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ORGANIZATIONS IN ECONOMIC ANALYSIS

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Abstract

- B1. Boyd, John H.; Prescott, Edward C. and Smith, Bruce D.
- B2. Organizations in Economic Analysis
- C2. Three economic environments are reviewed, and in each organizations play an essential role. For an adverse selection insurance economy, we find that when mutual insurance arrangements are permitted an equilibrium necessarily exists and is optimal. This example, and the two others, illustrate the problems that may result from imposing organizational structure on an environment rather than permitting the structure to be determined endogenously. Journal of Economic Literature classification numbers: 021, 026, 611.
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## I. Introduction

Firms, or more generally organizations, surely play an important role in production and distribution. Consequently, they have been widely studied by economic theorists. Little of this analysis, however, has been conducted in an equilibrium framework because in the economies of Arrow, Debreu, and McKenzie, organizations can play no important role. In that framework, firms are nothing more nor less than technologies--that is, subsets of the commodity space. These technologies along with the set of agents, their consumption possibility sets, utility functions, and endowments, define the economy. Absent monopoly power, it does not matter who manages the technologies. The same competitive equilibrium results if the workers manage the firm and rent capital, or if the owners of capital manage the firm and rent labor. Only if there are profits does ownership of the technology matter, and then only for income distribution.

The Arrow-Debreu-McKenzie framework, then, is simply not a good one for studying organizations. In this study, we used an alternative equilibrium concept based on the core to study three environments in which the organization arrangement is endogenously determined. First, we consider two adverse selection environments in which organizations arise naturally and are "important"--by which we mean they are a necessary part of an equilibrium arrangement. In these cases, the resulting equilibrium allocation is a Pareto optimum given resource and incentive constraints. In the third example private information is not a problem, but coalitions are still important. There, the technology is such that it cannot be traded in decentralized markets.

It can be argued that having "important" organizations is a significant advance in equilibrium theory. Potentially, this makes the theory easier to relate to data, since economic data are almost always gathered along organ-

izational lines. In addition, since our approach does not take organizational structure as given, it can in principle be used to predict the consequences of interventions to which arrangements are not invariant.

There may also be significant advantages to the use of equilibrium analysis in the study of organizations, themselves. In our examples the structure of efficient organizations depends crucially upon the characteristics of the environment. And, if technological and legal constraints permit the formation of such organizations, they will be formed. In every sense, organizations are endogenous here and there is no good basis for assuming a priori what their structure will turn out to be.

One example helps illustrate the problems that may result from imposing organizational structure on an environment. This is an insurance example, and as it turns out the preferred form is a "participating" organization, in which insurance contracts are conditioned on group loss experience. Such organizations are actually quite common among life and casualty insurers, but they have been overlooked in most previous studies of insurance markets. These studies have assumed, instead, insurance firms which write contracts conditional only on the state realization of one individual, the insured.

In the cases we consider where private information is an essential feature, we are hardly the first to notice that the structure of organizations is important. Miyazaki [1977], for instance, pointed out that in adverse selection environments the organization of production is potentially important in achieving efficiency. Smith and Stutzer [1987] point out that, in adverse selection environments with aggregate uncertainty, mutual insurance firms should co-exist with investor owned firms. However, earlier studies impose the presence of certain kinds of firms. The objective of this study is to predict the kinds of organizations that might emerge endogenously in these environments.

## II. Equilibrium Concept

For our purposes a core type definition of equilibrium is a natural choice, simply because we wish to predict what kinds of organizations, or coalitions of agents will form. It is true that in many environments, the set of competitive equilibrium allocations and core allocations coincide. In these cases, various organizational arrangements may support the equilibrium allocation(s), but so do market arrangements. For that reason, we would say that organizations are "not important" in such environments.<sup>1</sup>

Our examples are not in this class of environment, however. In each, there are elements which render competitive analysis and standard core theory inapplicable. Thus, we develop a modified concept of the core, not unlike that employed in economies with public goods.<sup>2</sup> The first two examples are characterized by private information prior to contracting--the adverse selection problem. Here individual rationality constraints for forming a coalition are part of the requirements for a potential coalition to block an arrangement.<sup>3</sup> The third example environment is a dynamic one. Here, the key feature is that future arrivals can not be part of coalitions blocking current arrangements. Only when these agents arrive can they join or form new organizations.

## III. Adverse Selection Examples: A Property of the Equilibrium Allocations

Our first example is the Rothschild-Stiglitz [1976] adverse selection environment. The second is the Boyd-Prescott [1986] intermediation environment, in which both adverse selection and moral hazard are present. Although these environments are quite different, they have important similarities, which lead to a significant finding regarding the equilibrium allocations.

Both environments feature two classes of agents, with different endowments, whose type is private information. "Poor" agents may have an incentive to misrepresent themselves as "rich," and if they do, an inefficient allocation of resources will result. In both cases, this problem may be overcome by a transfer payment, or subsidy, from rich to poor. The widely understood difficulty, however, is setting up an appropriate mechanism so that this transfer can occur.

Once we adopt the core-theoretic framework, one fact becomes quite clear. The rich have an important strategic advantage, for it is they who pay the subsidy, and they who can withhold it. The poor have no comparable option. This asymmetry is easily overlooked in the insurance case if one assumes the usual arrangement--that is, firms selling insurance contracts to individual agents whose only options are to buy, or not to buy. In our framework, there are no firms but rather coalitions of rational agents with common interests. They are fully aware of the asymmetry, and will organize in such a way as to take account of it. For this reason, in both these examples there is a unique equilibrium allocation. It is the Pareto optimal allocation which maximizes the utility of rich agents, subject to resource and incentive feasibility.<sup>4</sup>

We suspect that this result will also hold for other private information economies in which there is a role for subsidies between the two classes of agents. Although it is not proven here, it applies to the Spence signaling environment [1973] when there are two types of agents. Admittedly, however, we have had little success in generalizing the result to more than two agent types.

#### IV. First Example: Mutual Insurance Coalitions

Rothschild and Stiglitz [1976] studied an adverse selection environment in which insurance firms sold policies to a population of agents with heterogeneous risk characteristics. They showed that for some parameter values an equilibrium in policies failed to exist; and further, that even when one did exist it might not result in an efficient allocation. This environment has been much studied subsequently, and it has been shown that results are very sensitive to the equilibrium concept employed, and to assumptions concerning out-of-equilibrium behavior.<sup>5</sup> We have nothing new to say on that topic. Instead, we will show that when insurance organizations are not arbitrarily restricted, there exists an essentially unique (continuous) equilibrium arrangement, resulting in a Pareto optimal allocation. We find it somewhat reassuring that the arrangement looks something like "participating" insurance, a form of organization that is common in the life and casualty business.<sup>6</sup>

##### The Environment

There is a nonatomic measure space of agents, the real interval  $[0,1]$ . Agents in the economy can be partitioned into two types, with type indexed by  $i = 1, 2$ . Let  $\mu_i$  be the measure of agents of type  $i$ , and  $\sum \mu_i = 1$ . All agents of type  $i$  are identical ex ante, and each agent is faced with the probability of either of two states occurring,  $s = 1, 2$ . There is a single consumption good; an agent in state  $s$  receives an endowment of the good  $e_s$ . Let  $e_1 > e_2$ , so  $s = 2$  is what is commonly referred to as "the loss state." Realizations of  $s$  are independently distributed across agents with the distribution identical for all agents of the same type. Type  $i$  agents face a probability  $p_i$  that  $s = 1$ ;  $p_2 > p_1$ . Thus type 2 agents face "lower risk" than type 1 agents. We will periodically appeal to the law of large numbers, so that

there is no aggregate uncertainty.<sup>7</sup> Let  $c_{is}$  denote the consumption of a type  $i$  agent in state  $s$ . All agents have identical utility functions  $u(c)$  defined on  $R_+$ , with  $u'(c) > 0$ ,  $u''(c) < 0$  for all  $c \in R_+$ . Finally, the information structure is as follows. Each agent knows his own type prior to the realization of the state and prior to entering into any risk sharing arrangements. Type is private information, ex ante. All arrangements entered into are observable, however.

### Coalitions

Agents are viewed as forming coalitions for the purpose of pooling risk. The relevant aspect of a coalition is the measure (number, if finite) of agents of each type. This is denoted by  $\theta = (\theta_1, \theta_2)$ . The reason that only the measures of each type matter is that we require agents of the same type to be treated identically in an ex ante sense. With lotteries, ex post allocations for agents of the same type, of course, may differ. As the population measures are  $\mu = (\mu_1, \mu_2)$ , the space of possible coalitions (actually equivalence classes of coalitions) is  $\theta = \{\theta \in R^2: 0 \leq \theta \leq \mu\}$ .

Next, we define actions that are feasible for an arbitrary coalition  $\theta$ . Let  $c_i = (c_{i1}, c_{i2})$ , and let  $c = (c_1, c_2)$ . An allocation  $c$  is resource feasible for  $\theta$  if

$$(1) \quad \sum \theta_i [p_i(c_{i1} - e_1) + (1-p_i)(c_{i2} - e_2)] \leq 0.$$

Denote the set of allocations satisfying (1) for given  $\theta$  by  $R(\theta)$ .

Let  $v_i(c_j) \equiv p_i u(c_{j1}) + (1-p_i)u(c_{j2})$ . Then an allocation  $c$  is incentive feasible for  $\theta$  if for all  $i$  and  $j$

$$(2) \quad v_i(c_i) \geq v_i(c_j).$$

Denote the set of allocations satisfying (2) by  $I$ . An allocation is feasible for  $\theta$  if it is resource and incentive feasible. Let  $F(\theta) = R(\theta) \cap I$ .

### Arrangements

There are three periods. In period one the incumbent (grand) coalition specifies a rule for allocating resources. We call this rule an arrangement; arrangements specify allocations to be received by members of the coalition as functions of the composition of the coalition,  $\theta$ . After specification of an arrangement period two occurs, in which any potential blocking coalition may form and announce its own arrangement. Period three simply involves the blocking and residual coalitions consuming resources according to their announced arrangements and realized membership.

Formally, an arrangement  $a$  is a mapping  $a: \Theta \rightarrow R_+^2 \times R_+^2$  that specifies an allocation for a coalition as a function of its membership. Thus for a coalition  $\theta$  an arrangement specifies an allocation  $c = a(\theta)$ .<sup>8</sup> An arrangement is feasible if  $a(\theta) \in F(\theta)$  for all  $\theta \in \Theta$ .

### Core Arrangements

A feasible arrangement  $a$  is blocked by a coalition  $\beta$  and feasible arrangement  $b$  if

$$(3) \quad v_i[b_i(\beta)] > v_i[a_i(\mu)]$$

for some  $i$  such that  $\beta_i > 0$ ,

$$(4) \quad v_i[b_i(\beta)] \geq v_i[a_i(\mu - \beta)]$$

for all  $i$  such that  $\beta_i > 0$ , and

$$(5) \quad v_i[a_i(\mu - \beta)] \geq v_i[b_i(\beta)]$$

for all  $i$  such that  $\beta_i < \mu_i$ . If  $\beta = \mu$ , the block must specify the relative number of agent types in the measure zero residual coalition so that the utilities are defined. In this case, the relative number of any type in the residual coalition that strictly prefers membership in the blocking coalition must be less than or equal to its relative number in the population. Finally a core arrangement is any unblocked arrangement.

The interpretation of (3)-(5) is as follows. Condition (3) is the standard requirement for blocking to occur.<sup>9</sup> Condition (4) states that for all agents represented in positive measure in the blocking coalition,  $b_i(\beta)$  is weakly preferred to the allocation they would receive if they did not join the blocking coalition. Condition (5) requires that if some type  $i$  agents are left in the incumbent coalition, these agents must have no incentive to become members of the blocking coalition. Finally, note that conditions (4) and (5) require that a more favorable mix of agent types can be obtained only by creating incentives for agents to sort in the desired manner among coalitions.<sup>10</sup>

#### The Candidate Arrangement

We will prove that the arrangement  $a(\theta)$  that solves the following problem for each  $\theta$  is unblocked:

$$(P) \quad \max v_2(c_2) \text{ subject to}$$

$$(6) \quad c \in F(\theta)$$

$$(7) \quad v_1(c_1) \geq \bar{u}_1$$

where  $\bar{u}_1$  is the value of the program

$$\max v_1(c_1) \text{ subject to}$$

$$(8) \quad p_1 c_{11} + (1-p_1)c_{12} \leq p_1 e_1 + (1-p_1)e_2.$$

Denote the arrangement solving the problem above by  $a^*$ . This arrangement is feasible by construction, and it is also Pareto optimal given  $\theta$ .<sup>11</sup> In order to prove that  $a^*$  is an unblocked arrangement it will be helpful to produce a preliminary result about  $a^*$ . To begin, notice that the solution to (P) depends only on  $\eta = \theta_2/\theta_1$ . Then let us alternatively write the candidate arrangement as  $a^*(\eta)$ . We then have the following:

Lemma. If  $\eta' > \eta$  holds, then

$$(9) \quad v_2[a_2^*(\eta')] \geq v_2[a_2^*(\eta)]$$

and

$$(10) \quad v_1[a_1^*(\eta')] \geq v_1[a_1^*(\eta)].$$

Thus increasing the ratio of type 2 to type 1 agents makes no agents worse off under the candidate arrangement.

Proof. Result (9) is immediate since the allocation  $a^*(\eta)$  is feasible in the problem (P) when  $\eta' > \eta$ . Furthermore, result (10) is immediate if  $v_1[a_1^*(\eta)] = \bar{u}_1$ . It is also immediate if  $v_1[a_1^*(\eta')] = \bar{u}_1$ , since the allocation  $a^*(\eta')$  is the Pareto optimal allocation (satisfying  $v_1(c_1) \geq \bar{u}_1$ ) most preferred by type 2 agents for all  $\eta \in (0, \eta']$ . (See Rothschild-Stiglitz 1976 or Prescott-Townsend 1984.) Thus the only remaining case to be considered is that in which (7) does not bind.

Suppose, then, that (7) does not bind and that (10) is false. Then, since  $a^*$  specifies Pareto optimal allocations,  $v_2[a_2^*(\eta')] > v_2[a_2^*(\eta)]$  holds. Therefore, given the definition of  $a^*$  and  $v_2$ ,

$$(11) \quad v_2[a_2^*(\eta')] - v_2[a_2^*(\eta)] \equiv p_2\{u[c_{21}(\eta')] - u[c_{21}(\eta)]\} \\ + (1-p_2)\{u[c_{22}(\eta')] - u[c_{22}(\eta)]\} > 0.$$

Furthermore, equation (2) holds with equality in all solutions of (P), so that

$$(12) \quad v_1[a_1^*(\eta')] - v_1[a_1^*(\eta)] \equiv p_1\{u[c_{21}(\eta')] - u[c_{21}(\eta)]\} \\ + (1-p_1)\{u[c_{22}(\eta')] - u[c_{22}(\eta)]\} < 0,$$

where the latter inequality follows from the assumption that (10) is violated.

Since  $p_2 > p_1$ , (11) and (12) imply that  $c_{22}(\eta) > c_{22}(\eta')$ . Further, a well known property of solutions to (P) is that  $c_{11}(\eta) = c_{12}(\eta) \equiv c_1(\eta)$ . Then it is possible to show that any solution of the problem (P) must satisfy

$$(13) \quad \eta u'[c_1(\eta)]K(\eta) = \frac{p_1}{p_2} - \frac{1-p_1}{1-p_2},$$

where

$$K(\eta) \equiv u'[c_{22}(\eta)]^{-1} - u'[c_{21}(\eta)]^{-1}.$$

From (13)

$$\eta u'[c_1(\eta)]K(\eta) = \eta' u'[c_1(\eta')]K(\eta').$$

Now the assumption that (10) is violated implies  $u'[c_1(\eta')] > u'[c_1(\eta)]$ , and by assumption  $\eta' > \eta$ . Therefore  $K(\eta') > K(\eta)$ . ( $K(\eta) \leq 0$  holds for all  $\eta$ .) However, we know that  $c_{22}(\eta) > c_{22}(\eta')$ . Then satisfaction of (11) requires that  $c_{21}(\eta') > c_{21}(\eta)$ . Using these facts in the definition of  $K(\cdot)$  delivers  $K(\eta) > K(\eta')$ . We then have the desired contradiction, proving the lemma.  $\square$

Existence and Uniqueness of a Core Arrangement

We now state the following:

Theorem 1. The arrangement  $a^*$  is a core arrangement.

Proof. If  $a^*$  is blocked by  $\beta$  and  $b$ , there is a resulting feasible utility allocation

$$u_i = \max\{v_i[b_i(\beta)], v_i[a_i^*(\mu-\beta)]\}$$

for  $i = 1, 2$ . If  $a^*$  is blocked there are two exhaustive possibilities and we will show that both lead to a contradiction.

Case 1.  $u_2 > v_2[a_2^*(\mu)]$ . Note  $v_1[a_1^*(\mu-\beta)] \geq \bar{u}_1$  for all possible  $\mu - \beta$ . Thus necessarily  $u_1 \geq \bar{u}_1$ . This contradicts  $a^*(\mu)$  being the solution to program (P).

Case 2.  $u_1 > v_1[a_1^*(\mu)]$ . Clearly there can not be (weakly) relatively more of type 2 in the residual coalition than in the population. If there were, by the lemma  $v_2[a_2^*(\mu-\beta)] \geq v_2[a_2^*(\mu)]$  which would imply  $u_2 \geq v_2[a_2^*(\mu)]$ . Along with the assumption  $u_1 > v_1[a_1^*(\mu)]$ , this would contradict  $a^*(\mu)$  being a Pareto optimal allocation. Consequently, only the subcase of (strictly) relatively more of type 1 in coalition  $\mu - \beta$  than in the population remains. First, note that the measure of type 1 agents in the residual coalition must be zero, for by the lemma their utility is less than  $v_1[a_1^*(\mu)]$  which is strictly less than  $u_1 = v_1[a_1(\mu)]$ . Therefore  $\mu_2 - \beta_2$  must also be zero for otherwise the residual  $\eta = (\mu_2 - \beta_2)/(\mu_1 - \beta_1)$  would be infinite, contradicting the assumption that for the residual  $\eta < \mu_2/\mu_1$ . This leaves only one remaining subcase,  $\beta = \mu$ . It has already been established that type 1 agents must strictly prefer membership in the blocking coalition in the case being considered. By the condi-

tions for a block with  $\beta = \mu$ , the relative number of type 1 agents in the residual coalition must be less than or equal to that in the population. This contradicts the assumption that the residual's  $\eta < \mu_2/\mu_1$ .

We have ruled out the possibility of condition (3) being satisfied for any  $b$  and  $\beta$  that satisfy the other conditions for a block. This establishes the result.  $\square$

A coalition can be represented by its total measure  $\theta_1 + \theta_2$  and  $\eta$ , the number of type 2 relative to type 1. We say an arrangement is continuous if its utility allocation varies continuously with respect to the pair  $(\eta, \theta_1 + \theta_2)$  over the set  $[0, \infty] * [0, 1]$ .

Theorem 2. Let  $a$  be continuous and let  $a^*$  be a core arrangement. Then  $v_i[a_i(\mu)] = v_i[a_i^*(\mu)]$  for  $i = 1, 2$ .

Proof. We first establish  $v_1[a_1(\mu)] \geq \bar{u}_1$ . If it were not, some coalition  $\beta = (\beta_1, 0)$  which had only type 1 agents, and which set consumption for its members equal to the realized per capita endowment could block  $a$ . If  $v_1[a_1(\mu - \beta)] < \bar{u}_1$  for all  $\beta_1 \in [0, \mu_1]$ , then  $\beta_1 = \mu_1$  satisfies the sorting conditions. Otherwise, by continuity of  $a$ , there exists a  $\beta_1$  for which  $v_1[a_1(\mu - \beta)] = \bar{u}_1$ , again satisfying the sorting conditions.

We next establish  $v_2[a_2(\mu)] = v_2[a_2^*(\mu)]$ . If not,  $v_2[a_2(\mu)] < v_2[a_2^*(\mu)]$  given the already established fact that  $v_1[a_1(\mu)] \geq \bar{u}_1$ , and given that the  $a^*$  arrangement produces a Pareto optimal allocation. We now show that coalition  $\mu$  and arrangement  $a^*$  blocks. If under arrangement  $a$  with  $\eta = 0$  the utility of type 2 is less than or equal to  $v_2[a_2^*(\mu)]$ , the sorting conditions are satisfied for the block. If not, there exists an  $\eta$  between 0 and  $\mu_2/\mu_1$  such that  $v_2[a_2(\eta)] = v_2[a_2^*(\mu)]$ . This is possible given the continuity

of  $v_2$  under arrangement  $a$  with respect to coalition composition. At this point, by the lemma,  $v_1[a_1(\eta)] \leq v_1[a_1^*(\mu)]$ . Consequently all the conditions for  $\mu$  and  $a^*$  to block are satisfied for this  $\eta$ . This completes the proof.  $\square$

#### V. Second Example: Intermediary-Coalitions

This environment is one in which agents are endowed with private information about investment opportunities. In addition, more such information can be produced at cost. The problems of adverse selection and moral hazard are both present here. We show that financial intermediaries, which are coalitions of agents, arise endogenously and are part of the arrangement supporting the efficient equilibrium allocation. We also show that this allocation cannot be supported with a decentralized securities market. Intermediary coalitions exhibit the following characteristics: they borrow from and lend to a large group of agents; they produce information about investment projects; and they issue claims that have different state contingent payoffs than claims issued by ultimate borrowers. This environment was previously studied by Boyd and Prescott [1986], and therefore formal proofs are not reproduced here.<sup>12</sup>

#### The Environment

All agents live for two periods and have preferences ordered over expected consumption at the end of Period 2. Each agent is endowed with a unit of the investment good. In addition, some agents are endowed with a good type investment project,  $i = g$ , and others with a bad type,  $i = b$ . Project type signals (imperfectly) what a project's return  $r$  will be if it is actually funded. Each agent knows his own project type, which is private information. Except for these project endowments, all agents are identical and there are a countable infinity of them.

Agents may use their endowment of the investment good to evaluate a project, their own or someone else's. If a project is evaluated, a signal  $e = b$  or  $e = g$  is observed. This signal provides additional information about the rate of return on the project, and the evaluation result  $e$  is public. Both project type  $i$  and evaluation result  $e$  help predict a project's rate of return, and knowing both allows a better (but still imperfect) prediction than does knowing  $i$  or  $e$  alone.

Project scale  $\chi$  is large relative to any agent's unit endowment of the investment good. Thus, when a good project is obtained, efficiency requires that many agents invest their funds in it.

Obviously, this is an adverse selection environment due to the heterogeneous, and private project endowments. Depending on parameter values, it can also be a moral hazard environment, however. Suppose that, without the privateness of project type  $i$ , it would always pay to evaluate type  $i = g$  projects, and never pay to evaluate type  $i = b$  projects. In such environments, a sort of moral hazard may obtain, if type  $i = b$  endowed agents choose to misrepresent their project type and claim to have type  $i = g$ . If such projects are actually evaluated (and we shall show that that can occur in equilibrium), a deadweight loss will result.

We restrict parameter values so that, when all type  $i = g$  projects are evaluated and those that obtain a good evaluation are fully funded, some of the investment good will still remain. Resultantly, there will always be some investment in "marginal" type  $i = b$  projects. Finally, there are two resource constraints in this economy:

$$\begin{aligned} \text{Total investment per capita} + \text{Total number of evaluations per capita} \\ \leq \text{Total endowment per capita} \end{aligned}$$

$$\text{Per capita consumption} \leq \text{Per capital production of the consumption good}$$

### A Securities Market Equilibrium

One possible arrangement for this environment is a decentralized one in which some agents become "entrepreneurs," issue securities to other agents called "investors," and use the proceeds to fund their projects. Using standard definitions of securities (contingent claims) and of a decentralized securities market, we define such an equilibrium for this economy, and show that it exists. It has the following properties:

- All type  $i = g$  agents evaluate their projects and, if  $e = g$ , issue securities, each of which provides share  $1/\chi$  of the project's return, less the compensation of the entrepreneur. The entrepreneur's compensation is zero if  $r = b$ , and  $c_g$  if  $r = g$ .
- Some type  $i = b$  agents mimic the type  $i = g$ . That is, they evaluate their projects and, if  $e = g$ , issue shares. The other type  $i = b$  agents become investors.

Let  $m$  be the fraction of type  $i = b$  agents that choose to mimic, and let  $R$  be the expected rate or return earned by investors. Then, letting an asterisk denote equilibrium values,  $r^*$ ,  $m^*$ , and  $c^*$  are determined by the following equilibrium conditions.

$$(14) \quad r^* = \pi(e=g, r=g | i=b) c_g^*.$$

Type  $i = b$  agents are indifferent between mimicking and investing.

$$(15) \quad \chi [\pi(i=g, e=g) + m^* \pi(i=b, e=g)] = 1 - m^* \pi(i=b).$$

Demand for the investment good equals supply.

$$(16) \quad r^*\pi(i=b) + c_g^*\pi(i=g, e=g, r=g) = \pi(i=g, e=g)E\{\chi r | i=g, e=g\} \\ + m^*\pi(i=b, e=g)E\{\chi r | i=b, e=g\}.$$

Per capita consumption equals per capita output.

It should be clear that such mimicking behavior is costly, because it misdirects real resources into unproductive project evaluation. However, there is no way to avoid it with a decentralized market arrangement. To do so would require that some fraction of type  $i = b$  agents invest without evaluating, and that would constitute a perfect signal of their type. Potential investors would know for certain that the expected project return was  $E(r|i=b)$ , and would not finance the project.

#### An Equilibrium With Intermediary-Coalitions

There is, however, a form of organization which gets around the problem, and in so doing results in a Pareto Superior allocation--the feasible allocation which maximizes the utility of type  $i = g$  agents. We call this form of organization an "Intermediary-Coalition" and show that if such organizations are permitted to form, they will. If they do, of course, the securities market allocation described above is no longer an equilibrium allocation.

The basic structure of an Intermediary Coalition is as follows. It is composed of a large number  $n$  of type  $i = b$  agents, each of whom agrees to evaluate one project.  $n$  project owners or "entrepreneurs" contract with the coalition and will receive  $c_{gg}$  units of the consumption good if the project has evaluation  $e = g$  and return  $r = g$ , and zero units otherwise. The coalition also contracts with "depositors" who deliver their unit of the consumption good to it. In return, they are promised  $c_b$  units of the consumption good

in period 2. After paying off all entrepreneurs and depositors, coalition members are residual claimants and share equally in its profits.

The investment strategy of the Intermediary Coalition is to first invest fully in all the type  $i = g, e = g$  projects it discovers. Then, remaining funds are invested in the type  $i = b$  projects of depositors or evaluators.

We will not formally set out the conditions for equilibrium here. However, they require that type  $i = b$  agents be indifferent between being evaluators, depositors, and mimicking type  $i = g$  agents. Setting  $c_{gg}$  and  $c_b$  at the appropriate values will produce this result, and will also satisfy the two resource constraints. It is very important that each Intermediary Coalition be large, so that it obtains the expected fraction of type  $i = g, e = g$  projects. If, by chance, it were to obtain too many such projects, it could not fund all of them and the arrangement would be inefficient.

As residual claimants, the evaluators have no incentive to wastefully evaluate type  $i = b$  projects. Thus, these Intermediary Coalitions solve the "mimicking problem" in a way that a securities market cannot do. Indeed, as we show elsewhere, an arrangement with many such organizations supports a core equilibrium allocation which is essentially unique. These organizations have two key features, neither of which is present in the decentralized securities market. First, they pool many projects so that the law of large numbers can work. And second, they separate evaluators into a distinct class of agents who become equal residual claimants against the returns to a large pool of projects.

## VI. Third Example: Dynamic Coalitions

Coalitions were needed in the previous two examples to mitigate (as well as possible) the adverse selection problem. The final example is taken

from Prescott and Boyd [1987a,1987b]. In this environment dynamic production coalitions, or firms, play an important role. The reason coalitions are essential in this case is that the production technology is embodied jointly in the current coalition members; that is, in the organization. Because of this jointness, markets for human capital are inoperative. An individual's marginal product is higher if that member and other members remain in the coalition together. Technology is lost if the coalition is dissolved.

A member's productivity may depend not only on what he knows but also on what he knows about other coalition members' knowledge. The idea is that an organization is a mechanism for internalizing externalities associated with the production and use of information. In this sense organizations are playing the same role in this environment as they did in the financial intermediary coalition environment just reviewed. A final feature of this economy is that organizations have a life of their own which exceeds that of their members. In our example, people live but two periods with a new cohort being born every period. Organizations, on the other hand, live forever. The specific example, for which there is sustained growth without exogenous technological change, is as follows.

### The Environment

At each date an equal measure of two period lived agents are born. An agent born at date  $t$  has utility function

$$(17) \quad u_t = \ln y_t + \beta \ln z_{t+1}$$

where  $y_t$  is consumption when young,  $z_{t+1}$  consumption when old, and  $\beta$  is a parameter  $0 < \beta < 1$ .

At the initial date there are coalitions of old agents. Each coalition is endowed with common technology level  $k_0$ . On a per old member basis, a coalition at date  $t$  with technology level  $k_t$  faces the production constraint

$$(18) \quad c_t \leq k_t f(n_t) - k_{t+1} g(n_t)$$

where  $c_t$  is output of the consumption good and  $k_{t+1}$  is the new technology which is embodied in the  $n_t$  new members. The function  $f$  is increasing and concave while function  $g$  is increasing and convex. Further  $f'(1) - g'(1) < 0$ . The crucial condition is that the technology, embodied in the old or experienced coalition members, cannot be traded.

### A Constant Growth Equilibrium

Let  $u_t$  be the market utility level realized by date  $t$  young, and let  $\pi_{t+1}$  be the profit per coalition member per unit  $k_{t+1}$ . Old coalition members are the residual claimants, consuming per member profits of  $\pi k_t$ . The young, however, are free to join any coalition and consequently competition for new coalition members determines the distribution of product. Their compensation is current consumption  $y_t$  and embodied technology  $k_{t+1}$  which provides consumption  $z_{t+1} = \pi_{t+1} k_{t+1}$  when old. Thus the problem facing the operators of technology  $k_t$  is

$$(19) \quad \pi_t k_t = \max_{(n_t, k_{t+1}, y_t)} \{k_t f(n_t) - k_{t+1} g(n_t) - y_t n_t\}$$

subject to

$$\ln y_t + \beta \ln(\pi_{t+1} k_{t+1}) \geq u_t,$$

(those joining the coalition must realize the market utility level).

As all coalitions at date  $t$  have the same  $k_t$ , equilibrium requires that  $n_t = 1$ . A constant growth equilibrium exists with

$$\pi_t = \bar{\pi}$$

$$z_t = \bar{\pi} k_t$$

$$y_t = \bar{y}k_t$$

$$k_{t+1} = \bar{\gamma}k_t$$

$$n_t = 1$$

for all  $t$ . The  $\bar{\pi}$ ,  $\bar{y}$ ,  $\bar{\gamma}$  are uniquely determined by the first order conditions evaluated at the equilibrium value of  $n = 1$ . These are

$$(20) \quad -g(1)\gamma + \beta y = 0,$$

which is just the condition that the contract  $(k_{t+1}, y_t)$  results in the marginal rate of substitution between consumption when young and when old being equal to the marginal rate of transformation. Another condition is that the number of workers joining the coalition be optimal from the point of view of existing coalition members. This first order condition is

$$(21) \quad f'(1) - \gamma g'(1) - y = 0.$$

Equations (18) and (19) can be solved for  $\bar{\gamma}$  and  $\bar{y}$ , and in turn imply  $\bar{\pi}$  as

$$(22) \quad \bar{\pi} = f(1) - \bar{\gamma}g(1) - \bar{y}.$$

This equilibrium can be thought of as a sequential core. At the beginning of each period, coalitions gain new young workers and at the end of each period lose old members through retirement. In evaluating the desirability of joining a coalition, the young must take into account what core or equilibrium consumption they will realize in the subsequent period, conditional on the expertise they and co-workers receive in the current period.

Footnotes

<sup>1</sup>The concept of coalitions has been introduced into a general equilibrium framework by assigning a technology to each possible coalition. (See Ichiishi, 1983.) In the most interesting case, that in which a coalition's production possibility set depends only upon the number of members of each type, one could view these types as different factors of production. This expands the dimensionality of the commodity point and eliminates the need for coalitions.

<sup>2</sup>See, for example, Foley [1970], Richter [1974], and Starrett [1973].

<sup>3</sup>Private information does not necessarily imply that competitive analysis fails. Prescott and Townsend [1984] have shown how to extend competitive analysis to the study of environments with private information and unobservable actions. For those environments in which information is private at the time of contracting, however, their methods are not applicable. Neither are standard core concepts.

<sup>4</sup>There is also a so-called "participation constraint," requiring that poor agents' utility cannot be driven below a certain level (that which they could obtain on their own).

<sup>5</sup>Viewing the problem as a sequential Nash equilibrium, Cho and Kreps [1987] have shown that the stable equilibrium is the minimal signaling separating allocation, thus formalizing the intuition in Riley [1979]. Unlike the environment considered here, they assume agents are not together which rules out coalitional arrangements.

<sup>6</sup>Mutual insurance companies generally issue only "participating" policies, whereas stock insurance companies generally favor "nonparticipating" policies. With a participating arrangement, when total premium collections

exceed the amount actually needed to cover claims and administrative expenses, a portion of the premium is returned to each policy holder in the form of a "policy dividend." Under such arrangements, therefore, the terms of each individual policy are conditioned upon group loss experience. As will be shown, the efficient arrangement in our example environment exhibits this same structure.

Rothschild and Stiglitz mentioned the existence of participating insurance (1976, p. 276), but in their formal analysis did not allow such organizations to form. Rather, they assumed that insurance firms offered simple contracts, contingent only on the state realization of an individual agent. Most subsequent studies (excepting Smith and Stutzer [1987]) have made the same assumption.

<sup>7</sup>The applicability of the law of large numbers in this context is justified by Uhlig [1987].

<sup>8</sup>For technical reasons, an arrangement must specify allocations even when  $\theta = 0$ . As the feasible set of allocations  $F(\theta)$  depends only on  $\eta = \theta_2/\theta_1$ , the arrangement must also specify  $\eta$  when  $\theta = 0$ .

<sup>9</sup>Notice that we do not require that all types be made better off. If an arrangement can be found that makes one type better off and provides an incentive for agents of other types to defect to a blocking coalition, this is adequate for blocking to occur.

<sup>10</sup>Different than population mix of agent types cannot be obtained by excluding agents on the basis of (unobservable) type.

<sup>11</sup>Solutions to the problem (P) have been associated with Wilson equilibria by Miyazaki [1977] and Spence [1978]. However, notice that our formulation involves no dropping of policies in the event a blocking coalition forms, and in fact involves no "threat" of any kind. It merely involves the residual coalition reoptimizing after the formation of a blocking coalition.

<sup>12</sup>The Boyd-Prescott environment has more recently been used for several different applications. See Williamson [forthcoming], and Hargraves and Romer [1986].

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