1})$ dollars. The firm must compare the cost of reducing dividend payments by $P_t R_t$ dollars at the end of period t to the benefit of increasing dividend payments by $P_{t+1} f_K^*(K_{t+1}, z_{t+1} H_{t+1})$ dollars at the end of period $t+1$.

According to (10), it compares these alternatives by weighing them by shareholders' marginal value of a dollar. In particular, the period t utility cost to a shareholder of reducing dividends at the end of period t is the reduction in period $t+1$ utility that results from having to reduce period $t+1$ consumption by $1/P_{t+1}$: $\beta^* E_t u_{c,t+1}/P_{t+1}$. Similarly, the value to a shareholder of an extra dividend dollar at the end of period $t+1$ is $(\beta^*)^2 E_t u_{c,t+2}/P_{t+2}$. Using these weights to value the costs and benefits of a change from the firm's optimal plan leads to this condition:

$$(24)\quad R_t P_t E_t(\beta^* u_{c,t+1}/P_{t+1}) = E_t P_{t+1} f_K^*(K_{t+1}, z_{t+1} H_{t+1})\,[(\beta^*)^2 u_{c,t+2}/P_{t+2}].$$

If (24) did not hold, then firms could increase (10) by changing investment. Thus, (24) must hold at their optimal investment level.

The assumption that firms and households optimize implies that the efficiency conditions—(18), (19), (20), and (24)—hold not just in period t, but also in all future periods. In the Appendix, I show how these conditions, together with the household cash-in-advance and budget constraints, (1) and (2), and the market-clearing conditions, (14)–(16), can be used to actually compute the equilibrium quantities and prices for the model, conditional on a set of values being assigned to the model parameters.

Money Demand

The basic cash-in-advance model implies that, in equilibrium, a relationship exists among real balances, the nominal interest rate, and output which resembles an empirical money demand equation.

To see this, first combine the money market-clearing condition, (17), and the goods market-clearing condition, (16), to derive a convenient expression for the income velocity of money, V_t:

$$
\begin{aligned}
(25)\quad V_t &\equiv Y_t/(M_{t+1}/P_t)\\
&= Y_t/[Y_t + (W_t/P_t)H_t]\\
&= Y_t/[Y_t + (1-\alpha)Y_t/R_t]\\
&= 1/[1 + (1-\alpha)/R_t].
\end{aligned}
$$

Here, the second equality makes use of firms' production function, (5), and firms' demand for labor, (19). Expression

[9] For example, if $r_t = 1.03$ and $\pi_{t+1} = 1.04$, then $r_t \pi_{t+1} = 1.0712 \cong 1 + 0.03 + 0.04$.

(25) shows that money velocity and the nominal interest rate are positively related.

Now linearly expand the log of the last expression in (25) about $R_t - 1 = R - 1$, where R is the *nonstochastic steady-state* value of R_t. (By this I mean the constant value to which R_t eventually settles when the shocks, $\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$, are both held fixed at zero.) This expansion produces an expression that closely resembles the money demand equation in the literature:

$$(26)\quad \log(M_{t+1}/P_t) = a - b(R_t-1) + c\log(Y_t)$$

where

$$(27)\quad a = \log(1 + [(1-\alpha)/R]) + \{(1-\alpha)(R-1)/[R^2 + R(1-\alpha)]\}$$

$$(28)\quad b = -(1-\alpha)/[R^2 + (1-\alpha)R]$$

$$(29)\quad c = 1.$$

To fix units, suppose the nominal interest rate changes one percentage point, expressed at an annual rate, and output, Y_t, is held fixed. According to (26), real money demand then falls $b/4$ percent. (Here, b is divided by 4 because the time period of the model is one quarter, while we consider the impact of a change in the annualized rate of interest.) Since the only efficiency condition used to derive (26) is (19) and—as will become evident later—that equation holds in all three models, it follows that (26) does too. Because equation (26) resembles what is called a *money demand function* in the empirical literature, it is natural to refer to b and c as *money demand elasticities*.[10]

An important finding of the literature is that the lagged dependent variable in an empirical money demand equation enters statistically very significantly on the right side. For example, Goldfeld and Sichel (1990, Table 8.1) report t-statistics for the coefficient on lagged real balances that range from 12 to 67, depending on the data sample. It is readily verified that a version of (26) augmented to include a lagged dependent variable implies that the short- and long-run money demand elasticities differ. Since the right side of (26) has no lagged real balances, models of this paper cannot easily account for that feature of empirical money demand equations.[11]

Looking for a Dominant Liquidity Effect: Qualitatively . . .

Here I do a qualitative analysis: I try to determine what the models imply for the signs of the responses of the interest rate, employment, and output to a money growth shock. I reach definite conclusions about this for the basic cash-in-advance model, but not for the two modified models. It turns out that they require a quantitative analysis, which I will do in the next section.

In the Basic Model

To start the qualitative analysis, I use the efficiency and market-clearing conditions to establish that the basic cash-in-advance model cannot rationalize the widespread view that the liquidity effect is dominant: a money growth shock drives the nominal interest rate down and employment and output up, at least in the short run. If the money shock is temporary in this model, then it has no effect on these variables. If the shock is persistent, as is empirically plausible, then the variables respond by moving in the opposite directions than they're supposed to: the nominal interest rate rises, and employment and output fall.

□ *A Temporary Money Shock*

An unexpected change in the money growth rate, x_t, is temporary if it has no impact on future money growth rates. In (13), such a shock to x_t is given by $\varepsilon_{x,t}$, and it does not affect x_{t+j}, for $j > 0$, if $\rho_x = 0$. Of course, a temporary shock to the money growth rate corresponds to a permanent jump in the level of the money stock. This kind of monetary disturbance in the basic cash-in-advance model is known to be neutral (Greenwood and Huffman 1987, Sargent 1987b): it does not affect current or future consumption, investment, employment, and output; it results in an equiproportionate jump in current and future prices and wages, so that it does not affect the rate of inflation; and it does not affect the nominal and real rates of interest.

A key feature of the basic cash-in-advance model's neutrality property is that all agents increase their cash expenditures in equal proportion to the money shock. Thus, if $\varepsilon_{x,t} = 0.20$, so that the money stock jumps 20 percent, then households' consumption expenditures, $M_{t+j} - N_{t+j}$, and firms' employment and investment expenditures, $N_{t+j} + X_{t+j}$, also jump 20 percent, for $j \geq 0$. Obviously, if each agent's cash expenditures increase 20 percent and so do prices and wages, then agents can still afford the unshocked level of consumption, employment, and investment. In addition, it is easily confirmed (at least in the nonstochastic version of the model

[10] Other equilibrium models also imply an equation like (26); see, for example, Cooley and Hansen 1991, Lucas 1988, and Chari, Christiano, and Kehoe 1991. Lucas (1988) emphasizes the similarity between (26) and the money demand equations in the empirical literature. He also discusses the distinction between (26) and a demand equation in the price theory sense, which arises from the fact that one of the right-side variables, Y_t, is a choice variable from the point of view of firms.

[11] For a further discussion of this point, see Goodfriend 1985. One caveat to this conclusion arises from the fact that (26) contains no error term, whereas empirical money demand equations do. Conceivably, a plausible theory of the error term exists which, in conjunction with (26), would imply the statistical significance of the lagged dependent variable.

economy) that the efficiency conditions (18), (19), (20), and (24) continue to be satisfied with this response.

For later purposes I want to highlight the implications for N_{t+j}, for $j \geq 0$, of the result that all agents' cash expenditures increase in equal proportion to a temporary money growth shock. Suppose that without the shock, x_{t+j}, M_{t+j}, and N_{t+j} would have been 0, 100, and 50, respectively, for $j \geq 0$. With the shock, $x_t = 0.20$ and $x_{t+j} = 0$ for $j > 0$. Then an increase in all agents' cash expenditures of 20 percent in each of periods $t, t+1, t+2, \ldots$ requires that $N_t = 40$ and $N_{t+j} = 60$ for $j > 0$. Thus, households reduce the amount of money they send to financial intermediaries in the period of the shock by 10 dollars. In this way, they increase their nominal consumption spending by 10 dollars, which is a 20 percent increase over the 50 dollars they would have spent otherwise.

The reduction in cash supplied by households to financial intermediaries in the period of the shock also assures that cash available to intermediaries, $N_t + X_t$, does not rise more than 20 percent. For example, if households for some reason did not reduce the cash they supplied, then the amount of cash at financial intermediaries would jump 40 percent to 70 dollars. As long as $R_t > 1$, financial intermediaries would lend the money to firms, which would spend it. With some agents (firms) having to absorb a disproportionate share of the increased stock of money, a basic condition for the neutrality result fails.

□ *A Persistent Money Shock*

Now suppose a disturbance to money growth, $\varepsilon_{x,t}$, increases not just x_t, but also x_{t+j}, for $j > 0$. This would be true if $\rho_x > 0$ in (13). Greenwood and Huffman (1987) and Sargent (1987b) have shown in models closely related to mine that this leads to a rise in R_t and a fall in L_t and, hence, in output.

The intuition for this result is straightforward. Think of the experiment as a combination of two. First is an unexpected temporary jump in x_t, like the one just considered. That has no impact on R_t, L_t, or P_{t+1}/P_t. Second is an unexpected upward revision in the forecast of x_{t+1}. That, not surprisingly, exerts upward pressure on P_{t+1}/P_t. Because the Fisher relation holds in this model and because there is very little impact on the real rate of return, a jump in R_t results. But, by inspecting Chart 3, we can see that the jump in anticipated inflation shifts labor supply left and the jump in R_t shifts labor demand left. Thus, employment and output fall. The higher anticipated inflation acts like a tax on both sides of the labor market. To labor suppliers, the extra inflation means that dollars earned while working will buy less. To labor demanders, the inflation premium in the nominal interest rate means that labor costs more.[12]

In the basic cash-in-advance model, a persistent increase in the rate of money growth also acts like a tax on investment and thus discourages it.[13] Perhaps the simplest way to see this is to consider the long-run impact of a permanent increase in money growth arising from an increase in the value of the average money growth rate, x. In the long run, with the shocks held at zero—that is, in nonstochastic steady state—R_t and r_t settle to constants—say, R and r. Similarly, inflation, $K_t/(z_tL_t)$, and L_t settle to constants, $K/(zL)$ and L. Thus, in nonstochastic steady state, (24) is just

$$(30)\quad Rr = \beta^* f_K^*(K/(zL),1).$$

[Here I have exploited the fact that, when f^* is defined by (5), $f_K^*(K_t,z_tL_t) = f_K^*(K_t/(z_tL_t),1)$.] According to (30), a higher value of R—induced by a jump in the money growth rate, x—leads firms to operate at a point where the marginal product of capital is higher, since r is independent of x. (Formulas for R and r are given in note 19.) The reason for this is that the nominal rate of interest is part of the cost of investing, since firms must raise the cash in advance. To cover this cost, the marginal product of capital must be higher, which requires lowering the ratio of capital to labor. But the analysis of the labor market indicated that L_t falls with a persistent jump in x_t. Thus, the fact that $K/(zL)$ falls implies that K itself falls.

To summarize, a persistent jump in money growth raises anticipated inflation and, by the Fisher effect, the nominal interest rate. Because higher inflation rates and nominal interest rates act like a tax on market activity, the amount of that activity—employment, investment, and output—falls.

In Two Modified Models

□ *With Sluggish Household Saving*

So, the basic cash-in-advance model cannot rationalize the widely held view that a positive money growth shock—temporary or persistent—drives the nominal rate of interest down and the level of employment and output up. Why not? A literature, associated with Grossman and Weiss (1983), Rotemberg (1984), and Lucas (1990), has suggested that the key to understanding the economic impact of a money shock is to recognize that it does not impact equally on all economic agents. The basic cash-in-advance model assumes that it does.

To see that it does, recall what happens in the basic cash-in-advance model when there is a permanent increase in the money stock. There, a money growth shock is neutral in that it has no impact on the nominal or real interest rates, the inflation rate, output, employment, investment, or consump-

[12] For a further discussion of the impact of anticipated inflation on employment in a cash-in-advance economy, see Carmichael 1989.

[13] For discussions of the impact of inflation on the capital stock when there is a cash-in-advance constraint on investment, see Stockman 1981 and Abel 1985.

tion. A key requirement for this neutrality result is that cash expenditures by all agents—both households and firms—increase by the same proportion as the money injection in the period of the shock. In particular, for households this means reducing N_t, the money they lend to the financial intermediaries in the period of the shock. In the real world, this could be accomplished, for example, by reducing bank saving deposits or signaling firms during a shareholder meeting to increase dividend payments.

Recently, Fuerst (1990) and Lucas (1990) have argued that the ideas of Grossman and Weiss (1983) and Rotemberg (1984) could be captured in the basic cash-in-advance model by assuming that households have to set N_t before they know x_t and, hence, P_t, W_t, and R_t. Then, when a permanent jump in money occurs, N_t cannot be adjusted in the way that the neutrality result requires. The validity of this assumption rests in part on whether there is in fact some sluggishness in the way household portfolio, or saving, decisions are made in the real world.

As we will see below, there are values of the Fuerst-Lucas model parameters for which the nominal interest rate falls and employment and output expand in the period of a money growth shock. The intuition about why the model can produce such a result is straightforward.

Consider, for example, a temporary shock to the money growth rate. With N_t unable to fall in response, more of the extra cash (than in the basic cash-in-advance model) has to be absorbed by firms. What has to happen for this extra cash to be absorbed?

This question is easy to answer if only the nominal interest rate is assumed to change. With other variables fixed, equation (19) indicates that the only way to get firms to absorb more cash for employment and output purposes is to lower the rate of interest. (That is, for H_t to expand at a fixed W_t/P_t, R_t must fall.) Similarly, equation (24) indicates that the only way to get firms to invest more funds is also for R_t to fall.[14] This reasoning suggests that the Fuerst-Lucas model responds to a temporary injection of money by a fall in R_t.

The problem with this reasoning is that other variables do change. In particular, I will show that for plausible values of the model parameters, the nominal interest rate will rise and employment and output will fall in the period of a shock in the Fuerst-Lucas model. Thus, the signs of the responses to a money growth shock in the Fuerst-Lucas model are ambiguous. This result is consistent with the fact that a money growth shock also triggers an anticipated inflation effect in this model. The signs of the equilibrium interest rate, employment, and output responses depend on which is stronger: the anticipated inflation effect or the liquidity effect. This, in turn, depends on what values are assigned to the parameters.

That a money growth shock, especially a persistent one, could even in principle drive the nominal interest rate down in the Fuerst-Lucas model may be surprising in light of my discussion of the nominal interest rate in the basic cash-in-advance model. There I argued that the nominal interest rate is determined by its Fisherian fundamentals: anticipated inflation and the real interest rate. Focusing on the Fisher relation leads to two considerations which suggest that a money injection leads to, if anything, an increase in the nominal interest rate. First, a persistent rise in the money stock would, if anything, contribute to a rise in inflation. Second, in the Fuerst-Lucas model, such a money shock would drive down consumption if it caused the current price level to rise. Other things the same, this would tend to drive up the real interest rate as $u_{c,t}$ rose [as can be seen in (22)]. Both of these considerations suggest that the nominal interest rate ought to rise, not fall, with a positive money shock.

There is, however, no puzzle here. In the Fuerst-Lucas model, the connection of the nominal interest rate to Fisherian fundamentals is broken.

To see this, recall that the basic cash-in-advance model's implications for R_t were derived from (20), which only holds if households adjust N_t fully in the light of all period t information. But note: this condition is ruled out in the Fuerst-Lucas model. Here, households must make the N_t decision before they know x_t, θ_t, P_t, and R_t.

To derive the appropriate Fuerst-Lucas analog condition to (20), I retrace the reasoning that led to (20). Optimality of the households' choice of N_t implies that no feasible change generates an increase in utility. Consider a small positive disturbance in N_t. The cost of this is $E_{t-1}(u_{c,t}/P_t)$. The presence of the conditional expectation reflects that, at the time N_t is selected, households do not know what C_t, P_t, or L_t will be, since those values depend on the realization of θ_t and x_t. The benefit of the positive disturbance in N_t is $E_{t-1}R_t\beta^*(u_{c,t+1}/P_{t+1})$. Equality of costs and benefits requires that

$$(31)\quad E_{t-1}(u_{c,t}/P_t) = E_{t-1}R_t\beta^*(u_{c,t+1}/P_{t+1}).$$

This is the analog of (20) which holds in the Fuerst-Lucas model.

It is convenient to express this condition in a slightly different form. First, define $\Lambda_t = R_tE_t\beta^*(u_{c,t+1}/P_{t+1}) - (u_{c,t}/P_t)$, so that $E_{t-1}\Lambda_t = 0$.[15] Then solve this for R_t:

[14] In (24), after K_{t+1} is replaced by $I_t + (1-\delta^*)K_t$, it is easy to see that the right side of this equation is decreasing in I_t because of diminishing returns to capital. Thus, if other variables remain fixed, the only way for I_t to rise and absorb more funds is for R_t to fall.

[15] The condition $E_{t-1}\Lambda_t = 0$ uses the fact, known as the *law of iterated mathematical expectations*, that $E_{t-1}[E_tx_t] = E_{t-1}x_t$. For further discussion of this fact, see Sargent 1987a.

(32) $R_t = [\Lambda_t + (u_{c,t}/P_t)]/(\beta^* E_t u_{c,t+1}/P_{t+1})$

which is comparable to (21).

Fuerst (1990) calls the term Λ_t the *liquidity effect*. It measures the relative value of money in the loan market and in the consumption goods market. When $\Lambda_t < 0$, money is more valuable in the goods market since households would be willing to borrow at a higher rate than R_t if they had the opportunity to do so, while firms are willing to borrow at R_t exactly. For this reason, Fuerst says, when $\Lambda_t < 0$, the loan market is relatively liquid, whereas when $\Lambda_t > 0$, the goods market is. In the basic cash-in-advance model, where households and firms have equal access to financial intermediaries, $\Lambda_t = 0$ always. In the Fuerst-Lucas model, however, Λ_t is only zero on average because $E_{t-1}\Lambda_t = 0$ implies that $E\Lambda_t = 0$. Thus, in this model, the connection of R_t to Fisherian fundamentals holds only on average, not period by period. For example, if Λ_t is negative, then the nominal interest rate is low compared to what fundamentals dictate. In particular, if a money growth shock induces a sufficiently large fall in Λ_t, then R_t could jump even if anticipated inflation and the real rate of interest, $u_{c,t}/(\beta^* E_t u_{c,t+1})$, jump.

The efficiency conditions for the Fuerst-Lucas model are, then, (18), (19), (24), and (31). The Appendix shows how these conditions can be used to solve this model.

□ *Also With Sluggish Firm Investment*

Now let's modify the basic cash-in-advance model further, by adding one quite realistic assumption to the Fuerst-Lucas model: Firm investment decisions must be made before firms know the current-period values of the technology and money growth shocks, $\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$. This assumption captures the real-world idea that investment plans must be made in advance, that they are costly and time-consuming to change.

Because this sluggish capital model closes off investment as a potential outlet for an unexpected money injection, it can make employment and output respond positively to a money shock more easily than the other models can. Of course, the sluggish capital model does not make an unambiguous prediction about the employment or output response since a money shock may simultaneously induce other changes that shift the labor supply curve left.

The efficiency conditions associated with the sluggish capital model are (18), (19) (since labor supply and demand decisions are still made after θ_t and x_t are observed), (31), and a suitably modified version of (24):

$$(33)\quad E_{t-1}R_t P_t(\beta^* u_{c,t+1}/P_{t+1}) = E_{t-1}P_{t+1}f_K^*(K_{t+1},z_{t+1}H_{t+1})\,[(\beta^*)^2 u_{c,t+2}/P_{t+2}].$$

. . . And Quantitatively

In this section, I investigate some quantitative properties of the three models described above. First I explain how I chose values for the models' parameters. Then I report what the models say are the interest rate, employment, and output effects of money growth shocks.[16]

Parameter Values

The period in the models is assumed to be one quarter. Each model has 12 free parameters: β^*, ψ, θ, α, γ, δ^*, μ, x, ρ_θ, ρ_x, $\sigma_{\varepsilon,\theta}$, and $\sigma_{\varepsilon,x}$.

Three of these are set without reference to actual U.S. data. I set the discount rate, β^*, at $1.03^{-0.25}$. In the baseline experiments, I set the curvature parameter, ψ, to 0. This is the value used by Long and Plosser (1983) and Hansen (1985). However, results based on alternative values of ψ are reported too. The parameter θ, which is simply a scale variable, is arbitrarily set to 1.

The other parameters are set based on U.S. data for the inclusive period from the first quarter of 1959 to the first quarter of 1984. For Y_t, C_t, L_t, K_t, and I_t, I use the quarterly data used in Christiano 1988. For money, I use U.S. monetary base data, adjusted for reserve requirement changes, which are available from Citicorp's Citibase data bank.[17] The per capita consumption measure is the sum of private sector consumption of nondurables and services, the imputed rental value of the stock of consumer durables, and government consumption. The per capita hours-worked data are constructed from Hansen's (1984) hours-worked data, and the per capita capital stock data are the sum of the stock of consumer durables, producer structures and equipment, government and private residential capital, and government nonresidential capital. Data on per capita investment, I_t, are the flow data that match the capital stock concept. For further details on all these data, see Christiano 1987, 1988.

The depreciation rate, δ^*, is estimated to be 0.0212, the sample average of the depreciation rates implied by (8) and the data on K_t and I_t.[18] The estimate of the average growth rate of the state of technology, μ, is 0.0041, the sample

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[16] All calculations in this and the next section are based on model solutions obtained by a method that linearly approximates the efficiency conditions. That method is spelled out in the Appendix.

[17] The data mnemonic for the monetary base is FMFBA. It is the sum of total reserves (member bank reserve balances plus vault cash) and currency outside the U.S. Treasury, the Federal Reserve Banks, and commercial banks. These data are averages of daily figures. This may introduce some bias into the analysis since the models speak to beginning-of-the-quarter, point-in-time money data. As Friedman (1983) has emphasized, time-averaged money growth figures are less volatile than point-in-time observations. This has been confirmed by Baxter, Fisher, King, and Rouwenhorst (1990).

[18] In particular, from (8), the period t rate of capital depreciation is $\delta_t^* \equiv [(I_t - K_{t+1})/K_t] + 1$.

average of the growth rate of per capita output, Y_t. The average money growth rate, x, is set to 0.0119, the sample average of the growth rate of the monetary base.

I next consider the values of α and γ, the utility and technology parameters, and the remaining parameters of the shocks.

□ *Utility and Technology*

One way to select values for α and γ aligns the models' implications for the means of L_t and K_t/Y_t with the corresponding sample averages.

The models' mean implications for these variables correspond roughly to the values to which they converge when $\sigma_{\varepsilon,x} = \sigma_{\varepsilon,\theta} = 0$, or their steady-state values. The steady-state values of L_t and K_t/Y_t, denoted L and K/Y, are straightforward to compute given values for the models' parameters. The formulas for the computations are identical for the three models, since they are in fact the same model when there is no uncertainty.[19]

The formulas make it possible to compute α and γ given values for K/Y; the leisure-to-labor ratio, $(1-L)/L$; and the values already assigned to the other parameters. According to Christiano 1988 (Table 1), the sample averages of K/Y and per capita hours worked are 10.59 and 320.5, respectively. If households can devote a maximum of 16 hours per day to market activity, then the quarterly time endowment is 1,460 hours. This indicates that the empirical ratio of market-to-nonmarket activity averages 0.28, so that $(1-L)/L = 1/0.28$. Substituting these values into the steady-state formulas gives $\alpha = 0.56$ and $\gamma = 0.68$. [Because this procedure of assigning values to α and γ is based on matching sample averages (or first moments), I call it a *first-moment estimator*.] These estimates of α and γ, together with the already assigned parameter values, imply that $R = 1.0195$ (an 8 percent annual nominal interest rate) and that $C/Y = 0.73$, virtually the same as the sample average of C_t/Y_t reported in Christiano 1988 (Table 1).

A value of 0.56 for α is very high. It exceeds the highest that, according to Christiano 1988 (n. 3), can be rationalized based on the U.S. Commerce Department's National Income and Product Accounts (NIPA) data and the value generally used in the real business cycle literature. When average money growth, x, takes on a value which implies that $R = 1$, the steady-state efficiency conditions of the three models collapse into those of standard real business cycle models [for example, Hansen's (1985) divisible labor model]. Then my first-moment estimator implies standard values for these parameters: $\alpha = 0.35$ and $\gamma = 0.76$.

The models of this paper need higher values of α and γ to offset the depressive effects of inflation on the capital stock and labor supply. However, to preserve comparability with other studies, I work here with the standard values of $\alpha = 0.35$ and $\gamma = 0.76$. These values, given the other parameter values, imply that the capital-to-output ratio is 6.7, the ratio of market-to-nonmarket activity is 0.24, and $C/Y = 0.83$.

The fact that my first-moment estimator results in implausible values for α and γ is a count against the models' empirical plausibility. To gain insight into what is wrong with the models, let's measure and get some perspective on the magnitude of inflation's impact in them. Again, with the assigned values of μ, x, and ψ, $R = 1.0195$, or the nominal interest rate is roughly 8 percent at an annual rate. Suppose that money's annual growth rate were reduced eight percentage points. This would drive inflation's annual rate down eight percentage points and cause R to fall to 1. It is easily confirmed (using the formulas in note 19) that this policy results in a very large—58 percent—increase in the capital-to-output ratio, from 6.7 to 10.6. It also produces a 14 percent increase in labor effort.

These effects are substantially larger than those implied by the cash-in-advance model of Cooley and Hansen (1989). The difference reflects a difference in the impact of inflation in these models. In the models here, inflation distorts the capital investment decision directly and distorts labor demand as well as supply. In the Cooley-Hansen model, the cash-in-advance constraint is applied only to household purchases of con-

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[19] Formulas for the steady-state values of K/Y and $(1-L)/L$ may be obtained by solving the nonstochastic steady-state versions of the efficiency and market-clearing conditions and budget constraints. Accordingly, substitute out for W_t/P_t in (18) from (19) to get $-u_{L,t} = (f^*_{H,t}/R_t)\beta^* u_{c,t+1}(P_t/P_{t+1})$. From (20) this becomes $-u_{L,t}/u_{c,t} = f^*_{H,t}/R_t^2$, or $[\gamma/(1-\gamma)]C_t/(1-L_t) = (1-\alpha)(Y_t/L_t)/R_t^2$, which I'll call equation (†). Manipulating (16) gives $C_t/Y_t = 1 - (K_{t+1}/Y_t)[1 - (1-\delta^*)K_t/K_{t+1}]$. In the steady state, L_t, C_t/Y_t, K_t/Y_t, and K_t/K_{t+1} converge to constants: L, C/Y, K/Y, and $\exp(-\mu)$. From this, (†) becomes

$$(K/Y)\,[1 - (1-\delta^*)/\exp(\mu)] = (1/R)^2(1-\alpha)\,[(1-\gamma)/\gamma]\,[(1-L)/L].$$

Another equation that can be used to compute K/Y and L is (30), the nonstochastic steady-state version of (24):

$$Rr = \alpha(K/Y)^{-1} + 1 - \delta^*.$$

Here

$$R = (1+x)\exp[-\mu(1-\gamma)\psi]/\beta^*$$

$$r = \exp\{\mu[1-(1-\gamma)\psi]\}/\beta^*.$$

The variable r is the steady-state value of the real rate of interest, $u_{c,t}/[\beta^* u_{c,t+1}]$. (Note that the Fisher relation holds exactly here, since R is the product of r and the steady-state inflation rate.) The two equations above can be solved for K/Y and $(1-L)/L$ given values for the following model parameters: δ^*, μ, α, γ, x, β^*, and ψ. Alternatively, for fixed values of K/Y and $(1-L)/L$ [for example, the empirical sample averages of K_t/Y_t and $(1-L_t)/L_t$], these equations can be used to solve for α and γ given δ^*, μ, x, β^*, and ψ.

sumption goods. Therefore, inflation has no impact on the investment decision or the steady-state capital-to-output ratio. In addition, in the Cooley-Hansen model, a 14-percentage-point reduction in the annual rate of inflation engineered by an equal fall in the annual money growth rate, which gets the annualized nominal rate of interest down from 14 to 1 percent, increases labor effort by only 3.4 percent (Cooley and Hansen 1989, Table 2). Not surprisingly, this change produces only a very small welfare gain: 0.387 percent of gross output, Y. This is the constant increase in consumption, expressed as a fraction of steady-state Y, that would make households in the high-inflation environment indifferent between staying with high inflation and reducing annual inflation by 14 percentage points to get R down to 1. In the models here, the welfare costs of inflation are much higher. For example, the cost of having an inflation rate that produces an 8 percent annual nominal interest rate is 8.97 percent of Y.

Cooley and Hansen (1989) provide some empirical evidence which implies that the impact of inflation in my models may be implausibly large. They report cross-country evidence which suggests that a one-percentage-point drop in annual inflation raises average hours worked by at most 0.5 percent. In the models here, the boost to average hours would be close to 2 percent (that is, 14/8).

Thus, the implausibly high point estimates for α and γ produced by my first-moment estimator seem to reflect that the models assign a counterfactually large role to average inflation in determining average economic activity. Presumably, one way to improve the models' performance on this dimension would be to give agents more flexibility in deciding on the mix between cash and credit when they make transactions. By mitigating inflation's impact as a tax on market activity, this change should reduce its impact on the average levels of employment, output, and capital accumulation.

Charts 4 and 5

U.S. Data Used to Parameterize the Shocks

Quarterly, 1959:2–1984:1

Chart 4 Growth in the Monetary Base (x_t)

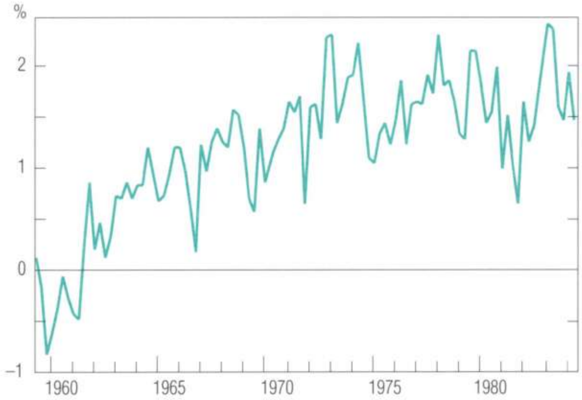
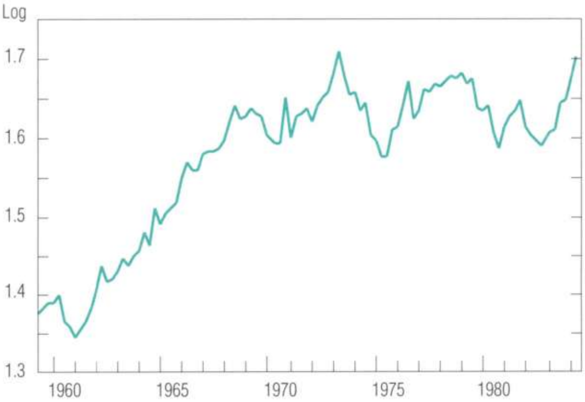


Chart 5 The State of Technology ($\log z_t$)



Sources: Citicorp's Citibase data bank and Christiano 1988

□ *Shocks*

Values for the parameters of the money growth process, ρ_x and $\sigma_{\varepsilon,x}$, are obtained from U.S. time series data on base money growth, $x_t = (M_{t+1} - M_t)/M_t$, for the inclusive period from the second quarter of 1959 to the first quarter of 1984; that data are plotted in Chart 4. Note how in the first half of the sample x_t seems to follow an upward trend, while in the second half it seems to fluctuate around a constant 1.5 percent quarterly growth rate. Not surprisingly, inference about the persistence of shocks to x_t is very sensitive to how this low-frequency behavior is accommodated.

One way to show this is to fit first-order autoregressive models [like (13)] to data on x_t using different subsamples. Results of doing that are reported in Table 1. When a first-order autoregressive model is fit to the entire sample, the coefficient ρ_x on lagged x_t is 0.80. This relatively high value reflects the autoregression's attempt to interpret the upward trend in the earlier part of the sample as a slow reversion to a stochastic mean of around 0.015. When the same calculation is done using data from the later part of the sample, the autoregressive coefficient, not surprisingly, is much smaller: 0.32. In light of these results, I use the persistence estimate obtained from the full sample as my benchmark, but I also consider the impact on my results of lower persistence.

Table 1

Estimated Money Growth Models†

$x_t = (M_{t+1} - M_t)/M_t$

$x_t = (1 - \rho_x)x + \rho_x x_{t-1} + \epsilon_{x,t}$

	Coefficients		Standard Deviation of Shock
Estimation Period	$(1-\rho_x)x$	ρ_x	$\sigma_{\epsilon x}$
Full Sample:			
1959:2–1984:1	.0025	.80	.0041
Subsamples:			
1959:2–1969:4	.0014	.81	.0037
1970:1–1984:1	.0110	.32	.0038

†In these models, M = U.S. base money. See note 17 for details.
Source of basic data: Citicorp's Citibase data bank

An estimate of the state of technology, z_t, is obtained using data on Y_t, K_t, and H_t; equation (5); and $\alpha = 0.35$. The result is plotted in Chart 5. These data exhibit the same trend behavior as do the money growth data in Chart 4. Not surprisingly, therefore, the same subsample instability appears when equations (6) and (12) are fit to that data.

A careful analysis along these lines is reported by Burnside, Eichenbaum, and Rebelo (1990). They fit a first-order autoregression to the linearly detrended logarithm of z_t over the same three subsamples reported in Table 1.[20] Using the whole sample, they find that $\rho_\theta = 0.9857$ and $\sigma_{\varepsilon,\theta} = 0.01369$. Over the first subsample, they get $\rho_\theta = 0.8624$ and $\sigma_{\varepsilon,\theta} = 0.00923$; over the second, $\rho_\theta = 0.8842$ and $\sigma_{\varepsilon,\theta} = 0.015538$. Clearly, if we insist on the simple autoregressive model with linear time trend for log z_t posited here (and used in the real business cycle literature), there is considerable uncertainty about what a plausible set of parameters for that model is. I will here take Burnside, Eichenbaum, and Rebelo's full sample results as the benchmark parameter values.

The Effects of a Money Shock

I now turn to an analysis of the dynamic properties of the models. I begin by investigating the models' ability to account for a dominant liquidity effect on interest rates and employment and output. My results are consistent with the earlier analysis: The basic cash-in-advance model does not exhibit a dominant liquidity effect in equilibrium. Whether or not the Fuerst-Lucas model can do so depends on parameter values; for plausible values, it does not. In contrast with the others, the sluggish capital model easily rationalizes a dominant liquidity effect.

I start with the immediate, or contemporaneous, impact in the three models of a shock to the money growth rate, shown in Table 2 for various settings of ψ, ρ_x, and δ^*. The other parameters are set at their benchmark values. In the table, R_x is the percentage-point change in the nominal interest rate associated with a one-percentage-point unexpected increase in money growth; L_x is the percentage change in labor effort associated with that increase. (Unless otherwise stated, all rates are quarterly.)

Let's start with the results for the basic cash-in-advance model. Note that whenever $\rho_x = 0$ in this model, $R_x = L_x = 0$. This reflects that, when $\rho_x = 0$, an unexpected change in money growth is purely temporary and so is neutral. In particular, there is a permanent, one-time jump in the money stock which leads to a contemporaneous, equiproportionate jump in current and anticipated price levels, leaving the anticipated inflation rate unaffected. Also, the nominal interest rate remains unchanged, as do employment and investment. Contrast this with what happens when there is positive persistence in money growth shocks, or $\rho_x > 0$. Then anticipated inflation increases and raises the nominal interest rate. In addition, by acting as a tax on labor effort, the jump in anticipated inflation produces a fall in labor supply and, thus, a fall in equilibrium employment.

Now turn to the Fuerst-Lucas model. The best case here for the dominant liquidity effect is when capital depreciates completely in one period and money growth has no persistence, in row (1). Then a one-percentage-point temporary increase in money growth produces a 91-basis-point fall in the nominal interest rate. At the same time, employment jumps by almost half a percent, as firms use the extra liquidity to expand employment and investment.

The remaining rows for the Fuerst-Lucas model show that moving away from these parameter values, in the direction of greater empirical plausibility, overturns the results. For example, row (2) indicates that when the depreciation rate is dropped to an empirically plausible level, the positive impact on employment observed in row (1) turns negative. The reason for this is that with less depreciation, the return on capital falls less rapidly with an expansion in investment. As

[20] Burnside, Eichenbaum, and Rebelo (1990) do not simply fit a first-order autoregression: they allow the possibility that there is classical measurement error in the hours-worked data used to construct z_t. The measurement error model they use is the one analyzed in Prescott 1986 and Christiano and Eichenbaum 1990.

Table 2

The Contemporaneous Impact of a Money Growth Shock in the Three Models

Percentage-Point Change in the Nominal Interest Rate (R_x)
and Percentage Change in Hours Worked (L_x)
in the Period of a One-Percentage-Point Surprise Increase in Money Growth†

	Parameters‡			Models					
				Basic Cash-in-Advance		Fuerst-Lucas		Sluggish Capital	
Rows	Utility ψ	Persist. ρ_x	Deprec. δ^*	R_x	L_x	R_x	L_x	R_x	L_x
(1)	0	0	1.00	0	0	–.910	.419	–4.35	2.010
(2)	0	0	.02	0	0	–.028	–.011	–3.11	1.500
(3)	0	.80	1.00	.699	–1.250	–.693	–.612	–2.97	.440
(4)	–4	0	1.00	0	0	–.899	–.166	–4.47	2.130
(5)	0	.80	.02	.290	–2.120	.200	–2.150	–2.26	–.944
(6)	0	.32	.02	.101	–.375	.064	–.390	–2.97	1.102
(7)	–4	.32	.02	.126	–.283	–.216	–.922	–2.93	1.129

†The derivatives, $L_x = d \log L/d\epsilon_x$ and $R_x = dR/d\epsilon_x$, are evaluated in nonstochastic steady state.

‡The parameter ψ is a curvature parameter on the utility function, (4); ρ_x is the autocorrelation of money growth; and δ^* is the rate of depreciation on capital in equation (8). The other parameters are set at $\beta^* = 1.03^{-0.25}$, $\mu = 0.0041$, $\theta = 1$, $x = 0.012$, $\rho_\theta = 0.9857$, $\alpha = 0.35$, and $\gamma = 0.76$.

a result, after a monetary injection, relatively more funds are absorbed into investment and less into employment.[21]

Row (3) in Table 2 indicates the marginal impact of increasing the persistence of the money growth shock. That also has the effect of making employment fall with a positive money shock. This reflects the effects of a phenomenon already observed in the basic cash-in-advance model. The persistent change in money growth pushes up the anticipated rate of inflation, producing a reduction in labor supply. In equilibrium, this reduction overwhelms the positive impact of the increased liquidity on labor demand.

Row (4) displays the marginal impact of increasing the curvature on the utility function. This also has the effect of making the employment response to a positive money growth shock negative. The reason for this is that, by driving up the price level, the money shock forces consumption to fall contemporaneously because of the cash constraint (1) and the fact that N_t cannot respond to the shock by assumption. It is readily confirmed from equation (4) that the fall in consumption drives up the marginal utility of leisure when $\psi < 0$. As a result, labor supply falls and, in equilibrium, so does employment.

Thus far, however, none of the changes from the row (1) parameterization have overturned the implication of the Fuerst-Lucas model that a money growth shock produces a fall in the nominal interest rate. In all of these cases, the liquidity effect on the interest rate dominates the anticipated inflation effect in equilibrium. When the changes are considered jointly, however, the liquidity effect is overwhelmed by the anticipated inflation effect.

This is the implication of the results in rows (5)–(7). Row (5) gives the values for R_x and L_x associated with the benchmark parameter values. Note that the anticipated inflation effect on the interest rate now swamps the liquidity effect. In addition, the reduction in labor supply from

[21] From (5), the marginal product of capital is $\alpha(z_t H_t/K_t)^{(1-\alpha)} + 1 - \delta^*$. Thus, dropping δ^* below unity introduces a linear term into the marginal product of capital, which makes it fall less quickly with expansions in K_t. The phenomenon identified here is also present in the real business cycle literature. For example, Long and Plosser's (1983) model assumes that $\delta^* = 1$ and, in equilibrium, investment is proportional to income. When the depreciation rate in that model is reduced, investment moves more than one-for-one with movements in income. The reason is that expansions in investment in response to a positive technology shock encounter diminishing returns less quickly when $\delta^* < 1$. Again, this reflects the addition of the linear term in the marginal product of capital.

anticipated inflation dominates any positive demand effect from a money injection. In fact, here the equilibrium effects of the money growth shock are not much different from what they are in the basic cash-in-advance model. Interestingly, dropping persistence in x_t to the not-unreasonable value of 0.32 [as in row (6)] does not change the qualitative result, but it does reduce the negative employment impact of a money shock. Row (7) indicates that increasing the curvature of the utility function has little impact.

Next, consider what happens in the sluggish capital model, which is shown in the final pair of columns in Table 2. Recall that, in this model, the role of investment in absorbing an infusion of liquidity is limited by the fact that the real level of investment is temporarily inflexible. Other things the same, this should enhance the positive impact on labor demand of a money infusion. The results in Table 2 indicate that for most parameterizations, it does: the equilibrium effect on employment of a money infusion is, indeed, positive. Only when a money growth shock is very persistent [as in row (5)] does the negative impact on labor supply occasioned by the shock dominate the positive impact on labor demand. Note, however, how very large the interest rate impact of a money shock is in this model. In all cases, a one-percentage-point surprise jump in the money growth rate produces a drop in the nominal interest rate greater than two percentage points. At an annual rate, this translates roughly into a drop of eight percentage points.

Now let's look at the longer-term impact of a money growth shock. Charts 6–10 show the contemporaneous and lagged responses of the models' key variables to a one-standard-deviation disturbance, $\varepsilon_{x,t}$, in the money growth rate. In all cases, the economy is assumed to start in nonstochastic steady state in period 1 and to experience the shock in period 10. Each chart shows how the variable responds in the three models. The parameterization underlying the impulse response functions in these charts correspond to that underlying row (6) in Table 2: $\beta^* = 1.03^{-0.25}$, $\mu = 0.0041$, $\theta = 1$, $x = 0.012$, $\rho_\theta = 0.9857$, $\alpha = 0.35$, $\gamma = 0.76$, $\psi = 0$, $\rho_x = 0.32$, and $\delta^* = 0.02$.

Five features on these charts are worth noting.

First, incorporating the Fuerst-Lucas assumptions into the basic cash-in-advance model has virtually no impact on the equilibrium employment and interest rate responses to a money growth shock (Charts 7 and 10).

Second, the liquidity effect in the sluggish capital model exists only in the period of the shock (Charts 7 and 10). In addition to being empirically implausible, this lack of persistence is a problem for another reason. In particular, the liquidity effect would likely disappear if the period length in the model were made shorter than the data sampling interval and the observed data were viewed as time-averaged. Such sensitivity is a significant shortcoming, since I have little confidence in the exact period length specification of my models.

Third, in all cases but one, the impulse responses starting the period after the shock virtually coincide across all models. The exceptional case is inflation (Chart 9), which is quite high in the sluggish capital model in the period after the shock, as prices catch up after the fall in price level that occurs in the period of the shock. This initial fall in the price level in this model reflects that the money growth shock has a relatively greater impact on the supply of output than on its demand.[22]

A fourth notable feature in the charts is the surge in investment that occurs in the period of the shock in the Fuerst-Lucas model (Chart 8). It is by suppressing this as an outlet for the money infusion that the sluggish capital model predicts a positive employment response to a money shock.

Fifth, note how the Fuerst-Lucas and sluggish capital models both imply that the consumption and price responses to a money growth shock are opposite in sign (Charts 6 and 9). This reflects the effect of the cash-in-advance constraint, equation (1). King (1990) presents evidence that suggests this is counterfactual.

To summarize, my results confirm that the basic cash-in-advance model cannot rationalize the widespread view that the liquidity effect overwhelms the anticipated inflation effect on the interest rate and on employment, at least in the short run. It also cannot rationalize a positive employment and output effect of a money infusion. Although in principle, the Fuerst-Lucas version of the model can rationalize this, in practice it fails to do so for plausible parameter values. For these values, the anticipated inflation effect associated with a money injection (which is emphasized by the basic cash-in-advance model) dominates in equilibrium.

A modification of the Fuerst-Lucas model which reduces the magnitude of the anticipated inflation effect on equilibrium employment would obviously help. One way to accomplish this may be to model the transaction motive for holding money in a way that gives agents more flexibility than does the cash-in-advance assumption. Another may be to allow certain purchases, like investment, to be made entirely on credit. It would be interesting to see whether the liquidity effect dominates in equilibrium in a version of the Fuerst-Lucas model modified in these ways.

Finally, the results in Table 2 suggest that—qualitatively, at least—the widespread view about the impact of a money

[22] Chart 9 highlights a distinction between the money transmission mechanism in the sluggish capital model and the one in other monetary rational expectations models. In those models (for example, Lucas 1973 and Sargent and Wallace 1975), a jump in the price level is instrumental in transmitting a surprise increase in money to an increase in output.

Charts 6–10

The Dynamic Effects of a Money Growth Shock in the Three Models

Responses of Model Variables to a One-Standard-Deviation Money Growth Shock in Period 10

Basic Cash-in-Advance Model
Fuerst-Lucas Model
Sluggish Capital Model

Chart 6 Consumption $[C_t \exp(-\mu t)]$

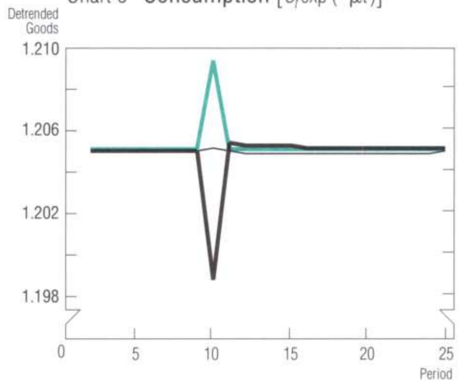


Chart 7 Time Worked (L_t)

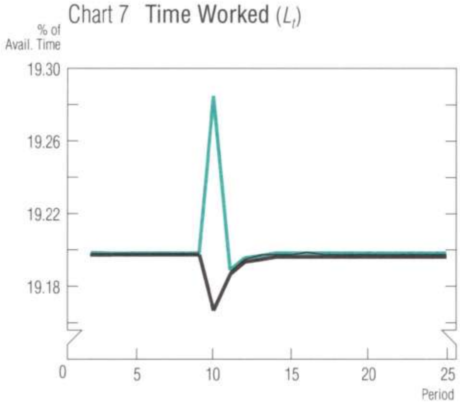


Chart 8 Investment $[I_t \exp(-\mu t)]$

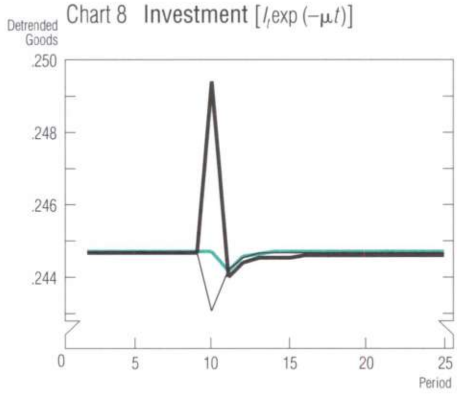


Chart 9 Inflation $\{[(P_t/P_{t-1}) - 1]100\}$

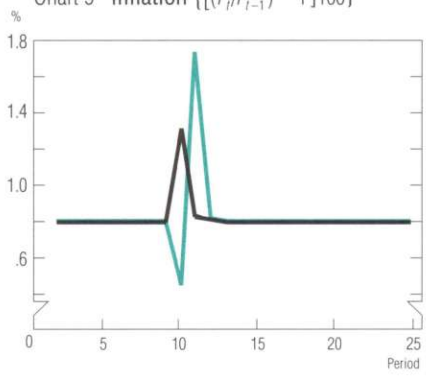
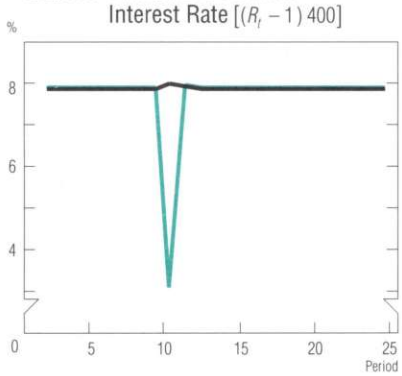


Chart 10 Annualized Nominal Interest Rate $[(R_t - 1)\,400]$



infusion is rationalized by a plausibly parameterized version of the sluggish capital model. However, the dominant liquidity effect shows little persistence in that model, according to the evidence in Charts 6–10; it plays a quantitatively significant role only in the period of the shock itself.

Looking at Other Model Implications

The modifications of the basic cash-in-advance model intended to capture the dominant liquidity effect would be questionable if they simultaneously resulted in drastically reduced empirical performance on other dimensions. Here I investigate the ability of the modified models to account for various other properties of the data, in particular, how volatile variables are, on average, and how much their movements are correlated with those of output. The results, reported in Tables 3–5, are mixed: the modifications help on some dimensions and hurt on others.

Volatility

Table 3 contains measures of the volatility of relevant variables in the three models and in actual U.S. data. All statistics shown there have been computed using data that were first logged and then detrended by the Hodrick-Prescott filter (described in Prescott 1986). Statistics in all rows except (2)–(4) are standard deviations. Statistics in rows (2)–(4) are standard deviations divided by the standard deviation of output.

Note that the U.S. data column in Table 3 includes results for several variables not yet mentioned. In particular, it includes two measures of the price level—the consumer price index (CPI) and the implicit price deflator of the gross national product—and the two measures of inflation corresponding to them. Also, three measures of the income velocity of money, V, are included. They correspond to the monetary base measure discussed above, as well as two of the Fed's broader monetary aggregates, M1 and M2. Finally, two short-term nominal interest rates are included: the yield on three-month Treasury bills (T-bills) and the effective yield on federal funds.

Consider first the real variables: Y, C, L, and I [in rows (1)–(4)].

One dramatic finding here is how much the rigidities in the modified models add to the relative volatility of consumption. In the basic cash-in-advance model, the relative volatility is already high compared to that in the data. Incorporating the rigidities of the Fuerst-Lucas and sluggish capital models adds substantially to that. This increased volatility in consumption reflects in part the effects of equation (1) and the assumption that households' portfolio decision, N_t, cannot respond contemporaneously to a shock. These two things imply that when the price level moves in response to an unexpected shock, consumption has to move in equal proportion, but in the opposite direction. These factors have the effect of exposing consumption to a considerable amount of extraneous variation. The models' households could feasibly control this variation by keeping extra cash balances on hand. However, that is unacceptably costly in these models, where the real return differential between money and bank loans averages eight percentage points, at an annual rate.

Offering households less costly ways to deal with price risk would probably reduce the models' counterfactually high implications for consumption volatility. A promising approach might be to model the transaction motive for holding money to give households more flexibility—say, by letting them get by with less cash than P_tC_t at the cost of some leisure time or consumption. This sort of flexibility might also eliminate the Fuerst-Lucas and sluggish capital models' implication that the effects on consumption and the price level of an unexpected shock are opposite in sign.

Another dramatic implication of the models in Table 3—also counterfactual—concerns employment volatility [row (3)]. In the data, the volatility of labor effort is about the same as that of output. In the models, though, employment's volatility is considerably less than output's. Interestingly, the sluggish capital model actually does better on this dimension than the other models. However, it still substantially undershoots the observed relative volatility of hours worked.

The poor performance of a nonmonetary version of this model has already been documented (Hansen 1985), and it is not surprising that a monetary version also underpredicts the volatility of work effort. Presumably, incorporating the assumption of indivisible labor, along the lines of Hansen 1985 and Rogerson 1988, would increase the models' volatility of labor.[23]

Finally, note that all the models account fairly well for the observed volatility of investment [row (4)].

Among the nominal variables, the most dramatic implication of the models lies with the nominal rate of interest, R [in rows (12)–(13)]. Interest rates simulated using the basic cash-in-advance and Fuerst-Lucas models are less volatile than their empirical counterparts. However, the reverse is true for interest rates simulated with the sluggish capital model: Rates from this model are far more volatile than those in the U.S.

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[23] Christiano (1988) and Burnside, Eichenbaum, and Rebelo (1990) argue that, in a real business cycle model, reducing ρ_θ, the serial correlation in the technology shock, reduces the relative volatility of consumption and increases the relative volatility of labor. This suggests that the models' difficulties on these dimensions could be ameliorated by reducing ρ_θ to 0.8842, the lowest of the several point estimates reported by Burnside, Eichenbaum, and Rebelo. Doing so turns out to have a negligible effect on the dynamic properties of the models here. For example, the relative volatility of consumption in the basic, Fuerst-Lucas, and sluggish capital models is 0.53, 0.88, and 1.03, respectively. The corresponding results for the standard deviation of labor effort relative to that of output are 0.29, 0.30, and 0.55, respectively. These results are very close to those reported for the $\rho_\theta = 0.9857$ models in Table 3.

Table 3

Volatility in the U.S. Data and in the Three Models†

Rows	Variables		U.S. Data 1959:1–1984:1	Models‡ Basic Cash-in-Advance	Fuerst-Lucas	Sluggish Capital
(1)	Real Output, Y		.017	.013	.013	.015
(2)	Consumption, C		.490	.630	.880	.990
(3)	Employment, L		1.000	.290	.300	.520
(4)	Investment, I		2.590	2.990	3.130	2.660
(5)	Price Level, P	CPI	.016	.015	.015	.018
(6)		Deflator	.010			
(7)	Inflation, P/P_{-1}	CPI	.005	.011	.011	.019
(8)		Deflator	.004			
(9)	Income Velocity of Money, V	MB	.0130	.0008	.0008	.0053
(10)		M1	.0130			
(11)		M2	.0190			
(12)	Nominal Interest Rate, R	T-Bills	.0033	.0022	.0021	.0140
(13)		Fed. Funds	.0046			

†Results in this table are based on standard deviations of data that have been first logged and then detrended using the Hodrick-Prescott filter. Actual standard deviations are reported in all but rows (2)–(4). Standard deviations in rows (2)–(4) have been divided by the standard deviation of output. Before logging and filtering, nominal interest rates were expressed at a gross quarterly rate.

‡Model results are averages across 20 simulations of 101 observations each.

Sources of basic data: Rows (1)–(4): See Christiano 1988.
Rows (5)–(13): Citicorp's data bank

data. This is not surprising, in light of the model's very strong interest rate impact of a money growth shock reported in Table 2.

Another result that stands out in Table 3 is how smooth the income velocity of money, V, is in the models compared to the data [rows (9)–(11)]. This is reminiscent of a result for an endowment economy reported by Hodrick, Kocherlakota, and Lucas (1991). Interestingly, the sluggish capital model is closer to the data on this dimension, but its implied volatility of V is still less than half that in the data.

The results on the models' implications for money demand, equation (26), can be used to gain insight into why V is so smooth in the sluggish capital model despite the high volatility of interest rates. Substituting $R = 1.0195$, $\alpha = 0.35$ into the formula for b [equation (28)], we find that the interest elasticity of money demand, $b/4$, implied by the models is 0.1. This is much smaller than the estimates in the empirical literature. For example, Lucas (1988, Table 4) and Stock and Watson (1991, Table 2) estimate this elasticity to be between 9 and 10.[24] If the sluggish capital model's interest elasticity had been empirically more plausible, then (other things unchanged) velocity would have been more volatile. This suggests that the model's empirical implications may be improved by incorporating a more flexible transaction model of money demand, since presumably this would increase the interest sensitivity of velocity.

Finally, note (in Table 3) that although the sluggish capital model captures reasonably well the volatility of the price level [rows (5)–(6)], it substantially overshoots the volatility of

[24]The Lucas and Stock-Watson numbers are long-run elasticities, which are the appropriate ones to compare with my models. The results in Goldfeld and Sichel's (1990) Tables 8.1 and 8.4 also support the view that my models' interest elasticity is considerably smaller than what is empirically plausible.

inflation [rows (7)–(8)]. Evidently, real-world prices are stickier than those anticipated by the model. Sims (1989) has made a similar observation in the context of a related model.

On net, it is hard to say whether the sluggish capital model is an improvement over the other models in terms of its implications for volatility. The sluggish capital modification hurts with consumption, but helps with employment. It hurts with inflation, but helps with velocity. Whether it helps or hurts with interest rates is not clear, since the sluggish capital model overshoots while the other models undershoot.

A question of independent interest is how much money growth shocks contribute to fluctuations in these models. One way to answer this question is to set σ_{ε_x}, the standard deviation of the unexpected part of money growth, to zero in the simulations. Doing so, I find that the basic cash-in-advance model, the Fuerst-Lucas model, and the sluggish capital model imply output standard deviations of 0.013, 0.013, and 0.014, respectively. These numbers are trivially less than those reported in Table 3 [row (1)]. Thus, in these models, money shocks contribute almost nothing to output fluctuations.

Correlations With Output

Tables 4 and 5 present measures of how movements in output are related to movements of other variables in the U.S. data and in the three models.

Table 4 reports these dynamic correlations for the real variables only.

Generally, the contemporaneous correlations in the models are somewhat higher than those in the U.S. data. The results for investment are a notable exception. The contemporaneous correlation between investment and output is substantially lower in both the Fuerst-Lucas and sluggish capital models than that in the data.

Even more interesting, these models imply that investment lags output by one quarter. This reflects the effects of the rigidities that inhibit households from financing an increase in investment in the period of a technology shock. In the Fuerst-Lucas model, some expansion in investment can nevertheless be financed in the period of a shock simply because a falling price level increases the value of the predetermined nominal loans made by households to the financial intermediaries. (The falling price level can be understood using standard textbook reasoning and the fact that the positive shock to technology generates an increase in output.) However, in the sluggish capital model, investment is assumed to be predetermined. As a result, there is no investment response to a technology shock until one period after the shock. Expanding investment then still makes sense since by assumption the shocks are highly persistent.

Table 5 reports dynamic correlations in the models and the data for the nominal variables.

Consider the detrended price level first. As pointed out by Kydland and Prescott (1990), the price level covaries negatively with output, whether measured by the CPI or by the gross national product deflator. The models have roughly the same implication. This is consistent with my earlier observation that technology shocks dominate in the models' dynamics.

Notice, however, how differently detrended inflation behaves. In the data, inflation is positively associated with output, both contemporaneously and at the one-quarter lag. In addition, inflation lags the cycle in the sense that it is most strongly correlated with detrended output one quarter earlier. By contrast, in all three models, inflation has a negative contemporaneous correlation with output. However, the models resemble the data in that detrended output is positively correlated with inflation one quarter later.

Next, consider the money data. The models imply that detrended money is roughly uncorrelated with output, while the data imply some correlation. Again, the very low correlation implied by the models is consistent with my earlier observation that money shocks play only a small role in the models' dynamics. Note that, in the data, the broader monetary aggregates lead the cycle; that is, detrended output is more highly correlated with their earlier values. This seems like an interesting fact that a richer version of these models should be able to confront.

The results for velocity are broadly similar to those for money.

Finally, consider the results for the nominal interest rate.

Note that in the data, both my measures of short-term rates lag the cycle. That is, detrended output is more highly correlated with future values of the interest rate than with contemporaneous values. In the models, by contrast, the nominal interest rate is more nearly coincident with the cycle. That is, the strongest cross-correlation is with the contemporaneous value of output.

In addition, although the nominal interest rate is positively correlated with the contemporaneous value of output in the basic cash-in-advance and Fuerst-Lucas models, it is negatively correlated with that value in the sluggish capital model. This reflects that, in the first two models, the impact of a positive technology shock on the nominal interest rate is positive, while in the sluggish capital model, it is negative. This negative impact presumably reflects that the sluggish capital model's assumption of a predetermined investment level in effect subtracts an important source of demand for funds from the loan market.

Overall, the dynamic correlations reported in Tables 4 and 5 seem to suggest that both the Fuerst-Lucas and sluggish capital models are a step backward from the basic cash-in-

Tables 4 and 5
Dynamic Correlations Between Output and Other Variables in the U.S. Data and in the Three Models

Table 4 The Real Variables

Variable, v_t	Data Source	Correlation of Real Output With v_{t-1}	v_t	v_{t+1}
Real Output, *Y*	U.S. Data	.81	1.00	.81
	Models:			
	Basic Cash-in Advance	.67	1.00	.67
	Fuerst-Lucas	.67	1.00	.67
	Sluggish Capital	.52	1.00	.52
Consumption, *C*	U.S. Data	.63	.68	.55
	Models:			
	Basic Cash-in Advance	.64	.98	.71
	Fuerst-Lucas	.57	.87	.37
	Sluggish Capital	.44	.90	.15
Employment, *L*	U.S. Data	.62	.75	.68
	Models:			
	Basic Cash-in Advance	.65	.91	.54
	Fuerst-Lucas	.64	.92	.56
	Sluggish Capital	.31	.82	.10
Investment, *I*	U.S. Data	.85	.90	.72
	Models:			
	Basic Cash-in Advance	.68	.98	.61
	Fuerst-Lucas	.49	.71	.78
	Sluggish Capital	.37	.57	.93

Sources of basic data: See Table 3.

advance model. This conclusion is based principally on the models' implications for the cyclical properties of investment and interest rates. The rigidities appear to make investment lag the cycle, by inhibiting a quick increase in financing for investment in response to a technology shock.[25] This factor may also account for the sluggish capital model's counterfactual implication that output covaries negatively with the rate of interest. These considerations suggest considering a modified version of the sluggish capital model in which household portfolio decisions and firm investment decisions are made after the technology shock, but still before the money growth shock. That assumption may be plausible if agents have some advance notice about disturbances to technology.

[25]Although aggregate investment is roughly coincident with the cycle, this masks interesting dynamics that occur at a more disaggregated level. According to Kydland and Prescott (1990, Table 2), the nonresidential part of business fixed investment lags the cycle, while the residential part leads it. The lag in nonresidential investment may reflect the effects of the precommitment captured by the sluggish capital model.

Table 5 **The Nominal Variables**

Variable, v_t	Data Source	Correlation of Real Output With v_{t-1}	v_t	v_{t+1}
Price Level, P	U.S. Data: CPI	−.67	−.55	−.32
	Deflator	−.65	−.57	−.38
	Models:			
	Basic Cash-in Advance	−.58	−.89	−.62
	Fuerst-Lucas	−.57	−.89	−.61
	Sluggish Capital	−.47	−.91	−.34
Inflation, P/P_{-1}	U.S. Data: CPI	.12	.38	.52
	Deflator	.04	.17	.26
	Models:			
	Basic Cash-in Advance	−.31	−.39	.33
	Fuerst-Lucas	−.29	−.39	.34
	Sluggish Capital	−.17	−.40	.51
Money, M	U.S. Data: MB	.39	.49	.48
	M1	.66	.65	.48
	M2	.70	.56	.32
	Models:			
	Basic Cash-in Advance	.05	.03	−.04
	Fuerst-Lucas	.06	.04	−.03
	Sluggish Capital	−.04	−.08	.02
Income Velocity of Money, V	U.S. Data: MB	.38	.65	.55
	M1	.09	.41	.44
	M2	−.16	.19	.31
	Models:			
	Basic Cash-in Advance	.08	.11	.06
	Fuerst-Lucas	.08	.13	.01
	Sluggish Capital	−.07	−.38	.24
Nominal Interest Rate, R	U.S. Data: T-Bills	.04	.30	.44
	Fed. Funds	.01	.30	.49
	Models:			
	Basic Cash-in Advance	.08	.11	.06
	Fuerst-Lucas	.08	.13	.01
	Sluggish Capital	.07	−.38	.24

Sources of basic data: See Table 3.

Summary and Directions for Further Research

Monetary versions of real business cycle models have great potential as laboratories for evaluating monetary policies which have real-world effects we do not yet understand. Before these models can be used with confidence for this, however, we need to be sure that they can at least replicate the effects of simple monetary policy experiments we think we do understand. I have focused on one simple experiment: an unanticipated change in the money growth rate, or a money growth shock. Many economists believe that a positive money shock drives the interest rate down and output and employment up, at least in the short run. Put differently, they think the liquidity effect dominates the anticipated inflation effect.

Simple monetary versions of real business cycle models predict the opposite. For example, when money is introduced by a cash-in-advance constraint, interest rates jump and output and employment fall after a money shock in the empirically plausible case that the shock triggers expectations of increased future money growth.[26] I argued that this reflects the role of the anticipated inflation effect and the absence of a liquidity effect.

I have explored ways of introducing a liquidity effect into this type of model, with mixed results. First, I used a device proposed by Lucas (1990), a rigidity in the household's nominal saving decision. With this modification, the model (the Fuerst-Lucas model) does produce a liquidity effect, but one too small relative to the anticipated inflation effect. Next, I changed the model in another way and managed to increase the magnitude of the liquidity effect enough so that it dominates the anticipated inflation effect. However, the dominant liquidity effect in the resulting model (the sluggish capital model) displays no persistence; it exists only in the period of the shock. Also, the model has implications for money demand and for several features of the U.S. business cycle which contradict the facts.[27] Obviously, more work needs to be done on monetary versions of real business cycle models.

Martin Eichenbaum and I have begun that work. We are exploring model specifications which enhance the likelihood that the Lucas rigidity will produce a dominant, persistent liquidity effect. We seek models that also avoid some of the counterfactual money demand and business cycle implications of the models in this paper.

To reduce the anticipated inflation effect, we are taking steps in the direction of realism by giving agents more flexibility in the way they finance their transactions than they have had here. In the paper I explained how, besides reducing the strength of the anticipated inflation effect, such modifications should help correct several counterfactual implications of the Fuerst-Lucas and sluggish capital models: their overprediction of the response of average employment to average inflation, their overprediction of the volatility of consumption, their implication that the impacts of a money growth shock on consumption and the price level must have opposite signs (which is empirically implausible, according to King 1990), and their low interest elasticity of money demand and consequent excess smoothness of money velocity.

To increase the liquidity effect, we plan to modify the assumption implicit in the models of this paper that the production period of firms and the decision period of the Federal Reserve coincide. More plausible is the assumption that production decisions take an appreciable amount of time to implement, while the Fed's decisions do not; they are implemented virtually instantaneously. Thus, when the Fed drains money from the financial system, at least some firms are likely to have already committed themselves to a production plan which they cannot easily adjust without suffering significant costs. Managers of such firms will be willing to pay a substantial premium to borrow the funds needed to continue financing their inputs and avoid interrupting production.[28] Under these circumstances, a negative money shock could produce a substantial rise in the interest rate.

To introduce persistence into the liquidity effect, we are exploring ways of making utility and production functions depend on lagged variables. One way to do this is to make the marginal utility of consumption an increasing function of lagged consumption, as in the *habit persistence* utility function.[29] Such a modification may help overcome another counterfactual implication of the models in this paper: that the short- and long-run money demand elasticities with respect to the interest rate coincide. In empirical money

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[26] My assumption about the nature of the money growth process, equation (13), deserves more attention. Although several other studies use this kind of model (Barro 1978, Barro and Rush 1980, Cooley and Hansen 1989, and King 1990), it is by no means uncontroversial. For example, Sims (1986) and Bernanke and Blinder (1990) criticize this specification of policy on empirical grounds and suggest alternatives. It would be worth investigating these to see how they affect the relative magnitude of the anticipated inflation and liquidity effects.

[27] My conclusions about the empirical plausibility of the various models and directions for future model modifications have been reached with relatively little use of the tools of formal statistical analysis. This reflects the preliminary stage of this work, in which mismatches between models and data are sufficiently blatant as to make a formal metric superfluous. In subsequent work, when comparisons between models and data involve greater subtleties, the tools of statistical sampling theory (for example, those discussed in Christiano and Eichenbaum 1990) will become necessary.

[28] Our assumption about the time period of production closely resembles the technology assumption of Diamond and Dybvig (1983). They cite Friedman and Schwartz (1963) for evidence consistent with their technology assumption (Diamond and Dybvig 1983, p. 403).

[29] One way to see why this modification might help is to recall the consumption response to a money shock in the sluggish capital model (Chart 6). It surges in the period of the shock and then immediately returns to its previous level. With a habit persistence utility function, consumption should instead come down slowly (as is empirically plausible, according to King 1990). Other things the same, a declining consumption trajectory implies a low interest rate.

demand functions, the long-term elasticity greatly exceeds the short-term elasticity.

Preliminary results of this further research are encouraging. Thus, there is reason to hope that we are making progress toward a model that can confidently be used to conduct monetary policy experiments.

Appendix
Finding Approximate Solutions to the Models

Here I describe the solution strategy underlying the results in the preceding paper. First I discuss the undetermined coefficient method used to obtain linearized decision rules. The basic idea behind this method is conveyed through a simple example. Then I show how to express the equations of the paper's models in a form suitable for applying the undetermined coefficient method. This involves, principally, handling complications that arise from the presence of sustained growth in the state of technology z_t, equation (6), and in the money supply M_t, equation (13).

An Undetermined Coefficient Method

Consider a one-sector neoclassical growth model in which there is only a saving/consumption decision to be made.

The competitive equilibrium solves the following planning problem: Maximize expected utility $E\Sigma_{t=0}^{\infty}\beta^t u(c_t)$ subject to the goods market-clearing condition $c_t + k_{t+1} - (1-\delta)k_t = f(k_t,\theta_t)$, where β is the discount rate, c_t is consumption, k_t is the capital stock, δ is the depreciation rate on capital, f is the production function, and θ_t is a technology shock. I assume that

$$\text{(A1)}\quad E[\theta_{t+1}\,|\,\theta_t,k_t] = \alpha_0 + \alpha_1\theta_t$$

where α_0 and α_1 are given constants.

The efficiency condition for capital investment requires that

$$\text{(A2)}\quad E[u_c(c_t) - \beta u_c(c_{t+1})\{f_k(k_{t+1},\theta_{t+1}) + 1 - \delta\}\,|\,\theta_t,k_t] = 0$$

for all $t \geq 0$, where f_k denotes the partial derivative of f with respect to its first argument and u_c denotes the marginal utility of consumption. After c_t is substituted out from the goods market-clearing condition,

$$\text{(A3)}\quad E[v(k_t,k_{t+1},k_{t+2},\theta_t,\theta_{t+1})\,|\,\theta_t,k_t] = 0.$$

Here

$$\begin{aligned}\text{(A4)}\quad v(k_t,k_{t+1},k_{t+2},\theta_t,\theta_{t+1}) \equiv{}& u_c[f(k_t,\theta_t) + (1-\delta)k_t - k_{t+1}]\\ &- \beta u_c[f(k_{t+1},\theta_{t+1})\\ &+ (1-\delta)k_{t+1} - k_{t+2}][f_k(k_{t+1},\theta_{t+1}) + 1-\delta].\end{aligned}$$

The exact solution to the problem is a function, $k_{t+1} = g(k_t,\theta_t)$, that satisfies (A3); that is,

$$\text{(A5)}\quad E\big[v(k_t,g(k_t,\theta),g[g(k_t,\theta),\theta_{t+1}],\theta_t,\theta_{t+1})\,|\,\theta_t,k_t\big] = 0$$

for all $k_t \geq 0$ and for all θ_t.

Determining g exactly can be computationally very costly. However, it may not be necessary. In several examples (Danthine, Donaldson, and Mehra 1989 and Christiano 1990a), Kydland and Prescott's (1982) suggestion that g be approximated by a linear function has been found to work well.

Following is a simple three-step procedure that delivers a linear approximation, G, to g:

1. Find the value of k that solves $v(k,k,k,\theta,\theta) = 0$, where $\theta \equiv E\theta_t$. The variable k is the value to which k_t tends in the nonstochastic version of the problem, where θ_t is held to its unconditional mean.
2. Compute V, the first-order Taylor series expansion of v about $k_t = k_{t+1} = k_{t+2} = k$ and $\theta_t = \theta_{t+1} = \theta$.
3. Define the following linear function:

$$\text{(A6)}\quad k_{t+1} = G(k_t,\theta_t) \equiv G_0 + G_1k_t + G_2\theta_t$$

where G_0, G_1, G_2 are (as yet) undetermined constants. Select values of G_0, G_1, G_2 so that the analog of (A5) is satisfied with v replaced by V:

$$\begin{aligned}\text{(A7)}\quad & E[V(k_t,G(k_t,\theta_t),G[G(k_t,\theta_t),\theta_{t+1}],\theta_t\theta_{t+1})\,|\,\theta_t,k_t]\\ &= \tilde{G}_0 + \tilde{G}_1k_t + \tilde{G}_2\theta_t = 0\end{aligned}$$

where $\tilde{G}_i$ are functions of G_0, G_1, G_2 for each $i = 1, 2, 3$. In (A7), the first equality follows by the linearity of V, G, and $E[\theta_{t+1}|\theta_t,k_t]$. The requirement that the last equality is satisfied for all $k_t \geq 0$ and all θ_t requires that

$$\text{(A8)}\quad \tilde{G}_0 = \tilde{G}_1 = \tilde{G}_2 = 0.$$

Equation (A8) represents three equations in the three unknowns, G_0, G_1, and G_2. These equations, in addition to a transversality-type condition, $|G_1| \leq \beta^{1/2}$, can be used to find unique values for G_0, G_1, and G_2.

To compute first- and second-moment implications like those analyzed in the paper, use the decision rule, G, and the time series model for θ_t to simulate artificial data on θ_t and k_t. The resource constraint can be used to compute the implied consumption data. Statistical analysis can then be performed on these data.

Three observations on the linearization procedure are in order.

First, a simple modification to the above procedure can be used to compute a log-linear decision rule: $\bar{k}_{t+1} = \bar{G}_0 + \bar{G}_1\bar{k}_t + \bar{G}_2\theta_t$, where $\bar{k}_t \equiv \log k_t$. Simply define

$$\text{(A9)}\quad \bar{v}(\bar{k}_t,\bar{k}_{t+1},\bar{k}_{t+2},\theta_t,\theta_{t+1}) = v[\exp(\bar{k}_t),\exp(\bar{k}_{t+1}),\exp(\bar{k}_{t+2}),\theta_t,\theta_{t+1}].$$

Then replace $\bar{v}$ by $\bar{V}$, the linear expansion of $\bar{v}$ about $\bar{k}_t = \bar{k}_{t+1} = \bar{k}_{t+2} = \log k$ and $\theta_t = \theta_{t+1} = \theta$. Finally, find the values of $\bar{G}_0$, $\bar{G}_1$, $\bar{G}_2$ which solve the analog of (A5) with v replaced by $\bar{V}$, subject to the condition $|\bar{G}_1| \leq \beta^{1/2}$.

Second, the undetermined coefficient procedure requires only that the efficiency conditions be satisfied as an equality, as they are in the illustration and in the models of the paper. In particular, the problem need not be expressible as a social planning problem, as it is in the illustration. For example, the models in the paper cannot easily be expressed as social planning problems.

Third, when applied to growth models like the one in the illustration, the method yields exactly the same solution as the linear-quadratic method proposed in Kydland and Prescott 1982. Also, the log-linear variant of the undetermined coefficient method yields the same solution as the log-linear-quadratic version of the Kydland-Prescott method used in Christiano 1988.

Applying the Undetermined Coefficient Method

To get the models of the paper in shape for the undetermined coefficient method, their efficiency conditions and other restrictions, such as the resource constraint, must be expressed in a form analogous to (A3). Here I primarily describe how to do that in the Fuerst-Lucas model. Then I show how to modify things to accommodate the basic cash-in-advance and sluggish capital models.

The Fuerst-Lucas Model

In the paper, the efficiency conditions for the Fuerst-Lucas model are equations (18), (19), (24), and (31). It is convenient to eliminate R_t from (19). Accordingly, substituting from (18) and (19) into (24) and (31) gives

$$\text{(A10)}\quad E_t\{(P_t/W_t)^2 f^*_{H,t}u_{L,t} - \beta^* P_{t+1} f^*_{K,t+1}(u_{L,t+1}/W_{t+1})\} = 0$$

$$\text{(A11)}\quad E_{t-1}\{(u_{c,t}/P_t) + (P_t/W_t) f^*_{H,t}(u_{L,t}/W_t)\} = 0.$$

For convenience, a slightly rearranged version of (18) is reproduced here:

$$\text{(A12)}\quad E_t\{u_{L,t} + W_t\beta^*(u_{c,t+1}/P_{t+1})\} = 0.$$

The efficiency conditions, (A10), (A11), and (A12), are not enough to solve the model. This is because these constitute only three restrictions, while we seek six objects: equilibrium decision rules for K_{t+1}, C_t, N_t, and L_t and market-clearing price rules for P_t and W_t. Three additional restrictions are given by the households' cash-in-advance constraint, (1); the loan market-clearing condition, (14); and the goods market-clearing condition, (16). I will show below how these restrictions can be used to substitute out for P_t, W_t, and C_t in (A10), (A11), and (A12). Then these three efficiency conditions, together with a transversality-type condition, will be enough to pin down approximate equilibrium decision rules for K_{t+1}, L_t, and N_t with a variant of the undetermined coefficient procedure just described. The equilibrium R_t rule can then be inferred from (19), while the rules for P_t, W_t, and C_t follow from (1), (14), and (16).

□ *Scaling the Variables*

All the variables in the model except L_t and R_t display growth in equilibrium. But the undetermined coefficient method requires that the variables display no growth. This is because the method involves approximating the efficiency conditions around a stationary point. (Recall step 1 above.) To meet the stationarity requirement of the solution method, I work with a version of the efficiency conditions, (A10)–(A12), and restrictions (1), (14), and (16) expressed in terms of variables that have been scaled appropriately to eliminate growth.

Define

$$\text{(A13)}\quad c_t = \exp(-\mu t)C_t$$

$$\text{(A14)}\quad k_{t+1} = \exp(-\mu t)K_{t+1}$$

$$\text{(A15)}\quad n_t = N_t/M_t$$

$$\text{(A16)}\quad w_t = W_t/M_t$$

$$\text{(A17)}\quad p_t = P_t\exp(\mu t)/M_t.$$

It turns out that c_t, k_{t+1}, n_t, w_t, and p_t converge to constants in the nonstochastic version of the model. This implies that C_t and K_t grow at the same rate, μ, as the state of technology, z_t, while W_t grows at the rate of money growth, x. Since K_t grows at the rate μ, it follows from the production function (5) that output, Y_t, does too. Finally, P_t's growth rate equals $x - \mu$, the rate of money growth less the rate of output growth.

Associated with the scaled variables in (A13)–(A17) are scaled marginal utilities and productivities. Consider the marginal utilities first. Replacing C_t by $\exp(\mu t)c_t$ in the partial derivative of (4) with respect to C_t gives

$$\begin{aligned}\text{(A18)}\quad u_{c,t} &\equiv u_c(C_t,L_t) = u_c(c_t,L_t)\exp\{\mu t[(1-\gamma)\psi-1]\}\\ &\equiv \bar{u}_{c,t}\exp\{\mu t[(1-\gamma)\psi-1]\}\end{aligned}$$

$$\begin{aligned}\text{(A19)}\quad u_{L,t} &\equiv u_L(C_t,L_t) = u_L(c_t,L_t)\exp\{\mu t(1-\gamma)\psi\}\\ &\equiv \bar{u}_{L,t}\exp\{\mu t(1-\gamma)\psi\}.\end{aligned}$$

Next consider the marginal productivities. Let

$$\begin{aligned}\text{(A20)}\quad f(k_t,\theta_t,L_t) &\equiv \exp(-\mu t)f^*(K_t,z_tL_t)\\ &= \exp(-\alpha\mu)k_t^{\alpha}[\exp(\theta_t)L_t]^{(1-\alpha)} + (1-\delta)k_t\end{aligned}$$

where $(1-\delta) \equiv (1-\delta^*)\exp(-\mu)$. If $f_{k,t}$ and $f_{H,t}$ denote the partial derivative of f with respect to its first and third arguments, then

(A21) $f^*_{H,t} = \exp(\mu t)f_{H,t}$

(A22) $f^*_{K,t} = \exp(\mu)f_{k,t}$.

(The variables H and L are interchangeable for my purposes at this point since the labor market-clearing condition requires that they be equal.) Substitute (A13)–(A22) into (A10)–(A12) and rearrange to get

(A23) $E_t\{(p_t/w_t)^2 f_{H,t}\bar{u}_{L,t} - \beta p_{t+1} f_{k,t+1}(\bar{u}_{L,t+1}/w_{t+1})\} = 0$

(A24) $E_{t-1}\{(\bar{u}_{c,t}/p_t) + (p_t/w_t)f_{H,t}(\bar{u}_{L,t}/w_t)\} = 0$

(A25) $E_t\{\bar{u}_{L,t} + w_t\beta^*(\bar{u}_{c,t+1}/[p_{t+1}(1+x_t)])\} = 0$.

Here $\beta \equiv \beta^*\exp[(1-\gamma)\psi\mu]$. Equations (A23)–(A25) are a version of the Fuerst-Lucas model's efficiency conditions in which all variables have been scaled.

Next consider the scaled version of restrictions (1), (14), and (16). Dividing (1) by C_t and using the goods market-clearing condition, (16), gives

(A26) $P_t = (M_t - N_t)/[f^*(K_t, z_t L_t) - K_{t+1}]$.

Multiplying both sides of (A26) by $\exp(\mu t)/M_t$ and using (A13)–(A17) yields

(A27) $p_t = (1 - n_t)/[f(k_t, \theta_t, L_t) - k_{t+1}] \equiv p(k_t, k_{t+1}, n_t, L_t, \theta_t)$.

Rearrange (14) as

(A28) $W_t = (N_t + X_t - P_t I_t)/L_t$.

Divide both sides of this by M_t to get

(A29) $w_t = [n_t + x_t - p_t\exp(-\mu t)I_t]/L_t$

$= \{n_t + x_t - p(k_t, k_{t+1}, n_t, L_t, \theta_t)[k_{t+1} - (1-\delta)k_t]\}/L_t$

$\equiv w(k_t, k_{t+1}, n_t, L_t, \theta_t, x_t)$.

In (A29), I have used (8), the definition of δ, and (A13)–(A17). Also, from the goods market-clearing condition,

(A30) $c_t = f(k_t, \theta_t, L_t) - k_{t+1} \equiv c(k_t, k_{t+1}, \theta_t, L_t)$.

□ *Solving the Scaled System*

The six restrictions (A23)–(A25), (A27), (A29), and (A30) can now be used to find the six objects: equilibrium decision rules for k_{t+1}, L_t, n_t, and c_t and equilibrium price rules for p_t and w_t. To translate these back into the unscaled counterparts that interest us involves a simple application of (A13)–(A17).

Replacing p_t, w_t, and c_t in the efficiency conditions, (A23)–(A25), by the functions $p(\cdot)$, $w(\cdot)$, and $c(\cdot)$, we can write

(A31) $W(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1})$

$= (p_t/w_t)^2 f_{H,t}\bar{u}_{L,t} - \beta p_{t+1} f_{k,t+1}(\bar{u}_{L,t+1}/w_{t+1})$

(A32) $q(k_t, k_{t+1}, L_t, n_t, s_t) = (\bar{u}_{c,t}/p_t) + (p_t/w_t)f_{H,t}(\bar{u}_{L,t}/w_t)$

(A33) $Q(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+2}, n_t, n_{t+1}, s_t, s_{t+1}) = \bar{u}_{L,t} + w_t\beta\bar{u}_{c,t+1}/[p_{t+1}(1+x_t)]$

where $s_t \equiv (\theta_t, x_t)'$ and obeys (12)–(13). In this notation, (A23)–(A25) can be written

(A34) $E[W(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1})|k_t, s_{t-1}, s_t] = 0$

(A35) $E[q(k_t, k_{t+1}, L_t, n_t, s_t)|k_t, s_{t-1}] = 0$

(A36) $E[Q(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1})|k_t, s_{t-1}, s_t] = 0$.

Define the following linear functions:

(A37) $k_{t+1} = k^0 + k^1 k_t + k^2 s_{t-1} + k^3 s_t$

(A38) $L_t = L^0 + L^1 k_t + L^2 s_{t-1} + L^3 s_t$

(A39) $n_t = n^0 + n^1 k_t + n^2 s_{t-1}$

where k^i, L^i, n^i are (undetermined) scalars for $i = 0$, 1 and 1×2 vectors for $i = 2, 3$.

These 16 undetermined coefficients can be computed by a suitably modified version of steps 1–3 above. In particular, first find the nonstochastic steady-state values of k_t, L_t, and n_t that obtain when x_t and θ_t are held fixed at x and θ. Then compute the first-order Taylor series expansion of W, q, and Q about the nonstochastic steady-state values of their arguments. (Here, it is understood that the nonstochastic steady-state values of x_t and θ_t are x and θ, respectively.) Finally, substitute (A37)–(A39) into the versions of (A34)–(A36) with W, q, and Q replaced by their linear expansions and solve for the undetermined coefficients subject to the transversality-type condition $0 \le k^1 \le \beta^{1/2}$. Here, it is useful to take advantage of the fact that, conditional on a value of k^1, the remaining undetermined coefficients may be found by solving a linear system of equations. For complete details of these computations, including steady-state formulas, see Christiano 1990b (sec. IX).

To simulate an artificial time series for the Fuerst-Lucas model, first draw a sequence of $\varepsilon_{\theta,t}$ and $\varepsilon_{x,t}$ from a random number generator. These, together with (12) and (13), can be used to generate a series of s_t's. Equations (A37)–(A39) can then be used to compute a sequence of k_t's, n_t's, and L_t's. Scaled prices, wages, and consumption may then be computed from $p(\cdot)$, $w(\cdot)$, and $c(\cdot)$ in (A27), (A29), and (A30). Unscaled variables may be obtained from (A13)–(A17). Finally, get R_t from (19).

The Other Cash-in-Advance Models

Solving the basic cash-in-advance and sluggish capital models requires only slight modifications to the preceding procedure. First, it is readily confirmed that those models' efficiency conditions are given by (A34)–(A36) with minor changes in the conditioning set for the expectations operator. Second, the linearized versions of W, q, and Q are identical to those of the Fuerst-Lucas model, since all models share the same nonstochastic steady state.

The efficiency conditions for the basic cash-in-advance model are

(A40) $E[W(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1}) \mid k_t, s_t] = 0$

(A41) $E[q(k_t, k_{t+1}, L_t, n_t, s_t) \mid k_t, s_t] = 0$

(A42) $E[Q(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1}) \mid k_t, s_t] = 0.$

Consider the following decision rules:

(A43) $k_{t+1} = k^0 + k^1 k_t + k^3 s_t$

(A44) $L_t = L^0 + L^1 k_t + L^3 s_t$

(A45) $n_t = n^0 + n^1 k_t + n^3 s_t$

where k^i, L^i, n^i are scalars for $i = 0, 1$ and 1×2 vectors for $i = 3$. Values for k^i, L^i, n^i, $i = 0, 1, 3$, may be obtained by substituting (A43)–(A45) into the version of (A40)–(A42) with W, q, Q replaced by their linear expansions and imposing $0 \leq k^1 \leq \beta^{1/2}$. For details, see Christiano 1990b (sec. X.B).

The efficiency conditions for the sluggish capital model are

(A46) $E[W(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1}) \mid k_t, s_{t-1}] = 0$

(A47) $E[q(k_t, k_{t+1}, L_t, n_t, s_t) \mid k_t, s_{t-1}] = 0$

(A48) $E[Q(k_t, k_{t+1}, k_{t+2}, L_t, L_{t+1}, n_t, n_{t+1}, s_t, s_{t+1}) \mid k_t, s_{t-1}, s_t] = 0.$

Consider the following decision rules:

(A49) $k_{t+1} = k^0 + k^1 k_t + k^2 s_{t-1}$

(A50) $L_t = L^0 + L^1 k_t + L^2 s_{t-1} + L^3 s_t$

(A51) $n_t = n^0 + n^1 k_t + n^2 s_{t-1}.$

Here, as before, the coefficients are found by substituting the decision rules into the efficiency conditions with W, q, Q replaced by their linear expansions and imposing the constraint $0 \leq k^1 \leq \beta^{1/2}$. Details may be found in Christiano 1990b (sec. X.A).

The linear decision rules just discussed, and those underlying the quantitative analysis in the paper, involve approximation error, since they solve the linearized, not the actual, efficiency conditions. Methods for increasing the accuracy of the solution to the models of the paper are discussed in detail in Christiano 1990b. It would be of interest to investigate these to determine whether there is significant approximation error in the solution analyzed here.

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