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[1]This paper builds on ideas first proposed in Luttmer [2012, 2013]. More results are available in Working Paper 748, which has the same title as this staff report.

Abstract

Most firms begin very small, and large firms are the result of typically decades of persistent growth. This growth can be understood as the result of some form of organization capital accumulation. In the US, the distribution of firm size k has a right tail only slightly thinner than $1/k$. This is shown to imply that incumbent firms account for most aggregate organization capital accumulation. And it implies potentially extremely slow aggregate convergence rates. A benchmark model is proposed in which managers can use incumbent organization capital to create new organization capital. Workers are a specific factor for producing consumption, and they require managerial supervision. Through the lens of the model, the aftermath of the Great Recession of 2008 is unsurprising if the events of late 2008 and early 2009 are interpreted as a destruction of organization capital, or as a belief shock that made consumers want to reduce consumption and accumulate more wealth instead.

Contents

1 Introduction

Between December 2007 and December 2009, the US civilian employment-population ratio fell sharply from 62.7% to 58.3%. As of December 2018, it has recovered to only 60.6%. If the gap between trough and peak is shrinking at a constant rate, this implies a half-life of about $9 \times \ln(2)/\ln(4.4/2.1) = 8.4$ years. Why is it taking so long for employment to recover? This paper argues that such a slow recovery is a robust implication of a model of the aggregate economy with heterogeneous firms that grow by accumulating some sort of organization capital.

The distribution of employment across US firms is very skewed. Although there are as many as 6 million employer firms, about half of aggregate employment is accounted for by the roughly 18,000 firms with more than 500 employees. And the 1,000 or so firms with more than 10,000 employees account for nearly a quarter of aggregate employment. To a first approximation, the distribution of employment size k of US firms is Pareto with a right tail that behaves like $k^{-\zeta}$, with $\zeta \approx 1.1$, just inside the $\zeta > 1$ region where the mean of a Pareto distribution is finite (the $\zeta \downarrow 1$ limit is known as Zipf's law). Most new firms start out with only a few employees, and it took the largest firms in the US economy decades of rapid and persistent growth to reach their current size.[1]

These facts are consistent with a very simple model of firm size: there is a constant flow f of new firms that start with size $k = 1$, grow at some rate g, and exit randomly at a mean rate $\varepsilon > g$. An easy calculation, reported in Section 3, shows that this yields $\zeta = \varepsilon/g > 1$. Furthermore, this process of firm entry, growth, and exit implies that the aggregate size K_t evolves according to $\mathrm{d}K_t = -\varepsilon K_t \mathrm{d}t + (gK_t + f)\mathrm{d}t$. That is, the aggregate mean reversion rate is $\varepsilon - g = (1 - 1/\zeta)\varepsilon$. Holding fixed ε, this implies slow aggregate convergence precisely when $\zeta > 1$ is close to 1, when the size distribution of firms is thick tailed. In particular, Zipf's law implies no aggregate convergence at all. In the US, firm entry and exit rates are around 10% per annum, and so the implied aggregate mean reversion rate is $\varepsilon - g \approx (1 - 1/1.1)0.1 \approx 0.01$. This implies a half-life of almost 70 years. Even longer half-lives emerge when not all exit is random (Luttmer [2011]).

Obviously, the US economy recovers more quickly from recessions than suggested by this simple calculation. But this account of firm growth and aggregate convergence conveys an important intuition: entry, exit, and non-stationarity at the level of individual firms very naturally lead to cross-sectional distributions that are stationary and thick-tailed, and to slow aggregate convergence.[2] This paper builds on this observation by

[1]These facts are well known. See Luttmer [2010] for sources and a survey of some explicit models.

[2]Not unlike in Penrose [1959], surviving firms in this economy keep growing. This is in contrast to, for example, Atkeson and Kehoe [2005] and much of the large literature that follows the tradition of Hopen-

interpreting firm growth as organization capital accumulation and allowing the flow of entrants f and the rate g at which firms choose to accumulate organization capital to respond to the state of the economy. The convergence rate of the economy is then characterized in terms of factor supply elasticities, factor share parameters, and curvature parameters.

In the Kydland and Prescott [1982] tradition and beyond, recessions are most often attributed to negative aggregate productivity shocks. Here instead, a recession is triggered by a destruction of organization capital, or by news that the stock of organization capital is below its long-run steady state. In a Cass-Koopmans economy with an elastic supply of labor and standard preferences, a low capital stock tends to result in high investment that is made possible by a combination of both low consumption and high employment. High employment is the opposite of what characterizes recessions. This paper avoids this familiar difficulty by making a distinction between managers and workers, and recognizing that they are not used with the same intensity in producing consumption and new organization capital. Workers are assumed to supply labor that can only be used to produce consumption goods, under the supervision of managers. This makes workers a specific factor for the consumption sector of the economy. Managers, on the other hand, can divide their time between two different tasks: overseeing workers and producing new organization capital. The need for managerial supervision means that fewer workers can be employed when managers find it more profitable to produce new organization capital.[3]

In the model, managers can produce new organization capital by replicating existing organization capital, which leads to incumbent firm growth, or by using a fixed factor (scientists, certain locations, say) to create new organization capital from scratch, interpreted as entry. The equilibrium f and g will be above their steady state levels when the organization capital stock, worker employment, and aggregate consumption are below their steady states. This endogenous response of organization capital accumulation to the level of the capital stock ensures that, even in the $\zeta \downarrow 1$ limit, the economy recovers at a strictly positive rate. The Zipf limit $\zeta \downarrow 1$ arises when labor and managerial services inputs are abundant relative to the fixed factor required for firm entry. Although entry rates remain positive in this limit, the contribution of entry to new employment becomes negligible relative to the contribution of incumbent firm growth. The Zipf limit produces

hayn [1992] and Hopenhayn and Rogerson [1993]. Without any post-entry dynamics at all, aggregate convergence rates would depend only on how entry and exit rates respond to the state of the economy.

[3] Many authors use the Greenwood, Hercowitz, and Huffman [1988] preference specification to eliminate the wealth effect on labor supply. Jaimovich and Rebelo [2009] add habit persistence to make this device consistent with balanced growth.

a good approximation for the observed firm size distribution, and the rate of convergence of the economy varies continuously as $\zeta \downarrow 1$. The question therefore arises: what governs the speed of adjustment of the $\zeta \downarrow 1$ economy?

Detailed analytical answers to this question can be given with the help of two strong parametric assumptions. One is the common simplifying assumption that flow utility is a logarithmic function of consumption and additively separable across consumption and leisure. The second assumption is that consumption is produced using organization capital and team services, with a technology for team services that is Leontief in labor and managerial services. Everything else is non-parametric. In particular, there are separate roles for factor share parameters and elasticities of substitution (curvature parameters).

A low aggregate stock of organization capital implies fewer opportunities to replicate organization capital, but replication will also be more profitable. There are non-generic (pivotal) combinations of parameters for which the aggregate supply of managerial services used to produce new organization capital turns out to be independent, locally near the steady state, of the aggregate stock of organization capital. This means that the quantity of managerial services used to replicate each unit of organization capital scales with the reciprocal of the aggregate stock of organization capital. It is easy to see that the rate of convergence is then equal to the product of the depreciation rate of organization capital and the factor share of managers in replicating organization capital—this factor share is just the elasticity of g with respect to managerial services used to replicate each unit of organization capital. A 10% depreciation rate and a factor share of 70% then imply a half-life of about 10 years. Not surprisingly, lowering the curvature of g relative to this benchmark scenario speeds up the economy.

If the technology for producing consumption happens to be Cobb-Douglas, then, apart from the subjective discount rate, the only additional parameter that matters for the speed of convergence is a residual factor supply elasticity: the elasticity of managerial services available to produce organization capital with respect to the managerial wage weighted by the marginal utility of consumption. In the model, this elasticity is determined by heterogeneity in ability and occupational choice (as in Roy [1951]), and by managers shifting between the task of supervising workers and the task of replicating organization capital. The effect of this elasticity on the speed of convergence interacts with the curvature of g. A more elastic residual supply of managerial services speeds up the economy if and only if the curvature of g is below 1. Intuitively, if there were an army of potential managers waiting in the wings to spring into action whenever the stock of organization capital takes a hit, then recoveries could well be quick. But that would only be the case if an incipient rise in managerial services available for replicating organization capital does not quickly lower the rewards from replicating organization capital.

Under an additional separability assumption about the supplies of labor and managerial services, replacing the Cobb-Douglas technology in the consumption sector with a technology that has a lower elasticity of substitution speeds up the economy.[4] Quantitative experiments reported in the paper are based on an elasticity of 0.6, making the factor share of organization capital a decreasing function of the ratio of capital to team services. This implies a high capital share when the capital stock is low, creating strong incentives to produce more organization capital. Relative to a Cobb-Douglas technology, this shortens half-lives by about two years.

To show that belief shocks in this type of economy can produce equilibrium trajectories that look like a recession, the economy is augmented with an outside asset. The outside asset never produces any dividends, but households hold the mistaken belief that it will produce significant dividends in the future. Households feel wealthy and choose to consume more than they would without the outside asset. This results in high real interest rates and crowds out the accumulation of organization capital. Managerial resources are directed towards overseeing workers as they produce the consumption goods that households want. Consumption and worker employment are high. When the mistaken household beliefs are corrected, the bubble bursts. Because of the earlier crowding out of investment, the stock of organization capital is now below its new steady state. Consumption drops and managers are redirected towards producing new organization capital. But workers are a specific factor for the consumption sector, and their employment drops along with consumption. Because workers account for most of the labor force, overall employment also drops. During the recovery that follows, consumption, employment, and worker and managerial wages all rise towards their new steady state.

Related Literature The connection between the thickness of the right tail of the firm size distribution and the aggregate convergence rate of an economy was first pointed out in Luttmer [2012]. Gabaix et al. [2016] use the same idea to explain why the simplest random growth model cannot account for the fairly rapid changes observed in the US earnings and wealth distributions. The current paper remains focused on firms, and on the speed with which an economy recovers from a recession. The distinction between individual and aggregate convergence rates is somewhat reminiscent of the Granger [1980] model of long memory. But Granger [1980] was aggregating a fixed population of heterogeneous stationary stochastic processes, while here firm histories are non-stationary, and

[4] This is also true in the usual Cass-Koopmans economy. Jones and Manuelli [1990] produce long-run growth (no convergence) by assuming a very high elasticity of substitution.

the population of firms is constantly changing as a result of entry and exit.

The observation that recent recessions seem to give rise to slow (some argue "jobless") recoveries has received significant attention.[5] In particular, organization capital is a key factor in the account of Koenders and Rogerson [2005]. But the underlying firm dynamics in their model is too simple to make contact with the evidence on how firms grow and the resulting thick-tailed stationary size distributions.

A recovery in this paper is not an exogenous improvement in productivity, but a period when the economy accumulates more organization capital. This is related to the idea that recessions are associated with a reallocation of resources from less productive producers to more productive producers, resulting in an improvement in the quality of the capital stock.[6] But this quality margin plays no role here, and the model does not predict high levels of reallocation across firms in the depths of a recession. Instead, when managers switch to the task of replicating existing organization capital, marginal workers who cannot contribute to this task are sidelined. They return to the labor force only gradually over time, as the stock of organization capital rises towards its steady state.

Outline Section 2 lays out the economy. Section 3 describes the steady state implications for the firm size distribution. The circumstances in which this size distribution approximates Zipf's law are given in Section 4. Section 5 characterizes the speed of convergence and contrasts the effect of a belief shock in this economy to what happens in conventional models of adjustment costs. Section 6 shows how an asset bubble that bursts can produce a recession followed by a slow recovery.

2 The Economy

Organization capital is taken to be a type of capital that can be used simultaneously to produce consumption and more organization capital. But the technologies for producing consumption and new organization capital are different. Labor is a specific factor for the consumption sector, while managerial services are used in both sectors. Heterogeneous ability and comparative advantage determine the aggregate supplies of labor and managerial services.

[5]Notable examples are Bachmann [2012], Berger [2016], Fukui, Nakamura and Steinsson [2018], Jaimovich and Siu [2015], and Koenders and Rogerson [2005].

[6]See Caballero and Hammour [1994], Davis and Haltiwanger [1992], and Hall [1991], to name only a few well-known examples. For a skeptical view, and more on the history of this idea, see Barlevy [2002] and Moscarini [2001].

2.1 Households

There is a size-Λ continuum of identical infinitely lived households who consume and supply primary factors of production. Each household is made up of a heterogeneous continuum of members with types indexed by $h = (h_c, h_u, h_v, h_w) \in \mathbb{R}^4_{++}$. The distribution of types in each household is assumed to be time invariant and denoted by Ψ. Time is continuous, and household preferences are recursive and additively separable across time. The contribution of a type-h household member to flow utility at time t is $h_c \ln(C_t(h)) + h_u(1 - \iota_t(h))$, where $C_t(h)$ is flow consumption and $\iota_t(h) \in \{0, 1\}$ is a labor market participation decision (some of the $\iota_t(h) = 0$ choices could also be interpreted as public-sector employment). It will be convenient to normalize the mean of h_c to be equal to 1, so that the marginal utility of household consumption is going to be $1/C_t$ when $C_t = \int C_t(h)\Psi(\mathrm{d}h)$. The distribution Ψ is also assumed to be sufficiently smooth that the employment lotteries proposed in Rogerson [1988] are not needed.[7] Household preferences over flows of consumption, labor, and managerial services are then

$$\mathcal{U}(C, L, M) = \int_0^\infty e^{-\rho t} \left(\ln(C_t) - V(L_t, M_t)\right) \mathrm{d}t,$$

where ρ is strictly positive, and where $V(L, M)$ is the household disutility from supplying L units of labor and M units of managerial services,

$$\begin{aligned} V(L, M) \quad = \quad & \min_{\substack{(\iota_v(\cdot),\iota_w(\cdot))\in\{0,1\}^2 \\ \iota_v(\cdot)+\iota_w(\cdot)\leq 1}} \int h_u[\iota_v(h) + \iota_w(h)]\Psi(\mathrm{d}h) \\ & \text{s.t.} \int h_w \iota_w(h)\Psi(\mathrm{d}h) \geq L, \quad \int h_v \iota_v(h)\Psi(\mathrm{d}h) \geq M. \end{aligned}$$

Households can earn $\widetilde{w}_t$ per unit of labor and $\widetilde{v}_t$ per unit of managerial services, both measured in units of consumption per unit of time.

Households are endowed with an equal share of the assets in the economy, and markets are complete. As a result, every household will consume the same amount of consumption C_t and supply the same amounts of labor and managerial services. The risk-free rate in this economy is related to per-capita consumption growth via the usual Euler condition $r_t = \rho + \mathrm{D}C_t/C_t$.

[7] The large family construct used here can be decentralized, with intricate risk-sharing arrangements. See Chang and Kim [2007] for an economy with smooth heterogeneity in labor productivities and incomplete markets.

2.1.1 Factor Supply Curves

Let λ_t denote the marginal utility of wealth of the typical household at time t. The potential earnings of a type-h household member are $\max\{\widetilde{v}_t h_v, \widetilde{w}_t h_w\}$, and it is optimal for this household member to participate in the labor market if and only if $\lambda_t \max\{\widetilde{v}_t h_v, \widetilde{w}_t h_w\} \geq h_u$. The smooth heterogeneity assumed here means that ties do not affect aggregate factor supplies. Since $\lambda_t = 1/C_t$, this can also be written as $\max\{v_t h_v, w_t h_w\} \geq h_u$, where

$$(v_t, w_t) = (\widetilde{v}_t, \widetilde{w}_t)/C_t$$

is the vector of marginal utility weighted factor prices. Throughout the rest of the paper, "factor prices" or "wages" will always refer to these marginal utility weighted prices. The resulting per-capita supplies of labor and managerial services are then

$$\mathcal{L}(v_t, w_t) = \int h_w \iota \left[w_t h_w > \max\left\{ h_u, v_t h_v \right\} \right] \Psi(\mathrm{d}h), \tag{1}$$

$$\mathcal{M}(v_t, w_t) = \int h_v \iota \left[v_t h_v > \max\left\{ h_u, w_t h_w \right\} \right] \Psi(\mathrm{d}h). \tag{2}$$

The fact that these supply curves only depend on the marginal utility weighted factor prices relies heavily on the assumption that households are identical. Without such an assumption, aggregate factor supplies would depend on the equilibrium distribution of wealth across households—a potentially important complication that is abstracted from here. The following assumption and lemma summarize the important properties of (1)-(2).

Assumption 1 *The type distribution Ψ has full support, a finite mean, and is sufficiently smooth to ensure that $V(L, M)$ is twice continuously differentiable.*

Lemma 1 *Suppose Assumption 1 holds. Then $\mathcal{L}(v, 0) = \mathcal{M}(0, w) = 0$ for all positive v and w. Furthermore, the slopes of these supply curves satisfy*

$$\mathrm{D}_1\mathcal{M}(v, w) \geq 0, \quad \mathrm{D}_2\mathcal{L}(v, w) \geq 0, \quad \mathrm{D}_2\mathcal{M}(v, w) = \mathrm{D}_1\mathcal{L}(v, w) \leq 0.$$

In addition $\mathcal{L}(\kappa v, \kappa w)$ and $\mathcal{M}(\kappa v, \kappa w)$ are both increasing in $\kappa > 0$. This implies that own price elasticities are larger in absolute value than cross price elasticities.

The symmetry follows because $\mathrm{D}^2 V(L_t, M_t)$ is symmetric and $L_t = \mathcal{L}(v_t, w_t)$ and $M_t = \mathcal{M}(v_t, w_t)$ solve $[w_t, v_t] = \mathrm{D}V(L_t, M_t)$. These per-capita factor supplies abstract from effort and are completely driven by the numbers of household members who are at the margins between not working, supplying labor, and supplying managerial services. The only household members who move in and out of the labor force with fluctuations in the state of the economy are those who have both low h_v/h_u and low h_w/h_u.

2.2 The Technology for Producing Consumption

Consumption is produced using capital and the services of a team of managers and workers. The technology for team services is Leontief. The input requirements for a unit of team services are one unit of labor and β units of managerial services. In any equilibrium, managerial and labor services will be used in exactly this proportion, and so $L_t = \mathcal{L}(v_t, w_t)$ measures both per-capita labor and team services. The per-capita output of consumption is then $C_t = F(K_t, L_t)$, where K_t is the per-capita capital stock, and where F is a constant returns to scale production function that is assumed to be smooth, strictly increasing in both factors, and concave. Because of the logarithmic utility assumption, the production function F will turn out to affect the dynamic properties of this economy only via the factor share[8]

$$A(k) = \frac{\mathrm{D}_2 F(k,1)}{F(k,1)}. \tag{3}$$

This is the factor share of team services when $K_t/L_t = k$. The Leontief technology for team services implies that the marginal utility weighted cost of a team is $\beta v_t + w_t$ per unit of team services. Equating the cost of a team with its marginal product and clearing the labor market gives

$$(\beta v_t + w_t)\mathcal{L}(v_t, w_t) = A\left(\frac{K_t}{\mathcal{L}(v_t, w_t)}\right). \tag{4}$$

This determines w_t given K_t and v_t. It is easy to see that a destruction of capital has a negative direct effect (that is, holding fixed v_t) on w_t and $\mathcal{L}(v_t, w_t)$ if $A(\cdot)$ is increasing. This will be the maintained assumption.

Assumption 2 *The production function F for consumption is strictly increasing in capital and team services, sufficiently smooth, concave, and exhibits constant returns to scale. The implied factor share of team services $A(\cdot) \in (0,1)$ is non-decreasing.*

The team factor share is increasing in K_t/L_t if F is a constant elasticity of substitution (CES) production function with an elasticity strictly below 1. The function $A(\cdot)$ is constant if F is Cobb-Douglas, and then the dependence of the right-hand side of (4) on K_t vanishes. The capital stock can still affect wages and worker employment in that case,

[8]What is very special here is that F depends on labor and managerial services only through the team services composite good. The Leontief assumption can be relaxed.

but only indirectly through its effect on the price v_t of managerial services. If $\beta > 0$, then there are two such indirect channels: an increase in v_t raises the cost of a team of managers and workers, and it lowers the supply of labor because marginal households switch from supplying labor to supplying managerial services. Only this second channel remains if $\beta = 0$, and then (4) implies that v_t and w_t co-move, weakly. If the cross price elasticities of $\mathcal{L}(v_t, w_t)$ and $\mathcal{M}(v_t, w_t)$ are also zero, then $w_t = \widetilde{w}_t / C_t$ and the supply of labor are both constant.

Given (K_t, v_t), the wage w_t is determined by (4). So one can define

$$S(K_t, v_t) = \mathcal{M}\left(v_t, w_t\right) - \beta \mathcal{L}\left(v_t, w_t\right).$$

Holding fixed K_t, this is a residual supply curve that determines the supply of managerial services that can be used for anything other than producing consumption.

Lemma 2 *The residual supply curve $S(K_t, \cdot)$ of managerial services is strictly upward sloping.*

This can be shown directly using the results of Lemma 1. A one-line proof follows from an alternative characterization of the equilibrium based on (27) below.

2.3 The Technology for Producing New Capital

New capital can be produced in two ways. Managerial services can be used to produce new capital from scratch. Using an aggregate of n_t units of managerial services generates an aggregate flow of $f(n_t)$ of new units of capital. The production function f is subject to decreasing returns to scale. For example, it could be that managerial services have to be combined with a special production location, and that these special production locations are heterogeneous and in fixed supply. Or there could be a fixed supply of experts whose inputs are needed for every start-up.

The second way capital can be produced is by replicating existing capital. One unit of capital can be replicated using m_t units of managerial services, at the average rate $g(m_t)$. Capital is assumed to be homogeneous, and so the per-capita output of new capital produced by replication is $K_t g(m_t)$. The production function g exhibits decreasing returns to scale. Recall that the measure of households is Λ. The per-capita stock of capital then evolves according to

$$\mathrm{D}K_t = (g(m_t) - \delta)K_t + \frac{f(n_t)}{\Lambda}. \tag{5}$$

The production functions f and g are taken to satisfy the following assumption.

Assumption 3 *The production functions f and g are strictly increasing, strictly concave, and smooth. Furthermore,*

(i) $f(0) = g(0) = 0$,

(ii) *the marginal products* $\mathrm{D}f(n)$ *and* $\mathrm{D}g(m)$ *range throughout* $(0, \infty)$.

Part (i) of this assumption implies that managerial services are essential inputs in producing capital. In particular, $g(0) = 0$ means that the type of autonomous growth of capital that occurs in the AK economies of Jones and Manuelli [1990] and Rebelo [1991] cannot happen here. Part (ii) serves to rule out corner solutions—there will always be some entry and some replication.

2.3.1 The Price of Capital

This economy features joint production: the same unit of capital is combined, simultaneously, with labor to produce consumption, and with managerial services to replicate capital. Both activities generate income that accrues to the owners of capital. Write $\widetilde{q}_t$ for the price of a unit of capital, measured in units of consumption. The usual asset pricing equation says that

$$r_t\widetilde{q}_t = \left(1 - A\left(\frac{K_t}{\mathcal{L}(v_t, w_t)}\right)\right)\frac{C_t}{K_t} + \max_m \left\{\widetilde{q}_t(g(m) - \delta) - \widetilde{v}_t m\right\} + \mathrm{D}\widetilde{q}_t.$$

That is, the required return on a unit of capital comes in the form of earnings from producing consumption, earnings from replicating capital, and capital gains.[9] The first-order condition for replicating capital is $\widetilde{v}_t = \widetilde{q}_t\mathrm{D}g(m_t)$, and the first-order condition for producing capital from scratch is $\widetilde{v}_t = \widetilde{q}_t\mathrm{D}f(n_t)$. Write $q_t = \widetilde{q}_t/C_t$ for the marginal utility weighted price of a unit of capital. The first-order conditions for m_t and n_t are then

$$v_t = q_t\mathrm{D}f(n_t), \quad v_t = q_t\mathrm{D}g(m_t). \tag{6}$$

Combining the asset pricing equation for $\widetilde{q}_t$ with the Euler condition $r_t = \rho + \mathrm{D}C_t/C_t$ then yields

$$\rho q_t = \left(1 - A\left(\frac{K_t}{\mathcal{L}(v_t, w_t)}\right)\right)\frac{1}{K_t} + q_t(g(m_t) - \delta) - v_t m_t + \mathrm{D}q_t. \tag{7}$$

As is standard, the optimality conditions for this economy also include a transversality condition that requires $e^{-\rho t}q_t K_t$ to go to zero as t becomes large.

[9] This asset pricing equation also applies when organization capital is accumulated in discrete unit increments, randomly at the rate $g(m)$, as in Klette and Kortum [2004] and Luttmer [2011].

2.4 Equilibrium

The remaining equilibrium condition is the market clearing condition for managerial services

$$\mathcal{M}(v_t, w_t) = \beta\mathcal{L}(v_t, w_t) + m_t K_t + \frac{n_t}{\Lambda}. \tag{8}$$

Fix a per-capita capital stock K_t and a price of capital q_t. Then the combination of the consumption-sector equilibrium condition (4), the two first-order conditions for managerial services inputs (6), and the market clearing condition (8) determines the factor prices v_t and w_t, as well as managerial inputs m_t and n_t. With these variables determined as a function of (K_t, q_t), the two differential equations (5) and (7) then govern the evolution of (K_t, q_t) over time. Given an initial value (K_0, q_0), this pins down the trajectory of (K_t, q_t). The initial value of K_0 is given. The transversality condition $\lim_{t\to\infty} e^{-\rho t} q_t K_t = 0$ can be used to determine q_0.

2.5 Alternative Formulations

The notion of "organization capital" adopted here is abstract. It can be made more explicit by taking capital to be discrete at the micro level. One unit of capital could then be a blueprint that can be used at the same time by only one team or a few teams of managers and workers, or in only a restricted number of geographical locations. And $g(\cdot)$ can then be interpreted as a Poisson arrival rate that describes the rate at which blueprints can be replicated. Models of customer capital (e.g., Steindl [1965], Luttmer [2006], and Gourio and Rudanko [2014]) have a very similar structure, as does the model of blueprint capital in Luttmer [2011].

Search and Matching Models of the Labor Market Or a unit of capital can be a job, as in Diamond-Mortensen-Pissarides models of search and unemployment. In conventional implementations of such models, the technology for producing this type of capital is most often one that uses consumption goods as an input. This is as if $g(m) = 0$ and $f(\cdot)$ takes consumption goods rather than managerial services as an input. The organization capital in this paper can be interpreted as "jobs" as well, but the labor market is frictionless. Delays come from creating jobs, not from finding the workers to do those jobs. As will become clear, $g(\cdot)$, not $f(\cdot)$, plays a central role in determining the dynamic properties of this economy.

Long-Run Growth A easy way to introduce long-run growth is to replace $C_t = F(K_t, L_t)$ by $C_t = z_t F(K_t, L_t)$ with z_t growing exponentially. The formulation of preferences ensures that the supplies of labor and managerial services are constant when consumption and factor prices grow at a common rate. Because z_t only affects the output of consumption goods, growth is balanced even though production functions need not be Cobb-Douglas. Much richer formulations, in which capital quality is heterogeneous and growth is endogenous, are possible, but beyond the scope of the current paper.

Monopolistic Competition It is possible to re-interpret K_t as the number of goods in an economy with monopolistic competition and a technology for producing differentiated commodities that is linear in team services. If C_t is a symmetric CES composite good of differentiated commodities, with an elasticity of substitution greater than 1, then the $A(\cdot)$ in (4) is a constant equal to 1 minus the reciprocal of the elasticity of substitution. The resulting economy is isomorphic to a competitive economy in which F is Cobb-Douglas. Everything that follows for the competitive Cobb-Douglas economy applies. For more general but still symmetric preferences over differentiated commodities, the elasticity of the demand curves faced by individual producers will depend only on the number of goods K_t. The function $A(\cdot)$ in (4) is then no longer a function of the capital-labor ratio $K_t/\mathcal{L}(v_t, w_t)$, but of the number of goods K_t only. This makes a difference for the dynamic properties of this economy. A detailed analysis is left to future work.

3 The Firm Size Distribution

This economy will be shown to have a unique steady state, with $(m_t, n_t) = (m, n)$, and both $f(n)$ and $\delta - g(m)$ positive. That is, existing capital is replicated at a lower rate than the depreciation rate δ, and capital produced from scratch makes up the difference.

The flow $f(n)$ of new capital produced from scratch can be interpreted as a flow of new firms, each with one unit of start-up capital. Firms then grow by replicating capital. Because the allocation of capital across firms does not matter, an arbitrarily small transaction cost is enough to keep all capital produced directly or indirectly from the initial unit of start-up capital within the same firm. From (5), note that $K = f(n)/[\Lambda(\delta - g(m))]$ in the steady state. The contribution of firm entry to aggregate capital accumulation is thus

$$\frac{f(n)/\Lambda}{g(m)K + f(n)/\Lambda} = 1 - \frac{g(m)}{\delta} \in (0, 1).$$

So the contribution of entry will be small precisely when $\delta - g(m) > 0$ is close to zero.

Of course, even though new firms contribute very little to aggregate capital accumulation upon entry, their subsequent contribution as incumbents can be very large.[10]

Suppose now that the capital embodied in firms can depreciate in two distinct ways: incumbent firm capital depreciates continuously at a rate $\delta_k \in [0, \delta)$, and all of the firm's capital is destroyed simultaneously and randomly at the complementary rate $\delta_f = \delta - \delta_k \in (0, \delta]$. That is, δ_f is a firm exit rate, and a firm's exit results in the destruction of all of its capital. In a steady state, this means that the age distribution of firms is exponential with mean $1/\delta_f$. Incumbent firms grow at the net rate $g(m) - \delta_k$ as long as the random exit shock does not hit, and so the size of a firm of age a will be $k = e^{(g(m)-\delta_k)a}$, measured in units of capital. Assume that $g(m) - \delta_k$ is positive, so that firms can grow beyond their start-up size. The distribution Φ of firm size will then be

$$\Phi(k) = 1 - e^{-\delta_f \ln(k)/(g(m)-\delta_k)} = 1 - k^{-\zeta}, \quad k \in [1, \infty).$$

This is a Pareto distribution on $[1, \infty)$, and

$$\zeta = \frac{\delta_f}{g(m) - \delta_k}$$

is the tail index of the distribution. The mean of this distribution is finite if and only if $\zeta > 1$. The steady state implies $0 < \delta - g(m) = \delta_f - (g(m) - \delta_k)$, and the assumption that firms grow beyond their start-up size says that $g(m) - \delta_k > 0$. Together, these inequalities imply that $\zeta > 1$. Moreover, as long as $g(m) - \delta_k > 0$ is bounded away from zero, $\delta - g(m) \downarrow 0$ is the same as the Zipf limit $\zeta \downarrow 1$.

Firm employment scales with firm capital because, in the steady state, capital-labor ratios are constant both in the production of consumption and in the replication of capital. In US data, the employment size distribution of firms has a tail index ζ of about 1.1 (Luttmer [2007]), and the interpretation given here means that $\delta - g(m) = (1-1/\zeta)\delta_f$ must be small. From (5), the per-capita capital stock K_t converges at the rate $\delta - g(m)$ when (m_t, n_t) is fixed at the steady state value (m, n). That is, slow aggregate convergence happens precisely when the tail index ζ of the firm size distribution is close to 1, as is the case in US data.

[10] It is tempting to compare the predicted contribution of new firms directly with gross employment flows in the Census data. But this highly stripped-down model of the firm size distribution does not account for the trial-and-error that occurs in reality and generates large numbers of very transitory firms (as in Jovanovic [1982]). Pries [2004] uses this argument to account for high turnover in the labor market.

Discrete Size Distributions In the economy outlined here, the capital stock of incumbent firms is replicated and depreciates continuously at the respective rates $g(m)$ and δ_k. Instead, if individual pieces of capital can be replicated at the Poisson rate $g(m)$ and depreciate randomly in one-hoss-shay fashion at the rate δ_k, then the resulting firm size distribution will not be Pareto but an analog of the Pareto distribution that has discrete support (a generalization of the Yule distribution associated with the special case $\delta_k = 0$). In particular, the right tail of that distribution will still behave like $k^{-\zeta}$. When the support of organization capital is discrete, firms losing their last unit of capital is another source of firm exit. This then implies aggregate firm exit and entry rates that lie in (δ_f, δ). As in Luttmer [2011], this can be used to accommodate the fact that firm exit rates are high (around 10% per annum in US data), and that the bulk of aggregate exit is accounted for by small firms. The aggregate properties of this economy are identical to the one with continuous depreciation.

4 The Steady State and the Zipf Asymptote

Steady states are defined by $\mathrm{D}K_t = 0$ in (5) and $\mathrm{D}q_t = 0$ in (7), together with the static equilibrium conditions implied by (4), (6) and (8). It will be convenient to write

$$n[m] = [\mathrm{D}f]^{-1}(\mathrm{D}g(m))$$

and define

$$m_\infty = \sup\{m : g(m) < \delta\}.$$

Since $g(m)$ is assumed to be strictly increasing, $m_\infty < \infty$ if and only if $g(m) \geq \delta$ for m large enough. In any case, the steady state supply of managerial services needed for replicating capital, $mK = mf(n[m])/[\Lambda(\delta - g(m))]$, explodes as m approaches m_∞ from below.

What follows proves the existence and uniqueness of a steady state by first establishing the result for the Cobb-Douglas case, where $A(k) = \alpha \in (0,1)$ identically. This Cobb-Douglas economy implies a capital-labor ratio $k(\alpha)$, resulting in a map $\alpha \mapsto k(\alpha)$. The steady state for an economy with a non-constant $A(\cdot)$ is simply a fixed point of the map $\alpha \mapsto A(k(\alpha))$. Assumption 2 is enough to guarantee a unique fixed point.

4.1 The Cobb-Douglas Case

Suppose the labor share in the consumption sector is equal to $A(k) = \alpha \in (0,1)$ identically. The steady state condition $\mathrm{D}q_t = 0$ then simplifies to $qK = (1-\alpha)/(\rho + \delta - [g(m) -$

$\mathrm{D}g(m)m])$. The first-order condition $v = q\mathrm{D}g(m)$ in turn implies $vK = (1-\alpha)\mathrm{D}g(m)/(\rho + \delta - [g(m) - \mathrm{D}g(m)m])$. The condition $\mathrm{D}K_t = 0$ says that $K = f(n[m])/[\Lambda(\delta - g(m))]$. Given any $m \in (0, m_\infty)$, this determines v and the quantity of managerial services $D_\Lambda(v) = mK + n[m]/\Lambda$ needed to maintain the steady state capital stock. Varying $m \in (0, m_\infty)$ traces out a demand curve $(v, D_\Lambda(v))$ for managerial services used to produce capital,

$$v \times \frac{f(n[m])}{\Lambda[\delta - g(m)]} = \frac{(1-\alpha)\mathrm{D}g(m)}{\rho + \delta - [g(m) - \mathrm{D}g(m)m]}, \tag{9}$$

$$D_\Lambda(v) = m \times \frac{f(n[m])}{\Lambda[\delta - g(m)]} + \frac{n[m]}{\Lambda}. \tag{10}$$

The left-hand side of (9) is strictly increasing in $m \in (0, m_\infty)$. Although $\mathrm{D}g(m)$ and $\rho + \delta - [g(m) - \mathrm{D}g(m)m]$ are both decreasing in m, an easy derivative calculation shows that the right-hand side of (9) is a strictly decreasing function of $m \in (0, m_\infty)$. It follows immediately that (9) defines v as a strictly decreasing function of $m \in (0, m_\infty)$. It is clear from (10) that $D_\Lambda(v)$ itself is strictly increasing in $m \in (0, m_\infty)$, and so (9)-(10) traces out a strictly decreasing demand curve for managerial services.

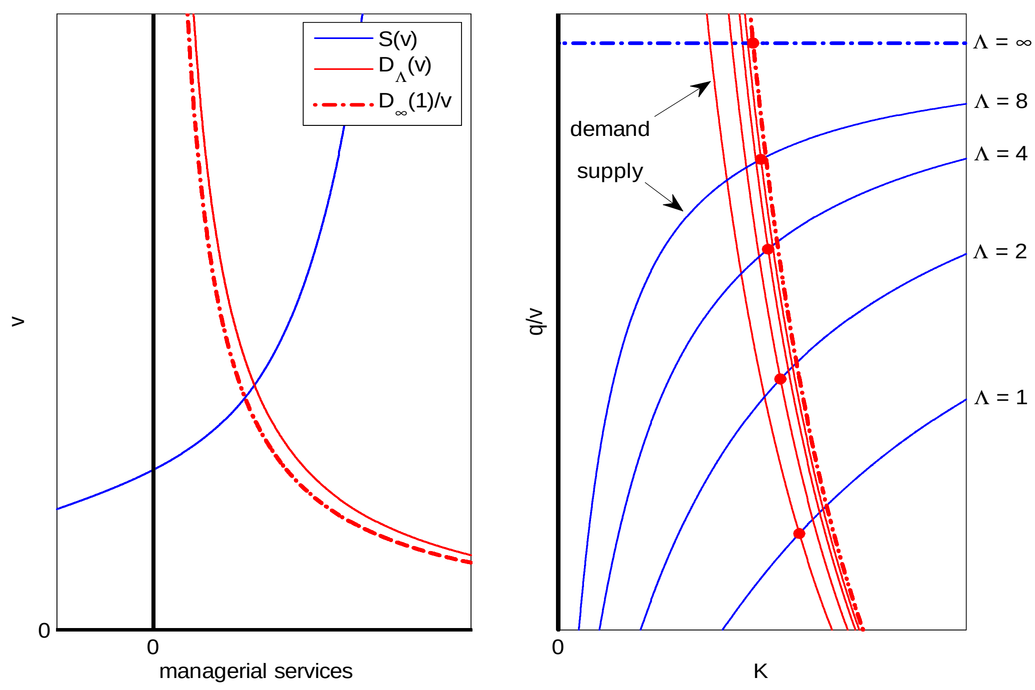

FIGURE 1 Taking the Zipf Limit

Managerial services are also used to produce consumption. The residual supply curve $S(\cdot)$ of managerial services available for producing capital is defined by

$$\alpha = (\beta v + w)\mathcal{L}(v, w), \tag{11}$$

$$S(v) = \mathcal{M}(v, w) - \beta\mathcal{L}(v, w). \tag{12}$$

Lemma 2 shows that this supply curve is strictly increasing. The left panel of Figure 1 shows an example. Since the supply and demand curves are strictly increasing and decreasing, respectively, it is immediate that the market clearing condition $D_\Lambda(v) = S(v)$ can have at most one solution, and so there can be at most one steady state. The existence of a steady state is guaranteed by the following proposition.

Proposition 1 *Given Assumptions 1 and 3 and a labor share $\alpha \in (0, 1)$, the Cobb-Douglas economy has a unique steady state, defined by (9)-(12), together with $K = f(n[m])/[\Lambda(\delta - g(m))]$ and $qK = (1-\alpha)/(\rho + \delta - (g(m) - \mathrm{D}g(m)m))$.*

Proof If $m_\infty < \infty$, then the left-hand side of (9) ranges throughout $(0, \infty)$ with $m \in (0, m_\infty)$. The right-hand side varies between $(1-\alpha)\mathrm{D}g(m_\infty)/(\rho + \mathrm{D}g(m_\infty)m_\infty)$ and ∞. It follows that (9) can solved for some $m \in (0, m_\infty)$ for every $v \in (0, \infty)$. And this m is decreasing in v. If $m_\infty = \infty$, then the left-hand side of (9) could be bounded above for $m \in (0, \infty)$ and a fixed v. But $m \downarrow 0$ implies $\mathrm{D}g(m) \uparrow \infty$, and so it is still the case that (9) can be solved for a unique m given any $v \in (0, \infty)$. It follows from (10) that $D_\Lambda(v)$ is well defined for all $v \in (0, \infty)$. It is decreasing in v, and taking $v \to \infty$ forces $m \downarrow 0$ and hence $D_\Lambda(v) \downarrow 0$. The supply curve $S(v)$ is upward sloping, and $S(v)$ will be positive for large enough v because $(\beta v + w)\mathcal{L}(v, w) = \alpha$ forces $\mathcal{L}(v, w) \downarrow 0$ as $v \to \infty$.■

In preparation for the general case, consider how the steady state of a Cobb-Douglas economy depends on the labor share α. Refer to Figure 1 and note from (9)-(10) that an increase in α shrinks the demand curve $D_\Lambda(v)$ toward the quantity axis. At the same time, it is easy to see from (11) that an increase in α, holding fixed v, raises w. By (12), this reduces the household supply of managerial services and raises the supply of labor, lowering the residual supply $S(v)$ on both counts. So an increase in α moves the supply curve $S(v)$ toward the price axis. It follows that an increase in α lowers $D_\Lambda(v) = S(v)$. The definition (10) makes $D_\Lambda(v)$ an increasing function of m, and so a reduction in $D_\Lambda(v)$ has to go together with a reduction in m. This lowers the steady state capital stock $K = f(n[m])/[\Lambda(\delta - g(m))]$. It turns out that $\mathcal{L}(v, w)$ is increasing in α, and so an increase in α lowers the capital-labor ratio. More detail on this last step is in Appendix A, proving the following lemma.

Lemma 3 *Given Assumptions 1 and 3, the steady state capital-labor ratio in the Cobb-Douglas economy is decreasing in the labor share α.*

The Demand and Supply of Capital The steady state must have $g(m) < \delta$, so that $K = f(n)/[\Lambda(\delta - g(m))]$ remains finite. To make the role of this asymptote more explicit, consider the right panel of Figure 1. It shows the steady state demand and supply of organization capital in terms of the relative price $s = q/v$. This price immediately pins down m and n via the first-order conditions $1 = s\mathrm{D}g(m)$ and $1 = s\mathrm{D}f(n)$. The implied steady state supply of capital at s is then simply $K = f(n)/[\Lambda(\delta - g(m))]$. This supply is zero at $s = 0$, obviously upward sloping as long as s implies $\delta > g(m)$, and it becomes infinitely elastic at the s that solves $\delta = g(m)$ and $1 = s\mathrm{D}g(m)$. The steady state demand for organization capital also follows from the first-order conditions for m and n, but now together with the steady state condition $\mathrm{D}q_t = 0$ and the market clearing condition for managerial services. The latter two conditions are $svK = (1-\alpha)/(\rho+\delta-[g(m)-\mathrm{D}g(m)m])$ and $S(v) = mK + n/\Lambda$, respectively. Eliminating v produces an equilibrium condition for K, the demand for capital at s. Since $\delta > g(m)$ implies $\rho+\delta > g(m) - \mathrm{D}g(m)m$, the supply of capital asymptotes before the demand curve becomes ill defined. As Figure 1 suggests, and as will be shown below for general F, the equilibrium approaches this asymptote when Λ becomes large.

4.2 The General Case

Consider a general production function F with a labor share $A(k)$ that is non-decreasing in k. The conditions for a steady state are then the Cobb-Douglas conditions (9)-(12) together with the requirement that $\alpha = A(k)$, where $k = K/\mathcal{L}(v, w)$ and $K = f(n[m])/[\Lambda(\delta - g(m))]$. To set up a fixed point condition for the labor share, start with any $\alpha \in (0, 1)$ and use (9)-(12) to construct a unique steady state for the associated Cobb-Douglas economy. This can be done by Proposition 1. Take the associated (v, w, m) to compute

$$\alpha' = A(k), \quad k = \frac{f(n[m])/[\Lambda(\delta - g(m))]}{\mathcal{L}(v, w)}. \tag{13}$$

This produces a well-defined mapping $\alpha \mapsto \alpha'$, from $(0, 1)$ into $(0, 1)$. If $\alpha' = \alpha$, then all the steady state equilibrium conditions for the general economy are satisfied. Proving the existence and uniqueness of a steady state now requires proving that $\alpha \mapsto \alpha'$ has precisely one fixed point. By Lemma 3, the mapping $\alpha \mapsto k$ implied by the Cobb-Douglas economy is decreasing. Assumption 2 requires $A(\cdot)$ to be weakly increasing. As a result,

$\alpha \mapsto A(k) = \alpha'$ is a weakly decreasing map. Such a map can cross the line $\alpha' = \alpha$ only once. Because $\alpha \mapsto \alpha'$ is well defined for any $\alpha \in (0, 1)$ and continuous, it follows that there is a unique fixed point.

Proposition 2 *Given Assumptions 1-3, the economy has a unique steady state.*

CES production functions with an elasticity of substitution greater than 1 have $A(\cdot)$ decreasing instead of increasing. This was used by Jones and Manuelli [1990] to construct a one-sector economy without a steady state, with $k_t \to \infty$ and $A(k_t) \downarrow 0$ over time. Here, the mapping $\alpha \mapsto A(k) = \alpha'$ becomes increasing, and it is no longer obvious that this mapping will have a fixed point. But F in this economy only determines $C_t = F(K_t, \mathcal{L}(v_t, w_t))$, and output of new capital is the maximum of $g(m_t)K_t + f(n_t)/\Lambda$ subject to the constraint $m_t K_t + n_t/\Lambda \leq S(K_t, v_t)$. Experiments in which F is CES with a finite elasticity of substitution deliver unique steady states even though $A(\cdot)$ is increasing.

4.3 The Large-Λ Limit

If $\mathrm{D}f(0)$ is finite, contrary to Assumption 2, then the steady state may very well have $f(n) = 0$ together with $\delta = g(m)$ and an exponentially declining number of ever larger firms. Such a steady state would not be able to account for the fact that entry and exit rates in the US are around 11% and 10% per annum, or for the implied stability of the per-capita number of firms, or for the stability of the firm size distribution. But the tail index $\zeta \approx 1.1$ of the US size distribution of firms does suggest that $\delta - g(m) = (1 - 1/\zeta)\delta_f$ must be very small. We therefore need to describe a scenario in which $\delta - g(m)$ is close to zero even though $f(n)$ is not. The dynamics (5) of the per-capita capital stock K_t and the right panel of Figure 1 indicate that these observations can be reconciled by taking Λ to be large.

As before, it is useful to first consider the Cobb-Douglas case.

Proposition 3 *Suppose Assumptions 1 and 3 hold and F is Cobb-Douglas with labor share $\alpha \in (0, 1)$. Assume that $g(m) > \delta$ for m large enough. Then $\Lambda[\delta - g(m)]$, v, and w converge to limits in $(0, \infty)$ as Λ becomes large. As a consequence, $g(m) \uparrow \delta$ and $f(n) \uparrow f(n[m_\infty])$, implying that the tail index of the firm size distribution converges to 1.*

Proof Recall the Cobb-Douglas steady state conditions (9)-(12). For any $u \in (0, \infty)$, define $m(u) \in (0, \infty)$ to be the solution to

$$u \times \frac{f(n[m])}{\delta - g(m)} = \frac{(1 - \alpha)\mathrm{D}g(m)}{\rho + \delta - [g(m) - \mathrm{D}g(m)m]}.$$

The demand curve is then determined by

$$D_\Lambda(v) = m(v/\Lambda) \times \frac{v}{\Lambda} \frac{f(n[m(v/\Lambda)])}{\delta - g(m(v/\Lambda))} \times \frac{1}{v} + \frac{n[m(v/\Lambda)]}{\Lambda}.$$

Write v_Λ for the solution to $D_\Lambda(v_\Lambda) = S(v_\Lambda)$. Taking $u \downarrow 0$ forces $m(u) \uparrow m_\infty$ and $g(m(u)) \uparrow \delta$. This implies

$$D_\infty(v) = \lim_{\Lambda\to\infty} D_\Lambda(v) = \frac{\mathrm{D}g(m_\infty)m_\infty}{\rho + \mathrm{D}g(m_\infty)m_\infty} \frac{1-\alpha}{v},$$

for any $v \in (0, \infty)$. This hyperbola is guaranteed to intersect the strictly increasing supply curve $S(v)$ precisely once, and so the limiting market clearing condition $S(v) = D_\infty(v)$ has a well-defined and unique solution $v_\infty \in (0, \infty)$. One can verify that $v_\Lambda \to v_\infty$ as $\Lambda \to \infty$.■

As the proof of Proposition 3 shows, the per-capita demand curve $D_\Lambda(v)$ for managerial services is unit elastic in the large-Λ limit. The underlying reason is that the steady state value $qK = (1-\alpha)/(\rho + \delta - [g(m) - \mathrm{D}g(m)m])$ of the capital stock converges to a well-defined limit as $m \uparrow m_\infty$. This means that $mK = qK\mathrm{D}g(m)m/v$ behaves like $1/v$ when m is close to m_∞, and that mK dominates the demand for managerial services when Λ is large.

The following proposition extends the large-Λ limit to general F.

Proposition 4 *Suppose Assumptions 1-3 hold, and that $g(m) > \delta$ for m large enough. Then the conclusions of Proposition 3 apply.*

Proof As in (13), the Cobb-Douglas economy generates a capital-labor ratio $\alpha \mapsto k_\Lambda[\alpha]$, and $\alpha = A(k_\Lambda[\alpha])$ has a unique fixed point $\alpha_\Lambda \in (0,1)$ by Proposition 1. Given Proposition 3, it is easy to check that $k_\Lambda[\alpha]$ converges to a large-Λ capital-labor ratio $k_\infty[\alpha]$ as $\Lambda \to \infty$. Along the lines of the proof of Proposition 1, one can verify that the mapping $\alpha \mapsto A(k_\infty[\alpha])$ produces a unique fixed point $\alpha_\infty \in (0,1)$ for the large-Λ economy. It is clear that $A(k_\Lambda[\alpha_\infty]) \to \alpha_\infty$ as $\Lambda \to \infty$. The assumption that $A(\cdot)$ is weakly decreasing can be used to argue that α_Λ must be close to α_∞ when $A(k_\Lambda[\alpha_\infty])$ is close to α_∞. This ensures that $\alpha_\Lambda \to \alpha_\infty$ as $\Lambda \to \infty$. Detailed versions of these steps are given in the online appendix.■

Robust Entry Note that n increases to the limit $n[m_\infty] \in (0, \infty)$ as Λ becomes large. So there will be robust entry in the limit, and the number of firms converges to $f(n[m_\infty])/\delta_f$. The steady state entry and exit rates are $\delta_f \in (0, \delta)$, but the per-capita number of firms goes to zero and the average size of firms, $\delta_f \Lambda K/f(n[m])$, diverges.[11] The contribution of new firms to the aggregate accumulation of capital will be negligible—both $(n/\Lambda)/(mK + n/\Lambda)$ and $(f(n)/\Lambda)/(g(m)K + f(n)/\Lambda)$ converge to zero as Λ becomes large. A large-Λ economy is an economy in which the managerial resources needed to produce capital are abundant, as is the supply of labor that can be combined with capital to produce consumption. Entry is one way in which these managerial resources could be used, but the marginal product of $f(n)$ would converge to 0 if n became large. In a steady state, this directs most of the abundant managerial resources toward the process of replicating existing capital.

The plausibility of all of this hinges on the interpretation of the fixed factor implicit in $f(n)/\Lambda$. The most natural interpretation is that this fixed factor is some type of inelastically supplied human input that is distinct from labor and managerial services. Perhaps the creativity involved in starting something new, as opposed to replicating something, is relatively scarce. If the households that supply this human factor also have additively separable logarithmic preferences over consumption, with the same subjective discount rate ρ, then Λ can be (or scale with) the number of households who cannot supply this factor relative to the number who can. This interpretation immediately allows for both populations to grow at some common rate, and so account for the fact that the number of firms tends to scale with population (Luttmer [2010] provides time-series evidence for the US).

The resulting interpretation of Zipf's law is, in essence, the one given in Luttmer [2011]. But there one type of labor is used to produce consumption and replicate capital, while another type of labor is used to create capital from scratch. The supply curves of both types of labor respond to factor prices. Here the explicitly modeled human factors of production that are used to replicate capital and create capital from scratch are perfect substitutes, distinct from the labor that is needed to produce consumption, and distinct from the implicit fixed factor needed to produce new capital from scratch.

5 Aggregate Dynamics

The pieces are now in place to characterize aggregate convergence rates when replication and entry rates depend on the state of the economy. For this characterization, it is

[11] Models with a negligible number of firms relative to the population are relatively common, but they usually involve similar firms that all employ a continuum of workers (see Kaas and Kircher [2015] for a recent example). Given that the modal firm in the data has at most a handful of employees, such models are impossible to calibrate. Here, every firm in the large-Λ economy has a finite number of workers and firms are not similar at all: the size distribution has a very thick tail.

convenient to write $\mathcal{S}_g = \mathrm{D}g(m)m/g(m)$ and $\mathcal{C}_g = -\mathrm{D}^2g(m)m/\mathrm{D}g(m)$ for the share and curvature parameters of $g(\cdot)$. The implied elasticity of substitution between managerial services and organization capital is $(1-\mathcal{S}_g)/\mathcal{C}_g$. The analogous share and curvature parameters for $F(1,l)$ are $\mathcal{S}_F$ and $\mathcal{C}_F$. Assumption 2 forces $\mathcal{C}_F \geq 1-\mathcal{S}_F$. The own price elasticities of labor and managerial services are $\mathcal{E}_L$ and $\mathcal{E}_M$.

5.1 Mechanical Examples

Suppose $\beta = 0$, and that the supply of managerial services is completely inelastic, equal to a constant $\mathcal{M}$. As long as the price of capital is positive, the net flow of new capital $\mathrm{D}K_t$ is then simply the maximum of $(g(m)-\delta)K_t + f(n)/\Lambda$ subject to the resource constraint $mK_t + n/\Lambda \leq \mathcal{M}$. Managerial services are good for nothing else. Their only use is to produce as much capital as possible. The first-order and envelope conditions immediately imply that

$$
\begin{aligned}
-\frac{\partial \mathrm{D}K_t}{\partial K_t} &= -\frac{\partial}{\partial K_t}\max_{m,n}\left\{(g(m)-\delta)K_t + \frac{f(n)}{\Lambda} : mK_t + \frac{n}{\Lambda} \leq \mathcal{M}\right\} \\
&= \delta - g(m_t) + g(m_t) \times \frac{\mathrm{D}g(m_t)m_t}{g(m_t)}. \qquad (14)
\end{aligned}
$$

In the Zipf limit, $\delta = g(m_\infty)$, and so (14) reduces to $\delta \times \mathrm{D}g(m_\infty)m_\infty/g(m_\infty) = \delta\mathcal{S}_g$. Because a low capital stock implies more inelastically supplied managerial services per unit of capital, the capital stock converges to the steady state, even in the Zipf limit. The rate at which this happens is $\delta\mathcal{S}_g$. Since $\mathcal{S}_g \leq 1$, a 10% depreciation rate implies half-lives of about 7 years or more, and these half-lives will be longer the lower is the factor share of managerial services. Outside the Zipf limit, the term $\delta - g(m_t)$ speeds up the economy, but the evidence on firm size and entry and exit rates suggests that this term is small.

This logic extends to some simple scenarios in which managerial task shifting plays a role. For example, suppose that $\beta > 0$, and that the supply of labor is also inelastic, at some $\mathcal{L} < \mathcal{M}/\beta$. In that case, the resource constraint is simply $mK_t + n/\Lambda \leq \mathcal{M} - \beta\mathcal{L}$, and (14) again applies. Alternatively, if the supply of labor can respond to wages but the technology for producing consumption is determined by the Leontief production function $F(K,L) = \min\{K, L\}$, then the resource constraint becomes $(\beta + m)K_t + n/\Lambda \leq \mathcal{M}$. The first-order and envelope conditions then imply $-\partial \mathrm{D}K_t/\partial K_t = (\delta - g(m_t)) + \mathrm{D}g(m_t)(\beta + m_t)$. The resulting speed is then $\mathrm{D}g(m_\infty)(\beta + m_\infty) = (1 + \beta/m_\infty)\delta\mathcal{S}_g$ in the Zipf limit. A capital stock below the steady state releases managerial services that are normally used to produce consumption, and this will speed up convergence.

More Generally Outside these special cases, the extent to which managerial services are used to produce new organization capital naturally depends on the state of the economy. Let $\mathsf{M}[K_t]$ be the resulting policy function. Replacing the constraint $mK_t + n/\Lambda \leq \mathcal{M}$ in (14) by $mK_t + n/\Lambda \leq \mathsf{M}[K_t]$ and considering the Zipf limit gives

$$-\frac{\partial \mathrm{D} K_t}{\partial K_t} = \left(1 - \frac{K_t \mathsf{DM}[K_t]}{\mathsf{M}[K_t]}\right) \delta \mathcal{S}_g.$$

As in (14), there would be an additional term $\delta - g(m_t)$ outside the Zipf limit. Not surprisingly, the rate of convergence is faster than $\delta \mathcal{S}_g$ when the policy function $\mathsf{M}[K_t]$ is decreasing in K_t. But there is no presumption that it is. If there is more capital that can be replicated, then it may well be optimal to use more managerial services to do so. To investigate, and to compare with the convergence speeds in more traditional models, it is useful to take a step back and consider a more general class of economies.

5.2 An Abstract Economy

Consider an economy with $\mathrm{D}K_t = K_t(g(m_t) - \delta)$ and flow utilities $U(K_t, m_t K_t)$. As in the Zipf limit of the economy with organization capital, the technology for producing new capital exhibits constant returns to scale in $(K_t, M_t) = (K_t, m_t K_t)$. The function $U(K_t, M_t)$ is assumed to be increasing in K_t, decreasing in M_t, concave, and sufficiently smooth.[12]

The Hamiltonian for this economy is

$$\mathcal{H}(K, q) = \max_{m} \left\{ U(K, mK) + qK(g(m) - \delta) \right\}. \tag{15}$$

This Hamiltonian is concave in K and convex in q. Along the optimal trajectory for (K_t, q_t), the first-order condition implied by (15) is

$$0 = \mathrm{D}_2 U(K_t, m_t K_t) + q_t \mathrm{D} g(m_t), \tag{16}$$

as long as the m_t that attains $\mathcal{H}(K_t, q_t)$ is positive. Clearly, an increase in q_t raises m_t. The dynamic optimality conditions for this economy require that (K_t, q_t) satisfy the differen-

[12] This formulation includes the special case $g(m) = m$. So this describes any economy with a single capital stock, $\mathrm{D}K_t = -\delta K_t + M_t$, and flow utility $U(K_t, M_t)$. A non-trivial $g(m)$ helps isolate the role of the technology for replicating capital.

tial equation

$$\begin{aligned}
\mathrm{D}K_t &= \mathrm{D}_2\mathcal{H}(K_t, q_t), \\
\mathrm{D}q_t &= \rho q_t - \mathrm{D}_1\mathcal{H}(K_t, q_t).
\end{aligned}$$

The fact that $\mathcal{H}(\cdot, q_t)$ is concave and $\mathcal{H}(K_t, \cdot)$ is convex implies that $\partial \mathrm{D}K_t/\partial q_t \geq 0$ and $\partial \mathrm{D}q_t/\partial K_t \geq 0$. Note also that $\partial \mathrm{D}K_t/\partial K_t + \partial \mathrm{D}q_t/\partial q_t = \rho$, provided that $\mathcal{H}$ is sufficiently smooth. See Cass and Shell [1976] for a discussion of these features of the Hamiltonian in a more general setting.

Observe that the envelope conditions for the Hamiltonian (15) imply that $\mathrm{D}_1\mathcal{H}(K_t, q_t) = \mathrm{D}_1U(K_t, m_tK_t) + \mathrm{D}_2U(K_t, m_tK_t)m_t + q_t(g(m_t) - \delta)$, and then the first-order condition (16) turns this into $\mathrm{D}_1\mathcal{H}(K_t, q_t) = \mathrm{D}_1U(K_t, m_tK_t) + q_t(g(m_t) - \delta - \mathrm{D}g(m_t)m_t)$. It is convenient to scale the first term by q_t and define $p_t = \mathrm{D}_1U(K_t, m_tK_t)/q_t$. This measures the profitability of capital from any use other than replication. The differential equation for (K_t, q_t) can then be restated as

$$\mathrm{D}K_t = K_t(g(m_t) - \delta), \tag{17}$$

$$\mathrm{D}q_t = \left(\rho + \delta - p_t - [g(m_t) - \mathrm{D}g(m_t)m_t]\right) q_t, \tag{18}$$

where (m_t, p_t) must solve

$$\begin{bmatrix} p_t & -\mathrm{D}g(m_t) \end{bmatrix} q_t = \mathrm{D}U(K_t, m_tK_t). \tag{19}$$

Imposing $\mathrm{D}K_t = 0$ and $\mathrm{D}q_t = 0$ gives $\delta = g(m)$ and $p = \rho + \delta \mathrm{D}g(m)m/g(m)$. The monotonicity of $g(\cdot)$ implies there can be only one pair (m, p) that solves these conditions. Any K that solves $p/\mathrm{D}g(m) = -\mathrm{D}_1U(K, mK)/\mathrm{D}_2U(K, mK)$ will then be a steady state capital stock.

5.2.1 Steady State Curvature Parameters

To characterize steady states and the dynamics of this economy near a steady state, it will be useful to introduce the following curvature parameters:

$$\mathcal{C}_{i,j} = -\frac{\mathrm{D}_{i,j}U(x)x_j}{|\mathrm{D}_iU(x)|}, \quad i, j \in \{1, 2\}, \quad (x_1, x_2) = (K, M).$$

The sign convention adopted in this definition, together with the concavity of $U(K, M)$, ensures that the diagonal curvature parameters are positive. Modulo a sign, the sum of these curvature parameters is just the scale elasticity of $\mathrm{D}_1 U(K,M)/\mathrm{D}_2 U(K,M)$,

$$-K \times \frac{\mathrm{D}_2 U(K,mK)}{\mathrm{D}_1 U(K,mK)} \frac{\mathrm{d}}{\mathrm{d}K} \frac{\mathrm{D}_1 U(K,mK)}{\mathrm{D}_2 U(K,mK)} = \mathcal{C}_{KK} + \mathcal{C}_{KM} + \mathcal{C}_{MK} + \mathcal{C}_{MM}, \tag{20}$$

where m is held constant. This can be used to describe how the steady state capital stock depends on the subjective discount rate ρ.

Proposition 5 *If (20) is non-zero at a steady state, then, near that steady state, the elasticity of the steady state capital stock with respect to the subjective discount rate is given by*

$$\frac{\rho}{K}\frac{\mathrm{d}K}{\mathrm{d}\rho} = -\frac{\rho}{\rho + \delta \mathcal{S}_g} \frac{1}{\mathcal{C}_{KK} + \mathcal{C}_{KM} + \mathcal{C}_{MK} + \mathcal{C}_{MM}}. \tag{21}$$

To see this, note that the steady state condition $\delta = g(m)$ does not depend on ρ. The elasticity of the steady state profit rate $p = \rho + \delta \mathcal{S}_g$ with respect to ρ is therefore simply $\rho/(\rho + \delta \mathcal{S}_g)$. Differentiating the efficiency condition $p/\mathrm{D}g(m) = -\mathrm{D}_1 U(K,mK)/\mathrm{D}_2 U(K,mK)$ then delivers (21).[13] Observe that the curvature of $g(\cdot)$ plays no role in (21).

5.2.2 Dynamics Near the Steady State

To examine the stability properties of this economy, we need to evaluate the Jacobian of the differential equation (17)-(19) at the steady state. One can verify that

$$\begin{bmatrix} \frac{\partial \mathrm{D}K_t}{\partial K_t} & \frac{q_t}{K_t}\frac{\partial \mathrm{D}K_t}{\partial q_t} \\ \frac{K_t}{q_t}\frac{\partial \mathrm{D}q_t}{\partial K_t} & \frac{\partial \mathrm{D}q_t}{\partial q_t} \end{bmatrix} = \delta \mathcal{S}_g \begin{bmatrix} 1 \\ -\mathcal{C}_g \end{bmatrix} \begin{bmatrix} \frac{K}{m}\frac{\partial m}{\partial K} & \frac{q}{m}\frac{\partial m}{\partial q} \end{bmatrix} - \begin{bmatrix} 0 \\ \rho + \delta \mathcal{S}_g \end{bmatrix} \begin{bmatrix} \frac{K}{p}\frac{\partial p}{\partial K} & \frac{q}{p}\frac{\partial p}{\partial q} \end{bmatrix}.$$

The elasticities of m and p with respect to (K,q) are those implied by (19), evaluated at the steady state. Not immediately apparent from this expression, but as already noted, $\partial \mathrm{D}K_t/\partial K_t + \partial \mathrm{D}q_t/\partial q_t = \rho$. It follows that the two eigenvalues of the Jacobian of the differential equation are given by $\lambda_\pm = (\rho/2) \pm \sqrt{(\rho/2)^2 + \mathcal{D}}$, where $\mathcal{D}$ is a determinant,

$$\mathcal{D} = \det \begin{bmatrix} \frac{K_t}{q_t}\frac{\partial \mathrm{D}q_t}{\partial K_t} & \frac{\partial \mathrm{D}q_t}{\partial q_t} \\ \frac{\partial \mathrm{D}K_t}{\partial K_t} & \frac{q_t}{K_t}\frac{\partial \mathrm{D}K_t}{\partial q_t} \end{bmatrix}.$$

If $\mathcal{D}$ is positive, then $\lambda_- < 0 < \lambda_+$, implying that the steady state is a saddle point. And $-\lambda_-$ and λ_+ are increasing functions of $\mathcal{D}$, with $\lambda_\pm \to 0$ as $\mathcal{D} \downarrow 0$. In fact, near zero, $\mathcal{D}/\rho$ is

[13] Together with the assumed smoothness of $U(K,M)$, this efficiency condition also imposes the restriction $\mathcal{C}_{KM} = \mathcal{C}_{MK} \times \delta \mathcal{S}_g/(\rho + \delta \mathcal{S}_g)$ on the curvature parameters.

a first-order approximation for $-\lambda_-$ (though an extremely poor one for reasonable values of ρ). So $\mathcal{D}/\rho$, if positive, is the approximate speed of convergence of this economy. One can verify that the slope of the resulting stable manifold is negative.

The determinant of the Jacobian does not change when a multiple of one row of the Jacobian is added to the other. For purposes of computing $\mathcal{D}$, we can therefore replace the coefficient $-\mathcal{C}_g$ in the above expression for the Jacobian by a zero. It is then immediate that

$$\mathcal{D} = (\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g \times \det \begin{bmatrix} \frac{K}{m}\frac{\partial m}{\partial K} & \frac{q}{m}\frac{\partial m}{\partial q} \\ \frac{K}{p}\frac{\partial p}{\partial K} & \frac{q}{p}\frac{\partial p}{\partial q} \end{bmatrix}. \tag{22}$$

Differentiating (19) gives

$$\begin{bmatrix} \frac{K}{m}\frac{\partial m}{\partial K} & \frac{q}{m}\frac{\partial m}{\partial q} \\ \frac{K}{p}\frac{\partial p}{\partial K} & \frac{q}{p}\frac{\partial p}{\partial q} \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \mathcal{C}_{KK} + \mathcal{C}_{KM} & 1 \end{bmatrix} + \begin{bmatrix} -1 \\ \mathcal{C}_{KM} \end{bmatrix} \frac{\begin{bmatrix} \mathcal{C}_{MM} + \mathcal{C}_{MK} & -1 \end{bmatrix}}{\mathcal{C}_g + \mathcal{C}_{MM}}. \tag{23}$$

Calculating the determinant of this matrix and combining the result with (22) proves the following proposition.

Proposition 6 *A steady state is a saddle point if and only if*

$$\mathcal{D} = (\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g \times \frac{\mathcal{C}_{KK} + \mathcal{C}_{KM} + \mathcal{C}_{MK} + \mathcal{C}_{MM}}{\mathcal{C}_g + \mathcal{C}_{MM}} \tag{24}$$

is strictly positive. Near such a steady state, the stable manifold has a negative slope.

Simply put, given $\mathcal{S}_g \in (0, 1]$ and $\mathcal{C}_g < \infty$, the steady state is a saddle point if and only if the sum of the curvature parameters of $U(K, M)$ is positive. By Proposition 5, this is precisely when the elasticity of the steady state capital stock with respect to ρ is negative.[14] In those circumstances, though, $\mathcal{D}$ can be made arbitrarily close to zero by taking $\mathcal{C}_g$ to be large enough—the definition of a steady state imposes no restrictions on this curvature. From (23), this makes m almost independent of (K, q), and then the zero convergence rate associated with Zipf's law and a constant $g(m)$ emerges.

Suppose now that the parameters happen to be such that (24) simplifies to $\mathcal{D} = (\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g$. Then the stable eigenvalue of this economy is $\lambda_- = -\delta\mathcal{S}_g$. This corresponds to the speed of convergence obtained in the Zipf limit of the basic example (14). So the factor that multiplies $(\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g$ on the right-hand side of (24) measures the extent to which the speed of convergence differs from that in an economy in which the input $M_t = m_t K_t$ used to replicate capital, whatever it is, happens to be completely inelastic at the steady

[14]This connection between saddle point stability and the dependence of a steady state on ρ is well known. For example, see Liviatan and Samuelson [1969] and Magill and Scheinkman [1979].

state. A direct calculation of the slope of M_t with respect to K_t, along the stable manifold, confirms that it is zero precisely when (24) implies $\mathcal{D} = (\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g$.

5.2.3 Two Further Decompositions of $\mathcal{D}$

The approximate speed $\mathcal{D}/\rho$ can be expressed in terms of two potentially observable elasticities. Note from (23) that $(q/m)\partial m/\partial q$ is equal to $1/(\mathcal{C}_g + \mathcal{C}_{MM})$. It follows that $\delta\mathcal{S}_g/(\mathcal{C}_g + \mathcal{C}_{MM})$ is the elasticity of $g(m)$ with respect to the shadow price q, holding fixed the capital stock. Propositions 5 and 6 can then be combined to imply

$$\frac{\mathcal{D}}{\rho} = \frac{q}{g(m)}\frac{\partial g(m)}{\partial q} \times \left(-\frac{\rho}{K}\frac{\mathrm{d}K}{\mathrm{d}\rho}\right)^{-1}. \tag{25}$$

This decomposes $\mathcal{D}/\rho$ into an instantaneous elasticity and a steady state elasticity. Intuitively, the approximate speed of convergence is slow if $g(m)$ is particularly inelastic with respect to the shadow price of capital, at a point in time, holding fixed the capital stock. It is also slow if small changes in the subjective discount rate imply large long-run changes in the stock of capital.

Observe that the first-order condition (16) for m_t can be written as $v_t = q_t\mathrm{D}g(m_t)$, where $v_t = -\mathrm{D}_2U(K_t, M_t)$. The latter condition defines a supply curve $M_t = S(K_t, v_t)$, with elasticities

$$\begin{bmatrix} \mathcal{E}_{S,K} & \mathcal{E}_{S,v} \end{bmatrix} = \frac{1}{\mathcal{C}_{MM}} \times \begin{bmatrix} -\mathcal{C}_{MK} & 1 \end{bmatrix}.$$

This can be used to rewrite the formula (24) for $\mathcal{D}$ as

$$\mathcal{D} = \frac{(\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g}{\frac{1}{1+\mathcal{E}_{S,v}} + \frac{\mathcal{E}_{S,v}}{1+\mathcal{E}_{S,v}} \times \mathcal{C}_g} \times \frac{\mathcal{C}_{KK} + \mathcal{C}_{KM} + \mathcal{C}_{MK} + \mathcal{C}_{MM}}{1 + \mathcal{C}_{MM}}. \tag{26}$$

In the two versions of this economy discussed next, Cobb-Douglas assumptions drastically simplify the second factor in this decomposition.

5.3 Back to the Specific Factor Economy

The economy introduced in Section 2 is a model of team production in which labor is a specific factor for the consumption sector. It corresponds to

$$U(K, M) = \max_{C,L} \left\{\ln(C) - V(L, \beta L + M) : C \leq F(K, L)\right\}. \tag{27}$$

This function is concave, increasing in K, and decreasing in M. The first-order and envelope conditions imply $\mathrm{D}_1U(K_t, m_tK_t) = \mathrm{D}_1F(K_t, L_t)/C_t$. Since q_t is the marginal utility weighted price of capital, this means that $p_t = \mathrm{D}_1U(K_t, m_tK_t)/q_t$ is the profit rate in the consumption sector—profit from the production of consumption, per unit of capital, and divided by the consumption price of capital. The price $v_t = -\mathrm{D}_2U(K_t, m_tK_t)$ is now the marginal utility weighted price of managerial services.

5.3.1 The Cobb-Douglas Case

If F is Cobb-Douglas, then (27) implies that $U(K, M) - (1 - \mathcal{S}_F)\ln(K)$ does not depend on K. This additive separability immediately yields $\mathcal{C}_{KM} = \mathcal{C}_{MK} = 0$ (which entails $\mathcal{E}_{S,K} = 0$), and the logarithmic term $(1 - \mathcal{S}_F)\ln(K)$ implies $\mathcal{C}_{KK} = 1$. So the first factor in (26) describes the speed of an economy with a Cobb-Douglas consumption sector. This first factor is automatically positive, and so the steady state is a saddle point.

When F is Cobb-Douglas, $\mathcal{C}_g \approx 1$ or a small $\mathcal{E}_{S,v}$ both imply $\mathcal{D} \approx (\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g$, resulting in the same speed as obtained for the Zipf limit of the basic example (14). If $\mathcal{C}_g < 1$, then the speed of convergence is increasing in the residual supply elasticity $\mathcal{E}_{S,v}$. But the reverse is true if $\mathcal{C}_g > 1$. In that case, an economy with an elastic supply of managerial services converges more slowly than the completely inelastic example described by (14). Whether a more elastic residual supply of managerial services helps or hurts the speed of convergence (in other words, whether it tends to make $M = mK$ decreasing or increasing in K along the stable manifold) all depends on whether the factor payments from replication that accrue to managers, $\mathrm{D}g(m)m$, are an increasing or a decreasing function of m.

5.3.2 Separable Factor Supplies

The curvature parameters of $U(K, M)$ simplify when $V(\cdot, \cdot)$ is additively separable. Equation (38) in Appendix B gives these curvature parameters in terms of $(\mathcal{S}_F, \mathcal{C}_F)$, the factor supply elasticities $\mathcal{E}_L$ and $\mathcal{E}_M$, the share $\beta v/(\beta v + w)$ of managers in the cost of a team producing consumption, and the fraction $\beta L/(\beta L + M) = \beta\mathcal{L}(w)/\mathcal{M}(v)$ of managerial services used for supervision.

If F is again assumed to be Cobb-Douglas (so that $\mathcal{C}_F = 1 - \mathcal{S}_F$), then an easy benchmark emerges when $\mathcal{E}_L = \infty$ and $\mathcal{E}_M = 0$. The residual supply elasticity $\mathcal{E}_{S,v}$ of managerial services is then driven entirely by managerial task shifting. Specifically, $\mathcal{E}_{S,v} = 1/\mathcal{C}_{MM}$ and (38) imply

$$\mathcal{E}_{S,v} = \frac{\beta v}{w + \beta v}\frac{\beta L}{M}.$$

This says that the residual supply of managerial services available to produce new capital is particularly elastic when managers are a significant cost component of a team producing consumption, and when overseeing workers is an important component of the total supply of managerial services. If $\mathcal{C}_g > 1$, this implies slow convergence.

More generally, the assumption that F has an elasticity of substitution below one is sufficient for saddle point stability, and one can prove the following proposition.

Proposition 7 *When the supplies of labor and managerial services are separable, $\mathcal{C}_F \geq 1 - \mathcal{S}_F$ implies that $\mathcal{D}$ is positive and increasing in $\mathcal{C}_F$.*

Intuitively, more curvature in F hastens the reallocation of managerial services from producing consumption to producing new capital when the capital stock is low. The proof is a somewhat elaborate calculation, based on the curvature parameters reported in (38), that shows that both $\mathcal{C}_{MM}$ and $(\mathcal{C}_{KK} + \mathcal{C}_{KM} + \mathcal{C}_{MK})/\mathcal{C}_{MM}$ are increasing in $\mathcal{C}_F \geq 1 - \mathcal{S}_F$. It is possible to have a steady state that is not a saddle point when, instead, $\mathcal{C}_F$ is close enough to zero.

5.4 Conventional Adjustment Costs

The conventional model of adjustment costs is $\mathrm{D}K_t = (g_t - \delta)K_t$ together with $C_t + a(g_t)K_t = F(K_t, L_t)$, where $a(\cdot)$ is increasing and convex (Lucas [1967], Hayashi [1982], Abel and Blanchard [1983]). Of course, this is the same as $\mathrm{D}K_t = (g(m_t) - \delta)K_t$ together with $C_t + m_t K_t = F(K_t, L_t)$ and $g(\cdot)$ increasing and concave. It will be of interest to also allow for the joint production of C_t and the investment good $M_t = m_t K_t$. To this end, consider

$$U(K, M) = \max_{C,L} \left\{ \ln(C) - V(L) : H(M, C) \leq F(K, L) \right\}, \tag{28}$$

where both $V(\cdot)$ and $H(\cdot,\cdot)$ are increasing and convex, and $H(\cdot,\cdot)$ exhibits constant returns to scale.[15]

The usual interpretation is that K is physical capital, but here it could be a form of organization capital that is jointly produced with consumption. In particular, organization capital can be discrete, with every unit replicated randomly at an average rate $g(m)$ when used together with l units of labor to produce c units of consumption, subject to the constraint $H(m, c) \leq F(1, l)$. If L is taken to be a composite of labor and managerial services, then the assumption implicit in (28) is that there is no special role for managerial services in producing organization capital. In the competitive equilibrium,

[15] Beaudry and Portier [2007] emphasize the importance of complementarities between M and C. More on this below. They trace this specification back to Sims [1989].

$\widetilde{v}_t = \mathrm{D}_1 H(M_t, C_t)/\mathrm{D}_2 H(M_t, C_t)$ is the price of the investment good in units of consumption, and so $\widetilde{v}_t/C_t = -\mathrm{D}_2 U(K_t, M_t) = v_t$ is the marginal utility weighted price of the investment good.

Write $\mathcal{S}_H$ for the expenditure share of consumption in intermediate output $H(M, C) = F(K, L)$, and $\mathcal{C}_H = \mathrm{D}_{22} H(M, C) C/\mathrm{D}_2 H(M, C)$ for the curvature of H with respect to consumption (this sign convention and the convexity of H imply that $\mathcal{C}_H$ is non-negative). The curvature parameters of $U(K, M)$ can be expressed in terms of $(\mathcal{S}_F, \mathcal{C}_F)$, $(\mathcal{S}_H, \mathcal{C}_H)$, and the Frisch labor supply elasticity $\mathcal{E}_L$. The result is (39) in Appendix B, and the decomposition (26) then becomes

$$\mathcal{D} = \frac{(\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g}{\frac{1}{1+\mathcal{E}_{S,v}} + \frac{\mathcal{E}_{S,v}}{1+\mathcal{E}_{S,v}} \times \mathcal{C}_g} \times \frac{\left(\frac{\mathcal{C}_F}{1-\mathcal{S}_F} + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}\right)\left(\frac{1}{\mathcal{E}_L} + 1\right)}{\frac{1}{\mathcal{S}_H}\left(1 + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}\right)\left(1 + \frac{1}{\mathcal{S}_F}\left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right)\right)}, \tag{29}$$

where $\mathcal{E}_{S,v} = 1/\mathcal{C}_{MM}$ is given by

$$\mathcal{E}_{S,v} = \frac{\frac{1}{\mathcal{S}_F}\left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right) + \frac{1+\mathcal{C}_H}{\mathcal{S}_H}}{\frac{1}{\mathcal{S}_F}\left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right)\frac{1}{\mathcal{S}_H}\left(1 - \mathcal{S}_H + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}\right) + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}}.$$

The speed parameter $\mathcal{D}$ given in (29) is automatically positive, without any assumptions on F other than that the labor share is strictly less than 1 and the curvature of $F(1, \cdot)$ is strictly positive. When H is linear, one can verify that more curvature in F speeds up the economy, as it does in the specific factor economy.

Imposing $\mathrm{D}K_t = 0$ and $\mathrm{D}q_t = 0$ in (17)-(19) gives a steady state condition that can be used to relate the consumption share $\mathcal{S}_H$ to the labor share $\mathcal{S}_F$ and the factor share of the investment good in replication $\mathcal{S}_g$,

$$\mathcal{S}_H = \frac{\rho + \delta\mathcal{S}_g\mathcal{S}_F}{\rho + \delta\mathcal{S}_g} \in (\mathcal{S}_F, 1). \tag{30}$$

When F is Cobb-Douglas, the second factor in (29) simplifies to $\mathcal{S}_H\mathcal{S}_F$. Using the steady state condition (30) to eliminate $\mathcal{S}_H$ then has the effect of replacing the factor $(\rho + \delta\mathcal{S}_g)\delta\mathcal{S}_g$ on the right-hand side of (29) by $(\rho + \delta\mathcal{S}_g\mathcal{S}_F)\delta\mathcal{S}_g\mathcal{S}_F$. This implies $\mathcal{D} = (\rho + \delta\mathcal{S}_g\mathcal{S}_F)\,\delta\mathcal{S}_g\mathcal{S}_F$ if $\mathcal{C}_g = 1$ as well, and then the exact speed is $\delta\mathcal{S}_g\mathcal{S}_F$. As in the specific factor economy, this matches the Zipf limit of the basic example (14), but with the labor share $\mathcal{S}_g\mathcal{S}_F$ of the integrated technology $Kg(F(K, L)/K)$ replacing the factor share $\mathcal{S}_g$.

5.4.1 The Cass-Koopmans Economy

In the Cass-Koopmans economy, $H(M,C) = M + C$ and $g(m) = m$.[16] This implies $\mathcal{C}_H = 0$, $\mathcal{C}_g = 0$ and $\mathcal{S}_g = 1$. Using (30), it is not difficult to show that (29) reduces to

$$\mathcal{D} = \frac{(\rho + \delta\mathcal{S}_F)\delta\mathcal{S}_F}{\frac{1}{1+\mathcal{E}_L} + \frac{\mathcal{E}_L}{1+\mathcal{E}_L} \times \mathcal{C}_F} \times \frac{1}{1-\mathcal{S}_H}\frac{\mathcal{C}_F}{1-\mathcal{S}_F}, \tag{31}$$

with a consumption rate given by $\mathcal{S}_H = (\rho + \delta\mathcal{S}_F)/(\rho + \delta) \in (\mathcal{S}_F, 1)$. The similarity between the leading factors of (26) and (31) is apparent. Suppose F is Cobb-Douglas in both economies, and $g(\cdot) = F(1,\cdot)$ in the specific factor economy. Then the two speed formulas (26) and (31) only differ by a factor $1/(1-\mathcal{S}_H) > 1$, and by the appearance of $\mathcal{E}_L$ in the formula for the Cass-Koopmans economy versus $\mathcal{E}_{S,v}$ in the formula for the specific factor economy. Since the Cobb-Douglas assumption means that $\mathcal{C}_F = 1 - \mathcal{S}_F \in (0,1)$, this implies a faster rate of convergence for the Cass-Koopmans economy if $\mathcal{E}_L > \mathcal{E}_{S,v}$.

A Back-of-the-Envelope Comparison An easy estimate of the speed of this economy is obtained by taking F to be Cobb-Douglas, so that $\mathcal{C}_F = 1 - \mathcal{S}_F$, and assuming that $\mathcal{E}_L = \infty$. This implies an exact speed of convergence $(\rho + \delta)\mathcal{S}_F/(1 - \mathcal{S}_F)$. The parameter values $\rho = 0.04$, $\delta = 0.10$, and $\mathcal{S}_F = 0.7$ then imply a half life of just over 2.1 years. The consumption rate is close to 80%, and so $1/(1-\mathcal{S}_H) \approx 5$. Omitting the factor $1/(1-\mathcal{S}_H)$ in (31) therefore drastically lowers $\mathcal{D}$. As a result, the half life for the corresponding specific factor economy with F Cobb-Douglas, $\rho = 0.04$, $\delta = 0.10$, $\mathcal{S}_g = 1 - \mathcal{C}_g = 0.7$ and $\mathcal{E}_M = \infty$ (to ensure $\mathcal{E}_{S,v} = 1/\mathcal{C}_{MM} = \infty$ for this comparison) is 4.9 years. From $\mathcal{E}_{S,v} = 1/\mathcal{C}_{MM}$ and (38), one can infer that this specific factor economy will have a still longer half-life when $\mathcal{E}_M < \infty$.

5.5 The Crucial Distinction: How C and L Respond to q

From Proposition 6 we know that taking $\mathcal{C}_g$ to be large can account for slow aggregate convergence rates. The details of $U(K,M)$ do not matter, as long as the sum of its curvature parameters is positive. In particular, it does not matter if the cost of accumulating more capital is managerial services or consumption goods. In either case, raising $\mathcal{C}_g$ slows

[16] Hopenhayn and Rogerson [1993] serves as the starting point for much of the quantitative literature on firm dynamics and the aggregate economy. It is worth emphasizing that their benchmark model, which has entry costs in units of final output, reduces to this Cass-Koopmans economy when there are no idiosyncratic productivity shocks and firms are assumed to exit randomly. In such a model, all investment in organization capital is via entry, not replication.

the economy down. But the details of $U(K, M)$ do very much matter for determining how employment and consumption respond to shocks.

For example, consider an unforeseen and permanent reduction in the subjective discount rate ρ. Proposition 5 says that this implies an increase in the steady state capital stock K. The steady state condition $\mathrm{D}K_t = 0$ does not depend on ρ. Saddle point stability implies that the stable manifold is downward sloping and must cut the $\mathrm{D}K_t = 0$ curve from above (with K on the horizontal axis and q on the vertical axis). It follows that q must jump up on impact. The first-order condition $q\mathrm{D}g(m) = -\mathrm{D}_2U(K, mK)$ then implies that $M = mK$ jumps up as well.

In the case of conventional adjustment costs, the effects, on impact, of such an increase in M are given by

$$\begin{bmatrix} \frac{M}{L}\frac{\partial L}{\partial M} \\ \frac{M}{C}\frac{\partial C}{\partial M} \end{bmatrix} = \frac{1-\mathcal{S}_H}{\mathcal{S}_H} \frac{1}{\frac{1}{\mathcal{S}_F}\left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right) + \frac{1+\mathcal{C}_H}{\mathcal{S}_H}} \begin{bmatrix} \frac{1}{\mathcal{S}_F}\left(1 + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}\right) \\ -\frac{1}{\mathcal{S}_F}\left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right) + \frac{\mathcal{C}_H}{1-\mathcal{S}_H} \end{bmatrix}. \tag{32}$$

These elasticities are determined by the version of $U(K, M)$ given in (28). If $H(M, C)$ is linear, the increase in M is accommodated in part by more employment, and in part by lower consumption. The increase in employment looks like a boom, but the associated decline in consumption looks like a recession. In a setting with a different type of news shock, Beaudry and Portier [2007] argued that the curvature of $H(M, C)$ must be positive for investment, employment and consumption to move together in this economy. The elasticities (32) confirm this: co-movement occurs only when the elasticity of substitution $(1 - \mathcal{S}_H)/\mathcal{C}_H$ between M and C is low enough—an intuitive but extreme example would be $H(M, C) = \max\{M, C\}$. The upward jump in q then produces an unambiguous and immediate boom: investment, employment and consumption all jump up. One can verify that the investment expenditures $\mathrm{D}_1H(M, C)M/\mathrm{D}_2H(M, C)$ jump up as well.

The core argument in this paper is, instead, that a shock that makes households act more patiently naturally produces a recession. This is what happens when labor is a specific factor for the consumption sector and requires managerial supervision. The effects of an upward jump in $M = mK$ on labor and consumption are given by

$$\begin{bmatrix} \frac{M}{L}\frac{\partial L}{\partial M} \\ \frac{M}{C}\frac{\partial C}{\partial M} \end{bmatrix} = -\frac{\frac{\beta v}{w+\beta v}\frac{M}{\beta L+M}\frac{1}{\mathcal{E}_M}}{\mathcal{S}_F + \mathcal{C}_F + \frac{w}{w+\beta v}\frac{1}{\mathcal{E}_L} + \frac{\beta v}{w+\beta v}\frac{\beta L}{\beta L+M}\frac{1}{\mathcal{E}_M}} \begin{bmatrix} 1 \\ \mathcal{S}_F \end{bmatrix}. \tag{33}$$

These elasticities follow from the version of $U(K, M)$ given in (27), with separable factor supplies. Because of this separability, an increase in M would have no effect on L and C if $\beta = 0$. But when $\beta > 0$, an increase in managerial services used to produce new organization capital comes at the cost of managerial services that can be assigned to supervise workers producing consumption. As expected, $\partial L/\partial M = -1/\beta$ when $\mathcal{E}_M = 0$. In any case, L and C automatically move together when $\beta > 0$, but in the opposite direction of investment in new organization capital. The fact that $v = -\mathrm{D}_2U(K, M)$ together with $\partial M/\partial q > 0$ and $\mathcal{C}_{MM} > 0$ implies that v rises with an increase in q. It follows that $\mathcal{M}(v)$ also rises with q and M, unless $\mathcal{M}(v)$ is completely inelastic. In contrast to worker employment, managerial employment does increase with q and M. But if the supply of managerial services is relatively inelastic and workers make up the bulk of employment, then consumption and aggregate employment will both decline.[17]

6 A Bursting Bubble Implies a Recession

An obvious objection to a belief shock interpretation of recessions along these lines is that recessions are invariably associated with declines in asset values—and by itself, an upward jump in q_t causes q_tK_t to jump up.[18] But observed asset values need not be a measurement of only q_tK_t, and a sudden drop in subject discount rates is not the only way in which households can be induced to act more patiently.

To describe a more plausible recession scenario, suppose households own not only organization capital but also an outside asset that initially produces no dividends. Households believe the asset will deliver a constant flow of consumption $X > 0$, indefinitely, starting at some random time that is exponentially distributed with mean $1/\theta \in (0, \infty)$. Then, unforeseen, the arrival rate θ drops to zero.[19] Such a bursting bubble can easily account for a decline in asset values if these asset values reflect what happens to portfolios of q_tK_t and the outside asset.

How does such a negative wealth shock affect consumption, employment, and wages? How does the economy converge to its new steady state? What follows is an answer to these questions for the Zipf limiting economy with separable supplies of labor and managerial services, augmented with an outside asset.

[17] If $\beta = 0$ and the disutility $V(L, M)$ is replaced by $V(L + M)$, then $\partial L/\partial M < 0$ and $\partial C/\partial M < 0$ as well. But this also implies $\partial(L + M)/\partial M > 0$ as long as $L > M$, and so aggregate employment and consumption would move in opposite directions.

[18] A destruction of organization capital, which also raises q_t but lowers K_t at the same time, can account for declining asset values if the elasticity of the stable manifold is in $(-1, 0)$. Recall also that the price of capital in units of consumption is q_tC_t, and consumption drops on impact.

[19] One can devise a signal structure in which Bayesian consumers foresee the possibility that θ is zero rather than positive, but come to believe with great certainty, and mistakenly, that θ is positive. The events that then unfold are exactly the same as what happens when, unforeseen, $\theta > 0$ drops to zero. See Luttmer [2013].

6.1 The Ex Post Equilibrium Conditions

Consider the ex post economy that arises when it is known that the outside asset produces a flow of $X \geq 0$ units of consumption forever. So consumption is now $C_t = F(K_t, L_t) + X$. Because of this, the labor share function $A(\cdot)$ of F no longer suffices for describing the equilibrium. The capital stock evolves according to $\mathrm{D}K_t = K_t(g(m_t) - \delta)$, as before. Given a capital stock K_t and a marginal utility weighted price Q_t of a unit of organization capital, the first-order and market clearing conditions are

$$\beta v_t + w_t = \frac{\mathrm{D}_2 F(1, l_t)}{F(1, l_t)K_t + X}, \qquad v_t = Q_t \mathrm{D}g(m_t), \tag{34}$$

$$\mathcal{L}(w_t) = l_t K_t, \qquad \mathcal{M}(v_t) = (\beta l_t + m_t)K_t. \tag{35}$$

These conditions determine the inputs (l_t, m_t) and the factor prices (v_t, w_t) as a function of (K_t, Q_t). The implied marginal utility weighted consumption-sector profits are $\mathrm{D}_1 F(1, l_t)/(F(1, l_t)K_t + X)$ per unit of capital. The asset pricing equation for Q_t is therefore

$$\rho Q_t = \frac{\mathrm{D}_1 F(1, l_t)}{F(1, l_t)K_t + X} + Q_t(g(m_t) - \delta) - v_t m_t + \mathrm{D}Q_t.$$

Together with a transversality condition, these conditions determine a stable manifold $Q_t = Q(K_t)$ for this ex post economy. Of course, these equilibrium conditions are exactly those described in Section 2 if $X = 0$.

Across steady states of different ex post economies, variation in X creates wealth effects with the usual implications for consumption and labor supply. But the relation between X and the capital stock is ambiguous. It depends on, among other things, how elastic the supply of managerial services is.

Proposition 8 *Suppose that $\mathcal{L}(\cdot)$ is strictly upward sloping. Comparing ex post steady states, an increase in $X \geq 0$*

(i) *raises C and lowers $\mathcal{L}(w)$;*

(ii) *raises K and lowers Q if $\mathcal{M}(\cdot)$ is completely inelastic;*

(iii) *lowers Q and $\mathcal{M}(v)$ if $\mathcal{M}(\cdot)$ has a positive but finite slope;*

(iv) *reduces K and leaves Q unchanged if $\mathcal{M}(\cdot)$ is perfectly elastic.*

The proof is in Appendix C. In an economy with conventional adjustment costs, it is easy to see that an increase in X always crowds out capital. Here, on the other hand, the negative wealth effect on $\mathcal{L}(w)$ of an increase in X immediately implies an increase in K when $\mathcal{M}(\cdot)$ is inelastic, via the market clearing condition $\mathcal{M}(v) = \beta\mathcal{L}(w) + m_\infty K$.

6.2 A Bubble Crowds Out Organization Capital

The outside asset produces nothing in the ex ante economy. Given a capital stock K_t and a price of capital q_t, the first-order and market clearing conditions are exactly as in Section 2 (or as in the ex post economy with $X = 0$). But the asset pricing equation (7) for q_t must be modified to allow for the capital gain $Q(K_t) - q_t$ that will occur when the outside asset begins to generate the consumption flow $X > 0$. Since this capital gain arrives randomly at the rate θ, the ex ante asset pricing equation becomes

$$\rho q_t = \frac{\mathrm{D}_1 F(1, l_t)}{F(1, l_t) K_t} + q_t(g(m_t) - \delta) - v_t m_t + \mathrm{D}q_t + \theta(Q(K_t) - q_t), \tag{36}$$

where (l_t, m_t) and (v_t, w_t) are determined by first-order and market clearing conditions (the $X = 0$ version of (34)-(35), with the ex ante price q_t replacing the ex post price Q_t). The capital stock evolves according to $\mathrm{D}K_t = K_t(g(m_t) - \delta)$. The initial capital stock K_0 is given and a transversality condition pins down the initial value q_0.

The ex ante steady state is determined by the first-order and market clearing conditions together with the two steady state requirements $\mathrm{D}K_t = 0$ and $\mathrm{D}q_t = 0$. Beliefs affect the steady state only via the appearance of the ex post stable manifold $Q(\cdot)$ in the equation for $\mathrm{D}q_t$. To determine how the ex ante steady state depends on beliefs, it is useful to start with the steady state requirement that does not depend on beliefs.

Lemma 4 *The $\mathrm{D}K_t = 0$ condition determines q as an increasing function of K. The implied capital-labor ratio K/L is increasing in K as well.*

Proof The condition $\mathrm{D}K_t = 0$ implies $m_t = m_\infty$, and the first-order condition (16) then gives $q = -\mathrm{D}_2 U(K, m_\infty K)/\mathrm{D}g(m_\infty)$, with $U(\cdot, \cdot)$ as defined in (27). The optimality conditions for the L that attains $U(K, m_\infty K)$ can be written as

$$\left(\beta v + \mathcal{L}^{-1}(L)\right) L = A\left(\frac{K}{L}\right), \quad \mathcal{M}(v) = \beta L + m_\infty K.$$

Viewed as functions mapping L into v, the first curve is downward sloping and the second is upward sloping. Increasing K shifts both curves upwards. So an increase in K will raise $v = -\mathrm{D}_2 U(K, m_\infty K)$, and hence $q = v/\mathrm{D}g(m_\infty)$ as well. An increase in K must raise K/L, or else $\beta v = (A(K/L)/L) - \mathcal{L}^{-1}(L)$ would decline with K.■

When it is possible to rank the price of organization capital not only across ex post steady states (as in Proposition 8) but also across different ex post stable manifolds, a definite prediction emerges for the ex ante steady state capital stock.

Proposition 9 *Consider two alternative ex ante economies with ex post stable manifolds $Q^{(1)}(\cdot)$ and $Q^{(2)}(\cdot)$. Suppose $Q^{(1)}(K) < Q^{(2)}(K)$ over a range of K that covers the respective ex ante steady state capital stocks $K^{(1)}$ and $K^{(2)}$. Then $K^{(1)} < K^{(2)}$.*

Proof Impose the steady state condition $\mathrm{D}q_t = 0$ in (36) and use $v = q\mathrm{D}g(m_\infty)$ to infer that in any steady state

$$(\rho + \theta + \mathrm{D}g(m_\infty)m_\infty)q - \left(1 - A\left(\frac{K}{L}\right)\right)\frac{1}{K} = \theta Q(K). \tag{37}$$

Lemma 4 says that $\mathrm{D}K_t = 0$ gives q as an increasing function of K, and profits $(1 - A(K/L))/K$ as a decreasing function of K. Imposing $\mathrm{D}K_t = 0$ therefore makes the left-hand side of (37) an increasing function of K. The result follows from shifting $\theta Q(K)$ along the upward-sloping left-hand side of (37).■

Not surprisingly, if an economy can transition to a state in which capital is cheap, then there are only weak incentives to accumulate capital before the transition. Here, this means that optimistic beliefs about an outside asset crowd out organization capital, in spite of the ambiguous effects, highlighted in Proposition 8, of X on K in ex post steady states.

6.3 Phase Diagrams and Equilibrium Trajectories

Figures 2 and 3 show the phase diagrams for two economies in which the premise of Proposition 9 is satisfied: a high X implies a low price of organization capital, not only in the ex post steady states (as asserted by Proposition 8) but also along the ex post stable manifolds. Because of this ranking of ex post stable manifolds, Proposition 9 ensures that the stock of organization capital will be lower in the ex ante steady state than in the ex post steady state with $X = 0$. This means a steady state stock of organization capital that is lower in the ex ante steady state than in the ex post steady state that will eventually emerge after consumers realize that $\theta = 0$. These phase diagrams prove that q_t must jump up when consumers suddenly realize that $\theta = 0$. The elasticities (33) for this economy (which apply at any combination of K and q) imply that this results in an immediate drop in consumption and labor.

Figure 2 describes an economy with an inelastic supply of managerial services. As predicted by Proposition 8, the steady state capital stock in the ex post economy with $X > 0$ is larger than it is in the $X = 0$ ex post economy. Figure 3 shows what happens when $\mathcal{E}_M = 0.3$. This is far from the perfectly elastic case mentioned in Proposition 7, but elastic enough to reverse the relation between X and K across ex post steady states.

The ex ante steady states and ex post equilibrium trajectories are shown in Figures 4 and 5, for a belief shock in year 10 that is augmented with a 5% destruction of organization capital—imagine that the confusion associated with the realization of an essentially unforeseen contingency also causes mistakes (in the context of this economy) that lead to the loss of some organization capital. It takes roughly 3 years to recover from this destruction of capital (as indicated), and the subsequent equilibrium trajectories can be interpreted as the equilibrium trajectories that are implied by a belief shock alone.

It is worth emphasizing the co-movement exhibited by this economy during the long recovery. Figures 4 and 5 show that K_t, C_t, L_t, w_tC_t and v_tC_t all rise.[20] Worker and managerial wages move in opposite directions only on impact. They would move together even on impact if the only shock were a destruction of organization capital.

The half-lives for the two economies are very similar: 8.0 years in the economy with $\mathcal{E}_M = 0$, and 7.8 years in the economy with $\mathcal{E}_M = 0.3$. A comparison of Figures 4 and 5 shows that the elasticity of $\mathcal{M}(\cdot)$ does play a significant role in determining what happens to consumption and labor supply in the long run. When $\mathcal{M}(\cdot)$ is inelastic, consumption barely recovers and labor never recovers after the (permanent) belief shock to θ. Labor still does not fully recover when $\mathcal{E}_M = 0.3$, but consumption rises significantly above the ex ante steady state. It does so by enough that worker wages in units of consumption also rise above the ex ante steady state, even though the marginal utility weighted wages of workers do not.

[20]The definition (27) of $U(K, M)$ implies $\partial L/\partial K > 0$ if and only if $\mathcal{C}_F > 1 - \mathcal{S}_F$, and that condition more than suffices for $\partial C/\partial K > 0$. Together with (32), this proves that K, L and C co-move along the stable manifold.

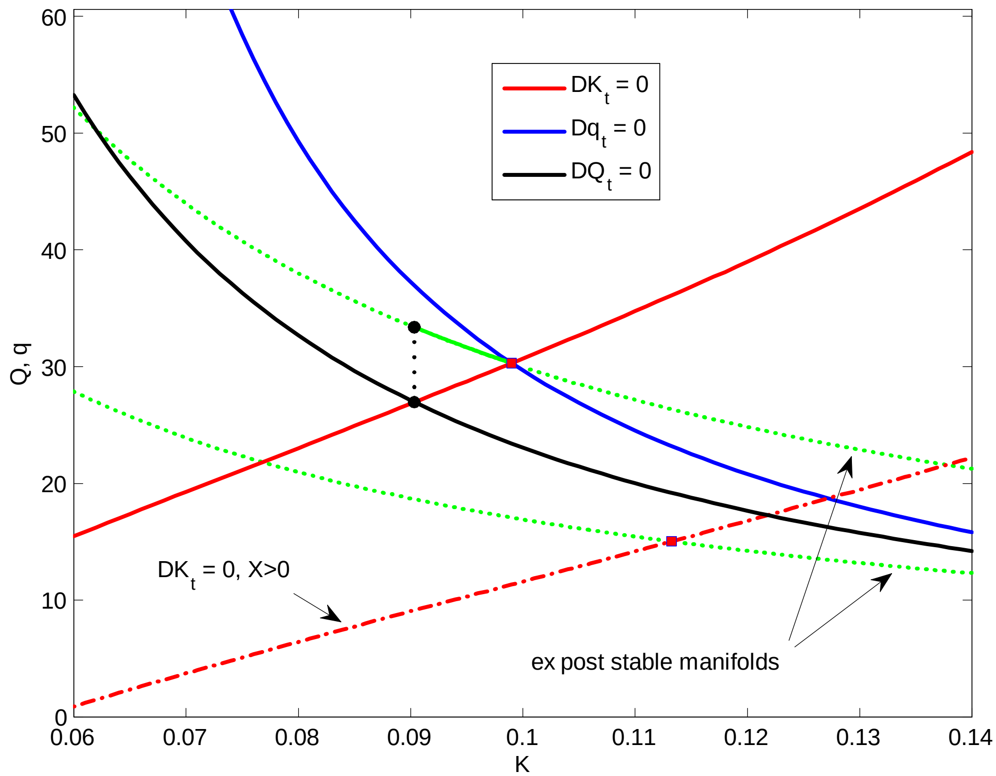

FIGURE 2 Phase Diagram for $\mathcal{E}_M = 0$

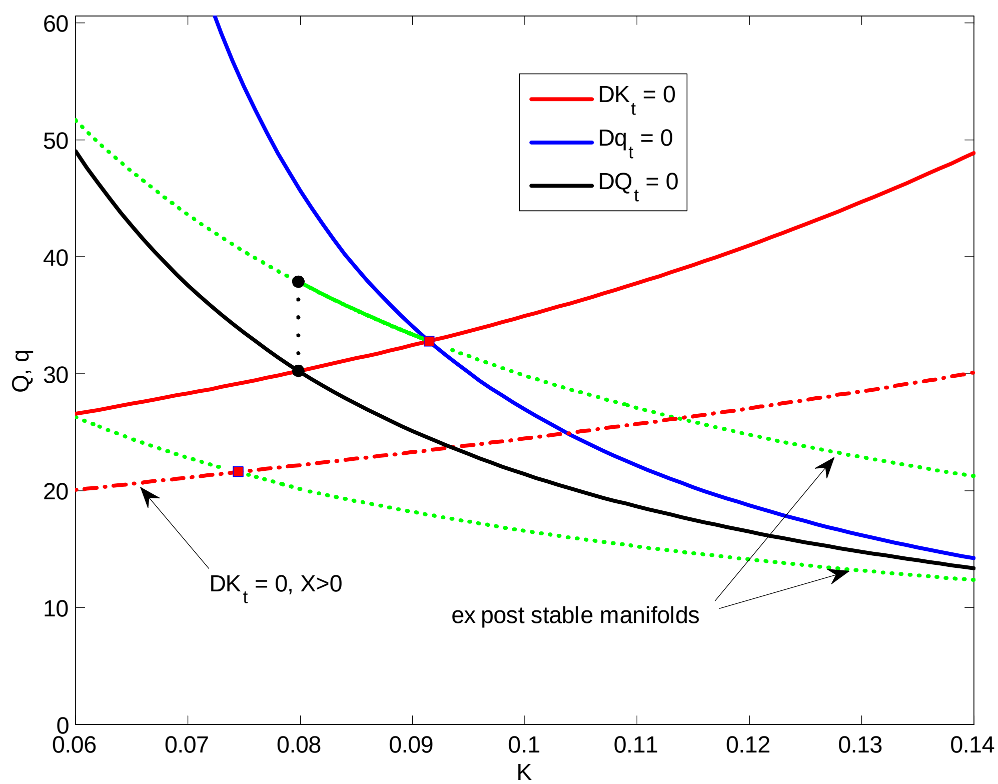

FIGURE 3 Phase Diagram for $\mathcal{E}_M = 0.3$

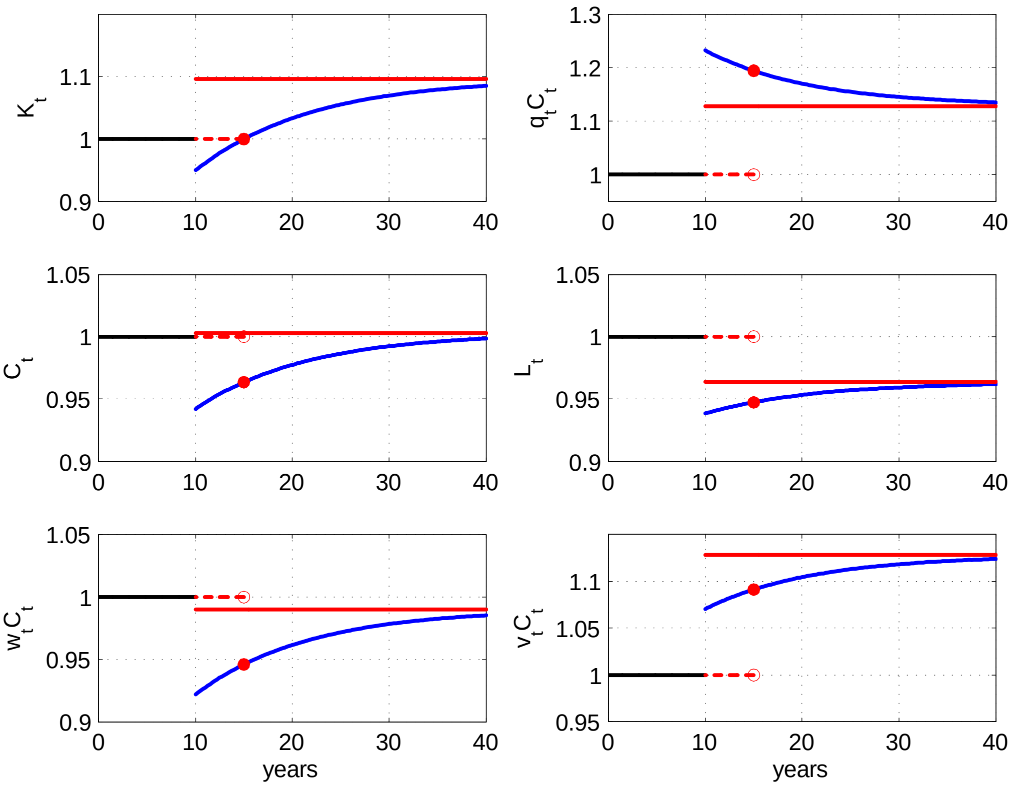

FIGURE 4 Equilibrium Trajectories for $\mathcal{E}_M = 0$

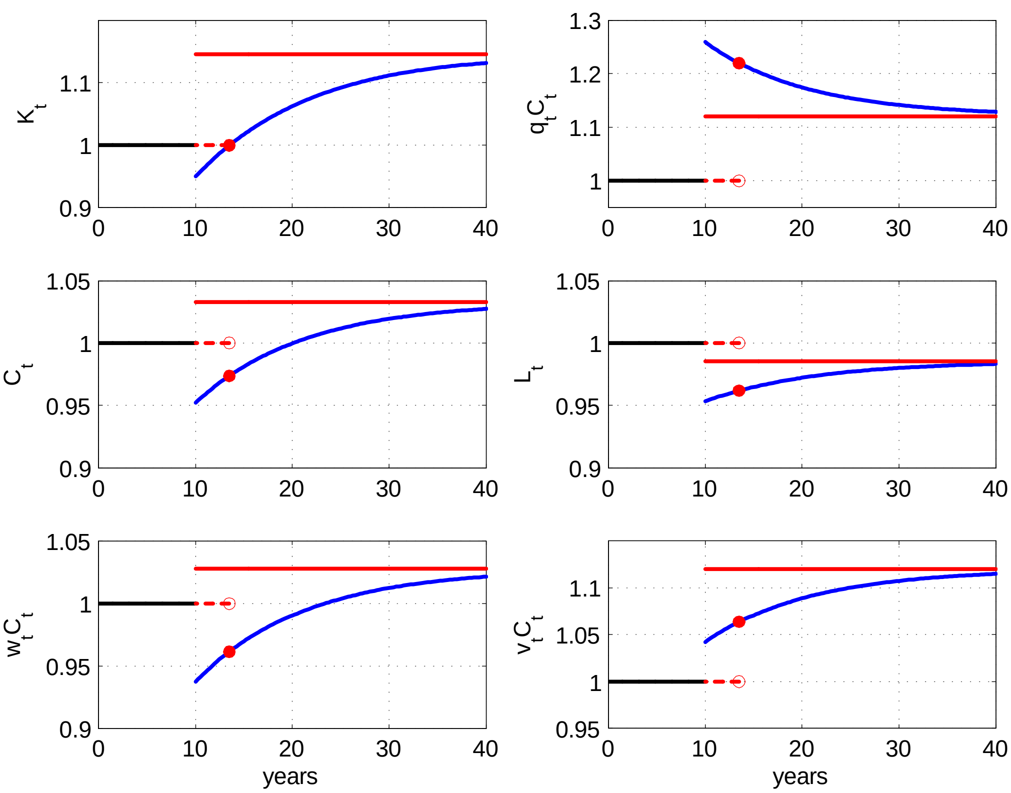

FIGURE 5 Equilibrium Trajectories for $\mathcal{E}_M = 0.3$.

6.4 Underlying Parameters and Further Implications

The subjective discount rate is assumed to be $\rho = 0.04$, and organization capital depreciates at the rate $\delta = 0.10$, per annum. Table 1 lists the other parameters that determine the speed of convergence in these two economies.

TABLE 1 Half-Life Parameters

$\mathcal{E}_L$	$\mathcal{E}_M$	$\mathcal{S}_F$	$\mathcal{C}_F$	$\mathcal{S}_g$	$\mathcal{C}_g$	$\frac{\beta L}{\beta L+M}$	$\frac{\beta v}{\beta v+w}$	`implied half-life`
3.0	$\{0.0, 0.30\}$	0.70	0.50	0.60	$0.\overline{66}$	0.70	0.60	$\{8.0, 7.8\}$

The implied half-lives are in line with what has happened in the US economy since 2008. Both $F(K, L)$ and $Kg(M/K)$ are taken to be CES production functions. The elasticity of substitution in the consumption sector is 0.6. Raising it to the Cobb-Douglas elasticity of 1 would add about two years to the half-lives of these economies. Note that supervision accounts for a large chunk of the supply of managerial services, and that supervision is an important component of the cost of a team of managers and workers. From (38), the residual supply elasticity of managerial services $\mathcal{E}_{S,v} = 1/\mathcal{C}_{MM}$ equals 1.05 at $\mathcal{E}_M = 0$, when managerial task shifting alone is what determines this elasticity. This residual supply elasticity rises to 2.05 in the economy with $\mathcal{E}_M = 0.3$. As Table 1 shows, the effect of this on the speed of convergence is very modest.

TABLE 2 The Decomposition of $\mathcal{D}/\rho$

		$\frac{q}{g(m)}\frac{\partial g(m)}{\partial q}$	$\frac{\rho}{K}\frac{\mathrm{d}K}{\mathrm{d}\rho}$
Specific factor	$\mathcal{E}_M = 0$	0.04	-0.14
	$\mathcal{E}_M = 0.3$	0.05	-0.18
Cass-Koopmans	$\mathcal{E}_L = 0$	0.37	-0.41
	$\mathcal{E}_L = \infty$	1.46	-0.49

Proposition 5, together with the curvature parameters reported in Appendix B, can be used to calculate the elements of the decomposition (25) of $\mathcal{D}/\rho$. The results are reported in Table 2, together with the corresponding elasticities for a Cass-Koopmans economy with the same ρ, δ, and $\mathcal{S}_F$, and with $\mathcal{C}_F = 1 - \mathcal{S}_F$. The elasticity of the long-run capital stock with respect to ρ in the specific factor economy is much smaller than in the Cass-Koopmans economy. In the specific factor economy, the effect of a capital income tax on the steady state capital stock would be only about a third of what it is in the standard model. The decomposition (25) suggests more rapid convergence in the specific factor economy too, but the elasticity of $g(m)$ with respect to q is also much smaller than in the Cass-Koopmans economy.

The arrival rate θ is taken to be 10%, and the outside source of consumption X is equal to 60% of the steady state level of consumption in the ex post economy with no outside source. Although there is an off setting effect on $F(K, L)$, this implies differences in ex post consumption levels between these two ex post economies that are very large. But because the pie-in-the-sky outside source of consumption is expected to arrive at only a 10% annual rate, the shocks to total wealth that happen when consumers realize that $\theta = 0$ are much more modest. Since utility is logarithmic, wealth is simply C_t/ρ, and so the magnitudes of these wealth shocks are implied by the consumption trajectories shown in Figures 4 and 5.

In both economies, the drop in θ lowers interest rates,[21] from around 6% ($6.4\,\%$ and 5.4%, respectively, when $\mathcal{E}_M = 0$ and $\mathcal{E}_M = 0.3$) to only slightly above the steady state value of 4%. The recovery in consumption is spread out over decades, and so it adds very little to interest rates during this recovery. Consumption in the ex ante steady state is constant, but consumers expect an improvement. An econometrician with limited data would be confronted with a Peso problem and could mistakenly infer a large drop in the subjective discount rate ρ when θ drops to zero.

The supplies of labor and managerial services are generated by assuming that 90% of the households can supply only labor, and that the remaining 10% can supply only managerial services. Both types of households make a discrete choice that is governed by distributions over (h_u, h_w) and (h_u, h_v) that are both independent Fréchet. For households who supply only labor, the probabilities of $\{h : h_u \geq h\}$ and $\{h : h_w \geq h\}$ behave like $h^{-\sigma}$, with $\sigma = 11$, and the scale parameters are restricted to ensure a labor force participation rate of 70%. This implies an aggregate labor supply elasticity equal to $\mathcal{E}_L = (1 - 0.7) \times (11 - 1) = 3.0$.[22] In the scenario of elastically supplied managerial services, the tail probabilities for households who can supply managerial services behave like h^{-4}. So abilities are more dispersed among households who can supply managerial services than among households who can supply only labor. The participation rate for potential managers is set at 90%. The resulting supply elasticity is then only $\mathcal{E}_M = (1 - 0.9) \times (4 - 1) = 0.3$.

In both economies, worker wages fall and managerial wages rise when θ drops to zero. This is similar to the Stolper-Samuelson relation between output and factor prices. In the economy with $\mathcal{E}_M = 0.3$, the increase in the marginal utility weighted factor price v_t implicit in Figure 5 is about 9.6% on impact and 8.5% in the long run. So the supply of

[21] In the ex ante steady state, the risk-free rate is $r = \rho + \theta\,(1 - C/C[K])$, where C and K are consumption and capital in the ex ante steady state, and $C[K]$ is consumption in the ex post economy with $X > 0$.

[22] The labor supply curve is of the form $\mathcal{L}(w) \propto [\mathcal{P}(w)]^{1-1/\sigma}$, where $\mathcal{P}(w) = (\mathcal{A}w)^\sigma/[1 + (\mathcal{A}w)^\sigma]$ is the participation rate and $\mathcal{A}$ is a scale parameter. This implies $\mathcal{E}_L = (1 - \mathcal{P}(w))(\sigma - 1)$.

managerial services rises by about 2.9% on impact and slightly less in the long run.

TABLE 3 Participation Rates

		ex ante	impact	long run
	workers	72.9	68.0	70.0
$\mathcal{E}_M = 0$	managers	100.0	100.0	100.0
	all	75.6	71.2	73.0
	workers	71.1	67.5	70.0
$\mathcal{E}_M = 0.3$	managers	86.7	90.3	90.0
	all	72.7	69.8	72.0

As shown in Table 3, because marginal managers are much less productive than the average employed manager, the participation rate among potential managers increases by more. But since this increase in participation applies to only 10% of all households, the effect on the aggregate participation rate is relatively small.[23] This effect is absent in the economy with $\mathcal{E}_M = 0$, resulting in the larger drop in aggregate participation reported in Table 3, one that matches the drop in the US participation rate seen during 2008-2009.

7 Concluding Remarks

Sims [1998] argued that many of the microeconomic stories underlying adjustment cost models are implausible. In the models he describes, capital accumulation amounts to opening a can of generic output, consuming some of it, and then using the rest to add to the capital stock. Adjustment costs arise because the amount of output it takes to augment the capital stock is increasing and strictly convex in the rate at which capital is being accumulated. It is hard indeed to tell plausible microeconomic stories for such adjustment costs.

As Prescott and Visscher [1980] suggested long ago, the evidence on how firms grow is hard to interpret without thinking about the time-consuming process of accumulating some form of organization capital. In the model presented here, organization capital cannot be created using cans of generic output. Instead, it takes managers, and these managers have to optimally allocate their time between two tasks: producing new organization capital, and overseeing workers as they produce consumption. This distinction is critical for how negative news about household wealth affects aggregate employment.

Along multiple dimensions, the model in this paper is highly rudimentary. It has no

[23] If households could also switch from supplying labor to supplying managerial services, as in non-separable versions of (1)-(2), then this positive effect on aggregate participation could be even smaller.

physical capital, no labor market frictions, no technical progress, and markets are complete. The contention is that some form of organization capital is the most important state variable that governs firm growth and aggregate convergence rates. The model captures the fact that firm growth is hard to predict based on size alone, and it generates a realistic distribution for the employment size of firms. A negative wealth shock generates a recession in aggregate consumption and employment. And the half-life of the subsequent recovery is similar to what has been observed following recent recessions. The fact that the firm size distribution is close to Zipf's law is a strong indication that most organization capital accumulation comes from incumbent firms expanding, and not from entry. Even if entry rates respond elastically to the state of the economy, the fact that entrants are small means that entry can do little to speed up a recovery. The model can easily be extended to allow for the costly mothballing of organization capital. Transitory shocks that are also perceived as such can then lead to speedy recoveries.

Because the economy has only one type of organization capital, there is only one aggregate state variable. As emphasized in Luttmer [2011], heterogeneity in the quality of organization capital is essential to account for the relatively young age of the very large firms that employ so much of the aggregate labor force. Large firms often have histories of persistent double digit growth lasting for multiple decades, and these growth rates do not appear to vary significantly across the business cycle.[24] The result is a type of systematic reallocation of employment that is abstracted from here. Understanding the aggregate dynamics of economies with richer forms of firm heterogeneity remains an important task for further research.

A Proof of Lemma 3

The conditions (9)-(12) for a steady state in the Cobb-Douglas economy imply a function $\Sigma : \alpha \mapsto S(v)$ that relates the labor share parameter α to the steady state value of the residual supply of managerial services. The argument given in the text leading up to Lemma 3 shows that this is a decreasing function. Given α and $\Sigma(\alpha)$, one can infer factor prices from

$$\alpha = (\beta v + w)\,\mathcal{L}(v, w), \quad \Sigma(\alpha) = \mathcal{M}(v, w) - \beta\mathcal{L}(v, w).$$

[24]See Luttmer [2012]. The persistent rapid growth of some firms led Birch [1979] to introduce the term "gazelles" to describe such firms. There are also many small firms that hardly grow (Hurst and Pugsley [2011]). Moscarini and Postel-Vinay [2012] examine the contributions of large and small employers to employment growth at different stages of the business cycle. Haltiwanger, Jarmin and Miranda [2013] emphasize the importance of firm age. In Luttmer [2011], size and age are imperfect indicators, and some measure of firm quality is essential for explaining how firms grow.

Differentiating this system with respect to α gives

$$\begin{aligned}
\frac{\alpha}{\mathcal{L}(v,w)}\frac{\mathrm{d}\mathcal{L}(v,w)}{\mathrm{d}\alpha} &= \begin{bmatrix} \mathcal{E}_{L,v} & \mathcal{E}_{L,w} \end{bmatrix} \begin{bmatrix} \frac{\alpha}{v}\frac{\partial v}{\partial \alpha} \\ \frac{\alpha}{w}\frac{\partial w}{\partial \alpha} \end{bmatrix} \\
&= \begin{bmatrix} \mathcal{E}_{L,v} & \mathcal{E}_{L,w} \end{bmatrix} \begin{bmatrix} \frac{\beta v}{\beta v+w} + \mathcal{E}_{L,v} & \frac{w}{\beta v+w} + \mathcal{E}_{L,w} \\ \frac{\mathcal{E}_{M,v} - \frac{\beta\mathcal{L}}{\mathcal{M}}\times\mathcal{E}_{L,v}}{1-\frac{\beta\mathcal{L}}{\mathcal{M}}} & \frac{\mathcal{E}_{M,w} - \frac{\beta\mathcal{L}}{\mathcal{M}}\times\mathcal{E}_{L,w}}{1-\frac{\beta\mathcal{L}}{\mathcal{M}}} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \frac{\mathrm{D}\Sigma(\alpha)\alpha}{\Sigma(\alpha)} \end{bmatrix},
\end{aligned}$$

where $[\mathcal{E}_{L,v}, \mathcal{E}_{L,w}]$ and $[\mathcal{E}_{M,v}, \mathcal{E}_{M,w}]$ are the elasticities of $\mathcal{L}(v,w)$ and $\mathcal{M}(v,w)$, respectively. Combining $\mathrm{D}\Sigma(\alpha) < 0$ with the fact that own price elasticities dominate cross price elasticities (Lemma 1), one can verify that this is positive. It follows that the steady state capital-labor ratio is decreasing in α.

B The Curvature Parameters of $U(K, M)$

Let $\mathcal{E}_A = \mathcal{C}_F - (1-\mathcal{S}_F)$, the elasticity of $A(\cdot)$. With an additively separable disutility $V(\cdot,\cdot)$, the curvature parameters of $U(K,M)$ as defined in (27) are

$$\begin{aligned}
\begin{bmatrix} \mathcal{C}_{KK} & \mathcal{C}_{KM} \\ \mathcal{C}_{MK} & \mathcal{C}_{MM} \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \left(\mathcal{E}_A + 1 + \frac{w}{w+\beta v}\frac{1}{\mathcal{E}_L} + \frac{\beta v}{w+\beta v}\frac{\beta L}{\beta L + M}\frac{1}{\mathcal{E}_M}\right)^{-1} \\
&\left\{\left(1 + \frac{w}{w+\beta v}\frac{1}{\mathcal{E}_L}\right)\begin{bmatrix} \frac{\mathcal{S}_F\mathcal{E}_A}{1-\mathcal{S}_F} & 0 \\ 0 & \frac{M}{\beta L+M}\frac{1}{\mathcal{E}_M} \end{bmatrix} + \frac{\mathcal{E}_A}{\mathcal{E}_M}\begin{bmatrix} \frac{\mathcal{S}_F}{1-\mathcal{S}_F}\frac{\beta v}{w+\beta v} \\ 1 \end{bmatrix}\begin{bmatrix} \frac{\beta L}{\beta L+M} & \frac{M}{\beta L+M} \end{bmatrix}\right\}. \quad (38)
\end{aligned}$$

The curvature parameters of $U(K,M)$ as defined in (28) are

$$\begin{aligned}
\begin{bmatrix} \mathcal{C}_{KK} & \mathcal{C}_{KM} \\ \mathcal{C}_{MK} & \mathcal{C}_{MM} \end{bmatrix} &= \frac{1}{\frac{1}{\mathcal{S}_F}\left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right) + \frac{1+\mathcal{C}_H}{\mathcal{S}_H}}\left\{\begin{bmatrix} \frac{1}{\mathcal{E}_L}\frac{\mathcal{C}_F}{1-\mathcal{S}_F} & 0 \\ 0 & \frac{\mathcal{C}_H}{1-\mathcal{S}_H} \end{bmatrix} + \right. \\
&\left. \frac{1}{\mathcal{S}_F\mathcal{S}_H}\begin{bmatrix} \left(\frac{1-\mathcal{S}_F}{\mathcal{E}_L} + \frac{\mathcal{C}_F}{1-\mathcal{S}_F}\right)(1+\mathcal{C}_H) & -\left(\frac{1}{\mathcal{E}_L} + \frac{\mathcal{C}_F}{1-\mathcal{S}_F}\right)(1-\mathcal{S}_H+\mathcal{C}_H) \\ -\left(\frac{1-\mathcal{S}_F}{\mathcal{E}_L} + \mathcal{C}_F\right)\left(1 + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}\right) & \left(\frac{1}{\mathcal{E}_L} + \mathcal{C}_F\right)\left(1 - \mathcal{S}_H + \frac{\mathcal{C}_H}{1-\mathcal{S}_H}\right) \end{bmatrix}\right\}, \quad (39)
\end{aligned}$$

where $\mathcal{E}_L$ is the Frisch elasticity implied by $V(\cdot)$.

C Proof of Proposition 8

The steady state requirement $\mathrm{D}K_t = 0$ implies $m_t = m_\infty$. So $v = Q\mathrm{D}g(m_\infty)$ transfers all statements about v into statements about Q. The steady state equilibrium conditions can be summarized as

$$\beta v + \mathcal{L}^{-1}(L) = \frac{F(k,1)A(k)}{F(k,1)L+X}, \quad (\beta + m_\infty k)L = \mathcal{M}(v),$$

together with

$$v = \frac{\delta \mathcal{S}_g}{\rho + \delta \mathcal{S}_g} \frac{F(k,1)}{F(k,1)L+X} \frac{1-A(k)}{m_\infty k}. \tag{40}$$

This last condition follows from $\mathrm{D}Q_t = 0$ and $v = Q\mathrm{D}g(m_\infty)$. Using (40) to eliminate v from the first two equilibrium conditions yields

$$\begin{aligned} \mathcal{L}^{-1}(L) &= \frac{F(k,1)}{F(k,1)L+X} \left(A(k) - \frac{\delta \mathcal{S}_g}{\rho + \delta \mathcal{S}_g} \frac{\beta}{m_\infty} \frac{1-A(k)}{k} \right), &(41)\\ L &= \frac{1}{\beta + m_\infty k} \times \mathcal{M}\left(\frac{\delta \mathcal{S}_g}{\rho + \delta \mathcal{S}_g} \frac{F(k,1)}{F(k,1)L+X} \frac{1-A(k)}{m_\infty k} \right). &(42) \end{aligned}$$

These are two equilibrium conditions that relate L to $k = K/L$. Both left-hand sides are increasing in L, and both right-hand sides, if positive, are decreasing in L. So each of these two conditions defines a function $k \mapsto L$. Observe that

$$\frac{\partial}{\partial k} \left\{ \frac{F(k,1)A(k)}{F(k,1)L+X} \right\} > 0, \quad \frac{\partial}{\partial k} \left\{ \frac{F(k,1)}{F(k,1)L+X} \frac{1-A(k)}{k} \right\} < 0,$$

because $A(\cdot)$ is non-decreasing and $X \geq 0$. It follows that the function $k \mapsto L$ defined by (41) must be increasing in k, and that the function $k \mapsto L$ defined by (42) must be decreasing. So there can be no more than one steady state. An increase in X shifts both functions $k \mapsto L$ down ((41) strictly and (42) weakly), and so an increase in X must lower L.

If $\mathcal{M}(\cdot)$ is completely inelastic, then the market clearing condition $\mathcal{M}(v) = \beta L + m_\infty K$ implies that a decline in L has to result in a rise in $K = kL$. It follows that $k = K/L$ must rise as well. Because of (41), the fact that $\mathcal{L}^{-1}(L)$ declines while k rises implies that $C = F(K,L) + X$ must increase. Together with (40), this implies that v declines. This proves the proposition for the case of inelastic managerial services.

Now suppose $\mathcal{M}(\cdot)$ is strictly increasing. Note that the equilibrium conditions for $(\beta v + \mathcal{L}^{-1}(L))L$ and βv can also be combined to yield (an instance of the restriction in footnote 13)

$$\frac{\beta v}{\beta v + \mathcal{L}^{-1}(L)} = \frac{\delta \mathcal{S}_g}{\rho + \delta \mathcal{S}_g} \frac{\beta}{m_\infty k} \frac{1-A(k)}{A(k)}. \tag{43}$$

The right-hand side of this equation is decreasing in k. Suppose now that an increase in X raises v. Since it certainly lowers L, this implies that $\beta v/(\beta v + \mathcal{L}^{-1}(L))$ rises. So then k must decline, and then so does $(\beta + m_\infty k)L = \mathcal{M}(v)$. This contradicts the presumed increase in v. So v must decline with an increase in X. It remains to investigate what happens to C. If k declines, then $(\beta + m_\infty k)L$ declines, and the fact that $F(1, 1/k)(1 - A(k))$ is a decreasing function then forces an increase in $C = F(k, 1)L + X$, by (42). On the other hand, if k increases, then the decline in $\mathcal{L}^{-1}(L)$ together with (41) again forces an increase in $C = F(k, 1)L + X$.

Finally, if $\mathcal{M}(\cdot)$ is perfectly elastic at some v, then the steady state is determined by (40) and (41). The condition (40) implies a decreasing function $k \mapsto L$ that shifts down with an increase in X. So we still have the conclusion that an increase in X lowers L. The condition (43) still applies, and so the decline in L implies that k must decline as well. Then (40) implies that an increase in X raises C.

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