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A NOTE ON LABOR CONTRACTS WITH PRIVATE INFORMATION AND HOUSEHOLD PRODUCTION

Ed Nosal*

University of Waterloo

Richard Rogerson*

Stanford University

Randall Wright*

University of Pennsylvania and
Federal Reserve Bank of Minneapolis

ABSTRACT

A classic result in the theory of implicit contract models with asymmetric information is that "underemployment" results if and only if leisure is an inferior good. We introduce household production into the standard implicit contract model and show that we can have underemployment at the same time that leisure is a normal good.

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I. Introduction

Labor contract theory, originating with the work of Azariadis (1975), Baily (1974) and others, has developed into a useful model of the allocation of resources between risk averse workers and risk neutral employers. These models have also incorporated asymmetric information, in the sense that while the contract can be made contingent on random economic conditions, only the employer, not the workers, sees the ex post realization of these conditions. Proponents of this framework hoped it might deliver the implication that workers were *underemployed*, ex post, in the sense that the marginal product of labor was above the marginal rate of substitution between consumption and leisure. A classic result, however, is that if leisure is a normal good, then underemployment cannot be part of an efficient contract.[1] As Green and Kahn put it, "Such a one-period implicit contracting model cannot, therefore, be used to 'explain' [underemployment] as a rational byproduct of risk sharing between workers and a risk-neutral firm under conditions of asymmetric information." (p. 173).

The goal here is to show that underemployment *can* occur in a model with risk neutral firms and risk averse workers who have normal leisure, once household production is explicitly introduced. We consider labor contracts with asymmetric information in a version of Becker's (1965) home production framework advocated by Gronau (1980, 1985), and used recently by Benhabib, Rogerson and Wright (1990a, 1990b) and by Greenwood and Hercowitz (1990).

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[1] See Green and Kahn (1983), Chari (1983) or Cooper (1983); the presentation here will follow Green and Kahn. Grossman and Hart (1981) and Azariadis (1983) do show how to overturn this result in models where the wealth effect on worker's leisure is zero, as long as the employer is sufficiently risk averse; in this paper, we stick with risk neutral firms.

Green and Kahn's analysis is extended to show that underemployment (overemployment) still requires hours of market labor increase (decrease) with an increase in exogenous wealth. However, in the home production model, market hours can increase with wealth and thus we can have underemployment even though leisure is a normal good in the sense that total (market plus household) labor hours decrease with wealth. Furthermore, even though market hours may increase with wealth, we show that our model is still consistent with the long run evidence on productivity and hours.

II. The Green and Kahn Model

There is a fixed number of homogeneous workers, with mass normalized to unity, who each have a von Neuman - Morgenstern utility function defined over consumption and hours worked, $u(c,h)$. We assume $u_1 > 0$, $u_2 < 0$ and strict concavity, and we let $\mu = -u_2/u_1$ denote the marginal rate of substitution. For future reference, note that leisure is a normal good if and only if $\eta \equiv \mu u_{11}+u_{12} < 0$. There is a single risk neutral firm with technology $y = \theta f(h)$, where $f' > 0$, $f'' < 0$, and θ is a random variable. For simplicity, we assume θ has a strictly positive p.d.f. $p(\theta)$ on its support $\Theta = [\underline{\theta},\bar{\theta}]\subset\mathbb{R}_{++}$. The distribution of θ is known by both the workers and the firm, but only the latter can observe the actual realization. An optimal contract $[h(\theta),c(\theta)]$ maximizes expected utility subject to the constraint that expected profit cannot fall below some reservation level, which we normalize to 0, and the incentive compatibility constraint that implies the firm will reveal the true state.

Let $\Pi(\alpha|\theta) = \theta f[h(\alpha)] - c(\alpha)$ denote profit when the true state is θ and the firm announces α, and let $\pi(\theta) = \Pi(\theta|\theta)$. The incentive compatibility constraint is that $\alpha = \theta$ maximizes $\Pi(\alpha|\theta)$ for all θ. Rather than impose this directly, we impose the first and second order conditions for $\alpha = \theta$ to maximize $\Pi(\alpha|\theta)$: for all θ,

$$\partial\Pi/\partial\alpha_{|\alpha=\theta} = \theta f'h' - c' = 0, \tag{1}$$

$$\partial^2\Pi/\partial\alpha^2{}_{|\alpha=\theta} = \theta f''h'^2 + \theta f'h'' - c'' \le 0. \tag{2}$$

Since (1) is an identity in θ, we can differentiate it with respect to θ, solve for c'', and insert the result into (2). Doing so implies that (2) simplifies to $h' \ge 0$. As in Green and Kahn, we will assume here that this constraint is not binding, so that $h' > 0$. Therefore, we simply maximize Eu subject to $E\pi \ge 0$ and (1).

This is a standard variational problem. The Lagrangian is

$$\mathcal{L} = \int\Big\{u[c(\theta),h(\theta)] + \lambda\theta f[h(\theta)] - \lambda c(\theta)$$
$$- \varphi(\theta)\theta f'[h(\theta)]h'(\theta) + \varphi(\theta)c'(\theta)\Big\}p(\theta)d\theta,$$

where λ is the multiplier on the constraint $E\pi \ge 0$, and $\varphi(\theta)$ is the multiplier on (1) in each state θ. Under standard regularity conditions, $h(\theta)$, $c(\theta)$ and $\varphi(\theta)$ will be smooth functions that are fully characterized by the Euler equations, transversality conditions and constraints. The Euler equations are $\mathcal{L}_{h(\theta)} = \frac{d}{d\theta}\mathcal{L}_{h'(\theta)}$ and $\mathcal{L}_{c(\theta)} = \frac{d}{d\theta}\mathcal{L}_{c'(\theta)}$, which simplify to

$$u_2 + \lambda\theta f' = -\varphi f' - \theta\varphi' f' \tag{3}$$

$$u_1(c,h) - \lambda = \varphi', \tag{4}$$

while the transversality conditions are $\varphi(\underline{\theta}) = \varphi(\overline{\theta}) = 0$.

Notice (3) and (4) together imply $\theta f' = \mu - \varphi f'/u_1$, where $\mu = -u_2/u_1$. Define *overemployment* to be the case where $\theta f' < \mu$, and *underemployment* to be the case where $\theta f' > \mu$. Then we have overemployment (underemployment) in a given state if and only if φ is positive (negative). We also have the following results.

Lemma 1: (a) If there exists θ such that $\varphi(\theta) < 0$, then there exists $\hat{\theta}\in(\underline{\theta},\overline{\theta})$ such that $\varphi(\hat{\theta}) < 0$ and $\varphi''(\hat{\theta}) \geq 0$. (b) If there exists θ such that $\varphi(\theta) > 0$, then there exists $\hat{\theta}\in(\underline{\theta},\overline{\theta})$ such that $\varphi(\hat{\theta}) > 0$ and $\varphi''(\hat{\theta}) \leq 0$.

Proof: If $\varphi(\theta) < 0$, then since $\varphi(\underline{\theta}) = \varphi(\overline{\theta}) = 0$, φ must achieve a local minimum at some $\hat{\theta}\in(\underline{\theta},\overline{\theta})$, where $\varphi(\hat{\theta}) < 0$, $\varphi'(\hat{\theta}) = 0$ and $\varphi''(\hat{\theta}) \geq 0$. This proves part (a); part (b) is symmetric. ∎

Proposition 1: (a) If $\eta = \mu u_{11}+u_{12} < 0$, then we cannot have underemployment in any state θ. (b) If $\eta > 0$, then we cannot have overemployment in any state θ.

Proof: Begin by differentiating condition (4) to yield $\varphi'' = u_{11}c' + u_{12}h' = (u_{11}\theta f'+u_{12})h'$. If we simplify further,

$$\varphi'' = \eta h' + (\theta f'-\mu)u_{11}h', \tag{5}$$

where $h' > 0$ as long as (2) is not binding, as we are assuming here. Suppose $\eta < 0$, which by (5) implies $\varphi'' < 0$ for all states in which we have underemployment. But Lemma 1 (b) tells us that if there is any state with underemployment then we must have some state with underemployment and $\varphi'' \geq 0$. This contradiction establishes part (a); part (b) is similar. ∎

Recalling $\eta < 0$ if and only if leisure is a normal good, Proposition 1 says that normal leisure and underemployment cannot coexist. The following corollary indicates that normal leisure implies workers prefer low θ states.

Corollary: Let $w(\theta) = u[c(\theta),h(\theta)]$; then $\eta < 0$ implies $w' < 0$, while $\eta > 0$ implies $w' > 0$.

Proof: Differentiation yields $w' = (c'-\mu h')u_1 = (\theta f'-\mu)u_1 h'$. By Proposition 1 (a), $\eta < 0$ implies $\theta f' < \mu$ and hence $w' < 0$, and vice-versa. ■

III. The Home Production Model

Now assume worker utility is defined over consumption of a market good, a home or nonmarket good, hours worked in the market, and hours worked in home or nonmarket activity, $U(c_m,c_n,h_m,h_n)$. We assume U_1, $U_2 > 0$, U_3, $U_4 < 0$, and strict concavity. The home production constraint is $c_n = g(h_n)$, where $g' > 0$. This constraint simply says that home produced goods must be produced in the home. Let $\mu_m = -U_3/U_1$ and $\mu_n = -U_4/U_2$ be the marginal rates of substitution between consumption and labor in the market and home, and let $\eta_m \equiv \mu_m U_{11}+U_{13}$ and $\eta_n \equiv \mu_n U_{12}+U_{14}$. The firm is the same as above and an optimal contract $[h_m(\theta),h_n(\theta),c_m(\theta),c_n(\theta)]$ still maximizes EU subject to $E\pi \geq 0$ and the incentive constraint. We again represent the incentive constraint by the first order condition $\theta f'(h_m)h_m' - c_m' = 0$, and assume that the second order condition does not bind, which means that $h_m' > 0$.

The Lagrangian is

$$\mathcal{L} = \int\Big\{U[c_m(\theta),g\circ h_n(\theta),h_m(\theta),h_n(\theta)] + \lambda\theta f[h_m(\theta)] - \lambda c_m(\theta) - \varphi(\theta)\theta f'[h_m(\theta)]h_m'(\theta) + \varphi(\theta)c_m'(\theta)\Big\}p(\theta)d\theta,$$

where we have substituted the constraint $c_n(\theta) = g \circ h_n(\theta)$ directly into the utility function. The Euler equations with respect to h_m, h_n and c_m are

$$U_3 + \lambda\theta f' = - \varphi f' - \theta\varphi' f' \tag{6}$$

$$U_2 g' + U_4 = 0 \tag{7}$$

$$U_1 - \lambda = \varphi', \tag{8}$$

while the transversality conditions are $\varphi(\underline{\theta}) = \varphi(\overline{\theta}) = 0$. Notice (7) implies $g' = \mu_n$, while (6) and (8) imply $\theta f' = \mu_m - \varphi f'/U_1$. As in the previous section, the latter indicates that we have overemployment (underemployment) in the market sector if and only if φ is positive (negative).

Lemma 1 in the previous section holds exactly as stated in this model. We also have the following new version of Proposition 1 and its corollary.

Proposition 2: (a) If $\eta_m h_m' + \eta_n h_n' < 0$, then we cannot have underemployment in any state θ. (b) If $\eta_m h_m' + \eta_n h_n' > 0$, then we cannot have overemployment in any state θ.

Proof: Differentiating (8) and simplifying yields

$$\varphi'' = \eta_m h_m' + \eta_n h_n' + (\theta f' - \mu_m) U_{11} h_m', \tag{9}$$

which should be compared with (5). The rest of the argument follows the proof of Proposition 1 exactly. ∎

Corollary: Let $W(\theta) = U[c_m(\theta), g \circ h_n(\theta), h_m(\theta), h_n(\theta)]$; then $\eta_m h_m' + \eta_n h_n' < 0$ implies $W' < 0$, while $\eta_m h_m' + \eta_n h_n' > 0$ implies $W' > 0$.

Proof: Differentiate and use Proposition 2. ∎

Although these natural extensions of the Green and Kahn results hold in the home production model, the relation between the sign of $\eta_m h'_m + \eta_n h'_n$ and normal leisure is not straightforward. Our goal is to show in our model that underemployment can coexist with normal leisure, where normal leisure means that total hours of work, $H = h_m + h_n$, decreases with an increase in exogenous wealth. Normal leisure does not require that h_m and h_n both decrease with wealth, only that the sum decreases.

We can make the case most efficiently using an indirect approach. To this end, define the reduced form utility function

$$V(c_m,h_m) = \max_{h_n} U[c_m(\theta), g\circ h_n(\theta), h_m(\theta), h_n(\theta)]. \tag{10}$$

By the envelope theorem, $V_1 = U_1 > 0$ and $V_2 = U_3 < 0$. It can also be shown that $V(\cdot)$ is strictly concave (see Benhabib, Rogerson and Wright 1990a). Therefore it defines a well-behaved preference ordering over (c_m,h_m). Clearly the solution to the contracting problem with home production is exactly the same as the solution to the standard Green and Kahn problem with no explicit home sector, but with $u(c,h)$ replaced by $V(c_m,h_m)$. Hence Proposition 1 tells us $\eta_V \equiv \mu_V V_{11} + V_{12} < 0$ (normal leisure according to V) implies overemployment in the sense that the contract entails $\theta f'(h_m) < \mu_m$, while $\eta_V > 0$ (non-normal leisure according to V) implies underemployment in the sense that $\theta f'(h_m) > \mu_m$.

If we could construct an example where the reduced form utility function $V(\cdot)$ displays non-normal leisure, even though the underlying utility function $U(\cdot)$ displays normal leisure, we will have accomplished the task of generating normal leisure and underemployment simultaneously. For one such example, consider a linear perturbation of the utility function that depends only on total consumption and total hours, and is separable in these totals,

$$U(c_m, c_n, h_m, h_n) = u(c_m + c_n) + v(h_m + h_n) - Bh_n, \tag{11}$$

where $B \geq 0$. This implies individuals get more disutility from an hour of house work than an hour of market work if $B > 0$. We claim that (11) entails normal leisure in the sense that an increase in exogenous wealth always lowers total hours, H, even though it may raise h_m, and therefore may give rise to a reduced form function $V(\cdot)$ that displays inferior leisure.

Consider the problem with no uncertainty of maximizing U subject to $c_m = f(h_m) + x$, where x is exogenous wealth, and $c_n = g(h_n)$. It is easy to show that the solution satisfies

$$\partial H/\partial x = -\Delta u' g'' \eta_m - \Delta u' f'' \eta_n < 0$$

$$\partial h_n/\partial x = -\Delta u' f'' \eta_n - \Delta(g' - f')u''v'' < 0$$

$$\partial h_m/\partial x = -\Delta u' g'' \eta_m + \Delta(g' - f')u''v'',$$

where $\eta_m < 0$, $\eta_n < 0$, Δ is a positive constant, and $g' > f'$ as long as $B > 0$. Thus, H and h_n necessarily decrease with wealth, while h_m may increase and will necessarily increase if $g'' = 0$. Preferences of this class always display normal leisure in the sense that $\partial H/\partial x < 0$, but if $g(\cdot)$ is linear, or, by continuity, close to linear, then the reduced form $V(\cdot)$ displays non-normal leisure in the sense that $\partial h_m/\partial x > 0$. In this case, the efficient contract entails underemployment and also $W'(\theta) > 0$.

As the above discussion emphasizes, an important implication of adding home production is that market hours can increase with exogenous wealth even though leisure is normal. At first glance it might seem that having market hours increase with wealth is inconsistent with the long run evidence. Over time there have been large increases in factor productivity that have not been accompanied by concomitant increases in average hours of market work per household (see Rios-Rull, 1990, for a discussion and references). In models without home production, since the substitution effect of productivity growth implies h_m increases, h_m must decrease due to the wealth effect if we are to match the data. A model with home production, however, can have h_m increase with exogenous wealth and still match the long run observations, as long as productivity growth in the nonmarket sector keeps pace with productivity growth in the market sector.

We demonstrate with a simple example.[2] Consider a representative agent economy with preferences

$$U = \ln(c_m+c_n) + v(h_m+h_n) - Bh_n, \tag{12}$$

where $v(\cdot)$ is a decreasing function and the technologies are $c_m = \theta f(h_m)$ and $c_n = \gamma g(h_n)$. Optimal hours (h_m,h_n) are the solution to:

$$\theta f'/(\theta f+\gamma g) + v'(h_m+h_n) = 0$$

$$\gamma g'/(\theta f+\gamma g) + v'(h_m+h_n) = B$$

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[2] We assume no uncertainty or private information, since the evidence in question concerns long run productivity changes that presumably can be observed by everyone. Also, this example can be extended to include capital accumulation, which then yields the result that along a balanced growth path h_m and h_n are constant.

Obviously, (h_m,h_n) depends only on the ratio θ/γ, and hence does not change when θ and γ both increase at the same rate. If we decentralize this allocation as a competitive equilibrium, the analogous statement is that (h_m,h_n) does not change when the market wage, home productivity, and profit income (or any other exogenous income) all increase at the same rate.

Nevertheless, since the preferences described by (12) are a special case of (11), for $g(\cdot)$ close to linear and $B > 0$, we know $dh_m/\partial x > 0$. Hence, the specification in (12) delivers underemployment in the efficient contract, while at the same time is consistent with normal leisure both in the sense that total hours decrease in exogenous wealth and in a sense that is consistent with the long run evidence.

IV. Conclusion

In the home production economy, underemployment and normal leisure may coexist, as demonstrated here by some simple examples. These examples do not seem at all contrived, although they do depend on the assumption that an individual would rather work in the market than at household production for a given amount of total work. It is not surprising that such an assumption does the trick. It is also worth noting that empirical studies of home production provide some support for this assumption (see, e.g., Juster and Stafford [1990], p. 31).

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