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Expectation Traps and Monetary Policy*

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ABSTRACT

Why is inflation persistently high in some periods and low in others? The reason may be absence of commitment in monetary policy. In a standard model, absence of commitment leads to multiple equilibria, or *expectation traps,* even without trigger strategies. In these traps, expectations of high or low inflation lead the public to take defensive actions, which then make accommodating those expectations the optimal monetary policy. Under commitment, the equilibrium is unique and the inflation rate is low on average. This analysis suggests that institutions which promote commitment can prevent high inflation episodes from recurring.

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1. INTRODUCTION

Many countries have experienced prolonged periods of costly, high inflation as well as prolonged periods of low inflation. The United States and other industrialized countries suffered through a high inflation episode in the 1970s and are now enjoying a low inflation episode. Why do such inflation episodes occur? What should be done to prevent high inflation episodes from recurring? These are two central questions in monetary economics.

One way to answer these questions builds on the time inconsistency literature pioneered by Kydland and Prescott (1977) and Barro and Gordon (1983). This literature points to absence of commitment in monetary policy as the main culprit behind high inflation. Static versions of the models in this literature have a unique equilibrium. Inflation rates can fluctuate only if the underlying fundamentals do. Often, however, it is difficult to identify the changes in the underlying fundamentals that could have generated the episodes of high and low inflation. In infinite-horizon versions of the Kydland-Prescott and Barro-Gordon models, trigger strategies can be used to produce the observed inflation outcomes. But such models have embarrassingly many equilibria. It is hard to know what observations would be ruled out by such trigger strategy equilibria.

Our work here does not include trigger strategies, but it is squarely within the tradition of the time inconsistency literature in pointing to absence of commitment as the main culprit behind the observed episodes of high and low inflation. We make three contributions. We show how the economic forces in the Kydland-Prescott and Barro-Gordon models can be embedded into a standard general equilibrium model. We find that once these forces have been so embedded, inflation rates can be high for prolonged periods and low for prolonged periods, even though trigger strategies are explicitly ruled out. And we think our model is a promising first step toward developing empirically plausible models of inflation in the United States and other countries.

In the Kydland-Prescott and Barro-Gordon models, the key trade-off is between the benefits of higher output from unexpected inflation and the costs of realised inflation. This is true in our general equilibrium model as well. In our model, unexpected inflation raises output because some prices are sticky. This rise in output has benefits because producers have monopoly power and the unexpected inflation reduces the monopoly distortion. However, realised inflation is costly in our general equilibrium model because households must use previously accumulated cash to purchase some goods, called *cash goods*. Higher realised inflation forces households to substitute toward other goods, called *credit goods*. This substitution tends to lower welfare. Thus, by design, the general equilibrium model captures the trade-off between the benefits of unexpected inflation and the costs of realised inflation in the Kydland-Prescott and Barro-Gordon framework.

This way of capturing the trade-off leads to multiple equilibria in our general equilibrium model. Private agents' expectations of high or low inflation can lead these agents to take defensive actions, which in effect trap the monetary authority. The agents' actions make validating their expectations the optimal monetary policy action. The main defensive action we focus on is that sticky price firms set high prices if they expect high inflation and low prices if they expect low inflation. Given these expectations and the associated defensive actions, the monetary authority chooses policy optimally by equating the marginal benefits of unexpected inflation to the marginal costs of realised inflation. We show analytically that marginal benefits equal marginal costs for at least two sets of policies and allocations. Our analytical procedure focuses only on necessary conditions for monetary authority optimality. In a large class of parameterisations, we use numerical methods to identify situations in which the necessary conditions are sufficient and those in which they are not.

In our basic model, the equilibrium interest rate is independent of shocks to technology and government consumption. Many researchers have presented evidence that the response of interest rates and other financial variables to shocks is very different in high and low inflation episodes. This evidence motivates us to develop a variant of our model with a variable payment technology in which this behaviour occurs. This variant provides a related, but different, channel which also leads to multiplicity of equilibria.

In this variable payment technology model, households can choose the fraction of goods purchased with cash and the fraction purchased with credit. If households expect high inflation, they defensively choose to purchase few goods with cash, so that the costs of realised inflation are low. Given the gains of inflation, the monetary authority then has an incentive to choose a high level of inflation. If households expect low inflation, however, they do not take defensive actions and choose instead to purchase many goods with cash, so that the marginal costs of realised inflation are high. Given the gains of inflation, the monetary authority then has an incentive to choose a low level of inflation. These considerations reinforce the sources of multiplicity in our original model with a fixed payment technology, so that the variable payment technology model also has multiple equilibria.

In the variable payment technology model, the interest rate responds to shocks. The interest rate response to a technology shock turns out to switch sign between the high and low inflation equilibria, while output increases in this shock in both equilibria. Our model also implies higher volatility of nominal variables in high inflation episodes than in low inflation episodes. The sign switch and volatility implications are supported by an examination of cross-country data (as in Albanesi, Chari and Christiano, 2002a). While a variety of other models might imply higher volatility, it is hard to see which models would generate the sign switch observation.

Following Chari, Christiano and Eichenbaum (1998), we call the kind of multiplicity identified here an *expectation trap* because the public's defensive actions induced by changes in expectations in effect trap policymakers into accommodating the expectations. Chari, Christiano and Eichenbaum rely on trigger strategies to generate expectation traps. The use of trigger strategies, however, is problematic because with them virtually any inflation outcome can be rationalized as an equilibrium. Here we instead restrict attention to Markov equilibria that rule out trigger strategies. Also, the Markov equilibrium of Chari, Christiano, and Eichenbaum is at a corner. One contribution of our work here is that it obtains an interior equilibrium. (See also Neiss, 1999.)

The notion of an expectation trap may shed light on the continuing debate about the interpretation of the successful and, thus far, sustained reduction in inflation since the 1970s in the United States and other industrialised countries. (See Sargent, 1999, for a discussion of this debate.) Our work here raises the possibility that the inflation of the 1970s was a high inflation expectation trap and that inflation may have declined simply because the public switched to a low inflation expectation trap. Since the structure of policymaking institutions has not fundamentally changed, we here raise the possibility that these countries could once again fall into a 1970s-style high inflation expectation trap.

In our model, without monetary policy commitment, the economy experiences spells of high and low inflation, somewhat like those experienced by many countries. With commitment, the equilibrium is unique, the nominal interest rate is zero and the inflation rate is low on average. Thus, our analysis points to absence of commitment as the chief culprit behind high and variable inflation. Our analysis suggests that institutions which promote the ability of central banks to commit to future actions can lead to welfare gains. Such institutions include laws that protect central bank independence and laws that provide appropriate incentive contracts for central bankers (as in, for example, Persson and Tabellini, 1993).

We proceed as follows. In Section 2, we describe the model with a fixed payment technology. In Section 3, we analyse the equilibria of this model and show that multiplicity is possible. In Section 4, we analyse an economy with a variable payment technology. In Section 5, we discuss the main forces behind the expectation traps we find, and in Section 6, we describe the relationship of our work to that of others. The final section concludes.

2. A MONETARY GENERAL EQUILIBRIUM ECONOMY

Our economy extends and modifies the Lucas and Stokey (1983) model with cash and credit goods in two ways. One modification is that, in our model, a subset of prices are set in advance by monopolistic firms. The other modification is that, as does Svensson (1985), we require households to use currency accumulated in the previous period to purchase cash goods in the current period. We assume that the monetary authority chooses monetary policy to maximize the welfare of the representative household. Our modifications imply that the trade-off the monetary authority confronts resembles that in the Kydland-Prescott and Barro-Gordon models. The sticky price modification implies that an unanticipated monetary expansion tends to raise output and welfare. The cash-in-advance modification implies that the inflation associated with a monetary expansion reduces welfare by reducing the consumption of cash goods relative to credit goods.

Our infinite-horizon economy is composed of a continuum of firms, a representative household and a monetary authority. The sequence of events within a time period t is as follows. First, the shock to the production technology, θ, and the shock to government consumption, g, are realised. We refer to $s = (\theta, g)$ as the *exogenous state* and assume that s follows a Markov process.[1] Then a fraction, μ of firms—the *sticky price firms*—set their prices. The remaining fraction, $1-\mu$, of firms are called *flexible price firms.* The average price set by sticky price firms is denoted $P^e(s)$. This price, as well as all other nominal variables, is scaled by the beginning-of-period aggregate stock of money.

Next, the monetary authority chooses the interest rate, R.[2] We denote the policy rule that the monetary authority is expected to follow by $R(s)$. The state of the economy after the monetary authority makes its choice, the *private sector's state,* is (s, R). Let $X(s, R)$ denote the money growth rate associated with (s, R). The production, consumption and employment decisions of the households and firms and the pricing decisions of the flexible price firms depend on the private sector's state.

In what follows, we first describe the problems of the household and firms in our economy given (s, R) and expected future monetary policy, $R(s)$. We then set up the monetary authority's problem and define a Markov equilibrium. The key part of a Markov equilibrium is that the monetary authority chooses policy optimally. To define the monetary authority's problem, we must specify the private equilibrium allocations as functions of the monetary authority's policy variable, R. We refer to these functions as a *private sector equilibrium.* A *Markov equilibrium* is a private sector equilibrium in which policy is set optimally.

2.1. *The representative household*

We begin with the household's problem. In each period, the household consumes a continuum of differentiated goods as in the work of Blanchard and Kiyotaki (1987) and supplies labour. The representative household's preferences are $\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$, where $0 < \beta < 1$,

$$c_t = \left[\int_0^1 c_t(\omega)^\rho d\omega\right]^{\frac{1}{\rho}}, \; u(c, n) = \frac{\left[c(1-n)^\psi\right]^{1-\sigma}}{1-\sigma},$$

$c_t(\omega)$ denotes consumption of the type ω good, n_t denotes labour time and $1-n_t$ leisure time, $0 < \rho < 1$, $\psi > 0$ and $\sigma > 0$.

Each good in this continuum is one of four types. A fraction μ of the goods are produced by sticky price firms, and a fraction $1-\mu$ are produced by flexible price firms. A fraction z of all goods consist of goods paid for with cash, and a fraction $1-z$ consist of goods paid for with credit. We refer to z as the *payment technology parameter*. The sticky and flexible price firms are randomly distributed over cash and credit goods. Thus, a fraction μz of goods are sticky price goods purchased with cash, a fraction $(1-\mu)z$ are flexible price goods purchased with cash, a fraction $\mu(1-z)$ are sticky price goods purchased with credit and a fraction $(1-\mu)(1-z)$ are flexible price goods purchased with credit. Since prices for goods within each type turn out to be the same, utility maximisation implies that the amounts purchased within each type of good are also the same. Let c_{11} and c_{12} denote the quantities of cash goods purchased from sticky and flexible price firms, respectively, and let c_{21} and c_{22} denote the quantities of credit goods purchased from sticky and flexible price firms, respectively. Then we have that

$$c = \left[\mu z c_{11}^{\rho} + (1-\mu) z c_{12}^{\rho} + \mu(1-z) c_{21}^{\rho} + (1-\mu)(1-z) c_{22}^{\rho}\right]^{\frac{1}{\rho}} . \tag{1}$$

Let A denote the nominal assets of the household, carried over from the previous period. In the asset market, the household trades money, M, and one-period bonds, B, with other households. The asset market constraint is

$$M + B \leq A. \tag{2}$$

Recall that nominal assets, money and bonds are all scaled by the aggregate stock of money. We impose a no-Ponzi constraint of the form $B \leq \bar{B}$, where $\bar{B}$ is a large, finite upper bound on bond holdings.

The household's cash-in-advance constraint is

$$P^e(s)\left[\mu z c_{11} + q(s,R)(1-\mu) z c_{12}\right] \leq M, \tag{3}$$

where $P^e(s)$, again, denotes the average price set by sticky price firms and $q(s,R)P^e(s)$ denotes that set by flexible price firms. Note that $q(s,R)$ is the relative price of flexible to sticky price goods. Nominal assets evolve over time as follows:

$$\begin{aligned} & zP^e(s)\left[\mu c_{11} + q(s,R)(1-\mu)c_{12}\right] + (1-z)P^e(s)\left[\mu c_{21} + q(s,R)(1-\mu)c_{22}\right] \\ & +X(s,R)A' \leq W(s,R)n + D(s,R) + \left[X(s,R) - 1\right] + M + RB. \end{aligned} \tag{4}$$

In (4), $W(s,R)$ denotes the nominal wage rate and $D(s,R)$ the profits after lump-sum taxes. Note that A' is multiplied by $X(s,R)$. This multiplication reflects that all nominal variables are scaled by the beginning-of-period aggregate stock of money, and A' is the household's nominal assets scaled by the next period's aggregate money stock. The next period's aggregate money stock is simply the current stock multiplied by the growth rate $X(s,R)$.

Our description of the asset market is somewhat different from that of Svensson (1985). In Svensson's model, each household sees itself as facing a cash-in-advance constraint in which only previously accumulated cash can be used for cash goods purchases. In our setup, an individual household does not face any such constraint; society as a whole faces it. This constraint manifests itself as an equilibrium condition that $M = 1$. The interest rate adjusts to ensure that the equilibrium condition is satisfied, so that the household optimally uses only previously accumulated cash for cash goods purchases. The analysis with Svensson's formulation leads to identical results.

Consider the household's asset, goods and labour market decisions. Given that the household expects the monetary authority to choose policy according to $R(s)$ in the future, the household solves the following problem:

$$v(A,s,R) = \max_{n,M,A',c_{ij};\ i,j=1,2} u(c,n) + \beta E_{s'}[v(A',s',R(s'))|s] \tag{5}$$

subject to (3), (4) and nonnegativity on allocations. Here we have substituted out for B in (4) using (2). The solution to (5) yields the household's decision rules, $d(A,s,R)$, where

$$d(A,s,R) = [n(A,s,R), M(A,s,R), A'(A,s,R), c_{ij}(A,s,R)], \tag{6}$$

for $i, j = 1, 2$.

2.2. *The firms and the economy's resource constraint*

Each of the differentiated goods in this economy is produced by a monopolist using the following production technology:

$$y(\omega) = \theta n(\omega),$$

where $y(\omega)$ denotes output and $n(\omega)$ denotes the employment level for the type ω good. Also, recall that θ is a technology shock that is the same for all goods. The household's problem yields demand curves for each good. The fraction, $1 - \mu$, of firms that are flexible price firms set their price to maximise profits subject to these demand curves. Because the household demand curves have constant elasticity, firms set prices as a fixed markup, $1/\rho$, above marginal cost, W/θ, so that the relative price of flexible to sticky price goods is given by

$$q(s, R) = \frac{W(s, R)}{P^e(s)\theta\rho}. \tag{7}$$

Sticky price firms set prices at the beginning of the period, after the exogenous shocks are realised. Here, as in the work of Blanchard and Kiyotaki (1987), sticky price firms must set their price in advance and must produce the amount of goods demanded at that price. These firms, like the flexible price firms, also wish to set their price as a markup, $1/\rho$, over marginal cost, W/θ. In order to do so, they need to forecast the wage rate, W. They do that by taking the wage rate as given by the private sector equilibrium. Thus, the wage they expect to prevail is $W(s, R(s))$. In equilibrium, then, the price set by sticky price firms satisfies this:

$$P^e(s) = \frac{W(s, R(s))}{\theta\rho}. \tag{8}$$

Turning to the government, we assume that there is no government debt, government consumption is financed with lump-sum taxes and government consumption is the same for all goods. As a result, the resource constraint for this economy is that

$$\theta n = g + z\left[\mu c_{11} + (1-\mu)c_{12}\right] + (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right],$$

where g denotes the exogenous shock to government consumption. Since there is no government debt, bond market-clearing requires that $B = 0$ and $A = 1$. Also, money market-clearing requires that $M = 1$.

2.3. *A private sector equilibrium*

We now define an equilibrium for each possible private sector state, (s, R), and future monetary policy rule, $R(s)$.

Definition 1. For each (s, R), given $R(s)$, a *private sector equilibrium* is a number, $P^e(s)$, and a collection of functions, $q(s, R)$, $W(s, R)$, $X(s, R)$, $v(A, s, R)$ and $d(A, s, R)$, such that the following hold:

1. The functions v and d solve (5).
2. Firms maximize profits; that is, $q(s, R)$ satisfies (7), and $P^e(s)$ satisfies (8).
3. The resource constraint is satisfied at $d(1, s, R)$.

4. The asset markets clear; that is, $A'(1, s, R) = M(1, s, R) = 1$.

Notice that a private sector equilibrium is defined for all values of R, not just for $R = R(s)$. We define a private sector equilibrium outcome as the allocations and prices that occur when $A = 1$ and actual policy, R, coincides with expectations of policy, $R(s)$:

Definition 2. For each s, a *private sector equilibrium outcome* is a collection of numbers, $P^e(s)$, $q(s, R(s))$, $W(s, R(s))$, $X(s, R(s))$, $v(1, s, R(s))$ and $d(1, s, R(s))$.

Combining (7) and (8), we have that in a private sector equilibrium outcome,

$$q(s, R(s)) = 1. \tag{9}$$

2.4. *The monetary authority's problem and a Markov equilibrium*

The monetary authority chooses R to maximize the representative household's discounted utility:

$$\max_{R} v(1, s, R), \tag{10}$$

where v is the value function in a private sector equilibrium. Recall that a private sector equilibrium takes as given the evolution of future monetary policy. Thus, in solving (10), the monetary authority implicitly does too.

We now have the ingredients needed to define a Markov equilibrium.

Definition 3. A *Markov equilibrium* is a private sector equilibrium and a monetary policy rule, $R(s)$, such that $R(s)$ solves (10).

Note that in a Markov equilibrium, the current money growth rate does not affect the household's discounted utility starting from the next period since that rate does not affect the next period's state. Therefore, the monetary authority faces the static problem of maximizing the current period's utility, and we only have to describe how the current interest rate affects current allocations. In a way parallel to that used for a private sector equilibrium outcome, we define a Markov equilibrium outcome as a Markov equilibrium in which actual policy, R, coincides with expectations of policy, $R(s)$:

Definition 4. For each s, a *Markov equilibrium outcome* is a collection of numbers, $P^e(s)$, $q(s, R(s))$, $W(s, R(s))$, $X(s, R(s))$, $v(1, s, R(s))$ and $d(1, s, R(s))$, where $R(s)$ is the monetary policy rule associated with a Markov equilibrium.

A useful benchmark for assessing Markov equilibrium outcomes is the commitment equilibrium. With commitment the monetary authority chooses policy for all periods and states at the beginning of period 0. It is straightforward to show that in such an equilibrium the nominal interest rate, R, equals unity in all periods and states. The reason is as follows. In our economy, decisions are distorted in two ways. One is that monopoly power creates a wedge between the marginal rates of substitution between consumption and leisure and the associated marginal rate of transformation. The other distortion is that a positive nominal interest rate induces households to consume an inefficiently low level of cash goods. Since prices are set after the realisation of shocks, they are not sticky in equilibrium; therefore, the monetary authority cannot reduce the monopoly wedge. Since the monetary authority can control the interest rate, it optimally eliminates the interest rate distortion by setting $R = 1$.

3. ANALYSIS OF EQUILIBRIUM

In our analysis, we decompose the first-order condition associated with the monetary authority's problem, (10), into the benefits and costs of inflation. To obtain these benefits and costs, we begin by characterising a private sector equilibrium. We then solve the monetary authority's problem. We show that, generically, at least two allocations satisfy the necessary conditions for a Markov equilibrium. We present numerical examples in which these allocations also satisfy the sufficient conditions for a Markov equilibrium.

3.1. *Characterising private sector equilibrium*

We first characterise a private sector equilibrium outcome. We use this characterisation to construct a private sector equilibrium.

With arguments of functions omitted for convenience, the first-order necessary conditions for household and firm optimisation are that

$$\begin{aligned}
\frac{u_{11}}{u_{12}} &= \frac{\mu}{(1-\mu)\,q}, && (11)\\
\frac{u_{21}}{u_{22}} &= \frac{\mu}{(1-\mu)\,q}, && (12)\\
\frac{u_{11}}{u_{21}} &= \frac{zR}{1-z}, && (13)\\
\frac{u_{12}}{u_{22}} &= \frac{zR}{1-z}, && (14)\\
-u_n &= \frac{\theta\rho u_{22}}{(1-\mu)(1-z)}, && (15)\\
\frac{Xu_{21}}{P^e\mu(1-z)} &= \beta E_{s'}[v_1(1,s',R(s'))|s], && (16)
\end{aligned}$$

where u_{ij} denotes the partial derivative of u with respect to c_{ij} and v_1 denotes the partial derivative of v with respect to its first argument. Equations (11) and (12) equate the marginal rate of substitution between sticky and flexible price goods to the relative price of these goods q, and equations (13) and (14) equate the marginal rate of substitution between cash and credit goods to their relative price, the interest rate. Equation (15) is obtained by noting that the marginal rate of substitution between labour and consumption of flexible price credit goods is equal to the ratio of the nominal wage to the price of flexible price goods. This ratio is simply the markup in (7). Finally, (16) is the intertemporal Euler equation for asset accumulation.

The cash-in-advance constraint can be written as

$$\mu z c_{11} + q(1-\mu) z c_{12} \leq \frac{1}{P^e}. \tag{17}$$

A necessary condition for the household problem to be well-defined is that

$$R \geq 1. \tag{18}$$

It is easy to show that the cash-in-advance constraint holds with equality if $R > 1$ and that if the cash-in-advance constraint is slack, then $R = 1$. These observations imply that the appropriate complementary slackness condition is that

$$\left\{ \frac{1}{P^e} - [\mu z c_{11} + q(1-\mu) z c_{12}] \right\} (R-1) = 0. \tag{19}$$

The resource constraint is that

$$g + z\,[\mu c_{11} + (1-\mu) c_{12}] + (1-z)\,[\mu c_{21} + (1-\mu) c_{22}] = \theta n. \tag{20}$$

We now can use the preceding equations to compute a private sector equilibrium outcome. Recall that a private sector equilibrium is conditioned on some given policy rule, $R(s)$. We fix the policy rule $R(s)$.We set $q = 1$, and for each s, we use (11)–(15), (19) and (20) to compute the six numbers $P^e(s)$, $n(1, s, R(s))$, $c_{ij}(1, s, R(s))$, for $i, j = 1, 2$. Notice that one of the equations in (11)–(14) is redundant and can be deleted. Thus, we can use these six independent equations to compute the six numbers of interest. The wage rate, money growth rate and value function in a private sector equilibrium outcome are straightforward to compute. For future use, note that $c(1, s, R(s))$ is obtained from (1) using $c_{ij}(1, s, R(s))$.

Given $P^e(s)$ from a private sector equilibrium outcome, we can compute a private sector equilibrium as follows. For each s and each R, we use (11)–(15), (19) and (20) to compute the functions $n(1, s, R)$, $c_{ij}(1, s, R)$, for $i, j = 1, 2$, and $q(s, R)$. As above, note that $c(1, s, R)$ is obtained from (1) using $c_{ij}(1, s, R)$.

3.2. *The monetary authority's problem*

The monetary authority's problem is static because we focus on Markov equilibria and the economy has no state variables. Recall that, in a Markov equilibrium, policymakers face dynamic problems only if their decisions affect future state variables. Without state variables, the monetary authority's problem is simply one of choosing current policy to maximise current period utility.

We let

$$U(s, R) = u\left[c(1, s, R), n(1, s, R)\right]$$

denote the utility associated with an interest rate, R, where $c(1, s, R)$, $n(1, s, R)$ are the private sector equilibria just constructed. The monetary authority's problem is now

$$\max_R U(s, R), \tag{21}$$

subject to $R \geq 1$.[3] Then the policy rule associated with a Markov equilibrium is the value of $R(s)$ for each s that solves (21).

3.3. *A Markov equilibrium*

We can think of constructing a Markov equilibrium in at least two ways. One is to treat (21) as defining an operator that maps the space of policy rules into the space of policy rules. The Markov equilibrium policy rule can be constructed by finding a fixed point of this operator. Another way to think of this construction is to think of (9), (11)–(20) and the first-order necessary condition associated with (21) as a system of equations used simultaneously to solve for a Markov equilibrium. The first-order condition is obtained by using (11)–(20) to obtain the derivatives of consumption and labour with respect to R, holding P^e fixed, and evaluating these derivatives at a point that solves (11)–(20) with $q = 1$. If the first-order condition for the monetary authority is also sufficient, then the two approaches are equivalent. We pursue the second approach here.

We show that, generically, at least two allocations satisfy the necessary condition associated with (21). In a large class of parameterisations for our economy, we verified numerically that this necessary condition is also sufficient. We also derive a relationship between the payment technology parameter z and the allocations and prices in a Markov equilibrium. We use this relationship when we discuss a Markov equilibrium with a variable payment technology.

The first-order condition associated with a solution to the monetary authority's problem, (21), is

$$U_R(s,R) = u_c c_R + u_n n_R \leq 0, \tag{22}$$

with equality if $R > 1$. In (22), U_R is the derivative of U with respect to R and u_c, u_n are derivatives of the utility function with respect to consumption and employment, respectively. Also, c_R, n_R are the derivatives of the private sector equilibrium functions, $c(1,s,R)$ and $n(1,s,R)$, with respect to R. If $R(s)$ is a Markov equilibrium policy rule, then it satisfies (22).

Now we show that (22) can be decomposed into a part that captures the incentives to increase inflation because of the presence of monopoly power and a part that captures the disincentives arising from the resulting reduction in cash goods consumption. Specifically, we prove the following proposition:

Proposition 1. *Suppose $R(s)$ is a Markov equilibrium policy rule. Then there exist a strictly positive function, $f(c_1,c_2)$, and a pair of functions, $\psi_{ID}(R)$ and $\psi_{MD}(R,z)$, given by*

$$\psi_{MD}\left(R,z\right) = -(1-\rho)R^{\frac{1}{\rho-1}} + \frac{R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}} + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}} + \frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}\left(\frac{z}{1-z}R^{\frac{\rho}{\rho-1}} + 1\right)}, \tag{23}$$

and

$$\psi_{ID}\left(R\right) = \left(R-1\right)R^{\frac{1}{\rho-1}}, \tag{24}$$

such that

$$U_R(s,R(s)) = f(c_1,c_2)\left[\psi_{MD}\left(R(s),z\right) - \psi_{ID}\left(R(s)\right)\right],$$

where $c_1 = c_{11}(1,s,R(s)) = c_{12}(1,s,R(s))$ and $c_2 = c_{21}(1,s,R(s)) = c_{22}(1,s,R(s))$. The *function, $f(c_1, c_2)$, is provided in the Appendix.*

Our notation emphasises the dependence of ψ_{MD} on z because this dependence plays an important role in our later discussion.

Before proving the proposition, we highlight three features. One is that in any interior equilibrium, $\psi_{ID}(R(s)) = \psi_{MD}(R(s), z)$, so that determining an equilibrium reduces to finding values of R for which the right side of (23) equals the right side of (24). Note also that, as we show below, the function $\psi_{MD}(R, z)$ can be interpreted as arising from the distortions induced by monopoly power and the function $\psi_{ID}(R)$ can be interpreted as the distortion arising from the inflation tax. This interpretation helps us to understand the costs and benefits that the monetary authority weighs in making its policy decision. Finally, note that the shocks, θ and g, do not enter into the functions, ψ_{MD} or ψ_{ID}. Thus, $R(s)$ does not depend on s.

Proof. We prove the proposition by proving a lemma. Consider first the function ψ_{MD}. To obtain this function, note that the efficient allocations in our economy satisfy this:

$$u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)} = 0. \tag{25}$$

The first term in (25) is the marginal disutility of labour associated with increasing labour input to credit goods production, say, and the second term is the marginal benefit from increased credit goods consumption. In our economy, the analogue of (25) is (15). Note that because of the presence of monopoly power, the right side of (15) is the same as the second term in (25) multiplied by $\rho < 1$. As a result, the net marginal benefit of increasing labour from its equilibrium value in our economy is positive. This distortion is due to monopoly power and suggests that the left side of (25) is a natural measure of the monopoly distortion in our economy. To isolate that measure in the monetary authority's problem, add and subtract $\theta u_{22} n_R / [(1-\mu)(1-z)]$ to and from (22), and rearrange terms, to obtain that

$$U_R = \left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)}\right] n_R + u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)}. \tag{26}$$

The term in brackets is our measure of the monopoly distortion.

In the Appendix, we prove the following lemma regarding the terms in (26):

Lemma 1. *In a Markov equilibrium,*

$$\left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)}\right] n_R = f(c_1, c_2)\psi_{MD}(R, z) \tag{27}$$

and

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1, c_2)\psi_{ID}(R), \tag{28}$$

where $\psi_{MD}(R, z)$ *and* $\psi_{ID}(R)$ *are as defined in* (23) *and* (24).

Proposition 1 then follows from Lemma 1. ‖

To see that ψ_{ID} is a measure of the inflation distortion, we use a simple consumer surplus type of analysis. In a monetary economy, let $D(r)$ denote the demand for real balances, m, with respect to the net interest rate, $r \equiv R - 1$. Let $g(m) = D^{-1}(r)$. Consumer surplus, S, is the area under the money demand function. A rise in the interest rate acts like a tax and reduces consumer surplus. We are interested in the marginal effects of this tax, namely, the derivative of S with respect to r:

$$\frac{dS}{dr} = \frac{dS}{dm}\frac{dm}{dr} = g(m)D'(r) = rD'(r).$$

In our economy,

$$D(r) = \frac{c}{1 + (1+r)^{\frac{1}{1-\rho}}},$$

where $c = c_1 + c_2$ denotes aggregate consumption. Therefore,

$$\frac{dS}{dr} = rD'(r) = -\frac{1}{1-\rho}\frac{c_2}{R}\left[R^{\frac{1}{\rho-1}}(R-1)\right] = -\frac{1}{1-\rho}\frac{c_2}{R}\psi_{ID}(R).$$

As we will see below, the key features of $\psi_{ID}(R)$ that deliver multiplicity are shared by $rD'(r)$. This result is one motivation for interpreting $\psi_{ID}(R)$ as the inflation distortion.

For another motivation, consider the following. Use $c_2/c_1 = R^{1/(1-\rho)}$ and the definition of ψ_{ID} to obtain $\psi_{ID}(R) = (R-1)c_1/c_2$. The net interest rate, $R-1$, measures the extent to which cash goods consumption is distorted relative to the efficient level. This distortion is akin to a tax. (See the work of Lucas and Stokey, 1983.) The base on which this tax is levied is the consumption of cash goods. Thus, one way to think of ψ_{ID} is as the product of a tax rate, $R-1$, and the base of taxation, c_1, scaled by a measure of the size of the economy, c_2. This reasoning provides an alternative motivation for using ψ_{ID} to measure the inflation distortion. In the efficient allocations, $R = 1$ and $\psi_{ID}(1) = 0$.

We now discuss some properties of the two distortions, ψ_{MD} and ψ_{ID}. From (23), the following is clear:

$$\psi_{MD}\left(R,z\right) \text{ is decreasing in } z, \text{ and } \lim_{R\to\infty}\psi_{MD}\left(R,z\right)=\frac{\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(\frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho}+\frac{\psi}{\rho}}>0. \tag{29}$$

Note that $\psi_{MD}\left(R,z\right)$ does not depend on the shocks θ and g. Next, inspecting (24), we have that $\psi_{ID}\geq 0$ and

$$\lim_{R\to\infty}\psi_{ID}(R)=\psi_{ID}(1)=0. \tag{30}$$

That is, there is no inflation distortion when the interest rate is high or low.

A numerical example helps illustrate the results in Proposition 1. We use $\mu=0.1$, $\rho=0.45$, $\psi=1$, $g=0.05$, $\theta=1$. Figure 1 displays the monopoly distortion, ψ_{MD}, and the inflation distortion, ψ_{ID}, for $R\in[1.0,4.5]$ and for $z=0.13$ and 0.15. The first-order necessary condition for monetary authority optimality is satisfied at the point where the two distortion functions intersect. The figure shows that the first-order necessary condition for monetary authority optimality is satisfied at $R=1.38$ and $R=2.07$ for $z=0.13$ and at $R=1.10$ and $R=3.17$ for $z=0.15$. For $z=0.15$, the inflation rate is somewhat below 10 percent in the low inflation equilibrium and just below 217 percent in the high inflation equilibrium.

From Proposition 1, (22) becomes

$$U_R=f(c_1,c_2)\psi(R,z)\leq 0, \tag{31}$$

with equality if $R>1$. Here $\psi(R,z)=-\psi_{ID}\left(R\right)+\psi_{MD}\left(R,z\right).$ Since $f(c_1,c_2)>0$, a solution to

$$\psi\left(R,z\right)\leq 0, \tag{32}$$

with equality if $R > 1$, satisfies the necessary condition for a Markov equilibrium. If (22) is also sufficient, then the interest rate, R, which solves (32) corresponds to a Markov equilibrium policy rule. Given an equilibrium value of the interest rate, we can solve for the allocations and other prices by setting $q = 1$ and using (11)–(15), (17) with equality and (20) for each value of θ and g.

We use the properties of the monopoly distortion function, ψ_{MD}, in (29) and the inflation distortion function, ψ_{ID}, in (30) to show that, generically, if this model has any Markov equilibria, it has at least two.

Proposition 2 (Generic Multiplicity). *Suppose that the monetary authority's first-order condition is sufficient for optimality. Then, generically, the model has either no Markov equilibria or at least two. Furthermore, the equilibrium interest rate does not depend on θ or g.*

Proof. A key property of the function $\psi(R, z)$ is that it is positive for R sufficiently large. This property follows from (29) and (30), which imply that

$$\lim_{R\to\infty} \psi(R, z) = \lim_{R\to\infty} [\psi_{MD}(R, z) - \psi_{ID}(R)] > 0.$$

Suppose first that $\psi(1, z) > 0$. Then, since $\psi(R, z)$ is positive at $R = 1$ and positive for large R, continuity implies that if $\psi(R, z)$ is ever zero, it must generically be zero at least twice. A nongeneric case occurs when the graph of $\psi(R, z)$ against R is tangent to the horizontal axis at a single value of R. Another nongeneric case is when $\psi(1, z) = 0$ and $\psi(R, z) > 0$ for $R > 1$. Both cases are nongeneric because for an arbitrarily larger value of z, the model must have multiple equilibria since $\psi(R, z)$ is strictly decreasing in z.

Suppose next that $\psi(1,z) < 0$. Then $R = 1$ satisfies (32) and corresponds to a Markov equilibrium. In addition, because $\psi(R,z) > 0$ for R sufficiently large, continuity implies that $\psi(R,z)$ must be equal to zero for at least one value of $R > 1$.

From (24), we have that ψ_{ID} does not depend on θ or g. Since ψ_{MD} does not depend on these variables either, the equilibrium interest rate, R, does not depend on θ or g. $\|$

We construct two examples to illustrate Proposition 2 and to compare outcomes between the low and high inflation equilibria. We also construct a third example to illustrate that the first-order condition of the monetary authority may not be sufficient for optimality. In all three examples, we use the values of μ, ρ, ψ, g and θ used in Figure 1. The first example has $z = 0.13$, the second has $z = 0.15$ and the third has $z = 0.152$. In Table 1, we display the candidate private sector equilibrium outcomes which satisfy the first-order condition of the monetary authority.

Note from Table 1 that c_1, R and P^e are quite different in the high and low inflation outcomes. The primary cost of high inflation is that it results in an inefficient level of cash goods consumption. For example, when $z = 0.13$, cash goods consumption (c_1) is more than 50 percent lower in the high inflation equilibrium than in the low inflation equilibrium. Note that credit goods consumption (c_2) changes little. Employment changes little either, because the bulk of labour is allocated to credit goods production.

We find that the first-order condition for monetary authority optimality is sufficient in the examples with $z = 0.13$ and $z = 0.15$. We determine sufficiency by examining the monetary authority's objective function, (21), at each value of P^e that corresponds to a private sector equilibrium. When $z = 0.13$, we find (in numerical results not reported here) that this objective function is concave for a range of values of R up to roughly 4, for both the high and low inflation values of P^e. When $z = 0.15$ or 0.152, this objective function is also concave for the low inflation value of P^e.

We illustrate this concavity by graphing the objective function when $z = 0.15$ for the low inflation value of P^e in Figure 2a. In Figure 2b we plot the corresponding objective function for the high inflation value of P^e. This figure shows that the objective function is locally, though not globally, concave. In addition, the figure shows that the high inflation candidate maximises the monetary authority's objective and is, therefore, an equilibrium.

In our third example, the low inflation candidate also turns out to be an equilibrium, but the high inflation candidate does not. In Figure 2c we plot the monetary authority's objective function at the high inflation value of P^e. This figure shows that, although $R = 3.27$ is a local maximum, the global maximum is $R = 1$.[4] This figure illustrates forcefully that the monetary authority's objective function must be checked globally, rather than just locally. Clearly, merely checking second-order conditions is not enough.

4. AN ECONOMY WITH VARIABLE PAYMENT TECHNOLOGY

Now we develop a version of our model with a *variable* payment technology. For convenience, we refer to the economy of the last section as the economy with a *fixed* payment technology. The variable payment version delivers a related but different channel by which monetary policy can be caught in an expectations trap. This version is also interesting in its own right as a model of financial intermediation. Finally, we use this model to analyse how equilibrium interest rates fluctuate in response to shocks.

4.1. *The alternative model*

In this version of the model, the fraction of goods purchased with cash, z, is chosen by the household at the beginning of the period, before the monetary authority chooses the interest rate, R. This timing assumption turns out to imply that we can characterise the equilibrium with two relationships. One relationship is between R and z for the fixed payment technology economy. The other relationship is obtained from the optimality condition associated with z.

Consider a version of the fixed payment technology economy in which each consumption good, $c(\omega)$, can be purchased with either cash or credit, with $\omega \in (0,1)$. For goods with $\omega > \bar{z}$ (where $\bar{z}$ is a parameter), the cost of purchasing with credit is zero. Purchasing goods with $\omega \leq \bar{z}$ on credit requires labour time. The household chooses the fraction $z \leq \bar{z}$ such that goods with $\omega < z$ are purchased with cash and goods with $\omega \geq z$ are purchased with credit. This cash-credit decision is made before the household knows which goods are produced by sticky or flexible price firms, so that the cash-credit good choice is independent of the type of firm.

The labour time required to purchase fraction z of goods with cash is given by $\eta(\bar{z} - z)^{1+\nu}/(1+\nu)$, where $\nu > 0$ is a parameter and $\eta > 0$ is a shock to the payment technology. Since this shock is realised at the beginning of the period, the exogenous state is now given by $s = (\theta, g, \eta)$. The household's labour time, l, is divided between time spent working in the market, n, and time spent on the payment technology as follows:

$$l = n + \frac{\eta(\bar{z} - z)^{1+\nu}}{1+\nu}. \tag{33}$$

Leisure time in the household's utility function is now given by $1 - l_t$, rather than $1 - n_t$.[5]

The decision problem of the household with respect to consumption, employment and asset accumulation described above is unchanged, except that now z is added to the state variables in (5) and (6) and labour is given in (33). The household chooses z to solve the following problem:

$$z(A,s) = \arg\max \; v(A,z,s,R(s)), \tag{34}$$

where v is the analogue of the value function in (5). Note that the choice of z depends on the household's expectations of the monetary authority's policy rule, $R(s)$, since z is chosen before R.

A Markov equilibrium, a private sector equilibrium and associated outcomes are defined in the obvious way. (For these definitions, see Albanesi, Chari and Christiano, 2002a.) We now characterise a Markov equilibrium for the variable payment technology economy. In addition to all the equilibrium conditions for the economy when z is fixed, this equilibrium must satisfy optimality of the choice of z.

4.2. *Equilibrium*

4.2.1. *Characterisation.* We analyse a Markov equilibrium for this economy by first establishing a relationship between the Markov equilibrium interest rate and the payment technology parameter, z, holding z fixed.

In Albanesi, Chari and Christiano (2002a), we show that Proposition 1 extends without change to the variable payment technology economy, namely, that the monetary authority's optimality condition can be written as $\tilde{f}(c_1,c_2)\psi(R,z) \leq 0$, where $\psi(R,z)$ is given in (31) and the function $\tilde{f}$ is strictly positive. Note that $\psi(R,z)$ does not depend on $s = (\theta, g, \eta)$. Thus, the equilibrium interest rate must satisfy the same conditions in the variable payment technology economy as in the fixed payment technology economy.

Consider the equilibrium interest rate holding z fixed, given by the solution to the analogue of (32). This solution depends on z, as can be seen from (23) and (24). We call this relationship between R and z the *interest rate policy correspondence* (or *policy correspondence,* for short). The following proposition establishes properties of this correspondence:

Proposition 3 (Interest Rate Policy Correspondence). *Suppose that the monetary authority's first-order condition is sufficient for optimality. Suppose also that for some* $z < \bar{z}$*, a Markov equilibrium exists. Then there is a critical value of* z*, say,* $\hat{z}$*, such that for* $z < \hat{z}$*, the economy has no Markov equilibria; for* $z = \hat{z}$*, it has at least one Markov equilibrium; and for* $z > \hat{z}$*, it has at least two Markov equilibria.*

Proof. First, we show that when z is sufficiently small, no interest rate less than $\bar{R}$ is an equilibrium, where $\bar{R}$ is arbitrarily large. Note from (23) that $\psi_{MD}(R, z) \to \infty$ as $z \to 0$ for all $R \in [1, \bar{R}]$ and from (24) that ψ_{ID} is bounded. Then, for some value of z, say, $\hat{z}_1$, $\psi(R, z)$ is strictly positive for all $z \leq \hat{z}_1$. Thus, no equilibrium interest rate is less than $\bar{R}$ for z sufficiently small.

Second, we show that no interest rate greater than $\bar{R}$ can be an equilibrium. We see from (24) that ψ_{ID} is bounded above by, say, k. Let $\hat{z}_2$ be defined by $\lim_{R \to \infty} \psi_{MD}(R, \hat{z}_2) = 2k$. Such a value of $\hat{z}_2$ exists from (29). Note also that for all $z \leq \hat{z}_2$, $\lim_{R \to \infty} \psi_{MD}(R, z) \geq 2k$. By definition of a limit, some interest rate $\bar{R}$ exists such that for all $R \geq \bar{R}$, $\psi_{MD}(R, \hat{z}_2) \geq 2k - \varepsilon$, where ε is, say, $k/2$. Therefore, for all $R \geq \bar{R}$, $\psi(R, \hat{z}_1) = -\psi_{ID}(R) + \psi_{MD}(R, \hat{z}_1) \geq k/2 > 0$. That is, no value of the interest rate greater than $\bar{R}$ is an equilibrium for $z = \hat{z}_2$. Since $\psi_{MD}(R, z)$ is decreasing in z, no value of the interest rate greater than $\bar{R}$ is an equilibrium for $z \leq \hat{z}_2$. We have established that if z is sufficiently small, then this economy has no equilibrium.

Next, $\psi_{MD}(R, z)$ is a continuous function of R and z. As z is increased from some arbitrarily low value, some first value of z exists so that $\psi(R, z) = 0$ for some R. Such a z — call it $\hat{z}$ — exists by our assumption that an equilibrium exists for some z. Consider increasing z above $\hat{z}$. Since ψ_{MD} is strictly decreasing, the graph of $\psi(R, z)$ against R must intersect the horizontal axis at two points, at least. Thus, for $z > \hat{z}$, the economy has at least two Markov equilibria. $\|$

Consistent with our theoretical findings, Figure 1 has shown that the inflation distortion does not depend on the payment technology parameter, z, while the monopoly distortion is decreasing in this parameter. We graph the policy correspondence in Figure 3.[6] Notice that when z is below $\hat{z} = 0.127$, an equilibrium does not exist. The reason, of course, is that when z is sufficiently small, the monopoly distortion lies above the inflation distortion, and the economy has no equilibrium. As z increases, the monopoly distortion declines. At a critical value of z, equal to $\hat{z}$, the economy has a unique equilibrium, and for values of z larger than this critical value, the economy has two equilibria. Notice that as z increases, the equilibrium interest rate falls in the low inflation equilibrium and rises in the high inflation equilibrium.

We now develop the second relationship between the equilibrium interest rate, R, and the payment technology parameter, z. We obtain this relationship from the first-order condition associated with the household's choice of z:

$$\left(1-\frac{1}{\rho}\right)\frac{1-R^{\frac{\rho}{1-\rho}}}{z+(1-z)R^{\frac{\rho}{1-\rho}}}=\frac{\psi\eta(\bar{z}-z)^{\nu}}{1-n-\frac{\eta(\bar{z}-z)^{1+\nu}}{(1+\nu)}}. \tag{35}$$

We can use the equations that define a private sector equilibrium, (9), (11)–(15), (17) with equality and (20) to substitute for labour, n, in (35). Doing so (in Lemma 2 in the Appendix), we obtain that

$$\frac{(\frac{1}{\rho}-1)(1-R^{\frac{\rho}{\rho-1}})}{z\left[(R^{\frac{1}{\rho-1}}-1)+\frac{\psi}{\rho}(R^{\frac{\rho}{\rho-1}}-1)\right]+(1+\frac{\psi}{\rho})}=\frac{\rho\eta(\bar{z}-z)^{\nu}}{\left(1-\frac{\eta(\bar{z}-z)^{1+\nu}}{1+\nu}\right)-\frac{g}{\theta}}. \tag{36}$$

For each z, at most one R solves (36). To see this result, note that the left side of (36) is increasing in R, while the right side does not depend on R. Let $R_p(z,g,\theta,\eta)$ denote the value of R that solves (36). We refer to this function as the *payment technology function,* or *payment function*, for short.

We develop the set of payment technology parameters z for which this function is defined as follows. As $R\to\infty$, the left side of (36) converges to $(1-\rho)/[(\rho+\psi)(1-z)]$, which at $z=0$ becomes $(1-\rho)/(\rho+\psi)$. The right side of (36) at $z=0$ is $\rho\eta\bar{z}^{\nu}/\{1-[\bar{z}^{1+\nu}\eta/(1+\nu)]-g/\theta\}$. If

$$\frac{1-\rho}{\rho+\psi}<\frac{\rho\eta\bar{z}^{\nu}}{1-[\bar{z}^{1+\nu}\eta/(1+\nu)]-g/\theta},$$

then the function $R_p(z,g,\theta,\eta)$ goes to infinity at some critical value of z, say z^*. Then the function is defined for $(z^*,\bar{z}]$. If not, then the function is defined for $(0,\bar{z}]$. Let the domain of the function be $(\tilde{z},\bar{z}]$, where $\tilde{z}=z^*$ if the above inequality holds and $\tilde{z}=0$ otherwise.

We know from (36) that R_p is decreasing in z, since the left side of (36) is increasing in z, while the right side is decreasing in z. We also know that R_p is increasing in g/θ and η since an increase in g/θ or η increases the right side of (36) and so increases R for a given value of z.

Each R, z which satisfies the policy correspondence, (31), and the payment function, (36), corresponds to a Markov equilibrium. The other allocations, prices and the monetary authority's policy rule can be obtained by solving (9), (11)–(16), (17) with equality and (20).

Next, we prove a proposition that under certain conditions on the policy correspondence, the economy has two Markov equilibria. We say that the policy correspondence is *horseshoe-shaped* if it satisfies the following conditions: (i) there exist two continuous functions, $R_c^1(z)$ and $R_c^2(z)$, which map $[\hat{z}, \bar{z}]$ into the space of interest rates with $R_c^1(z) < R_c^2(z)$, for $z \in (\hat{z}, \bar{z}]$, $R_c^1(\hat{z}) = R_c^2(\hat{z})$, and (ii) for all $z \in [\hat{z}, \bar{z}]$, the solution to (32) is either $R_c^1(z)$ or $R_c^2(z)$, where $\hat{z}$ is defined as in Proposition 2.

Proposition 4. *Suppose the policy correspondence is horseshoe-shaped. Then, generically, the economy with a variable payment technology satisfies the necessary conditions for a Markov equilibrium twice, if at all.*

Proof. Suppose to begin with that $\tilde{z} < \hat{z}$. Recalling that $R_p(\bar{z}) = 1$ and $R_c^1(\bar{z})$, $R_c^1(\bar{z}) \geq 1$, we can divide the proof into two cases: when $R_p(\bar{z}) < R_c^1(\bar{z})$ and when $R_p(\bar{z}) = R_c^1(\bar{z}) = 1$.

Consider the first case, that is, when $R_p(\bar{z}) < R_c^1(\bar{z}) \leq R_c^2(\bar{z})$. Here, if $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$, then since R_p is below R_c^1 and R_c^2 at $\bar{z}$ and above R_c^1 and R_c^2 at $\hat{z}$, by continuity, R_p must intersect at least once with each R_c^1 and R_c^2. Each of these intersections corresponds to a Markov equilibrium. If $R_p(\hat{z}) < R_c^1(\hat{z}) = R_c^2(\hat{z})$, then since R_p is below R_c^1 at both $\bar{z}$ and $\hat{z}$, R_p and R_c^1 intersect twice, if at all. The case when $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$ is clearly nongeneric.

Now consider the second case, that is, when $R_p(\bar{z}) = R_c^1(\bar{z}) = 1$. Then the policies and allocations associated with an interest rate of unity constitute an equilibrium. Generically, there must also be one other equilibrium. To see this, note that, generically, if $R_c^1(\bar{z}) = 1$, then some neighbourhood of $\bar{z}$ exists such that for all z in this neighbourhood, $R_c^1(z) = 1$. Since R_p is strictly decreasing, it follows that for z in this neighbourhood, $R_p(z) > 1 = R_c^1(z)$. Suppose that $R_p(\hat{z}) < R_c^1(\hat{z})$. Then since R_p is above R_c^1 in a neighbourhood of $\bar{z}$ and below R_c^1 at $\hat{z}$, by continuity R_p and R_c^1 must intersect at least once. Now suppose that $R_p(\hat{z}) > R_c^1(\hat{z}) = R_c^2(\hat{z})$. Then since R_p is below R_c^2 at $\bar{z}$ and above R_c^2 at $\hat{z}$, by continuity R_p must intersect at least once with R_c^2. We have established that in this second case, generically, the necessary conditions for equilibrium must be satisfied twice, if at all.

Suppose, finally, that $\tilde{z} > \hat{z}$. Then for z near $\tilde{z}$, R_p is arbitrarily large and must be larger than R_c^2. Exactly the same arguments used above can then be used to conclude that the necessary conditions for a Markov equilibrium must be satisfied twice, if at all. $\|$

The restriction that the policy correspondence be horseshoe-shaped is not severe. In Proposition 3 we have shown that for each $z > \hat{z}$, at least two interest rates belong to the policy correspondence. Using the implicit function theorem, we can represent these interest rates as continuous functions of z. Thus, the assumption that the correspondence is horseshoe-shaped only rules out the possibility that three or more interest rates belong to the correspondence. Extending the proof of Proposition 4 to this case is straightforward, but tedious. Furthermore, in all the numerical examples we have considered, the correspondence is horseshoe-shaped.

4.2.2. *Properties.* Now we describe the properties of the interest rate policy correspondence and the payment function for various realisations of shocks to the production technology and the payment technology in a numerical example. We do this in Figure 4. Figure 4a shows the interest rate correspondence and the payment function for two realizations of the production technology shock, θ, with the other shock held at its mean value. Figure 4b shows an analogous graph for the payment technology shock, η.

These graphs display four properties:

- The policy correspondence does not depend on the shocks, as we saw in Proposition 1.
- The payment function is decreasing in the interest rate, as discussed above.
- The payment function is increasing in η and decreasing in θ, as also discussed above.
- There are two Markov equilibria. The *low inflation equilibrium* is the one associated with the lower intersection of the interest rate correspondence and payment function; the *high inflation equilibrium* is the one associated with the higher intersection.

Figure 4 displays an interesting *sign switch* phenomenon; the interest rate response to a shock switches sign between the high and low inflation equilibria. For example, from Figure 4a, we see that the interest rate is increasing in the technology shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. We verified, for our numerical example, that in both equilibria, output is increasing in the technology shock. If technology shocks were the dominant shocks, then the correlation between output and the interest rate would be positive in the low inflation equilibrium and negative in the high inflation equilibrium. From Figure 4b, we see the sign switch for the payment shock: the interest rate is decreasing in this shock in the low inflation equilibrium and increasing in this shock in the high inflation equilibrium. In our numerical example, output is increasing in the payment shock in the low inflation equilibrium and decreasing in this shock in the high inflation equilibrium. So, if payment shocks were the dominant shocks, then the correlation would be negative in both equilibria. Therefore, in an economy with both shocks, the correlation of output and the interest rate is negative in the high inflation equilibrium and larger (perhaps even positive) in the low inflation equilibrium. We call this finding the *decreasing correlation implication*.

Our numerical examples also show that the volatility of interest rates is substantially smaller in the low inflation equilibrium than in the high inflation equilibrium. The reason is that the policy correspondence is flatter at the low inflation equilibrium than at the high inflation equilibrium. We call this finding the *increasing volatility implication.*

Elsewhere we find that both of these implications are supported by data for high and low inflation episodes in a cross-section of countries (Albanesi, Chari and Christiano, 2002a).

5. KEY FEATURES BEHIND EXPECTATION TRAPS

Now we ask which features of our model are crucial for generating expectation traps. We focus on six features and find that three of them play essential roles, one plays a convenient role and two play a subsidiary role. We also briefly discuss extensions of the analysis.

One feature essential for expectation traps to occur is the assumption that some prices are preset. To see the importance of this assumption, suppose that all prices are flexible. Then the monetary authority cannot reduce the monopoly distortion by making inflation higher than expected because monopolists simply raise their prices in response to expansionary monetary policy; the monopoly wedge is thus invariant to monetary policy. Furthermore, an expansionary policy is costly because it raises the price level, reduces consumption of cash goods and thereby reduces welfare. Indeed, these forces imply that the monetary authority gains by pursuing a contractionary policy, as long as $R > 1$. Thus, when all prices are flexible, the unique Markov equilibrium has $R = 1$. Technically, this result can be seen by setting $\mu = 0$ in (23). After some manipulation, we see that $\psi_{MD}(R, z) < 0$ for all R, so that the equilibrium has $R = 1$.

Another essential feature for expectation traps is the assumption that some prices are flexible. To see the importance of this assumption, suppose that all prices are fixed. Then expansionary monetary policy is welfare-enhancing because it reduces the monopoly distortion. Such a policy is not costly because with the price level fixed, cash goods consumption is also fixed. These forces imply that the monetary authority always gains by pursuing an expansionary monetary policy. As a result, no equilibrium exists. Technically, this result can be seen from (23), which implies that $\psi_{MD} \to \infty$ as $\mu \to 1$. Since ψ_{ID} is bounded, no equilibrium exists.

The final essential feature for expectation traps is that firms have monopoly power. Again, since the only benefit of expansionary monetary policy is to reduce the monopoly distortion, and since realised inflation is costly, the equilibrium without monopoly power has $R = 1$. Technically, suppose that $\rho = 1$ in (23). Then $\psi_{MD}(R, z) = 0$ for all R. And the unique equilibrium has $R = 1$.

The feature of our model that is convenient for expectation traps to occur is our timing assumption under which monetary injections cannot be used to purchase cash goods in the same period. This assumption implies that a monetary expansion, by raising prices, directly reduces consumption of cash goods. This reduction in the consumption of cash goods lowers welfare. An alternative timing assumption is that of Lucas and Stokey (1983), under which households can use the current monetary injection for current cash goods purchases. Mechanically, Lucas-Stokey timing amounts to adding current money growth to the right side of the cash-in-advance constraint. Since a monetary injection can then be used to purchase current cash goods, a greater than expected expansion does not directly change the mix of cash and credit goods consumption. Induced movements in the interest rate could change this mix, and that possibility is worth exploring.

The subsidiary features of our model in generating expectation traps concern the specification of preferences and that of money demand. These features are subsidiary in the sense that altering preferences and the nature of money demand eliminates the multiplicity of equilibria in the fixed payment technology model but not in the variable payment technology model.

In Albanesi, Chari and Christiano (2002b), we consider utility functions of the following form:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - an,$$

where a is a parameter. We show that the fixed payment technology economy has a unique equilibrium, but the variable payment technology economy may have multiple equilibria. Roughly, the reason is as follows. The monetary authority's first-order condition can be decomposed into two distortion functions: a monopoly distortion and an inflation distortion. Here the monopoly distortion is negative for $R = 1$ and positive for R sufficiently large. The inflation distortion function is similar to that in Figure 1. The two functions intersect only once, so the fixed payment technology economy has a unique equilibrium. The interest rate policy correspondence that is analogous to the one in Figure 3 becomes a downward-sloping graph. Nevertheless, since the payment function is also downward-sloping, there can be multiple intersections and multiple equilibria.

In terms of money demand, in Albanesi, Chari and Christiano (2002b) we present a model which shares many of the features of our model here, except for the specification of money demand. Interestingly, in that model, the monetary authority's first-order condition can be decomposed into terms similar to the inflation and monopoly distortion functions. Indeed, the form of the inflation distortion function turns out to be identical to the form here. In particular, substituting $c_2/c_1 = R^{1-\rho}$ into (24), we see that

$$\psi_{ID} = (R-1)\frac{c_1}{c_2}. \tag{37}$$

In both our 2002b work and here, the inflation distortion has the form given in (37). It is easy to see that when R is sufficiently large, ψ_{ID} is approximately given by Rm/c, where m denotes real balances and c denotes total consumption. In this study, as $R \to \infty$, m approaches zero faster than R approaches infinity, so that $\psi_{ID} \to 0$ as $R \to \infty$. In our 2002b work, however, ψ_{ID} does not go to zero because m approaches zero at the same rate as R approaches infinity. The key difference in the two studies lies in the interest elasticity of money demand. In the 2002b work, money demand is inelastic, and the inflation distortion function increases monotonically; and the inflation and monopoly distortion functions intersect only once. Thus, the fixed payment technology model has at most one equilibrium. With a variable payment technology, however, multiple equilibria are possible. In the fixed payment technology model here, money demand is elastic, the inflation distortion function is U-shaped and the inflation and monopoly distortion functions intersect twice.

In terms of extensions, it would be useful to ask whether these equilibria are stable under various learning schemes. In our numerical examples, including the one associated with Figure 1, the inflation distortion has a single-peaked Laffer curve shape, while the monopoly distortion is relatively flat. This shape is reminiscent of the shape of the monetary Laffer curve in analyses in which governments rely on inflation to finance expenditures. (See, for example, the work of Sargent and Wallace, 1981.) In those analyses, there are two steady-state levels of inflation, but only one of them is stable under a large class of learning schemes. In Albanesi, Chari and Christiano (2002a), we examine the stability properties of the equilibria in our model under a simple learning scheme. We find that both equilibria are stable. Exploring stability under a broader class of learning schemes would be of interest.

It would also be useful to analyse nonstationary equilibria in our model. Here we have focused on Markov equilibria which are stationary in the sense that they cannot depend on time. If we add calendar time as a state variable, then our model has other Markov equilibria as well. For example, one such equilibrium has the economy moving to the low inflation equilibrium in even-numbered periods and to the high inflation equilibrium in odd-numbered periods. More interesting is the possibility of sunspot-driven Markov equilibria in which a sunspot at the beginning of each period coordinates private agents' expectations and induces agents to pick the high or the low inflation equilibrium depending on the realisation of the sunspot. Such sunspot equilibria clearly exist and lead to volatility in inflation rates.

6. RELATED LITERATURE

Our work adds to a small literature in which the monetary authority explicitly chooses policy without commitment and without trigger strategies. Dedola (2002) and Khan, King and Wolman (2001) also generate multiple equilibria in such models. The mechanism for generating multiplicity in Dedola (2002) is similar to ours here. Khan, King and Wolman (2001) have a finite-horizon model in which in every period, one-third of firms choose the prices that they will charge for the next three periods. When firms expect high inflation, they choose high prices. The cost of not validating firms' expectations is that relative prices become distorted, and output falls. The staggered setting in the Khan, King and Wolman model plays the same analytic role as the Svensson (1985) timing assumption in our model. Both features have the effect that realised inflation is costly. In some of the literature using sticky prices, firms are allowed effectively to choose different prices for each period (though are not allowed to make these prices contingent on shocks). We conjecture that with such a formulation, the equilibrium in the Khan, King and Wolman model would be unique. The Khan, King and Wolman model also simply imposes money demand by adding an equation to the equilibrium of the model which requires consumption to equal real balances. This additional equation is not the same as a cash-in-advance constraint on households because firms and households will not accept money for the goods they receive in the last period. It would be interesting to ask whether in an infinite-horizon version of the Khan, King and Wolman model the interest elasticity of money demand would matter for multiplicity.

It is increasingly standard in monetary economics to characterise equilibria without commitment in stochastic economies by studying linear-quadratic approximations around a steady state. (See, for example, the work of Clarida, Galí and Gertler, 1999.) This literature simply assumes the steady-state values of policy variables like inflation. The difficulty is that in determining steady-state policy, the policymaker needs to forecast how private agents will respond to alternative policies. That is, an analysis like ours here is necessary in order to determine steady states before one knows around what point to conduct the approximation. If the linear-quadratic method yields deviations from the state which are independent of the value of that state, then the method may be a good approximation of equilibria that remain close to steady state. In economies with multiple steady states, like ours, however, the method would entirely miss any equilibria in which the economy switches from one steady state to another.

7. CONCLUSION

Here we have asked and answered two questions. One is, Why do economies experience persistent episodes of high and low inflation? The answer, according to a standard model, is that these episodes are expectation traps that arise due to the absence of commitment in monetary policy. The main force driving expectation traps is defensive actions taken by the public to protect themselves from inflation. Those actions reduce the costs of inflation for a benevolent monetary authority and induce the authority to supply the expected level of inflation. Our other question is, What should be done to prevent the high inflation episodes from recurring? The answer, according to our model, is to somehow make the monetary authority commit itself to a policy, for then the economy has a unique equilibrium with low inflation on average. That is, our study suggests that there are gains from setting up institutions which increase commitment to future monetary policies.

Notes

[1]Notice that we do not include the beginning-of-period aggregate stock of money in our states. In our economy, all equilibria are neutral in the usual sense that if the initial money stock is doubled, an equilibrium exists in which real allocations and the interest rate are unaffected and all nominal variables are doubled. This consideration leads us to focus on equilibria which are invariant with respect to the initial money stock. We are certainly mindful of the possibility of equilibria which depend on the money stock. For example, if multiple equilibria in our sense exist, then trigger strategy–type equilibria which are functions of the initial money stock can be constructed. In our analysis, we exclude such equilibria.

[2]In Albanesi, Chari and Christiano (2002a), we show that this specification of the monetary authority's choice variable is equivalent to one in which the monetary authority chooses the money growth rate.

[3]Technically, the set of interest rates should also be limited to those in which (11)–(15) and (17)–(20) have a solution. Our analysis of the monetary authority's problem uses a first-order condition approach which only asks whether small deviations are optimal. The implicit function theorem can be used to show that in some neighbourhood of an equilibrium, (11)–(15) and (17)–(20) have a solution. Thus, we will not have to deal with whether the allocation functions are well-defined for arbitrary interest rates.

[4]Of course, $R = 1$ is not a Markov equilibrium, because $P^e = 171.6$ and $R = 1$ is not part of a private sector equilibrium outcome.

[5]For payment technology models with similar features, see Cole and Stockman (1992), Schreft (1992), Ireland (1994), Dotsey and Ireland (1996), Lacker and Schreft (1996), Aiya-

gari, Braun and Eckstein (1998), and Freeman and Kydland (2000).

[6]In all the numerical examples we have studied, the necessary conditions also turn out to be sufficient.

APPENDIX

Proofs of Lemmas 1 and 2

Lemma 1. *In a Markov equilibrium,*

$$\left[u_n + \frac{\theta u_{22}}{(1-\mu)(1-z)}\right] n_R = f(c_{1,}c_2)\psi_{MD}\left(R,z\right) \tag{38}$$

and

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1, c_2)\psi_{ID}\left(R\right), \tag{39}$$

where $\psi_{MD}\left(R,z\right)$ *and* $\psi_{ID}\left(R\right)$ *are as defined in* (23) *and* (24).

Proof. To prove Lemma 1, we use the necessary and sufficient conditions for an interior private sector equilibrium. Using our functional form assumptions, we can reduce (11)–(15) to

$$\begin{aligned} c_{12} &= c_{11} q^{\frac{-1}{1-\rho}} && (40)\\ c_{21} &= c_{11} R^{\frac{1}{1-\rho}} && (41)\\ c_{22} &= c_{21} q^{\frac{-1}{1-\rho}} && (42)\\ \frac{\psi}{\rho} c^{\rho} c_{22}^{1-\rho} &= \theta(1-n). && (43) \end{aligned}$$

We have omitted (14) because (11)–(14) include only three linearly independent equations. These expressions together with (19) and (20) are necessary and sufficient conditions for a private sector equilibrium.

Lemma 1 is established using (40)–(43), (17) with equality and (20) to construct functions $c_{ij}(s, P^e, R)$, $q(s, P^e, R)$ and $n(s, P^e, R)$, differentiating these functions with respect to R and evaluating the derivatives at $q = 1$. Mechanically, we first drop n from the system by substituting out for n in (43) using (20). Then we differentiate (40)–(42) and simplify to obtain one equation containing the derivatives of c_{11} and q with respect to the interest rate, $c_{11,R}$ and q_R. We use (17) to obtain another equation in these derivatives. We can then evaluate all the other derivatives. We prove the lemma in two parts

Lemma 1a. *In a Markov equilibrium,*

$$\frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} = f(c_1, c_2)\psi_{MD}(R, z), \tag{44}$$

where $f(c_1, c_2) > 0$ *for* $c_1, c_2 > 0$ *and* $\psi_{MD}(R, z)$ *is given in* (23).

Proof. We substitute for n from (20) and for c from (1) into (43) to obtain that

$$\begin{aligned} & \frac{\psi}{\rho}\left[z\mu c_{11}^{\rho} + z(1-\mu)c_{12}^{\rho} + (1-z)\mu c_{21}^{\rho} + (1-z)(1-\mu)c_{22}^{\rho}\right] c_{22}^{1-\rho} \\ = \quad & \theta - g - z\left[\mu c_{11} + (1-\mu)c_{12}\right] - (1-z)\left[\mu c_{21} + (1-\mu)c_{22}\right]. \end{aligned}$$

Differentiating with respect to R, we get that

$$\begin{aligned} & z\left[\mu c_{11,R} + (1-\mu)c_{12,R}\right] + (1-z)\left[\mu c_{21,R} + (1-\mu)c_{22,R}\right] \\ & +\psi\left[z\mu c_1^{\rho-1}c_{11,R} + z(1-\mu)c_1^{\rho-1}c_{12,R} + (1-z)\mu c_2^{\rho-1}c_{21,R} + (1-z)(1-\mu)c_2^{\rho-1}c_{22,R}\right] c_2^{1-\rho} \\ & +\frac{\psi}{\rho}(1-\rho)c^{\rho}c_2^{-\rho}c_{22,R} = 0, \end{aligned} \tag{45}$$

where all derivatives are evaluated at a value of P^e such that $q = 1$. Here, when $q = 1$,

$c_1 = c_{11} = c_{12}$ and $c_2 = c_{21} = c_{22}$. Now we differentiate (40)–(42) with respect to R to obtain that

$$
\begin{aligned}
c_{12,R} &= c_{11,R} - \frac{c_1}{1-\rho} q_R && (46)\\
c_{21,R} &= c_{11,R} R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho} && (47)\\
c_{22,R} &= c_{21,R} - \frac{c_2}{1-\rho} q_R. && (48)
\end{aligned}
$$

Differentiating (17) with equality and substituting for $c_{12,R}$ from (46), we obtain that

$$
\mu z c_{11,R} + (1-\mu) z \left(c_{11,R} - \frac{c_1}{1-\rho} q_R \right) + (1-\mu) z c_1 q_R = 0.
$$

Simplifying, we obtain that

$$
q_R = \frac{1-\rho}{\rho(1-\mu)c_1} c_{11,R}. \qquad (49)
$$

From (46)–(49), using $(c_2/c_1)^{1-\rho} = R$, we obtain that

$$
\mu c_{11,R} + (1-\mu) c_{12,R} = c_{11,R} - \frac{(1-\mu)c_1}{1-\rho} q_R = c_{11,R}[1-(1/\rho)], \qquad (50)
$$

$$
\begin{aligned}
\mu c_{21,R} + (1-\mu) c_{22,R} &= c_{21,R} - \frac{(1-\mu)c_2}{1-\rho} q_R && (51)\\
&= c_{11,R} R^{\frac{1}{1-\rho}} \left(1 - \frac{1}{\rho}\right) + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho} && (52)
\end{aligned}
$$

$$
c_{22,R} = c_{11,R} R^{\frac{1}{1-\rho}} \left(1 - \frac{1}{\rho(1-\mu)}\right) + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}. \qquad (53)
$$

Substituting from (49)–(53) into (45), we obtain that

$$
\begin{aligned}
& zc_{11,R}[1-(1/\rho)] + (1-z)\left[c_{11,R}[1-(1/\rho)]R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}\right] \\
& +\psi z c_1^{\rho-1} c_2^{1-\rho} c_{11,R}[1-(1/\rho)] + \psi(1-z)\left[c_{11,R}[1-(1/\rho)]R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}\right] \\
& +\frac{\psi}{\rho}(1-\rho)c^{\rho}c_2^{-\rho}\left\{c_{11,R}R^{\frac{1}{1-\rho}}\left[1-\frac{1}{\rho(1-\mu)}\right] + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}\right\} = 0.
\end{aligned}
$$

Grouping terms, we obtain that:

$$
\begin{aligned}
& \frac{c_{11,R}}{c_1}\left[z + (1-z)R^{\frac{1}{1-\rho}} + \psi z R + \psi(1-z)R^{\frac{1}{1-\rho}} - \psi\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}\left(1-\frac{1}{\rho(1-\mu)}\right)\right] \\
= \quad & -\frac{\rho}{\rho-1}\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{\rho}{1-\rho}}.
\end{aligned}
$$

Finally, we obtain this:

$$
\frac{c_{11,R}}{c_1} = \frac{\frac{\rho}{1-\rho}\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{\rho}{1-\rho}}}{\left(z + (1-z)R^{\frac{1}{1-\rho}} + \psi z R + \psi(1-z)R^{\frac{1}{1-\rho}}\right) + \psi\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}\left(\frac{1}{\rho(1-\mu)} - 1\right)}. \tag{54}
$$

We use these derivatives to obtain c_R and n_R. Differentiating (1) with respect to R, we obtain that

$$
c_R = c^{1-\rho}\left[z\mu c_1^{\rho-1}c_{11,R} + z(1-\mu)c_1^{\rho-1}c_{12,R} + (1-z)\mu c_2^{\rho-1}c_{21,R} + (1-z)(1-\mu)c_2^{\rho-1}c_{22,R}\right]. \tag{55}
$$

Substituting from (50) and (51), we obtain that

$$
\frac{c_R}{c^{1-\rho}} = c_1^{\rho-1} z c_{11,R}[1-(1/\rho)] + (1-z)c_2^{\rho-1}\left[c_{11,R}[1-(1/\rho)]R^{\frac{1}{1-\rho}} + \frac{c_1 R^{\frac{\rho}{1-\rho}}}{1-\rho}\right].
$$

Collecting terms, we get that

$$c_R = c^{1-\rho} c_2^{\rho-1} c_1 \left[\frac{c_{11,R}}{c_1} \left(zR + (1-z) R^{\frac{1}{1-\rho}} \right) \left(1 - \frac{1}{\rho} \right) + \frac{1-z}{1-\rho} R^{\frac{\rho}{1-\rho}} \right]. \tag{56}$$

Differentiating the resource constraint, we obtain n_R:

$$\theta n_R = z \left[\mu c_{11,R} + (1-\mu) c_{12,R} \right] + (1-z) \left[\mu c_{21,R} + (1-\mu) c_{22,R} \right]$$

or, after substituting from (50) and (51) and collecting terms,

$$\theta n_R = c_{11,R} \left(1 - \frac{1}{\rho} \right) \left(z + (1-z) R^{\frac{1}{1-\rho}} \right) + (1-z) \frac{c_1}{1-\rho} R^{\frac{\rho}{1-\rho}}. \tag{57}$$

From (57), using $(c_2/c_1)^{1-\rho} = R$, we obtain that

$$\begin{aligned} \theta n_R &= \frac{(1-\frac{1}{\rho})}{1-\rho} c_{11,R} z \left[(1-\rho) \left(1 + \left(\frac{1-z}{z} \right) R^{\frac{1}{1-\rho}} \right) + \frac{(1-z)/z}{(1-\frac{1}{\rho})} \frac{c_1}{c_{11,R}} R^{\frac{\rho}{1-\rho}} \right] \\ &= \frac{c_2}{c_1} \frac{(1-\frac{1}{\rho})}{1-\rho} c_{11,R} z \left[(1-\rho) \left(R^{\frac{1}{\rho-1}} + \frac{1-z}{z} \right) + \frac{(1-z)/z}{(1-\frac{1}{\rho})} \frac{c_1}{c_{11,R}} R^{-1} \right]. \end{aligned}$$

Substituting in (27) and using the result that for our functional forms $u_{22}/(1-\mu)(1-z) = u_c \left(\frac{c}{c_2} \right)^{1-\rho}$, we obtain that

$$\begin{aligned} \frac{(1-\rho)\theta u_{22} n_R}{(1-\mu)(1-z)} &= f(c_1, c_2) \left[-(1-\rho) R^{\frac{1}{\rho-1}} - \left(\frac{1-z}{z} \right) \left((1-\rho) - \frac{\rho}{(1-\rho)} \frac{c_1}{c_{11,R}} R^{-1} \right) \right] \\ &= f(c_1, c_2) \psi_{MD}(R, z), \end{aligned}$$

where

$$f(c_1, c_2) = u_c c_2 \left(\frac{c}{c_2}\right)^{1-\rho} \left(\frac{1}{\rho} - 1\right) \frac{c_{11,R}}{c_1} z$$

$$\psi_{MD}(R, z) = -(1-\rho) R^{\frac{1}{\rho-1}} + \left(\frac{1-z}{z}\right) \left[\frac{\rho}{(1-\rho)} \frac{c_1}{c_{11,R}} R^{-1} - (1-\rho)\right]. \tag{58}$$

Consider the term in brackets in (58). Using (54), this term is

$$\begin{aligned}
& \left(\frac{1}{R}\right) \frac{z + (1-z) R^{\frac{1}{1-\rho}} + \psi z R + \psi (1-z) R^{\frac{1}{1-\rho}} + \psi \left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}} \left(\frac{1}{\rho(1-\mu)} - 1\right)}{\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{\rho}{1-\rho}}} - (1-\rho) \\
= \quad & \frac{z + \psi z R + \psi \left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}} \left(\frac{1}{\rho(1-\mu)} - 1\right) - (1-\rho)\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}}{\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{1}{1-\rho}}} \\
= \quad & \frac{z(1+\psi R) + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho} R^{\frac{1}{1-\rho}}}{\left[(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}\right] R^{\frac{1}{1-\rho}}}.
\end{aligned}$$

Substituting for c/c_2 in this expression and then substituting in (58), we obtain that

$$\psi_{MD}(R, z) = -(1-\rho) R^{\frac{1}{\rho-1}} + \left\{ \frac{(1-z)\left(R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}}\right) + \left(\frac{1-z}{z}\right)\frac{\mu}{1-\mu}\frac{\psi}{\rho}\left[z R^{\frac{\rho}{\rho-1}} + 1 - z\right]}{(1+\psi)\frac{1-z}{1-\rho} + \frac{\psi}{\rho}\left(z R^{\frac{\rho}{\rho-1}} + 1 - z\right)} \right\}.$$

Dividing the numerator and denominator of the term in braces by $1-z$ and rearranging, we obtain that

$$\psi_{MD}(R, z) = -(1-\rho) R^{\frac{1}{\rho-1}} + \frac{\left(R^{\frac{1}{\rho-1}} + \psi R^{\frac{\rho}{\rho-1}}\right) + \frac{\mu}{1-\mu}\frac{\psi}{\rho}\left(R^{\frac{\rho}{\rho-1}} + \frac{1-z}{z}\right)}{\frac{1+\psi}{1-\rho} + \frac{\psi}{\rho}\left(\frac{z}{1-z} R^{\frac{\rho}{\rho-1}} + 1\right)}.$$

We have proved the first part of Lemma 1. ‖

Lemma 1b. *In a Markov equilibrium,* (28) *holds; that is,*

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = -f(c_1, c_2)\,(R-1)\,R^{\frac{1}{\rho-1}}. \tag{59}$$

Proof. Using our functional forms, we obtain that

$$u_c c_R - \frac{\theta u_{22} n_R}{(1-\mu)(1-z)} = u_c \left[c_R - \theta \left(\frac{c}{c_2}\right)^{1-\rho} n_R\right]. \tag{60}$$

Substituting for θn_R from (57) and c_R from (56) into (60), we obtain that

$$\begin{aligned} u_c \left[c_R - \theta \left(\frac{c}{c_2}\right)^{1-\rho} n_R\right] &= u_c \left[\frac{c_{11,R}}{c_1}\left(zR + (1-z)R^{\frac{1}{1-\rho}}\right)\left(1-\frac{1}{\rho}\right) + \frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}} \right. \\ &\quad \left. -\frac{c_{11,R}}{c_1}\left(1-\frac{1}{\rho}\right)\left(z+(1-z)R^{\frac{1}{1-\rho}}\right) - \frac{1-z}{1-\rho}R^{\frac{\rho}{1-\rho}}\right] c_1 \left(\frac{c}{c_2}\right)^{1-\rho} \\ &= u_c \frac{c_{11,R}}{c_1} c_2 z \left(1-\frac{1}{\rho}\right)\left(\frac{c}{c_2}\right)^{1-\rho}(R-1)\,\frac{c_1}{c_2} \\ &= -f(c_1,c_2)(R-1)R^{\frac{1}{\rho-1}}, \end{aligned}$$

where

$$f(c_1, c_2) = u_c \frac{c_{11,R}}{c_1} c_2 z \left(\frac{1}{\rho} - 1\right)\left(\frac{c}{c_2}\right)^{1-\rho}.$$

We have proved the lemma. ‖

Lemma 2. *Equation* (35) *reduces, in a private sector equilibrium, to* (36):

$$\frac{\left(\frac{1}{\rho}-1\right)\left(1-R^{\frac{\rho}{1-\rho}}\right)}{z\left[\left(R^{\frac{1}{\rho-1}}-1\right)+\frac{\psi}{\rho}\left(R^{\frac{\rho}{1-\rho}}-1\right)\right]+\left(1+\frac{\psi}{\rho}\right)} = \frac{\rho\eta(\bar{z}-z)^{\nu}}{\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-\frac{g}{\theta}}.$$

Proof. The necessary and sufficient conditions for a private sector equilibrium are (40)–(42) and the following slightly modified version of (43):

$$\frac{\psi}{\rho}c^{\rho}c_{22}^{1-\rho} = \theta\left(1 - n - \frac{\eta(\bar{z}-z)^{1+\nu}}{1+\nu}\right). \tag{61}$$

Using (61) in (35), we obtain that

$$\left(1-\frac{1}{\rho}\right)\frac{1-R^{\frac{\rho}{1-\rho}}}{z+(1-z)R^{\frac{\rho}{1-\rho}}} = \frac{\theta\rho\eta(\bar{z}-z)^{\nu}}{(c/c_2)^{\rho}\,c_2}. \tag{62}$$

We use the resource constraint, (20), and (61) to obtain an expression for c_2 in terms of c_1/c_2 and z. Rearranging (61), we obtain that

$$\theta n = \theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\,\eta}{1+\nu}\right) - \frac{\psi}{\rho}\left(\frac{c}{c_2}\right)^{\rho}c_2.$$

Substituting this equation into the resource constraint, taking into account $c^{\rho} = zc_1^{\rho}+(1-z)c_2^{\rho}$, and rearranging, we obtain that

$$c_2 = \frac{\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-g}{z\frac{c_1}{c_2}+\frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho}+(1-z)(1+\frac{\psi}{\rho})}.$$

Substituting for c_2 in (62), we obtain that

$$\left(1-\frac{1}{\rho}\right)\frac{1-R^{\frac{\rho}{1-\rho}}}{z+(1-z)R^{\frac{\rho}{1-\rho}}} = \left[\frac{\theta\rho\eta(\bar{z}-z)^{\nu}}{z\left(\frac{c_1}{c_2}\right)^{\rho}+1-z}\right]\left[\frac{z\frac{c_1}{c_2}+\frac{\psi z}{\rho}\left(\frac{c_1}{c_2}\right)^{\rho}+(1-z)(1+\frac{\psi}{\rho})}{\theta\left(1-\frac{(\bar{z}-z)^{1+\nu}\eta}{1+\nu}\right)-g}\right].$$

After rearranging and making use of $R=(c_1/c_2)^{\rho-1}$, we obtain (36). ‖

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TABLE 1

Candidate equilibrium outcomes in three numerical examples

Fraction of Goods Purchased With Cash z	Inflation	Outcomes: Consumption: Cash Goods c_1	Credit Goods c_2	Interest Rate R	Employment n	Average Goods Price P^e
.130	Low	.17	.31	1.38	.339	49.1
	High	.08	.32	2.07	.337	99.2
.150	Low	.25	.30	1.10	.342	26.3
	High	.04	.33	3.17	.336	165.0
.152	Low	.26	.30	1.08	.343	25.4
	High	.04	.33	3.27	.336	171.6

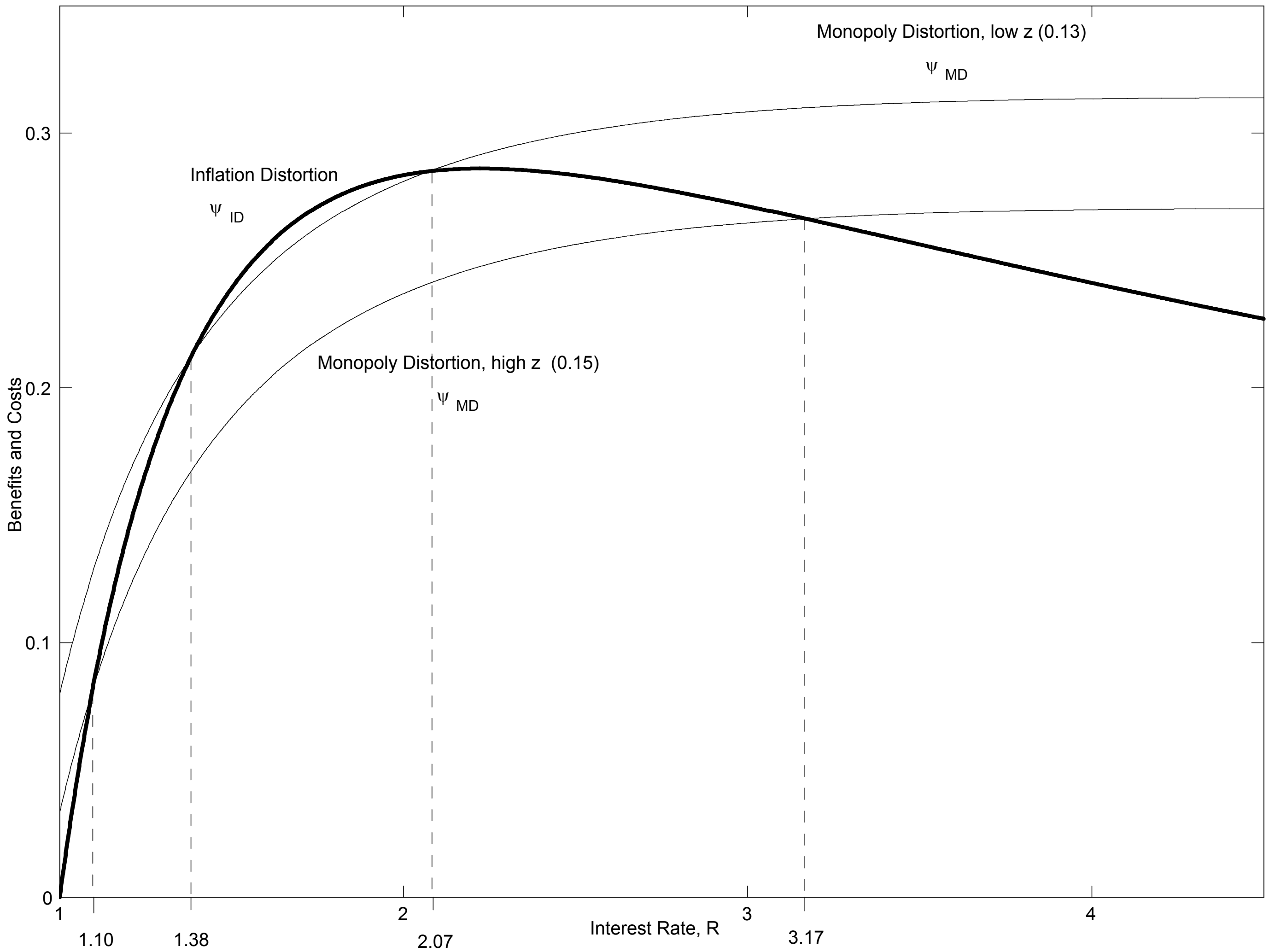

Figure 1: The Monetary Authority's Marginal Benefits and Marginal Costs

Figure 2

The Monetary Authority's Objective Function

for Two Fractions of Goods Purchased With Cash

(z = 0.15 and z = 0.152)

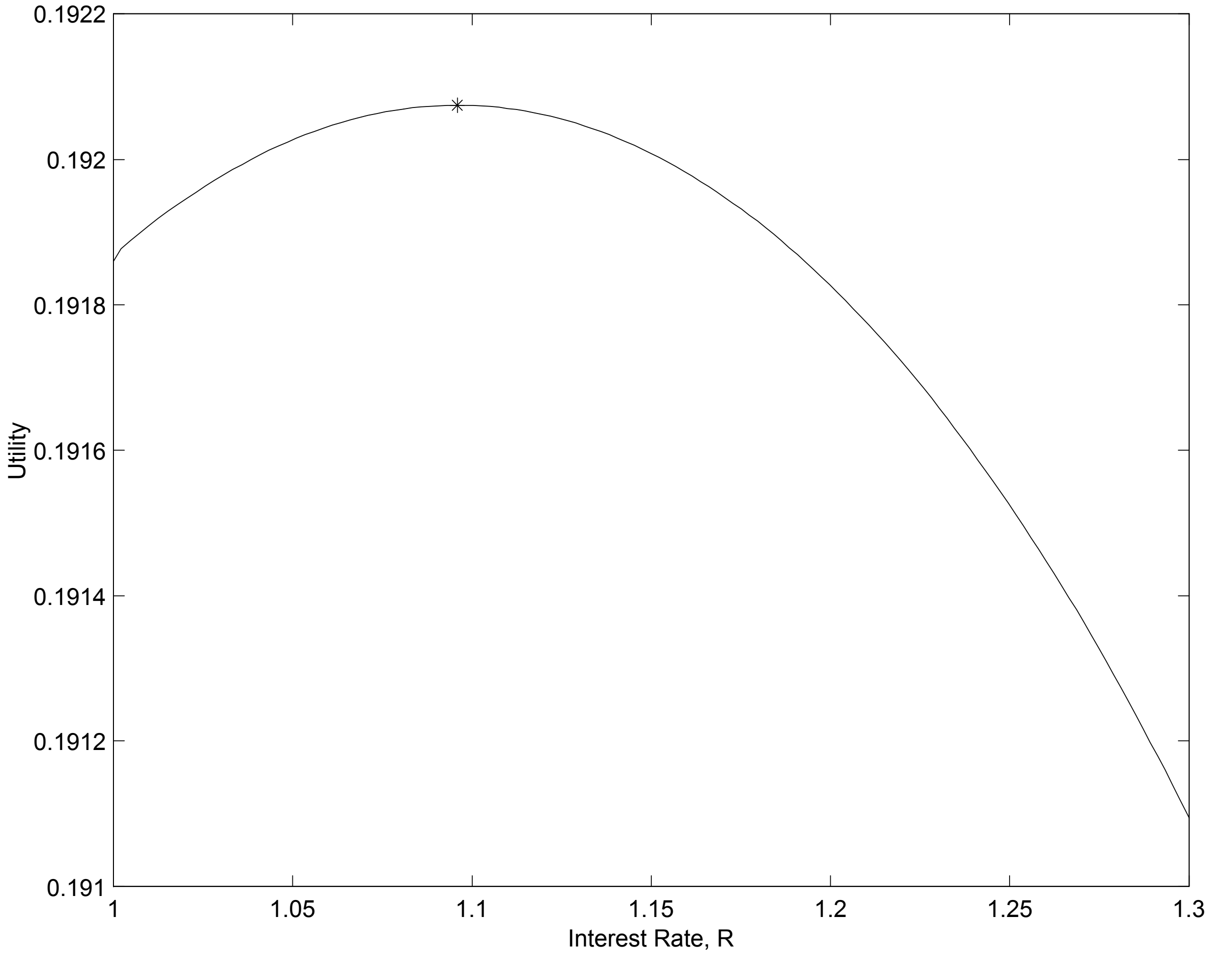

Figure 2a: Objective Function in Low Inflation Equilibrium, $z = 0.15$

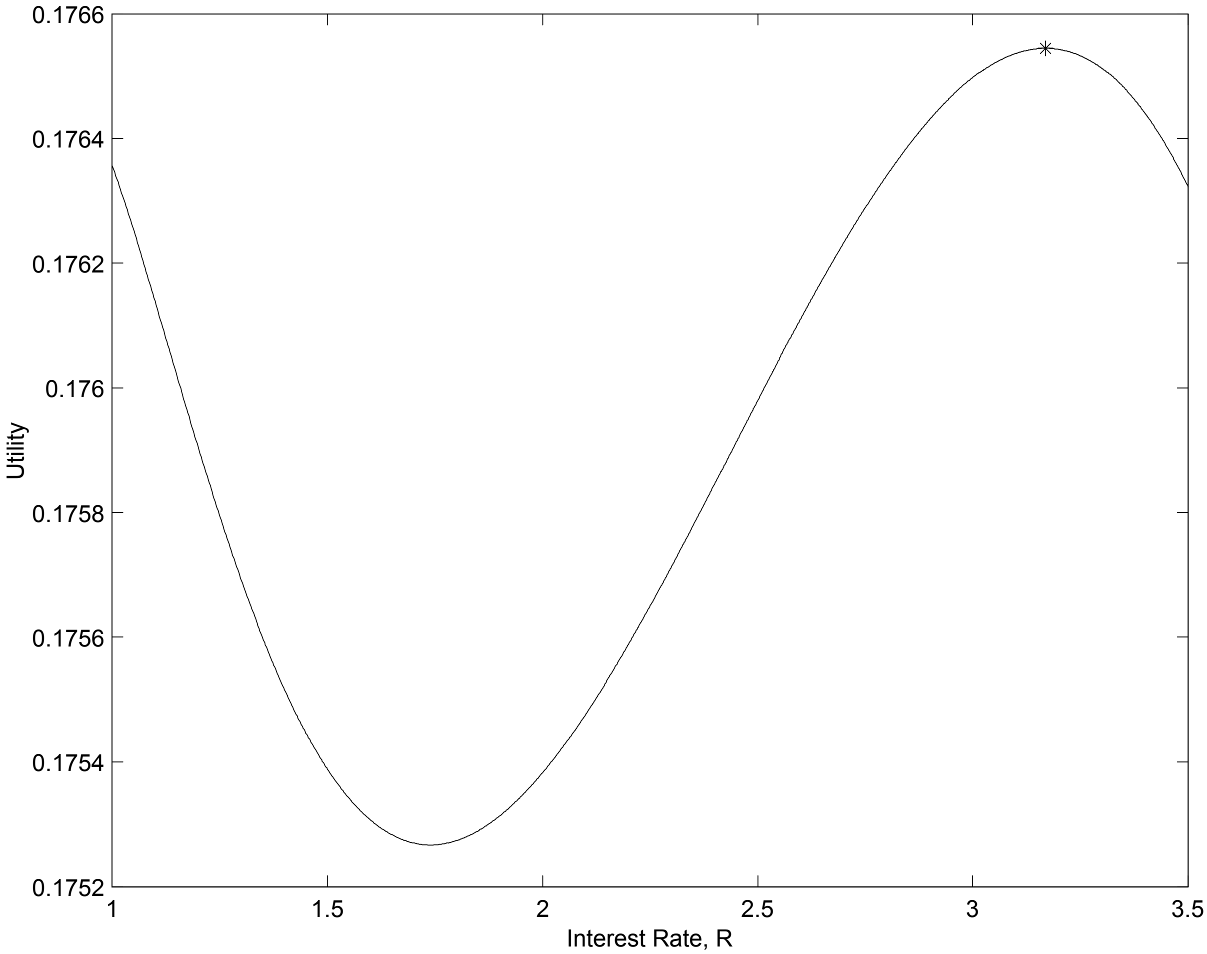

Figure 2b: Objective Function in High Inflation Equilibrium, $z = 0.15$

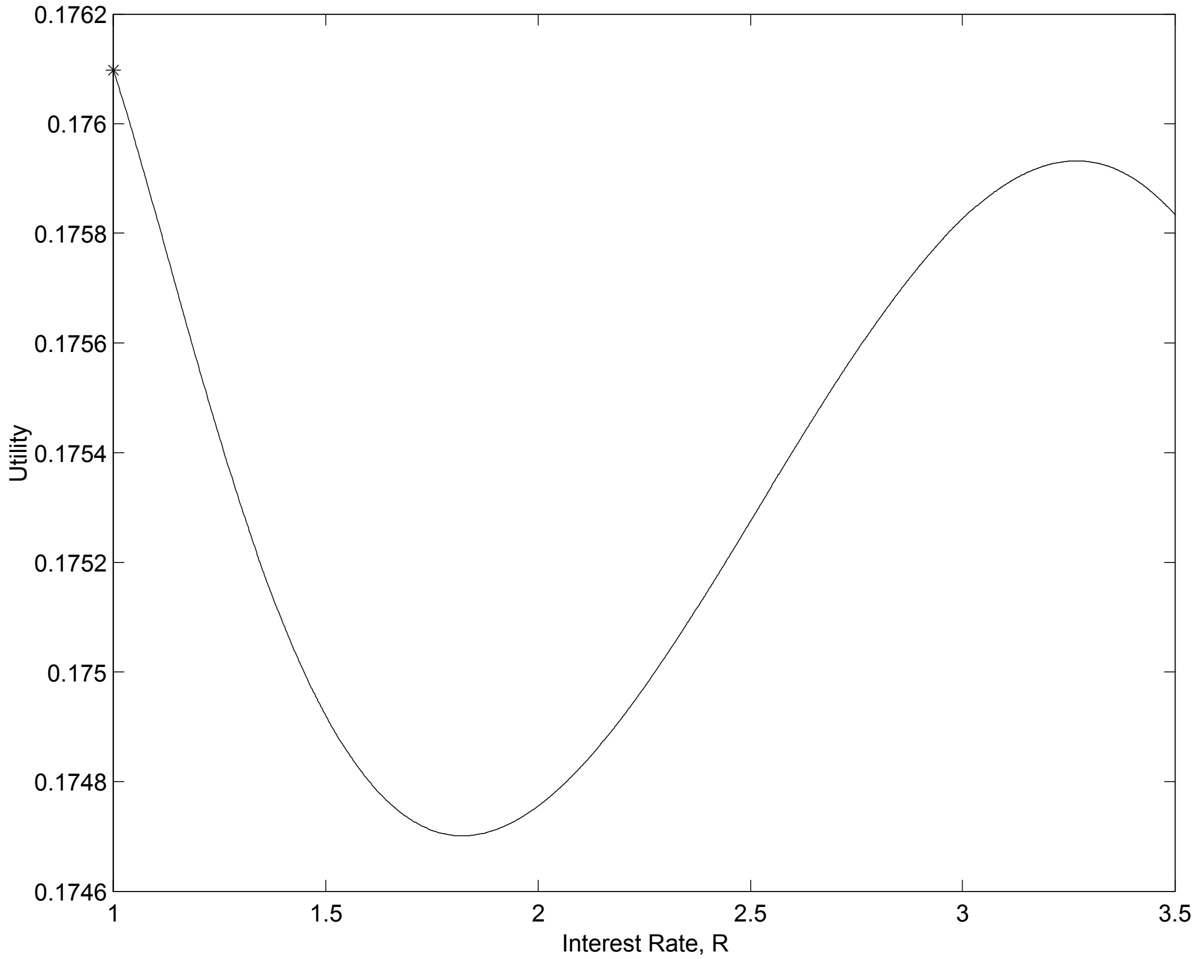

Figure 2c: Objective Function in Candidate High Inflation Equilibrium, $z = 0.152$

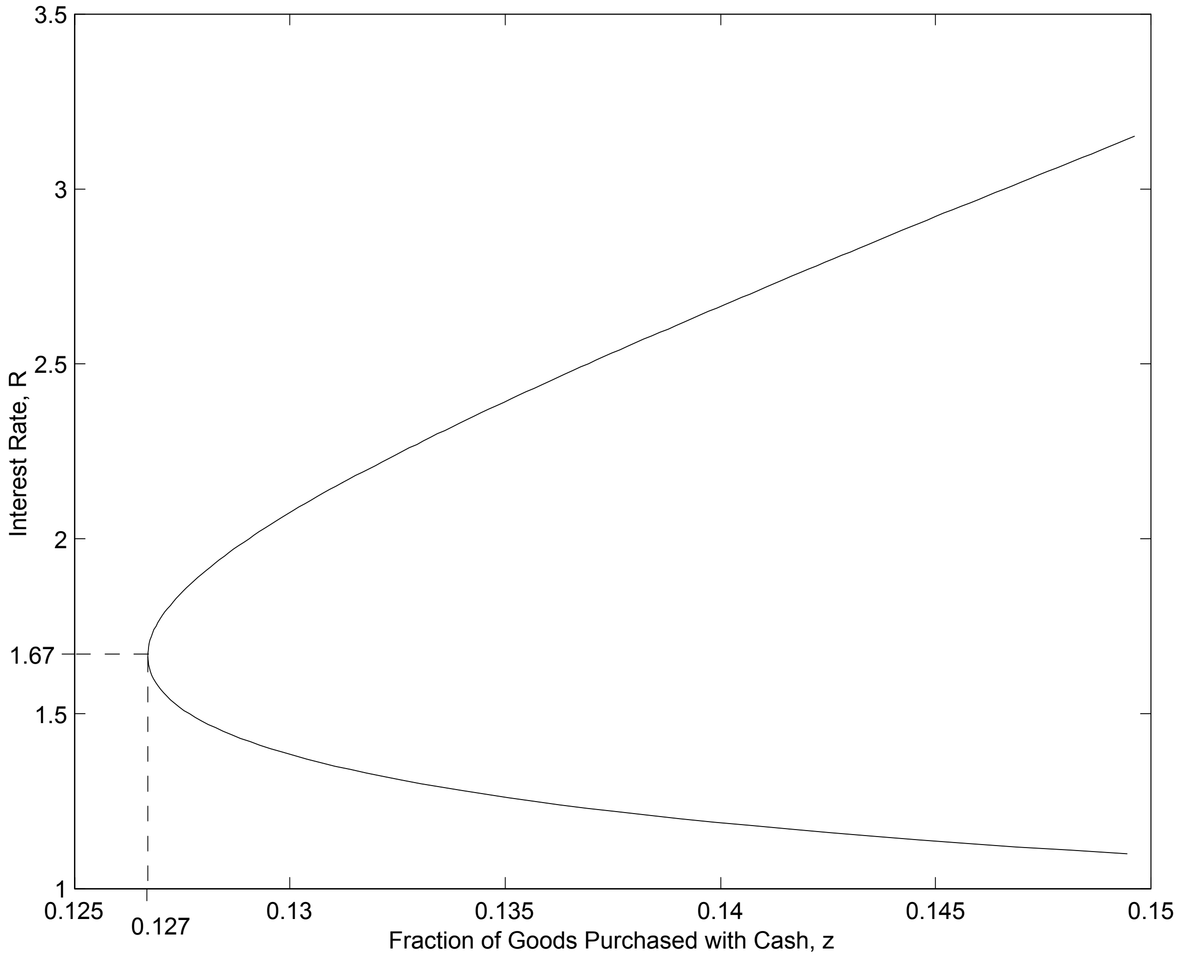

Figure 3: Interest Rate Policy Correspondence

Figure 4

Markov Equilibrium Interest Rate Policy Correspondence and Payment Technology Functions With Two Types of Shocks

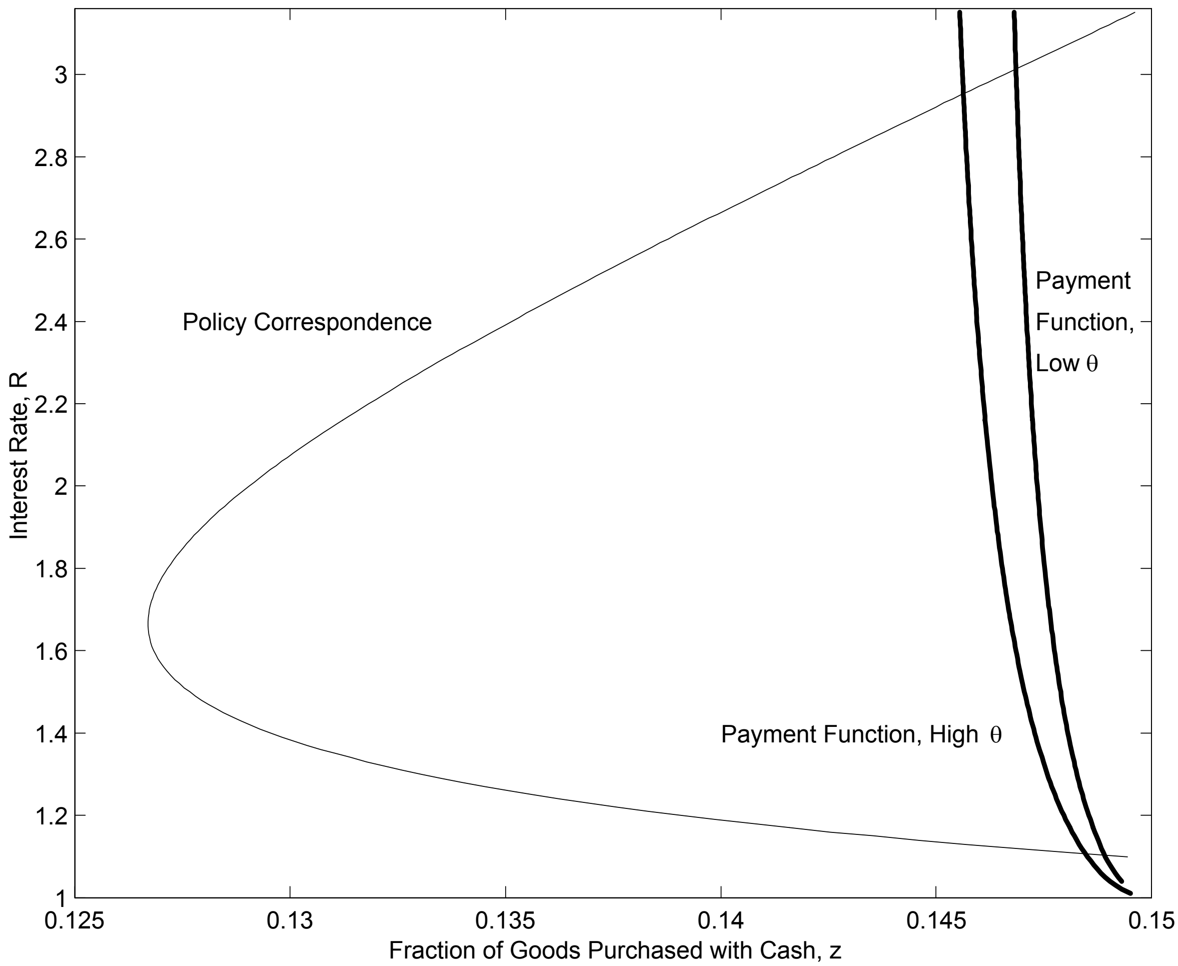

Figure 4a: Markov Equilibrium With Production Technology Shock, θ

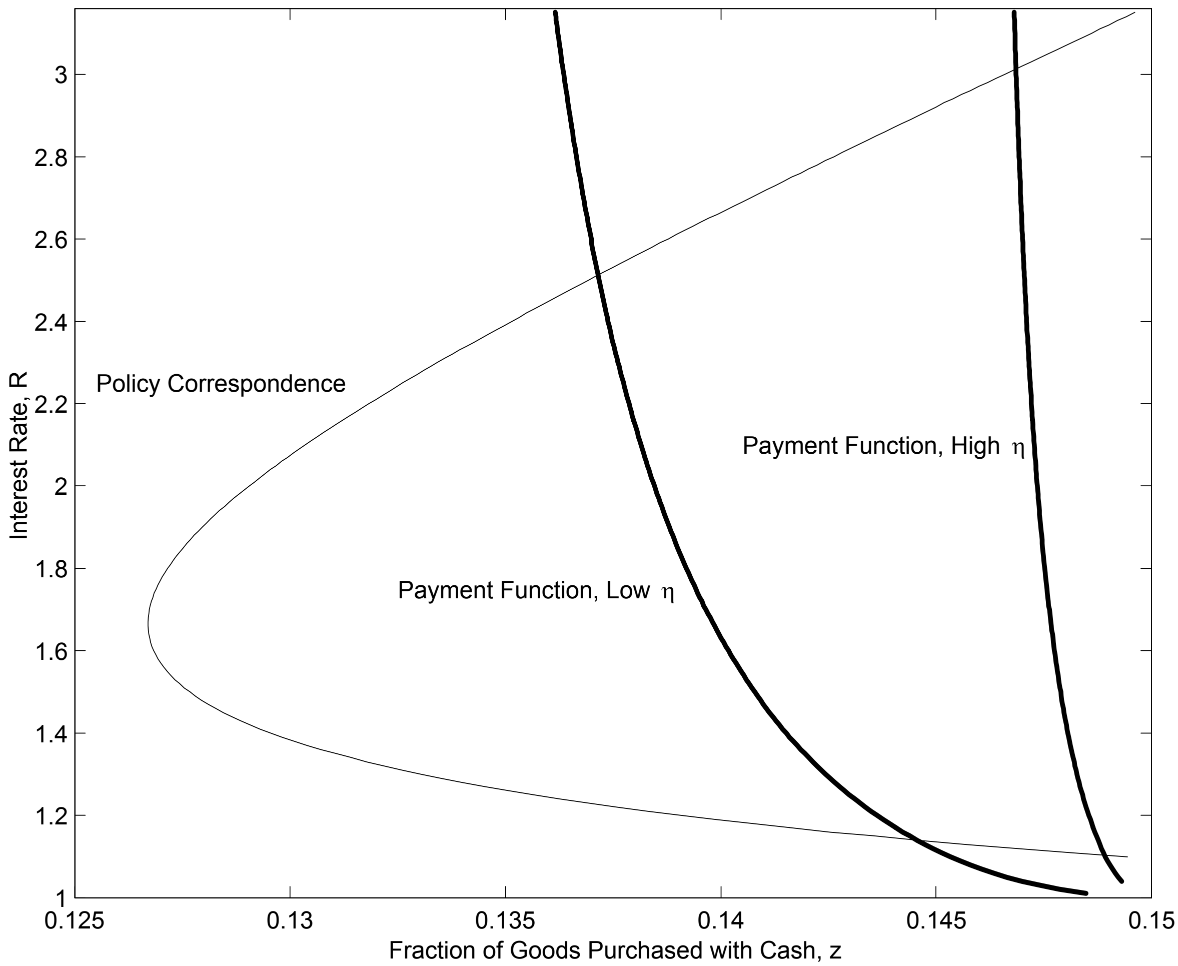

Figure 4b: Markov Equilibrium With Payment Technology Shock