Perfect Substitution in Models of the CD Market

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January 1979

Staff Report #: 41

As the CD market has become an important source of bank funds, it has also become an important market for policymakers to understand. But so far model builders have not recognized the significance of assuming that new and old CDs are perfect substitutes. Therefore, they have misused the assumption, discarded relevant data, and ignored evidence inconsistent with perfect substitution. This study shows that models of the CD market should not treat new and old issues as perfect substitutes and that they should not drop observations when market rates are above the Regulation Q ceiling.

The views expressed herein are solely those of the author and do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.
1. Introduction and Conclusions

Since their introduction in 1961, large-denominated negotiable certificates of deposit (CDs) have become a significant source of bank funds. Banks have been better able to manage their liabilities by offering attractive rates of return on CDs, something they cannot do explicitly with demand deposits. And because these certificates are negotiable in a secondary market, banks have been able to easily attract large amounts of funds whenever they are needed. As a result, CDs have grown dramatically over the years and now account for over 10 percent of all bank deposits.

These developments have had some important policy implications. In fact, to engineer an appropriate monetary policy today, policymakers must consider the influence of the CD market. When the Federal Reserve attempts to slow the economy by reducing reserves, for example, commercial banks can partially offset this action by raising funds in this market. And when it attempts to stimulate economic activity by creating reserves, banks can readily reduce these liabilities.

Modelling this market properly is thus important for economic policymakers. But a close look at model builder's current approach to this market reveals some serious problems. The two most serious stem from the assumption that at the same price, investors are indifferent between buying a CD newly issued by a commercial bank and buying a CD in the secondary market, that is, the assumption that new and old CDs are perfect substitutes.

One problem is a logical inconsistency. In some parts of the theorizing, model builders assume that new and old CDs are perfect substitutes, but in other parts they do not. Model builders estimate a single aggregate demand equation, thus treating both CD issues as one. But they
drop observations for periods when the secondary market rate is above the maximum primary rate (the Regulation Q ceiling), as though new and old issues are not perfect substitutes when Regulation Q is effective.

The other serious problem with the current approach to modelling the CD market is the shakiness of the perfection substitution assumption itself. That is, it appears to be inconsistent with the data. If old and new CDs were perfect substitutes, we would expect their rates of return to be roughly equal, but the secondary market rate has consistently been higher than the primary rate. We would also expect that when the secondary market rate rose well above Regulation Q, few, if any, new issues would be sold, yet a significant number are.

In light of these problems, if policymakers want to continue to rely on models of the CD market to help them design appropriate policies, the use of the perfect substitution assumption must be reconsidered. That is the purpose of this study. In Section 2 we describe a typical model which assumes new and old CDs are perfect substitutes and use it to critique some previously estimated CD demand equations. We then estimate a new equation based on this critique, but the results are mixed and only weakly support such a model. In Section 3, therefore, we develop an alternative model which treats new and old CDs as different assets. This model has a variety of testable implications, and the empirical results generally support it. According to these results, to correctly model the CD market Q-ceiling observations should not be discarded and new and old CDs should not be treated as perfect substitutes.

2. The Current Model: New and Old CDs as Perfect Substitutes

2.1 Description

Financial sectors of large-scale econometric models of the U.S. economy usually contain linear asset demand equations which are based on a
standard portfolio theory. The theory assumes agents are maximizing a nonlinear utility function where utility is an increasing function of the rate of return on financial wealth. Financial wealth is known at the time of the portfolio decision, while interest rates are assumed to be stochastic. Knowing the probability distributions, agents maximize expected utility subject to their wealth constraint. The first- and second-order maximization conditions yield linear asset demand equations with expected interest rates as the independent variables and with these properties:

1. Equations are homogeneous of degree one in wealth.
2. Expected own-rates enter their equation with a positive sign.
3. Across all asset demand equations, the constant terms sum to one and the coefficients on each expected interest rate sum to zero.
4. A symmetry condition holds which is analogous to the symmetry condition in consumer theory.
5. The parameters of these equations are functions of the known probability distribution of interest rates.

To illustrate, these are the demand curves for the three-asset case:

\[
\begin{align*}
A_1/W &= a_{10} + a_{11} \hat{R}_1 + a_{12} \hat{R}_2 + a_{13} \hat{R}_3 \\
A_2/W &= a_{20} + a_{21} \hat{R}_1 + a_{22} \hat{R}_2 + a_{23} \hat{R}_3 \\
A_3/W &= a_{30} + a_{31} \hat{R}_1 + a_{32} \hat{R}_3 + a_{33} \hat{R}_3 \\
W &= A_1 + A_2 + A_3
\end{align*}
\]

(1)

The \( A_i \)'s are the assets and the \( \hat{R}_i \)'s the corresponding expected own-rates of return. The parameters \( a_{ij} \)'s have the following properties:
\[ a_{ij} > 0 \quad (i=j) \]
\[ a_{1j} + a_{2j} + a_{3j} = \begin{cases} 0 & j=1,2,3 \\ 1 & j=0 \end{cases} \]
\[ a_{ij} = a_{ji} \quad (j \neq 0) \]
\[ a_{ij} = r^{ij}(\Sigma) \]

\( \Sigma \) is the known 3x3 variance-covariance matrix of interest rates.

Some of the constraints across equations are a result of the balance sheet identity. Any change in wealth must be divided among the existing assets. And any change in the demand for one asset must be exactly offset by an opposite change in demand for the other assets. The other constraints are due to the symmetry condition; for example, the change in demand for \( A_1 \) due to a unit rise in \( \hat{R}_2 \) must equal the change in demand for \( A_2 \) due to a unit rise in \( \hat{R}_1 \).

It's important to note that together balance sheet and symmetry constraints imply that the interest rate coefficients within an equation sum to zero. Thus, by estimating only one equation and testing for the zero-sum constraint, we can jointly test both symmetry and balance sheet constraints for the entire model.

To see how this theory is used in practice, we have reproduced the CD equations from two prominent econometric models: the Wharton and the MIT-PENN-SSRC.\(^2\)

**Wharton Equation**

\[
\frac{CD_t}{Y} = .0160 + .0121 R_2 - .0121 R_{tb} - .0024 D66.3 - .0013 D66.4 + .0006 D68.2 \\
(10.4) \quad (-4.2) \quad (-1.3) \quad (-.60) \quad (.30)
\]

\( \bar{R}^2 = .43 \quad S.E.E. = .0021 \quad D.W. = 1.3 \)

**PERIOD FIT:** 1963.4-1968.3
MIT-PENN-SSRC Equation

\[ \frac{CD_t}{Y} = .0164 + .0223 R_s - .0140 R_{tb} - .0083 R_{cp} \]

(12.5) (-5.3) (-1.7)

\[ R^2 = .62 \quad \text{S.E.E.} = .0019 \quad \text{D.W.} = .97 \]


where the numbers in parentheses below the regression coefficients are the t-statistics and where

\[ CD_t = \text{Total outstanding stock of CDs issued before period t plus all newly issued CDs in period t} \]

\[ Y = \text{Gross National Product (GNP)} \]

\[ R_{tb} = \text{Treasury bill rate (3-month)} \]

\[ R_{cp} = \text{Commercial paper rate (4-6-month)} \]

\[ R_s = \text{Secondary market rate on CDs (3-month)} \]

\[ D66.3, D66.4, D68.2 = \text{Dummy variables for quarters 1966.3, 1966.4, 1968.2} \]

These equations are similar in form and yield roughly the same statistical results. Both equations are linear, both define the dependent variable as the ratio of CDs to income, both assume Treasury bills and CDs are substitutes, and both constrain the sum of the interest rate coefficients to zero.\(^3\) Except for the first observation, the equations are estimated over the same data period, a period that excludes observations generated when the secondary market rate was above Regulation Q. Coefficients on variables appearing in each equation are of the same sign, and the standard errors of estimates are almost identical.

2.2 Critique

Because the parameter estimates appear consistent with the theory and because the equations fit the data reasonably well, these equations have
been accepted as good estimates of the underlying behavioral relationships between the demand for CDs, the CD rate, and competing rates. A closer look, however, suggests that they have been accepted too readily.

The theory underlying these equations posits that new and old CDs are the same asset. That is why a single aggregate equation combining new and old issues is estimated. But then why are Q-ceiling observations dropped from the estimation period?

The standard argument goes something like this. During periods when Regulation Q is effective, the public cannot rid themselves of CDs; that is, the quantity of CDs is predetermined. Therefore, investors are off their demand curves in Q-ceiling periods, and coefficients in any demand equation estimated over the entire sample period will be distorted.\textsuperscript{11/}

This reasoning is faulty. While the quantity of CDs supplied is clearly predetermined in Q-ceiling periods, it does not follow that investors are off their demand curves. When bank rates hit the Q ceiling, investors—if new and old issues are perfect substitutes—are just as willing to go to the secondary market and buy old CDs at higher rates; the demand schedule for CDs is not affected by the ability of banks to offer competitive rates on new issues. Dropping Q-ceiling observations is thus inconsistent with the underlying model.

Even if the Q-ceiling observations had been used, however, these demand equations would still not have been estimated properly. Recall that the portfolio theory yielded linear asset demand equations with expected interest rates. Expected rates, not being observed, were replaced by actual rates in estimation. In general this implies that the independent variables will be correlated with the residual so that the Ordinary Least Squares procedure (OLS) gives inconsistent and biased estimates of the true
parameters. Although a simultaneous estimator is therefore more appropriate, both the Wharton and MIT-PENN-SSRC model builders used OLS.

This examination of the current approach to modeling the CD market suggests that both a more consistent and efficient estimation procedure exists. First, to correct for inconsistency, an instrumental variable estimator can be used. Second, to take advantage of the exogeneity of the quantity of CDs during Q-ceiling periods, the demand equation can be estimated with the CD rate as the dependent variable and the quantity as an independent variable. The quantity of CDs can then be used as an instrument during Q-ceiling periods.\textsuperscript{5/}

2.3 Reestimating the Aggregate CD Equation

Using monthly data over the period 1967-1975 and the procedure described above, we estimated and tested a new equation.\textsuperscript{6/} Besides providing more degrees of freedom, the advantage of these data over those previously used is that they contain longer and more significant periods when Q ceilings were effective.\textsuperscript{7/} The tests include an F-statistic for evaluating the difference between equations estimated in ceiling and nonceiling periods and a t-statistic for evaluating the zero-sum restriction on the interest rate coefficients. The variables used in the monthly regressions are those in the quarterly regressions reported above except for income; we used personal income instead of GNP.

Table 1 shows the results of these tests. Unrestricted estimation results are presented for the ceiling period, the nonceiling period, and the total sample period. And in order to examine these equations in a more familiar way, they are renormalized with the quantity as the dependent variable. The F-statistic below the renormalized equations tests for structural change between ceiling and nonceiling periods.\textsuperscript{8/} Equations are
### Table 1

**AGGREGATE EQUATION: DEPENDENT VARIABLE $R_s$**

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>$CD_t/Y$</th>
<th>$R_{cp}$</th>
<th>$R_{tb}$</th>
<th>$1-\Sigma R$</th>
<th>$R^2$</th>
<th>D.W.</th>
<th>Rho</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nonceiling Period</strong>&lt;br&gt;(7/67-11/68, 6/70-12/75)</td>
<td>-.637</td>
<td>3.17</td>
<td>.999</td>
<td>.101</td>
<td>-.101</td>
<td>.99</td>
<td>2.3</td>
<td>.38</td>
</tr>
<tr>
<td></td>
<td>(-5.0)*</td>
<td>(1.2)</td>
<td>(21.1)*</td>
<td>(1.5)</td>
<td>(-3.4)*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ceiling Period</strong>&lt;br&gt;(11/68-5/70)</td>
<td>-2.22</td>
<td>19.8</td>
<td>1.05</td>
<td>.202</td>
<td>-.257</td>
<td>.99</td>
<td>1.9</td>
<td>.56</td>
</tr>
<tr>
<td></td>
<td>(-1.11)</td>
<td>(0.6)</td>
<td>(5.0)*</td>
<td>(1.4)</td>
<td>(-1.3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL PERIOD</strong>&lt;br&gt;(7/67-12/75)</td>
<td>-.680</td>
<td>5.48</td>
<td>.983</td>
<td>.106</td>
<td>-.089</td>
<td>.99</td>
<td>1.9</td>
<td>.41</td>
</tr>
<tr>
<td></td>
<td>(-5.4)*</td>
<td>(3.4)*</td>
<td>(22.6)*</td>
<td>(1.9)*</td>
<td>(-3.8)*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**RENORMALIZED EQUATION: DEPENDENT VARIABLE $CD_t/Y$**

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>$R_s$</th>
<th>$R_{cp}$</th>
<th>$R_{tb}$</th>
<th>$R_s + R_{cp} + R_{tb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nonceiling Period</strong></td>
<td>.201</td>
<td>.315</td>
<td>-.315</td>
<td>-.032</td>
<td>-.032</td>
</tr>
<tr>
<td><strong>Ceiling Period</strong></td>
<td>.112</td>
<td>.050</td>
<td>-.165</td>
<td>-.013</td>
<td>-.128</td>
</tr>
<tr>
<td><strong>TOTAL PERIOD</strong></td>
<td>.124</td>
<td>.183</td>
<td>-.180</td>
<td>-.018</td>
<td>-.015</td>
</tr>
</tbody>
</table>

Test for structural change: $F_{4,93} = 1.13$

*Significant at 95 percent level of confidence*
corrected for first-order serial correlation; a correlation coefficient (Rho) and a Durbin-Watson (D.W.) statistic are reported. Other statistics shown are the adjusted $R^2$, the regression coefficients, one minus the sum of interest rate coefficients, and the corresponding t-statistics.²/²

At first glance the results look quite reasonable. The coefficients in the total sample period equation all have their expected signs, negative on competing rates and positive on the CD variable. The equation fits the data period quite well, and the Durbin-Watson statistic suggests that first-order serial correlation is no problem.

But as support for the standard portfolio theory and the perfect substitution assumption, the results are mixed. As the theory implies, the own-rate coefficient is positive in the total sample period equation and in each subperiod equation. Contrary to the theory, however, the sum of the interest rate coefficients is significantly different from zero in two of the three equations. Only in the ceiling period is the sum consistent with the theory. Furthermore, under the perfect substitution assumption, we expect a demand equation estimated over a nonceiling period to have the same coefficients as one estimated over a ceiling period, and we could not reject this hypothesis for the complete set of estimated coefficients. But equality does not hold when we limit the test to just the coefficients on the CD variable.

3. An Alternative Model: New and Old CDs as Different Assets

3.1 Rationale and Development

Those who have used the perfect substitution model can take some comfort in the results reported in Table 1. While not all the restrictions implied by the theory are found in the data, the coefficients are appropriately signed and the equation is stable between ceiling and nonceiling periods. Anyone accepting this model, though, would be ignoring some
important countervailing evidence. If agents are indifferent between holding new and old issues, the primary and secondary rates should be approximately equal. And during Q-ceiling periods, when the primary rate can't be raised and is significantly lower than the secondary rate, no new CDs should be sold. But this is not what we observe.

The primary rate has been consistently lower than the secondary rate, and when the difference becomes quite substantial, a large number of new issues are still sold. During 1967-1975, not including periods when market rates were above Regulation Q ceilings, the three-month secondary CD rate averaged 30 basis points higher than the two- to three-month primary rate (New York) and 15 basis points higher than the three- to six-month rate (New York), and it rarely fell below a new issue rate. And when market rates were well above the Q ceiling in 1969 (on average 200 basis points higher), new issues were still being sold at an average of close to $3 billion a month. Although this was down 40 percent from the previous nonceiling year, it was still much higher than one would predict if new and old issues are perfect substitutes.\textsuperscript{10/}

So while an aggregate CD demand equation may explain some of the data, it cannot explain why new issues are sold during Q-ceiling periods or why secondary rates are generally higher than primary rates. One way to explain these observations is to assume that new and old CDs are not perfect substitutes and that some investors prefer new CDs if they are priced the same as old. To develop and test such a model we posit two types of investors: the general public—those who treat primary and secondary issues as perfect substitutes—and homeowners—those who, other things being equal, prefer new issues of hometown banks to secondary market issues.
Hometown investors exist for at least two reasons. First, bank customers usually buy more than one product from their banks. By supporting a local bank's CD sale today, for example, customers may expect better financing privileges in the future. Second, state and local governments may invest in local bank CDs because they feel obliged to do business in their home territory—and in some cases this obligation is legal. Many states restrict the investment powers of state and local governments, forcing them to deposit idle funds in local commercial banks.  

To explicitly test this model, we postulate the following set of asset demand equations:

**Hometowners**

\[
\begin{align*}
\frac{x^h}{w^h} &= a_{10} + a_{11}\hat{R}_s + a_{12}\hat{R}_n + a_{13}\hat{R}_o \\
\frac{x^h}{w^h} &= a_{20} + a_{21}\hat{R}_s + a_{22}\hat{R}_n + a_{23}\hat{R}_o \\
\frac{x^h}{w^h} &= a_{30} + a_{31}\hat{R}_s + a_{32}\hat{R}_n + a_{33}\hat{R}_o \\
W^h &= x^h + x^h + x^h \\
\end{align*}
\]

(2)

where

\[a_{ij} > 0 \quad i=j\]

\[a_{1j} + a_{2j} + a_{3j} = \begin{cases} 0 & j=1,2,3 \\ 1 & j=0 \end{cases}\]

\[a_{ij} = a_{ji} \quad j \neq 0\]

It follows that \(a_{11} + a_{12} + a_{13} = 0 \) (i=1,2,3).

**General Public**

\[
\begin{align*}
\frac{x^g}{w^g} &= b_{10} + b_{11}\hat{R}_s + b_{13}\hat{R}_o \\
\frac{x^g}{w^g} &= 0 \\
\end{align*}
\]
\[
\begin{align*}
X_o^{G} / W^G &= b_{30} + b_{31} \hat{R}_s + b_{33} \hat{R}_o \\
W^G &= \chi_s^G + \chi_n^G + \chi_o^G
\end{align*}
\]

where

\[
\begin{align*}
b_{ij} &> 0 \quad i = j \\
b_{1j} + b_{3j} &= \{0 \quad j = 1, 3 \\
b_{1j} &= b_{j1} \quad j \neq 0
\end{align*}
\]

It follows that \( b_{11} + b_{13} = 0 \) \((i = 1, 3)\).

The homeowners divide their wealth \((W^h)\) among three assets: secondary CD issues \((X_s^h)\), new CD issues \((X_n^h)\), and others \((X_o^h)\). The properties of their asset demand equations (2) are the same as discussed above for equations (1). The demand for an asset is a positive function of its own expected rate and is homogeneous of degree one in wealth. Coefficients across equations sum to zero except for constants, which sum to one. And balance sheet and symmetry constraints imply that interest rate coefficients within an equation sum to zero.

Similarly, the general public divide their wealth among the three assets \((X_s^G, X_n^G, X_o^G)\), and their asset demand equations (3) have the properties listed above. As long as the secondary rate is greater than the primary rate, the general public never purchase new issues.

Estimating these two sets of equations over a sample period that includes Q-ceiling observations will let us test the general portfolio theory as well as the assumption that new and old issues are not perfect substitutes. The restrictions on interest rate coefficients provide a direct test of the portfolio theory. However, testing the assumption that
old CDs are not perfect substitutes for new CDs is more subtle. Recall that the portfolio theory implies that the parameters of the model are functions of the known probability distribution of interest rates. The Q-ceiling period is a change in the distribution, a significant reduction in the primary rate variance. Parameters of the hometown equation, therefore, are different in ceiling and non-ceiling periods, and we would expect the estimated coefficients to fail a test of stability. We would also expect sharper estimates of the hometown parameters when estimated only over the Q-ceiling period, because the collinearity between the primary and secondary rates is much lower over this period than over the non-ceiling period.\textsuperscript{12/}

But the model postulated above cannot be directly estimated because the data are not disaggregated by investors, only by markets. To see how this affects the testable implication discussed above, we aggregate (2) and (3) to get market demand equations.

**New Issues**

\[
\frac{X_n}{W} = w_1 \left( \frac{X_n}{W} \right) = a_{20}w_1 + a_{21}w_1^R_s + a_{22}w_1^R_n + a_{23}w_1^R_o
\]  

(4)

where

\[
w_1 = \frac{w^h}{W} \text{ and } W = w^h + w^g
\]

**Secondary Issues**

\[
\frac{X_s}{W} = w_1 \left( \frac{X_s}{W} \right) + w_2 \left( \frac{X_s}{W} \right) =
\]

\[
(a_{10}w_1^h + b_{10}w_2^h) + (a_{11}w_1^h + b_{11}w_2^h)^R_s + a_{12}w_1^hR_n + (a_{13}w_1^h + b_{13}w_2^h)R_o
\]

(5)

where

\[
w_1 = \frac{w^h}{W}, \ w_2 = \frac{w^g}{W}, \text{ and } W = w^h + w^g
\]
Most of the testable implications still hold. Balance sheet and symmetry constraints are intact. In particular, interest rate coefficients sum to zero, and the coefficient of the secondary rate in the new-issue equation equals the coefficients of the primary rate in the secondary equation. Since the new-issue equation contains only structural parameters from the hometown equation weighted by \( w_1 \), parameter estimates should still change during ceiling periods, assuming \( w_1 \) remains constant or does not exactly offset the change in the hometown parameters. The secondary issues equation, however, should not change. Although the parameters in this equation are functions of the hometown coefficients, in most cases the general public coefficients will dominate (that is, \( w_2 \) is likely to be significantly greater than \( w_1 \)), and the parameters will appear stable.

3.2 Estimation and Test Results

We estimate the new- and old-issue equations (4) and (5) with few modifications. Our proxy for wealth is personal income (Y). As a proxy for homeowners' future commitment to buying local bank CDs, we include in the new-issue equation the amount of CDs maturing (Mat).\(^{13}\) And for competing interest rates we use those in current models: the three-month Treasury bill rate (\( R_{tb} \)) and the four- to six-month commercial paper rate (\( R_{cp} \)).

Our data period and estimation procedures are similar to those used for reestimating and testing the aggregate equation in Section 2. Observations are again monthly and cover the period 1967-1975. The instrumental variable technique discussed earlier is used again for estimating the asset demand equations.\(^{14}\)

Tables 2 and 3 show the estimation and test results for the new- and secondary-issue equations, respectively. These tables are similar in
Table 2

NEW ISSUE EQUATION: DEPENDENT VARIABLE $\frac{100.0 \text{ } C_n}{Y}$

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>$R_n$</th>
<th>$R_s$</th>
<th>$R_{cp}$</th>
<th>$R_{tb}$</th>
<th>Mat</th>
<th>$\Sigma R$</th>
<th>$\bar{R}^2$</th>
<th>D.W.</th>
<th>Rho</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonceiling Period (7/67-11/68, 6/70-12/75)</td>
<td>.264</td>
<td>-.051</td>
<td>.342</td>
<td>-.164</td>
<td>-.083</td>
<td>.060</td>
<td>.004</td>
<td>.95</td>
<td>1.9</td>
<td>.14</td>
</tr>
<tr>
<td>Ceiling Period (11/68-5/70)</td>
<td>-.246</td>
<td>.106</td>
<td>-.445</td>
<td>.403</td>
<td>.008</td>
<td>.008</td>
<td>.072</td>
<td>.85</td>
<td>2.4</td>
<td>.14</td>
</tr>
<tr>
<td>TOTAL PERIOD (7/67-12/75)</td>
<td>.528</td>
<td>.210</td>
<td>.342</td>
<td>-.420</td>
<td>-.138</td>
<td>.063</td>
<td>-.005</td>
<td>.96</td>
<td>2.0</td>
<td>.24</td>
</tr>
</tbody>
</table>

Test for structural change: $F_{6,89} = 3.1^*$

*Significant at 95 percent level of confidence
**Table 3**

**SECONDARY ISSUE EQUATION: DEPENDENT VARIABLE $R_s$**

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>$C_{D_s}/Y$</th>
<th>$R_n$</th>
<th>$R_{cp}$</th>
<th>$R_{tb}$</th>
<th>$1-\Sigma R$</th>
<th>$\bar{R}^2$</th>
<th>D.W.</th>
<th>Rho</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nonceiling Period</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7/67-11/68, 6/70-12/75)</td>
<td>-.527</td>
<td>3.86</td>
<td>.161</td>
<td>.845</td>
<td>.086</td>
<td>-.091</td>
<td>.99</td>
<td>2.0</td>
<td>.38</td>
</tr>
<tr>
<td></td>
<td>(-4.7)*</td>
<td>(1.9)*</td>
<td>(.74)</td>
<td>(3.8)*</td>
<td>(1.3)</td>
<td>(2.7)*</td>
<td></td>
<td></td>
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<tr>
<td><strong>Ceiling Period</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(11/68-5/70)</td>
<td>-1.97</td>
<td>30.7</td>
<td>-.120</td>
<td>1.10</td>
<td>.202</td>
<td>-.182</td>
<td>.98</td>
<td>1.7</td>
<td>.46</td>
</tr>
<tr>
<td></td>
<td>(-.87)</td>
<td>(1.2)</td>
<td>(-.62)</td>
<td>(6.5)*</td>
<td>(1.4)</td>
<td>(-.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL PERIOD</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(7/67-12/75)</td>
<td>-.683</td>
<td>6.87</td>
<td>.040</td>
<td>.956</td>
<td>.101</td>
<td>-.100</td>
<td>.99</td>
<td>1.9</td>
<td>.43</td>
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<tr>
<td></td>
<td>(-4.7)*</td>
<td>(1.9)*</td>
<td>(0.8)</td>
<td>(14.4)*</td>
<td>(1.7)*</td>
<td>(4.1)*</td>
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</table>

**RENORMALIZED EQUATION: DEPENDENT VARIABLE $C_{D_s}/Y$**

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>$R_s$</th>
<th>$R_n$</th>
<th>$R_{cp}$</th>
<th>$R_{tb}$</th>
<th>$R_s+R_n+R_{cp}+R_{tb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nonceiling Period</strong></td>
<td></td>
<td>.137</td>
<td>.259</td>
<td>-.042</td>
<td>-.219</td>
<td>-.022</td>
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<tr>
<td><strong>Ceiling Period</strong></td>
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<td>.064</td>
<td>.033</td>
<td>.004</td>
<td>-.035</td>
<td>-.007</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL PERIOD</strong></td>
<td></td>
<td>.099</td>
<td>.146</td>
<td>-.005</td>
<td>-.139</td>
<td>-.014</td>
</tr>
</tbody>
</table>

Test for structural change: $F_{5,91} = .95$

*Significant at 95 percent level of confidence*
format to Table 1. Each shows the estimated coefficients and t-statistics for the total sample period, the Q-ceiling period, and the nonceiling period. When the secondary rate is the dependent variable, we report a set of renormalized equations, with the quantity as the dependent variable. And we report an F-statistic testing for structural change between ceiling and nonceiling periods.

Although the results are somewhat mixed, they tend to support the standard portfolio model and the hypothesis that new and old CDs are not perfect substitutes.

The new-issue equation gives them the most support (Table 2). As predicted, it fails the Chow test with an F-statistic significant at the 95 percent level of confidence. Also as expected, the substitution effect is more significant in the ceiling period. In fact, the coefficients on both the new and secondary rates are appropriately signed and statistically significant in the ceiling period while statistically zero in the nonceiling period. In all periods the balance sheet constraint appears to hold as the sums of the interest rate coefficients are close to zero.

The results from the secondary-issue equation are not quite as strong but are still consistent with the model (Table 3). As predicted, the F-statistic is not significant so we can use the total period coefficients as the "best" parameter estimates. In this equation the interest rate coefficients are all appropriately signed and all but the new-issue rate are statistically significant. The sum of the interest rate coefficients, however, is statistically different from zero.

Finally, the equations satisfy the symmetry condition. Taking the ceiling period results as our "best" estimate of the new-issue equation and the total period results as our "best" estimate of the secondary-issue
equation, the coefficient of the secondary rate in the new-issue equation (-.0045) is not significantly different from the coefficient of the new-issue rate in the secondary equation (-.0054).

4. Summary

Our examination of the perfect substitution assumption in models of the CD market suggests that an aggregate CD demand equation is not good enough to be used by policymakers. An aggregate equation fits the data period reasonably well when estimated correctly (using all observations and an instrumental variable estimator), but it is generally not consistent with the restrictions implied by the standard portfolio model or the perfect substitution assumption. Moreover, this approach cannot explain why the primary rate has always been lower than the secondary rate or why new issues have been sold when the secondary rate is above the Regulation Q-ceiling.

Our alternative approach explains this evidence by assuming that new and old CDs are not perfect substitutes, and it seems to hold up much better. The test results are generally consistent with both standard portfolio restrictions and the assumption that new and old CDs are different assets. To construct models of the CD market that policymakers can use to design appropriate policies, therefore, model builders should neither drop Q-ceiling observations from the sample period nor treat new and old CDs as perfect substitutes.
Footnotes

Earlier versions of this paper were presented at the September 1976 meetings of the Econometric Society in Atlantic City and at the May 1978 Committee on Financial Analysis at the Federal Reserve Bank of Cleveland. The views expressed herein do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. For their criticisms and suggestions, I am indebted to Neil Wallace and Tom Sargent. Any errors, however, are my own.

1/ For other applications of this theory see Parkin (1970); Parkin, Gray, and Barrett (1970); and Gramlich and Kalchbrenner (1970).

2/ The Wharton equation comes from a version of the model described in McCarthy (1972). The MIT-PENN-SSRC equation comes from a version of the model described in Federal Reserve Board (1975).

3/ Although these equations are normalized on income instead of wealth, if income is a good proxy for wealth and if the income-wealth relationship is independent of the interest rates appearing in the CD equation, then the balance sheet and symmetry constraints still hold.


5/ This approach was suggested to me by Neil Wallace.

6/ The right-side endogenous variables were the Treasury bill rate, the commercial paper rate, and the CD-to-income ratio in nonceiling periods. The instrumental variables were current and lagged values of the federal funds rate, the Aaa corporate bond rate and income, lagged values of the Treasury bill rate, the commercial paper rate, and the current value of the CD-to-income ratio in ceiling periods.
During quarters excluded from the Wharton and MIT-PENN-SSRC equations, the secondary rate exceeded the primary rate (on two- to three-month issues) by only 40 basis points on average. The mean difference over nonceiling periods was about 30 basis points. In our data the Q-ceiling period is defined as from November 1968 to July 1970. During this period the mean difference between the secondary and primary rate was over 200 basis points.

This is the well-known Chow test. See Johnston (1972, p. 201).

In equations where the secondary rate is the dependent variable, the interest rate coefficients must sum to one to satisfy the zero-sum restriction.

Because foreign official institutions are exempt from Regulation Q ceilings, they explain part of the demand for new issues during Q-ceiling periods. However, their purchases were relatively small over the data period. The new-issue variable used in the empirical work in this paper is net of foreign official institution purchases. We got estimates of these purchases from the Federal Reserve Board of Governors, Washington, D.C.


In the Q-ceiling period, the simple correlation coefficient between the primary and secondary rate is less than .30, while in the non-ceiling period it is .97.

The amount of CDs maturing in the homeowners' portfolio at least partly represents a long-term commitment to local banks. An increase in maturing issues should increase demand for new issues. Because we do not have a breakdown by investor type, we use the total stock of maturing issues as a proxy.
14/ The right-side endogenous variables were the Treasury bill rate, the commercial paper rate, the secondary-issue rate, and the new-issue rate in nonceiling periods. The instrumental variables were current and lagged values of the federal funds rate, the Aaa corporate bond rate and income, lagged values of the Treasury bill rate, the commercial paper rate, and the current value of the stock of old CDs-to-income ratio.
References


