

FEDERAL RESERVE BANK OF MINNEAPOLIS

QUARTERLY REVIEW

QR

July 2010

Measurement with Minimal Theory

Ellen R. McGrattan

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and Monetary Policy  
in the Search Theory of Money

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FEDERAL RESERVE BANK OF MINNEAPOLIS

## Quarterly Review Vol. 33, No. 1

ISSN 0271-5287

This publication primarily presents economic research aimed at improving policymaking by the Federal Reserve System and other governmental authorities.

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# Asset Prices, Liquidity, and Monetary Policy in the Search Theory of Money\*

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Many financial assets are held not only for the intrinsic value of the stream of consumption that they yield, but also for their usefulness in facilitating exchange. Consider a buyer who cannot commit or be forced to honor debts, and who wishes to make a purchase from a seller. This buyer would find any asset that is valuable to the seller (e.g., an equity share, a bond, money) helpful in carrying out the transaction. For example, the buyer could settle the transaction on the spot by using the asset directly as a means of payment. In some modern transactions, oftentimes the buyer would use a financial asset to enter a repurchase agreement with the seller, or as collateral to borrow the funds needed to pay the seller. Once stripped from the subsidiary contractual complexities, the essence of these transactions is that the asset helps the untrustworthy buyer to obtain what he wants from the seller. In this sense, many financial assets are routinely employed in the exchange process and play a role akin to a medium of exchange. That is, they provide *liquidity*—the term that monetary theorists use to refer to the usefulness of an asset in facilitating transactions.

Financial assets are subject to price fluctuations resulting from aggregate shocks, so to the extent that these assets serve as a source of liquidity, shocks to their

prices will translate into aggregate liquidity shocks that disrupt the mechanism of exchange and the ensuing allocations. Recent developments in financial markets have renewed economists' interest in the idea that fluctuations in asset prices can disrupt the exchange process in some key markets, and through this channel, propagate to the macroeconomy.

Much of the policy advice offered to central banks is framed in terms of simple interest-rate feedback rules loosely motivated by a particular class of models where the preeminent friction is a specific type of reduced-form nominal rigidity. Such policy recommendations are based on the premise that the primary goal of monetary policy is to mitigate the effects of these rigidities. With no room or role for a notion of liquidity (and typically even no meaningful role for money), this conventional view that dominates policy circles has failed to offer relevant policy guidance in the midst of the recent financial crisis. I interpret this failure as an indication that the consensus stance toward monetary policy, with its theoretical focus on sticky-price frictions and its imple-

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\*This article was prepared for the *Proceedings of the Thirty-fourth Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis*. Financial support from the C.V. Starr Center for Applied Economics at New York University is gratefully acknowledged.

mentation emphasis on ad hoc feedback interest-rate rules, is too narrow in that it neglects the fundamental frictions that give rise to a demand for liquidity.

In this article I present a dynamic equilibrium microfounded monetary asset-pricing framework with multiple assets and aggregate uncertainty regarding liquidity needs, and discuss the main normative and positive policy implications of the theory. The broad view that emerges from explicitly modeling the role of money and other liquid assets in the exchange process is that of a monetary authority that seeks to provide the private sector with the liquidity needed to conduct market transactions. Specifically, I state and explain three propositions that answer the following questions: How should monetary policy be conducted in order to mitigate the adverse effects of shocks to the valuations of the financial assets that provide liquidity to the private sector? What are the implications for asset prices of deviating from the optimal monetary policy? Are such deviations capable of causing real asset prices to be above their fundamental values for extended periods of time?

## Model

In order to address the questions just posed, in this section I outline a bare-bones model that encompasses the key economic mechanisms.<sup>1</sup> The model combines elements of the asset-pricing model of Lucas (1978) with elements of the model of monetary exchange of Lagos and Wright (2005). Time is discrete and the horizon infinite. There is a  $[0,1]$  continuum of infinitely lived agents. Each time period is divided into two subperiods where different activities take place. There are three non-storable and perfectly divisible consumption goods at each date: fruit, general goods, and special goods. Fruit and general goods are homogeneous goods, whereas special goods come in many varieties. The only durable commodity in the economy is a set of Lucas trees. The number of trees is fixed and equal to the number of agents. Trees yield a random quantity  $x_t$  of fruit in the second subperiod of every period  $t$ . Each tree yields the same amount of fruit as every other tree, so  $x_t$  is an aggregate shock. The realization of  $x_t$  becomes known at the beginning of period  $t$  (when agents enter the first subperiod). Production of fruit is entirely exogenous: no resources are utilized, and it is not possible to affect the output at any time. The motion of  $x_t$  will be taken to follow a Markov process, defined by its transition function

$F(x', x) = \Pr(x_{t+1} \leq x' | x_t = x)$ . For each fixed  $x$ ,  $F(\cdot, x)$  is a distribution function with support  $\Xi \subseteq (0, \infty)$ .

In each subperiod, every agent is endowed with  $\bar{n}$  units of time which can be employed as labor services. In the second subperiod, each agent has access to a linear production technology that transforms labor services into general goods. In the first subperiod, each agent has access to a linear production technology that transforms his own labor input into a particular variety of the special good that he himself does not consume. This specialization is modeled as follows. Given two agents  $i$  and  $j$  drawn at random, there are three possible events. The probability that  $i$  consumes the variety of special good that  $j$  produces but not vice versa (a single coincidence) is denoted  $\alpha$ . Symmetrically, the probability that  $j$  consumes the special good that  $i$  produces but not vice versa is also  $\alpha$ . In a single-coincidence meeting, the agent who wishes to consume is the *buyer*, and the agent who produces, the *seller*. The probability that neither wants what the other can produce is  $1 - 2\alpha$ , with  $\alpha \leq 1/2$ . Fruit and general goods are homogeneous and hence consumed (and, in the case of general goods, also produced) by all agents.

In the first subperiod, agents participate in a *decentralized market* where trade is bilateral (each meeting is a random draw from the set of pairwise meetings), and the terms of trade are determined by bargaining (a take-it-or-leave-it offer by the buyer, for simplicity). The specialization of agents over consumption and production of the special good, combined with bilateral trade, gives rise to a double-coincidence-of-wants problem in the first subperiod. In the second subperiod, agents trade in a *centralized market*. Agents cannot make binding commitments, and trading histories are private in a way that precludes any borrowing and lending between people, so all trade—in both the centralized and decentralized markets—must be quid pro quo.

Each tree has outstanding one durable and perfectly divisible equity share that represents the bearer's ownership and confers him the right to collect the fruit dividends. There is a second financial asset, money, which is intrinsically useless (it is not an argument of any utility or production function), and unlike equity, ownership of money does not constitute a right to collect any resources. Money is issued by a "government" that

<sup>1</sup>The analysis that follows is based on Lagos (2009, 2010). See also Lagos (2006).

at  $t=0$  commits to a *monetary policy* represented by a sequence of positive real-valued functions,  $\{\mu_t\}_{t=0}^\infty$ . Given an initial stock of money,  $M_0 > 0$ , a monetary policy induces a *money supply process*,  $\{M_t\}_{t=0}^\infty$ , via  $M_{t+1} = \mu_t(x^t)M_t$ , where  $x^t$  denotes a history of realizations of fruit dividends through period  $t$ , i.e.,  $x^t = (x_t, x_{t-1}, \dots, x_0)$ . The government injects or withdraws money via lump-sum transfers or taxes in the second subperiod of every period, i.e., along every sample path,  $M_{t+1} = M_t + T_t$ , where  $T_t$  is the lump-sum transfer (or tax, if negative). All assets are perfectly recognizable, cannot be forged, and can be traded among agents in both the centralized and decentralized markets. At  $t=0$  each agent is endowed with  $a_0^s$  equity shares and  $a_0^m$  units of fiat money.

Let the utility function for special goods,  $u: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ , and the utility function for fruit,  $U: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ , be continuously differentiable, bounded by  $B$  on  $\Xi$ , increasing, and strictly concave, with  $u(0) = U(0) = 0$ . Let  $-n$  be the utility from working  $n$  hours in the first subperiod. Also, suppose there exists  $q^* \in (0, \infty)$  defined by  $u'(q^*) = 1$ , with  $q^* \leq \bar{n}$ . Let the utility for general goods, and the disutility from working in the second subperiod, be linear. The agent prefers a consumption and labor sequence  $\{q_t, n_t, c_t, y_t, h_t\}_{t=0}^\infty$  over another sequence  $\{\tilde{q}_t, \tilde{n}_t, \tilde{c}_t, \tilde{y}_t, \tilde{h}_t\}_{t=0}^\infty$  if

$$\liminf_{T \rightarrow \infty} E_0 \left\{ \sum_{t=0}^T \beta^t [u(q_t) - n_t + U(c_t) + y_t - h_t] - \beta^t [u(\tilde{q}_t) - \tilde{n}_t + U(\tilde{c}_t) + \tilde{y}_t - \tilde{h}_t] \right\} \geq 0,$$

where  $\beta \in (0, 1)$ ,  $q_t$ , and  $n_t$  are the quantities of special goods consumed and produced in the decentralized market,  $c_t$  denotes consumption of fruit,  $y_t$  consumption of general goods, and  $h_t$  the hours worked in the second subperiod. Here,  $E_t$  is an expectations operator conditional on the information available to the agent at time  $t$ , defined with respect to the matching probabilities and the probability measure induced by  $F$ .

In the remainder of this section, I describe the individual optimization problem that each agent faces, and then define an equilibrium for this economy. It is convenient to analyze the decisions that an agent has to make in any given period by working backward from the end of the period: I first consider the agent's problem in

the centralized market, I then explain the determination of the terms of trade in a bilateral meeting, and finally, I provide an expression for the agent's value function when he enters the decentralized market before the round of bilateral trade. Let  $\mathbf{a}_t = (a_t^s, a_t^m)$  denote the portfolio of an agent who holds  $a_t^s$  shares and  $a_t^m$  units of money. Let  $W_t(\mathbf{a}_t)$  and  $V_t(\mathbf{a}_t)$  be the maximum attainable expected discounted utility of an agent when he enters the centralized and decentralized markets, respectively, at time  $t$  with portfolio  $\mathbf{a}_t$ . Then,

$$\begin{aligned} (1) \quad W_t(\mathbf{a}_t) &= \max_{c_t, y_t, h_t, \mathbf{a}_{t+1}} \{U(c_t) + y_t - h_t + \beta E_t V_{t+1}(\mathbf{a}_{t+1})\} \\ &\text{s.t. } c_t + w_t y_t + \phi_t^s a_{t+1}^s + \phi_t^m a_{t+1}^m \\ &\quad = (\phi_t^s + x_t) a_t^s + \phi_t^m (a_t^m + T_t) + w_t h_t \\ &\quad 0 \leq c_t, 0 \leq h_t \leq \bar{n}, \mathbf{0} \leq \mathbf{a}_{t+1}. \end{aligned}$$

That is, when he enters the centralized market of period  $t$  with portfolio  $\mathbf{a}_t$ , the agent chooses consumption of fruit ( $c_t$ ), consumption of general goods ( $y_t$ ), labor supply ( $h_t$ ), and an end-of-period portfolio ( $\mathbf{a}_{t+1}$ ) in order to maximize his expected discounted utility. Fruit is used as the numéraire:  $w_t$  is the relative price of the general good,  $\phi_t^s$  is the (ex-dividend) price of a share, and  $1/\phi_t^m$  is the dollar price of fruit.

Next, consider a meeting in the decentralized market of period  $t$  between a buyer with portfolio  $\mathbf{a}_t$  and a seller with portfolio  $\tilde{\mathbf{a}}_t$ . Let  $[q_t, (\mathbf{a}, \tilde{\mathbf{a}}), \mathbf{p}_t(\mathbf{a}, \tilde{\mathbf{a}})]$  denote the terms at which a buyer who owns portfolio  $\mathbf{a}$  trades with a seller who owns portfolio  $\tilde{\mathbf{a}}$ , where  $q_t(\mathbf{a}, \tilde{\mathbf{a}}) \in \mathbb{R}^+$  is the quantity of special good traded, and  $\mathbf{p}_t(\mathbf{a}, \tilde{\mathbf{a}}) = [p_t^s(\mathbf{a}, \tilde{\mathbf{a}}), p_t^m(\mathbf{a}, \tilde{\mathbf{a}})] \in \mathbb{R}^+ \times \mathbb{R}^+$  is the transfer of assets from the buyer to the seller (the first argument is the transfer of equity). As mentioned earlier, the terms of trade,  $(q_t, \mathbf{p}_t)$ , are determined by the buyer, who makes a take-it-or-leave-it offer to the seller. In order to choose his offer, the buyer solves

$$\begin{aligned} &\max_{q_t, \mathbf{p}_t \leq \mathbf{a}_t} [u(q_t) + W_t(\mathbf{a}_t - \mathbf{p}_t) - W_t(\mathbf{a}_t)] \\ &\text{s.t. } W_t(\tilde{\mathbf{a}}_t + \mathbf{p}_t) - q_t \geq W_t(\tilde{\mathbf{a}}_t). \end{aligned}$$

The constraints  $\mathbf{p}_t \leq \mathbf{a}_t$  specify that the buyer cannot spend more assets than he owns, and the other constraint ensures that the buyer's offer is acceptable to the

seller. Let  $\lambda_t = (\lambda_t^s, \lambda_t^m)$ , with  $\lambda_t^s \equiv (1/w_t)(\phi_t^s + x_t)$  and  $\lambda_t^m \equiv (1/w_t)\phi_t^m$ . Intuitively,  $\lambda_t^s$  is the period  $t$  real cum-dividend value of equity, and  $\lambda_t^m$  is the period  $t$  real value of a unit of money, both expressed in terms of period  $t$  general goods. The bargaining outcome is as follows. If  $\lambda_t \mathbf{a}_t \geq q^*$ , the buyer buys  $q_t = q^*$  in exchange for a vector  $\mathbf{p}_t$  of assets with real value  $\lambda_t \mathbf{p}_t = q^* \leq \lambda_t \mathbf{a}_t$ . Otherwise, the buyer pays the seller  $\mathbf{p}_t = \mathbf{a}_t$  in exchange for  $q_t = \lambda_t \mathbf{a}_t$ . Hence, the quantity of special good exchanged is  $\min(\lambda_t \mathbf{a}_t, q^*) \equiv q(\lambda_t \mathbf{a}_t)$ , and the real value (expressed in terms of general goods) of the portfolio used as payment is  $\lambda_t \mathbf{p}_t(\mathbf{a}_t, \tilde{\mathbf{a}}_t) = q(\lambda_t \mathbf{a}_t)$ .

With the bargaining solution, the value of search to an agent who enters the decentralized market of period  $t$  with portfolio  $\mathbf{a}_t$  can be written as

$$V_t(\mathbf{a}_t) = \Gamma(\lambda_t \mathbf{a}_t) + W_t(\mathbf{a}_t),$$

where  $\Gamma(x) \equiv \alpha \{u[q(x)] - q(x)\}$  is the expected gain from trade in the decentralized market. Substitute the budget constraint (1) and  $V_t(\mathbf{a}_t)$  into the right side of  $W_t(\mathbf{a}_t)$  to arrive at

$$(2) \quad W_t(\mathbf{a}_t) = \lambda_t \mathbf{a}_t + \tau_t + \max_{c_t \geq 0} \left[ U(c_t) - \frac{c_t}{w_t} \right] \\ + \max_{\mathbf{a}_{t+1} \geq 0} \left\{ -\phi_t \mathbf{a}_{t+1} / w_t \right. \\ \left. + \beta E_t [\Gamma(\lambda_{t+1} \mathbf{a}_{t+1}) + W_{t+1}(\mathbf{a}_{t+1})] \right\},$$

where  $\tau_t = \lambda_t^m T_t$  and  $\phi_t = (\phi_t^s, \phi_t^m)$ . This expression provides a simple recursive representation of the maximum attainable expected discounted utility of an agent who enters the centralized market of period  $t$  with portfolio  $\mathbf{a}_t$ .

Given a process  $\{M_t\}_{t=0}^\infty$ , a (symmetric) equilibrium is a plan  $\{c_t, \mathbf{a}_{t+1}\}_{t=0}^\infty$ , pricing functions  $\{w_t, \phi_t\}_{t=0}^\infty$ , and bilateral terms of trade  $\{q_t, \mathbf{p}_t\}_{t=0}^\infty$  such that: (i) given prices and the bargaining protocol,  $\{c_t, \mathbf{a}_{t+1}\}_{t=0}^\infty$  solves the agent's optimization problem on the right side of (2); (ii) the bilateral terms of trade are  $q_t = \min(\lambda_t \mathbf{a}_t, q^*)$  and  $\lambda_t \mathbf{p}_t = q_t$ ; and (iii) the centralized market clears, i.e.,  $c_t = x_t$ , and  $a_{t+1}^s = 1$ . The equilibrium is *monetary* if  $\phi_t^m > 0$  for all  $t$ , and in this case the money-market-clearing condition is  $a_{t+1}^m = M_{t+1}$ . The market-clearing conditions imply  $\{c_t, a_{t+1}^s, a_{t+1}^m\}_{t=0}^\infty = \{x_t, 1, M_{t+1}\}_{t=0}^\infty$ ,  $w_t = 1/U'(x_t)$ , and once  $\{\phi_t\}_{t=0}^\infty$  has been found,  $\{q_t\}_{t=0}^\infty = \{\lambda_t \mathbf{p}_t\}_{t=0}^\infty = \{\min(\Lambda_{t+1}, q^*)\}_{t=0}^\infty$ , where  $\Lambda_{t+1} \equiv \lambda_{t+1}^s +$

$\lambda_{t+1}^m M_{t+1}$  is the real value of the equilibrium portfolio that each agent brings into the decentralized market of period  $t+1$ . Therefore, given a money supply process  $\{M_t\}_{t=0}^\infty$ , and letting  $L(\Lambda_{t+1}) \equiv [1 + \Gamma'(\Lambda_{t+1})]$ , a monetary equilibrium can be summarized by a sequence  $\{\phi_t\}_{t=0}^\infty$  that satisfies the following necessary and sufficient conditions for individual optimization:

$$U'(x_t)\phi_t^s = \beta E_t[L(\Lambda_{t+1})U'(x_{t+1})(\phi_{t+1}^s + x_{t+1})]$$

$$U'(x_t)\phi_t^m = \beta E_t[L(\Lambda_{t+1})U'(x_{t+1})\phi_{t+1}^m]$$

$$\lim_{t \rightarrow \infty} E_0[\beta^t U'(x_t)\phi_t^s] = 0$$

$$\lim_{t \rightarrow \infty} E_0[\beta^t U'(x_t)\phi_t^m M_{t+1}] = 0.$$

The last two conditions are the transversality conditions for equity and money. The first and second conditions are the Euler equations for equity and money, respectively. The fact that both assets can be used as a medium of exchange in bilateral trades implies that the usual stochastic discount factor is augmented by the liquidity factor  $L(\Lambda_{t+1})$ , which captures the liquidity value of the asset, i.e., the degree to which the asset is useful as a medium of exchange, at the margin.

### Normative Results: Optimal Policy and Implementation

The Pareto optimal allocation in this environment can be found by solving the problem of a social planner who wishes to maximize average (equally weighted across agents) expected utility. The planner chooses a plan  $\{c_t, q_t, n_t, y_t, h_t\}_{t=0}^\infty$  subject to the feasibility constraints, i.e.,  $0 \leq c_t \leq d_t$ ,  $y_t \leq h_t$ , and  $0 \leq q_t \leq n_t$  for those agents who are matched in the first subperiod of period  $t$  and  $q_t = n_t = 0$  for those agents who are not. Under these constraints, the planner's problem consists of finding a feasible plan  $\{c_t, q_t\}_{t=0}^\infty$  such that for all feasible plans  $\{\tilde{c}_t, \tilde{q}_t\}_{t=0}^\infty$ ,

$$\liminf_{T \rightarrow \infty} E_0 \left\{ \sum_{t=0}^T \beta^t \{ \alpha [u(q_t) - q_t] + U(c_t) \} \right. \\ \left. - \beta^t \{ \alpha [u(\tilde{q}_t) - \tilde{q}_t] + U(\tilde{c}_t) \} \right\} \geq 0.$$

Here,  $E_0$  denotes the expectation with respect to the

probability measure over sequences of dividend realizations induced by  $F$ . The solution to the planner's problem is  $\{c_t, q_t\}_{t=0}^{\infty} = \{x_t, q^*\}_{t=0}^{\infty}$ .

In equilibrium, market clearing implies  $c_t = x_t$ , i.e., equilibrium consumption of fruit is at the efficient level. However, the equilibrium allocation has  $q_t \leq q^*$ , which may hold with strict inequality in some states. That is, equilibrium consumption and production in the decentralized market may be below their efficient levels. In this context, the role of monetary policy is to ensure that the value of money is large enough to allow for  $q_t = q^*$  with probability one, for all  $t$ .

To state the results that follow, it will be convenient to introduce the following notion of *nominal interest rate*. Imagine that there existed an additional asset in this economy, an *illiquid nominal bond*, i.e., a one-period risk-free government bond that pays a unit of money in the centralized market and which cannot be used in decentralized exchange. Let  $\phi_t^n$  denote the price of this asset. In equilibrium, this price must satisfy  $U'(x_t)\phi_t^n = \beta E_t[U'(x_{t+1})\phi_{t+1}^m]$ . Since  $\phi_t^n / \phi_t^m$  is the money price of a nominal bond, the (net) nominal interest rate in a monetary equilibrium is  $i_t = \phi_t^m / \phi_t^n - 1$ , or equivalently,

$$(3) \quad i_t = \frac{E_t[L(\Lambda_{t+1})\lambda_{t+1}^m]}{E_t(\lambda_{t+1}^m)} - 1.$$

**PROPOSITION 1.** *Equilibrium quantities in a monetary equilibrium are Pareto optimal if and only if  $i_t = 0$  with probability one for all  $t$ , in the monetary equilibrium.*

Proposition 1 establishes the optimality of the *Friedman rule*—Milton Friedman's (1969) prescription that monetary policy should induce a zero nominal interest rate in order to lead to an optimal allocation of resources. The proof is as follows. The equilibrium allocation is efficient if and only if  $q_t(\Lambda_t) = q^*$ , and this equality holds if and only if  $\Lambda_t \geq q^*$ , i.e., if and only if the real value of the equilibrium portfolio,  $\Lambda_t$ , is at least as large as the real liquidity needs, represented by  $q^*$ . The nominal interest rate,  $i_t$ , is zero if and only if  $L(\Lambda_{t+1}) = 1$ , and this equality holds if and only if  $\Lambda_t \geq q^*$ . Hence,  $q_t(\Lambda_t) = q^*$  if and only if  $i_t = 0$ . Intuitively, the cost of producing real balances is zero to the government, so the optimum quantity of real balances should be such that the marginal benefit—which in equilibrium equals the marginal cost,  $i_t$ —is zero to the economic agents.

I next turn to the question of implementation: Which monetary policies are consistent with a monetary equilibrium in which the nominal interest rate is at its optimal target level of zero? The following result addresses the issue of (weak) implementation by characterizing a family of monetary policies that are consistent with an equilibrium with  $i_t = 0$  for all  $t$ .

**PROPOSITION 2.** *Let  $\phi_t^{s*} = E_t \sum_{j=1}^{\infty} \beta^j [U'(x_{t+j})/U'(x_t)] x_{t+j}$ ,  $\lambda_t^{s*} = U'(x_t)(\phi_t^{s*} + x_t)$ , and  $\mathcal{T}$  be the set of dates for which  $q^* - \lambda_t^{s*} > 0$  holds with probability  $\pi_t > 0$ . Assume that  $\inf_{t \in \mathcal{T}} \pi_t > 0$ . A monetary equilibrium with  $i_t = 0$  with probability one for all  $t$  exists under a deterministic money supply process  $\{M_t\}_{t=0}^{\infty}$  if and only if the following two conditions hold:*

- (4)  $\lim_{t \rightarrow \infty} M_t = 0$
- (5)  $\inf_{t \in \mathcal{T}} M_t \beta^{-t} > 0$  if  $\mathcal{T} \neq \emptyset$ .

Conditions (4) and (5) are rather unrestrictive asymptotic conditions. The first one requires that the money supply converges to zero. The second condition requires that asymptotically, on average over the set of dates  $\mathcal{T}$  when fiat money plays an essential role, the growth rate of the money supply must be at least as large as the discount factor. Versions of this result have been proven by Wilson (1979) and Cole and Kocherlakota (1998) for deterministic competitive economies with cash-in-advance constraints that are imposed on agents every period with probability one.

Proposition 2 has several implications. First, even though liquidity needs are stochastic in this environment (because equity, whose value is stochastic, can be used alongside money as a means of payment), a deterministic money supply sequence can suffice to implement a zero nominal rate in every state of the world. Second, even within the class of deterministic monetary policies, there is a large family of policies that can implement the Pareto optimal equilibrium. Finally, it would be impossible for someone with access to a finite time series for the path of the money supply to determine whether an optimal monetary policy is being followed. On the other hand, a single observation of a positive nominal rate constitutes definitive evidence of a deviation from an optimal monetary policy.

### Positive Results: Interest-Rate Targets and Asset Prices

In this section I consider perturbations of the optimal monetary policy that consist of targeting a constant positive nominal interest rate. I also discuss some of the positive implications of changes in the nominal interest-rate target for the inflation rate, equity prices, and equity returns. To this end, it is convenient to focus on a recursive formulation in which prices are invariant functions of the aggregate state  $s_t = (x_t, M_t)$ , i.e.,  $\phi_t^s = \phi^s(s_t)$ ,  $\phi_t^m = \phi^m(s_t)$ ,  $\lambda_t = (\lambda^s(s_t), \lambda^m(s_t))$ , where  $\lambda^s(s_t) = U'(x_t)[\phi^s(s_t) + x_t]$  and  $\lambda^m(s_t) = U'(x_t)\phi^m(s_t)$ , and  $\Lambda_t = \lambda^s(s_t) + \lambda^m(s_t)M_t$ . Also, restrict attention to stationary monetary policies, i.e.,  $\mu: \Xi \rightarrow \mathbb{R}^+$ , so that  $M_{t+1} = \mu(x_t)M_t$ . To illustrate the main ideas as simply as possible, the following proposition specializes the analysis to the case of independently and identically distributed dividends and liquidity constraints that would bind with probability one at every date in the absence of money.

**PROPOSITION 3.** *Assume  $dF(x', x) = dF(x)$ . Let  $l(\delta) = 1 - \alpha + \alpha u'(\delta q^*)$  and  $\underline{\delta}$  be defined by  $l(\underline{\delta}) = 1/\beta$ . Let  $\delta_0 \in (\underline{\delta}, 1)$  be given, and suppose that  $B \leq [1 - \beta l(\delta_0)]\delta_0 q^*$ . Then for any  $\delta \in [\delta_0, 1]$ , there exists a recursive monetary equilibrium under the monetary policy*

$$(6) \quad \mu(x; \delta) = \frac{\delta q^* - \frac{1}{1 - \beta l(\delta)} \int x' U'(x') dF(x')}{\delta q^* - \frac{\beta l(\delta)}{1 - \beta l(\delta)} \int x' U'(x') dF(x') - x U'(x)}.$$

The equilibrium prices of equity and money are

$$(7) \quad \phi^s(x; \delta) = \frac{\beta l(\delta)}{1 - \beta l(\delta)} \frac{\int x' U'(x') dF(x')}{U'(x)}$$

$$(8) \quad \phi^m(s; \delta) = \frac{\delta q^* - \frac{\beta l(\delta)}{1 - \beta l(\delta)} \int x' U'(x') dF(x') - x U'(x)}{U'(x)M}$$

Together with (3), the asset prices (7) and (8) imply that the monetary policy (6) induces an equilibrium gross

nominal interest rate that is constant (independent of  $s$ ) and equal to  $l(\delta) \geq 1$  (with equality only if  $\delta = 1$ ). The function  $\mu(\cdot; \delta)$  defines a class of monetary policies indexed by the parameter  $\delta$ , which effectively determines the level of the constant nominal interest rate implemented by the policy. According to (8), real money balances and the value of money are decreasing in the nominal interest-rate target (increasing in  $\delta$ ). According to (7), under the proposed policy, the real price of equity is increasing in the nominal interest-rate target (decreasing in  $\delta$ ). As  $\delta \rightarrow 1$ ,  $l(\delta) \rightarrow 1$ , and therefore (according to Proposition 1) the policy  $\mu(x; \delta)$  approaches an optimal policy under which the recursive monetary equilibrium decentralizes the Pareto optimal allocation.

Notice that  $\phi^s(x; 1)$  is the equilibrium equity price that would result in a Lucas (1978) economy with no liquidity needs. Therefore, the fact that  $\phi^s(x; 1) < \phi^s(x; \delta)$  for all  $x$  and any  $\delta \in [\delta_0, 1)$  implies that deviations from the optimal policy raise real asset prices above the value that a financial analyst would calculate based on the expected stream of dividends discounted by the stochastic discount factor of Lucas' model,  $\beta U'(x_{t+1})/U(x_t)$ .

On average, liquidity considerations generate a negative relationship between the nominal interest rate (and the inflation rate) and equity returns: if the target nominal rate,  $l(\delta) - 1$ , is higher, the average inflation rate is higher, real money balances are lower, and the liquidity return on equity rises, which causes its price to rise and its measured real rate of return to fall. Intuitively, a higher nominal interest-rate target implies that buyers are on average short of liquidity, so equity becomes more valuable as it is used by buyers to relax their trading constraints. This additional liquidity value causes the real financial return on equity to be lower, on average, at a higher interest rate.

Proposition 3 also shows explicitly how monetary policy must be conducted in order to support a recursive monetary equilibrium with a constant nominal interest rate (with the Pareto optimal equilibrium in which the nominal rate is zero as a special case): the growth rate of the money supply must be relatively low following states in which the real value of the equilibrium equity holdings is below average. Equivalently, the implied inflation rate will be relatively low between state  $x$  and a next-period state  $x'$ , if the realized real value of the equilibrium equity holdings in state  $x$  is below the state- $x$  conditional expectation of its value next period.

## Conclusion

I have presented a simple version of a prototypical search-based monetary model in which money coexists with a financial asset that yields a risky real return. In this formulation, money is not assumed to be the only asset that must, nor the only asset that can, play the role of a medium of exchange: nothing in the environment prevents agents from using equity along with money, or instead of money, as a means of payment. Since the equity share is a claim to a risky aggregate endowment, the fact that agents can use equity to finance purchases implies that they face aggregate liquidity risk, in the sense that in some states of the world, the value of equity holdings may turn out to be too low relative to what would be needed to carry out the transactions that require a medium of exchange. This seems like a natural starting point to study the role of money and monetary policy in providing liquidity to lubricate the mechanism of exchange in modern economies.

In this context, I characterized a large family of optimal monetary policies. Every policy in this family implements Friedman's prescription of zero nominal interest rates. Under an optimal policy, equity prices and returns are independent of monetary considerations. I have also studied a class of monetary policies that target a constant but nonzero nominal interest rate. For this perturbation of the family of optimal policies, I found that the model articulates the idea that, to the extent that a financial asset is valued as a means to facilitate transactions, its real rate of return will include a liquidity return that depends on monetary considerations. As a result of this liquidity channel, persistent deviations from the optimal monetary policy will cause the real prices of assets that can be used to relax borrowing or other trading constraints to exhibit persistent deviations from their fundamental values.

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