A Comparison of Gasoline Sales Taxes and Automobile Efficiency Taxes as Methods for Reducing Gasoline Consumption

John Danforth

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The views expressed herein are solely those of the author and do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

Ed Foster offered valuable comments regarding the formulation of the problem considered here. N. J. Simler and Craig Swan also made helpful suggestions. Any errors which remain are my own.
Members of the present administration have recently resurrected
the notion of an automobile efficiency tax, i.e., an excise tax which is
negatively related to mileage ratings, as a means for reducing gasoline
consumption. This proposal has been offered either as an alternative
to, or in conjunction with, gasoline sales taxes. It is clear to most
economists that a sales tax is a more direct method of affecting gasoline
consumption than an efficiency tax and, hence, likely to be more effective
in achieving that end. There is evidently some disagreement with this
position within the economics profession though, since many of those
policy makers advocating efficiency taxes are economists.

In this paper I argue for the widely held view that more
direct methods are more efficient. I compare sales tax and automobile
efficiency tax schemes within a very simplified general equilibrium
model of an economy. Firms are assumed to operate with one input constant
coefficient production functions, and all individuals are assumed to
have identical utility functions and initial endowments. I find that
within this simple model, gasoline sales taxes will definitely lead to a
reduction in gasoline consumption, while automobile efficiency taxes may
actually increase gasoline consumption. Furthermore, I show that for
any efficiency tax program which does reduce fuel consumption, the sales
tax program which accomplishes the same reduction in fuel consumption
yields a Pareto superior allocation of other resources.

What alternative specification of the economy would favor
efficiency taxes over sales taxes? It is my impression that the appeal
of efficiency taxes is based on the observation that individual endowments
differ, contrary to the assumptions of the present model, and this tax
redistributes wealth from rich Cadillac owners to poor Vega owners. If
this impression is accurate, then one should ask if an automobile efficiency
tax is an efficient way to redistribute wealth, and I believe that
question is even more easily answered than the one considered in this
text.

The Model

We assume that there are \( I \) individuals with identical strictly
quasi-concave and twice continuously differentiable utility functions,
\( V \). \( V \) has as its arguments \( \alpha_A Q_A, \alpha_B Q_B, H-L, \) and \( Q_C \); where \( \alpha_i \) is the
number of miles driven per automobile of type \( i \), \( Q_i \) is the number of
automobiles of type \( i \) purchased, \( H \) is the total hours available for work
or leisure, \( L \) is the hours worked, and \( Q_C \) is the quantity of a composite
good consumed. The partial derivatives of \( V \), with respect to each of
its arguments, are assumed to be strictly positive.

Individual budget constraints prior to any tax-cum-subsidy
programs are of the form

1) \[ wL \geq P_A Q_A^i + P_B Q_B^i + P_C Q_C^i + P_G Q_G^i \]

2) \[ Q_G^i \geq \alpha_A^i Q_A^i b_A + \alpha_B^i Q_B^i b_B \]

\( Q_A^i = 0 \) or 1, \( Q_B^i = 0 \) or 1, \( Q_C^i \geq 0 \), and \( Q_G^i \geq 0 \)

where \( w \) is the wage rate and \( P_A, Q_A^i, P_B, Q_B^i, P_C, Q_C, P_G, \) and \( Q_G^i \) are the
price and quantity of automobiles of type \( A \), automobiles of type \( B \), the
consumption good, and gasoline, respectively. \( b_A \) and \( b_B \) are the different
gallons of gasoline used per mile of travel in automobiles of type \( A \) and
of type \( B \).
Production of each commodity in the economy is accomplished by means of a constant returns to scale technology. In particular,

\[ Q_A = aL_A, \quad Q_B = bL_B, \quad Q_C = cL_C, \quad \text{and}, \quad Q_G = gL_G \]

where \( L_j, j = A, B, C, \) and \( G \) is the quantity of labor devoted to the production of \( Q_j \). Production of \( Q_j \) is carried out by \( N_j \) firms. Each firm, \( n_j \), seeks to maximize \( P_j Q_j^{n_j} - wL_j^{n_j} \). The total labor demand,

\[
\sum_{n_A=1}^{N_A} L_A^{n_A} + \sum_{n_B=1}^{N_B} L_B^{n_B} + \sum_{n_C=1}^{N_C} L_C^{n_C} + \sum_{n_G=1}^{N_G} L_G^{n_G},
\]

will equal the total labor supply, \( IL^I \), in equilibrium.

The absolute price level in a nonmonetary economy such as this one is indeterminate. As a consequence it will prove convenient to designate one commodity as numeraire and to express all prices in units of that commodity. We have chosen labor services as the numeraire for the present analysis, and prices hereafter are interpretable as the rate at which any particular market good can be acquired in exchange for labor services.

We assume that the economy is at an initial competitive equilibrium with positive \( Q_A, Q_B, Q_C, \) and \( Q_G \) being produced. Recall that a competitive equilibrium is a market clearing allocation of goods and services resulting from consumer utility maximization and firm profit maximization at prices which are viewed as parameters by all market participants. Prices and quantities associated with this initial competitive equilibrium are denoted with a single asterisk. (Thus, the aggregate outputs and prices in the no tax equilibrium are: \( Q_A^*, Q_B^*, Q_C^*, Q_G^*, L^*, P_A^*, P_B^*, P_C^*, P_G^*, 1 \).)
Gasoline Sales Tax

Instituting a sales tax on gasoline and rebating the proceeds to consumers leads to a new competitive equilibrium. We denote the prices and quantities associated with this equilibrium with two asterisks. Aggregate outputs and prices with the sales tax are thus $Q_A^{**}$, $Q_B^{**}$, $Q_C^{**}$, $P_A^{**}$, $P_B^{**}$, $P_C^{**}$, and $l$. Our first theorem asserts that aggregate gasoline consumption will decline in response to the gasoline sales tax levy.

Theorem 1: $Q_C^{**} > Q_C^*$. 

Proof:

There are four possibilities for equilibrium automobile production, since all individuals are identical and individual automobile demand is a zero-one variable:

1) $Q_A^{**} = 0$ and $Q_B^{**} = 0$
2) $Q_A^{**} = I$ and $Q_B^{**} = 0$
3) $Q_A^{**} = 0$ and $Q_B^{**} = I$
4) $Q_A^{**} = I$ and $Q_B^{**} = I$.

The theorem's validity is obvious for Case 1. There are only minor differences in the proof of the theorem for Cases 2, 3, and 4, so only Case 2, $Q_A^{**} = I$ and $Q_B^{**} = 0$, is dealt with here.

Notice first that profit maximizing firms with the production functions given on page 2 will produce positive but finite quantities of $Q_A$, $Q_B$, $Q_C$, and $Q_G$ only if $P_A = 1/a$, $P_B = 1/b$, $P_C = 1/c$, and $P_G = 1/g$. Therefore, these must be the pre-tax competitive equilibrium prices.
But this implies that the sales tax equilibrium individual consumption bundles must have been feasible at the pre-tax equilibrium prices. That is,

\[(I) \quad L^{**} = P^*Q_A^{**} + P^*Q_C^{**} + P^*Q_G^{**},\]

since the right-hand side is merely the sum of the labor demands in the three active industries and all individuals are identical so the \((Q_A^{**}, Q_C^{**}, Q_G^{**}, L^{**})\) must be feasible and equal for all \(i\). Thus, by revealed preference

\[(II) \quad V(\alpha_A^{**}Q_A^{**}, 0, H-L^{**}, Q_C^{**}) \leq V(\alpha_A^{**}Q_A^{**}, \alpha_B^{**}Q_B^{**}, H-L^{**}, Q_C^{**}).\]

Since \(Q_A^{**}\) and \(Q_G^{**}\) are positive and finite, we must again have \(P_A^{**} = 1/a\) and \(P_G^{**} = 1/g\). Also, \(Q_C^{**}\) and \(Q_B^{**}\) are finite, so \(P_C^{**} \leq 1/c\) and \(P_B^{**} \leq 1/b\). \(P_B^{**}\) is net of taxes though, and hence each individual's budget constraint in the sales tax equilibrium has a different slope than in the initial equilibrium. This fact, together with (II) and the continuous differentiability of \(V\), imply

\[(III) \quad P^*Q_G^{**} + L^* < P^*Q_A^{**} + P^*Q_B^{**} + P^*Q_C^{**} + (1+t)P^*Q_G^{**}.\]

Subtracting (I) from (III) we obtain

\[(IV) \quad (1+t)P^*(Q_G^{**}-Q_A^{**}) + (L^*-L^{**}) < P^*Q_A^{**} + P^*(Q_C^{**}-Q_G^{**}),\]

since \(P_A^{**} = P_G^{**} = P_C^{**}\) with inequality holding only if \(Q_C^{**} = 0\).

We know from (I) that

\[L^*-L^{**}) + P_G^{**}(Q_G^{**}-Q_A^{**}) = P_B^{**}Q_B^{**} + P_C^{**}(Q_C^{**}-Q_G^{**}),\]

but this implies

\[(V) \quad (L^*-L^{**}) + (1+t)P_G^{**}(Q_G^{**}-Q_A^{**}) \geq P_B^{**}Q_B^{**} + P_C^{**}(Q_C^{**}-Q_G^{**}).\]
if \((Q_G^{**} - Q_G^*) \geq 0\), since \(t > 0\). However,

\[ P_B^* \geq P_B^{**} \text{ and } P_C^*(Q^*-Q^{**}) \geq P_C^{**}(Q^*-Q^{**}) \]

because \(P_C^* = P_C^{**}\) whenever \(Q^{**} > 0\).

Thus, replacing the right-hand side of (V) with the smaller quantity

\[ \frac{P_B^*Q_B^* + P_C^*(Q^*-Q^{**})}{P_B^{**}Q_B^{**} + P_C^{**}(Q^*-Q^{**})} \]

contradicts (IV), hence (V) cannot be true, and we must have

\[ Q_G^{**} < Q_G^* \].

**Automobile Efficiency Tax**

We may next analyze the effects of the gallons per mile based excise tax or "efficiency tax" scheme. As in the preceding case, a new equilibrium may be associated with this scheme. We shall denote the prices and quantities existing in such an equilibrium by the symbol \(\hat{\cdot}\).

Thus, the vector of equilibrium aggregate quantities and prices is

\[ (\hat{Q}_A, \hat{Q}_B, \hat{Q}_C, \hat{Q}_G, \hat{L}, \hat{P}_A, \hat{P}_B, \hat{P}_C, \hat{P}_G, \hat{1}) \].

Each consumer will be maximizing \(V\) subject to

\[ \frac{b_A \hat{Q}_A + b_B \hat{Q}_B}{L} + L_i = (\hat{P}_A + b_A \tau)Q_A^i + (\hat{P}_B + b_B \tau)Q_B^i + \hat{P}_C Q_C^i + \hat{P}_G Q_G^i \]

and

\[ Q_G^i = \alpha_{A}^i \hat{Q}_A + \alpha_{B}^i \hat{Q}_B \]

where \(\tau\) is the efficiency tax factor. Each producer will be maximizing profits at the prices, \(\hat{P}_A, \hat{P}_B, \hat{P}_C, \hat{P}_G\), and \(\hat{w} = 1\).
Theorem 2:

The institution of an automobile efficiency tax and offsetting rebate may increase, decrease, or leave unchanged equilibrium gasoline consumption.

Proof (by example):

In order to simplify the following examples we shall assume labor supply is fixed for each individual.\textsuperscript{1/}

Example 1—Increase in Gas Consumption

\[
V(\alpha^*_A, \alpha^*_B, Q^*_A, Q^*_B, Q^*_C) = (\alpha^*_A Q^*_A)^{1/2} + \alpha^*_B Q^*_B/40 + 10\ln Q^*_C + [H-L^i+1]^{1/2}/100 - 1/100
\]

\[b_A = .2, \quad b_B = .1, \quad a = (19)^{-1}, \quad b = (10)^{-1}, \quad c = 1, \quad g = 1, \quad H = 129, \quad I = 100.\]

Equilibrium prior to the institution of the tax is:

\[Q^*_A = 100, \quad Q^*_B = 100, \quad \alpha^*_A = 100, \quad \alpha^*_B = 400, \quad Q^*_C = 4000, \quad Q^*_G = 6000, \]

\[P^*_A = 19, \quad P^*_B = 10, \quad P^*_C = 1, \quad P^*_G = 1, \quad \text{and} \quad w^* = 1.\]

Let \(\tau = 50\). Since all consumers are alike and \(Q^*_A\) and \(Q^*_B\) are zero-one decision variables, there are only four possibilities for zero-net-revenue equilibrium individual rebates, 0, \(b_A\tau = 10\), \(b_B\tau = 5\), and \(b_B\tau + b_B\tau = 15\).

Also note that because of our simple production functions, we need only consider \(P^*_A = 19, \quad P^*_B = 10, \quad P^*_C = 1, \quad P^*_G = 1, \quad \text{and} \quad w = 1.\)

Table I indicates maximum consumer utility associated with auto purchases indicated in the column headings and rebate magnitudes indicated in the left-hand column.
Only when the largest row element appears on the main diagonal will the economy be in a zero-net-government revenue competitive equilibrium. This occurs when the rebate magnitude is 5 per household. The equilibrium values for the various outputs are:

\[ \hat{Q}_A = 0, \hat{Q}_B = 100, \hat{Q}_C = 4000, \hat{Q}_G = 7900, \hat{\alpha}_B = 790. \]

Notice that gasoline consumption has increased by 1,900 gallons.

**Example 2—Decrease in Gasoline Consumption**

\[
\begin{align*}
V(\alpha_{A'}^{i-j}, \alpha_{B'}^{i-j}, H-L, Q_{C'}) &= (\alpha_{A'}^{i-j})^{3/2} + 2.5 \log \alpha_{B'}^{i-j} + 10 \log Q_{C'} + \\
&\quad [H-L+1]^{1/2}/100 - 1/100
\end{align*}
\]

\[
b_A = .2, \ b_B = .1, \ a = (40)^{-1}, \ b = (25)^{-1},
\]

\[
c = 1, \ g = 1, \ H = 135, \ I = 100.
\]

Equilibrium prior to the institution of the tax is:

\[
Q_A^* = 100, Q_B^* = 100, \alpha_A^* = 100, \alpha_B^* = 100, Q_C^* = 4000, Q_G^* = 3000,
\]

\[
P_A^* = 40, P_B^* = 25, P_C^* = 1, P_G^* = 1, \text{ and } w^* = 1.
\]

Let \( \tau = 50 \). Again, we may construct a table indicating the maximum consumer utility associated with each possible auto purchase scheme and government rebate magnitude (see Table II). Once again a zero-net-government revenue equilibrium is attained with a rebate of 5. Equilibrium values for the various outputs are:

\[
\hat{Q}_A = 0, \hat{Q}_B = 100, \hat{Q}_C = 8800, \hat{\alpha}_B = 220, \hat{Q}_G = 2200, \hat{P}_A = 40,
\]

\[
\hat{P}_B = 25, \hat{P}_C = 1, \hat{P}_G = 1, \text{ and } \hat{w} = 1.
\]
Gasoline consumption has fallen by 800 gallons as a result of the efficiency tax-cum-rebate.

Example 3--No Change in Gasoline Consumption

\[ V(\alpha_A^{1\frac{1}{4}}, \alpha_B^{1\frac{1}{4}}, H-L^{1\frac{1}{4}}, Q_C) = \ln \alpha_A + 2.5 \ln \alpha_B Q_B + 10 \ln Q_C + \frac{[H-L^{1\frac{1}{4}} + 1]^{1\frac{1}{4}}}{100} - 1/100 \]

\[ b_A = .2, b_B = .1, a = (40)^{-1}, b = (25)^{-1}, \]

\[ C = 1, G = 1, H = 135, I = 100. \]

The competitive equilibrium for this economy has:

\[ Q_A^* = 100, Q_B^* = 100, \alpha_A = 25.9, \alpha_B = 129.7, Q_C^* = 5188, \]

\[ Q_G^* = 1815, P_A^* = 40, P_B^* = 25, P_C^* = 1, P_G^* = 1, \text{ and } w^* = 1. \]

The form of the utility function insures that for any level of income greater than the cost of one type A and one type B car, both will be purchased. Thus, letting \( \tau = 50 \) once again, it is clear that a zero-net-government revenue competitive equilibrium will obtain only when the rebate is 15. The equilibrium in this situation is precisely that which existed in the absence of the efficiency tax scheme.

**Welfare Comparisons of Sales and Efficiency Taxes**

It was demonstrated in the preceding section that an automobile efficiency tax may or may not succeed in reducing the consumption of gasoline in our simple economy. In this section we show that even if an efficiency tax will reduce gasoline consumption, a gasoline sales tax having the same impact on gasoline consumption would be strictly preferred by the consumers in our economy.
Again, we use the symbols *, **, and ^ to designate equilibrium quantities and prices in the economy when there are no taxes, when there is a gasoline sales tax rate \( \bar{\tau} \) and offsetting subsidy, and when there is an automobile efficiency tax of rate \( \bar{\tau} \) and offsetting subsidy, respectively.

**Theorem 3:**

If \( 0 < Q_G^* = \hat{Q}_G < Q_G^\hat{\ } \)

then

\[
V(\hat{Q}_A^i, \hat{Q}_B^i, \hat{Q}_C^i; H-L^i, H-L^i, H-L^i, H-L^i, H-L^i, H-L^i) > V(\hat{Q}_A^i, \hat{Q}_B^i, H-L^i, H-L^i).
\]

**Proof:**

Note first that either \( \hat{Q}_A \) or \( \hat{Q}_B \) equals zero, since if both are positive, the pre- and post-tax equilibria will be identical, violating the conditions of the theorem. We assume, without loss of generality, that \( \hat{Q}_A = 1 \) and \( \hat{Q}_B = 0 \).

Finally, since net taxes are zero for all individuals under both schemes and \( \hat{Q}_G = Q_G^* \), one can manipulate the post-tax equilibrium budget constraints to obtain

\[
-L^* + P^{**}Q_A^{**} + P^{**}Q_B^{**} + P^{**}Q_C^{**} = P^{**}Q_A^i + P^{**}Q_C^i - \hat{L}.
\]

Consumers, however, will be facing budget constraints with differing slopes under the two tax schemes, since \( P^{**} = \hat{P}_A, P^{**} = \hat{P}_C \) but \( (l+\bar{\tau})P_G^{**} \neq \hat{P}_G \) and \( P^{**} \neq \hat{P}_A + b_A\bar{\tau} \). Our assumption on the continuous differentiability of \( V \) thus rules out identical utility maximizing consumption bundles under the two tax schemes. Therefore, \((\hat{Q}_A^i, \hat{Q}_C^i, \hat{Q}_G^i)\) is feasible but not chosen at prices \((P^{**}, P^{**}, P^{**}, P^{**}, l)\), and the theorem follows by revealed preference/.
Conclusions

In this note we have compared the efficiency of automobile efficiency taxes and gasoline sales taxes as means for reducing domestic consumption of gasoline. We found that within a very simple general equilibrium model the sales tax is to be preferred to the efficiency tax for at least two reasons. First, an automobile efficiency tax with offsetting income subsidy can not be counted on to reduce gasoline consumption (Theorem 2), whereas a gasoline sales tax and rebate will definitely achieve a reduction in fuel use (Theorem 1). Second, if gasoline consumption can be reduced by the same amount with either an automobile efficiency tax-cum-rebate or a gasoline sales tax-cum-rebate, the latter program will yield an allocation of resources, which is Pareto superior to the allocation which is attained with the former program (Theorem 3).
Footnotes

1/ This is not inconsistent with our assumptions regarding individual preferences, since the marginal utility of leisure could be made positive but small so that the maximum possible labor supply always would be forthcoming.

2/ Each individual would be maximizing utility along the same budget hyperplane he faced in the no tax equilibrium if $\hat{Q}_A$ and $\hat{Q}_B$ were both positive.
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