



## **Sovereign Risk Contagion**

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# Sovereign Risk Contagion \*

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## Abstract

We develop a theory of sovereign risk contagion based on financial links. In our multi-country model, sovereign bond spreads comove because default in one country can trigger default in other countries. Countries are linked because they borrow, default, and renegotiate with common lenders, and the bond price and recovery schedules for each country depend on the choices of other countries. A foreign default increases the lenders' pricing kernel, which makes home borrowing more expensive and can induce a home default. Countries also default together because by doing so they can renegotiate the debt simultaneously and pay lower recoveries. We apply our model to the 2012 debt crises of Italy and Spain and show that it can replicate the time path of spreads during the crises. In a counterfactual exercise, we find that the debt crisis in Spain (Italy) can account for one-half (one-third) of the increase in the bond spreads of Italy (Spain).

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# 1 Introduction

Sovereign debt crises occur in multiple countries at the same time. The Russian crisis of 1998, the Mexican crisis of 1994, and the Latin American debt crisis of the 1980s, among others, spilled over to other emerging markets. During these crises, the sovereign risk of many countries rose and fell in tandem. More recently, during the 2012 European debt crisis, many countries also experienced sizable spikes in their sovereign risk.<sup>1</sup> Yet, despite sovereign debt crises occurring in multiple countries simultaneously, theoretical work on sovereign risk has mainly studied countries in isolation.

We develop a multi-country model where default in one country triggers default in other countries, generating comovement in sovereign risk. This *contagion* in sovereign risk arises because countries borrow, default, and renegotiate with common lenders. A foreign default lowers the home country's bond prices and the future home recovery rates in the event of a renegotiation. These two forces can induce a home default as rolling over the debt is more difficult and renegotiating the defaulted debt in the future is more attractive. We apply our model to the 2012 debt crises of Italy and Spain and show that it can replicate the time path of spreads observed. In a counterfactual exercise, we assess the contribution of the foreign debt crisis on home bond spreads and find that about one-half of the bond spread in Italy and about one-third of the bond spread in Spain can be attributed to the debt crisis of the other country.

The model economy is dynamic and consists of two symmetric countries that borrow, default, and renegotiate their debt with competitive lenders that are risk averse. Borrowing countries are subject to fluctuations in their output, and bond prices reflect the risk-adjusted compensation for the loss that lenders face in case of default. Default entails costs for borrowing countries in terms of access to financial markets and direct output costs. After default, countries can renegotiate with a committee of lenders through Nash bargaining and pay the debt recovery. When multiple countries renegotiate with lenders, they do it simultaneously.

Countries are connected because bond prices and recovery rates for *each* country are schedules that depend on *both* countries' choices to default, borrow, and renegotiate, as well as their output shocks. Borrowing countries are strategic players and understand that their choices affect all bond prices and recoveries. We analyze an intraperiod game where default, borrowing, and renegotiation decisions are best responses in the context of a dynamic recursive Markov equilibrium.

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<sup>1</sup>Reinhart and Rogoff (2011) document not only that sovereign risk is highly correlated across countries, but also that default events are clustered historically.

Countries' default incentives depend on one another through the *strategic renegotiation* channel and the *pricing kernel* channel. A foreign default lowers the expected recovery rates for the home country reducing the cost of a default for the home country. This strategic renegotiation channel can induce a default in the home country. A foreign default also makes borrowing more expensive for the home country by increasing the bond pricing kernel which increases the incentives to default for the home country. The dependency of home default on foreign default arises during fundamental foreign defaults, where the foreign country defaults because of high debt and low income, and also during self-fulfilling defaults, where both countries default only because the other is defaulting. Such dependencies generate ex-ante correlated bond spreads.

Renegotiations with multiple borrowing countries lead to lower recoveries because countries exert a large effective bargaining power when renegotiating simultaneously. Lenders are willing to accept these lower recoveries because during these events they solve two default events simultaneously. We show that this empirical implication of the model is borne out in the historical cross-country data. In the data, recovery rates are about 16% lower on average in years when many countries are renegotiating. We use these data to discipline this force in our quantitative exercise.<sup>2</sup>

In our model, as in many dynamic models of borrowing and default, the shape and responses of the bond price schedule are crucial for equilibrium spreads dynamics and co-movements. The bond price schedule for each country incorporates a common pricing kernel and the country specific default probabilities and expected recovery rates. As is standard in these models, the bond price for the home country worsens with larger home borrowing and lower home output because default probabilities rise with large debt and low output.<sup>3</sup> The novelty in our model is that the foreign country's choices for default and borrowing as well as its output also affect the bond price schedule for the home country. Foreign defaults worsen the bond price schedule for the home country because they increase the lenders' pricing kernel as they lower lenders' wealth and consumption. This tighter home bond price schedule can induce a home default. Foreign defaults also increase the home country's future default probabilities and lower the home country's expected recovery rates because the pricing kernel effects from a foreign default last until the foreign country renegotiates and resolves the default.

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<sup>2</sup>The Brady plan of the early 1990s is a specific example in which many Latin American countries renegotiated together and received an unusually good deal. These countries were able to exchange their defaulted debt for new Brady bonds with principal collateralized by the U.S. government.

<sup>3</sup>See Aguiar and Gopinath (2006) and Arellano (2008) for an extensive discussion on the properties of the bond price schedules in dynamic models of borrowing and default.

Not only do actual foreign default events affect the home country bond price schedule through its effect on the pricing kernel, but also the foreign country's choices for borrowing and its output affect the schedule because they change the home country's future default probabilities. When the foreign country is in a recession or borrows a large amount, its own default probabilities in the future rise. Higher future foreign default probabilities raise future home default probabilities. This higher likelihood of future joint defaults tightens the home country's current bond price schedule. In response to the tight bond price schedule, the home country reduces its borrowing or defaults, if the effect is strong enough. The equilibrium comovement of sovereign spreads across countries depends on the sensitivity of the bond price schedule to foreign conditions relative to the strength of the endogenous borrowing response.

We parameterize the model to Italy and Spain. The model predicts that country bond spreads and default probabilities comove because foreign recessions increase both foreign and home spreads. We show through impulse response functions that absent any changes in home output, a 3% decline in foreign output leads to a 40 basis-point increase in foreign spreads. The increase in home spreads in the impulse response function is about 40% that of foreign spreads. The comovement of bond spreads across countries is a hallmark of the historical sovereign debt crises in the world.

To compare the model to the European experience, we perform an event analysis. We feed into the model the paths for output observed in Italy and Spain from 2006 to 2015, which feature sizable declines, and compare the predictions of the model for bond spreads. In the model, bond spreads in the event increase in response to the decline in output in both countries as well as to the contagion in sovereign risk. The bond spreads in the model can replicate the paths in the data. Spreads in the model and data are very low in the beginning of the event, increase starting in 2009, and peak in 2012 at about 5% for Italy and about 4% for Spain. We then assess the contribution of the foreign debt crisis for the increase in bond spreads in each country through a counterfactual exercise that eliminates the debt crisis in one country at a time. We find that contagion from the debt crisis from one country to another is strong and accounts for more than one-third of the increase in spreads during 2012.

We also show that the prediction of our model for the patterns of defaults, renegotiations, and recoveries is consistent with the cross-country historical data for 77 countries since 1970. In our model, as in the data, the probability of default increases when others default, and the probability of renegotiation increases when others renegotiate and decreases when others default. The patterns of recovery rates in our model also mirror the patterns in the data. Recoveries not only are lower when other countries are renegotiating but also are higher when

other countries are defaulting.

Finally, we compare our benchmark results to two stripped-down models to quantify the importance of the pricing kernel and strategic renegotiation channels in our benchmark model. The stripped-down models have a risk neutral pricing kernel and zero debt recovery, respectively. The correlation of home and foreign spreads in these stripped-down models is substantially lower than in the benchmark, indicating that the results are driven mainly by the interaction between the two channels. In our model, strategic renegotiations not only generate differential recovery rates, which lead to correlated defaults, but also magnify the pricing kernel channel of contagion. Debt recovery after renegotiations allows the model to generate larger debt levels, which in turn amplifies the importance of debt dynamics and default on the pricing kernel, increasing the comovement of sovereign risk.

The model in this paper builds on the benchmark model of equilibrium default with incomplete markets analyzed in Aguiar and Gopinath (2006) and Arellano (2008), and in a seminal paper on sovereign debt by Eaton and Gersovitz (1981). These papers analyze the case of risk-neutral lenders, abstract from recovery, and focus on the default experiences of single countries. Borri and Verdelhan (2009) and Lizarazo (2013) study the case of risk-averse lenders, and Pouzo and Presno (2011) study the case of lenders with uncertainty aversion. They show that deviations from risk neutrality allow the model to generate spreads that are larger than default probabilities, which is a feature of the data. Borri and Verdelhan also show empirically that a common factor drives a substantial portion of the variation observed.

Our paper is related to the literature on contagion through a common investor, such as Kyle and Xiong (2001) and Calvo and Mendoza (2000). Kyle and Xiong (2001) analyze the wealth channel of contagion and show that if investors' preferences exhibit decreasing absolute risk aversion, they have incentives to reduce the riskiness of their investment when their wealth is lower. In our work, we build on this insight and provide a framework where sovereign default lowers investors' wealth. Calvo and Mendoza (2000) focus on how common investors of multiple countries have incentives to reduce the gathering of country-specific information during crises and retrench their loans. While we abstract from information frictions as a source of contagion, we share their implication of reductions in capital flows during episodes of crises.

In the context of sovereign debt, our paper is related to the contemporaneous work by Park (2013). He studies contagion in a model similar to ours in which multiple borrowers trade with risk-averse lenders who are subject to capital requirements. His model can also generate comovement in spreads across borrowing countries; however, he abstracts from any debt recovery and strategic interactions.

Yue (2010) and Benjamin and Wright (2009) study debt renegotiation in a model with risk-neutral lenders. They find that debt renegotiation allows the model to better match the default frequencies and the debt-to-output ratios. Muller, Storesletten, and Zilibotti (2015) also focus on debt renegotiation and show that a renegotiation protocol where creditors make take-it-or-leave-it offers can alleviate the inefficiencies that arise during sovereign defaults.

Our model also presents new types of self-fulfilling equilibria that lead to sovereign defaults. Coordination failures have been popular explanations for sovereign debt crises. The main channel analyzed in the literature, however, emphasizes coordination failures among lenders, whereas we focus on coordination issues among borrowers. Cole and Kehoe (2000) and Lorenzoni and Werning (2013) develop models with multiple equilibria in which defaults are self-fulfilling: lenders refuse to completely roll over the country's debt because they think that countries will default on the debt, which in turn leads to default. In contrast, the self-fulfilling equilibria of our model arise because of strategic interactions among borrowers, which we view as also relevant for the case in which sovereign countries borrow from international lenders.

Our application to the European debt crisis is most similar to Bocola and Dovis (2016). They develop a model with fundamental and self-fulfilling sovereign defaults in the context of a time-varying pricing kernel to study the Italian experience. They decompose the sources of high Italian spreads and find that although fundamental domestic shocks can account for a large fraction of the variation, sunspot shocks in the context of time-varying risk premia also account for a sizable fraction of the variation. We view our model with strategic borrowing countries as complementary and providing a rationale for time-varying risk premia.

## 2 Model

Consider an economy in which two symmetric countries, Home and Foreign, borrow from a continuum of foreign lenders. Countries are strategic players who borrow, default, and renegotiate their debt. Lenders are risk averse and competitive. Countries that default receive a bad credit standing, are excluded from borrowing, and suffer a direct output cost. Countries in bad credit standing can renegotiate their debt with a committee of lenders and bargain over the debt recovery. After renegotiation is complete, countries regain their good credit standing.

The lifetime payoff to each borrowing country  $i$  is  $E \sum_{t=0}^{\infty} \beta^t u(c_{it})$ , and the payoff to lenders is  $E \sum_{t=0}^{\infty} \delta^t u(c_{Lt})$ , where  $c_{it}$  is the consumption of the representative household in each country,  $c_{Lt}$  is the consumption of lenders, and the function  $u(\bullet)$  is increasing and

concave. Borrowing countries are more impatient than lenders:  $0 < \beta < \delta < 1$ .

Each borrowing country receives a stochastic endowment each period. Let  $y = \{y_i\}_{\forall i}$  be the vector of endowments for each country in a period. These shocks follow a Markov process with transition matrix  $\pi(y', y)$ . We assume that lenders face no additional shocks. The endogenous aggregate states consist of the vector of countries' debt holdings  $b = \{b_i\}_{\forall i}$  and their credit standing  $h = \{h_i\}_{\forall i}$ . The economy-wide state  $s$  incorporates the endogenous and exogenous states:  $s = \{b, h, y\}$ .<sup>4</sup>

## 2.1 Borrowing Countries

The government of each country is benevolent, and its objective is to maximize household utility. The government trades one-period discount bonds with foreign lenders, decides whether to repay or default on its debt, and after a default, decides whether or not to renegotiate the debt. The government rebates back to households all the proceedings from its credit operations in a lump-sum fashion. We label country  $i$  as Home and country  $-i$  as Foreign. Below we describe in detail the problem for the home country. The problem for the foreign country is symmetric.

We consider a Markov equilibrium where the governments take as given future decisions. The *current strategy* for the government at Home incorporates its repayment or renegotiation decision  $d_i$  and its borrowing decision  $b'_i$ . When the country is in good credit standing  $h_i = 0$ , it decides to repay the debt by setting  $d_i = 0$ . Only after deciding to repay can the country choose its new borrowing  $b'_i$ . If the government decides to default by setting  $d_i = 1$ , the government cannot borrow and its credit standing changes to bad the following period. When the home government is in bad credit standing  $h_i = 1$ , it decides to renegotiate by setting  $d_i = 0$ . Renegotiation changes the government's credit standing to good the next period. After renegotiation the government starts with zero debt,  $b'_i = 0$ . The current strategy for both countries is summarized by  $\{b', d\} = \{b'_i, d_i\}_{\forall i}$ .

The home bond prices  $q_i(s, b', d)$  and recoveries  $\phi_i(s, b', d)$  are functions that depend on the current strategies for both countries as well as the aggregate state. In making decisions, the governments take as given the bond price and recovery functions. The bond price function compensates the lender for the risk-adjusted loss in case of default and depends on the strategies of both countries and the aggregate states because the lenders' pricing kernel, as well as future defaults, renegotiations, and recoveries, depend on all of these variables. The recovery function is the result of a bargaining process, the outcome of which depends on

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<sup>4</sup>Although we focus on two symmetric countries for computational tractability, the forces we emphasize also apply for environments with additional and asymmetric borrowing countries.

the countries' strategies and the aggregate state. Below we specify how the bond price and recovery functions are determined.

The current home consumption depends on the aggregate state and the current strategies of both countries  $c_i(s, b', d)$ . Consider the case of the home country having good credit standing,  $h_i = 0$ , and an arbitrary strategy to repay  $d_i = 0$  and to borrow  $b'_i$ . Consumption in this case is

$$c_i = y_i - b_i + q_i(s, b', d)b'_i. \quad (1)$$

Consumption for country  $i$  also depends on the state and strategy of the other country by their effect on the price  $q_i$ . Now consider consumption with a strategy to default, such that  $d_i = 1$ . Default results in exclusion from trading international bonds and output costs  $y_i - y_i^d$ , with  $y_i^d \leq y_i$ . Consumption equals output during these periods:

$$c_i = y_i^d. \quad (2)$$

Finally, consider the case when country  $i$  is in bad credit standing such that  $h_i = 1$ . When renegotiation is chosen,  $d_i = 0$ , the country pays the recovery  $\phi_i(s, b', d)$ , starts tomorrow with zero debt,  $b'_i = 0$ , and consumption is

$$c_i = y_i - \phi_i(s, b', d). \quad (3)$$

Here, the state and strategy of the other country also affect home consumption by their effect on the recovery. If the home country does not renegotiate, then consumption satisfies (2).

We represent the home country's payoffs as a dynamic programming problem. The government today takes as given all the decisions of future governments, which are summarized by the continuation value function from tomorrow on  $v_i(s')$  when the state tomorrow is  $s'$ . The lifetime payoff of the home country today when the state is  $s$  for arbitrary current strategies  $(b', d)$  is

$$w_i(s, b', d) = \{u(c_i(s, b', d)) + \beta \sum_{y'} \pi(y', y)v_i(s')\}. \quad (4)$$

Tomorrow's state  $s' = \{b', h', y'\}$  depends on the current strategy of both countries. Specifically, the future credit standing and debt tomorrow depend on the default and renegotiation of each country, as follows:

$$h'_i = \begin{cases} 1 & \text{if } d_i = 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } i \quad (5)$$

$$b'_i = \begin{cases} b'_i & \text{if } h_i = 0 \text{ and } d_i = 0 \\ b_i & \text{if } d_i = 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } i \quad (6)$$

In our model, each borrowing country internalizes the effects its strategies have on bond prices and recoveries. We consider an intraperiod game between the two countries that has two stages. In the first stage, countries make their default and renegotiation decisions. In the second stage, if countries chose to repay in the first stage, they make their borrowing decisions.<sup>5</sup>

To develop the intraperiod game, we start with the second borrowing stage after default and renegotiation decisions  $d$  have been made. The nature of this subgame depends on the credit standing of countries and their repayment decisions. When all countries are in good credit standing and repay,  $\{d_i = 0\}_{\forall i}$ , equilibrium borrowing strategies  $B(s, d) = \{B_i(s, d)\}_{\forall i}$  are Nash in that  $\{B_i = x_i^b(B_{-i}, s, d)\}_{\forall i}$ , where  $x_i^b(b'_{-i}, s, d)$  is the borrowing best response of each country  $i$  for arbitrary borrowing strategies  $b'_{-i}$ , given states  $s$  and repayment choices  $d$ ,

$$x_i^b(b'_{-i}, s, d) = \{b'_i : \max_{b'_i} w_i(s, b', d; v_i(s'))\} \text{ for all } i. \quad (7)$$

When each country starts with a bad credit standing or it defaults, it cannot borrow and hence does not enter the second borrowing stage of the game. Here, the remaining country  $i$  chooses its borrowing to satisfy (7), where  $b'_{-i}$  equals  $b_{-i}$  or 0 according to the default and renegotiation choices given by (6).

In the first stage of the game, each country  $i$  chooses its repayment strategy  $d_i$  taking as given the equilibrium borrowing strategies of the second stage. The equilibrium repayment strategies  $D(s) = \{D_i(s)\}_{\forall i}$  are Nash in that  $\{D_i = x_i^d(D_{-i}, s, B(s, D))\}_{\forall i}$ , where  $x_i^d(d_{-i}, s, B(s, d))$  is the repayment best response of each country  $i$  for arbitrary repayment strategies  $d_{-i}$ , given states  $s$  and taking into account the outcome of the second borrowing stage  $B(s, d)$ :

$$x_i^d(d_{-i}, s, B(s, d)) = \{d_i : \max_{d_i} w_i(s, B(s, d), d; v_i(s'))\} \quad \text{for all } i. \quad (8)$$

The resulting outcome of the intraperiod game is summarized by the repayment and borrowing functions  $\{D(s)\}$  and  $\{B(s) = B(s, D(s))\}$ , as well as the consumptions  $c(s) = \{c_i(s)\}_{\forall i}$  and values  $v(s) = \{v_i(s)\}_{\forall i}$ .

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<sup>5</sup>We subdivide the intraperiod game between the two countries into a repayment and borrowing stage because it substantially simplifies our computational algorithm as explained in the Appendix.

**Definition 1.** A Markov partial equilibrium takes as given bond price functions  $\{q_i(s, b', d)\}_{\forall i}$  and recovery functions  $\{\phi_i(s, b', d)\}_{\forall i}$  and consists of equilibrium strategies  $\{B(s), D(s)\}$  and payoffs  $c(s)$  and  $v(s)$  such that

- (1) Given future value functions  $v(s')$ , period equilibrium strategies  $\{B(s), D(s)\}$  are the solution of the intraperiod game such that they satisfy (6), (7), and (8).
- (2) Equilibrium payoffs  $v(s)$  implied by equilibrium strategies  $\{B(s), D(s)\}$  are a fixed point

$$v_i(s) = w_i(s, B(s), D(s); v_i(s')) \quad \text{for all } i.$$

## 2.2 Lenders

Competitive lenders trade bonds with the two borrowing countries. Every period lenders receive a constant payoff from the net operations of other loans  $r_L L$  and deposits  $r_d D$ , which we summarize by  $y_L = r_L L - r_d D$ . We assume that lenders honor all financial contracts.

Lenders take as given the evolution of the aggregate state,

$$s' = H(s) \tag{9}$$

and the corresponding decision rules for debt, default, and renegotiation,  $\{B(s), D(s)\}$ . Lenders choose optimal consumption  $c_L$  and loans to the borrowing countries  $\ell' = \{\ell'_i\}_{\forall i}$ , taking as given the prices of bonds  $Q = \{Q_i\}_{\forall i}$  and recoveries  $\Phi = \{\Phi_i\}_{\forall i}$ . The value function for lenders is given by

$$v^L(\ell, s) = \max_{\{c_L, \ell'_i \text{ if } h_i = h'_i = 0\}_{\forall i}} \{u(c_L) + \delta \sum_{y'} \pi(y', y) v^L(\ell', s')\}. \tag{10}$$

Lenders maximize their value subject to their budget constraint that depends on the credit standing of each borrowing country and whether they repay,

$$c_L = y_L + \sum_i (1 - D_i(s)) \left( (1 - h_i)(\ell_i - Q_i \ell'_i) + h_i \frac{\Phi_i \ell_i}{b_i} \right), \tag{11}$$

the evolution of the endogenous states when they do not trade with each country,

$$\ell'_i = \begin{cases} \ell_i & \text{if } h'_i = 1 \\ 0 & \text{if } (h_i = 1 \text{ and } h'_i = 0) \end{cases} \quad \text{for all } i, \tag{12}$$

and the evolution of the aggregate state (9).

Using the first order conditions and envelope conditions for the lenders' problem, one can

show that bond prices satisfy

$$Q_i = \sum_{s'} m(s', s)(1 - D_i(s')(1 - \zeta_i(s'))) \quad \text{for all } i, \quad (13)$$

where  $\zeta_i(s')$  is the present value of recoveries and is defined recursively by

$$\zeta_i(s) = \sum_{s'} m(s', s) \left[ (1 - D_i(s')) \frac{\Phi_i(s')}{b'_i} + D_i(s') \zeta_i(s') \right] \quad \text{for all } i. \quad (14)$$

and  $m(s', s)$  is the lenders' stochastic discount factor or pricing kernel,

$$m(s', s) = \frac{\delta \pi(y', y) u'(c_L(s'))}{u'(c_L(s))},$$

where  $c_L(s)$  is the equilibrium consumption in state  $s$ .

The bond prices in (13) and the values of recoveries in (14) are easily interpretable. The bond price contains two elements: the payoff in nondefault states  $D_i(s') = 0$  and the payoff in default states  $D_i(s') = 1$ . The lender discounts cash flows by the pricing kernel  $m(s', s)$ , and hence states are weighted by  $m(s', s)$ . For every unit of loan  $\ell'_i$ , the lender gets one unit in the nondefault states and the value of recovery  $\zeta_i(s')$  in default states. The recovery value is the expected payoff from defaulted debt the following period. It also contains two parts. If the country renegotiates next period,  $D_i(s') = 0$ , and the value of recovery for every unit of loan is  $\frac{\Phi_i(s')}{b'_i}$ . If the country does not renegotiate,  $D_i(s') = 1$ , and the present value of recovery is the discounted value of future recovery given by  $\zeta_i(s')$ . These future recovery values are weighted by the pricing kernel  $m(s', s)$ , which implies that recovery values are weighted more heavily for states  $s'$  that feature a higher pricing kernel.

The bond price compensates the lender for any covariation between its kernel and the bond payoffs. If default happens in states when  $m(s', s)$  is high, the price contains a positive risk premia for the default event. Moreover, if the value of recovery is low when  $m(s', s)$  is high, the price also contains positive risk premia for the covariation of recovery. The pricing kernel tends to be high when borrowing countries borrow large amounts and when they default because these choices tend to lower  $c_L$ .

### 2.3 Renegotiation Protocol

During renegotiation, countries renegotiate their debt with a committee of lenders. The renegotiation protocol we consider is one in which the committee of lenders bargains simul-

taneously with all the countries using Nash bargaining.<sup>6</sup>

To build the Nash bargaining problem, consider each borrowing country  $i$  paying a candidate recovery value  $\hat{\phi}_i$  to lenders. The payoff for this country  $w_i$  depends on the aggregate state  $s$  and is defined as in (4) over arbitrary strategies  $(b', d)$ . The payoff for this borrowing country from renegotiation is then  $w_i(s, b', d; \hat{\phi}_i)$  for this candidate value of recovery  $\hat{\phi}_i$ . The payoff for lenders from renegotiating and receiving candidate recoveries  $\{\hat{\phi}_i, \hat{\phi}_{-i}\}$  given aggregate state  $s$  and arbitrary strategies  $(b', d)$  equals the value of the representative lender evaluated at the aggregate debt values and arbitrary strategies  $V_L(s, b', d; \hat{\phi}_i, \hat{\phi}_{-i}) \equiv v_L(b, s, b', d; \hat{\phi}_i, \hat{\phi}_{-i})$ .

If the parties do not reach an agreement, each country receives the value of renegotiation failure denoted by  $v_i^f(s)$ . The value for renegotiation failure for a country with bad credit  $h_i = 1$  is permanent financial autarky with  $y_i = y_i^d$  such that  $v_i^f(s_{\{h_i=1\}}) = u(y_i^d) + \beta E v_i^f(s'_{\{h_i=1\}})$ . The value for renegotiation failure for countries with good credit  $v_i^f(s_{\{h_i=0\}})$  is the value arising in an environment with only one country borrowing from lenders. This value is characterized in the Appendix in the single-country Markov equilibrium. The value of renegotiation failure for lenders is consistent with that of the borrowing countries, and hence it depends on aggregate state  $s$ . When both countries have bad credit standing  $h_i = h_{-i} = 1$ , the value of renegotiation failure for lenders is permanent autarky such that  $V_L^f(s_{\{h_i=1\}\forall i}) = u(y_L)/(1-\delta)$ . If only one borrowing country has bad credit, then the value from renegotiation failure for lenders also arises from a single-country Markov equilibrium in the Appendix.

The Nash bargaining problem solves for the recovery values  $\{\phi_1, \phi_2\}$  that maximize the product of the surpluses of all parties such that

$$\max_{\phi_1, \phi_2} \left[ V_L(s, b', d; \phi_1, \phi_2) - V_L^f(s) \right]^{\theta_L} \prod_{i=1,2} \left[ w_i(s, b', d; \phi_i) - v_i^f(s) \right]^{\theta_i} \quad (15)$$

subject to all parties receiving positive weighted surplus,  $\theta_i(w_i(s, b', d; \phi_i) - v_i^f(s)) \geq 0$  and  $\theta_L(V_L(s; \phi_1, \phi_2) - V_L^f(s)) \geq 0$ , and the law of motion for aggregate states (9). The bargaining weights for borrowing countries depend on the credit standing,  $\theta_i = \theta h_i$ , such that only countries with bad credit and with the option to renegotiate have positive weights. For simplicity, we also impose zero recoveries for countries with good credit  $\phi_i(h_i = 0) = 0$ .<sup>7</sup>

An important aspect of the renegotiation protocol we consider is the simultaneity in bargaining between the committee of lenders and all countries. This renegotiation protocol is reminiscent of the Brady renegotiation episode that included many countries in Latin

<sup>6</sup>This *strict simultaneous bargaining* protocol has often been used in industrial organization models of multifactors. See Dobson (1994) and Horn and Wolinsky (1988) for details.

<sup>7</sup>This assumption is without loss. Any recovery payment from countries with  $h_i = 0$  does not affect lenders' consumption, as seen from lenders' budget constraint 11.

America that were in default.

## 2.4 Functions for Bond Prices and Recoveries

The lenders' problem and the renegotiation protocol determine the functions for bond prices and recoveries. First consider the case when both countries are in good credit standing,  $\{h_i = 0\}_{\forall i}$ . Here, bond price functions  $q(s, b', d) = \{q_i(s, b', d)\}_{\forall i}$  solve the demand system determined by lenders' first order conditions:

$$q_i = \sum_{s'} [m(s', s; q, b', d)(1 - D_i(s')(1 - \zeta_i(s')))] \text{ for all } i, \quad (16)$$

where the state tomorrow  $s' = \{b', h', y'\}$  depends on countries' current strategies  $(b', d)$  and the lenders' pricing kernel  $m(s', s; q, b', d)$  is itself a function of prices, countries' strategies, and current and future states.

Now consider the case when country  $i$  is in good credit standing and country  $-i$  is in bad credit standing,  $h_i = 0$  and  $h_{-i} = 1$ . The bond price function for country  $i$  and the recovery function derived from (15) for country  $-i$ ,  $\{q_i(s, b', d), \phi_{-i}(s, b', d)\}$  solve

$$q_i = \sum_{s'} [m(s', s; q, b', d)(1 - D_i(s')(1 - \zeta_i(s')))] \quad (17)$$

$$\frac{\theta_{-i} u'(y_{-i} - \phi_{-i})}{\left[ w_{-i}(s, b', d; \phi_{-i}) - v_{-i}^f(s) \right]} = \frac{\theta_L u'(c_L(s, b', d, q_i, \phi_{-i}))}{\left[ V_L(s, b', d; q_i, \phi_{-i}) - V_L^f(s) \right]},$$

where the lender's consumption and values are evaluated for every strategy and corresponding price and recovery.

Finally, when both countries are in bad credit standing,  $\{h_i = 1\}_{\forall i}$  recovery functions  $\phi(s, b', d) = \{\phi_i(s, b', d)\}_{\forall i}$  solve

$$\frac{\theta_i u'(y_i - \phi_i)}{\left[ w_i(s, d; \phi_i) - v_i^f(s) \right]} = \frac{\theta_L u'(c_L(s, d; \phi_i, \phi_{-i}))}{\left[ V^L(s, d; \phi_i, \phi_{-i}) - V_L^f(s) \right]} \text{ for all } i. \quad (18)$$

The risk-free rate is defined in a standard way as the inverse of the expected pricing kernel. We can define a risk-free rate function over arbitrary strategies  $(b', d)$  given state  $s$  as

$$r^f(s, d, b') = \frac{1}{E m(s, s')}, \quad (19)$$

where the prices and recoveries  $\{q_i, \phi_i\}_{\forall i}$  solve equations (16), (17), and (18).

## 2.5 Equilibrium

We focus on recursive Markov equilibria in which all decision rules are functions only of the state variable  $s$ .

**Definition 2.** *A recursive Markov equilibrium for this economy consists of (i) countries' policy functions for repayment, borrowing, and consumption,  $\{B(s), D(s), C(s)\}$ , and values  $v(s)$ ; (ii) lenders' policy functions for lending choices and consumption  $\{\ell'(\ell, s), c_L(\ell, s)\}$  and value function  $v^L(\ell, s)$ ; (iii) the functions for bond prices and recoveries  $\{q(s, b', d), \phi(s, b', d)\}$ ; (iv) the equilibrium prices of debt  $Q(s)$  and recovery rates  $\Phi(s)$ ; (v) the evolution of the aggregate state  $H(s)$ ; and (vi) the lenders' and borrowers' values in the case of renegotiation failure  $V_L^f(s)$  and  $v^f(s)$  such that given  $b_0 = \ell_0$ :*

1. *Taking as given the bond price and recovery functions, the policy and value functions for countries satisfy the Markov partial equilibrium in Definition 1.*
2. *Taking as given the bond prices  $Q(s)$ , recoveries  $\Phi(s)$ , and the evolution of the aggregate states  $H(s)$ , the policy functions and value functions for the lenders  $\{\ell'(\ell, s), c_L(\ell, s), v^L(\ell, s)\}$  satisfy their optimization problem.*
3. *Taking as given countries' policy and value functions, bond price and recovery functions  $\{q(s, b', d), \phi(s, b', d)\}$  satisfy (16), (17), and (18).*
4. *The prices of debt  $Q(s)$  clear the bond market for every country,*

$$\ell'_i(s) = B_i(s) \text{ for all } i.$$

5. *The recoveries  $\Phi(s)$  exhaust all the recovered funds,*

$$\phi_i(s, B(s), D(s)) = \Phi_i(s) \text{ for all } i.$$

6. *The goods market clears,*

$$c_1 + c_2 + c_L = y_1 + y_2 + y_L.$$

7. *The law of motion for the evolution aggregate states (9) is consistent with countries' decision rules and shocks.*

8. The values in the case of renegotiation failure for lenders and borrowers  $\{V_L^f(s), v_i^f(s)\forall i\}$  arise from the single-country Markov equilibrium in Definition 3 in Appendix I.

### 3 Joint Defaults

In this section, we develop a simple two-period example to illustrate why countries have incentives to default together.

Consider a two-period version of our model with no uncertainty, where countries have identical endowment paths  $y$  and  $y'$ . The lenders' payoff function is  $u(c_L) = \frac{c_L^{1-\sigma}-1}{1-\sigma}$ . In period 1, the two countries with debt  $b_i$  and  $b_{-i}$  are in good credit standing and are deciding whether to repay their current debt or default on it. If countries repay their debt, they choose to borrow. In period 2, countries either repay their debts if they borrowed in period 1 or pay the recovery  $\phi'$  if they defaulted in period 1. In this example without uncertainty, in period 2 countries with good credit always repay and countries with bad credit always renegotiate,  $\{d'_i = 0\}_{\forall i}$ . Default does not happen in equilibrium in period 2 because default would be perfectly foreseen and the price of such a loan would be zero. Default incentives in period 2, however, limit the borrowing possibilities for period 1. In particular, in period 1 countries effectively face a borrowing limit  $\bar{b}$ , which is the maximum repayment that countries would be willing to make and equals the default penalty in period 2,  $\bar{b} = y' - y^d$ , where  $y^d < y'$  is the income in case of default.

In this example, we assume that  $\beta$  is sufficiently less than  $\delta$  such that it is optimal for countries to borrow to the limit in period 1. Hence, we abstract from the interdependence across countries in the borrowing decisions and focus on the interdependence in their repayment/default decisions. In this simplified environment, the relevant states for bond prices are the debt states  $b$  and the default decisions of both countries  $d$ ,  $\{q_i(b, d)\}_{\forall i}$ . The relevant states for recovery tomorrow are the credit standing of both countries  $h'$ , which are determined by  $d$ ,  $\{\phi'_i(h')\}_{\forall i}$ . This example has these reduced states because we are assuming that endowments are constant for the countries. Here again, we label  $i$  as Home and  $-i$  as Foreign.

In period 1, each country repays and sets  $d_i = 0$  if the value of repayment is greater than the value of default:

$$u(y - b_i + q_i(b, d)\bar{b}) + \beta u(y' - \bar{b}) \geq u(y^d) + \beta u(y' - \phi'_i(h')) \text{ for all } i. \quad (20)$$

It is apparent that default is more likely for country  $i$  when debt  $b_i$  is high, the price  $q_i$  is low, and the recovery tomorrow  $\phi'_i$  is low. The default decisions of the two countries are

linked because bond prices today and recoveries tomorrow depend on the decisions of both countries through the lenders' problem.

It is useful to derive the home country's default best response conditional on the foreign country's default decision,  $x_i^d(d_{-i}, b)$ . The foreign default decision affects the home country's future recovery  $\phi_i'$  and current debt price  $q_i$ . We show that a foreign default today decreases the home recovery  $\phi_i'$  because the effective bargaining power when two countries renegotiate simultaneously with lenders is larger than when each country renegotiates alone.

**Proposition 1.** *When two countries renegotiate simultaneously, recovery is smaller than when one country renegotiates alone:  $\phi_i'(h'_{-i} = 1) \leq \phi_i'(h'_{-i} = 0)$ .*

Proof. See Appendix II. We label this effect the *strategic renegotiation* force. Intuitively, this result arises because lenders are willing to accept lower recoveries in multiple-country renegotiations to solve the two default episodes simultaneously.<sup>8</sup> An important assumption for this result is that each country will not renegotiate with lenders by itself in case of renegotiation failure in the multiple-country renegotiation. This implicit agreement is optimal for the borrowing countries because it allows them to pay lower recoveries.

This result implies that a foreign default  $d_{-i} = 1$  increases the right-hand side of equation (20) and thus increases the incentive to default for the home country.

The second effect to consider is how a foreign default affects price  $q_i$  through its impact on the pricing kernel. This effect depends on the net capital flows that lenders forgo with the foreign default,  $b_{-i} - q_{-i}\bar{b}$ . The larger the foreign forgone capital flows, the higher the increase in the pricing kernel with a foreign default. We label this effect the *pricing kernel* effect. The following proposition shows that capital flows are increasing with  $b_{-i}$ , and the effect of a foreign default is increasingly detrimental for  $q_i$  the higher is  $b_{-i}$ .

**Proposition 2.** *Home bond prices increase with the foreign country's debt when the foreign country repays:  $q_i(b, d)$  is increasing in  $b_{-i}$  when  $d_{-i} = 0$ .*

Proof. See Appendix II.

As in single-country default models, the home country will default when its current debt  $b_i$  is sufficiently high. It is useful to consider two home debt cutoffs,  $\hat{b}(b_{-i}, d_{-i} = 0)$  and  $\hat{b}(b_{-i}, d_{-i} = 1)$ , which depend on the foreign state and default decision. Home defaults when its debt level is above these two cutoffs.

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<sup>8</sup>The outcome that default costs are lower when many borrowers are in default is also present in environments with bailouts increasing the number of borrowers in default, as in Schneider and Tornell (2004), and when there are limits on the ability to liquidate when many borrowers are in default, as in Arellano and Kocherlakota (2014).

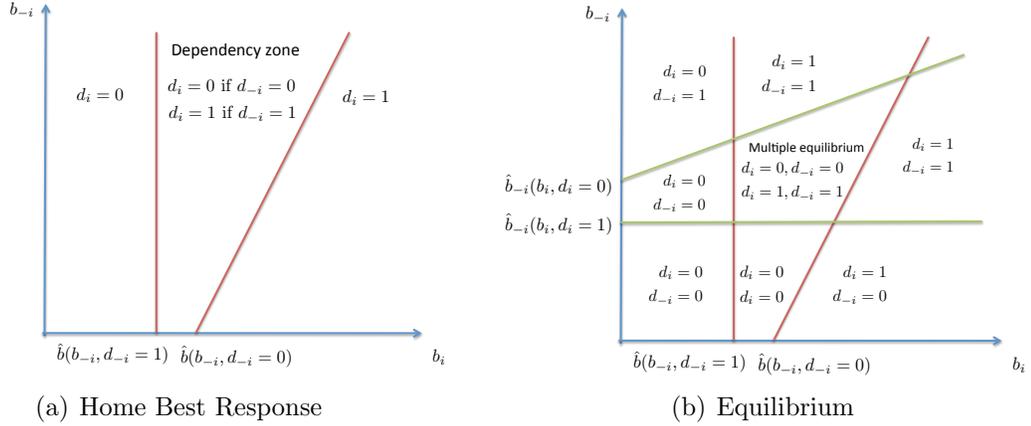


Figure 1: Financial Contagion

The effects of a foreign default on the price  $q_i$  and the future recovery  $\phi'_i$  imply that  $\hat{b}(b_{-i}, d_{-i} = 0)$  is increasing in  $b_{-i}$  and that  $\hat{b}(b_{-i}, d_{-i} = 1)$  is independent of  $b_{-i}$ . The ranking of  $\hat{b}(b_{-i}, d_{-i} = 0)$  and  $\hat{b}(b_{-i}, d_{-i} = 1)$  at  $b_{-i} = 0$  depends on the details of the utility of lenders. We assume that the strategic renegotiation force is strong enough such that  $\hat{b}(b_{-i} = 0, d_{-i} = 0) > \hat{b}(b_{-i} = 0, d_{-i} = 1)$ .

To summarize this analysis, Figure 1(a) plots the home best responses for default as a function of its own debt level  $b_i$  and the foreign country's debt level  $b_{-i}$  conditional on the foreign default decision  $d_{-i}$ . For sufficiently low (or high) levels  $b_i$ , the home country always repays (or defaults) independently of the foreign decision. For intermediate levels of  $b_i$ , however, the home country repays only if the foreign country repays. We label this region the *dependency zone*. By symmetry, the best response of the foreign country is identical to that of the home country, such that for intermediate levels of debt, the foreign country repays only if the home country repays.

Figure 1(b) illustrates the equilibrium in this example by considering both best response functions. The figure shows that in the dependency zones, both countries have joint repayments and joint defaults. Consider the dependency zone for country 1. When the foreign debt is low enough, the foreign repayment guarantees a home repayment. For high foreign debt, a foreign default guarantees a home default. When the foreign debt is in the intermediate region, our model features multiple equilibria: either both countries default or both countries repay. Nevertheless, even in this region, the equilibrium features either joint defaults or joint repayments.

This example has highlighted the two forces that in our model lead to joint defaults due to a common lender. The main idea is that foreign defaults lead to home defaults because

foreign defaults lead to lower future recoveries and tighter current bond prices for the home country due to strategic renegotiation and the pricing kernel effects. Joint defaults and joint repayments occur for fundamental and self-fulfilling reasons. In this example, however, we have abstracted from debt dynamics, bond spreads, and have considered an arbitrary level of initial debt. In practice, the level of debt is endogenous to countries' decisions, and their choices interact with defaults and renegotiations. In the following section, we analyze the general dynamic model with endogenous borrowing and bond spreads.

## 4 Quantitative Analysis

This section presents the quantitative analysis of the model. We first describe the parametrization of the model and discuss the main mechanisms by analyzing how the bond price function depends on the other country's states and choices. We show that our model delivers comovement between countries' bond spreads, defaults, and recoveries. We then perform an event analysis applied to Italy and Spain and find that our model can replicate the paths of bond spreads seen in the data and the high spreads during 2012. In a counterfactual analysis, we assess the contribution of contagion to the debt crisis in each country. Finally, we present a comparative static exercise to decompose the forces driving the results.

### 4.1 Parametrization

Here we discuss the functional forms and parameter values used in the quantitative analysis. We parameterize the model to reproduce data from Italy and Spain. We also use cross-country data on recovery rates to inform other parameters.

**Recovery Rates** An important ingredient in our model is that recovery rates can vary with the number of countries renegotiating. The parameters that control the mean and differential recoveries in multiple relative to single-country renegotiations are the bargaining parameters and the output costs from nonrenegotiation and default. We measure the empirical variation of recoveries with the dataset in Cruces and Trebesch (2013), which compiles recovery rates across 182 sovereign restructures for the period 1970-2010.

To examine the relation between recovery rates and the number of countries renegotiating, we group the years in the dataset based on the number of countries renegotiating in the year. The number of countries renegotiating in each year ranges from one country to seven

countries.<sup>9</sup> Figure 2 plots the average recoveries as a function of the number of countries renegotiating. The figure shows that, historically, recoveries have been lower when many countries renegotiate ranging from about 0.7 when only one country renegotiates to about 0.5 when seven countries renegotiate.

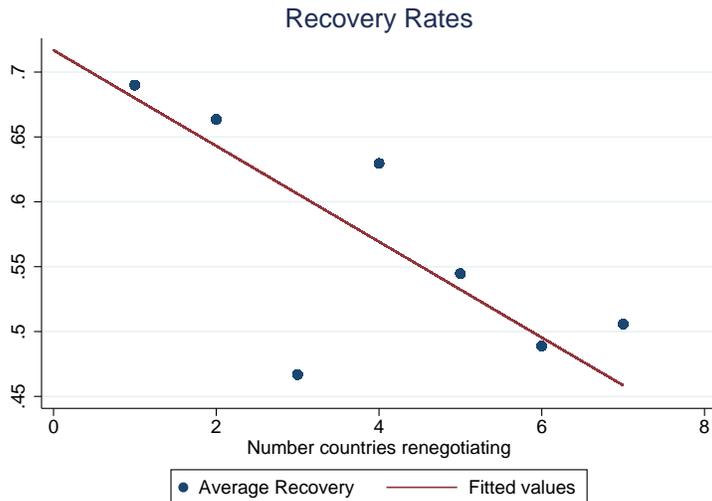


Figure 2: Recovery Rates and Countries

In our model, we can only distinguish recoveries in periods with single renegotiations from recoveries in periods with multiple-country renegotiations. Using the cross-country data, we define multiple-country renegotiation periods as those years with two or more countries renegotiating and single-country renegotiation periods as those years with only one country renegotiating. The difference in recoveries in the data between multiple and single-country renegotiations is 16%, while the overall average recovery rate is 60%.<sup>10</sup>

**Preferences and Technology** We set the length of a period to one year. We assume that the stochastic process for output for each borrowing country follows a log-normal AR(1) process:  $\log(y_{t+1}) = \rho \log(y_t) + \varepsilon_{t+1}$  with  $E[\varepsilon^2] = \eta^2$ . The shock processes for the two countries are identical and uncorrelated between each other. We discretize the shocks into a nine-state Markov chain using a quadrature-based procedure (Tauchen and Hussey, 1991). To set the volatility and persistence of this process, we use linearly detrended real GDP for Italy and Spain for the period 1960 to 2015 and set these parameters to be the average

<sup>9</sup>Cruces and Trebesch (2013) document multiple renegotiations for some of the default episodes. For these we consider only the final renegotiation of the episode.

<sup>10</sup>We found similar results using an alternative dataset of renegotiations provided by Benjamin and Wright (2009). In this dataset, recovery rates are 13% lower in years with multiple-country renegotiations.

ones across the estimated values for the two countries.<sup>11</sup> We normalize the mean output for borrowing countries and lenders to 1.

The utility function for the borrowing countries and lenders is  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ . We set the risk aversion coefficient  $\sigma$  to 2, which is a common value used in real business cycle studies.

For the output cost of default, we follow Chatterjee and Eiygungor (2012) in assuming that it is controlled by two parameters  $\{\lambda_1, \lambda_2\}$  as follows

$$y^d = y - \max\{0, \lambda_1 y + \lambda_2 y^2\}.$$

This state-dependent cost of default allows the model to deliver a differential recovery between single and multicountry renegotiations as well as to generate reasonable levels of bond spreads, as discussed by Chatterjee and Eiygungor (2012).

**Moment Matching** With these assumptions, the numerical specification of the model requires values for five parameters: the lenders' and borrowers' discount rates  $\{\delta, \beta\}$ , the default costs  $\{\lambda_1, \lambda_2\}$ , and the borrower's bargaining parameter  $\theta$ . We choose these five parameters in a moment-matching exercise to best fit five moments: an average risk-free rate of 4%, a mean and standard deviation for bond spreads of 1.0% and 1.3%, an average recovery of 60%, and a difference between recoveries in multiple-country renegotiations and single-country renegotiations of 16%. Table 1 summarizes the parameter values. Spreads in the model and data are defined as the difference between the country yield and the risk-free rate  $spr = 1/q - r_f$ . Recovery rates in the model are defined as the recovery relative to the debt in default  $\phi/b$ .

The mean risk-free rate is calculated from the series for German five-year bond yields since 1980. For spreads, we use annual series of Italian and Spanish five-year Euro bonds since 2001. The mean and standard deviation for Italian spreads are 1.1 and 1.4, respectively. The corresponding values for Spanish spreads are 0.9 and 1.2. The spreads moments we target are the average ones across the values for the two countries.

We solve the model as the limit of a finite horizon model in which each period the borrowing countries' equilibrium strategies are Nash, taking as given the future decisions that are encoded in the future values. As in the simple example, for a certain region of the state space, our model features multiple equilibria. We select the equilibrium that maximizes the joint values for the two borrowing countries,  $v_1 + v_2$ . The numerical algorithm is explained

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<sup>11</sup>To isolate the magnitude of the contagion channel, we consider an uncorrelated output process for the benchmark. In the event analysis of the next section, however, we feed in the observed output comovement and perform our main counterfactual.

Table 1: Parameter Values

	Value	Target
<i>Assigned parameters</i>		
Shocks process	$\rho = 0.935, \eta = 0.027$	Italy and Spain output
Risk aversion	$\sigma = 2$	Standard from literature
<i>Parameters from moment matching</i>		
Output cost after default	$\lambda_1 = -0.168$ $\lambda_2 = 0.186$	Italy and Spain spread: mean and standard deviation
Bargaining power	$\theta = 0.466$	Recovery:
Borrowers' discount factor	$\beta = 0.845$	mean and difference
Lenders' discount factor	$\delta = 0.96$	Mean risk-free rate

in detail in the Appendix.

Table 2 reports the results from our moment-matching exercise. Overall, the model fits the data fairly well. In the model, the mean and volatility of the spread of 1.0% and 1.4% are similar to those in the data of 1.0% and 1.3%. The average recovery in the model of 60% is equal to the average in the data. Recoveries in the model during multiple-country renegotiations are 17% smaller than single-country renegotiations, close to the 16% difference found in the data. Finally, the risk-free rate of 4% is similar in the model to the data.

Table 2: Moment-Matching Exercise

	Data (%)	Model (%)
Mean spread	1.0	1.0
Volatility spread	1.3	1.4
Mean recovery	60	60
Multiple-country recovery	-16	-17
Mean risk-free rate	4.0	4.1

Although the calibrated moments are jointly controlled by all parameters, certain parameters affect certain moments more. The mean and volatility of spreads are mainly controlled by the borrowers' discount factor and the output costs of default. The mean recovery and the recovery difference are controlled by the bargaining power and also by the output costs of default because these are also the costs from renegotiation failure. The mean risk-free rate is mostly determined by the lenders' discount factor.

## 4.2 Model Mechanisms

Before comparing our model to other aspects of the European debt crisis, we explore the model's mechanisms for contagion. We focus on how the states and choices of one country, the *foreign* country, affect the spreads, default, and recovery rates of the other country, the *home* country.

We first illustrate the spread schedules for the home country as a function of its borrowing and the foreign country's output, borrowing, and default choice. We then show how average spreads, default probabilities, and recovery rates vary for the home country when the foreign country has low spreads, when it has high spreads, when it defaults, or when it renegotiates. Finally, we present impulse response functions to a decrease in output in the foreign country and show that both foreign and home spreads increase.

**Spread Schedules.** Each borrowing country  $i$  faces bond price schedules  $q_i(s, b', d)$  that depend on the aggregate states  $s$  as well as the borrowing and default choices for both countries  $(b', d)$  because these variables affect country  $i$ 's default probability and recovery rate, and also the lenders' pricing kernel. We define the *spread schedule* for each country as the difference between the country's yield and the risk-free rate  $1/q_i(s, b', d) - r_f(s, b', d)$ .

We plot the spread schedules for the home country and show that foreign choices and states affect the home schedules. The schedule for the home country is tight when the foreign country defaults, borrows a large amount, or is in a recession.

Figure 3 plots the spread schedules for the home country as a function of its borrowing level  $b'_i$ , for states when both countries are in good credit  $h_i = h_{-i} = 0$ . For the baseline schedule, we set the level of the output and debt states for both countries to be at the mean level, and the borrowing level for the foreign country to be at the optimal level  $b'_{-i} = B(s, d)$ . As is standard in dynamic models of debt and default, home spreads increase with large home borrowing because default is more likely when debt is high.

We also graph the spread schedules for alternative states and choices of the foreign country, keeping the other states as in the baseline. As the figure shows, when the foreign country borrows a larger amount, 20% larger than optimal, the schedule at home tightens. When the foreign country defaults, the schedule further tightens. When the foreign country is in a recession, with output 5% lower than the baseline, the schedule at home is also tighter.

To illustrate the magnitudes of these changes, consider the home country borrowing 8% of output. For the baseline setting, the spread is less than 1%; if the foreign country defaults, the spread jumps to 20%; if the foreign country enters a recession, the spread increases to about 7%.

The foreign country's states and choices affect the home spread schedule because they change the home country's probability of defaulting the next period as well as its recovery conditional on defaulting. When the foreign country borrows a lot or is in a recession, its next period's default probability rises, which increases the home default probability because of the strategic renegotiation and pricing kernel channels. Moreover, the fact that recoveries are lower during multiple-country renegotiations further increases the spreads at home ex-ante.

An actual foreign default also tightens the home schedule for two reasons: a rise in the lenders' pricing kernel and a higher home default probability the next period. A foreign default lowers lenders' consumption, which increases the lenders' kernel and hence the home spread. A foreign default this period can also lead to a higher home default probability the next period because the foreign country can choose to delay renegotiation such that both countries exploit the lower recovery rate arising from joint renegotiations.<sup>12</sup>

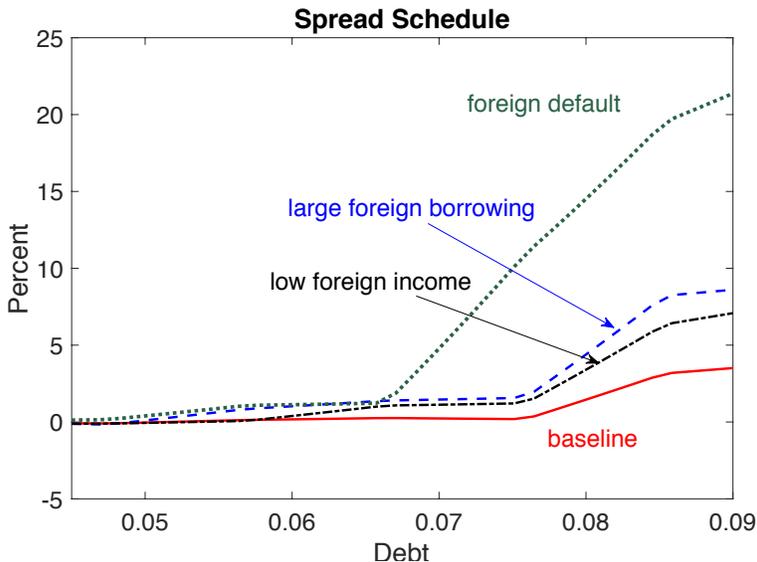


Figure 3: Spread Schedule

Of course, the optimal borrowing and default choices for the home country change in response to different foreign strategies and states. As is typical in models of borrowing and default, tight spread schedules lead to less borrowing. This endogenous borrowing response weakens the comovement of equilibrium spreads in the model. In fact, home spreads in equilibrium can increase or decrease in response to high foreign spreads depending on the strength of the endogenous borrowing response relative to the shift in the schedule. Very

<sup>12</sup>Broner et al. (2006) and Gonzales and Levy (2008) find empirical support for these mechanisms in cross country data. They document that a change in the risk appetite of the investors, which in our model maps into changes in the lenders' kernel, have important effects on sovereign bond spreads and on the portfolio allocation of investors.

tight spread schedules can also induce an actual default because they make it harder to roll over any existing debt. We analyze these equilibrium responses next.

**Spreads, Default Probabilities, and Recoveries.** We now report simulation results across the limiting distribution of states in our model. In Table 3, we report spreads, default probabilities, and recovery rates for the home country in the limiting distribution conditional on whether the foreign country is in good credit or bad credit. We divide the good credit foreign states into states with low foreign spreads, defined as below the 25th percentile, which equals 0.1%, high foreign spreads, defined as above the 75th percentile, which equals 1.6%, and foreign default. Home spreads and probabilities of default are only observed in states when the home country is in good credit standing, while recovery rates are only observed in states when the home country is in bad credit standing.

Table 3: Comovement in Limiting Distribution

Home	Foreign good credit			Foreign bad credit
	Low spreads	High spreads	Default	
Spread	0.7	1.8	1.3	1.0
Default prob.	0.8	1.4	47	2.2
Recovery	67	71	78	43

Table 3 shows that when the foreign country has low spreads, spreads at home are also low and on average equal to 0.7%. When foreign spreads are high, home spreads are also high and on average equal to 1.8%. Default probabilities at home are also lower when the foreign country has low spreads relative to when the foreign country has high spreads, 0.8% and 1.4%, respectively. The default probability, however, jumps up to 47% when the foreign country actually defaults. Although contagion in sovereign risk and default is sizable, a large fraction of variation in home spreads and default probabilities results from changing domestic conditions. Consider, for example, the case when the foreign probability of default is 100%, which by construction is the case in the column labeled Default in Table 3. Here, the home country is more likely to repay than to default: 53% relative to 47% of the time.

These comovements arise in our model due to the shapes of the functions for spreads and recoveries. As illustrated above, the spread schedules are tight when the foreign country borrows a large amount or is in a recession. These cases are associated with high foreign

spreads in equilibrium. In response to the tight schedules, the home country has higher spreads and default probabilities in equilibrium. The very tight spread schedules during foreign defaults also induce home defaults.

Foreign states also have large effects on the recoveries that the home country pays. In states when the foreign country is repaying the debt and has low spreads, recoveries are 67%; they are somewhat higher, equal to 71%, when the foreign country is repaying and has high spreads. Recovery rates are the highest, equal to 78%, when the foreign country defaults, and they are the lowest, equal to 51%, when the foreign country is in bad credit and renegotiates together with the home country. The probability of renegotiation is 100% for the home country, except during periods when the foreign country defaults; then it collapses to 1%. Renegotiation is delayed with a foreign default because the home country prefers to wait and renegotiate together with the foreign country to pay a lower recovery.

The recovery functions play an important role for the comovements. The recovery function is most lenient when the foreign country is also renegotiating because here the countries exert a higher bargaining power, which leads to low recoveries. The recovery function for the home country is tightest when the foreign country defaults because this default increases the lenders' marginal utility, which increases the lenders' marginal surplus from the renegotiation with the home country.

The model implications on the ranking of home recovery rates based foreign defaults and foreign renegotiations and on the simultaneity of default and renegotiations across countries are borne out in the cross-country data. As we will see in Section 5, the historical cross-country data feature recovery rates that are not only low when many countries renegotiate but also high when many countries default. These data also feature simultaneous defaults and renegotiations across countries.

**Impulse responses.** Having presented how spreads for one country depend on the other country on average, we now study these links by presenting impulse responses to shocks. We consider the responses of home and foreign spreads to a negative shock to the foreign output.

We construct the impulse response functions in our nonlinear model following Koop et al. (1996). We simulate 1,000 paths for the model for 1,000 periods. From periods 1 to 500, the aggregate shocks follow their underlying Markov chains. In period 501, the impact period, we reduce each of the values of foreign output  $y_{-i}$  across the paths by 3%, which is about half of the standard deviation of the shock. From period 501 on, the aggregate shocks follow the conditional Markov chains. The impulse responses plot the average, across the 1,000 paths, of the variables from period 501 to 512 conditional on countries being in good credit

standing  $h_i = h_{-i} = 0$  and not defaulting  $d_i = d_{-i} = 0$ . The output values for the foreign and home countries are reported in percentage deviations from their value at  $t = 500$ , while the foreign and home spreads are in percentage points.

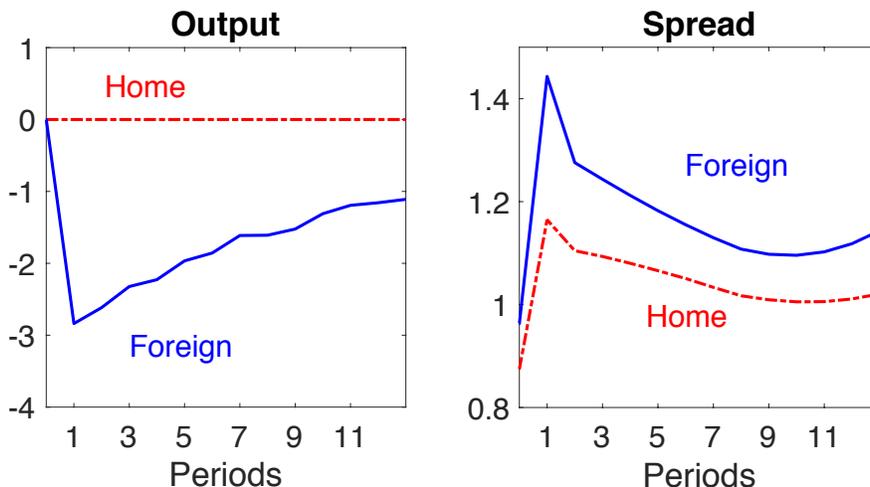


Figure 4: Impulse Responses

Figure 4 graphs the impulse response functions. In the left panel, we plot the output values for the foreign and home countries, which are our shocks. We label the impact period as 1 in the figure. In the impact period, foreign output drops by about 3%. From then on, output returns to its mean following its Markov chain. The home output remains unchanged throughout. In the right panel, we plot the spreads for the foreign and home countries. In response to a decline in foreign output of 3%, foreign spreads spike 0.45% (45 basis points) on impact. After the impact, foreign spreads fall substantially and return to their average level as output recovers. This negative comovement between output and spreads is typical in debt and default models.

Most importantly, this figure shows that home spreads also rise in response to a decline in foreign output even though the home country faces no domestic shock. In the impact period, home spreads increase about 0.2% (20 basis points) and then return to their average level. The increase in home spreads is about 40% that of the rise in spreads in the foreign country where the shock originated.

### 4.3 Event Analysis

We have shown that our model features contagion across sovereign spreads, as spreads at home rise when foreign spreads rise even if home conditions do not change. We now turn to compare the model predictions for the spread comovement during the debt crises in Italy

and Spain. Unlike the impulse response exercise, during the event both countries experienced recessions. We feed into the model a sequence of shock realizations that replicate the time paths of output in these two countries and then compare the resulting model's time paths of spreads to data. In this event analysis, the resulting correlation in spread paths arises because of contagion and also because of a deterioration in domestic conditions in both countries.

In Figure 5 we plot the data and model paths for output from 2006 to 2015.<sup>13</sup> In the beginning of the event, output for Italy and Spain was above trend, peaking at about 5% and 10% above trend in 2007 and 2008. Starting in 2009, output for both countries fell substantially. From peak to trough, output in Italy and Spain fell close to 10% and 15%, respectively.

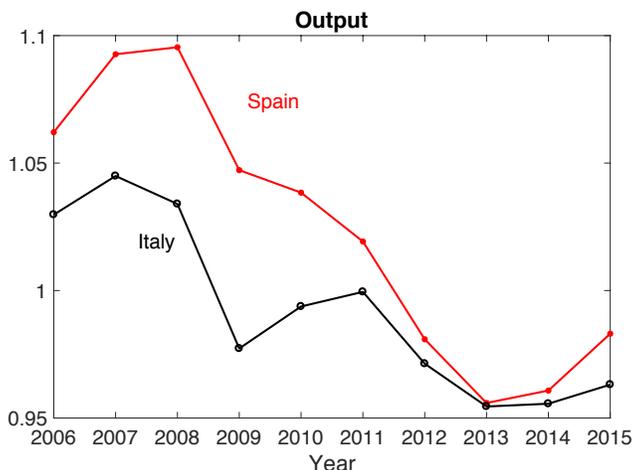


Figure 5: Event Output

For the model series in Figure 5, we follow a procedure similar to the one we used for the impulse responses and feed in shocks such that the model replicates the output series in the data. We simulate 1,000 paths for 1,000 periods. For the first 500 periods the aggregate shocks follow their underlying Markov chains. Period 501 is the first period of the event, which corresponds to 2006. From 2006 on, we feed into the model the sequence of shocks that best approximate the paths of output. Specifically for each period and country, we feed in the two grid points that are closest to the data, namely the closest grid point above and below the data. The fraction of draws for each of these two grid points is based on the linear distance to the data point. The model output paths equal the average output across the simulated paths, which by construction replicate the data.

We now analyze the predictions for spreads. The left panel of Figure 6 plots the data and model time paths for spreads in Italy, and the right panel plots those for Spain. The model

<sup>13</sup>Output in the data is reported relative to a 2% trend.

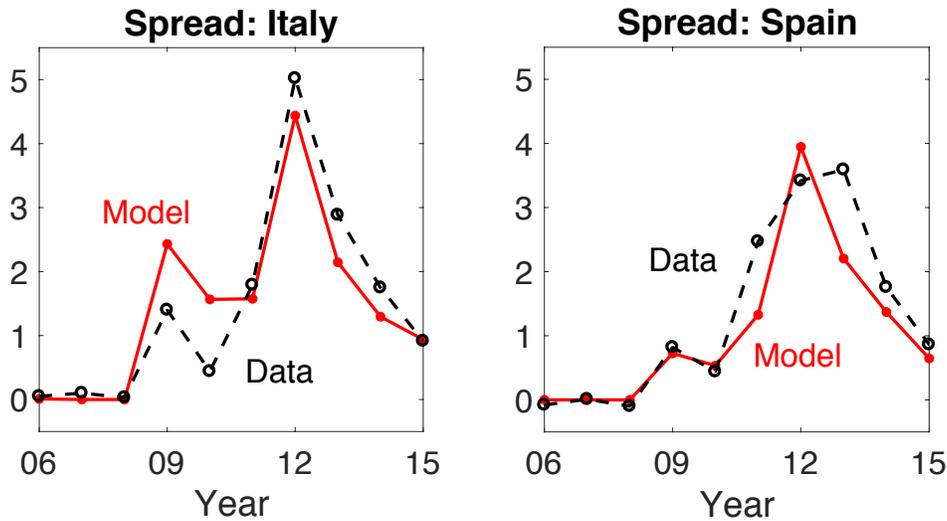


Figure 6: Event Spread

paths are also averages across the simulations. The data paths illustrate the large increases in spreads for both countries from about 0 to about 5% for Italy and 4% for Spain during 2012. Spreads decrease starting in 2013 to less than 1% in 2015.

The model paths for spreads replicate the data paths fairly well. During 2006 and 2007, spreads are close to zero for both countries in the model and in the data. In 2009 spreads rise in the model as in the data, although for the case of Italy, the magnitude of the increase in the model is larger than that observed in the data. Spreads in the model peak in 2012 as in the data. The model predicts a spread of about 4.5% for Italy and 4% for Spain, close to the magnitudes observed in the data. Finally, the model matches the data during 2014 and 2015 as spreads decreased to similar magnitudes as in the data.

As the figure shows, spreads across these two countries are correlated in the model and the data. The model predicts a correlation of spreads during this event of 0.9, close to the data correlation of 0.95. The comovement in spreads arises in the event because of two reasons. First, domestic output fell substantially for both countries during this period, as seen in Figure 5. This positive comovement in output naturally generates comovement in spreads because default probabilities rise for both countries when their outputs fall. Second, the contagion mechanisms in our model make spreads correlated as increases in default probability in one country induce a higher default probability in the other country.

We now turn to assess the contribution of contagion during this event by performing counterfactual experiments with our model. In these counterfactuals, we evaluate how spreads in Italy (or Spain) would have evolved during the event absent a debt crisis in Spain (or Italy). To eliminate the debt crisis in Spain (or Italy) in the counterfactual, we feed in a flat output

path for Spain (or Italy) that does not contain a recession.

More precisely, for the first counterfactual, we feed in the event the exact same sequence of shocks to Italy as in the benchmark analysis, but hold the Spanish output at its mean throughout the event. In the second counterfactual, we feed in the shocks for Spain as in the benchmark while holding constant Italy's output.

We then evaluate the resulting spreads for Italy and Spain and compare them with the benchmark model paths. The difference between the counterfactual spreads and the benchmark spreads provides a measurement for the contribution of the Spanish (Italian) debt crisis to the Italian (Spanish) spreads due to contagion.

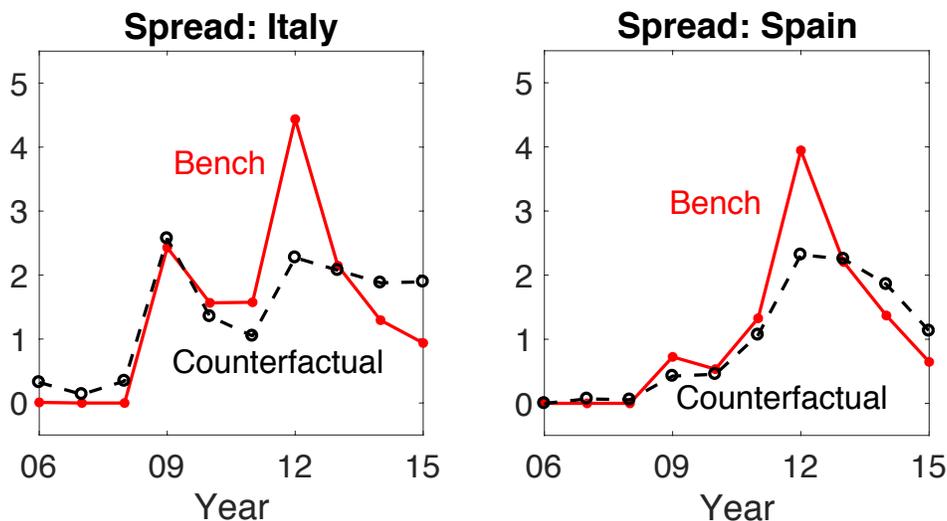


Figure 7: Event Spread: Counterfactual

In Figure 7 we compare the path of spreads in the benchmark with the path in the counterfactuals. Consider first the left panel, where we plot the counterfactual spread path for Italy and compare it with the benchmark path. The rise in spreads for 2009 in the counterfactual is very similar to the benchmark. In contrast, the counterfactual spread rises more modestly in 2012, reaching only 2%, about half of the increase in the benchmark. This result implies that the recession and debt crisis in Spain had a negative impact on Italy's spreads during 2012, accounting for about half of the 2012 spreads. Finally, during 2015, Spain's recovery helps Italy; absent Spain's recovery, spreads in Italy would have remained quite elevated.

The right panel in Figure 7 presents the counterfactual exercise for Spain. The increase in Spanish spreads in the counterfactual is also more muted, reaching about 2.5% in 2012, or about two-thirds of the level in the benchmark. This result implies that the recession and debt crisis in Italy account for about one-third of the elevated spreads in Spain. Italy's

smaller contribution to Spain can be explained by the fact that Italy experienced a smaller output decline than Spain during the event.

In sum, our model implies that bond spreads during the debt crises in Italy and Spain responded both to the deterioration in domestic conditions and to contagion through financial links. The debt crisis in each country contributed to the debt crisis in the other country, and this contribution accounted for a sizable fraction of the observed increase in spreads.

## 4.4 Comparative Statics

The comovement in spreads arises in our benchmark model because of two main channels: the strategic renegotiation channel and the pricing kernel channel. Recall that the strategic renegotiations among borrowing countries generate correlated recoveries that lead to correlated defaults. Also, lenders' concavity generates a varying pricing kernel, which is common for the borrowing countries and depends on their choices. In this section, we quantify the importance of these channels in generating our results and find that the interaction between strategic renegotiations and the pricing kernel accounts for much of the comovement in bond spreads.

To understand these two channels, we analyze two stripped-down versions of our model. We compute a *linear* model with risk neutral lenders where the pricing kernel is constant. The interactions between borrowing countries in this model arise only because of the strategic renegotiations and correlated recoveries. We also compute a *zero recovery* model where we shut down the strategic renegotiations and assume zero recovery after default. The results in the benchmark model relative to the stripped-down models highlight how the two channels interact.

Table 4 reports the sensitivity results for the two versions of our model as well as the benchmark results. The table reports comovements and means of the variables of interest in the limiting distribution for each model.

Consider first the results for the linear model. The linear model is identical to the benchmark, except that the risk aversion parameter for lenders is 0. This model generates a positive correlation between home and foreign spreads of 0.13, which is about one-fifth of the correlation in the benchmark. The differences in recoveries in multiple renegotiations relative to single renegotiations of -0.18 are similar to the benchmark. These results indicate that the strategic renegotiation force by itself leads to a moderate correlation in spreads of the two countries. In terms of means, the linear model generates larger mean spreads and default probabilities, yet very comparable mean recoveries. The higher default probabilities in the linear model arise because the bond price schedules for the two countries are more lenient

here, as more borrowing does not increase the pricing kernel absent default.

Now consider the results from the zero recovery model. This model is identical to the benchmark, except that the recovery function is replaced with zero. As Table 4 shows, in this model the correlation of bond spreads between the home and foreign country in the limiting distribution is small and equal to 0.02, while the mean spread and default probability are much larger than in the benchmark. The zero recovery model generates bond price schedules that are much more tight and steep than in the benchmark. Equilibrium debt levels in this model drop by a factor of 6, from about 6% of output in the benchmark to less than 1% when recovery is zero. This property implies that the zero recovery model not only eliminates the strategic renegotiation force but also largely eliminates the varying pricing kernel force, as with very small debt levels, defaults barely affect the pricing kernel. In fact, the standard deviation of the lenders' consumption drops from 1.3% in the benchmark to 0.6% in the model with zero recovery.

As the table shows, the cross-country bond spread correlation of 0.63 in the benchmark model arises mainly from the interaction of strategic renegotiation and the time varying pricing kernel, as each channel by itself gives at most a modest positive correlation. The reason is that the recoveries present with strategic renegotiation allow for larger debt levels in the benchmark, which in turn magnifies the pricing kernel channel of contagion.

Table 4: Decomposing Mechanism

<i>Limiting Distribution (%)</i>	Benchmark	Linear	Zero Recovery
Correlations home and foreign spreads (%)	63	13	2
Recovery Multiple-Single	-17	-18	0
Means			
Spread	1.0	2.9	4.5
Default probability	2.1	6.8	3.9
Recovery	60	63	0

## 5 Broader Empirical Results

The analysis above focused on the experiences of Italy and Spain during the recent European debt crisis and on the comovement in bond spreads of these countries. In this section, we provide empirical support for additional implications of our model using a broader dataset on defaults, renegotiations, and recoveries. As described above and illustrated in Table 3, our model contains the following additional empirical predictions:

1. Default probabilities are higher when other countries are defaulting and lower when other countries are renegotiating.
2. Renegotiation probabilities are lower when other countries are defaulting and higher when other countries are renegotiating.
3. Recovery rates are lower when other countries are renegotiating and higher when other countries are defaulting.

We show that these empirical implications are consistent with the historical experiences of countries. We assemble a panel dataset of 77 developing countries from 1970 to 2011, which includes all the countries that have experienced a default event as defined by Standard and Poor's (S&P) or are in the Cruces and Trebesch (2013) dataset in addition to all emerging market countries.

We measure whether the fraction of countries that are in default and the fraction of those that are renegotiating correlate with the probability that any one country  $i$  defaults or renegotiates at time  $t$ . Specifically, we run the following two linear probability regressions:

$$\text{default}_{it}[\text{renegotiation}_{it}] = \alpha_i + \beta_D \text{Frac Default}_{it} + \beta_R \text{Frac Renegotiate}_{it} + \beta_{dy} \text{Debt/GDP}_{it} + \varepsilon_{it}.$$

The variable  $\text{default}_{it}$  is a binary and equals 1 if the country is in default according to S&P and zero otherwise. The variable  $\text{renegotiation}_{it}$  equals 1 if a country that is in default renegotiates the debt and is no longer in default according to S&P and equals zero if it is in default without renegotiating the debt. The variables  $\text{Frac Default}_{it}$  and  $\text{Frac Renegotiate}_{it}$  are the fraction of countries, not including  $i$ , that are in default or are renegotiating in the dataset. To smooth discrete changes in these variables, we use five-year moving averages. Finally, we include as additional controls country fixed effects and the level of debt to output for each country and year. The variable  $\text{Debt/GDP}_{it}$  is equal to the external debt to GDP ratio and is taken from the World Development Indicators database.

The first additional implication of our theory predicts that in the default regression,  $\beta_D > 0$  and  $\beta_R < 0$ . Moreover, as in standard default models, we expect  $\beta_{dy} > 0$ . The second additional implication of the theory predicts that in the renegotiation regression,  $\beta_D < 0$  and  $\beta_R > 0$ . The country fixed effects absorb the average default and renegotiation frequency for each country.

A main channel in our model for the default/renegotiation comovement is the variation in recoveries. As already shown in the calibration of the model, in the data recoveries are lower on average in years with multiple renegotiations. Here we extend this analysis and examine

how recovery varies continuously with the two variables,  $\text{Frac Default}_{it}$  and  $\text{Frac Renegotiate}_{it}$ , as well as with  $\text{Debt/GDP}_{it}$ . We run a similar regression as follows:

$$\text{recovery}_{it} = \alpha + \gamma_D \text{Frac Default}_{it} + \gamma_R \text{Frac Renegotiate}_{it} + \gamma_{dy} \text{Debt/GDP}_{it} + \varepsilon_{it}.$$

The variable  $\text{recovery}_{it}$  equals the recovery rate estimates from the Cruces and Trebesch (2013) dataset.<sup>14</sup> Our theory predicts that  $\gamma_D > 0$  and  $\gamma_R < 0$ . Moreover, as in other models of renegotiation, our model predicts that  $\gamma_{dy} < 0$ .

Table 5: Cross-Country Regressions

	Default	Renegotiation	Recovery
Fraction in $\text{Default}_{it}$	1.36***	-0.88***	0.92***
Fraction $\text{Renegotiating}_{it}$	-2.13*	4.60**	-7.39***
$\text{Debt/GDP}_{it}$	0.11**	-0.03*	-0.21***
Country fixed effects	Yes	Yes	No
Adjusted $R^2$	0.28	0.06	0.34
Observations	2682	552	139

Table 5 reports the regression results. All standard errors are clustered at the country level. The coefficients on all the independent variables of interest have the sign predicted by the theory and are significant. The results in the default regression indicate that a 1% increase in the fraction of other countries in default increases the default probability of any one country by 1.4%, whereas an increase of 1% in the fraction of other countries renegotiating decreases the default probability by 2.2%. More indebted countries are also more likely to be in default. An increase of 10% in  $\text{Debt/GDP}_{it}$  increases the default probability by 1%. In the renegotiation equation, an increase of 1% in the fraction of countries in default decreases the renegotiation probability by 0.9%, whereas a 1% increase in the fraction of countries renegotiating increases the renegotiation probability by 4.6%. The debt to GDP ratio has no effect on the renegotiation probability. The results in the recovery equation say that a 1% increase in the fraction of countries in default increases recovery by a bit less than 1%, whereas an increase in the fraction of renegotiators increases recovery by 7.4% from an

<sup>14</sup>We use the recoveries in Cruces and Trebesch (2013) during the final renegotiations. The results are similar if we use a weighted average recovery based on the partial renegotiations.

average of 60%. The Appendix contains additional robustness results as well as descriptive statistics for all the variables. It shows that the main results are maintained when controlling for world GDP as well as for selection issues.

The historical patterns across countries documented in this section are consistent with our theory. Countries default together and renegotiate together because recoveries are more favorable during multiple country renegotiations.

## 6 Conclusion

We have developed a multi-country dynamic model of borrowing and default that generates sovereign risk contagion across countries. Countries are linked to one another because they borrow, default, and renegotiate with common lenders. A foreign default worsens the home country's bond prices and lowers the home country's future debt recovery rates in the event of a renegotiation. These two forces can trigger a home default because rolling over the debt is more difficult and renegotiating the defaulted debt is more attractive. In a quantitative application to the 2012 European debt crisis, we find that these forces are quantitatively important for generating the observed correlation of bond spreads.

Understanding the reasons for the simultaneity of costly sovereign debt crises across countries is important for the policy approaches to these crises. Our work points to strategic complementarities in borrowing, default, and renegotiation as powerful drivers for the simultaneity of crises. In this context, financial policies such as bailout guarantees and international bankruptcy courts can potentially prove useful improving the efficiency of sovereign debt markets.<sup>15</sup> Future work exploring in detail policies aimed at reducing sovereign risk contagion seems fruitful.

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<sup>15</sup>In the context of self-fulfilling debt crises, Roch and Uhlig (2016) show that bailout guarantees improve the efficacy of defaultable sovereign debt markets.

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# Appendix I. Auxiliary Models

## One Large Country Model

Let  $v_{i,fail}^L(\ell, s_i)$  be the value to the lender when trading only with country  $i$ :

$$v_{i,fail}^L(\ell, s_i) = \max_{\{d_L, \ell' \text{ if } h_i = h'_i = 0\}} \{u(c_L) + \delta \sum_{y'_i} \pi(y'_i, y_i) v_{i,fail}^L(\ell', s'_i)\}, \quad (21)$$

subject to its budget constraint,

$$c_L = y_L + [1 - D_i(s)] \left( (1 - h_i)(\ell - Q_i \ell') + h_i \frac{\Phi_i \ell}{b_i} \right),$$

the evolution of the endogenous states akin to equation (12), and a law of motion of aggregate states for the case that country  $i$  is dealing alone with lenders  $s'_i = H_{fail}(s_i)$ . The optimal solution of the lender is given by  $c_{L,fail}(\ell, s_i)$  and  $\ell'_{fail}(\ell, s_i)$ .

The problem for country  $i$  in the case when it trades alone with the lenders is similar to one described in Section 2.1 with three main differences. First, its aggregate states are only  $s_i = \{b_i, h_i, y_i\}$ . Second, the price function  $q_{i,fail}(s_i, b'_i, d_i)$  and recovery  $\phi_{i,fail}(s_i, b'_i, d_i)$  depend only on its own states and its own strategies. Third, the intraperiod Nash game between countries is absent. The decision rules for this problem are labeled  $B_{i,fail}(s_i)$  for borrowing and  $D_{i,fail}(s_i)$  for repayment. These decisions in turn determine the evolution of the aggregate state  $s'_i = H_{fail}(s_i)$ .

When  $h_i = 0$ , the price function  $q_{i,fail}(s_i, b'_i, d_i)$  solves

$$q_{i,fail} = \sum_{s'} m_{fail}(s'_i, s_i; q_{i,fail}, b'_i, d_i) [1 - D_{i,fail}(s'_i)(1 - \zeta_{i,fail}(s'_i))]. \quad (22)$$

Here, the decision rules of the country and the lender's kernel are those corresponding to the problem when country  $i$  trades alone with the lender.

When the country is in bad credit standing and chooses to renegotiate, the recovery function  $\phi_{i,fail}(s_i, d_i)$  solves

$$\frac{\theta u'(y_i - \phi_{i,fail})}{[v_i(s_i; \phi_{i,fail}) - v_{i,aut}(y_i)]} = \frac{(1 - \theta)u'(s_i, \phi_{i,fail}, d_i)}{[V^L(s_i, \phi_{i,fail}, d_i) - V_{aut}^L]}. \quad (23)$$

We now describe the equilibrium

**Definition 3.** A single-country recursive Markov equilibrium consists of (i) country  $i$ 's policy functions for repayment, borrowing, and consumption,  $\{B_{i,fail}(s_i), D_{i,fail}(s_i), C_{i,fail}(s_i)\}$ , and

values  $v_{i, fail}(s_i)$ ; (ii) lenders' policy functions for lending choices and dividends  $\{\ell'_{fail}(\ell, s_i), c_{L, fail}(\ell, s_i)\}$  and value function  $v_{i, fail}^L(\ell, s_i)$ ; (iii) the functions for bond prices and recoveries  $\{q_{i, fail}(s_i, b'_i, d_i), \phi_{i, fail}(s_i, d_i)\}$ ; (iv) the equilibrium prices of debt  $Q_{i, fail}(s_i)$  and recovery rates  $\Phi_{fail}(s_i)$ ; (v) the evolution of the aggregate state  $H_{fail}(s_i)$  such that given  $b_0 = \ell_0$ :

- Taking as given the bond price and recovery functions, country  $i$ 's policy functions for repayment, borrowing, and consumption,  $\{B_{i, fail}(s_i), D_{i, fail}(s_i), C_{i, fail}(s_i)\}$ , and values  $v_{i, fail}(s_i)$  solves country  $i$ 's problem when it trades alone with the lenders.
- Taking as given the bond prices  $Q_{fail}(s_i)$ , recoveries  $\Phi_{fail}(s_i)$ , and the evolution of the aggregate states  $H_{fail}(s_i)$ , the policy functions and value functions for the lenders  $\{\ell'_{fail}(\ell, s_i), c_{L, fail}(\ell, s_i)\}$ ,  $v_{i, fail}^L(\ell, s_i)$  satisfy lenders' optimization problem in (21).
- Taking as given countries' policy and value functions, bond price and recovery functions  $\{q_{i, fail}(s_i, b'_i, d_i), \phi_{i, fail}(s_i, d_i)\}$  satisfy (22) and (23).
- The prices of debt  $Q_{fail}(s_i)$  clear the bond market,  $\ell'_{i, fail}(b_i, s_i) = B_{i, fail}(s_i)$ .
- The recoveries  $\Phi_{i, fail}(s_i)$  exhaust all the recovered funds:  $\phi_{i, fail}(s_i, D_{i, fail}(s_i)) = \Phi_{fail}(s_i)$ .
- The law of motion for the evolution aggregate states  $H_{fail}(s_i)$  is consistent with country  $i$ 's decision rules and shocks.

## Small Country Model

The model for the small country is a one-country competitive version of the benchmark model. This model is studied in Yue (2010), but here the risk-free rate is time varying and depends on the evolution of the aggregate states. The recursive problem for the small country takes as given the law of motion of aggregate states (9). Given the individual state  $(b_s, y_s, h_s)$  and aggregate state  $s$ , the small country's problem is given by

$$v_s(b_s, y_s, h_s = 0, s) = \max_{d_s \in \{0, 1\}} \{(1 - d_s)v_s^0(b_s, y_s, h_s = 0, s) + d_s v_s^1(b_s, y_s, h_s = 0, s)\}.$$

If it repays, the small country chooses optimal consumption and savings:

$$v_s^0(b_s, y_s, h_s = 0, s) = \max_{c_s, b'_s} \{u(y_s - b_s + q_s(b'_s, y_s, s)b'_s) + \beta E v_s(b'_s, y'_s, h'_s = 0, s')\}.$$

If it defaults, the small country's value is given by

$$v_s^1(b_s, y_s, h_s = 0, s) = \{u(y_s^d) + \beta E v_s(b_s, y'_s, h'_s = 1, s')\}. \quad (24)$$

If the country is in bad credit standing, it chooses whether to renegotiate according to

$$v_s(b_s, y_s, h_s = 1, s) = \max_{d_s \in \{0,1\}} \{(1 - d_s)v_s^0(b_s, y_s, h_s = 1, s) + d_s v_s^1(b_s, y_s, h_s = 1, s)\}.$$

Its renegotiation value depends on the recovery  $\phi_s(b_s, y, s)$  and is given by

$$v_s^0(b_s, y_s, h_s = 1, s) = u(y_s - \phi_s(b_s, y, s)) + \beta E v_s(0, y'_s, h'_s = 0, s').$$

Without renegotiation, its value is the same as the default value given by equation (24).

In equilibrium, bond price and recovery functions for the small country satisfy the following equations:

$$\begin{aligned} q_s &= E [1 - d'_s(b'_s, y'_s, h'_s, s')(1 - \zeta_s(b'_s, y'_s, h'_s, s'))] E m(s', s), \\ \zeta_s(b_s, y_s, h_s, s) &= E[(1 - d'_s(b'_s, y'_s, h'_s, s')) \frac{\phi_s(b_s, y'_s, h'_s, s')}{b_s} + d'_s(b'_s, y'_s, h'_s, s') \zeta_s(b_s, y'_s, h'_s, s')] E [m(s', s)], \\ 1 - \theta &= \frac{\theta u'(y_s - \phi_s)}{[v_s^0(b_s, y_s, h_s = 1, s; \phi_s) - v_{aut}(y_s)]}, \end{aligned}$$

where  $m(s', s)$  is the equilibrium pricing kernel from the two-big-country problem.

## Appendix II. Proofs

*Proof for Proposition 1.* Let us call  $\phi_2^i$  and  $\phi_2^{-i}$  the recovery values for country  $i$  and  $-i$  respectively when the two countries renegotiate jointly with lenders, and  $\phi_1^i$  be the recovery value when country  $i$  renegotiates alone with lenders. Nash bargaining implies that  $\phi_2^i$  satisfies

$$\frac{\theta u'(y_2^i - \phi_2^i)}{u(y_2^i - \phi_2^i) - u(y^d)} = \frac{(1 - \theta)u'(y_L + \phi_2^i + \phi_2^{-i})}{u(y_L + \phi_2^i + \phi_2^{-i}) - u(y_L)} \leq \frac{(1 - \theta)u'(y_L + \phi_2^i + \phi_2^{-i})}{u(y_L + \phi_2^i + \phi_2^{-i}) - u(y_L + \phi_2^{-i})}.$$

The inequality holds because  $u$  is an increasing function and  $\phi_2^{-i} \geq 0$ . Suppose the following condition holds:

$$\frac{(1 - \theta)u'(y_L + \phi_2^i + \phi_2^{-i})}{u(y_L + \phi_2^i + \phi_2^{-i}) - u(y_L + \phi_2^{-i})} \leq \frac{(1 - \theta)u'(y_L + \phi_2^i + \bar{b})}{u(y_L + \phi_2^i + \bar{b}) - u(y_L + \bar{b})}. \quad (25)$$

Then, the recovery under two borrowing countries  $\phi_2^i$  satisfies

$$\frac{\theta u'(y_2^i - \phi_2^i)}{u(y_2^i - \phi_2^i) - u(y^d)} \leq \frac{(1 - \theta)u'(y_L + \phi_2^i + \bar{b})}{u(y_L + \phi_2^i + \bar{b}) - u(y_L + \bar{b})},$$

where recovery alone  $\phi_1^i$  satisfies

$$\frac{\theta u'(y_2^i - \phi_1^i)}{u(y_2^i - \phi_1^i) - u(y^d)} = \frac{(1 - \theta)u'(y_L + \phi_1^i + \bar{b})}{u(y_L + \phi_1^i + \bar{b}) - u(y_L + \bar{b})}.$$

It is easy to show by contradiction that  $\phi_2^i \leq \phi_1^i$  because  $u$  and  $g$  are increasing and concave. Note that concavity is not necessary to guarantee  $\phi_2^i \leq \phi_1^i$ .

We still need to show that inequality (25) holds. Given  $\bar{b} \geq \phi_2^{-i}$ , we need to show that the function  $f(x) = \frac{u'(y_H+x)}{u(y_H+x)-u(y_L+x)}$  with  $y_H = y_L + \phi_2^i \geq y_L$  weakly increases with  $x$ . Under the assumption that  $u(c) = c^{1-\sigma}/(1-\sigma)$ , we can write  $f(x)$  as

$$f(x) = \frac{-\sigma(1-\sigma)}{\Delta^2} \left\{ 1 - \left( \frac{y_H+x}{y_L+x} \right)^{\sigma-1} + \frac{1-\sigma}{\sigma} \left( 1 - \left( \frac{y_H+x}{y_L+x} \right)^\sigma \right) \right\},$$

where  $\Delta = (y_H+x)^\sigma [(y_H+x)^{1-\sigma} - (y_L+x)^{1-\sigma}]$ . It is easy to show that  $f'(x) \geq 0$ . Q.E.D.

*Proof for Proposition 2.* Conditional on repaying, country  $i$ 's net capital flow to lenders increases with its initial debt holding  $b_i$ . To see this, let  $\omega_L(b_{-i}, d_{-i})$  and  $\omega'_L(d_{-i})$  be the lenders' wealth from trading with the other country  $-i$  in period 1 and period 2, respectively. In particular,

$$\omega_L(b_{-i}, d_{-i}) \equiv y_L + (1 - d_{-i})TB(b_{-i}).$$

We can define the net capital flow from country  $i$  as  $TB_i = b_i - q_i \bar{b}$ , where  $q_i$  solves

$$q_i = \frac{\delta u'[\omega'_L(d_{-i}) + \bar{b}]}{u'[\omega_L(b_{-i}, d_{-i}) + b_i - q_i \bar{b}]},$$

It is easy to show that

$$\partial TB_i / \partial b_i = \frac{u'[\omega_L(d_{-i}) + b_i - q_i \bar{b}]}{u'(\omega_L(b_{-i}, d_{-i}) + b_i - q_i \bar{b}) - q_i u''(\omega_L(b_{-i}, d_{-i}) + b_i - q_i \bar{b}) \bar{b}} \geq 0.$$

Higher  $b_{-i}$  therefore leads to higher net capital flow  $TB_{-i}$  and so higher lenders' wealth from country  $-i$  since  $\omega_L(b_{-i}, d_{-i})$ . The bond price of country  $i$  thus increases with  $b_{-i}$  conditional on country  $-i$  repaying.

## Appendix III. Computational Algorithm

We first discretize the endowment space  $y = (y_1, y_2)$  into 81 pairs using the Tauchen-Hussey (1991) method and the debt space  $b = (b_1, b_2)$  into 225 pairs. We then compute the model as the limit of a finite horizon model with  $T$  periods. We start with a large enough  $T$  and solve the problem backwardly until the value functions and decision rules converge. In each period, we compute two models: a single-country model and a two-country model. We need to compute the first model, since its equilibrium values are used in solving for the Nash bargaining allocations of the second model.

We now describe the algorithm for a generic period  $t \leq T$ .

### 1. Single-country model.

In this computation, we take as given the following functions from period  $t+1$ : country  $i$ 's value function and default decision  $\{v_{i,t+1}^1(s), D_{i,t+1}^1(s)\}$ , discounted value of future recovery  $\zeta_{i,t+1}^1(s)$ , and lenders' consumption and value function when dealing with country  $i$  alone  $\{c_{i,t+1}^L(s), V_{i,t+1}^L(s)\}$  for  $i = 1, 2$  and  $s = \{(b_i, h_i, y_i)\}_{i=1,2}$ .<sup>16</sup> We then update these function for period  $t$  using the optimal decisions from this period.

Let us first construct expected future value function  $W$  and expected repayment function  $\psi$  on the grids of  $(b', s)$ . They both depend on the current state  $s$  and are a function of debt choice  $b'$ :

$$W_{i,t+1}^1(b', s) = \sum_{y'} \pi(y'|y) v_{i,t+1}^1(s')$$

$$\psi_{i,t+1}^1(b', s) = \sum_{y'} \pi(y'|y) u'[c_{i,t+1}^L(s')] \{(1 - D_{i,t+1}^1(s')) + D_{i,t+1}^1(s') \zeta_{i,t+1}^1(s')\}.$$

With these two functions, we can solve the single-country model at period  $t$ . In particular, we solve it in two cases: when the country has good credit standing and when the country has bad credit standing.

For the country in good credit standing, we solve its problem in two steps. In the first step, we find the optimal borrowing decision conditional on repaying. In the second step, we find the optimal default decision taking as given the optimal borrowing decision and repaying value from the first step.

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<sup>16</sup>For convenience of notation, we write the state space of the single-country model the same as that of the two-country model. Of course, in the single-country model, country  $-i$ 's state  $(b_{-i}, h_{-i}, y_{-i})$  does not affect country  $i$ 's problem.

1.1 Borrowing decision. Taking as given expected future value function  $W$  and expected repayment function  $\psi$ , we solve the following optimization problem:

$$\begin{aligned} v_{i,t}^{1,r}(s) &= \max_{b'} u(y_i - b_i + qb') + \beta W_{i,t+1}^1(b', s) \\ \text{s.t.} \quad & qu'[y_L + b_i - qb'] = \delta \psi_{i,t+1}^1(b', s). \end{aligned} \quad (26)$$

Note that we do not use the grid-search method to solve for the optimal  $b'$ . Instead, we solve  $b'$  continuously by interpolating the functions of  $W_{i,t+1}^1(b', s)$  and  $\psi_{i,t+1}^1(b', s)$ . Let  $B_{i,t}^{1,r}(s)$  and  $Q_{i,t}^1(s)$  be the optimal borrowing decision and the corresponding equilibrium bond price satisfying equation (26) when  $b' = B_{i,t}^{1,r}(s)$ , respectively.

1.2 Default decision. It is a zero-one choice depending on which value is larger, repaying or defaulting:

$$v_{i,t}^1(s) = \max_d (1-d)v_{i,t}^{1,r}(s) + d v_{i,t}^{1,d}(s),$$

with the default value given by  $v_{i,t}^{1,d}(s) = u(y_i^d) + \beta W_{i,t+1}^1(s)$ . Let  $D_{i,t}^1(s)$  be the optimal default decision.

For the country in bad credit standing, we solve it in two steps as well. In the first step, we solve the optimal recovery for each grid  $s$ . We then figure out the optimal renegotiation decision taking as given the optimal recoveries.

1.3 Recovery function. For each grid  $s$  with  $h_i = 1$ , the optimal recovery  $\phi$  satisfies the following equation:

$$\frac{(1-\theta)u'(y_L + \phi)}{u(y_L + \phi) + \delta EV_{i,t+1}^L(s') - V_{L,t}^{aut}} = \frac{\theta u'(y_i - \phi)}{u(y_i - \phi) + \beta W_{i,t+1}^1(0, s) - v_{i,t}^{aut}(y)}$$

where the autarky value for lenders is given by  $V_{L,t}^{aut} = u(y_L) + \delta V_{L,t+1}^{aut}$  and the autarky value for the country is given by  $v_{i,t}^{aut}(y) = u(y_i^d) + \beta \sum_{y'} \pi(y'|y) v_{i,t+1}^{aut}(y')$ . The optimal recovery is denoted as  $\phi_{i,t}^1(s)$ .

1.4 Renegotiation decision. Taking as given the recovery schedule  $\phi_{i,t}^1(s)$ , the country makes a zero-one choice over renegotiation:

$$v_{i,t}^1(s) = \max_d (1-d)[u(y_i - \phi_{i,t}^1(s)) + \beta W_{i,t+1}^1(0, s)] + d[u(y_i^d) + \beta W_{i,t+1}^1(b, s)].$$

Let  $D_{i,t}^1(s)$  be the optimal non-renegotiating decision.

We can now evaluate lenders' consumption at period  $t$  according to

$$c_{i,t}^L(s) = y_L + (1 - D_{i,t}^1(s)) [(1 - h_i) (b_i - Q_{i,t}^1(s)B_{i,t}^{1,r}(s)) + h_i\phi_{i,t}^1(s)].$$

The value of the lender is given by

$$V_{i,t}^L(s) = u(c_{i,t}^L(s)) + \delta \sum_{y'} \pi(y'|y) V_{i,t+1}^L(s'),$$

where the future debt in  $s'$  for country  $i$  is  $B_{i,t}^1(s)$ . In particular,  $B_{i,t}^1(s) = B_{i,t}^{1,r}(s)$  if  $h_i = 0$  and  $D_{i,t}^1(s) = 0$ ,  $B_{i,t}^1(s) = 0$  if  $h_i = 1$  and  $D_{i,t}^1(s) = 0$ , and  $B_{i,t}^1(s) = b_i$  if  $D_{i,t}^1(s) = 1$ .

The discounted value of future recovery at period  $t$  is given by

$$\zeta_{i,t}^1(s) = \delta \sum_{y'} \pi(y'|y) u'(c_{i,t}^L(s')) [(1 - D_{i,t}^1(s'))\phi_{i,t}^1(s') + D_{i,t}^1(s')\zeta_{i,t+1}^1(s')].$$

## 2. Two-country model.

We take as given the following functions from period  $t+1$ : country  $i$ 's value function and default decision  $\{v_{i,t+1}(s), D_{i,t+1}(s)\}$ , discounted future value of recovery  $\zeta_{i,t+1}(s)$ , and lenders' consumption and value function when dealing with country  $i$  alone  $\{c_{L,t+1}(s), V_{L,t+1}(s)\}$  for  $i = 1, 2$  and  $s = \{(b_i, h_i, y_i)\}_{i=1,2}$ . We then update these functions for period  $t$  using the optimal decisions from this period.

We first need to construct the expected value function  $W$  and the expected repayment function  $\psi$  for any pair of  $b' = (b'_1, b'_2)$  on the grid:

$$W_{i,t+1}(b', s) = \sum_{y'} \pi(y'|y) v_{i,t+1}(s')$$

$$\psi_{i,t+1}(b', s) = \sum_{y'} \pi(y'|y) u'[c_{L,t+1}(s')] \{(1 - D_{i,t+1}(s')) + D_{i,t+1}(s')\zeta_{i,t+1}(s')\}.$$

We solve this model in two steps. In the first step, taking as given default/renewal choices of the two countries, we solve the optimal borrowing decisions and update the value functions for repaying, defaulting, renegotiating, and non-renegotiating. In the second step, we find the optimal default/renewal decision taking as given the optimal borrowing decisions in the first step.

2.1 Borrowing decisions and value functions. We solve three cases in this step.

- Case 1. Both countries are in good credit standing. We solve three sub cases.
  - Case 1.1. Both choose not to default.

In this case, taking as given  $\{W_{i,t+1}, \psi_{i,t+1}\}_{i=1,2}$ , we look for the fixed point  $\{B_{i,t}(s, d), B_{-i,t}(s, d)\}$  that satisfies for each  $i = 1, 2$

$$B_{i,t}(s, d) = \operatorname{argmax}_{\{b'_i, q_i, q_{-i}\}} w_{i,t}(s, (b'_i, B_{-i,t}(s, d)), d),$$

subject to the following conditions:

$$\begin{aligned} w_{i,t}(s, b', d) &= u(y_i - b_i + q_i b'_i) + \beta W_{i,t+1}(b', s), \\ q_i u'[y_L + (b_i - q_i b'_i) + (1 - d_{-i})(b_{-i} - q_{-i} b'_{-i})] &= \delta \psi_{i,t+1}(b', s), \\ q_{-i} u &= u'[y_L + (b_i - q_i b'_i) + (1 - d_{-i})(b_{-i} - q_{-i} b'_{-i})] = \delta \psi_{-i,t+1}(b', s). \end{aligned}$$

Let the equilibrium bond prices be  $Q_{i,t}(s, d)$  for  $i = 1, 2$ .

- Case 1.2. Country  $i$  repays but country  $-i$  defaults.

We only need to solve country  $i$ 's optimal debt:

$$\begin{aligned} B_{i,t}(s, d) &= \operatorname{argmax}_{\{b'_i, q_i\}} w_{i,t}(s, b'_i, d) \\ \text{s.t. } w_{i,t}(s, b', d) &= u(y_i - b_i + q_i b'_i) + \beta W_{i,t+1}(b', s), \\ q_i u'[y_L + (b_i - q_i b'_i)] &= \delta \psi_{i,t+1}(b', s). \end{aligned}$$

The value of country  $-i$  is given by

$$w_{-i,t}(s, B_{i,t}(s, d), d) = u(y_{-i}^d) + \beta W_{-i,t+1}((B_{i,t}(s, d), b_{-i}), s).$$

Let the equilibrium bond prices be  $Q_{i,t}(s, d)$  for  $i = 1, 2$ .

- Case 1.3. Both choose to default.

The value functions of default are given by, for each  $i$ ,

$$w_{i,t}(s, b, d) = u(y_i^d) + \beta W_{i,t+1}(b, s). \quad (27)$$

- Case 2. Country  $i$  is in good credit standing and country  $-i$  is in bad credit standing. We solve four sub cases here.

- Case 2.1. Both choose to repay.

In this case,  $d_{i,t} = d_{-i,t} = 0$ . We only need to solve country  $i$ 's optimal

debt with  $b'_{-i} = 0$ :

$$\begin{aligned}
B_{i,t}(s, d) &= \operatorname{argmax}_{\{b'_i, q_i, \phi_{-i}\}} w_{i,t}(s, b'_i, d) \\
s.t. \quad w_{i,t}(s, b', d) &= u(y_i - b_i + q_i b'_i) + \beta W_{i,t+1}((b'_i, 0), s), \\
q_i u'[y_L + (b_i - q_i b'_i) + \phi_{-i}] &= \delta \psi_{i,t+1}((b'_i, 0), s) \\
\frac{\theta u'(y_{-i} - \phi_{-i})}{v_{-i}(s; \phi_{-i}) - v_{-i, aut}(y_{-i})} &= \frac{(1 - \theta) u'(s, q_i, \phi_{-i}, b', d)}{V_t^L(s, q_i, \phi_{-i}, b', d) - V_{fail}^L(s_i)}.
\end{aligned}$$

Country  $-i$ 's value is given by

$$w_{-i,t}(s, (B_{i,t}(s, d), 0), d) = u(y_{-i} - \phi_{-i}) + \beta W_{-i,t+1}((B_{i,t}(s, d), 0), s).$$

Let the optimal recovery be  $\phi_{-i,t}(s, d)$  and the equilibrium bond price be  $Q_{i,t}(s, d)$ .

- Case 2.2. Country  $i$  repays but country  $-i$  chooses not to renegotiate. Country  $i$ 's optimal debt and value solve the following problem. We only need to solve country  $i$ 's optimal debt with  $b'_{-i} = b_{-i}$ :

$$\begin{aligned}
B_{i,t}(s, d) &= \operatorname{argmax}_{\{b'_i, q_i\}} w_{i,t}(s, b'_i, d) \\
s.t. \quad w_{i,t}(s, b', d) &= u(y_i - b_i + q_i b'_i) + \beta W_{i,t+1}((b'_i, b_{-i}), s), \\
q_i u'[y_L + (b_i - q_i b'_i)] &= \delta \psi_{i,t+1}((b'_i, b_{-i}), s)
\end{aligned}$$

Country  $-i$ 's value is given by

$$w_{-i,t}(s, (B_{i,t}(s, d), b_{-i}), d) = u(y_i^d) + \beta W_{i,t+1}((B_{i,t}(s, d), b_{-i}), s).$$

Let the equilibrium bond prices be  $Q_{i,t}(s, d)$ .

- Case 2.3. Country  $i$  defaults but country  $-i$  renegotiates.

The recovery function  $\phi_{-i}$  solves the following equation:

$$\frac{\theta u'(y_{-i} - \phi_{-i})}{v_{-i}(s; \phi_{-i}) - v_{-i, aut}(y_{-i})} = \frac{(1 - \theta) u'(s, \phi_{-i}, b', d)}{V_t^L(s, \phi_{-i}, b', d) - V_{fail}^L(s_i)}.$$

Let the optimal recovery be  $\phi_{-i,t}(s, d)$ . With  $b' = (b'_i, b'_{-i})$ ,  $b'_i = b_i$  and

$b'_{-i} = 0$ , the value functions of the two countries are given by

$$\begin{aligned} w_{i,t}(s, b', d) &= u(y_i^d) + \beta W_{i,t+1}(b', s) \\ w_{-i,t}(s, b', d) &= u(y_{-i} - \phi_{-i,t}(s, d)) + \beta W_{-i,t+1}(b', s). \end{aligned}$$

– Case 2.4. Both choose not to repay.

The values of the two countries are updated according to equation (27).

- Case 3. Both countries are in bad credit standing.

The two recovery functions solve the Nash bargaining problem jointly. Otherwise, the two recovery functions are independent of each other.

## 2.2 Default/renewal decisions.

Taking as given the optimal borrowing decisions and value functions from Step 2.1, we find the equilibrium default/renewal decisions  $\{D_{i,t}(s), D_{-i,t}(s)\}$  that solve jointly

$$D_{i,t}(s) \in \operatorname{argmax}_{\{d_{i,t}\}} w_{i,t}(s; d_{i,t}, D_{-i,t}(s), B(d_{i,t}, D_{-i,t}(s)))$$

$$D_{-i,t}(s) \in \operatorname{argmax}_{\{d_{-i,t}\}} w_{-i,t}(s; D_{i,t}(s), d_{-i,t}, B(D_{i,t}(s), d_{-i,t})).$$

If there are multiple pairs of  $(D_{i,t}, D_{-i,t})$  as equilibrium for a state  $s$ , we take the pair that maximizes  $w_{i,t}(s, D_{i,t}(s), B_{i,t}(s, D_{i,t}(s))) + w_{-i,t}(s, D_{-i,t}(s), B_{-i,t}(s, D_{-i,t}(s)))$ . We use these equilibrium default/renewal decisions to update the functions for period  $t$ .

## 2.3 We finally update the period $t$ value for each country $i$ :

$$v_{i,t}(s) = w_{i,t}(s, D_{i,t}(s), B_{i,t}(s, D_{i,t}(s))),$$

lenders' consumption

$$\begin{aligned} c_{L,t}(s) &= y_L + \sum_{i=1}^2 [(1 - h_{i,t})(1 - D_{i,t}(s)) [b_i - Q_{i,t}(s, D_{i,t}(s)) B_{i,t}(s, D_{i,t}(s))] \\ &\quad + \sum_{i=1}^2 [h_{i,t}(1 - D_{i,t}(s)) \phi_{i,t}(s, D_{i,t}(s))], \end{aligned}$$

and the expected discounted recovery  $\zeta$

$$\zeta_{i,t}(s) = \delta \sum_{y'} \pi(y'|y) u'(c_{L,t}(s')) [(1 - D_{i,t}(s')) \phi_{i,t}(s') + D_{i,t}(s') \zeta_{i,t+1}(s')].$$

The value of the lender is given by

$$V_{L,t}(s) = u(c_{L,t}(s)) + \delta \sum_{y'} \pi(y'|y) V_{L,t+1}(s'),$$

where the future debt in  $s'$  for country  $i$  is  $B_{i,t}^*(s)$ . In particular,  $B_{i,t}^*(s) = B_{i,t}(s, D_{i,t}(s))$  if  $h_i = 0$  and  $D_{i,t}(s) = 0$ ,  $B_{i,t}^*(s) = 0$  if  $h_i = 1$  and  $D_{i,t}(s) = 0$ , and  $B_{i,t}^*(s) = b_i$  if  $D_{i,t}(s) = 1$ .

## Appendix IV. Empirical Robustness

This appendix provides descriptive statistics and robustness of the empirical results in Section 5. Figure 8 plots the five-year moving average of the fraction of countries in default and

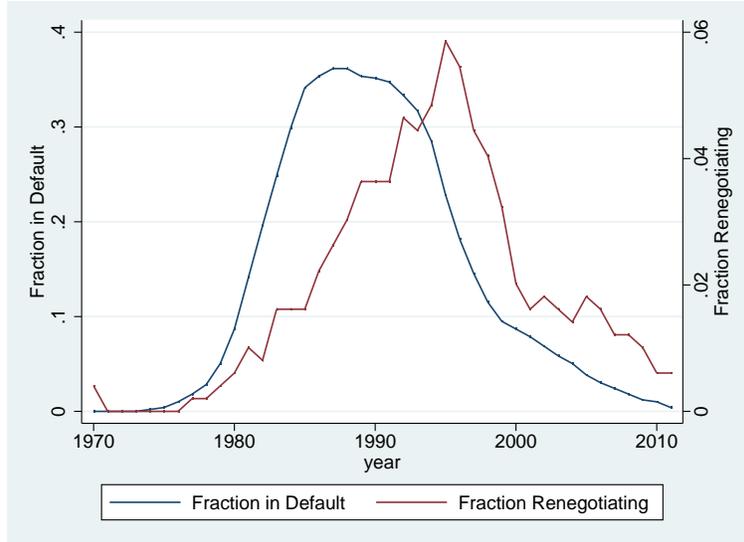


Figure 8: Historical Defaults and Renegotiations

the fraction of countries renegotiating over time. The figure illustrates that default rose in the early 1980s and remained elevated until the mid-1990s. Such an inverted hump shape mainly reflects the debt crises of the 1980s across Latin America. The fraction of countries

renegotiating the debt rose to almost 0.06 in the mid-1990s.<sup>17</sup>

We provide two sets of robustness analysis for the regression results. We first address the concern that a common world shock might be driving the fraction of countries in default and the fraction of countries renegotiating. In Table 6 we add linearly detrended world GDP to all the regressions. World GDP is significant in the renegotiation and recovery regressions. World booms are associated with fewer renegotiations and higher recovery rates. The variables  $\text{Frac Default}_{it}$  and  $\text{Frac Renegotiate}_{it}$  continue to be significant and with the expected sign in all specifications. All standard errors continue to be clustered at the country level.

Table 6: Cross-Country Regressions with World GDP

	Default	Renegotiation	Recovery
Fraction in Default $_{it}$	1.36***	-0.44**	0.44**
Fraction Renegotiating $_{it}$	-2.18*	3.14**	-5.28***
Debt/GDP $_{it}$	0.11**	-0.04*	-0.21***
World GDP $_t$	0.03	-2.42***	2.52***
Country fixed effects	Yes	Yes	No
Adjusted $R^2$	0.28	0.06	0.38
Observations	2682	552	139

We also estimate the renegotiation and recovery regressions, taking into account the inherent selection of these observations. Being in a state where renegotiation and recovery are nonmissing observations requires the country to be in a default state. The default state equals 1 in years where each country is in default or renegotiates. Table 7 presents the maximum likelihood estimation results from this specification with clustered errors. We estimate the selection equation with a probit and use lags of the independent variables as regressors. The coefficients on  $\text{Frac Default}_{it}$ ,  $\text{Frac Renegotiate}_{it}$ , and  $\text{Debt/GDP}_{it}$  in the renegotiation and recovery equations continue to be significant and with the expected sign. Economically, the coefficients are somewhat smaller than in the benchmark specification.

<sup>17</sup>Recall that in the regressions, the main independent variables  $\text{Frac Default}_{it}$  and  $\text{Frac Renegotiate}_{it}$  are of the fraction of countries, not including  $i$ , that are in default or are renegotiating. In this figure, however, we simply illustrate overall fractions.

Table 7: Cross-Country Regressions with Heckman Selection Estimates

	Renegotiation	Recovery
Fraction in Default $_{it}$	-0.55**	1.44***
Fraction Renegotiating $_{it}$	2.45**	-6.83***
Debt/GDP $_{it}$	-0.03**	-0.19***
Selection Eq.	State default	State default
Fraction in Default $_{it-1}$	5.13***	5.12***
Fraction Renegotiating $_{it-1}$	-6.43	-6.23
Debt/GDP $_{it-1}$	0.39*	0.40*
Observations	2682	2279