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CONTRACTS, CONSTRAINTS, AND CONSUMPTION

Edward J. Green*

University of Minnesota and
Federal Reserve Bank of Minneapolis

Soo-Nam Oh*

Federal Reserve Bank of Minneapolis,
University of Minnesota, and
University of Pittsburgh

ABSTRACT

The paper compares implications of three kinds of models of households' consumption behavior: the basic permanent-income model, several models of liquidity-constrained households, and a model of an informationally-constrained efficient contract. These models are distinguished in terms of implications regarding the present discounted values of net trades to households at various levels of temporary income, and the households' marginal rates of substitution. Martingale consumption is studied as an approximation to the predicted consumption process of the efficient-contract model.

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1. Introduction

In this paper we compare the implications of three models of households' consumption behaviour: the basic permanent-income model, several models of liquidity-constrained households, and a model of an informationally-constrained efficient contract.¹

The main implications of the permanent-income model are that the consumption of a household is a martingale and that the level of consumption is always equal to the annuitized value of the lifetime wealth of the household.² During the past fifteen years, data sets have been examined with a variety of statistical techniques to see how this implication fares. Many researchers would interpret the statistical evidence to indicate that a modest, systematic deviation from the martingale-consumption prediction does exist.³

Our main goal is to make some prototypical comparisons among the potentially observable implications of schematic versions of the three models. The interpretation of data (and even the choice of which data to collect and to analyze) needs to be related explicitly to comparisons regarding the theoretical predictions of various alternative models. Our

¹ The permanent-income model was introduced by Friedman (1957) and reformulated by Hall (1978). A wide variety of liquidity-constraint models have been proposed; recent surveys include Hall (1989), Hayashi (1987), and King (1985). Townsend (1982) suggested that credit allocation to households might be modelled as being implemented by a long-term contractual arrangement rather than by a sequence of markets for a debt security. This idea was developed further by Green (1987) and by Thomas and Worrall (1990). (Atkeson and Lucas (1991), Banerjee and Newman (1991), Gertler (1988), Marimon (1988), and Taub (1989) have explored related ideas.) All three models concern the kinds of asset—canonically, bank-intermediated credit and debt—that are most relevant to consumption smoothing at approximately a business-cycle frequency. The overlapping-generations structure of the economy, which would be very salient in the context of life-cycle allocation of consumption but less so in the context of a shorter-horizon problem, is not modelled.

² Friedman declined to identify permanent income precisely with the annuitized value of lifetime wealth in the sense of the expected present discounted value of future income, but this definition of permanent income (which Hall used) is a close proxy for Friedman's concept. Hall derived that it is the marginal utility of consumption, rather than consumption itself, that is a martingale. However, he emphasized the case of quadratic utility where his characterization of the "law of motion" for consumption implies Friedman's characterization.

³ One intuitive expression of such a finding, due to Hall and Mishkin (1982), is eighty percent of aggregate household consumption seems to be due to households that the permanent-income model describes well, while the remaining twenty percent is due to consumption by households that the model does not fit well. This assessment of the evidence is not unanimous, though, nor do those who endorse it necessarily have a unanimous view of precisely how the data deviates from the prediction of the permanent-income model. In contrast to one widely held view, for example, Campbell and Deaton (1989) argue that consumption varies too little to be explained by the permanent-income hypothesis. They focus on consumption of nondurable goods, which varies less over the business cycle than do purchases of consumer durables and services.

purpose in drawing such comparisons is not to accept or reject models on the basis of what we currently understand about the data, but rather to help applied macroeconomists to refine their further study of the data.

It is important to understand that the deviations of actual consumption from the permanent-income prediction are modest, even if they are systematic. That is, the permanent-income model provides a much more adequate account of individual households' consumption behaviour than do polar models such as a complete-markets model or a model of autarkic consumption. The inability of these polar models to explain how the permanent-income model coheres fairly well with the data constitutes a severe limitation of the polar models themselves. In general, the capacity to explain the relatively high degree of success of the permanent-income model is a prerequisite for any alternative model to be taken seriously. In effect, then, a serious alternative model must not have predictions that diverge radically from those of the permanent-income model—including the prediction of martingale consumption. Especially in the case of the efficient-contract model, the degree of approximation to a martingale-consumption prediction has been an open question.⁴ We derive such an approximation in this paper.

The plan of the paper is as follows. In Section 2, we describe an infinite-horizon economy in the context of which explicit time-series predictions regarding households' consumption paths can be formulated. In Section 3, we define the three kinds of models that we will study and we argue that they all permit consumption to be approximately a martingale. In Section 4, we describe a two-period environment with respect to which it is easy to derive schematic versions of the observable time-series features implied by the various models. In Section 5, we derive such implications regarding the present discounted values of net trades to households at various levels of temporary income, and the households' marginal rates of substitution. Section 6 provides a tabular summary of these

⁴ Green (1987) compared predicted consumption to a random walk with drift but did not bound the magnitude of the drift.

implications, in terms of which all of the models that we consider are distinguishable from one another.

2. Modelling the Household Sector

In our model of an economy, households are given a stylized, but detailed, description, and the production sector is just detailed enough to define the equilibria of various institutional arrangements. Although business-cycle theory provides an important motivation for the study of households' consumption decisions, we simplify the study itself by abstracting from the existence of aggregate economic disturbances.⁵ This abstraction seems justifiable—Lillard and Willis (1978) and Pischke (1990) have shown that idiosyncratic factors account for far more of the variability in a typical household's consumption than does the business cycle. Finally, although the random events confronting an actual household typically might have more to do with its employment opportunities than with its receipt of income *simpliciter*, the simplest model will abstract from labor-supply questions and treat income as exogenous.

Consider an economy consisting of a large number of households that are identical *ex ante*. These households have independent, identically distributed random endowments of a perishable composite commodity which is consumed at each date $t = 0, 1, 2, \dots$. Each household's endowment is i.i.d. as a sequence of random variables indexed by time as well. (That is, the first sense in which endowments are random is as infinite-dimensional endowment vectors indexed by households.) Assume for simplicity that a household receives one unit of the composite commodity at any given date with probability p , and 0 units with probability $1 - p$. We will often refer to the household's endowment quantity of the good at date t as its income at t . A crucial issue is the specification of the timing and publicity of information regarding endowments. Alternative theories make incompatible

⁵ This abstraction is made as a transitional step in formulating a version of consumption theory that will take business-cycle considerations explicitly into account—Taub (1989) has already made some progress in this respect.

assumptions regarding this feature of the environment. We will specify this informational aspect of the models when we discuss each of the theories.

The law of large numbers implies that, at each date, the per capita average endowment will be arbitrarily close to p units with very high probability, if the population is sufficiently large. Imagine that this average endowment is exactly p units, with probability 1. In the ensuing discussion, $X = \langle X_0, X_1, \dots \rangle$ will denote the random sequence of consumption levels enjoyed by a representative household, and $Y = \langle Y_0, Y_1, \dots \rangle$ will denote the random endowment of that household. That is, $X - Y$ will be the net trade of the household.

Suppose that the household has utility function

$$U(\langle X_t, X_{t+1}, \dots \rangle) = E \left[\sum_{j=0}^{\infty} \beta^j W(X_{t+j}) \right]. \quad (1)$$

That is, the household has additively separable, time-stationary expected utility. It is assumed that W is strictly concave. The domain of W , which represents the set of feasible consumption levels for the household at each date, will be taken to be either \mathbb{R} or else \mathbb{R}_+ .

Assume that aggregate consumption can be shifted over time by the use of a linear technology that can convert 1 unit of the consumption good endowed at date t to $\beta^{t-\tau}$ units of the good for consumption at date τ (for any τ before or after t). By equating the rate of pure time preference to the marginal rate of transformation in this way, we are in effect formulating a partial-equilibrium model. Note that the assumption that future commodities can be transformed for immediate consumption cannot be taken literally (in a closed economy, at least). It may be made more palatable by thinking in terms of the economy having a large, steady-state capital stock that must be maintained by investment. This investment can be foregone for a while in order to have higher-than-steady-state consumption until the effects of capital depreciation begin to be felt. Allowing the capital stock to depreciate in this way, with the intention of replenishing it eventually from endowment, is

tantamount to conversion of future endowment to present consumption. Another possible interpretation of negative investment would be external borrowing by the inhabitants of an open economy.

3. Three Models of Allocative Institutions

3.1. The Permanent-Income Model

Because of its relevance to the efficient-contract model, we begin by describing Friedman's (1957) permanent-income model. Then we will describe Hall's (1978) reformulation of the model.

Friedman envisions households as trading in a sequence of competitive markets for a debt security, but as lacking access to markets for contingent claims. He does not formally analyze an infinite-horizon model such as we have formulated here. Rather, Friedman bases his argument on the intuitive proposition that a household with a concave, time-separable utility function would attempt—for a reason analogous to risk aversion—to minimize the deviation of its consumption from long-run trend. To do so, it would calculate its wealth and treat this wealth as a long-term investment from which it would consume the interest income.

To be specific, let the debt security be a consol bond.⁶ Suppose that such a bond can be purchased for one unit of consumption and that it pays interest of $R - 1$ units of consumption at every subsequent date. Thus, purchasing a bond and selling it ex-dividend at the next date exchanges a unit of current-date consumption good for R units of the good at the next date. To avoid an arbitrage between bond-holding and use of the investment technology, it must be that $R = \beta^{-1}$.

Suppose that the household observes its income at each date, and then decides how

⁶ In this and other technical respects, our exposition does not exactly coincide with Friedman's. These expositions are equivalent, though.

much to consume at that date. Friedman assumes that the household discounts its present and future endowment at the pure rate of time preference. Thus the expected value of its endowment from date $t + 2$ onward, discounted to $t + 1$, is $\frac{p\beta}{1-\beta}$. In addition let B_t denote end-of-period bond holdings that the household carries over a portfolio of B_t bonds to the beginning of date $t + 1$. Therefore in Friedman's sense, its wealth at that point is $Y_{t+1} + B_t + \frac{p\beta}{1-\beta}$. It will consume the net interest rate $R - 1$ times this wealth. That is, it will consume

$$X_{t+1} = (\beta^{-1} - 1) \left[Y_{t+1} + B_t + \frac{p\beta}{1-\beta} \right] \quad (2)$$

at $t + 1$. (Since possibly $Y_{t+1} + B_t + \frac{p\beta}{1-\beta} < 0$, it is implicitly assumed that negative consumption levels are feasible.) The difference between income Y_{t+1} and consumption X_{t+1} will be saved by purchasing bonds (or financed by selling bonds). That is,

$$B_{t+1} - B_t = Y_{t+1} - (\beta^{-1} - 1) \left[Y_{t+1} + B_t + \frac{p\beta}{1-\beta} \right]. \quad (3)$$

Note that $1 - \beta$ is Taylor's approximation of $\beta^{-1} - 1$. Therefore (3) implies that, for $\beta \approx 1$, $B_{t+1} - B_t \approx Y_{t+1} - p$.⁷ Taking the expectation of both sides of this approximation conditional on B_t yields

$$E \left[B_{t+1} \middle| B_t \right] - B_t \approx 0. \quad (4)$$

That is, $\langle B_t \rangle$ is approximately a martingale. This, together with (2), is the prediction of Friedman's permanent-income theory if income is independently and identically distributed.

Hall (1978) models the household's optimization problem more explicitly than does Friedman. He considers the following perturbation of the household's consumption plan:

⁷ In making these approximations, we are going to set terms such as $(1 - \beta)B_t$ equal to zero, on the grounds that $\beta \approx 1$. Clearly such a step is unwarranted if B_t is huge, and B_t will be huge at some time t if it is approximately a random walk. Therefore the approximation is only valid as a local approximation from initial conditions in which B_t is sufficiently small.

reduce consumption infinitesimally at date t (in the event of a particular income history prior to that date), invest the amount for one period, then consume the interest proceeds from sale of the investment (at the ex-dividend price) at date $t + 1$. If the consumption plan is optimal, then this perturbation must not affect the household's utility level. The first-order necessary condition for utility not to be affected is that

$$\langle W'(X_t) \rangle \text{ is a martingale.} \tag{5}$$

If W is quadratic, then (5) implies that $\langle X_t \rangle$ is a martingale—a conclusion that is also an implication of (2) and (4).

3.2. Liquidity-Constraint Models

Macroeconomists use the term 'liquidity-constraint model' in a precise way. A liquidity-constraint model posits a specific form of constraint in the context of an institutional structure that apparently would otherwise implement a permanent-income allocation (for example, in the sense of (5)).⁸ Hayashi (1987) identifies two broad categories of such models that have been most often studied in macroeconomics: credit-rationing models and differential-rates models.

- **Credit-rationing models** assume that the interest rate is set such that aggregate excess demand for credit would be positive in the absence of a constraint. Borrowing is allowed at this interest rate, but the size of allowable borrowing is constrained directly to a limit that equates aggregate borrowing to the amount of credit offered to the market by lenders.
- **Differential-rates models** assume that borrowers must repay loans at a higher interest rate than lenders receive. If the market-clearing rate is between the borrowing and lending rates, then it is possible for aggregate supply of credit and aggregate

⁸ We emphasize the qualification *apparently*, in view of the difficulty that we have noted regarding spot-market enforcement of the budget constraint.

demand for credit to be equally far below the competitive level, allowing the market to clear without explicit rationing.

Market outcomes in these models are specified by an interest rate and a credit ceiling, or a pair of interest rates for borrowing and lending, respectively. In each of these specifications, the decision problem of a representative household is well defined, so equilibrium can be defined in terms of decentralized optimization by households and aggregate materials balance. It is clear that some equilibrium allocations in these models are not supportable as equilibria of a permanent-income model. Hall's version of the permanent-income model is a specialization (or trivialization) of these models, though. A credit-rationing model with a high ceiling or a differential-rates model with only a small gap between borrowers' and lenders' rates would therefore account for the capability of the permanent-income model to provide an approximate description of actual consumption patterns.

3.3. The Efficient-Contract Model

Now we turn to the efficient-contract model. We will show that it shares the capacity of the credit-rationing and differential-rates models to explain the approximate accuracy of the permanent-income model.

In the environment described in Section 2, make the following assumptions about the information structure and the legal structure of the economy. A household learns about its own income at each date when the income is received, and it takes this information into account in making its decisions at that date. However, no household (or other agent in the economy) is able to observe directly the income or consumption levels of any other household. Households can communicate about these quantities, but their reports are unverifiable. As in the case of liquidity-constraint models, it is essential to assume that households lack (or can legally be denied) direct access to the intertemporal-transformation technology. Otherwise they would make use of the technology to arbitrage the contract

(cf. Allen (1985)). Finally, it must be assumed that households can bind themselves to long-term participation in a contractual arrangement that has the potential to dictate an ex post welfare level lower than autarky under some circumstances.⁹

If households could enter into long-term contracts that were completely enforceable, then they would agree to pool their endowments so that each household would consume p units at every date. Such an allocation would not be implementable, though, because all households would have a dominant strategy of always claiming to have received zero income in order to claim a transfer of p units rather than having to make a transfer of $1 - p$ units.¹⁰ A household could always make such a claim, regardless of whether or not it were true, because no other household can observe its income. With every household demanding a net transfer from the others in this way, the prescribed allocation would be infeasible.

If complete risk sharing and time smoothing of income cannot be implemented, then how close can the economy come to achieving these welfare-enhancing goals? To define exhaustively the feasibility conditions for allocations in this economy, including the conditions regarding incentive compatibility in the presence of private information, is complicated. It is fairly easy to explain the incentive-compatibility condition and its implications in an intuitive way, though. We do so now, following the formal argument of Green (1987) and of Thomas and Worrall (1990). A contract is a sequence of rules, one for each date, that determines a net trade for each household as a function of its past and current reports about its income. Because income and consumption are unobservable, these reports—without verification—are all that the contract could possibly be based on. Since the endowment

⁹ It would be very desirable to have a version of an efficient-contract model that embodies a sequential individual-rationality constraint. Thomas and Worrall (1990) have already taken a step towards this goal. However, it should not be assumed a priori that adopting such a constraint is the best way to model actual markets for intermediated credit. Keep in mind that the empirical counterpart of autarky in the present model is an allocation in which households earn income which have to be consumed immediately. If a household's income can be garnished, then an intermediary can indeed hold the household to a utility level below autarky.

¹⁰ Here we appeal to the revelation principle. Townsend (1988) has derived the principle for an environment that resembles this one.

processes of distinct households are statistically independent, there is no way to use one household's reports to affect net trades to other households in any useful way.

Consider a household that receives zero income at date 0. Let b_0 denote the amount of consumption that will be transferred to the household if it reports its zero endowment, and let b_1 be the amount (presumably negative) that will be transferred to the household if it reports (falsely) having received one unit of endowment. Let $Z_0 = \langle Z_1^0, Z_2^0, \dots \rangle$ and $Z_1 = \langle Z_1^1, Z_2^1, \dots \rangle$ be the best random consumption streams that the household can obtain from date 1 forward by responding strategically to the contract after having initially reported income of 0 or 1 respectively at date 0. Then, by the additive separability and stationarity assumed in (1), the household will obtain utility level $W(b_0) + \beta U(Z_0)$ if it reports 0 as its date 0 income, and it will obtain utility level $W(b_1) + \beta U(Z_1)$ if it reports 1 as its date 0 income. Therefore in order for this household that has actually received income 0 at date 0 to be willing to tell the truth, it is necessary that

$$W(b_0) + \beta U(Z_0) \geq W(b_1) + \beta U(Z_1). \quad (6)$$

Since the contract cannot be made contingent on actual (as opposed to reported) initial income, and since future income is independent of initial income, the quantities b_0, b_1, Z_0 , and Z_1 have the same significance for a household with initial income 1 as for a household with initial income 0. The only distinction is that consumption at date 0 will include the 1 unit of income as well as the transfer. Of course, the inequality between the expected utility of reporting income 0 and reporting income 1 must be reversed if truthful reporting is to be optimal for a household that has actually received income at date 0. That is,

$$W(1 + b_0) + \beta U(Z_0) \leq W(1 + b_1) + \beta U(Z_1). \quad (7)$$

Subtracting (7) from (6) formally establishes two facts about incentive-compatible allocations (that is, those satisfying both (6) and (7)). The first fact, which is intuitive, is that households with income at date 0 transfer consumption at that date to households

without income.¹¹ Condition (7) then shows that the households receiving income must be induced to make these transfers by being rewarded with a prospect of subsequent consumption that will yield higher discounted expected utility than what households without income will experience.

The second fact that follows from this subtraction argument is that it is impossible for both incentive constraints to be satisfied with equality, except in the degenerate case that the date 0 transfer to households is independent of their reports. Since the efficient contract is structured to achieve as much mutual insurance among households as possible, but fails to provide full insurance, the constraint must be binding for the households that receive income rather than for the households that do not receive it. If one thinks of these as rich and poor households, respectively, then this feature of the contract may not seem plausible. However, it should be kept clearly in mind that whether the constraint binds depends on the household's current receipt of temporary income—not on its wealth. Rich households do not spend their lives posing as poor ones, but households that have “a good week” do sometimes try to conceal their windfall from their creditors.

Consider next the problem of designing the contract that maximizes the ex ante expected utility of the representative household. Recall that we have posited a linear technology that can transform consumption from one date to another and that we have made the special assumption that the marginal rate of transformation for this technology is equal to households' rate of pure time preference. When we make this assumption, we have in mind an economy in which households can participate in a market for illiquid assets (so that the equation between time preference and marginal rate of transformation (MRT) is motivated by an equilibrium condition), but in which they must rely on intermediated contractual arrangements for liquidity within the business-cycle time horizon. In order for households not to have arbitrage opportunities against intermediaries who

¹¹ More accurately, $b_1 \leq b_0$. It is conceivable that, by use of the intertemporal-transformation technology, b_1 could be made positive.

would attempt to approximate in an actual economy the efficient contract modelled here, it would be essential that they should be unable to make liquid investment in the asset market. We represent this constraint formally by supposing that only the contract designer or intermediary has access to the intertemporal-transformation technology.

Now suppose that the endowment of a representative household is $Y = \langle Y_0, Y_1, \dots \rangle$ and that this household is supposed to receive consumption $X = \langle X_0, X_1, \dots \rangle$. If the differences between consumption and income at all dates were transformed into the date 0 commodity using the technology of the economy, then the expected output of this process would be $E \left[\sum_{t=0}^{\infty} \beta^t (X_t - Y_t) \right] = E \left[\sum_{t=0}^{\infty} \beta^t X_t \right] - \frac{p}{1-\beta}$. This output quantity is the net cost of providing X to the household.

In a closed economy, the problem of designing the ex ante efficient contract can be formulated as: maximize $U(X)$ subject to the incentive-compatibility constraints (6) and (7), and subject also to the materials-balance constraint that the net cost of X must not exceed zero.

For any u in the range of U , let $C(u)$ denote the minimum net cost (in the sense just defined) of providing the household with a consumption stream X such that $U(X) = u$ and such that the incentive-compatibility conditions (6) and (7) hold, and such that their analogues hold at all dates and after all possible histories of income reports.¹² (For example, $C^{-1}(0)$ is the utility level that an efficient contract achieves for the representative household.) C will play an analogous role in this theory to B in the permanent-income model. Suppose that $U(X) = u$, that X is implemented by an incentive-compatible contract, and that the net cost of X is $C(u)$. Then, in the notation that has been defined above, C satisfies the functional equation

$$C(u) = E \left[b_{Y_0} + \beta C(U(Z_{Y_0})) \right]. \quad (8)$$

¹² We will often assume implicitly that $U(X) = u$, and that quantities derived by use of this assumption are implicitly functions of u .

This same analysis applies also at every later date. At any date t , and for any sequence $y = \langle y_0, \dots, y_{t-1} \rangle$, define Z_y to be the random stream $\langle X_t, X_{t+1}, \dots \rangle$, under its distribution conditional on the event that, for all $\tau < t$, $Y_\tau = y_\tau$.

Define b_y analogously to Z_y . That is, if y is a sequence of length $t + 1$, then b_y is the transfer that a household would receive at date t if it had reported having income y_τ at each date $\tau \leq t$. Then (8) generalizes to¹³

$$C(U(Z_{(Y_0, \dots, Y_{t-1})})) = E \left[b_{(Y_0, \dots, Y_t)} + \beta C(U(Z_{(Y_0, \dots, Y_t)})) \middle| Y_0, \dots, Y_{t-1} \right]. \quad (9)$$

Equation (9) is an instance of a more streamlined functional equation:

$$C(u) = p \left[b_1(u) + \beta C(v_1(u)) \right] + (1 - p) \left[b_0(u) + \beta C(v_0(u)) \right]. \quad (10)$$

In particular, equation (8) is obtained from (10) by setting $b_{Y_0} = b_{Y_0}(u)$ and $U(Z_{Y_0}) = v_{Y_0}(u)$. By making analogous substitutions, incentive-compatibility conditions corresponding to (6) and (7) would be defined.

As a specific example, assume that $W(x) = -\exp(-rx)$ in (1). That is, the household has additively separable, time-stationary expected utility that exhibits constant absolute risk aversion (with parameter r which we take to be positive) at each date. The utility function U is then defined on all of \mathbb{R}^∞ , which we take to be the household's consumption set.

Given a value of u and a candidate function C , candidate values for the quantities $b_0(u), v_0(u), b_1(u)$, and $v_1(u)$ are obtained as the unique solution to the problem of minimizing the right side of (10) subject to the expected-utility constraint that $E[-\exp(-r(Y_0 + b_{Y_0}(u))) + \beta v_{Y_0}(u)] \geq u$ and the two incentive-compatibility constraints. When this is done for all values of u and when the candidate values of the solution are

¹³ This equation holds almost surely. From this point onward, this qualification will apply implicitly throughout the analysis. Henceforth we will refer to this equation even when $t = 0$, in which case it is to be understood as actually referring to (8).

substituted in the right side of (10), then the resulting expression defines a new candidate function for C . This iterative process carries the class of functions of the form

$$C(u) = c - r^{-1}(1 - \beta)^{-1} \ell n(-u) \quad (11)$$

into itself, and there is a unique value of c for which the corresponding function is a fixed point of the process. A dynamic-programming argument shows that this fixed point is the economically meaningful cost function for the efficient contract. Given the functional form (11), constrained minimization of the right side of (10) is achieved by setting

$$-\exp(-r(k + b_k(u))) = \phi_k u \text{ and } v_k(u) = \psi_k u \text{ for } k \in \{0, 1\}. \quad (12)$$

The constants ϕ_k and ψ_k do not depend on u . If X is the allocation implemented by the efficient contract and if $U(X) = u$, then it can be shown (by induction) that for all t ,

$$U(Z_{(Y_0, \dots, Y_{t-1})}) = \left[\prod_{\tau < t} \psi_{Y_\tau} \right] u. \quad (13)$$

Therefore after substituting $U(Z_{(Y_0, \dots, Y_{t-1})})$ for u in the first equation of (12) and then applying (13), we see that $-\exp(-r(Y_t + b_{(Y_0, \dots, Y_t)})) = \phi_{Y_t} \left[\prod_{\tau < t} \psi_{Y_\tau} \right] u$. Multiplying this equation by -1 and taking the logarithm on both sides, and then rearranging terms, yields

$$X_t = Y_t + b_{(Y_0, \dots, Y_t)} = -r^{-1} \left[\ell n(\phi_{Y_t}) + \left[\sum_{\tau < t} \ell n(\psi_{Y_\tau}) \right] + \ell n(-u) \right]. \quad (14)$$

Thus we have derived the “law of motion” for the consumption of a representative household. Now we examine how closely this stochastic process of consumption corresponds to a random walk (that is, a process with independent increments) and to a martingale (that is, a process for which the current value is equal to the conditional expectation of the next value). Since the variance of a random walk grows to infinity at a constant rate, there is a clear sense in which a process that differs from a random walk by an i.i.d. sequence of random variables with finite variance provides a good approximation to the random

walk.¹⁴ (This is essentially the idea of cointegrability in econometrics.) Erasing the term $\ln(\phi_{Y_t})$ in (14), which are i.i.d. with finite variance, yields such an approximation:

$$X_t \approx -r^{-1} \left[\left[\sum_{\tau < t} \ln(\psi_{Y_\tau}) \right] + \ln(-u) \right]. \quad (15)$$

Using (13) to make a substitution in (11) yields the conclusion that

$$C(U(Z_{(Y_0, \dots, Y_{t-1})})) \approx -r^{-1}(1 - \beta)^{-1} \left[\left[\sum_{\tau < t} \ln(\psi_{Y_\tau}) \right] + \ln(-u) \right]. \quad (16)$$

By comparing (15) with (16) and noting that $(1 - \beta)$ is Taylor's approximation of $\beta^{-1} - 1$, we see that $X_t \approx (\beta^{-1} - 1)C(U(Z_{(Y_0, \dots, Y_{t-1})}))$. Note that $\beta^{-1} - 1$ would be the net interest rate on 1-period debt, if there were a competitive market for debt claims in this environment and if an agent with access to the intertemporal-transformation technology were to behave competitively.

Again, since we are interested in approximate comparison of random quantities having variances that increase with bound over time, we may add a term which is i.i.d. with finite variance and a constant to only the right side of the previous equation. In particular we can add $(\beta^{-1} - 1)(Y_t + \frac{p\beta}{1-\beta})$. That is analogously to (2),

$$X_t \approx (\beta^{-1} - 1) \left[C(U(Z_{(Y_0, \dots, Y_{t-1})})) + Y_t + \frac{p\beta}{1-\beta} \right]. \quad (17)$$

The first term in the brackets on the right side of (17) is the expected discounted value of the transfers that the household will receive according to the contract, the second term is the income received at date t , and the third term is the expected discounted value of the household's income at $t + 1$ and subsequent dates. That is, the expression in brackets is the household's wealth at date t . Thus, (17) states that the household consumes approximately the net interest (in a neoclassical model such as Irving Fisher's

¹⁴ The consumption process of the representative household is actually an IMA (0, 1, 1) process. See Box and Jenkins (1976).

or Friedman's) on its wealth. This idea that the household consumes the annuitized value of its wealth at each date is the core of Friedman's permanent-income theory. It follows in the efficient-contract theory, just as in Friedman's, that consumption is (approximately, in this case) a martingale. Consumption is approximately a martingale because wealth is a martingale and consumption is approximately proportional to wealth, not because marginal utility is a martingale (as in Hall's permanent-income model). Formally, (17) is equivalent to $b_{(Y_0, \dots, Y_t)} \approx (\beta^{-1} - 1) [C(U(Z_{(Y_0, \dots, Y_{t-1})})) + Y_t + \frac{p\beta}{1-\beta}] - Y_t$, and making this substitution in (9) yields

$$C(U(Z_{(Y_0, \dots, Y_{t-1})})) \approx E \left[(\beta^{-1} - 1) [C(U(Z_{(Y_0, \dots, Y_{t-1})})) + Y_t + \frac{p\beta}{1-\beta}] - Y_t + \beta C(U(Z_{(Y_0, \dots, Y_t)})) \middle| Y_0, \dots, Y_{t-1} \right]. \quad (18)$$

Using the Taylor's-approximation equivalence of $\beta^{-1} - 1$ and $1 - \beta$, and also the fact that $E[Y_t] = p$, (18) reduces to the martingale equality for C :

$$C(U(Z_{(Y_0, \dots, Y_{t-1})})) \approx E \left[C(U(Z_{(Y_0, \dots, Y_t)})) \middle| Y_0, \dots, Y_{t-1} \right]. \quad (19)$$

This is the analogue of (4) in Friedman's permanent-income model. Thus, the efficient-contract model satisfies approximate versions of all of the main predictions of Friedman's model when the discount factor β is sufficiently close to 1.

Despite the approximate similarity of the efficient-contract allocation to the permanent-income allocation (when β is close to 1), though, the efficient-contract allocation violates Hall's martingale condition (5) for marginal utility in a systematic way. The first-order condition for (12) to minimize the right-hand side of (10) is that $\langle C'(U(Z_{(Y_0, \dots, Y_{t-1})})) \rangle$ must be a martingale (cf. Thomas and Worrall (1990, §5)). Since C' is increasing and strictly convex (by (11)), it follows from Jensen's inequality that

$$U(Z_{(Y_0, \dots, Y_{t-1})}) > E \left[U(Z_{(Y_0, \dots, Y_t)}) \right]. \quad (20)$$

Given the functional form (1) of U and the negative-exponential specification of W adopted in this section (which makes $W(x)$ and $W'(x)$ proportional), this inequality is clearly inconsistent with (5).¹⁵ It follows that the efficient-contract allocation cannot be supported as an equilibrium allocation of Hall's (1978) permanent-income model, or indeed as an equilibrium of any incomplete-markets model.¹⁶

4. Towards a Comparison of Direct Predictions: A Two Period Model

In order to compare the alternative theories in the most intuitive way possible, we now present a schematic, two-period version of the environment defined in Section 2.¹⁷ The key to this simplification is the observation that if a contract determines X as a function of Y , then

$$\mathbb{E} \left[U(X) \middle| Y_0 \right] = W(X_{Y_0}) + \beta \mathbb{E} \left[U(Z_{Y_0}) \right]. \quad (21)$$

Because expectations have been taken in (21), $W(X_{Y_0})$ is the only term that varies across states of nature. Thus (21) suggests thinking in terms of an economy where the only endowment uncertainty concerns income at date 0. Once this simplification has been made, evidently there is little loss in supposing that there is only a single date subsequent to the date at which uncertainty is resolved. Moreover, we can formulate the model in terms of the utility $w(x)$ of consumption x enjoyed at date 0, and of expected utility (discounted to date 0) of v enjoyed at date 1, rather than in terms of consumption at date 1:¹⁸

¹⁵ Thomas and Worrall (1990) provide a proof that is valid for a larger class of functions W .

¹⁶ The question of whether efficient-contract allocations coincide with equilibrium allocations in some system of incomplete markets was raised by Green (1987), who also pointed out the close relationship between efficient-contract allocations and the no-surplus allocations defined by Ostroy (1980).

¹⁷ While the model is schematic in one respect, it is more satisfactory than the infinite-horizon model of Section 2 in several respects. The assumption that there are only two income levels is relaxed (as in Thomas and Worrall (1990)). Finite lower bounds on consumption are imposed. The analysis of the efficient contract is freed from the assumption of negative-exponential utility.

¹⁸ The function w will be assumed to be strictly increasing, strictly concave, and twice differentiable.

$$u(x, v) = w(x) + v. \tag{22}$$

We may suppose that each household receives an endowment of initial income Y (independent across households), that there is a linear technology for converting 1 unit of date 0 consumption to R units of date 1 consumption, and that $RC(v)$ is the amount of the consumption good that a household must receive at date 1 (net of whatever endowment each household receives at date 1) in order to achieve utility level v .¹⁹ Thus, the amount of endowment needed for date 0 consumption together with use as an input to production for date 1, if the household is to be offered the net trade x at date 0 and the welfare level v at date 1, is $x + C(v)$. Since v in (22) corresponds to $\beta E[U(Z_{Y_0})]$, the expected marginal utility of consumption at date 1 is $(RC'(v))^{-1}$. To state this in terms of the intertemporal marginal rate of substitution (MRS):

$$\begin{aligned} &\text{The MRS of consumption at date 1 for} \\ &\text{consumption at date 0 is } RC'(v)w'(x). \end{aligned} \tag{23}$$

We make the assumptions that there are minimum consumption levels at both dates (the date 1 constraint being represented implicitly by a lower bound on v), and that materials balance must be satisfied in both periods with consumption being transformed from date 0 to date 1 but not vice versa (that is, irreversibility of technology). Assume that 0 is the minimum consumption level at date 0, and let $v^- \in \mathbb{R} \cup \{-\infty\}$ denote the utility level (discounted to date 0) that the household will receive at date 1 if it has the minimum feasible level of consumption at that date.²⁰ Suppose that Y can take n possible values, $y_1 < y_2 < \dots < y_n$, and that a household receives income at level y_i with probability π_i . As in the infinite-horizon model, a law of large numbers is supposed to hold exactly. The constraints are formalized respectively by the following conditions:

¹⁹ From our assumptions regarding w , it follows that C is strictly increasing, strictly convex, and twice differentiable.

²⁰ Nothing depends essentially on these assumptions. The minimum consumption level at date 0 could be set at $-\infty$, if it were desired to abstract from this constraint, and the analysis to be presented in the next section would still be valid.

$$\forall i \left[b_i \geq -y_i \text{ and } v_i \geq v^- \right], \quad (24)$$

and

$$\sum_{i=1}^n \pi_i b_i \leq 0 \text{ and } \sum_{i=1}^n \pi_i \left[b_i + C(v_i) \right] \leq 0. \quad (25)$$

5. Comparing Predicted Allocations

The purpose of presenting this two-period model has been to facilitate comparing the allocations predicted by the various models of allocation to households. The goal is to identify potentially observable features of actual allocations, and to derive predictions regarding these features. We will consider two attributes of each household to be observable. The first attribute is the present discounted value of the net trade that it receives ex post in the allocation. The second attribute is its marginal rate of substitution of future consumption for present consumption at each income level. The permanent-income model has precise implications about both of these attributes. Specifically, the present discounted value of every household's net trade is 0, and the MRS of every household is R .²¹ We will now indicate how these well-known implications are represented within the present model, and subsequently we will derive the corresponding implications of liquidity-constraint models and of the efficient-contract model. It will be seen that all of the models differ systematically from one another.

Before we proceed, a caveat is in order. What we are asserting is that longitudinal panel data could be gathered that would allow discounted present values and the marginal rates of substitution to be observed or estimated. We are not claiming that currently existing data is necessarily adequate for this purpose. To the contrary, we do not know whether any data set contains all of the information on households' income, consumption and asset balances that would probably be required. One of the functions of theory is to

²¹ Here we ignore households that receive boundary consumption bundles.

guide the collection of data, and we suspect that new data may have to be assembled to conduct the sort of study that we envision.

5.1. The Permanent-Income Model

In the two-period environment, the permanent-income model reduces to a model of a competitive market at date 0 for lending and borrowing between dates 0 and 1, which opens after households have already learned their income levels. Therefore an interior permanent-income allocation is characterized by the feasibility conditions (24) and (25), and by the following equilibrium conditions:²²

$$\forall i \left[b_i + C(v_i) = 0 \right], \quad (26)$$

and

$$\forall i \left[C'(v_i)w'(y_i + b_i) = 1 \right] \quad (\text{that is, MRS} = \text{MRT}). \quad (27)$$

Conditions (26) and (27) respectively imply the two comparisons across households (that is, in terms of present discounted values and of marginal rates of substitution) that we have asserted the model to imply.

5.2. Liquidity-Constraint Models

Liquidity-constraint models add three features to the permanent-income model. First, the interest rate need not be identical to the MRT determined by technology. Second, the interest rate paid to savers may be less than the interest rate paid by borrowers. Third, there may be a non-budget constraint on the amount of indebtedness that is allowed. Let γ , δ , and λ be the interest rate paid to savers, the interest rate paid by borrowers, and the

²² This characterization could be extended in the usual way to equilibria in which some households consume at the boundary of their feasible sets, by replacing (26) with a pair of inequalities involving Lagrange multipliers (that is, one inequality and one multiplier for each date) with complementary slackness conditions imposing equality except at the consumption-set boundaries.

rationing constraint on indebtedness respectively. Besides the feasibility conditions (24) and (25), the following characterization of optimization by households defines equilibrium.

$$\forall i [b_i \leq \lambda], \quad (28)$$

$$\forall i [b_i < 0 \implies b_i + \gamma C(v) = 0 \text{ and } RC'(v)w'(y_i + b_i) = \gamma], \quad (29)$$

$$\forall i [b_i = 0 \implies C(v) = 0 \text{ and } \gamma \leq RC'(v)w'(y_i + b_i) \leq \delta], \quad (30)$$

$$\forall i [0 < b_i < \lambda \implies b_i + \delta C(v) = 0 \text{ and } RC'(v)w'(y_i + b_i) = \delta], \quad (31)$$

and

$$\forall i [b_i = \lambda \implies b_i + \delta C(v) = 0 \text{ and } RC'(v)w'(y_i + b_i) \geq \delta]. \quad (32)$$

It has already been pointed out that the assumptions that households optimize relative to the constraints of this model and that markets clear determine an equilibrium concept formally. But note that the equilibrium concept is incomplete, because it is not specified what kind of optimization or strategic interaction occurs among intermediaries. Now we will consider three kinds of assumption that have been used elsewhere to complete this model.²³ All three of these modelling strategies require further assumptions beyond the ones that we have made, in order to fully endogenize the allocations that we will propose. Thus we will describe these models informally, but characterize precisely the aspects of their equilibrium allocations with which we are directly concerned.

- **The usury-law model** posits that intermediaries are competitive in the sense that they take their competitors' decisions to be parametric and that free entry imposes

²³ The reader is referred to Hayashi (1987) for a discussion of this literature.

a zero-profit condition on equilibrium. It is assumed that there is a usury interest-rate ceiling below what is implied by the MRT and that savings and investment are imperfect substitutes for households (possibly because investment is risky and a household cannot hold a diversified portfolio, or possibly for the simple reason that households do not have direct access to investment) so that some households save a positive amount at this ceiling rate. It is also assumed that intermediaries are subject to a portfolio constraint, that the savings they receive must be lent rather than invested. The interest rate to be paid by borrowers is set at the usury ceiling—as high as is permitted. By the zero-profit constraint, the interest rate paid to lenders must also be set at the ceiling. If the demand to borrow exceeds the supply of savings at this rate, then credit will be rationed. Since the interest rate for all households is below R , the households that borrow more will have higher present discounted values of net trade. Borrowing will be inversely related to income at date 0, with strict monotonicity above the income level where notional demand to borrow is equal to the credit constraint. Since it is the low-income households that will be constrained, the MRS of date 1 consumption for date 0 consumption is higher for low-income (at date 0) households than for high-income households. In the range of income where the credit constraint is binding, this MRS will be strictly decreasing in income.

- **The default model** posits that some borrowing households are unable (perhaps because of a “spatial-separation” constraint) to repay their debts at date 1, or that they become exempt from the legal consequences of nonpayment and therefore decide not to repay. We assume that a proportion $1 - \theta$ of households would default if they were to borrow, that this proportion is identical for all income levels, and that no household has private information regarding its propensity to default. For simplicity, we will assume that the defaulting households make no repayment whatsoever and

that they consume their entire income at date 1.²⁴ Thus competitive intermediaries set the interest rate for savers at R , and they set the interest rate for borrowers at a level such that the borrowers who repay provide R times the total amount that was borrowed: that is, R/θ . Borrowing will be decreasing in income and strictly decreasing at some levels if the lowest-income households borrow a positive amount (since the highest-income households must consequently save a positive amount). Households that save will have net trades with a present discounted value of zero, while households that borrow and repay will have net trades with present discounted values that become more negative the higher the level of borrowing. Since a borrowing household pays interest at rate R/θ with probability θ and at rate 0 with probability $1 - \theta$, the expected interest rate is R . Therefore the expected interest rate is R for all households, and therefore (neglecting boundary consumption) the MRS is the same for all households except those that default.

- **The monopoly model** assumes that the households can only save through the intermediary, and that the intermediary is a monopolist. Thus the interest rate for creditors will be below R , and the interest rate for borrowers will be above R . Thus all households except those that consume their endowments will have net trades with negative discounted present value. Lower-income households will be the borrowers, and they will therefore have higher MRS than do the higher-income households that lend (since the MRS of each household will be equal to the interest rate that it faces).

5.3. The Efficient-Contract Model

A contract in this environment is a function Γ that assigns a pair (b_i, v_i) to each income level y_i . As before, the argument of Γ should be conceived as an unverifiable report about income rather than as actual income. The image of Γ represents a net trade

²⁴ We are going to focus on the consumption bundles of savers and of non-defaulting borrowers. The qualitative conclusions that we derive will be robust to changing this specific assumption.

at date 0 and a guaranteed level of expected utility that the household will receive from consumption at date 1.

$$\forall i \forall j \left[b_j \geq -y_i \implies w(y_i + b_i) + v_i \geq w(y_i + b_j) + v_j \right]. \quad (33)$$

Note that, if a household with actual income y_i would receive an infeasible net trade by reporting income y_j , then the question of utility comparison is irrelevant. Therefore the incentive compatibility condition (33) has the logical form of an implication.²⁵

Define

$$U(\Gamma) = \sum_{i=1}^n \pi_i u(y_i + b_i, v_i). \quad (34)$$

(No confusion with the infinite-horizon utility function $U(X)$ of (1) will arise.) Now the efficient-contract allocation can be characterized as the solution of the problem:

$$\text{Maximize } U(\Gamma) \text{ subject to (24), (25), and (33)}. \quad (35)$$

The qualitative features of the solution are well known, if all households' consumption is strictly above the minimum level at both dates and if investment is strictly positive (that is, $\sum_{i=1}^n \pi_i b_i < 0$). In particular, the following three conditions hold:²⁶

$$\forall i < n \left[b_i + C(v_i) > b_{i+1} + C(v_{i+1}) \right], \quad (36)$$

$$\forall i < n \left[C'(v_i) w'(y_i + b_i) < 1 \right], \quad (37)$$

²⁵ This situation of immediate non-survival as a consequence of misrepresentation contrasts sharply with the potential for misrepresentation of endowments in models where a household is able to borrow by overstating *future* income about which it has private information, and where an intermediary is later left "holding the bag" because the contractually specified repayment is infeasible. This latter kind of informational environment is usually used (for example, by Farmer (1988) and by Gale and Hellwig (1985)) to derive liquidity constraints in an equilibrium setting.

²⁶ Analogues of these conditions, but with weak inequality rather than strict inequality, are proved by Hart (1983), for example. Oh and Green (1991) have proved the strict-inequality versions. Besides strengthening the results in this way, they have given a direct proof that avoids asserting a constraint-qualification condition. This assertion is required for the validity of the usual proof, but to the best of our knowledge it has not yet been established.

and

$$C'(v_n)w'(y_n + b_n) = 1. \tag{38}$$

Since $b_i + C(v_i)$ is the present discounted value of the net trade received by households with income y_i at date 0, (36) shows that the efficient contract succeeds in providing some insurance in present discounted value terms. That is, households with low temporary income (and thus *ceteris paribus* with lower present discounted value of endowment) will receive net trades with higher present discounted value. An obvious way to effect such insurance would be to have a lump-sum tax, the proceeds of which would be used to subsidize borrowing at an interest rate below the MRT. Condition (37) is consistent with the existence of such a scheme. However, (38) imposes the condition that $MRS = MRT$ on households at the highest income level. This is another way, in addition to the failure of Hall's martingale condition (5), to see that the optimal contract cannot be implemented by a budget constraint alone. The best description of the efficient-contract allocation may be that lower-income households are being rationed in their access to subsidized credit. This is a substantially different sort of rationing than is contemplated by any of the liquidity-constraint models discussed above.

6. Summary and Conclusions

The following table summarizes the comparisons that we have made in the preceding section. This table supposes that all households consume at levels strictly above their minimum consumption levels at both dates, and that investment is strictly positive.²⁷ In the labels for this table, MRS and PDV refer to the marginal rate of substitution and the present discounted value of net trade for a household having a particular status in the model (for example, borrowers' PDV). Where we refer to lenders, in the case of the

²⁷ Strictly positive investment is sufficient for the first-order conditions (27) and (38) to hold with equality. Note that the equality between MRS and MRT is contrary to the assumption of the usury-law model.

efficient-contract model we mean specifically the households at the highest income level y_n . Where we refer to borrowers, in the case of the default model we mean specifically borrowers who repay their debt at date 1, and in the case of the efficient-contract model we mean specifically households at the lowest income level. By Δ PDV (resp. Δ MRS), we refer to the difference between the PDV (resp. MRS) of the highest-income and lowest-income households. In the entries of the table, the symbols 0, +, and - indicate whether the relevant quantity is equal to, greater than or less than, respectively, the analogous quantity in the permanent-income model. The symbol ? in an entry indicates that the quantity is indeterminate.

Comparisons among the Model

MODEL	Permanent Income	Usury Law	Default	Monopoly	Efficient Contract
Lenders' PDV	0	-	0	-	-
Borrowers' PDV	0	+	-	-	+
Δ PDV	0	-	+	?	-
Lenders' MRS	0	-	0	-	0
Borrowers' MRS	0	?	0	+	-
Δ MRS	0	-	0	-	+

This table makes it clear that, in principle, the equilibrium allocations of all five of the models can be distinguished from one another on the basis of observable features. One caveat might be that the intertemporal MRT of the economy plays an essential role in assessing the features (especially for PDV) and that the imprecision of measuring the MRT would make the comparisons envisioned here untrustworthy. We believe that this problem may have the least significance for the comparisons in terms of Δ PDV and Δ MRS. That is, we would expect measurements of systematic differences between households with disparate net trades to be more robust than the corresponding measurements of the net

trades of the individual households (or homogeneous groups of households). If this is so, then it is appropriate to ask how completely the various models can be distinguished from one another in terms of Δ PDV and Δ MRS alone. Most of the models can be distinguished from one another in terms of these two features; the exception is that the usury-law model and the monopoly model are indistinguishable. The efficient-contract model, in particular, can be distinguished from all of the liquidity-constraint models in this way. The possibility of such discrimination among models is the main thesis that we have argued here.

References

- Allen, F. (1985), "Repeated Principal-Agent Relationships with Lending and Borrowing," *Economics Letters*, **17**, 27-31.
- Atkeson, A. and Lucas R. E., Jr. (1991), "On Efficient Distribution with Private Information," (mimeo, University of Chicago).
- Banerjee, A. and Newman, A. (1991), "Risk-Bearing and the Theory of Income Distribution," *Review of Economic Studies*, **58**, 211-235.
- Box, G.E.P. and Jenkins, G.M. (1976), *Time Series Analysis: Forecasting and Control*, (San Francisco: Holden-Day).
- Campbell, J. and Deaton, A. (1989), "Why Is Consumption So Smooth?," *Review of Economic Studies*, **56**, 357-373.
- Farmer, R. E. A. (1988), "Money and Contracts," *Review of Economic Studies*, **55**, 431-446.
- Friedman, M. (1957), *A Theory of the Consumption Function* (Princeton: Princeton University Press).
- Gale, D. and Hellwig, M. (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem," *Review of Economic Studies*, **52**, 647-663.
- Gertler, M. (1988), "Financial Capacity and Output Fluctuations in an Economy with Multiperiod Financial Relationships," (mimeo, University of Wisconsin).
- Green, E. J. (1987), "Lending and the Smoothing of Uninsurable Income," in Prescott, E. and Wallace, N. (eds.) *Contractual Arrangements for Intertemporal Trade* (Minneapolis: University of Minnesota Press), 3-25.
- Hall, R. E. (1978), "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, **86**, 971-987.
- Hall, R. E. (1989), "Consumption," in Barro, R. J. (ed.) *Modern Business Cycle Theory* (Cambridge: Harvard University Press), 153-177.

- Hall, R. E. and Mishkin, F. S. (1982), "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," *Econometrica*, **50**, 461-481.
- Hart, O. D. (1983), "Optimal Labor Contracts under Asymmetric Information: An Introduction," *Review of Economic Studies*, **50**, 3-35.
- Hayashi, F. (1987), "Tests for Liquidity Constraints: A Critical Survey and Some New Observations," in Bewley, T. (ed.) *Advances in Econometrics, Fifth World Congress*, Vol. 2, (Cambridge: Cambridge University Press), 91-120.
- King, M. (1985), "The Economics of Saving: A Survey of Recent Contributions," in Arrow, K. J. and Honkapohja, S. (eds.), *Frontiers of Economics* (New York: Blackwell), 227-294.
- Lillard, L. A. and Willis, R. J. (1978), "Dynamic Aspects of Earning Mobility," *Econometrica*, **46**, 985-1012.
- Marimon, R. (1988), "Wealth Accumulation with Moral Hazard," (mimeo, University of Minnesota).
- Oh, S.-N. and Green, E. J. (1991), "A Reexamination of Efficient Contracts," (mimeo, University of Minnesota).
- Ostroy, J. M. (1980), "The No-Surplus Condition as a Characterization of Perfectly Competitive Equilibrium," *Journal of Economic Theory*, **22**, 183-207.
- Pischke, S. (1990), "Individual Income, Incomplete Information and Aggregate Consumption," (mimeo, Princeton University).
- Taub, B. (1989), "Aggregate Fluctuations, Interest Rates and Repeated Insurance under Private Information," (mimeo, University of Illinois).
- Thomas, J. and Worrall, T. (1990), "Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem," *Journal of Economic Theory*, **51**, 367-390.
- Townsend, R. M. (1982), "Optimal Multiperiod Contracts and the Gain from Enduring Relationships under Private Information," *Journal of Political Economy*, **90**, 1166-

1186.

Townsend, R. M. (1988), "Information Constrained Insurance: The Revelation Principle Extended," *Journal of Monetary Economics*, **21**, 411-450.